Symmetry TFTs
for Continuous Spacetime Symmetries

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Based on 2509.07965
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Simons Collaboration on
Global Categorical Symmetries

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As an example, an ordinary symmetry measures (in the path integral formulation) the presence of point-like operators, which are 0-dimensional. So we call these symmetries "0-form symmetries" and we associate them to $(d-1)$-manifolds.

Symmetries and the symmetry theory

Symmetries are *categorical*

Symmetries = topological operators

This topological subsector of the theory can often be very nicely characterised as coming from a topological theory in one dimension higher, which is sometimes called the symmetry theory[1] or SymTFT. [Gaiotto, Kapustin, Seiberg, Willett '14], [Freed, Teleman '18], [Ji, Wen '19], [Kong, Lan, Wen, Zhang, Zheng '20], [Gaiotto, Kulp '20], [Apruzzi, Bonetti, IGE, Hosseini, Schäfer-Nameki '21], [Burbano, Kulp, Neuser '21], [Apruzzi '22], [Freed, Moore, Teleman '22], [Kaidi, Ohmori, Zheng '22], [. . .]

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Heuristically, the symmetry theory is the (non-invertible $\approx$ quantum) TFT that arises from gauging, in one dimension higher, the global symmetries of the theory of interest.

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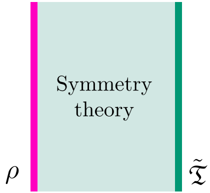
Separating global structure from local dynamics



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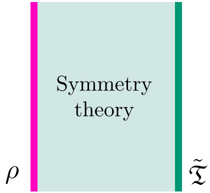

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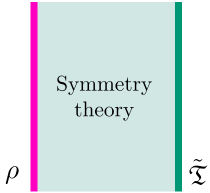

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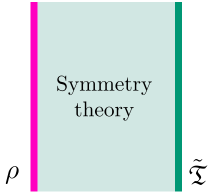

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Different choices of ρ correspond to different global structures for the theory with local dynamics $\tilde{\mathfrak{T}}$.

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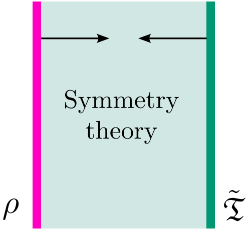

From this point of view, we obtain a standard four dimensional theory $\mathcal{T}$ by collapsing ρ and $\tilde{\mathfrak{T}}$ together, and the symmetries of $\mathcal{T}$ are those topological operators that survive the sandwiching.

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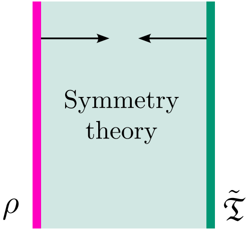

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As an example: $\mathcal{N}=4$ SYM in $d=4$ with gauge algebra $\mathfrak{su}(N)$ comes in different global forms: $SU(N)$, $PSU(N) := SU(N)/\mathbb{Z}_N$, … differing by their 1-form symmetries [Aharony, Seiberg, Tachikawa '13]

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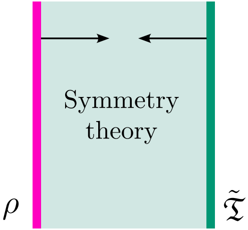

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This classification can be obtained from the SymTFT by picking $\mathbb{Z}_N$ gauge theory as the SymTFT and classifying the choices for ρ.

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But all work so far in the context of the SymTFT has been on *internal* symmetries. Our paper is a first step towards incorporating ***continuous spacetime*** symmetries (such as translations, or rotations) as gauged bulk symmetries into the SymTFT picture.

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But all work so far in the context of the SymTFT has been on *internal* symmetries. Our paper is a first step towards incorporating ***continuous spacetime*** symmetries (such as translations, or rotations) as gauged bulk symmetries into the SymTFT picture.

(The problem of how to incorporate discrete spacetime symmetries, such as time reversal, into the SymTFT as bulk gauged symmetries remains open, but see for instance [Pace, Aksoy, Lam '25] for how to incorporate *ungauged* discrete spacetime symmetries into the SymTFT picture.)

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Although the proposal I will present for how to incorporate spacetime symmetries into the SymTFT framework does not rely on having a tractable holographic description, it can be motivated by holography, so let me start there.

Holography and global structure

What is the holographic interpretation of the possible variants listed in [Aharony, Seiberg, Tachikawa '13] for the $\mathfrak{su}(N)$ $\mathcal{N} = 4$ theory in 4d?

