

Hyper-K potential to study the Leptonic Unitarity Triangle

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Akira Konaka (TRIUMF)

- Unitarity condition: $U^\dagger U = 1$

- Normalization

$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1$$

$$|U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 = 1$$

$$|U_{\tau 1}|^2 + |U_{\tau 2}|^2 + |U_{\tau 3}|^2 = 1$$

- Unitarity triangle

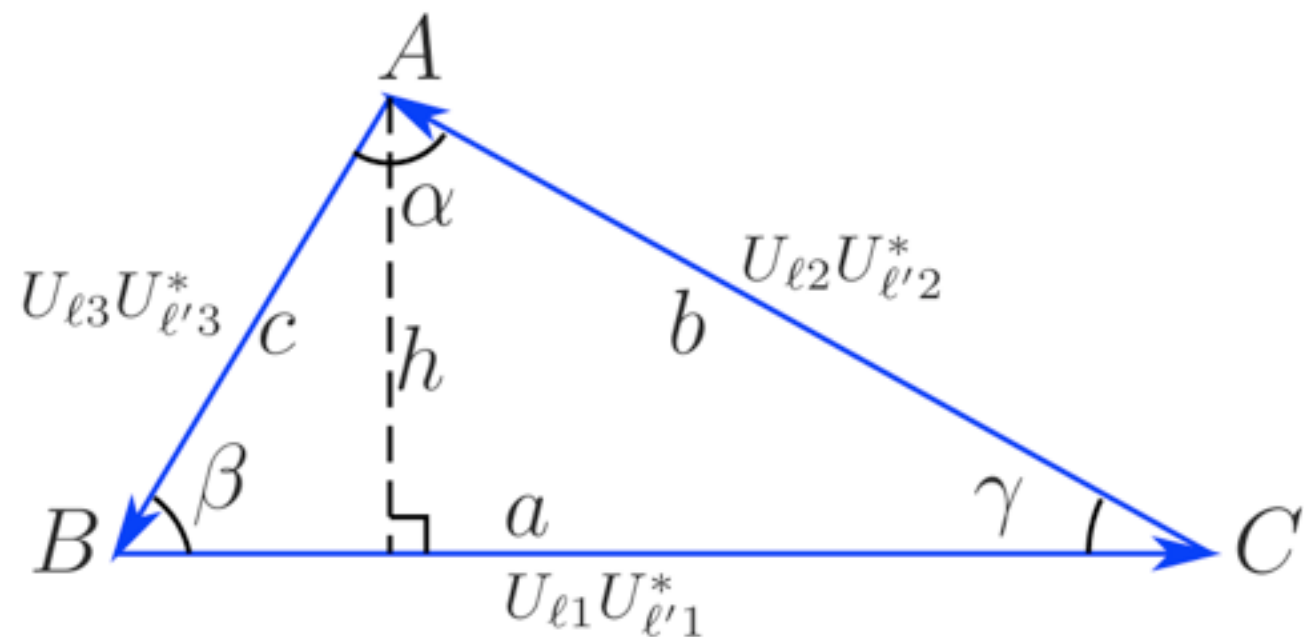
$$U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$$

$$U_{\mu 1}U_{\tau 1}^* + U_{\mu 2}U_{\tau 2}^* + U_{\mu 3}U_{\tau 3}^* = 0$$

$$U_{\tau 1}U_{e1}^* + U_{\tau 2}U_{e2}^* + U_{\tau 3}U_{e3}^* = 0$$

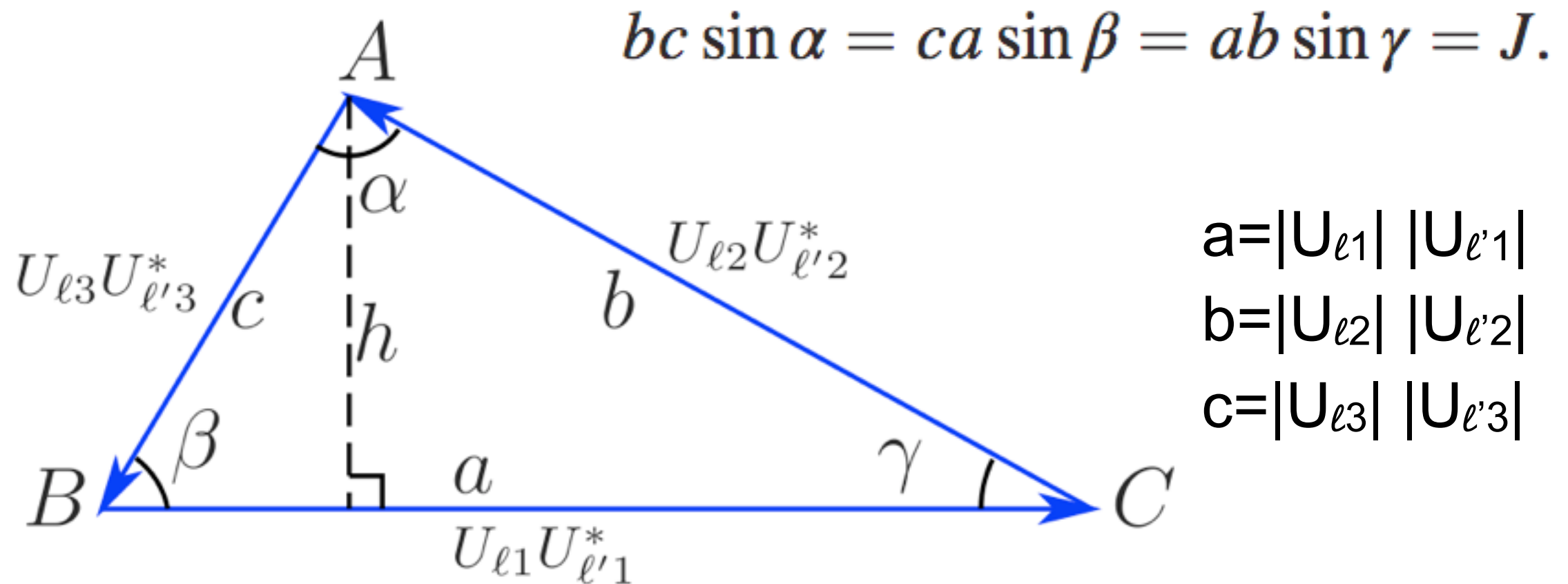
- Unitarity of PMNS is broken due to ν_R

- 4th generation or new interactions, can also do it.



Lesson from CKM (quarks)

- **Normalization:** $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
 - $|V_{ud}|^2 < 1$ lead to the idea of Cabibbo angle: strange quark implication in nuclear β decay
- **Flavour Changing Neutral Current (FCNC)**
 - Unitarity prohibits FCNC
 - Already tested for charged leptons: e.g. $\mu \rightarrow e \gamma$
- **Unitarity triangle:**
 - Unitarity condition including CP phase is tested in B, K decays: works very well
 - Over-constrain the unitarity triangle to explore physics beyond Standard Model



$$\begin{aligned} a &= |U_{\ell 1}| |U_{\ell' 1}| \\ b &= |U_{\ell 2}| |U_{\ell' 2}| \\ c &= |U_{\ell 3}| |U_{\ell' 3}| \end{aligned}$$

Appearance

$$\begin{aligned} P_{\ell \rightarrow \ell'} &= 4ab \sin(\Delta_{12} \pm \gamma) \sin \Delta_{12} \\ &\quad + 4bc \sin(\Delta_{23} \pm \alpha) \sin \Delta_{23} \\ &\quad + 4ac \sin(\Delta_{31} \pm \beta) \sin \Delta_{31} \end{aligned}$$

or

$$\begin{aligned} P_{\ell \rightarrow \ell'} &= a^2 + b^2 + c^2 - 2ab \cos(2\Delta_{12} \pm \gamma) \\ &\quad - 2bc \cos(2\Delta_{23} \pm \alpha) - 2ca \cos(2\Delta_{31} \pm \beta) \end{aligned}$$

$$\Delta_{12} = \Delta m_{12}^2 L / 4E$$

$$\Delta_{23} = \Delta m_{23}^2 L / 4E$$

$$\Delta_{31} = \Delta m_{31}^2 L / 4E$$

- Disappearance gives sides of the triangle:

$$\begin{aligned}
 P_{\text{disapp}} &= 1 - P_{\ell \rightarrow \ell} \\
 &= 4|U_{\ell 1}|^2 |U_{\ell 2}|^2 \sin^2 \Delta_{12} \\
 &\quad + 4|U_{\ell 2}|^2 |U_{\ell 3}|^2 \sin^2 \Delta_{23} \\
 &\quad + 4|U_{\ell 3}|^2 |U_{\ell 1}|^2 \sin^2 \Delta_{31}
 \end{aligned}$$

At 1st oscil. max.

$$\begin{aligned}
 \Delta_{23} &\sim \Delta_{13} \\
 &\sim 0.03 \Delta_{12}
 \end{aligned}$$

- Different L/E scale to untangle each contrib.
 - solar (Δ_{12}) and atmospheric (Δ_{23}, Δ_{31}) scales
 - level crossing due to matter effect provides an additional handle

e-μ unitarity in PMNS

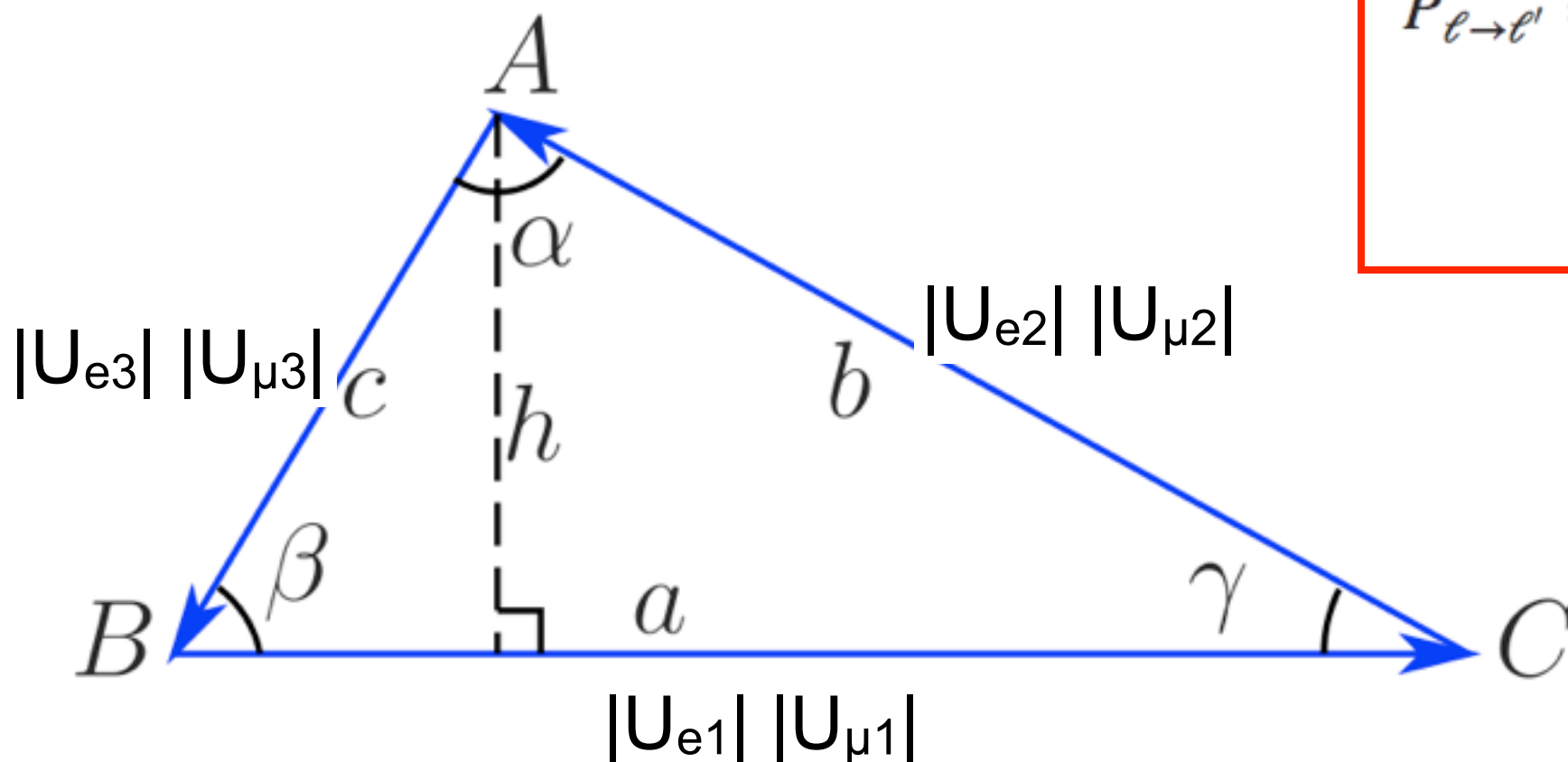
- e-μ part of unitarity can be studied best:

