Symmetry-Breaking in Quantum Gravity

Based on work with:

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- Physics and Mathematics of Generalized Symmetries -
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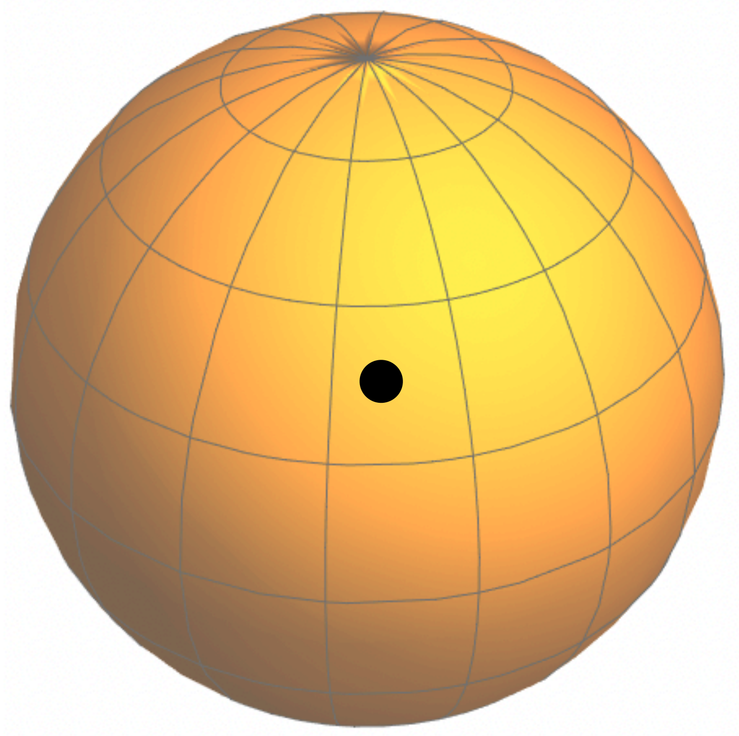
Global Symmetries in QFT

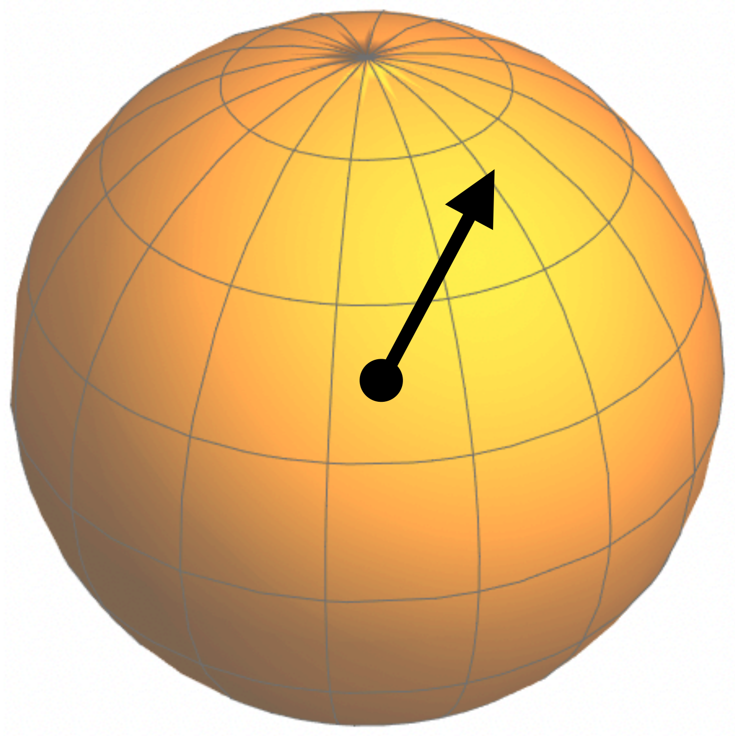
Universal **guiding principle: Organizing phases** (Landau paradigm)



$$\langle\vec{\phi}\,\rangle = 0$$

symmetry unbroken

(unique groundstate)



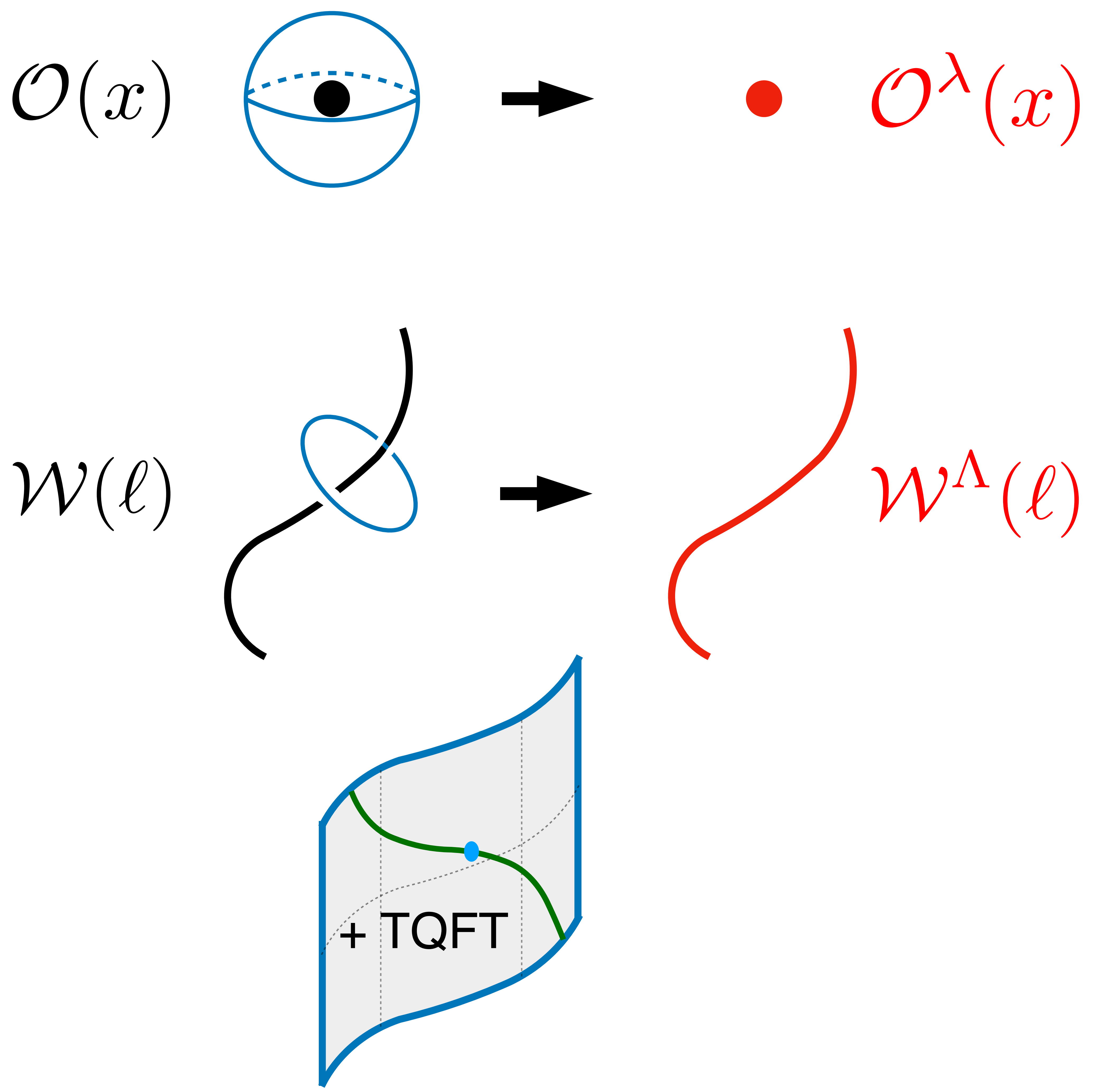
$$\langle\vec{\phi}\,\rangle = \vec{v}$$

symmetry broken (spontaneously)

(ground state degeneracy; Goldstones)

Insights into **strong coupling regions** (dualities, anomaly matching, …)