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Answered in [Witten '98]. The key insight is that the reduction of IIB on S^5 gives a term

$$L_{\text{CS}} = 2\pi i N \int_{X_5} B_2 \wedge dC_2$$

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L_{CS} is a Lagrangian for the $d=5$ $\mathbb{Z}_N$ theory (recall that this is the SymTFT in this case), and its set of boundary conditions reproduces the possible global forms for the $\mathfrak{su}(N)$ theory.

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More involved examples are much less straightforward, but happily the same basic intuition holds in every example with a holographic dual that has been checked so far:

> The SymTFT arises from the limiting behaviour of the bulk theory near the conformal boundary of AdS.

The proposal

In the case of (anomaly-free) 0-form internal symmetries given by a group G, the SymTFT is a BF theory

$$S_{BF} = \int \mathrm{tr}(B \wedge F_A)$$

where A is a G-connection, B is an adjoint valued form, and $F_A \coloneqq dA + A \wedge A$. Here A and B are to be integrated over, so this is as a very simple topological G gauge theory. [Horowitz '89], ..., [Bonetti, Del Zotto, Minasian '25]

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Appropriate generalisations of this statement hold if G is anomalous, or if the symmetry structure is more general than a 0-form symmetry.

In this talk I will present evidence that (at least in some cases) the SymTFT for a spacetime symmetry group G_{st} is also a BF theory for G_{st}.

Where is this BF theory in AdS/CFT?

Correlation functions in the CFT$_d$ (on $\mathcal{B}^d := \partial\mathrm{AdS}_{d+1}$) correspond to partition functions of the bulk string theory with prescribed boundary conditions: [Gubser, Klebanov, Polyakov '98], [Witten '98]

$$\left\langle e^{\int_{\mathcal{B}^d} \phi_0(x)\mathcal{O}(x)d^dx} \right\rangle_{\mathsf{CFT}} = Z_{\mathsf{string}}[\phi|_{\mathcal{B}^d} = \phi_0]\,.$$

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For continuous internal 0-form symmetries of the CFT, we have [Witten '98]

$$\left\langle e^{\int_{\mathcal{B}^d} A_0 \wedge j} \right\rangle_{\mathsf{CFT}} = Z_{\mathsf{string}}[A|_{\mathcal{B}^d} = A_0]\,,$$

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From here, we have that insertion of a symmetry operator

$$U_\alpha(\Sigma) := e^{\alpha \int_{\Sigma^{d-1}} j}$$

in the CFT corresponds to choosing a singular boundary value $A_0 = \alpha\delta(\Sigma^{d-1})$ for the supergravity fields.

Internal symmetries and holography

If we choose $\Sigma^{d-1} = \partial D^d$ then $A_0 = \alpha\delta(\Sigma^{d-1})$ can be understood as the result of acting on $A_0 = 0$ with a gauge transformation $A_0 \to A_0 + d\lambda$, where

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(Here when talking about Gauss law generators I am thinking of the boundary conditions as a state in the Hilbert space of the bulk theory, quantised taking the radial direction in AdS as the time direction.)

Internal symmetries and holography

Maxwell as a deformed BF theory

A useful way to understand the generator of gauge transformations explicitly is to start from the following formulation of Maxwell theory:

$$S_{\text{Maxwell}} = \frac{e^2}{4\pi}\int B \wedge \star B + 2\pi i \int B \wedge dA\,.$$

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(Alternatively, we could integrate A out, which imposes $B = dC$, so this becomes Maxwell theory with connection C and coupling $1/e^2$. [Buscher '88], [Roček, Verlinde '92], [Witten '95])

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(Alternatively, $B \sim \star F$, and this becomes the usual statement in electromagnetism that $d \star F$ generates gauge transformations.)

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This discussion extends, mutatis mutandis, to non-abelian internal symmetries [Cattaneo, Rossi '02], [Jia, Luo, Tian, Wang, Zhang '25], [Apruzzi, Dondi, IGE, Lam, Schäfer-Nameki '25], [Bonetti, Del Zotto, Minasian '25], [Bah, Bonetti, Chitoto, Leung '25].