$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1$$

$$|U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 = 1$$

Normalization terms and sides of the triangle can be determined by disappearance

$$U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$$



$$P_{\ell \rightarrow \ell'} = 4ab \sin(\Delta_{12} \pm \gamma) \sin \Delta_{12} \\ + 4bc \sin(\Delta_{23} \pm \alpha) \sin \Delta_{23} \\ + 4ac \sin(\Delta_{31} \pm \beta) \sin \Delta_{31}$$

$$\Delta_{13} : \beta \sim \delta_{cp}$$

$$\Delta_{12} : \gamma \sim \delta_{\text{subGeV-atm}}$$

$$\Delta_{23} : \alpha \sim \delta_{\text{nth-max}}$$

$$|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2 = 1$$

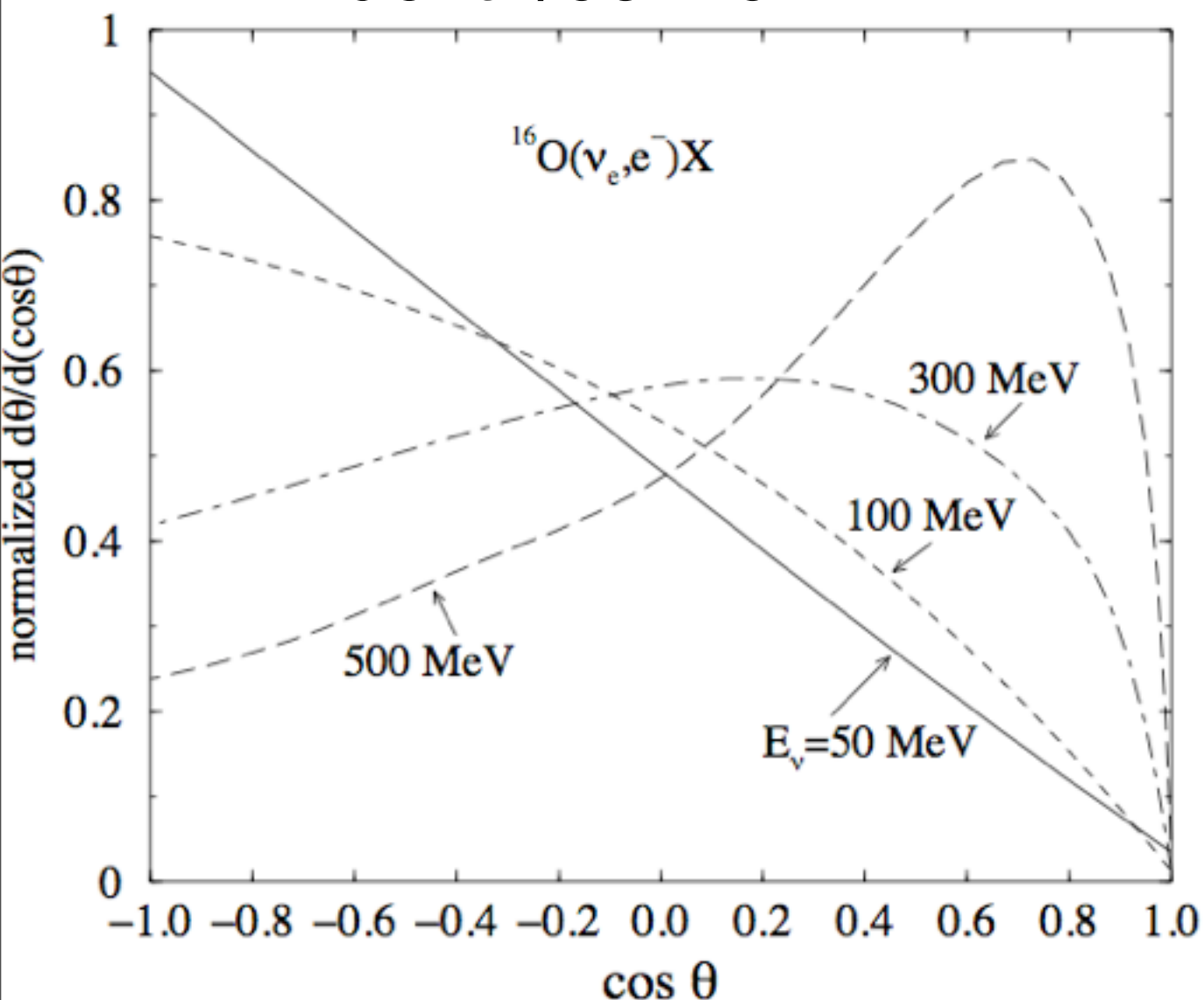
- Solar neutrino: $|U_{e2}|^2$
 - MSW conversion in the sun: $\nu_e \rightarrow \nu_2$ and detect ν_e
- KamLand: $|U_{e1}|^2 |U_{e2}|^2$
 - Reactor ν_e disappearance at Δ_{12} scale of L/E
- Reactor θ_{13} : $|U_{e3}|^2 (|U_{e1}|^2 + |U_{e2}|^2)$
 - Reactor ν_e disappearance at Δ_{12} scale of L/E
- All the parameters are measured. Better precision desired for KamLand measurement
 - JUNO experiment will achieve this in future

$$|U_{\mu 1}|^2 + |U_{\mu 2}|^2 + |U_{\mu 3}|^2 = 1$$

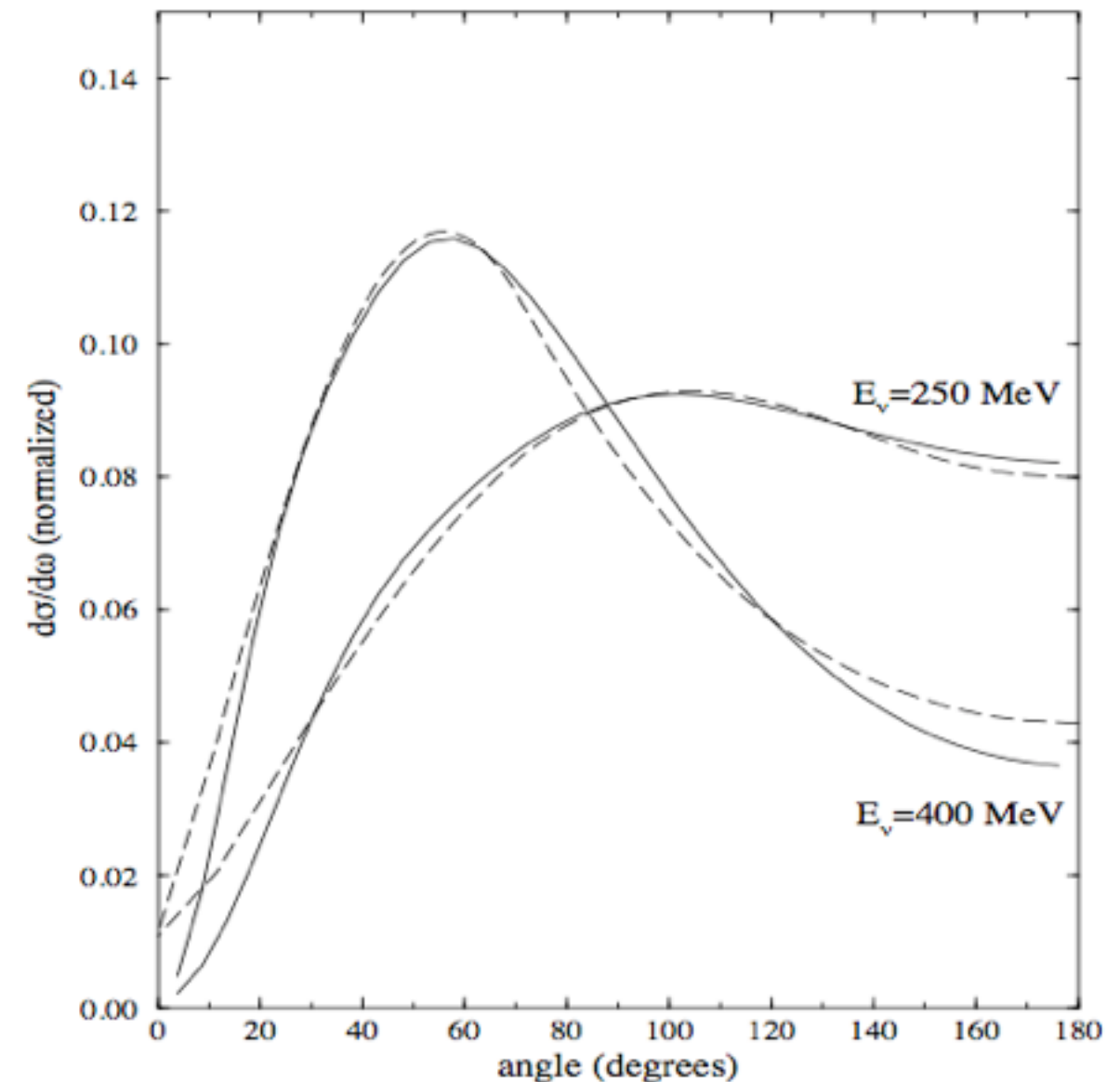
- LBL ν_{μ} disappearance: $|U_{\mu 3}|^2 (|U_{\mu 1}|^2 + |U_{\mu 2}|^2)$
 - This is what T2K/MINOS/NovA measure
- $\nu_{\mu} \rightarrow \nu_e$ atm. matter resonance: $(r|U_{\mu 3}|^2 - 1)$
 - Amplitude of the atmospheric mass hierarchy study [HK]
- Solar scale atm. ν_{μ} disapp. : $|U_{\mu 1}|^2 |U_{\mu 2}|^2$
 - 0.4–0.8 GeV up-going atm. ν_{μ} disapp. [HK]
 - zenith angle measurement: anti- ν_{μ} , proton energy cut
- 1–3 GeV atm. ν_{μ} disapp. (4–6th max.) can untangle $|U_{\mu 1}|^2$ and $|U_{\mu 2}|^2$ terms in LBL disapp. [HK]
 - Δ_{13} and Δ_{23} show 30–20% difference
 - 2nd max. LBL detector could also do this.

Lepton from neutrino reaction goes backward at low energy

nucl-th/0311022



nucl-th/9901027

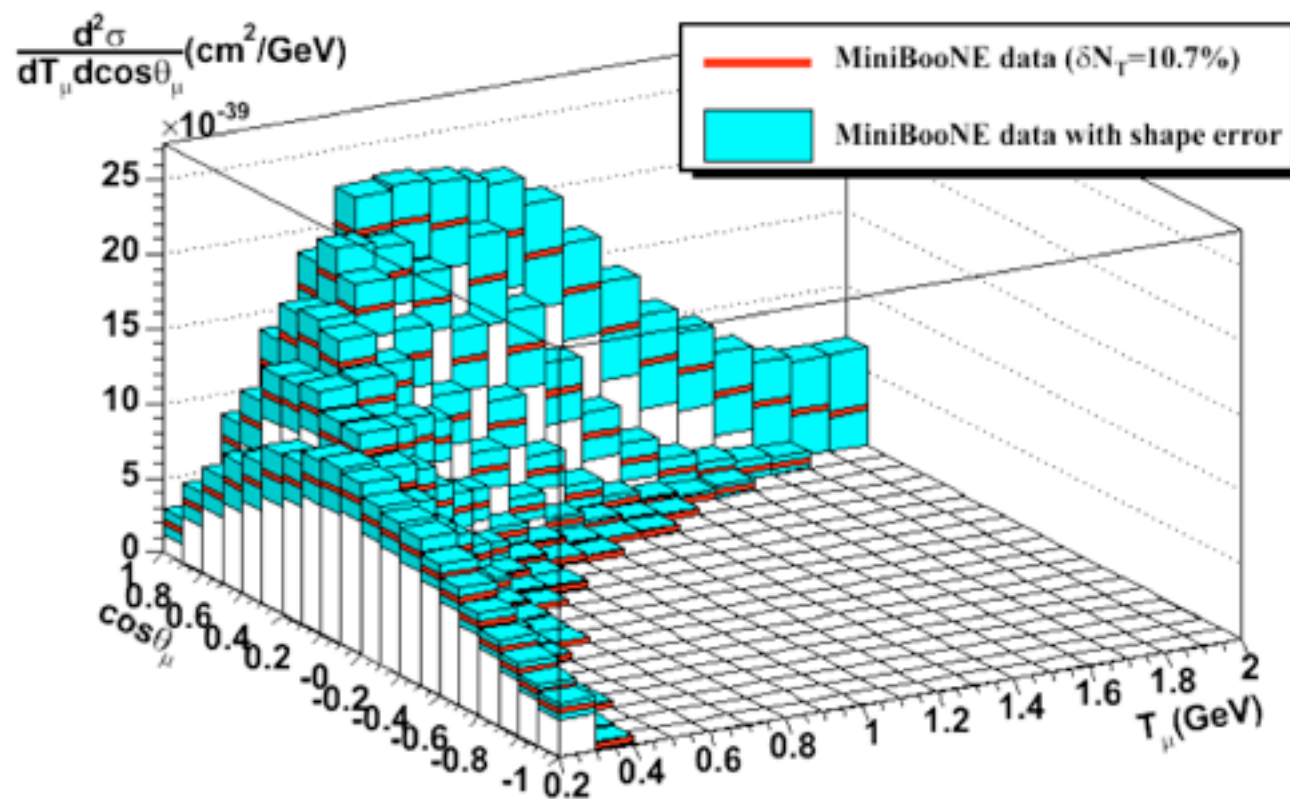


Neutrino direction measurement possible for $E_\nu > 400\text{--}500\text{ MeV}$
 Cross section study by vPRISM is important.