Global Symmetries in QFT



Especially including
generalized symmetries

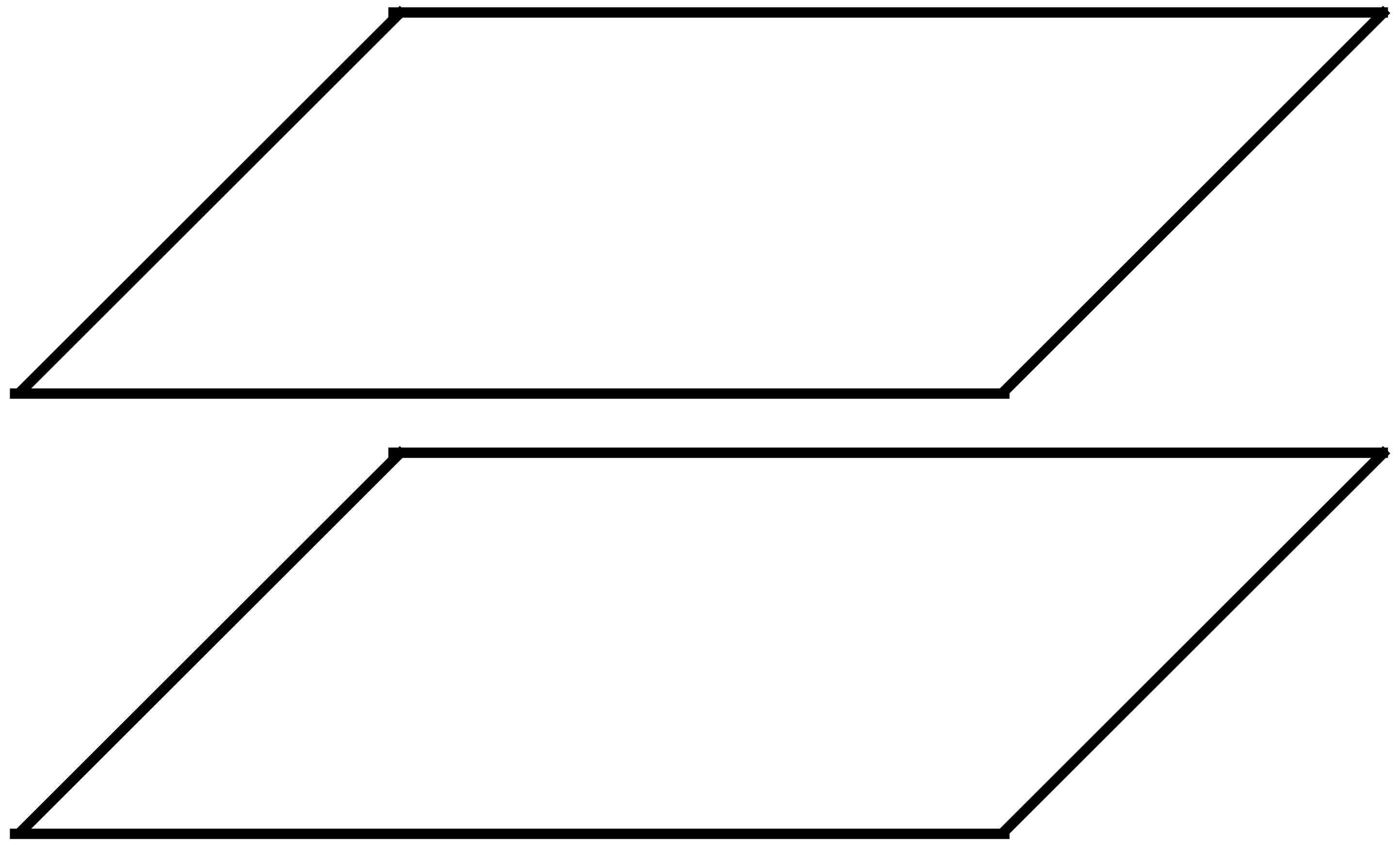
$\updownarrow$

Topological symmetry operators

Why do we like it so much?

Tells you about **conserved quantities (currents and charges; Noether)**

$$Q' = \int_{\text{space}} J^0(t')$$

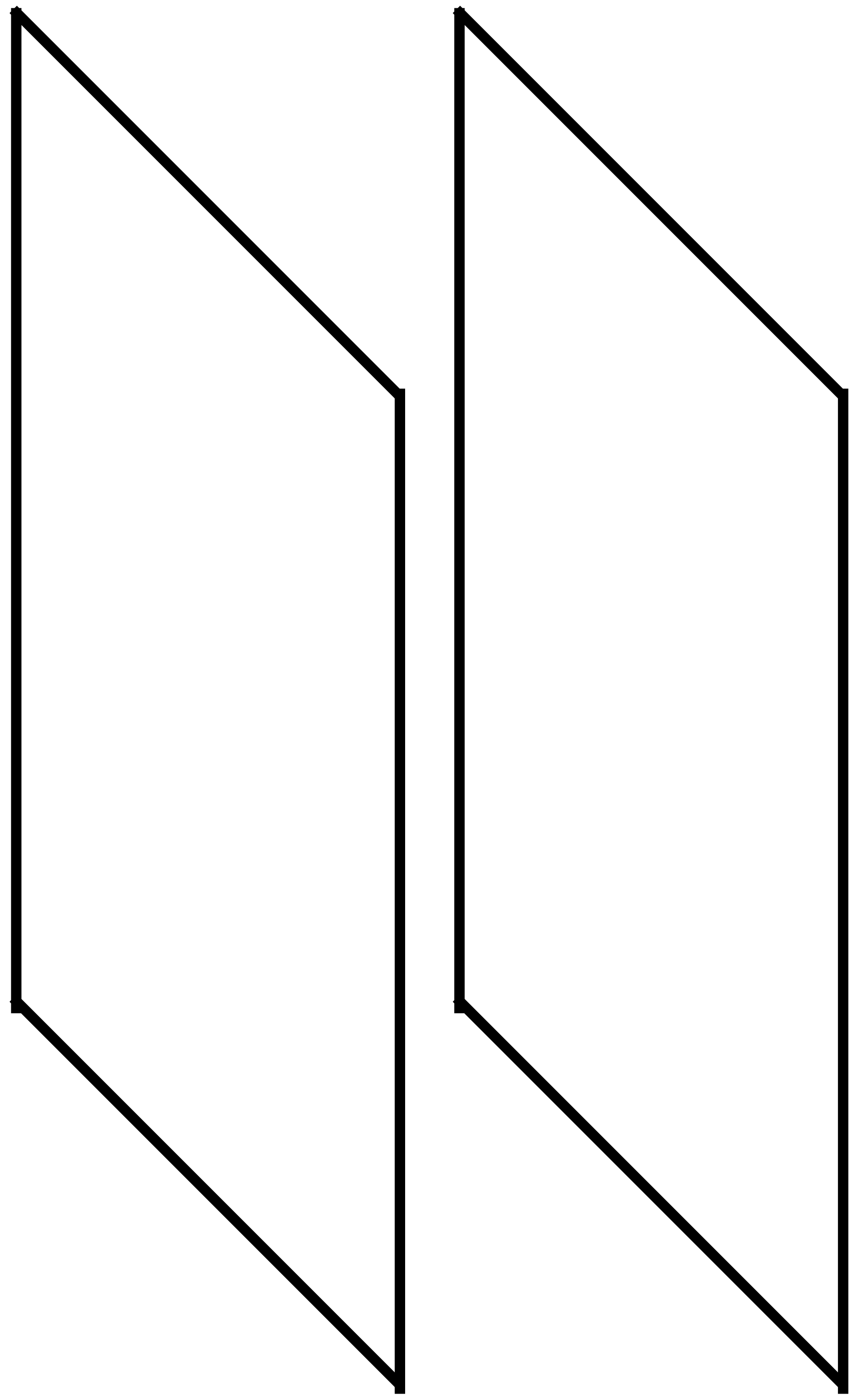


$$Q = \int_{\text{space}} J^0(t)$$

$$\partial_\mu J^\mu = 0$$

$$Q = Q'$$

not necessarily space

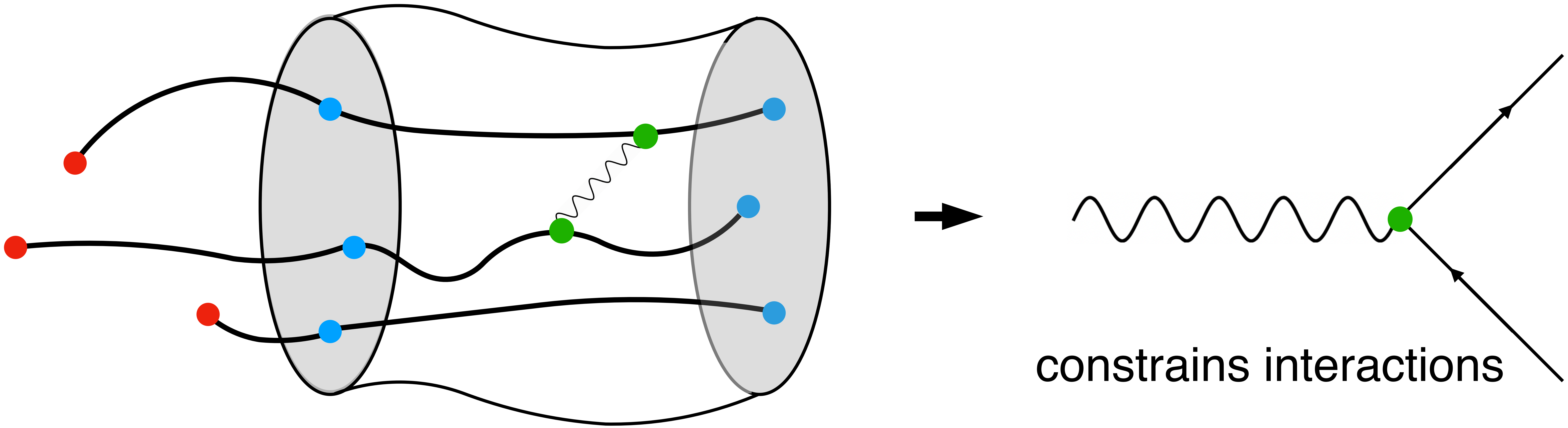


$$\int_\Sigma *J$$

Label for different states in theory (no energy cost)

Charge conservation

Imagine a system with a global symmetry, e.g., **U(1) baryon number**



● is a local operator that is **charged under the global symmetry**

(e.g. creation operator of a baryon)