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Famously, the conformal group on the boundary arises from the boundary behaviour of the bulk gravitational sector. [Brown, Henneaux '86]

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- For even $d > 0$ there's a conformal anomaly, so this case should requite extra ingredients (a CS term). More details in the paper. (We don't understand this case well enough.)
- For $d = 1$ we famously have a bulk described by JT gravity, which is indeed an $SO(2,1)$ BF gauge theory, at a Lagrangian level at least.

Spacetime symmetries and holography

MacDowell-Mansouri gravity

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$$S_{\mathsf{MM}} = \int_{\mathcal{M}^4} B^{IJ} \wedge F_{IJ} - \frac{1}{2} \int_{\mathcal{M}^4} B_{IJ} \wedge B_{KL} \epsilon^{IJKLM} v_M \,,$$

where $B \in \Omega^2(\mathcal{M}^4, \mathfrak{so}(4,1))$, $F := dA + A \wedge A$ and A a connection on a principal $SO(4,1)$ bundle. The $I, \ldots, M$ indices are $SO(4,1)$ vector indices, and v_M is a (dimensionless) fixed $SO(4,1)$ vector.

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MacDowell-Mansouri gravity

$$S_{\text{FSS-MM}} = \int_{\mathcal{M}^4} B^{IJ} \wedge F_{IJ} - \frac{1}{2}\int_{\mathcal{M}^4} B_{IJ} \wedge B_{KL}\epsilon^{IJKLM} v_M \,,$$

If we choose $v = (0,0,0,0,\Lambda G_N/6)$, and integrate B out, we end up with the MacDowell-Mansouri action

$$S_{\text{MM}} = \frac{3}{2\Lambda G_N}\int_{\mathcal{M}^4} F_{IJ} \wedge F_{KL}\epsilon^{IJKL5} = \frac{3}{2\Lambda G_N}\int_{\mathcal{M}^4} \hat{F}_{mn} \wedge \hat{F}_{pq}\epsilon^{mnpq}$$

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with m, n, p, q now $SO(4)$ indices, and $\hat{F}_{mn} := F_{mn}$.

To get ordinary (first-order) gravity set $e^m := \ell A^{5m}$, $\omega^{mn} := A^{mn}$:

$$S = \frac{-1}{2G_N}\int_{\mathcal{M}^4} \epsilon_{mnpq}\left(R(\omega)^{mn} \wedge e^p \wedge e^q - \frac{\Lambda}{6} e^m \wedge e^n \wedge e^p \wedge e^q + \dots\right)$$

where I have omitted a term proportional to the Euler density, and $\ell^2 := 3/\Lambda$. (See [Wise '06], [Wise '09] for more.)

Spacetime symmetries and holography

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The boundary CFT couples to the boundary value A_0 via the following decomposition [Fradkin, Tseytlin '85]

$$A_0 = \frac{1}{2} w^{ab} L_{ab} + e^a P_a + f^a K_a + bD \,,$$

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Although the way this background couples to the CFT is very different from an ordinary gauge background, all the SymTFT needs to do is to transform the metric structure as the spacetime symmetry would. I'll show that this can be achieved by shifting A_0 appropriately via a bulk B operator, so it is enough to think about the BF term only.

Spacetime symmetries and holography

So at least in the case of CFT$_3$ / 4d gravity the $v \to 0$ limit of gravity suggests that we can indeed take a BF theory for the conformal group $SO(4,1)$ as our spacetime SymTFT.

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- But for $v = 0$ we can think of the full group of diffeomorphisms in terms of $SO(4,1)$ gauge transformations.

Testing the conjecture beyond holography

One of the advantages of the SymTFT, compared to full-fledged holography, is that the SymTFT is just as tractable for theories without a holographic limit. In the rest of the talk, I will work in terms of the SymTFT, without assuming the existence of a holographic limit.

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Let me start by identifying the conformal symmetry generators in terms of BF theory with the expected symmetry generators in a conformal field theory. I will do this in flat space, for simplicity.

Symmetry operators from the BF theory

From the action

$$S = \int \mathrm{tr}(B \wedge F_A)$$

with $F_A = dA + A \wedge A$ we read an equation of motion for A of the form $d_A B = dB + [A, B] = 0$.

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In general, we postulate that the geometric interpretation of the boundary value of A is as in [Fradkin, Tseytlin '85]

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We expand

$$B = \frac{1}{2} j^{ab} L^*_{ab} + t^a P^*_a + s^a K^*_a + \phi D^*\,,$$

where $*$ denotes the dual algebra elements.