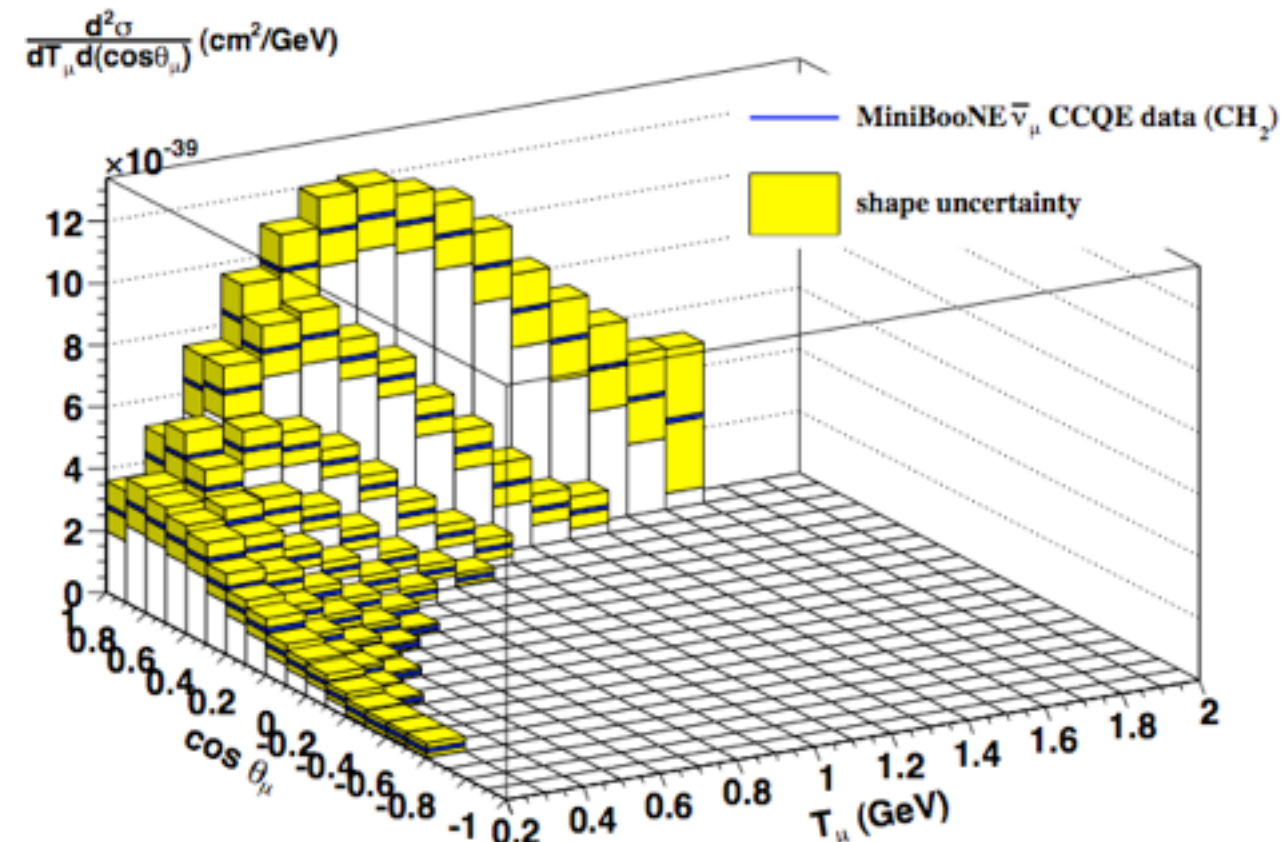
MiniBooNE differential x-sec

- Forward peak for anti- ν even for neutrino energy at 200MeV
 - $\bar{\nu}_e$ tagging by WbLs proton veto, Gd neutron tag?
 - $\bar{\nu}_e$ cross section is reasonably large at low E_ν

neutrino CCQE



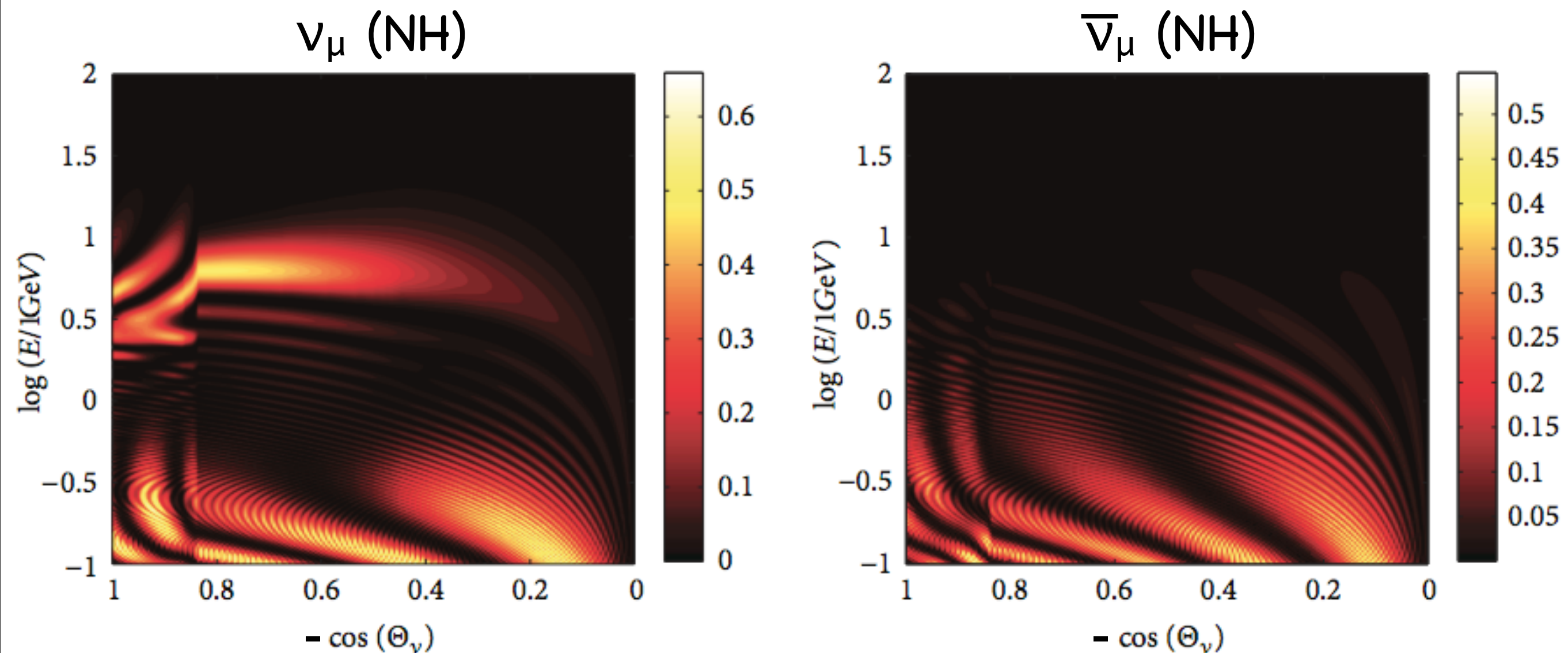
anti-neutrino CCQE



- CP study at Δ_{12} scale using sub-GeV atmospheric neutrinos

Atmospheric $\nu_\mu \rightarrow \nu_e$

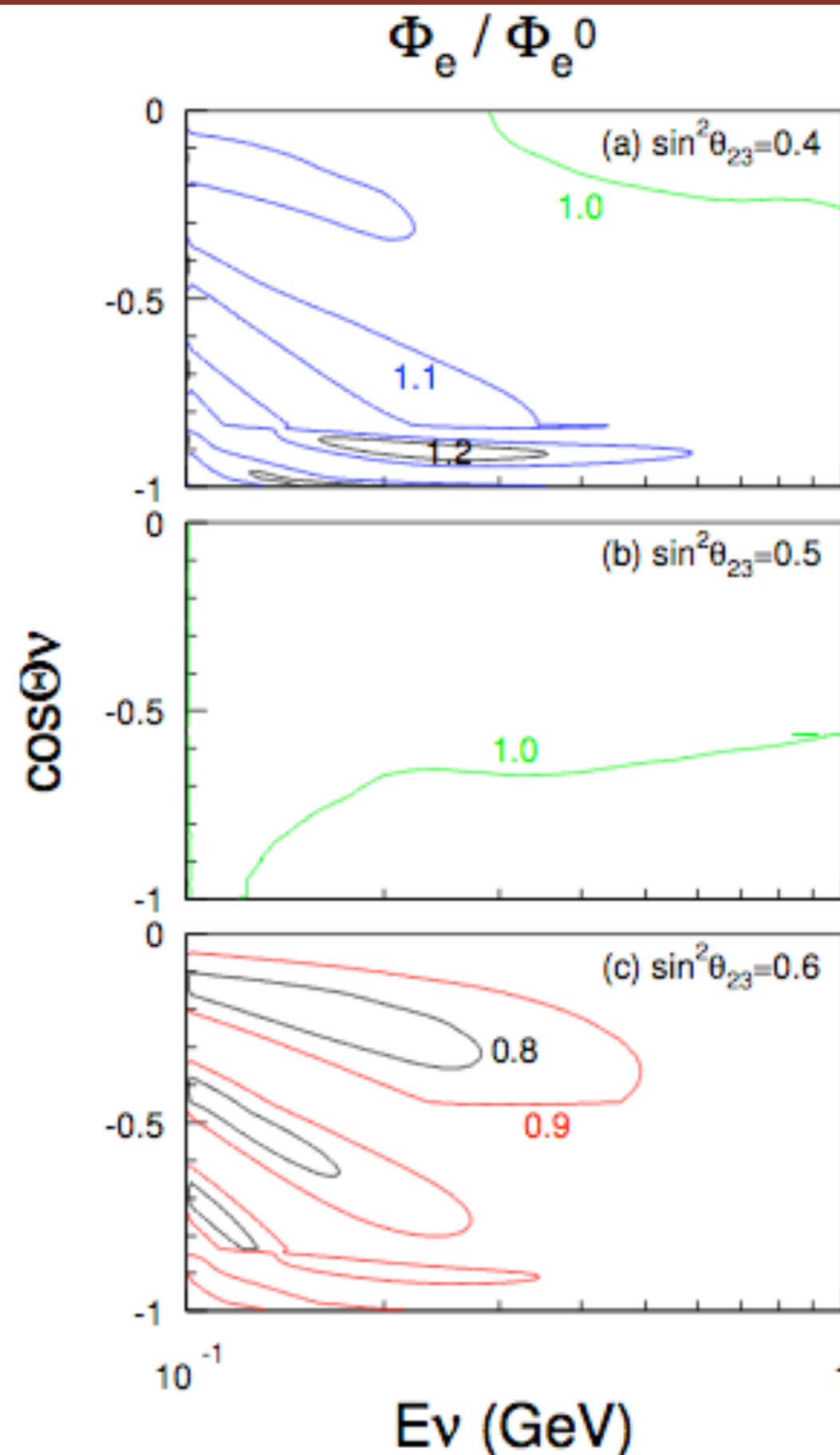
- Δ_{12} oscillation below 1GeV: large θ_{12} oscillation effect
 - Above matter resonance ($E_R=0.1\text{GeV}$), $\nu_\mu \rightarrow \nu_e$ level crossing
 - phase difference between ν and $\bar{\nu}$ due to matter effect



Can we measure this pattern?

Compensation due to $r=2$

- Observed ν_e is from
 - $\Phi(\nu_\mu) \times P_{\mu e} = \Phi(\nu_\mu) \times \sin^2\theta_{23}P_{ee}$
 - $\Phi(\nu_e) \times (1-P_{ee})$
- $r=\Phi(\nu_\mu)/\Phi(\nu_e)=2$, $\sin^2\theta_{23}=0.5$, they cancel: no effect!
 - $r=2$ for $E < 3\text{GeV}$
 - even cancels matter effect
- Some cases that avoid $r=2$
 - $E_\nu > 3\text{GeV}$ ($r > 2$): mass hierarchy
 - $\sin\theta_{23} \neq 0.5$: constrained by T2K
 - **CP violation!**



- Oscillogram formula (arXiv:0804.1466)

$$P_{\mu e} = c_{23}^2 |A_{e\tilde{2}}|^2 + s_{23}^2 |A_{e\tilde{3}}|^2 + 2 s_{23} c_{23} \operatorname{Re}(e^{-i\delta} A_{e\tilde{2}}^* A_{e\tilde{3}}),$$

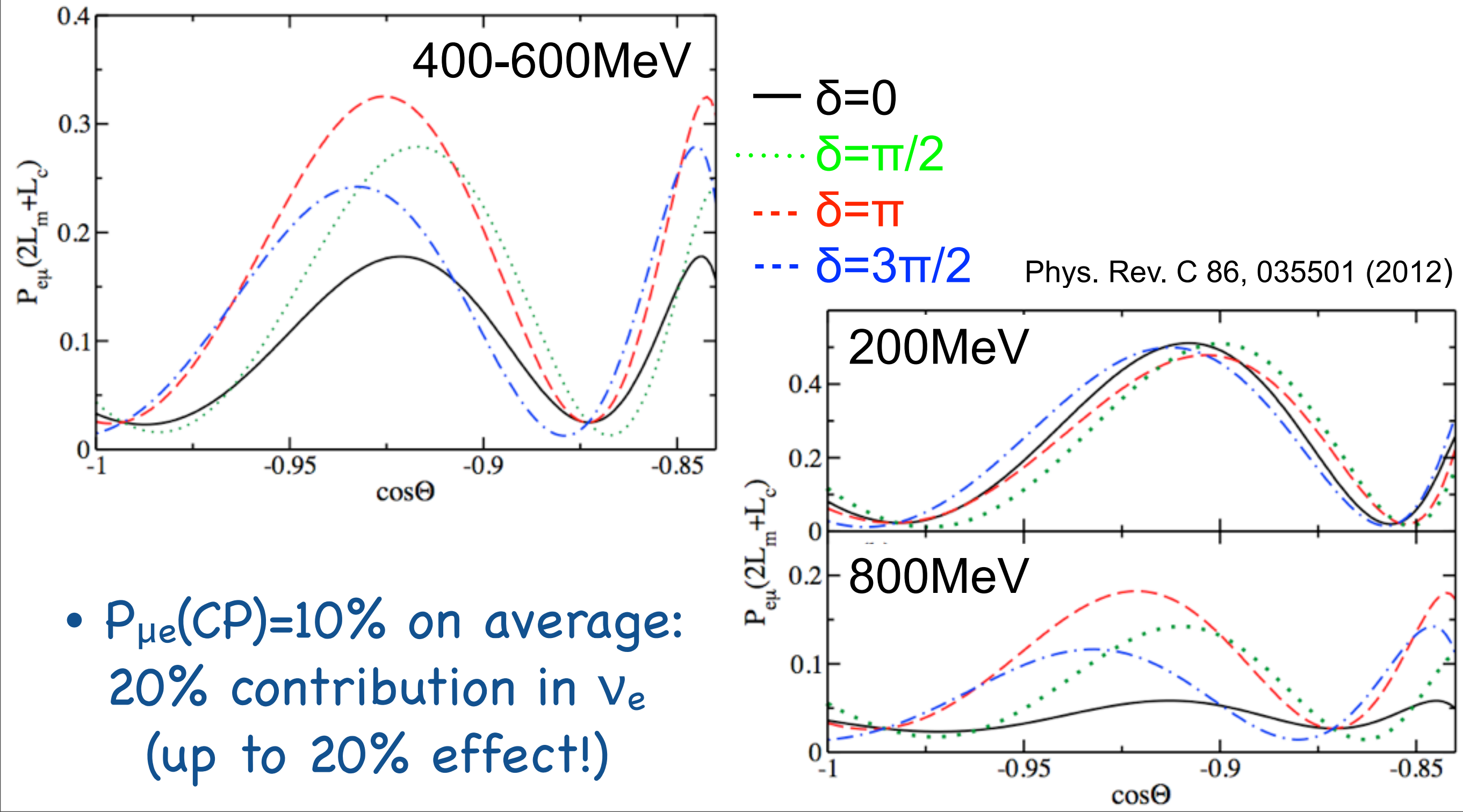
$$P_{\tau e} = s_{23}^2 |A_{e\tilde{2}}|^2 + c_{23}^2 |A_{e\tilde{3}}|^2 - 2 s_{23} c_{23} \operatorname{Re}(e^{-i\delta} A_{e\tilde{2}}^* A_{e\tilde{3}}),$$

$$1 - P_{ee} = P_{\mu e}(-\delta) + P_{\tau e}(-\delta) = |A_{e2}|^2 + |A_{e3}|^2$$

$$2 \times P_{\mu e}(\delta) - (1 - P_{ee}) \sim \operatorname{Re}(e^{i\delta} A_{e2}^* A_{e3})$$