All of them should be trivial in Quantum Gravity

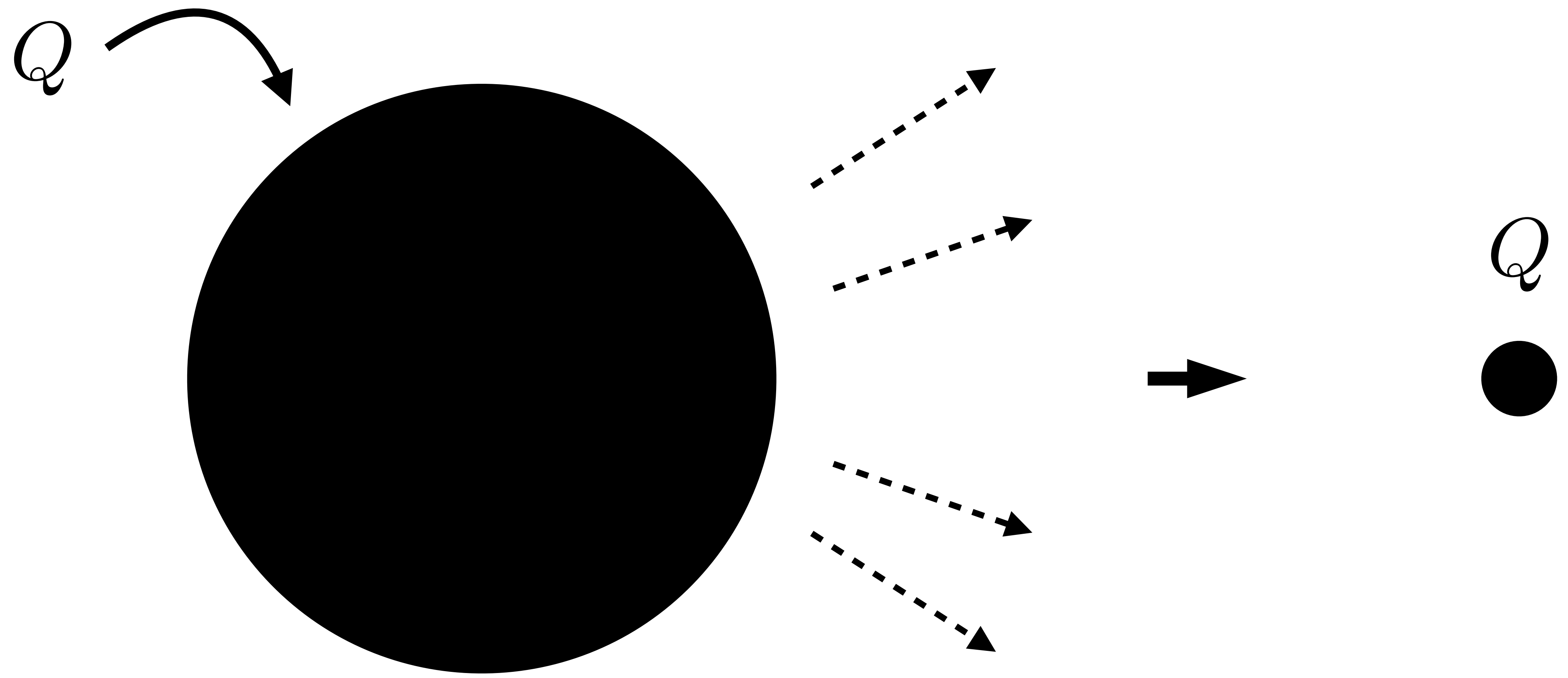
So rather than to explore **what we learn from their presence**

we want to explore **what we learn from their absence**

Evidence for: No global symmetries

See e.g. [Banks, Dixon ’88; Banks, Seiberg ’11]

Global charge labels different states but **costs no energy**



Remnants with arbitrary global charge (includes generalized symmetry charges)

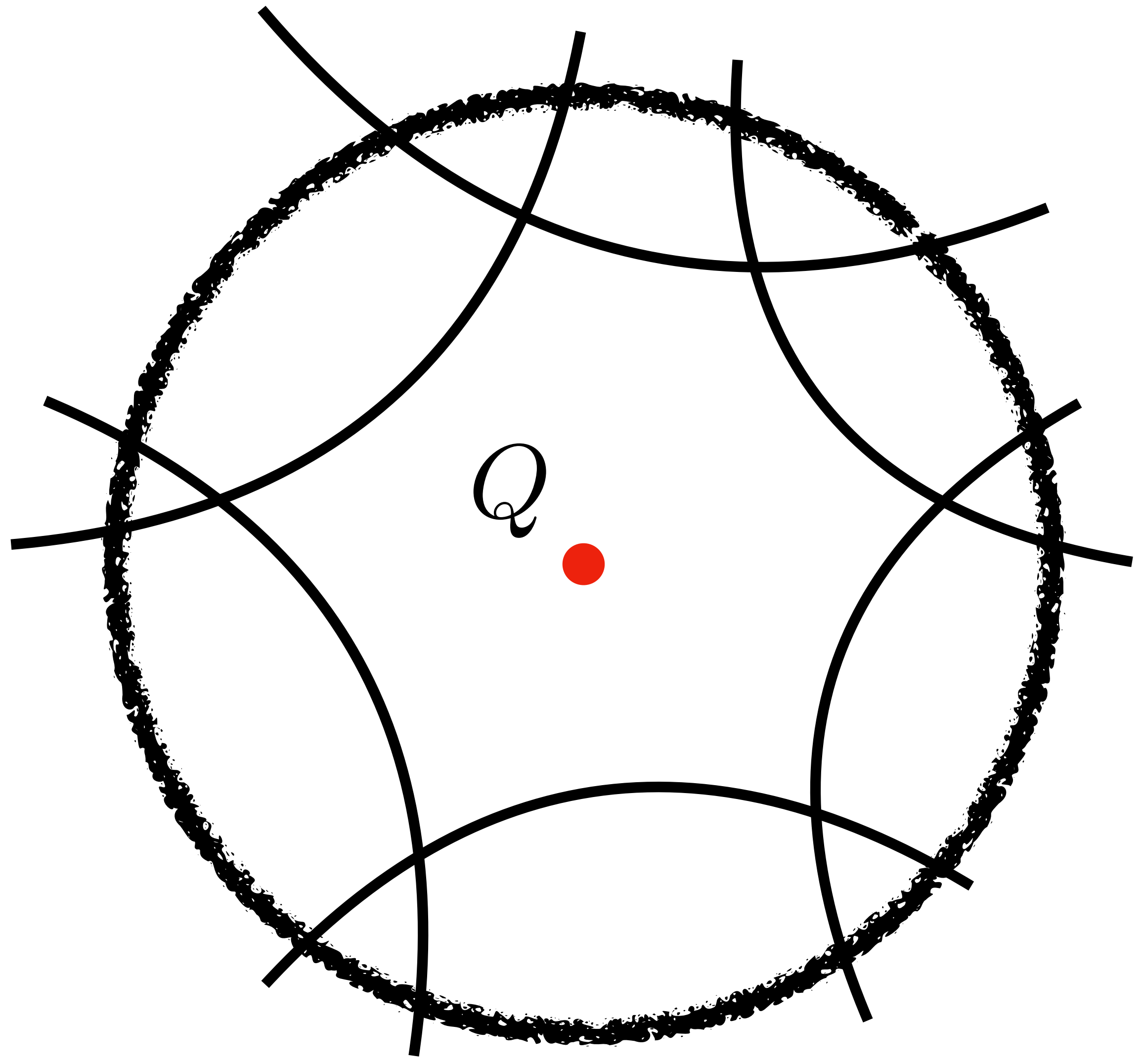
↓

Violation of **entropy bounds**

(e.g. baryon charge of collapsing neutron star)

Evidence for: No global symmetries

Remember: **Global charge labels different states** but **costs no energy**



Also leads to violation of
Holographic principle

[Harlow, Ooguri '18]

& violation in AdS/CFT

[Harlow, Shaghoulian '20],
[Bah, Chen, Maldacena '22],…

Violation typically **at least** $e^{-M_{Pl}^2/\Lambda^2}$

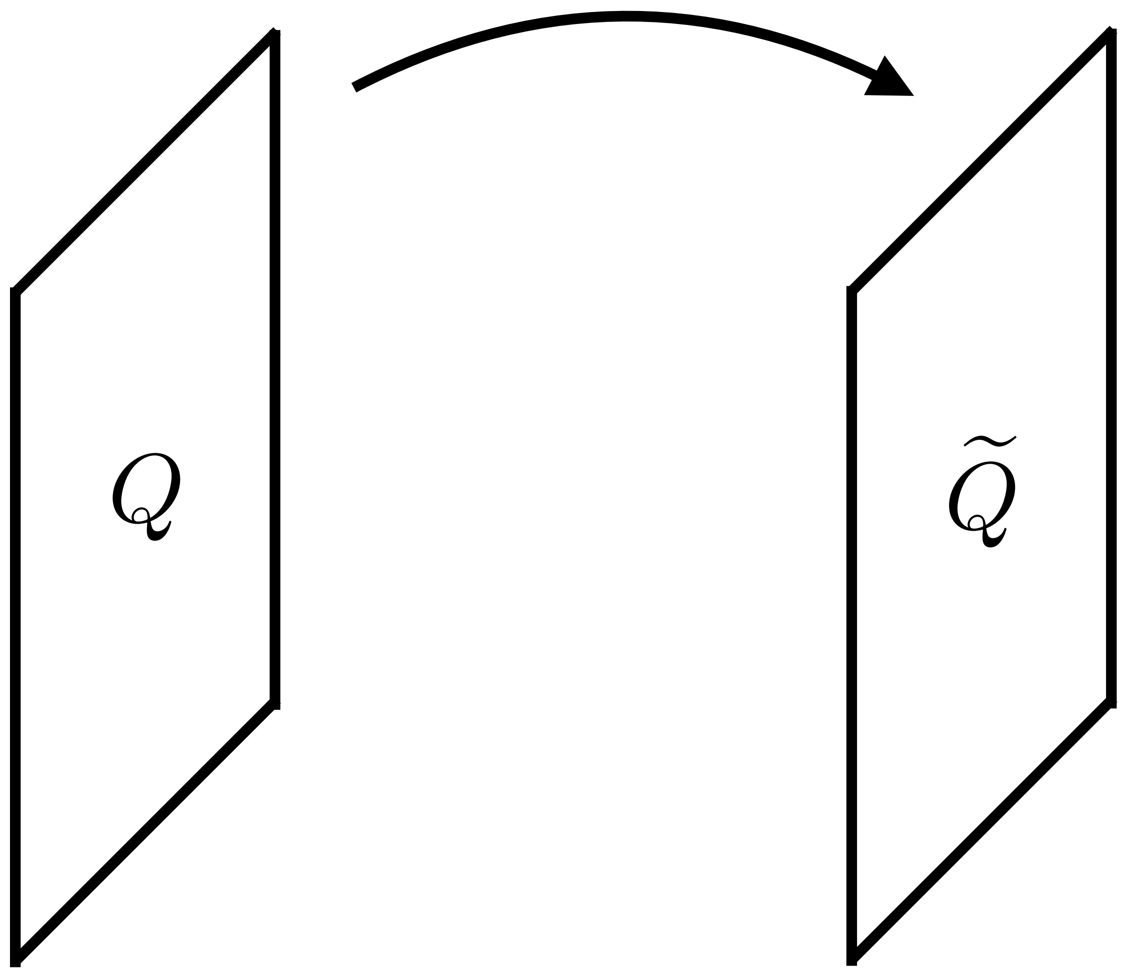
see also [Daus, Hebecker, Leonhardt, March-Russell '20],
[Cordova, Ohmori, Rudelius '22]