Symmetry operators from the BF theory

The $d_A B = 0$ equation of motion implies

$$
\begin{aligned}
0 &= \mathrm{d}j^{ab} + w^{[a}{}_c \wedge j^{cb]} - 2e^{[a} \wedge t^{b]} - 2f^{[a} \wedge s^{b]} \ , \\
0 &= \mathrm{d}t^a + w^a{}_b \wedge t^b + f^b \wedge j_b{}^a + f^a \wedge \phi - b \wedge t^a \ , \\
0 &= \mathrm{d}s^a + w^a{}_b \wedge s^b + e^b \wedge j_b{}^a - e^a \wedge \phi + b \wedge s^a \ , \\
0 &= \mathrm{d}\phi - 2e^a \wedge t_a + 2f^a \wedge s_a \ .
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The equations above then simplify to

$$\begin{aligned}
0 &= \mathrm{d}(j^{ab} - 2x^{[a}t^{b]}) \ , \\
0 &= \mathrm{d}t^a \ , \\
0 &= \mathrm{d}(s^a + x^b {j_b}^a - x^a\phi - x^b x_b t^a + 2x^a x^b t_b) \ , \\
0 &= \mathrm{d}(\phi - 2x^a t_a) \ .
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Symmetry operators from the BF theory

Out of these conserved currents we can reproduce the familiar generators of the conformal algebra

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\begin{aligned}
\mathcal{P}^a &= \oint t^a \ , \\
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(More precisely, we obtain the usual generators if we identify t^a with $\star T^a$, where $T^a \coloneqq e^{a\mu} T_{\mu\nu} dx^\nu$.)

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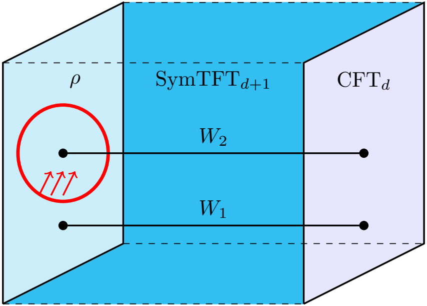
Note that these generators depend explicitly on the coordinates, so they can be invariant under deformations while failing to commute with the energy-momentum tensor. ($dX/dt = [X, H] + \partial X / \partial t$.)

How to move a point?

Consider for example the $\mathcal{P}^a = \oint t^a$ generator, which should generate translations. How does it achieve this in practice, in the following configuration?

$\mathcal{U}_{e^{x^a \mathcal{P}_a}}[\Sigma_{d-1}]$



Commutation relations

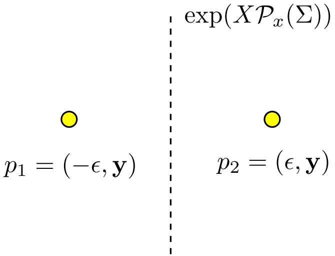
Since A and B are canonically conjugate variables, and $e \otimes P \in A$, $t \otimes P^* \in B$, we have

$$\{e^b, \mathcal{P}_a(\Sigma_{d-1})\} = \delta^b{}_a\, \delta^{(1)}(\Sigma_{d-1})\,,$$

which implies that the translation $\exp(X\mathcal{P}_a(\Sigma_{d-1}))$ acts as a shift of the vielbein $e^b \to e^b + X\delta^b{}_a\, \delta^{(1)}(\Sigma_{d-1})$.

Introducing space

Take, for simplicity, $\mathcal{M}^d = \mathbb{R}^d$ with the flat metric, parameterised by $(x, y^1, \ldots, y^{d-1})$. We take $\Sigma_{d-1} = \{x = 0\}$ and choose to generate translations in the x direction.



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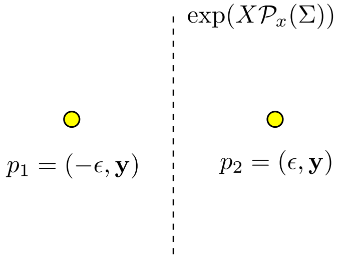

The effect of this operator is to modify the vielbein

$$e^x \to e^x + X\delta(x)\mathrm{d}x\,.$$

which modifies the distance $\int e$ between the points, as expected for a translation.