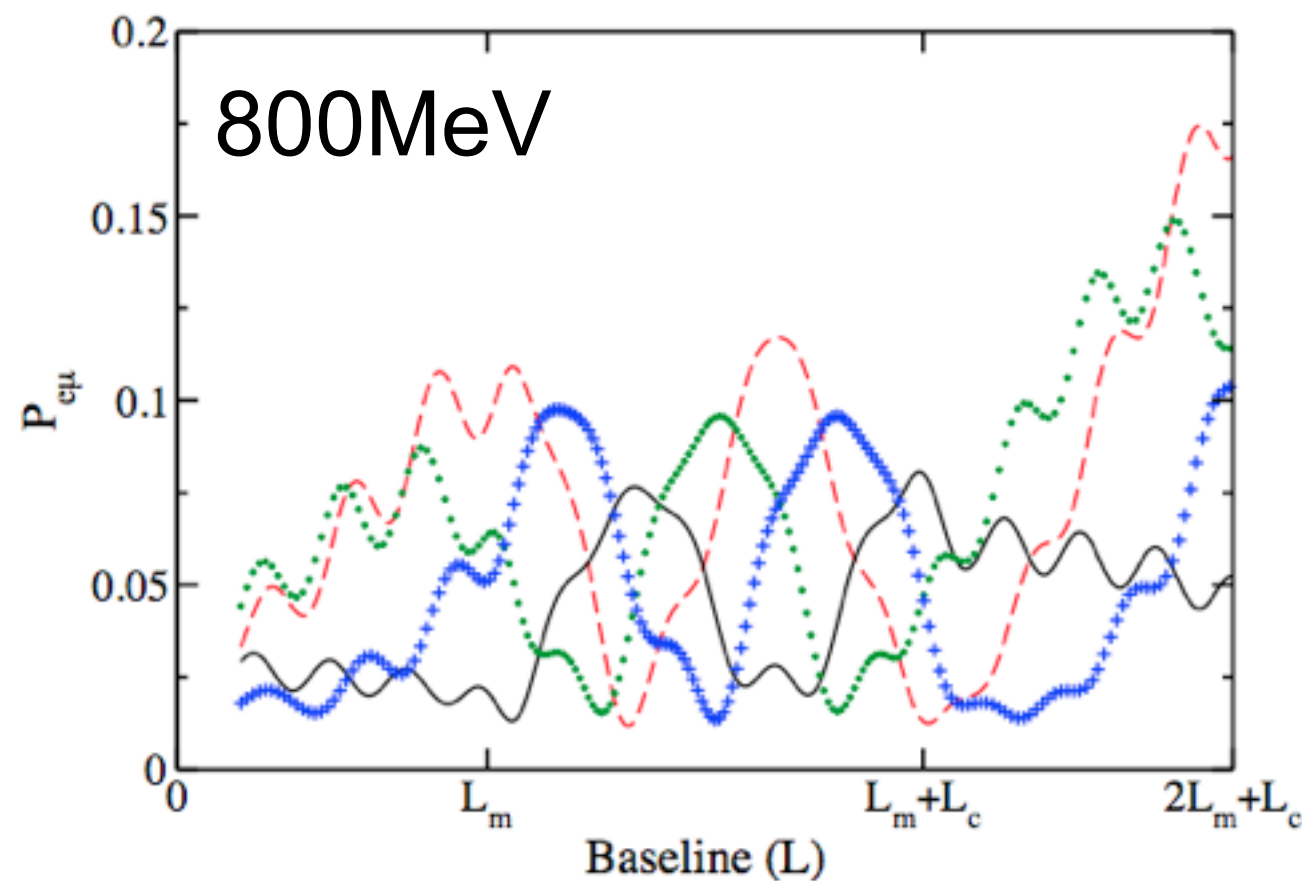
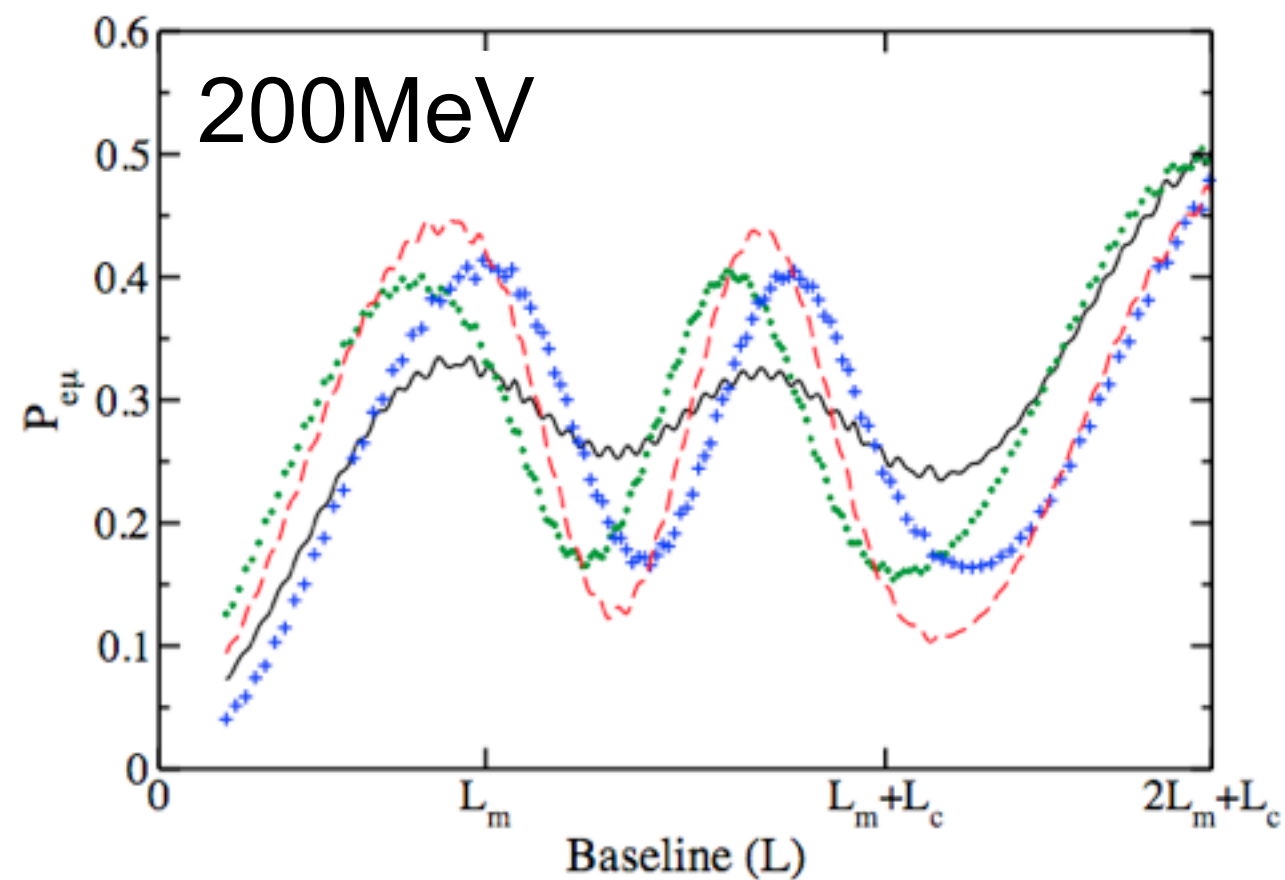
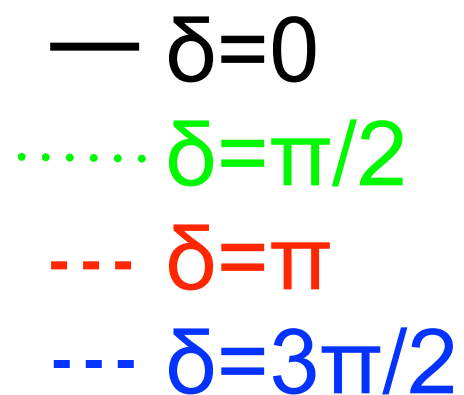
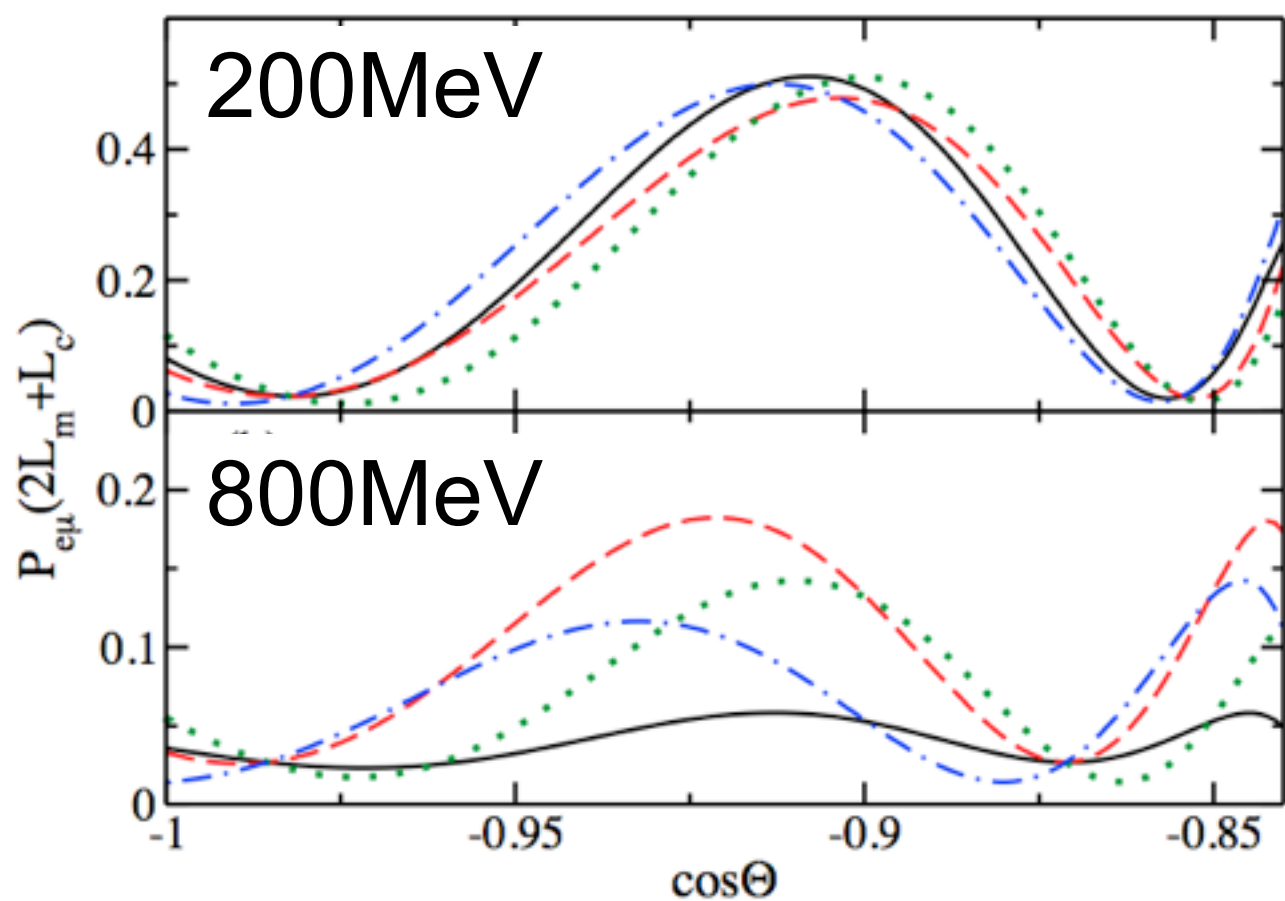
- $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ differs by T (CP) violation
 - All the oscillation due to Δ_{12} cancels including matter effects due to $r=2$, **except for T violation**
 - Opposit sign for $\bar{\nu}$, but luckily we mainly see ν_e :
 $\sigma(\nu_\mu \rightarrow \nu_e) \sim 5 \sigma(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$
 - **Very sensitive test of T-violation !**