Non-perturbative
in **gravity**

No global symmetries

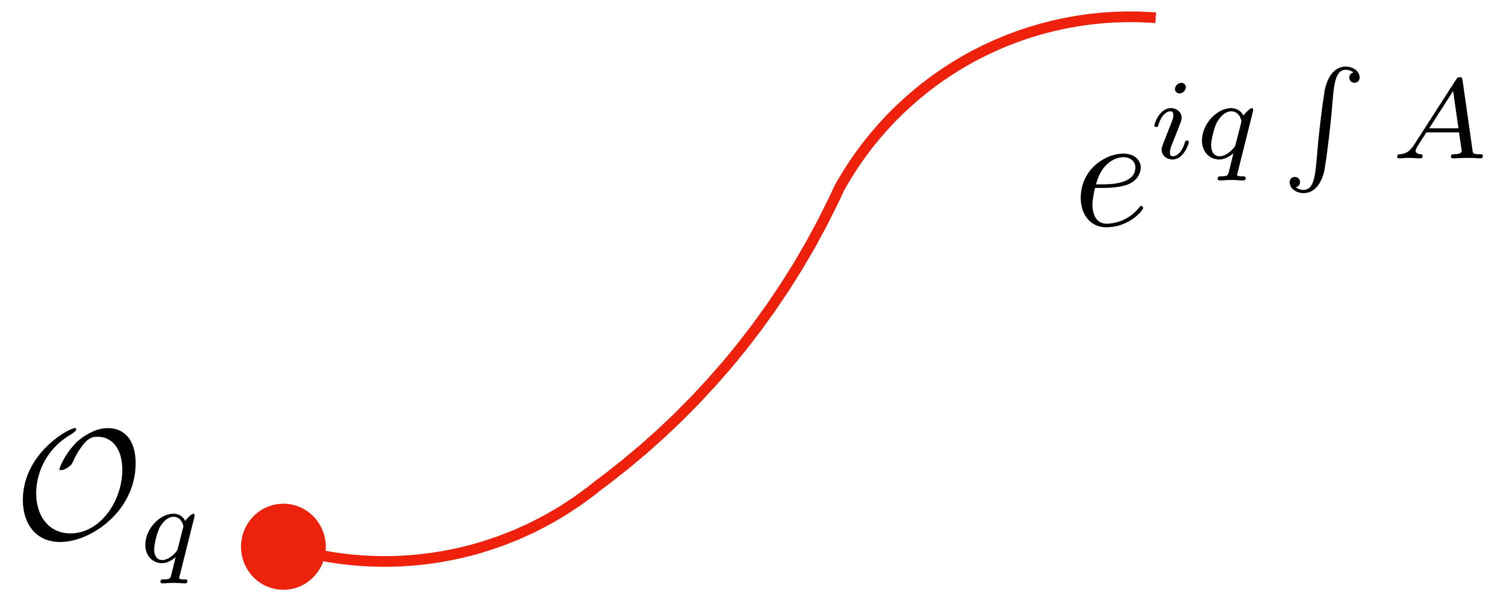
If no global symmetries they **need to be:**

broken



charge not conserved

gauged



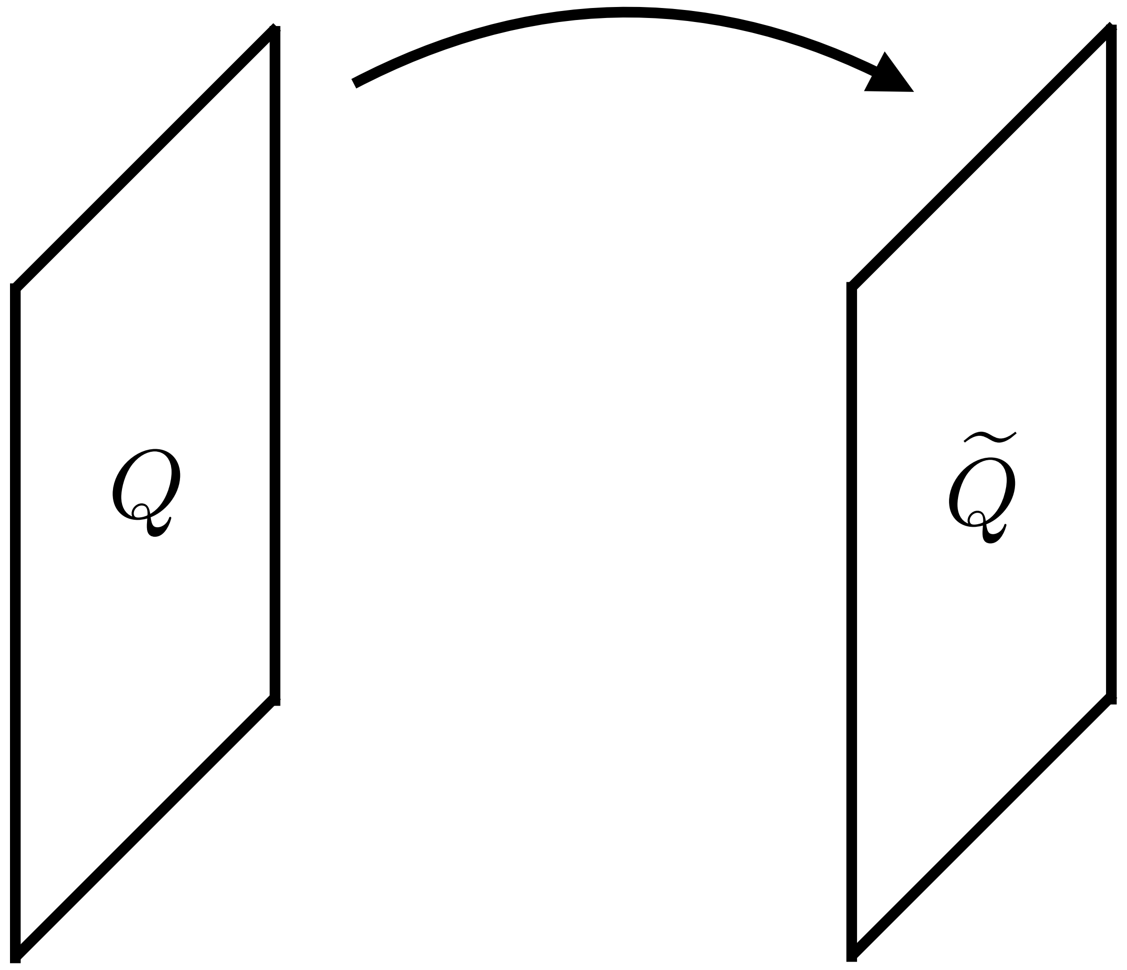
no anomalies (incl. gravity)

(breaking by quantum corrections)