Translations as gauge transformations

There is a nicer interpretation of the previous calculation: rewrite

$$\mathcal{P}_x(\Sigma) = \int_\Sigma t_x = \int_\Theta dt_x$$

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where $\lambda := \theta(x)\delta^a{}_x$ and $d\theta = \delta(x)$, which reproduces $\{e^b, \mathcal{P}_a(\Sigma_{d-1})\} = \delta^b{}_a\, \delta^{(1)}(\Sigma_{d-1})$.

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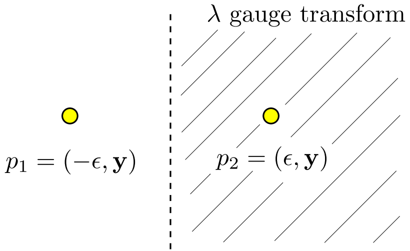

How to move a point

A beautiful property of BF theory is useful at this point: consider the action of a diffeomorphism ξ on A:

$$\mathcal{L}_\xi A = \iota_\xi dA + d\iota_\xi A = \iota_\xi F_A + d_A \iota_\xi A \,.$$

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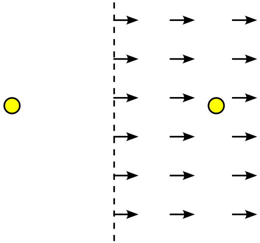
In our case the converse statement holds: there is a diffeomorphism that undoes the gauge transformation generated by $\mathcal{P}_x(\Sigma_{d-1})$, and takes A back to the flat metric. This diffeomorphism separates the points as expected.



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- Undo the action of Φ^{-1} on A_0 by acting with (a "passive" diffeomorphism) Φ on the metric and the operator insertions.
- Effectively, this moves all operator insertions according to Φ, keeping the background A_0 invariant, so the overall effect is:

$$\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_n(x_n) \rangle_{A_0} \to \langle \mathcal{O}_1(\Phi(x_1)) \cdots \mathcal{O}_n(\Phi(x_n)) \rangle_{A_0} \ .$$

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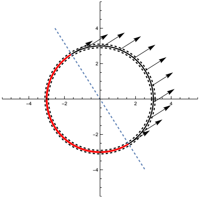

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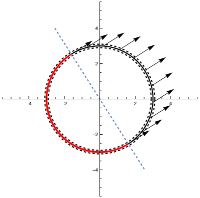

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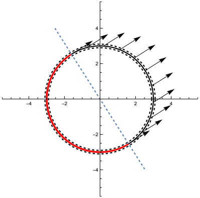

The issue only arises if we start from the localised symmetry generators, instead of smooth vector fields defining diffeomorphisms. To me there seems to be no good reason *not* to consider such localised symmetry generators:

How does the CFT react to these regions?

Ghost in the Shell 2

(Preliminary) work in progress with Michelangelo Tartaglia

Take the natural coupling of QFT fields to the first order formalism:

$$\mathcal{L} = \det(e)\eta^{ab}e_a^\mu e_b^\nu \partial_\mu\phi\partial_\nu\phi + \dots$$

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Is this a problem?

Pauli-Villars as ghosts

The Pauli-Villars regularisation method is typically formulated by replacing (scalar, say) propagators with substracted ones:

$$\frac{1}{p^2+m^2} \to \frac{1}{p^2+m^2} - \frac{1}{p^2+M^2}$$

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['t Hooft and Veltman '81] point out that this regularisation can be achieved at the Lagrangian level by the replacement:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - V(\phi)$$

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So what's happening in our case is heuristically a spatially localised version of the Pauli-Villars mechanism.

SymTFTs for SSB

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I am abusing language here: by SSB I really mean non-linear realisation, in the style of [Callan, Coleman, Wess, Zumino '69]. True SSB would require studying SymTFTs with corners, as in [Copetti '24], [Cordova, Holfester, Ohmori '24], [Cvetič, Donagi, Heckman, Hübner, Torres '04], [Copetti, Cordova, Komatsu '24], [IGE, Huertas, Uranga '24], [Choi, Rauhaun, Zheng '24], [Das, Moline, Vilaplana, Saura-Bastida '24], [Bhardwaj, Copetti, Pajer, Schäfer-Nameki '24], [Heyman, Quella '24], . . . , [Bonetti, Del Zotto, Minasian '25]

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The photon can be seen as a Goldstone mode of an spontaneously broken 1-form symmetry. [Kovner, Rosenstein '92], [Gaiotto, Kapustin, Seiberg, Willett '14]