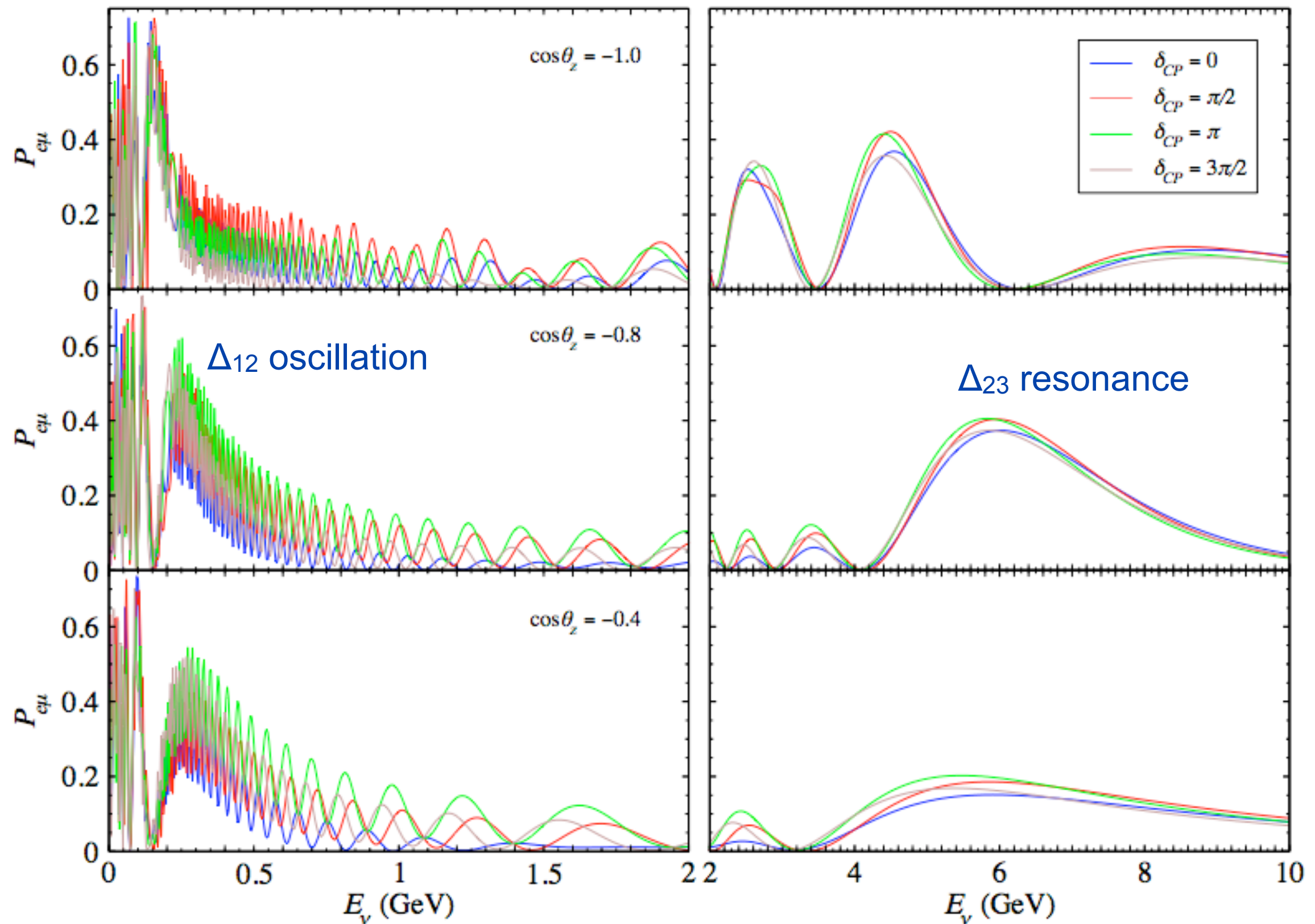
- $P_{\mu e}=10-30\%$ with significant CP violation,
 $P_{\mu e}(\text{CP})\sim 10-20\%$, for $E_\nu=400-800\text{MeV}$ ($E_e=300-700\text{MeV}$)



- $P_{\mu e}(\text{CP})=10\%$ on average:
 20% contribution in ν_e
 (up to 20% effect!)

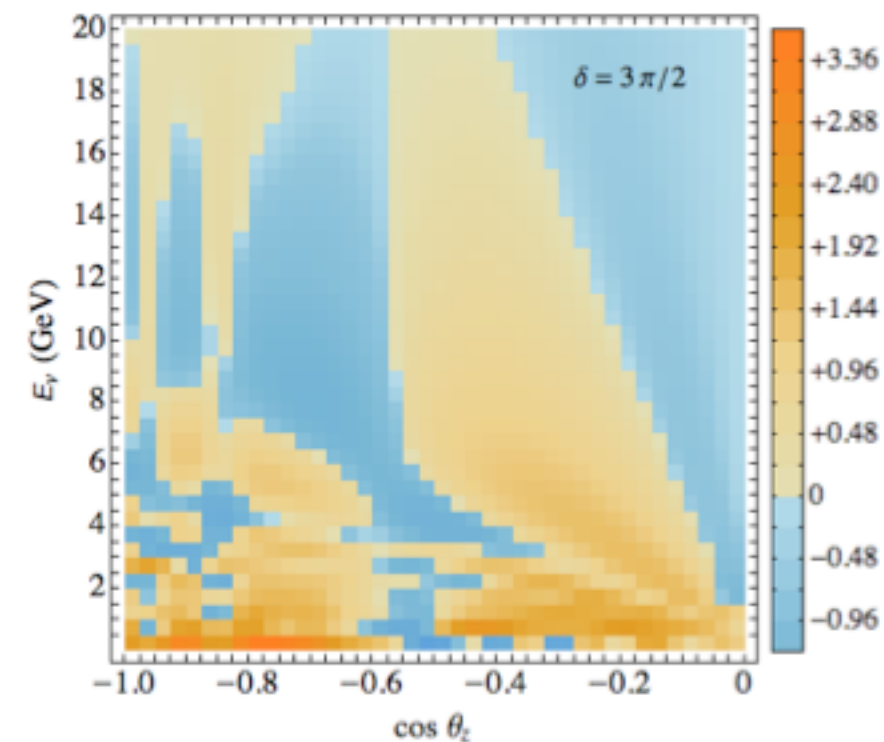
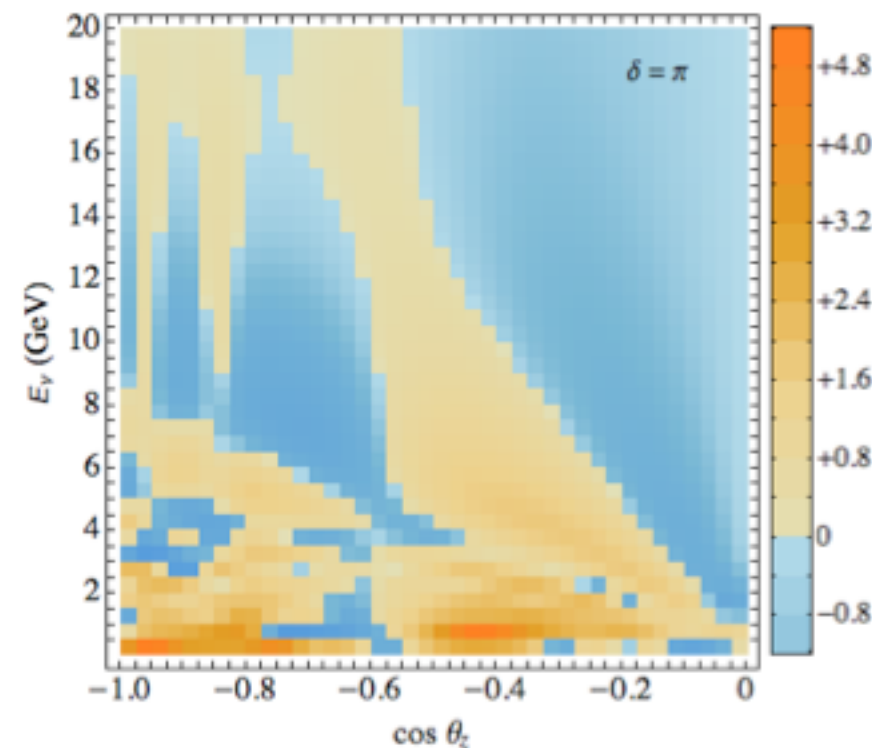
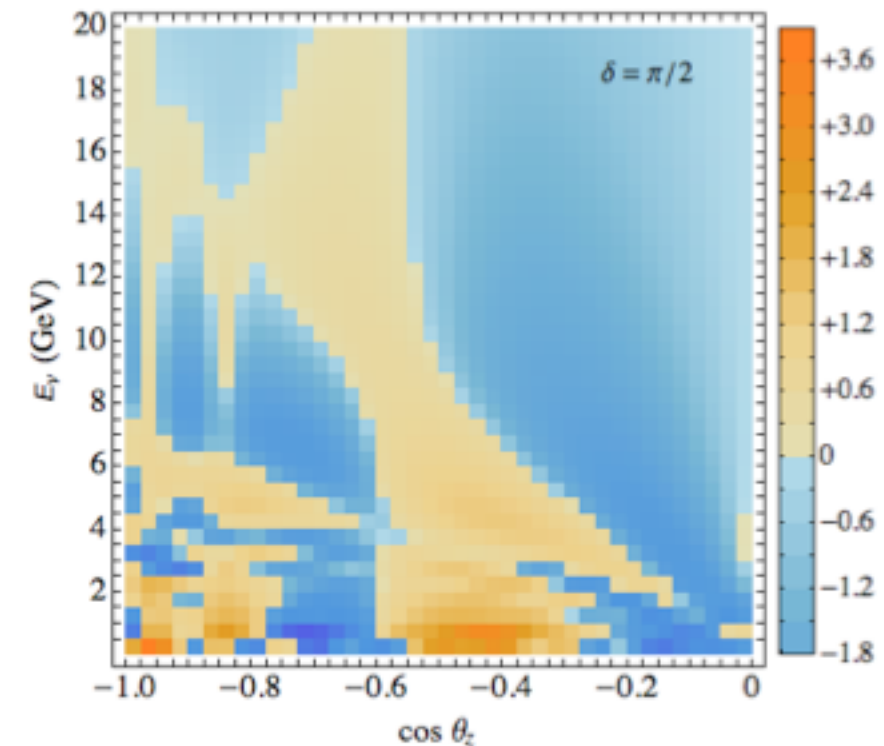
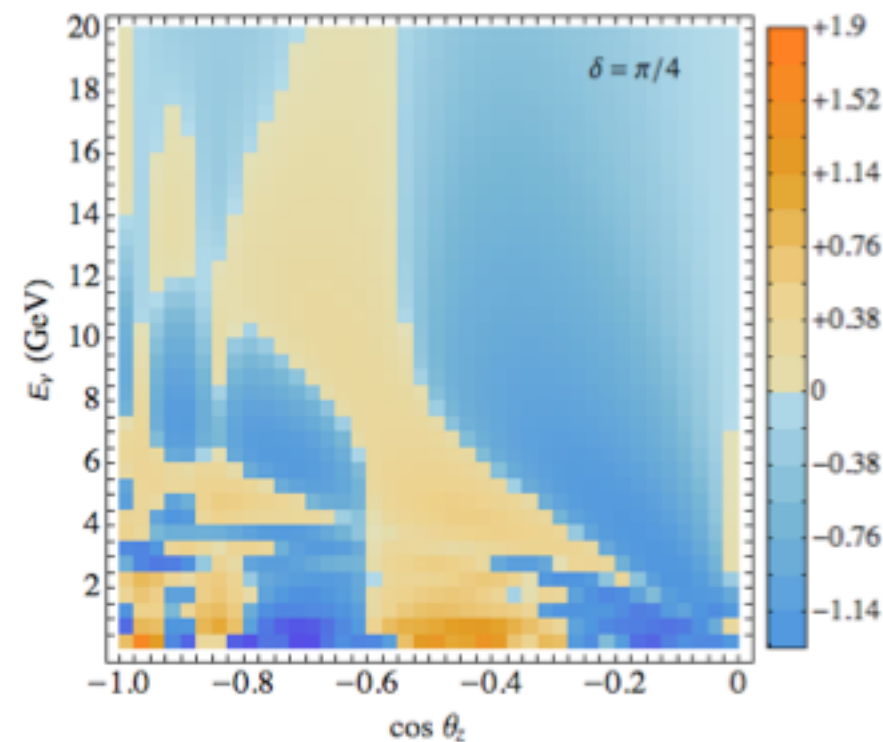
Oscillation through the earth





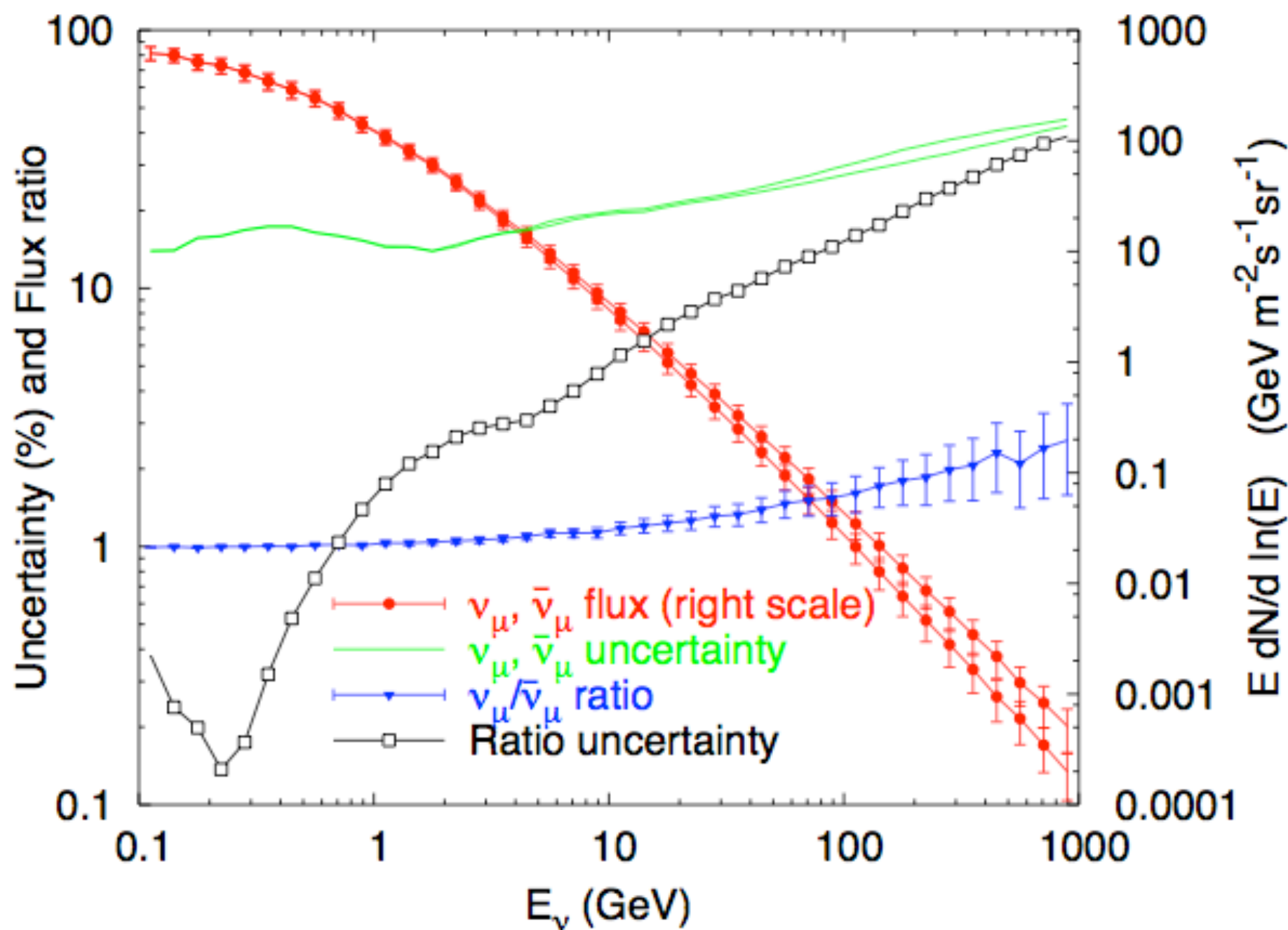
1-3GeV region is also sensitive (4th-7th max of Δ_{13})