No global symmetries

If no global symmetries they **need to be:**

broken



charge not conserved

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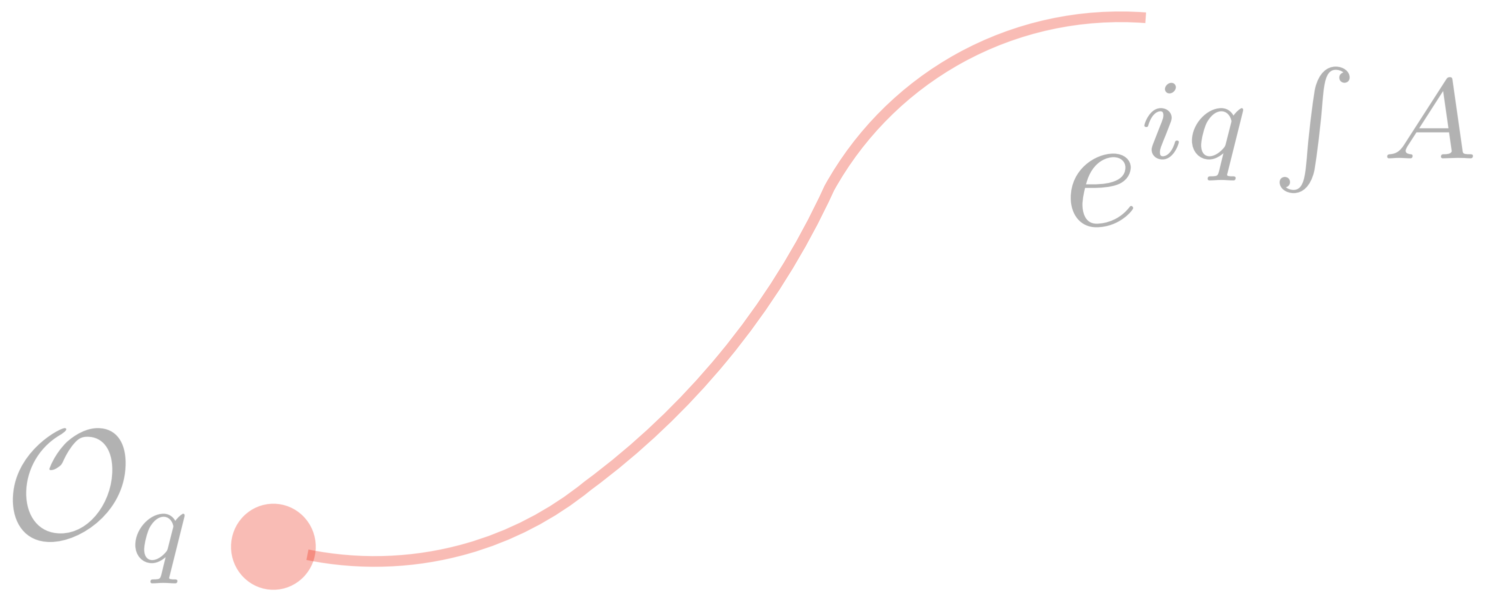
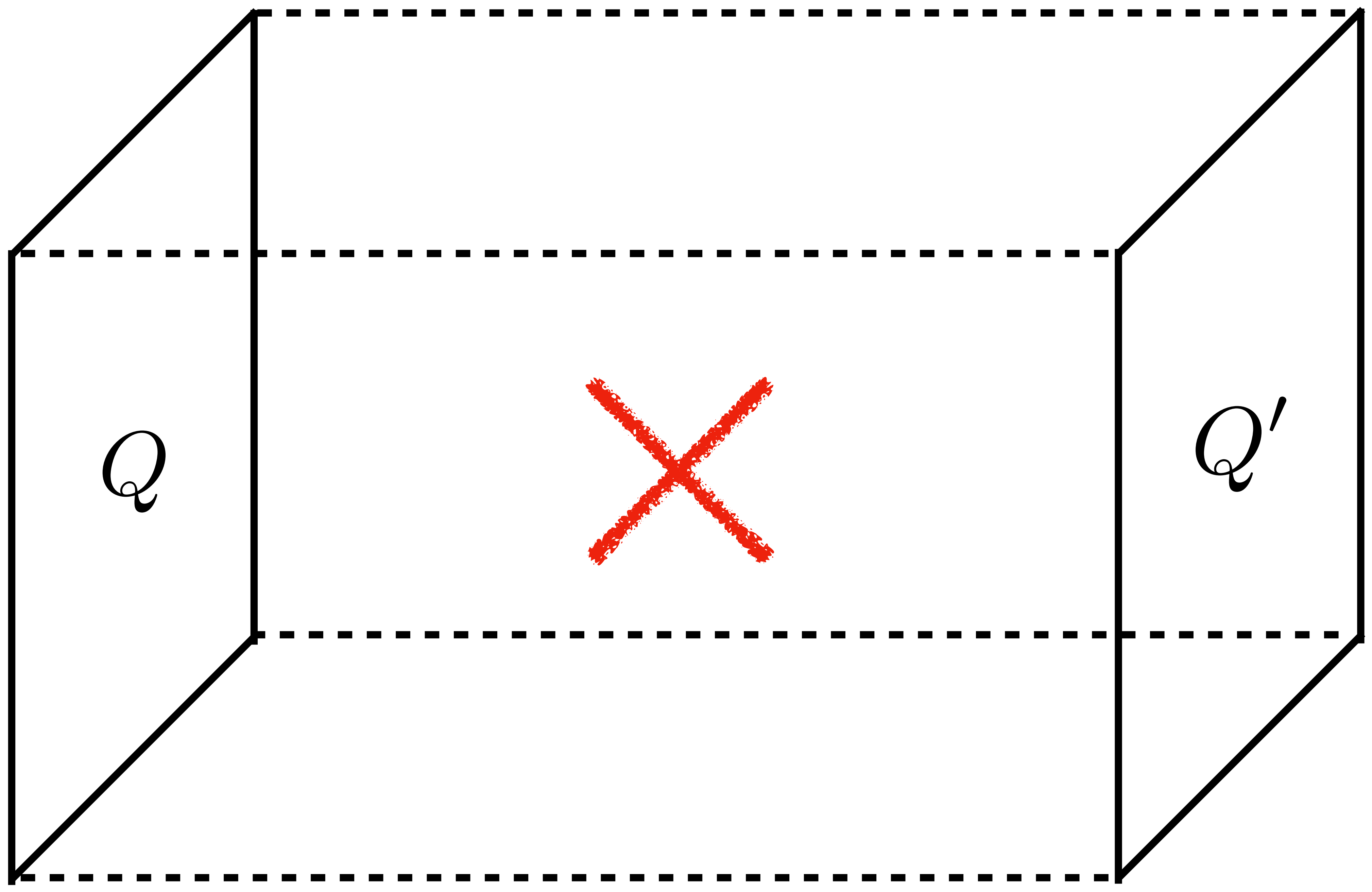


no anomalies (incl. gravity)

(breaking by quantum corrections)

Charges as obstructions

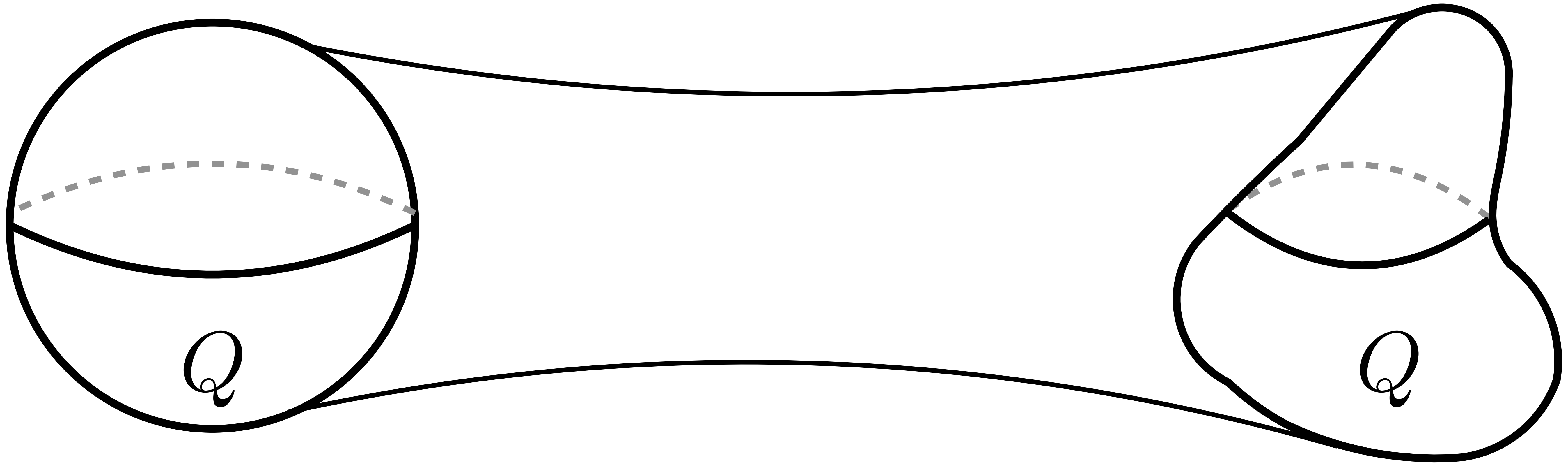
Conservation of charge restricts the transitions in theory



Defines disconnected sectors (deformation classes)

Obstructions as charges

One can turn the logic around: **Sectors that cannot be connected**



Non-trivial deformation classes $\longleftrightarrow$ **conserved charges**

$\updownarrow$

global symmetries

Obstructions as charges

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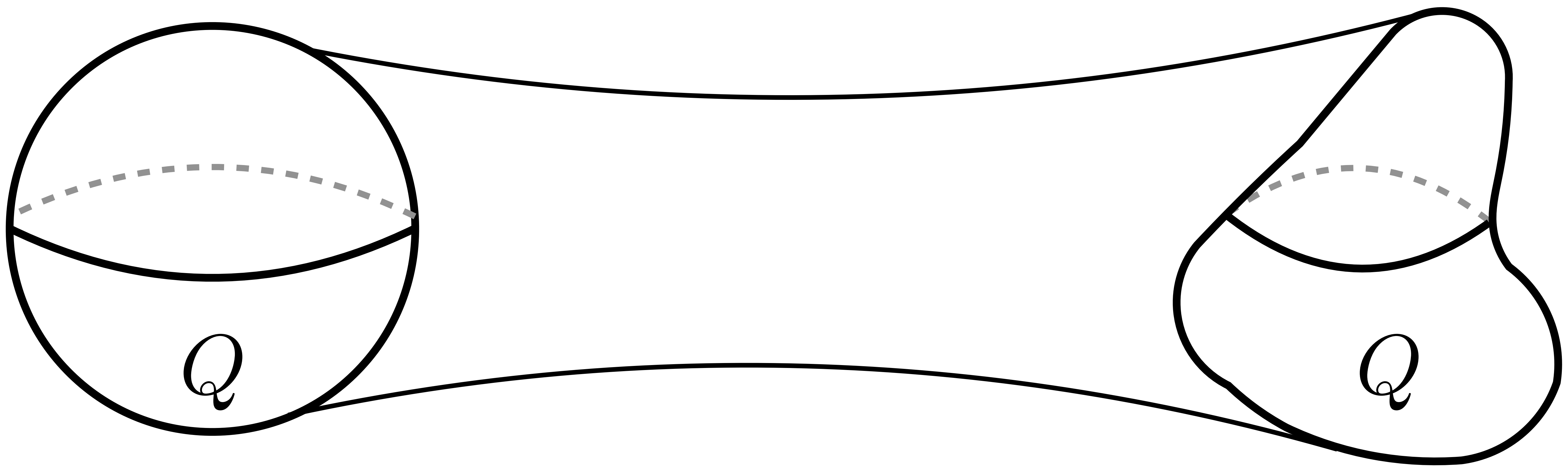