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- A modified Neumann boundary condition on the right boundary [Maldacena, Moore, Seiberg '01]

$$\frac{1}{2e^2}\int_{\text{R-bdry}} B|_R \wedge \star B|_R$$

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Defining $B|_L - B|_R = da$, we end up with the expected effective action:

$$\frac{1}{2e^2}\int (da - B_L) \wedge \star (da - B_L)\,.$$

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Additionally, we need to give meaning to the Hodge star "$\star$" in the (non-abelian analogue of the) boundary term leading to the Goldstone kinetic term:

$$\frac{1}{2e^2}\int_{\text{R-bdry}} A \wedge \star A\,.$$

Conformal symmetry breaking

Hodge stars without metrics

Recall that the geometric structures the CFT couples to appear in the decomposition of the A field on the Dirichlet boundary:

$$A_L = \frac{1}{2} w^{ab} L_{ab} + e^a P_a + f^a K_a + bD \,,$$

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Whenever the vielbeine e^a arising from A_L are non-degenerate (in the SymTFT sandwich this follows from a suitable choice of Dirichlet boundary) we define a linear $\mathrm{Hod}(A, \cdot)$ operation acting on the basis of p-forms (and then, by linearity, on every p-form) as

$$\mathrm{Hod}(A, e^{a_1} \wedge \cdots \wedge e^{a_p}) = \frac{1}{(d-p)!} \eta^{a_1 b_1} \cdots \eta^{a_p b_p} \epsilon_{b_1 \ldots b_d} e^{b_{p+1}} \wedge \cdots \wedge e^{b_d} \,.$$

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Using this "relative" Hodge star, we obtain the expected action for the dilaton [Monin, Pirtskhalava, Rattazzi, Seibold '16]

$$S_{\mathsf{Sandwich}} = \frac{f^2}{2} \int_{\mathcal{M}^d} \mathrm{d}^d x \sqrt{g}\, e^{-(d-2)\sigma} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \,, \quad g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu \,.$$

Conclusions

To construct a SymTFT description of continuous spacetime symmetries, we need to understand first SymTFTs for continuous non-abelian symmetries, and there has been remarkable progress on this during the last couple of years, in the non-abelian case building on work by Cattaneo and Rossi in 2000 and 2002 [Brennan, Sun '24], [Antinucci, Benini '24], [Apruzzi, Bedogna, Dondi '24], [Arbalestrier, Argurio, Tizzano '24], [Bonetti, Del Zotto, Minasian '24], [Argurio, Collinucci, Galati, Hulik, Paznokas '24], [Jia, Luo, Tian, Wang, Zhang '25], [Apruzzi, Dondi, IGE, Lam, Schäfer-Nameki '25], [Bonetti, Del Zotto, Minasian '25], [Bah, Bonetti, Chitoto, Leung '25].

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Spacetime symmetries bring in additional conceptual complications:

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- What is the connection of operators in the SymTFT to diffeomorphisms?

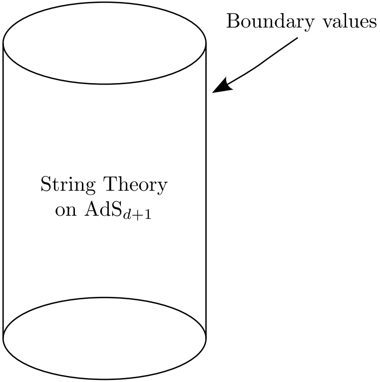
Building on the work above, we proposed a SymTFT for spacetime symmetries, where we can give partial answers to these questions, and which passes additional checks.

Open problems

There are a lot of interesting open directions, here are some:

- We should strengthen the connection to gravity, particularly for QFTs in even dimensions, which have conformal anomalies.
- We encountered an interesting twist in the usual issue of (non-)invertibility of vielbeine in first order formulations of gravity. Can we give meaning in field theory to the generically unavoidable "non-geometric" regions created by localised symmetry generators?
- Other phases of spacetime symmetry breaking? [Nicolis, Penco, Piazza, Rattazzi '15]
- Extension to supersymmetry. [Wang, Zhang '23], [Ambrosino, Luo, Wang, Zhang '24], [Grassi, Penati '25]
- Mixed spacetime/internal symmetries.
- We need a better mathematical understanding of the relevant SymTFTs. [Weis '22], [Marín-Salvador '25], [Stockall, Yu '25]
- Finite spacetime symmetries!