- ν_e appearance (normal hierarchy)
- Super-PINGU
 - 2.2Mton-year
 - 4-year of HK
- $\Delta N/\sqrt{N}$
- Good sensitivity
 - $E < 1\text{GeV}$
 - $\cos\theta < -0.6$



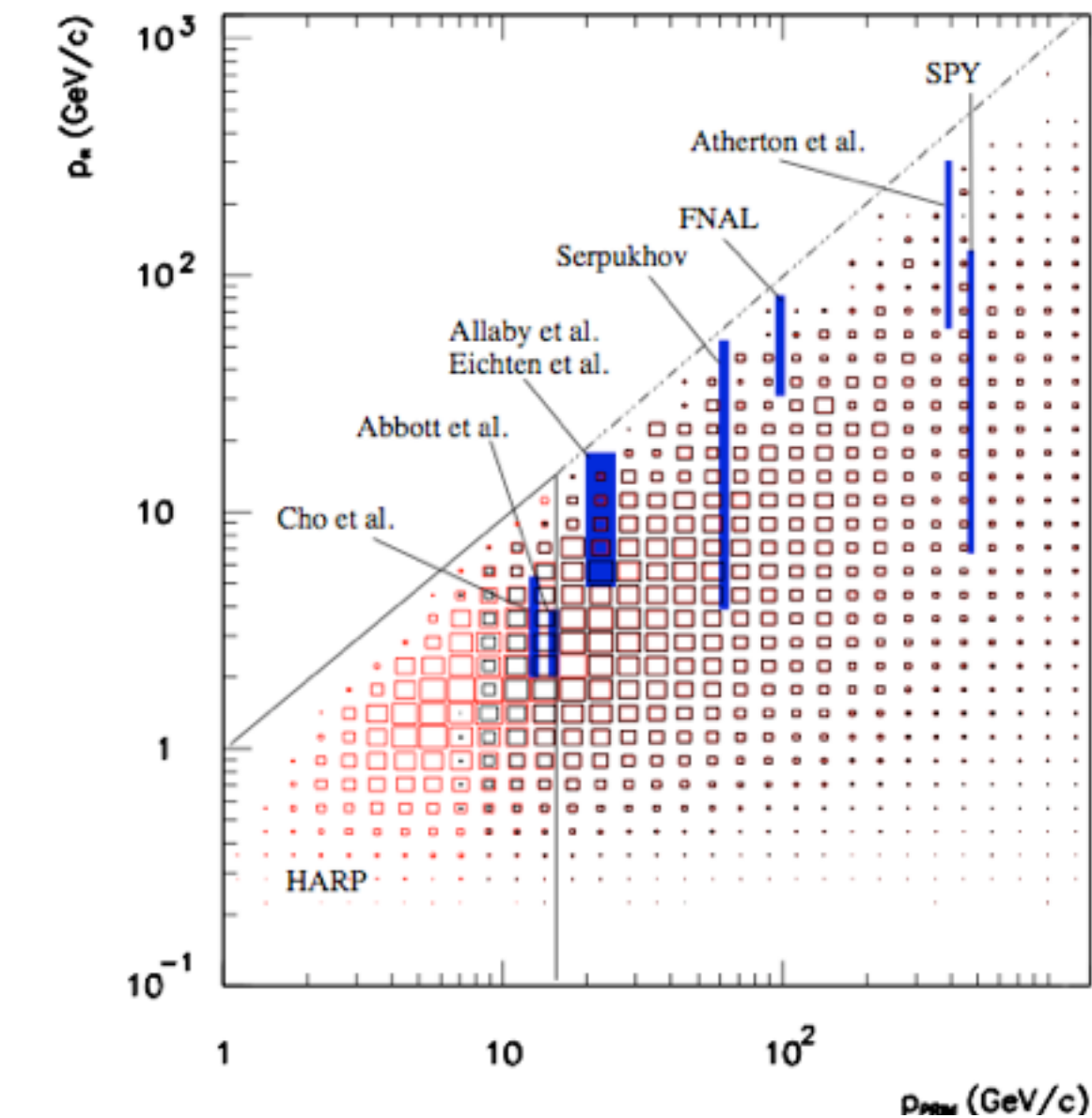
Slow dependence in energy and zenith angle:
modest subGeV angular and energy resolution is OK

- Neutrino flux (astro-ph/0611266)
 - Primary cosmic ray flux (AMS, BESS, etc.)
 - Hadron production (HARP, etc.): dominant flux error
 - Follow-up of the HARP experiment is needed!
 - Geomagnetic field effect (east-west effect)
- Ratio measurements reduce systematics to 2–3%
 - ν_e/ν_μ
 - up/down, up/horizontal
- Neutrino cross section
 - nuPRISM solves this $E_\nu=400\text{--}800\text{MeV}$ range



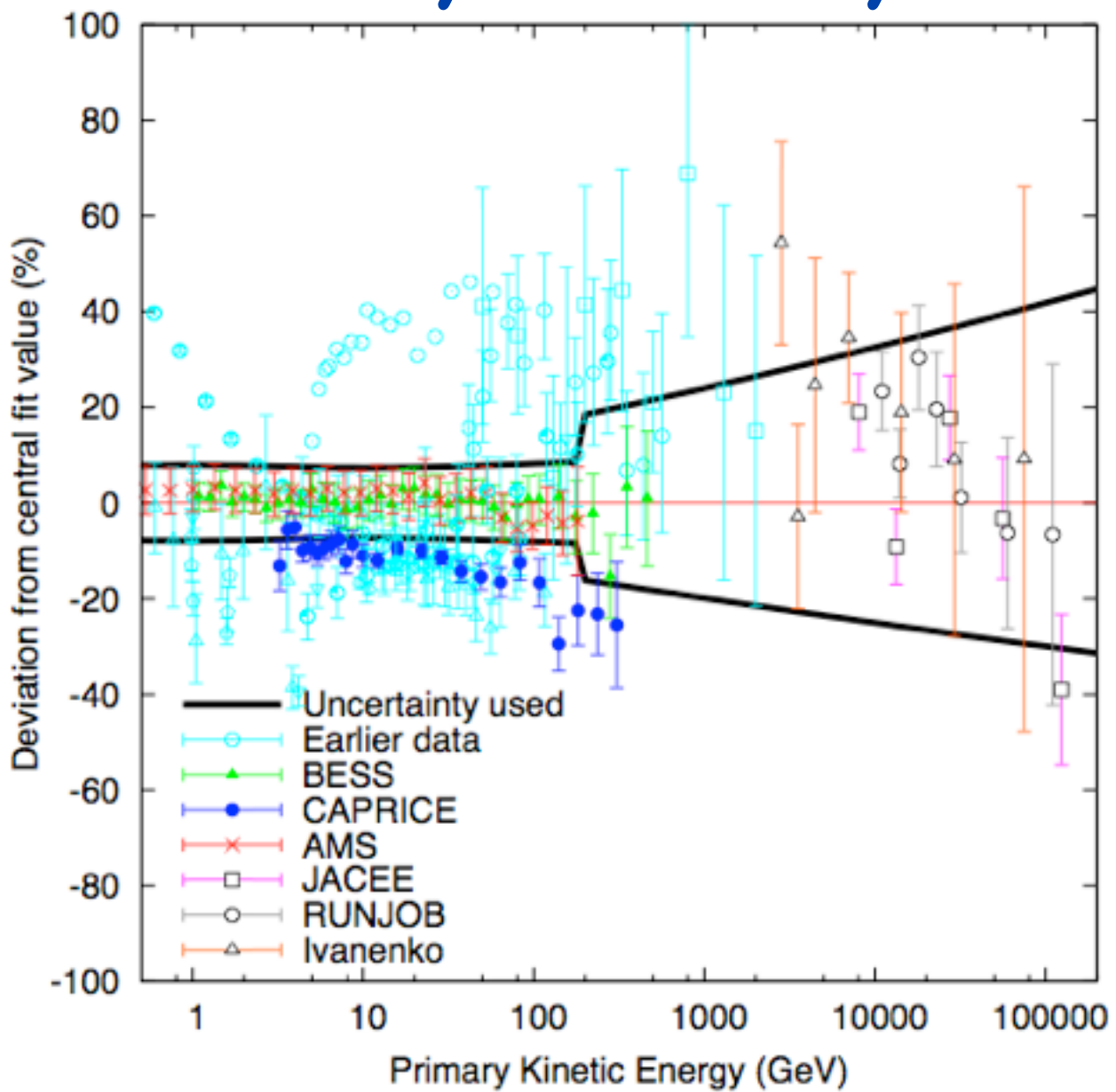
- Normalization error is $\sim 12\%$

Hadron production



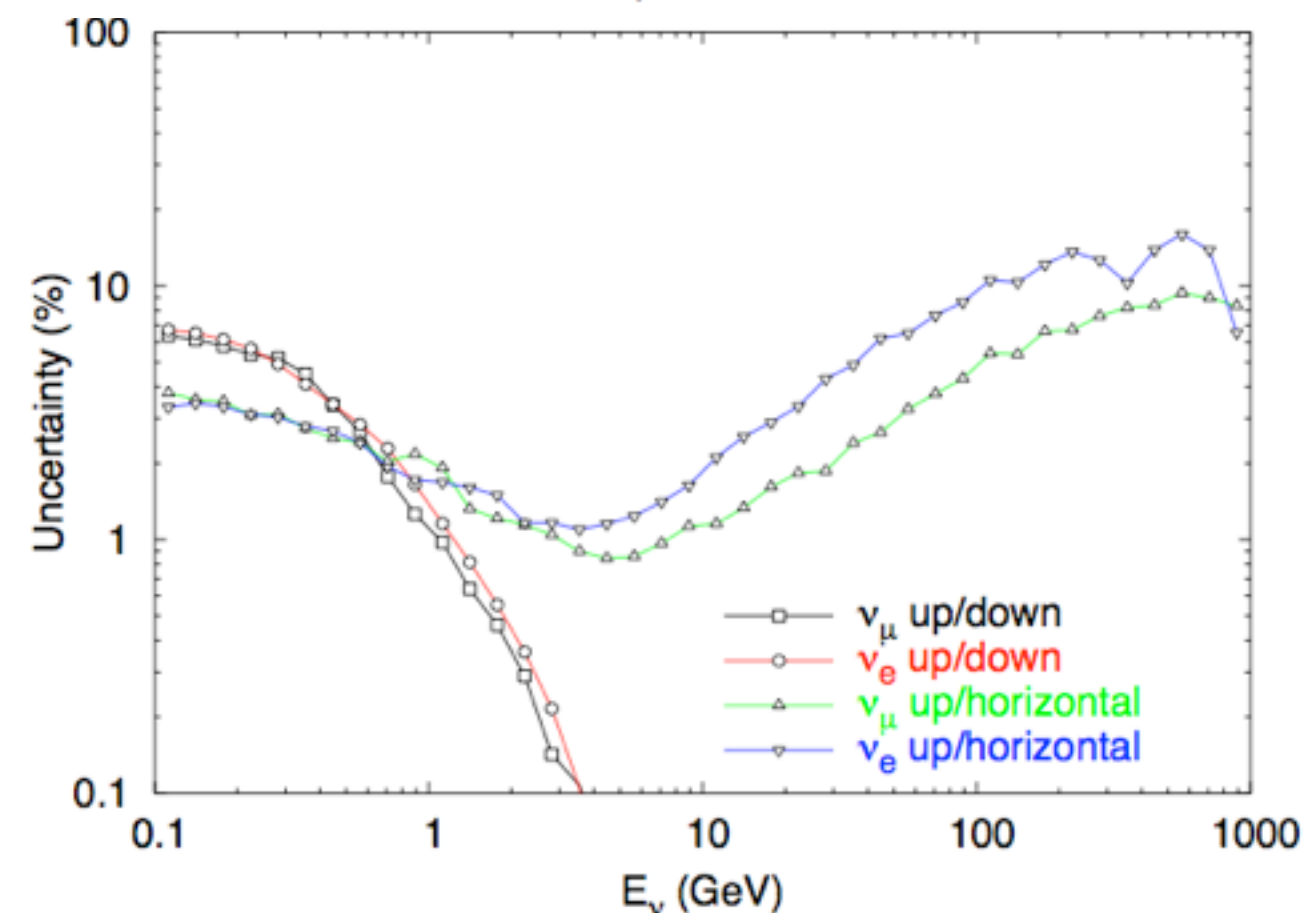
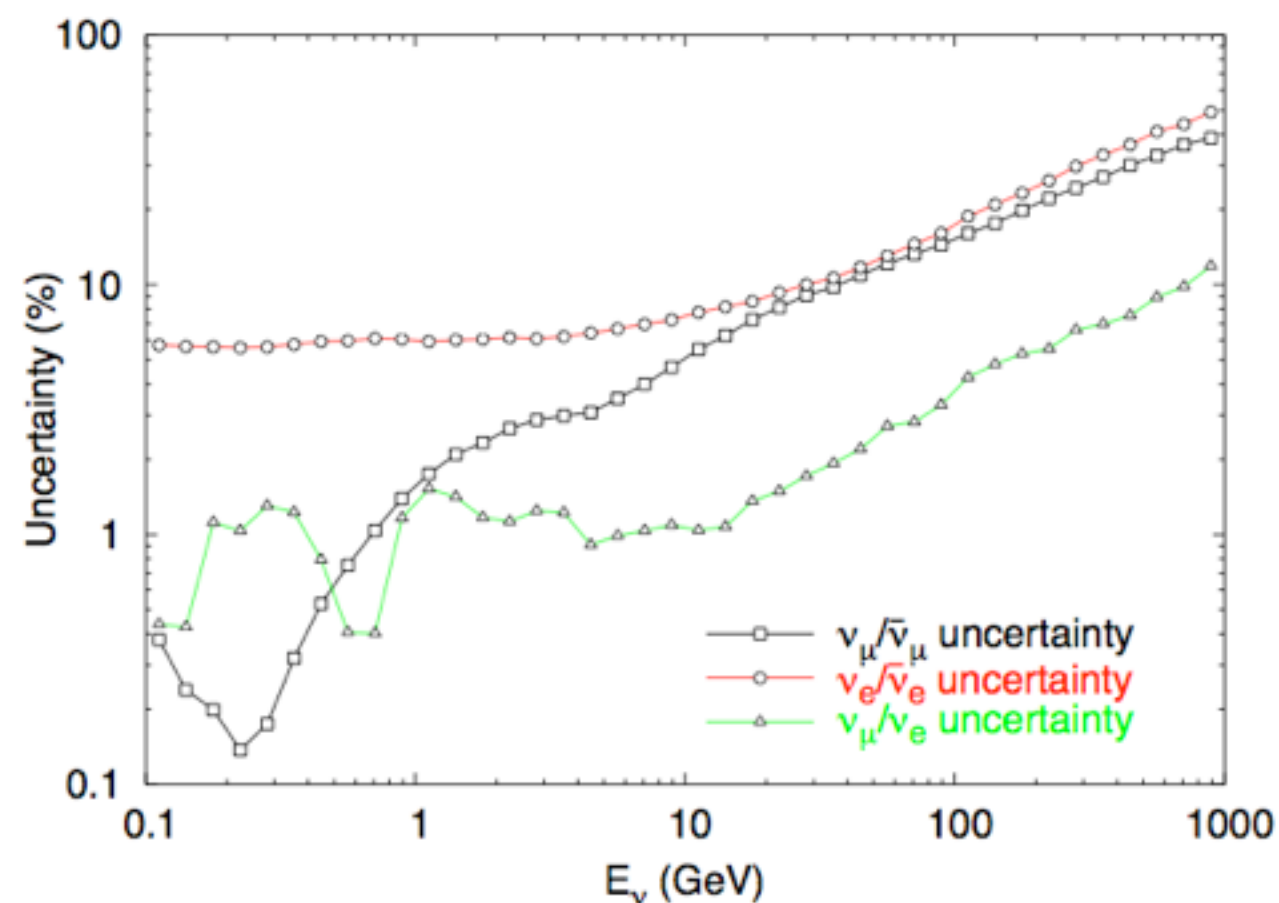
E_i (GeV)	Pions			Kaons		
	10%	30%		40%		
<8	10%	30%		40%		
8–15	30%	10%	30%	40%		
15–30	30	10	5%	10%	30	20
30–500	30	15%		30%	40	
>500	30	15%+Energy dep.		30%+Energy dep.	40	
	0	0.5	x_{LAB}	0	0.5	x_{LAB}