Non-trivial deformation classes ⟷ **conserved charges**

⟷

global symmetries

Global symmetries

How can one be sure that **there are no global symmetries?**

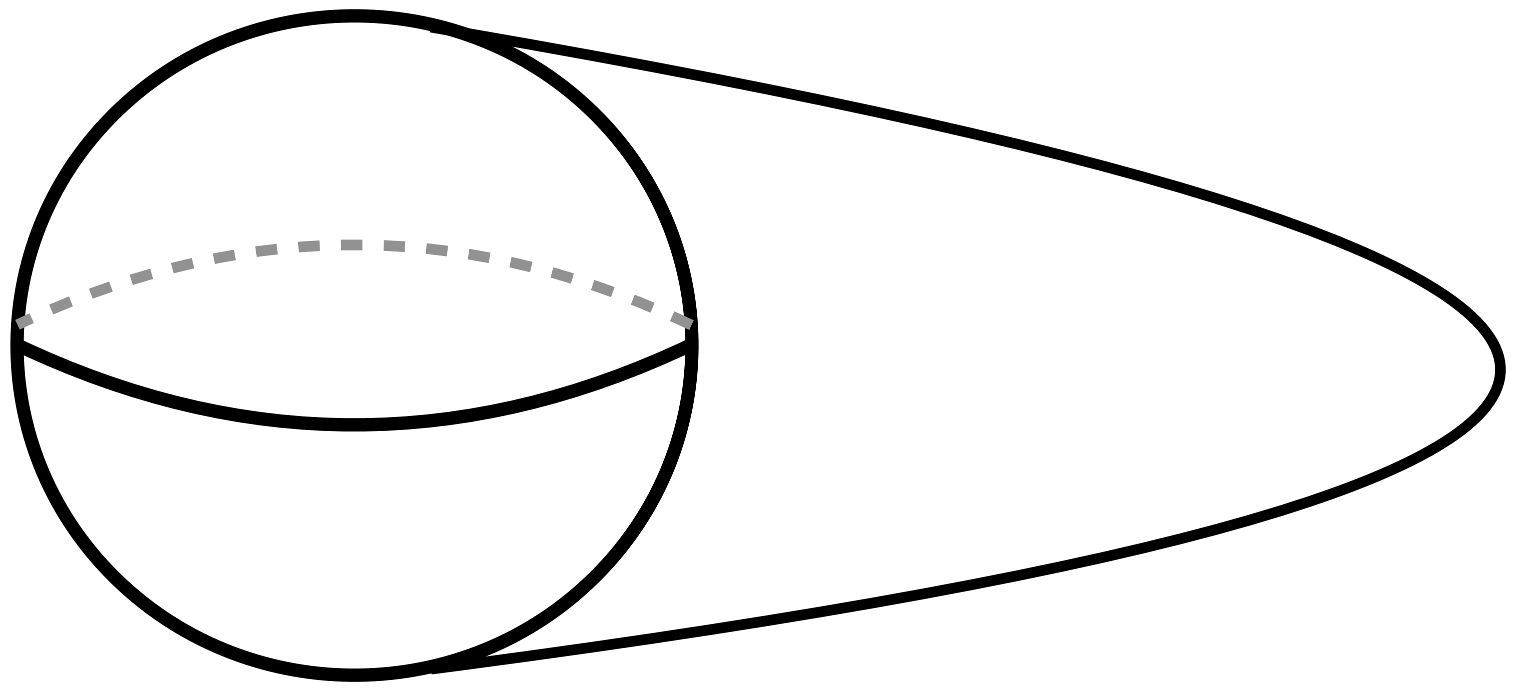
(no Lagrangian description, non-perturbative effects, …)

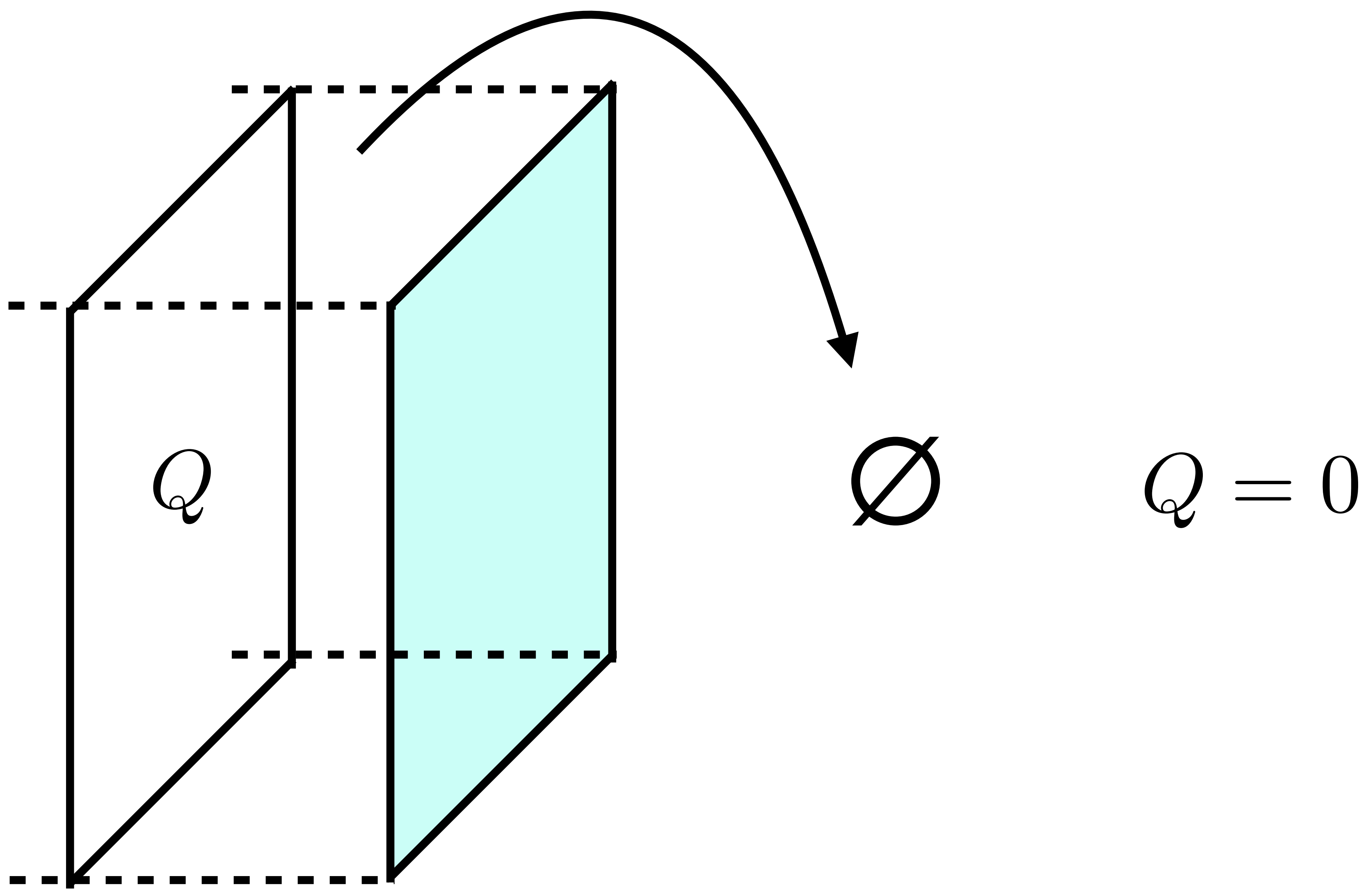
One state guaranteed carry **no charges:**

$$\varnothing$$

Global symmetries

If theory admits **transition to nothing** there are **no global symmetries**:





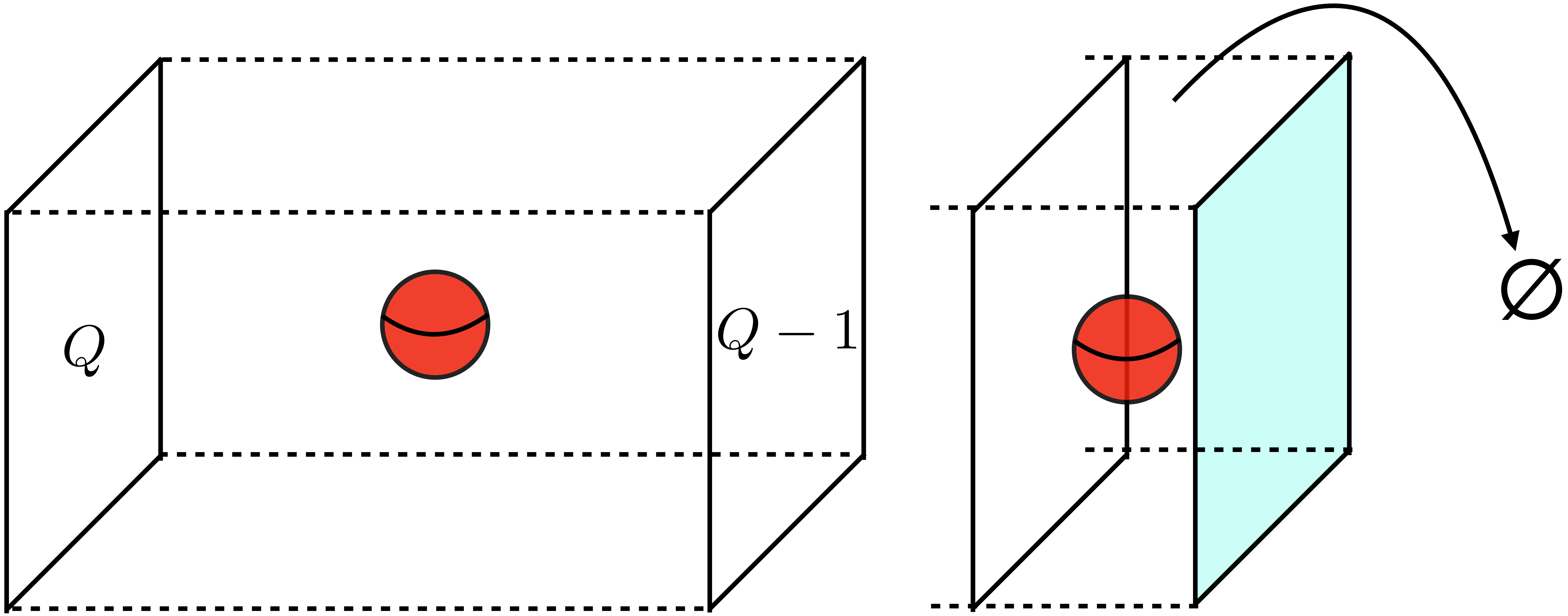
more mathematical: no non-trivial deformation classes captured by **bordisms**

$$\Omega_d^{\mathrm{QG}} = 0\,, \quad d < D$$ **Cobordism Conjecture**

[McNamara, Vafa '19], see also [Montero, Vafa '20], [MD, Heckman '20]

Symmetry-breaking objects

This can require **new symmetry-breaking defects**



Can look **singular** at **low-energies**, but have **finite mass** (tension)

(sub-Planckian) [Nevoa, Raman, Vafa ’25]

Strategy

Define an **approximation to a Quantum Gravity theory in the IR**

Determine:

- **Requirement on spacetime** ξ
- **Gauge degrees of freedom** BG
- **Their mixing**

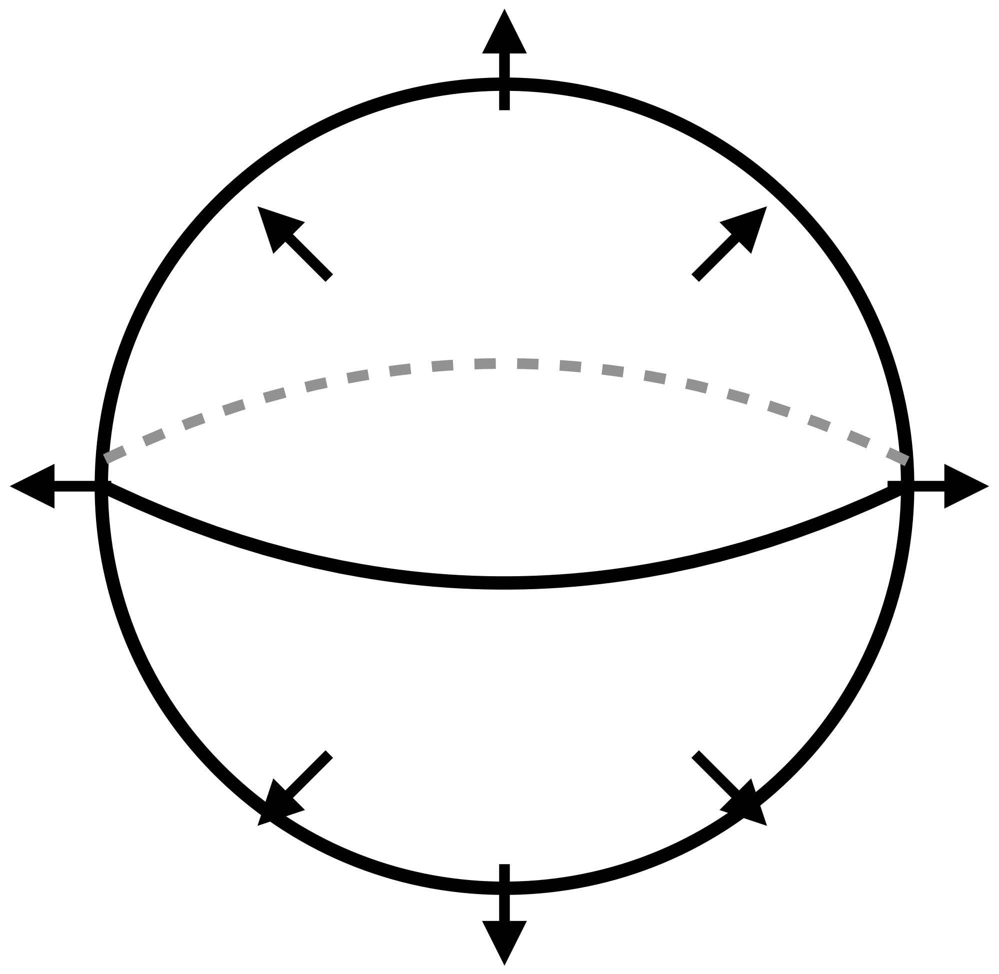
$$\rightarrow \boxed{\Omega_k^{\xi}(BG)} \qquad 0 < k < D$$

Bordism groups determine **conserved charges**

Add symmetry-breaking (UV) objects

Global charges

Example: U(1) gauge sector with fermions



$\not\to \ \varnothing$

$dF = 0$

$\oint F \neq 0$ threaded by magnetic flux

deformation classes of Spin manifolds
with principal U(1) bundle

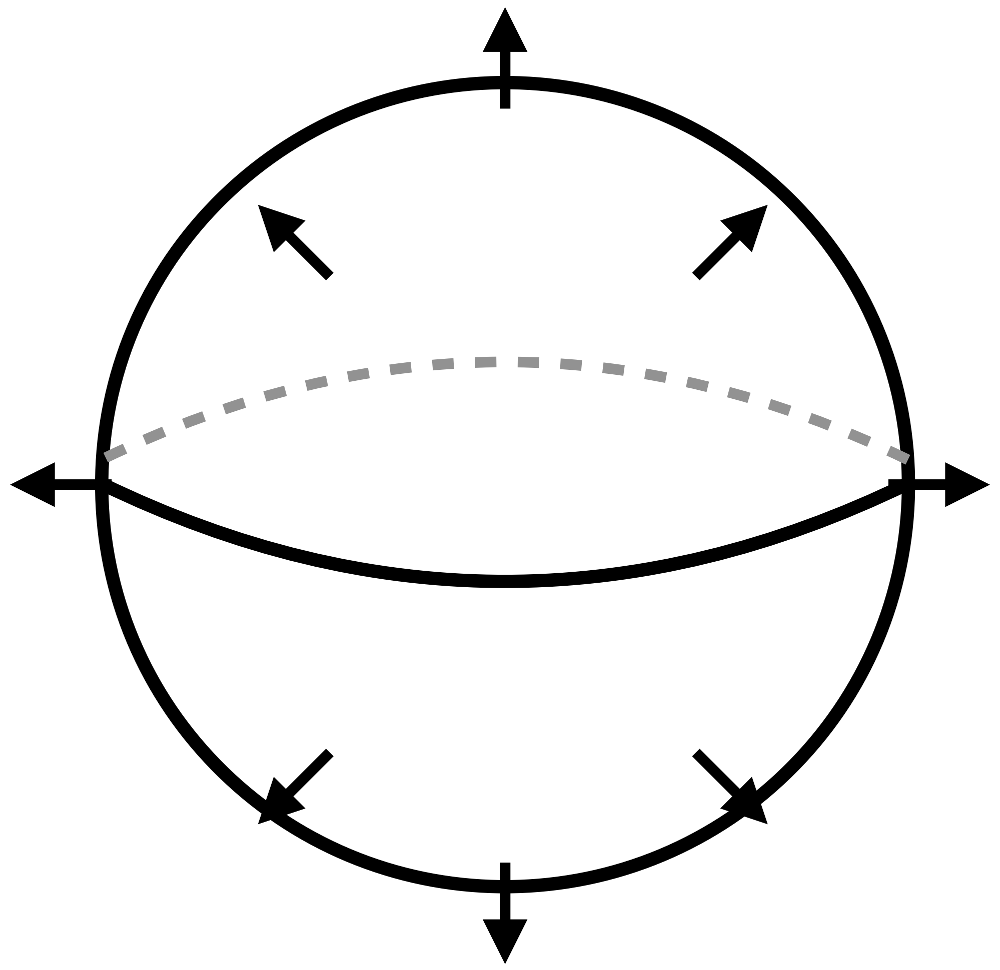
$$\Omega_2^{\mathrm{Spin}}(B\mathrm{U}(1)) \supset \mathbb{Z} \neq 0$$