The SymTFT from holography



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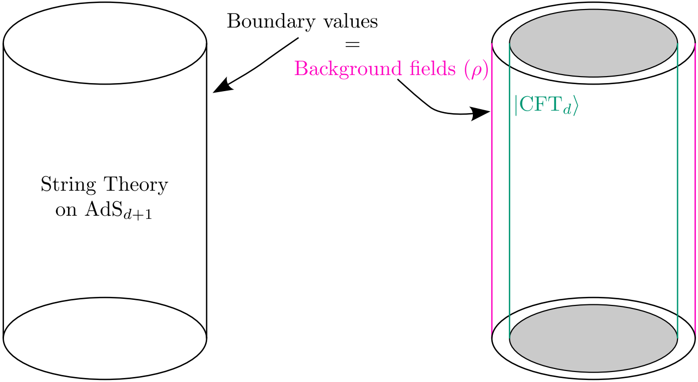

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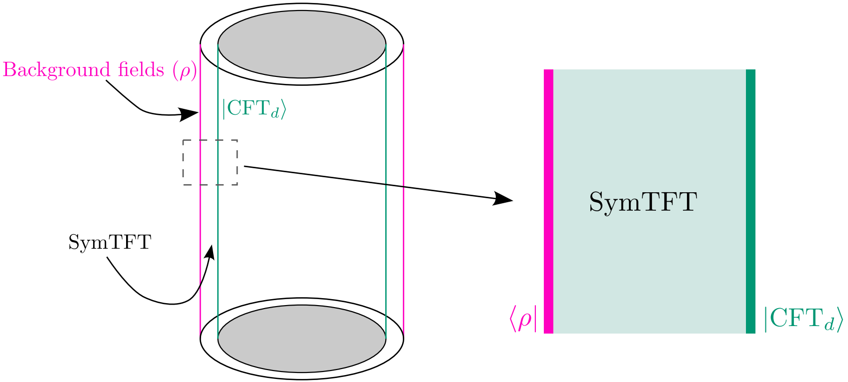

Symmetries as topological operators

An abelian symmetry $\hat{U}_\alpha = e^{i\alpha\hat{Q}}$ acts on operators $\mathcal{O}_q$ of charge q as

$$\hat{U}_\alpha \mathcal{O}_q \hat{U}_\alpha^{-1} = e^{i\alpha q}\mathcal{O}_q \, .$$

(This is the exponentiated version of $[\hat{Q}, \mathcal{O}_q] = q\mathcal{O}_q$.)

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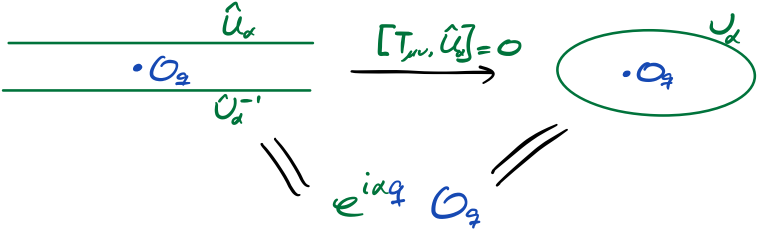
Internal symmetries commute with the energy-momentum tensor, so they can be deformed, as long as we don't encounter a charged operator. In the path integral:



B holonomies in the non-abelian BF theory

From [Cattaneo, Rossi '02]

$$\mathcal{U}_{d-1}^{[e^X]}[\Sigma_{d-1}] \coloneqq \int \mathcal{D}\alpha_0 \mathcal{D}\beta_{d-2} \,\mathsf{exp}\left\{ i \oint_{\Sigma_{d-1}} \langle \alpha_0, \mathrm{d}_A \beta_{d-2} + B_{d-1} \rangle \right\} .$$

where

$$\alpha_0 \in \Omega^0(\Sigma_{d-1}, [X]) ,$$
$$\beta_{d-2} \in \Omega^{d-2}(\Sigma_{d-1}, \mathfrak{g}^*) .$$

Example: $\mathcal{N} = 4$ SYM

Consider for instance the case of $\mathcal{N} = 4$ super-Yang-Mills theory with gauge algebra $\mathfrak{su}(2)$ in $d = 4$. For fixed coupling, there are three four dimensional theories with this algebra (modulo SPTs), known as $SU(2)$, $SO(3)_+$ and $SO(3)_-$. [Gaiotto, Moore, Neitzke '10], [Aharony, Seiberg, Tachikawa '13].