Primary cosmic ray flux

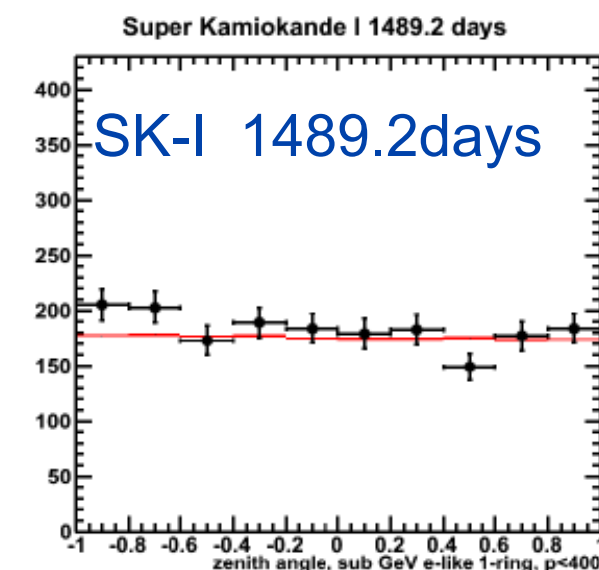
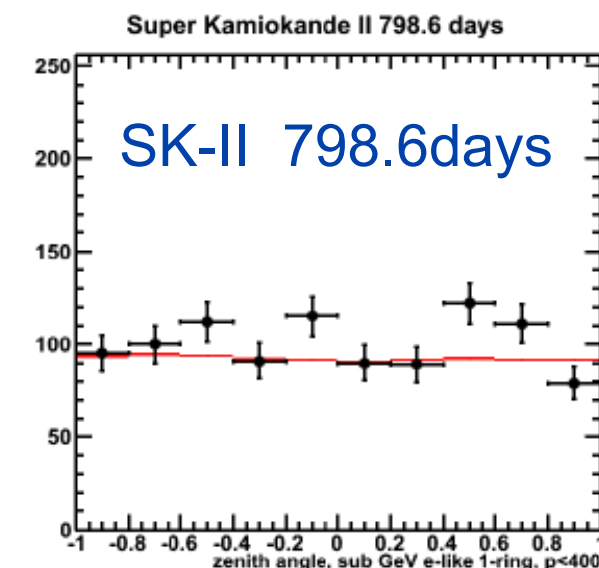
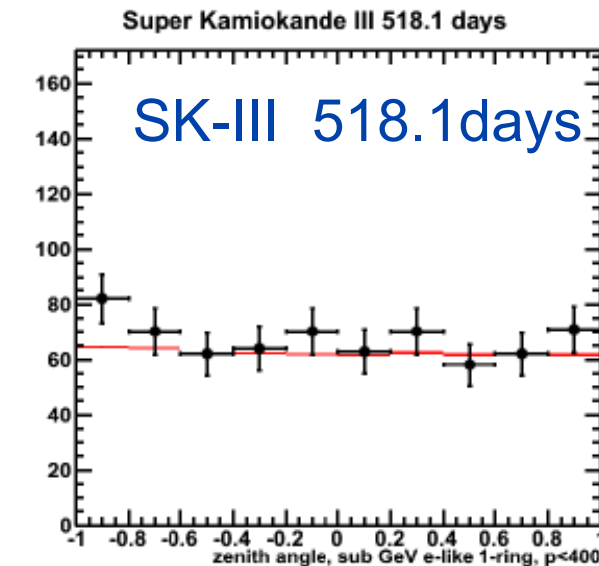
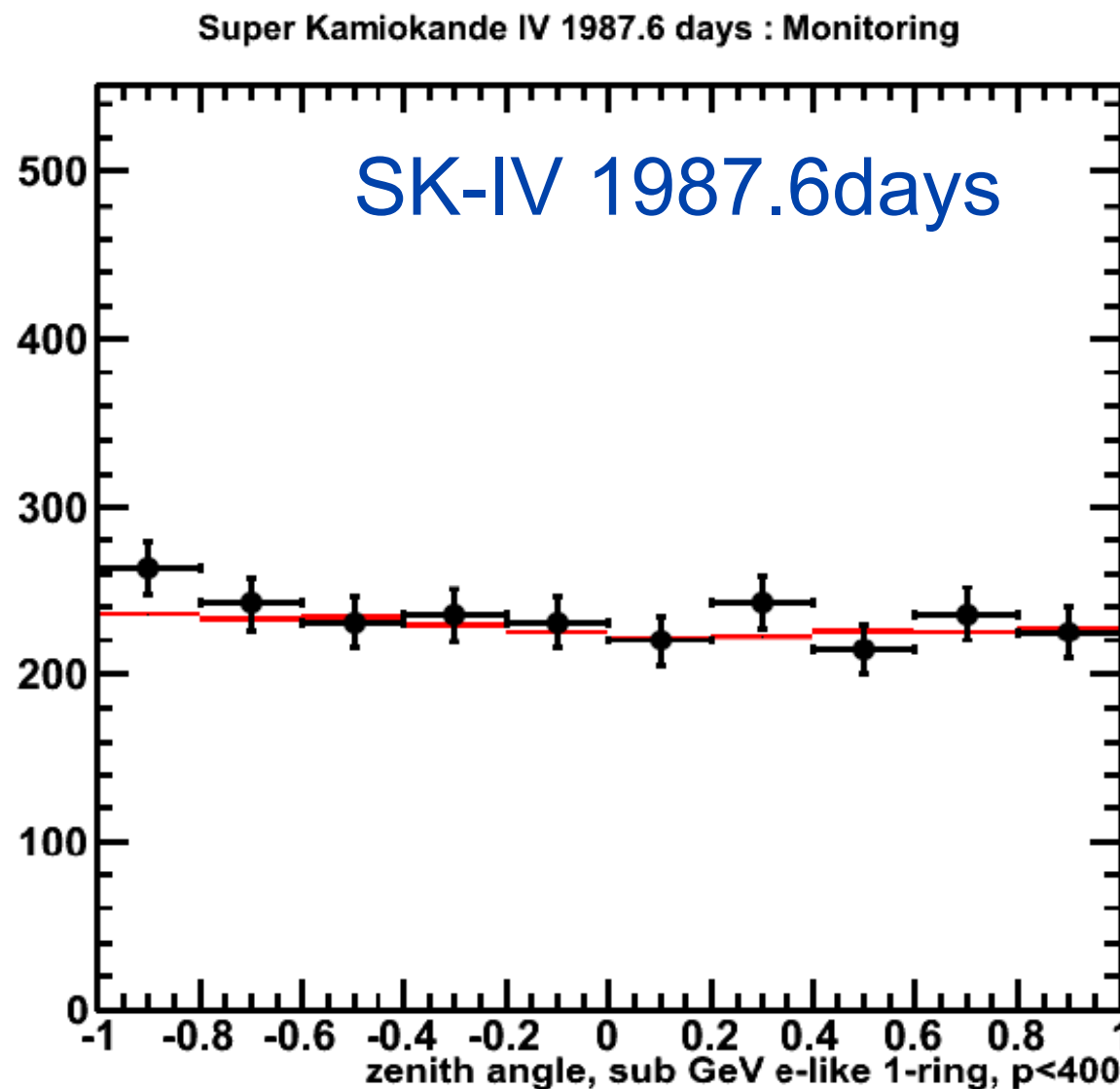


Parameter	Proton fluxes	Nuclear fluxes
a (normalization)	1.49 ± 0.10	0.060 ± 0.004
b	2.15 ± 0.025	1.25 ± 0.03
c	-2.21 ± 0.02	-0.14 ± 0.02
d (index) < 200GeV/n	2.74 ± 0.01	2.64 ± 0.02
> 200GeV/n	2.74 ± 0.03	2.64 ± 0.04