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Ignoring the R-symmetry (and its anomaly) and charge conjugation for simplicity, the symmetry theory in this case reduces to a higher discrete $\mathbb{Z}_2$ gauge theory, which can be presented in BF form: [Maldacena, Moore, Seiberg '01], [Banks, Seiberg '10]

$$S_{BF} = 2\pi i N \int_{X^5} B_2 \wedge dC_2 \,,$$

where I am normalising B_2 and C_2 to have period 1. (Here $N=2$, but working in general makes the discussion easier to follow.)

Example: $\mathcal{N}=4$ SYM (continued)

The equations of motion in this theory are $dB_2 = 0$ and $dC_2 = 0$. The non-trivial operators are

$$U(\Sigma_2) = \exp\left(2\pi i \int_{\Sigma_2} B_2\right) \qquad V(\Theta_2) = \exp\left(2\pi i \int_{\Theta_2} C_2\right)$$

subject to the constraint $U^N = V^N = 1$. Because B_2 is canonically conjugate to $-2\pi i N C_2$:

$$[B_2(x), C_2(y)] = -\frac{1}{2\pi i N}\delta(x-y)$$

so in the Hamiltonian picture, on a 4d spatial slice, these operators have commutation relations

$$U(\Sigma_2)V(\Theta_2) = e^{-2\pi i\, \Sigma_2\cdot\Theta_2/N} V(\Theta_2)U(\Sigma_2)\,.$$

Example: $\mathcal{N} = 4$ SYM (continued)

We would like to understand the gapped boundary conditions for the BF theory. These arise from giving some fields Dirichlet boundary conditions on the 4d boundary, which I call $\mathcal{M}^4$.

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We cannot just set $B_2 = C_2 = 0$ in general, since this would imply $U(\Sigma_2) = V(\Theta_2) = 1$ for all Σ_2 and Θ_2. If $H_2(\mathcal{M}^4; \mathbb{R}) \neq 0$ this leads to an inconsistency with the commutation relation

$$U(\Sigma_2)V(\Theta_2) = e^{-2\pi i\, \Sigma_2 \cdot \Theta_2 / N} V(\Theta_2)U(\Sigma_2)\,.$$

Example: $\mathcal{N} = 4$ SYM (continued)

We would like to understand the gapped boundary conditions for the BF theory. These arise from giving some fields Dirichlet boundary conditions on the 4d boundary, which I call $\mathcal{M}^4$.

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The right approach is as usual in quantum mechanics when we have non-commuting operators: choose a maximal subset of commuting operators, and specify a basis of states by giving the eigenvalues for this subset. [Kapustin, Saulina '10]

Example: $\mathcal{N} = 4$ SYM (continued)

For $N = 2$ there are three non-trivial operators (up to scaling): U, V and UV, and the following "universal" (independent of $\mathcal{M}^4$) choices of a maximal commuting set L: $\{U\}$, $\{V\}$ and $\{UV\}$.

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The operator $U(\Sigma_2) = \exp(2\pi i \int_{\Sigma_2} B_2)$ then becomes a trivial operator when pushed to the boundary. It can nevertheless *end* on the boundary, since for this boundary condition we don't sum over gauge related configurations.

Example: $\mathcal{N} = 4$ SYM (continued)

We have chosen the maximal commuting set $\mathsf{L} = \{U\}$, such that the U operators become trivial (constant) when pushed to the boundary.

Thinking of the U and V operators as discrete (exponentiated) versions of position and momentum, we are choosing an eigenstate $|\Phi_2\rangle$ of position. This implies that it is a superposition of momentum eigenstates:

$$|\Phi_2\rangle \propto \sum_{\Psi_2} (-1)^{\Phi_2 \cdot \Psi_2} |\Psi_2\rangle \quad \text{with} \quad V(\Theta_2)\, |\Psi_2\rangle = (-1)^{\Theta_2 \cdot \Psi_2} |\Psi_2\rangle \ .$$

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So the path integral with this choice of boundary conditions involves a sum over $|\Psi_2\rangle$, and $V(\Theta_2)$ remains a non-trivial (1-form symmetry) operator when pushed to the boundary:

$$\langle V(\Theta_2)\rangle_{\Phi_2}=\frac{Z[\Phi_2+\Theta_2]}{Z[\Phi_2]}\neq 1$$