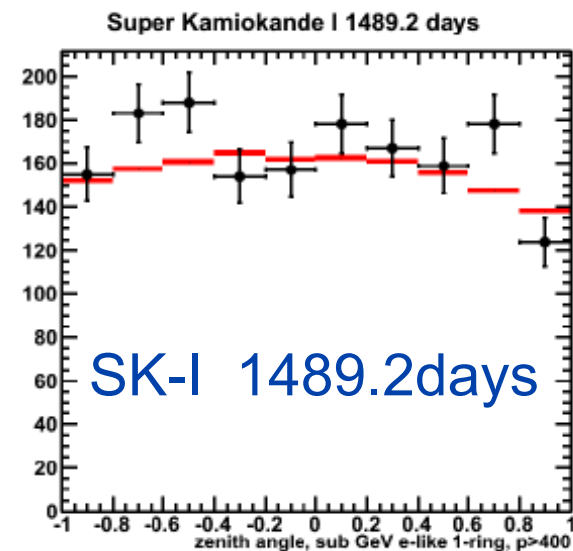
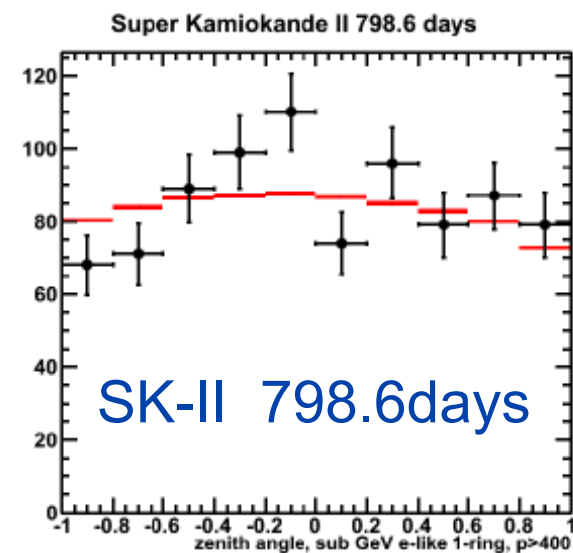
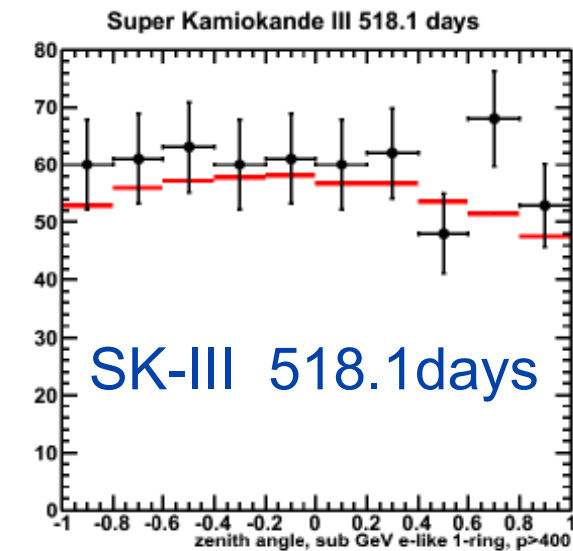
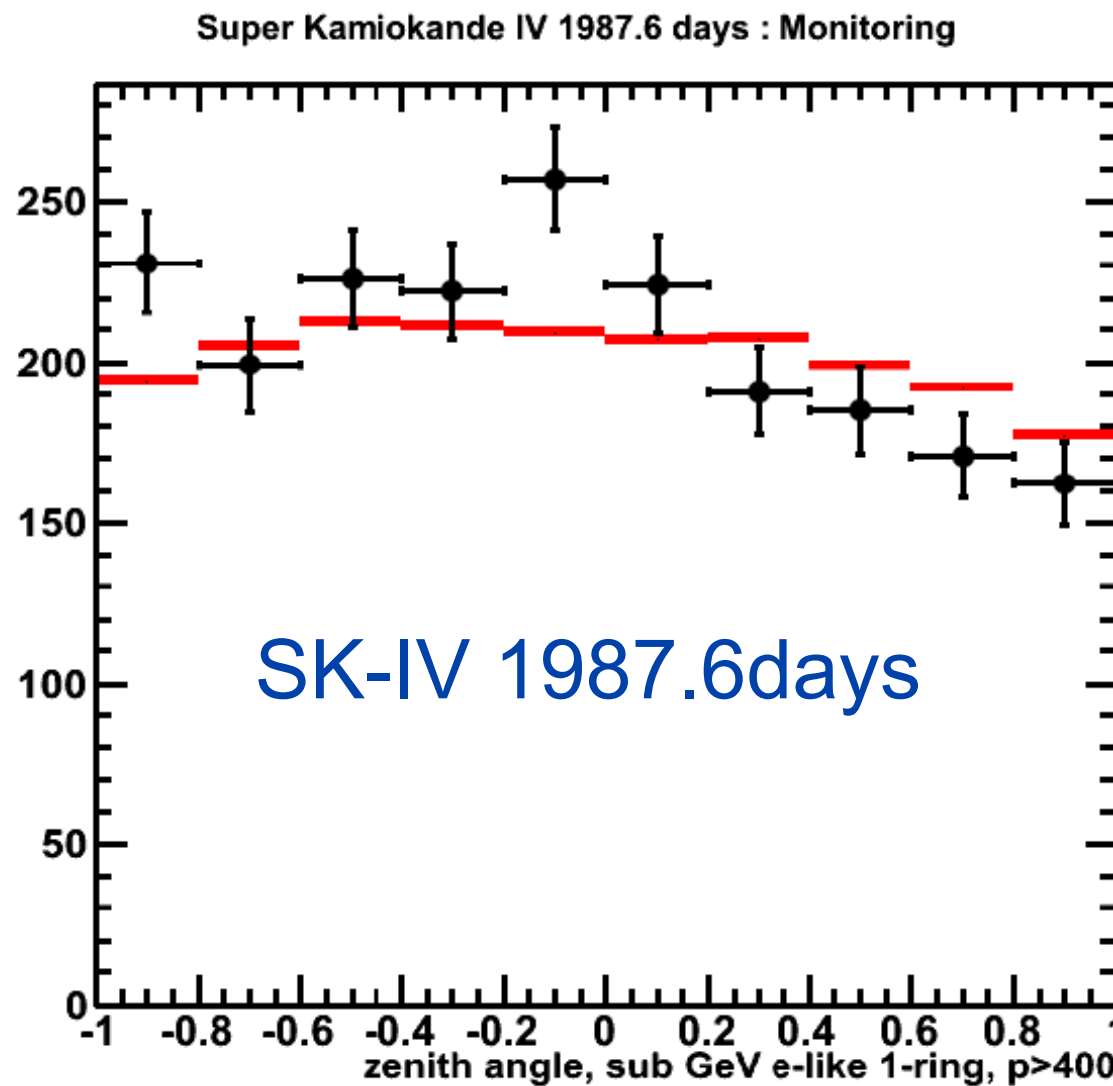
- Systematics cancels for the ratios:
 - ν_e/ν_μ ($\sim 1\%$)
 - charge asymmetry in $\nu_e/\bar{\nu}_e$ need improvement
 - hadron production study like HARP helps
 - up/down ($\sim 2-5\%$)
 - up/horizontal ($\sim 2-3\%$)
 - up/horizontal $\sim 2-3\%$
 - East-west effect to test geomagnetic field



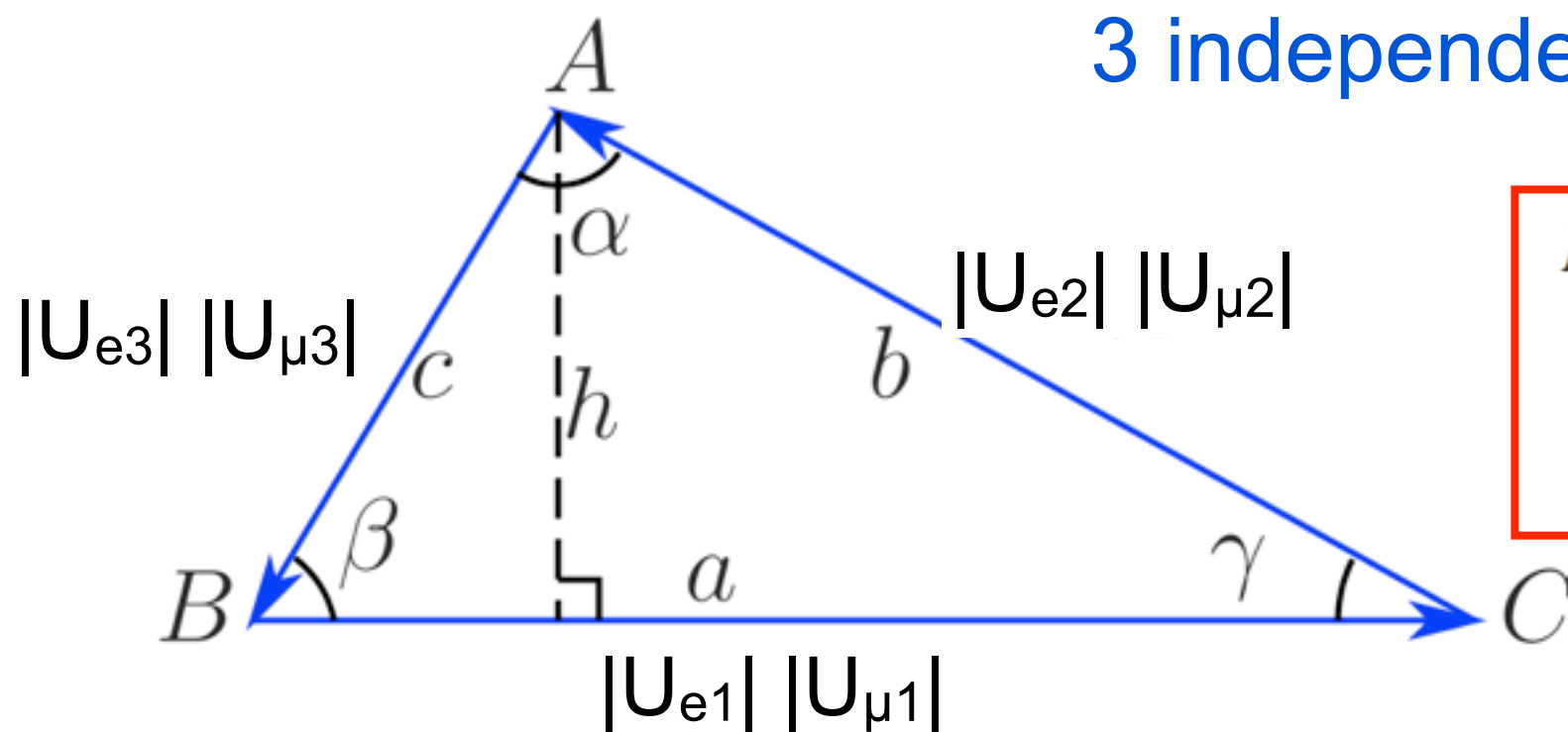
(Expected to be flat without signal)



Hint of of enhancement
@ $\cos\theta < -0.4$



$$U_{e1}U_{\mu 1}^*+U_{e2}U_{\mu 2}^*+U_{e3}U_{\mu 3}^*=0$$



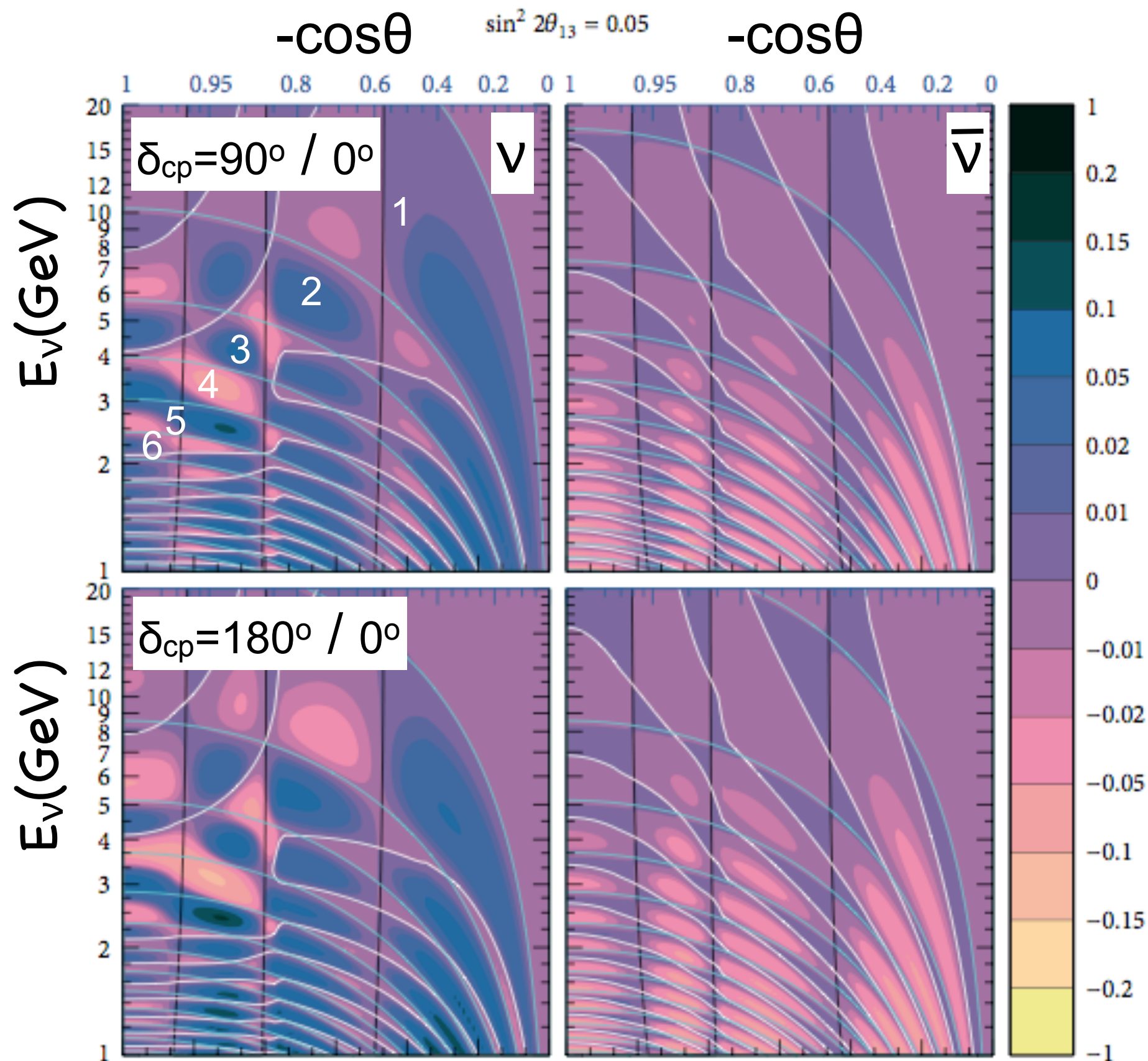
3 independent parameters

$$P_{\ell \rightarrow \ell'} = 4ab \sin(\Delta_{12} \pm \gamma) \sin \Delta_{12} \\ + 4bc \sin(\Delta_{23} \pm \alpha) \sin \Delta_{23} \\ + 4ac \sin(\Delta_{31} \pm \beta) \sin \Delta_{31}$$

- a, b, c can be determined by disappearance measurements
 - amplitude of $\nu_{\mu} \rightarrow \nu_e$ appearance also constrains $a+b$
- α, β from LBL $\nu_{\mu} \rightarrow \nu_e$ appearance, γ from atm. $\nu_{\mu} \rightarrow \nu_e$
 - comparison between ν and anti- ν : phase changes signs
 - T2K/T2HK determines β
 - γ is determined by the 0.5–0.8 GeV atm. $\nu_{\mu} \rightarrow \nu_e$
 - 1–3 GeV atm. $\nu_{\mu} \rightarrow \nu_e$ helps separating $\beta(\Delta_{31})$ and $\alpha(\Delta_{23})$ terms

ΔP_{CP} oscillogram

CP effect:
 $E \sim 2-5 \text{ GeV}$
 $\cos\theta \sim 0.9$
 (4-6th
 Δ_{32} max)



- Sub-GeV neutrino for CP study may be possible
 - Peak of atmospheric ν event rate
 - Directional information available @ $E > 0.4\text{--}0.5\text{GeV}$
 - $r = \Phi(\nu_\mu)/\Phi(\nu_e) = 2$ cancels all the oscillation signal except for CP violation.
 - Large CP violation $\nu_\mu \rightarrow \nu_e$ (10–20%) possible at 400–800 MeV
 - **If Oscillation pattern is seen, it is likely CP violation!**
 - ν_μ disappearance ($|U_{\mu 1}|^2 - |U_{\mu 2}|^2$) is also possible
- Systematic errors can be handled by
 - taking ratios: up/horizontal, up/down, ν_μ/ν_e
 - hadron production studies (more precise HARP exp.)
 - Sub-GeV neutrino cross section study is essential (nuPRISM)

More provocative note

- We could access all the sides and angles of the lepton unitarity triangle: **HyperK leads this!**
- The current SK data may already be sensitive to these parameters:
 - $\cos\theta_\nu < -0.4$ for sub-GeV ν_e (hint already there?)
 - Further improvements:
 - improved fitQun reconstruction
 - zoom in to sensitive energy region: $E_e=0.3-0.7\text{GeV}$
 - anti-neutrino study: opposite CP, good angular resolution
 - update oscillation simulation Prob3++
- ν PRISM sub-GeV cross section study and precise hadron production study like HARP would be essential for this precision measurement.

Energy and angular resolution