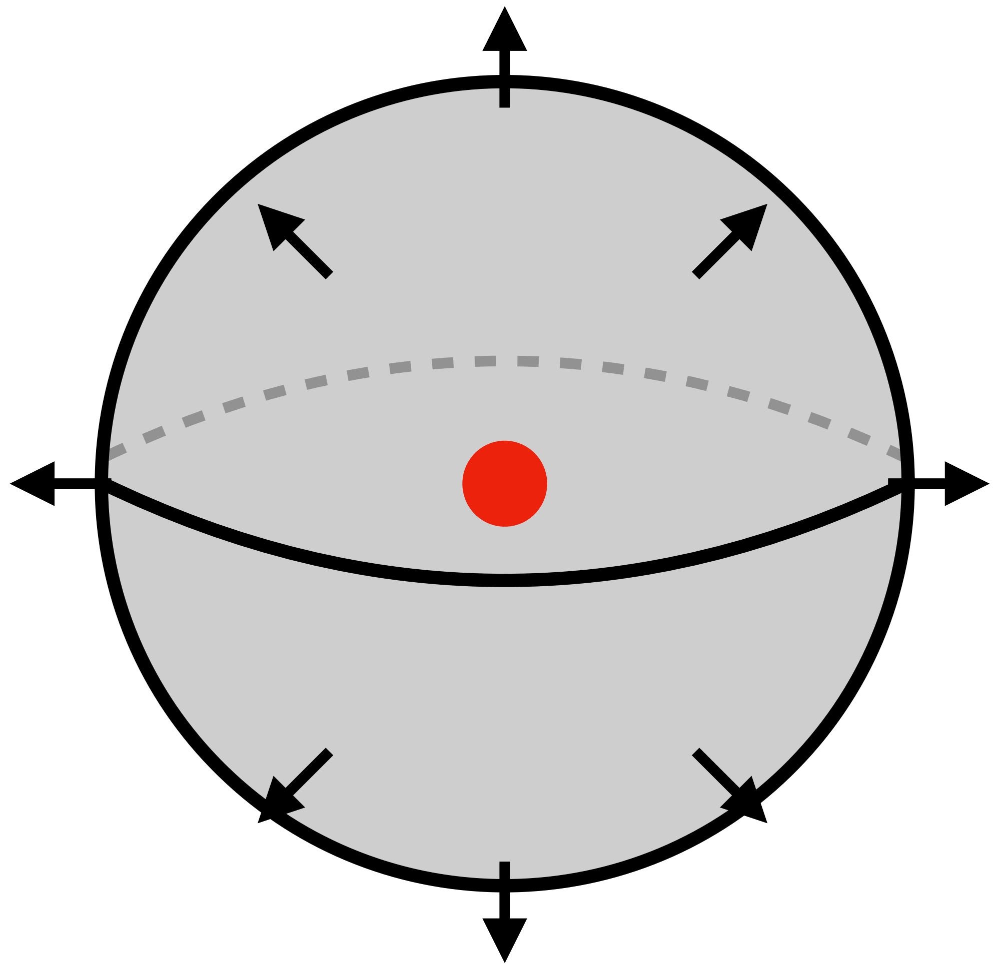
(D-3)-form symmetry

One needs to **violate** the Bianchi identity: $\quad dF = 0$

How does a charged state look like?

Has the deformation class as **boundary**, can look **singular in IR**





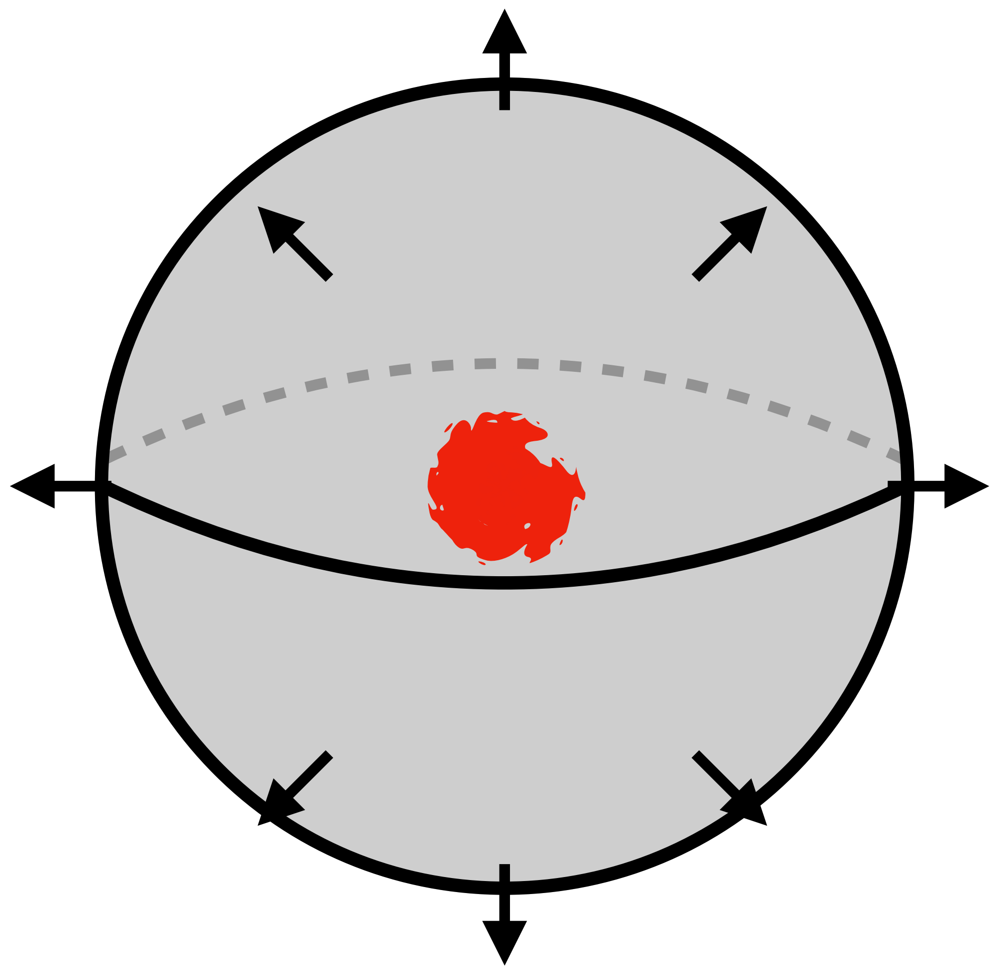
$$dF \neq 0$$

This defect is a **magnetic monopole**; if **dynamical** it **breaks symmetry**

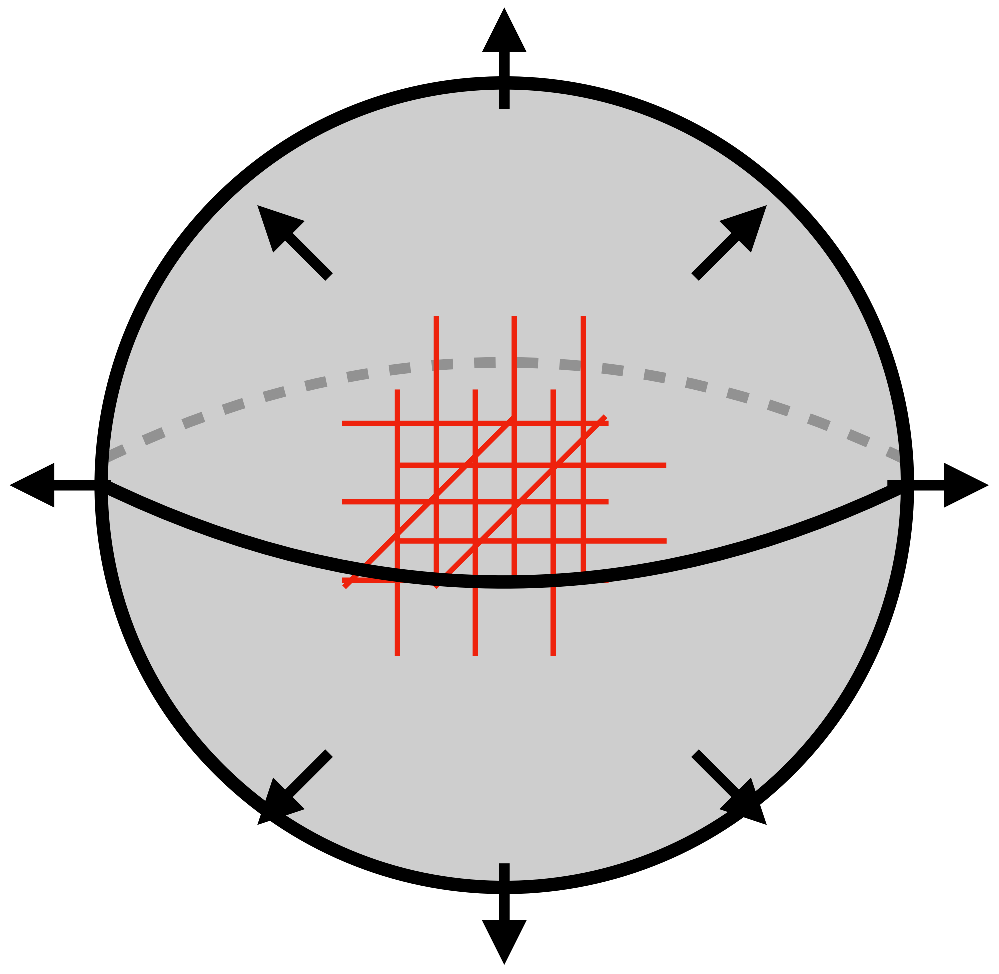
→ **predicted by quantum gravity** to avoid global symmetries

UV completion?

This approach is **not sensitive to UV properties**



$$\mathrm{U}(1) \hookrightarrow \mathrm{SU}(2)$$



Could have UV completions in **quantum field theory or gravity**

Cannot be smooth spacetime (Spin) with smooth U(1) bundle

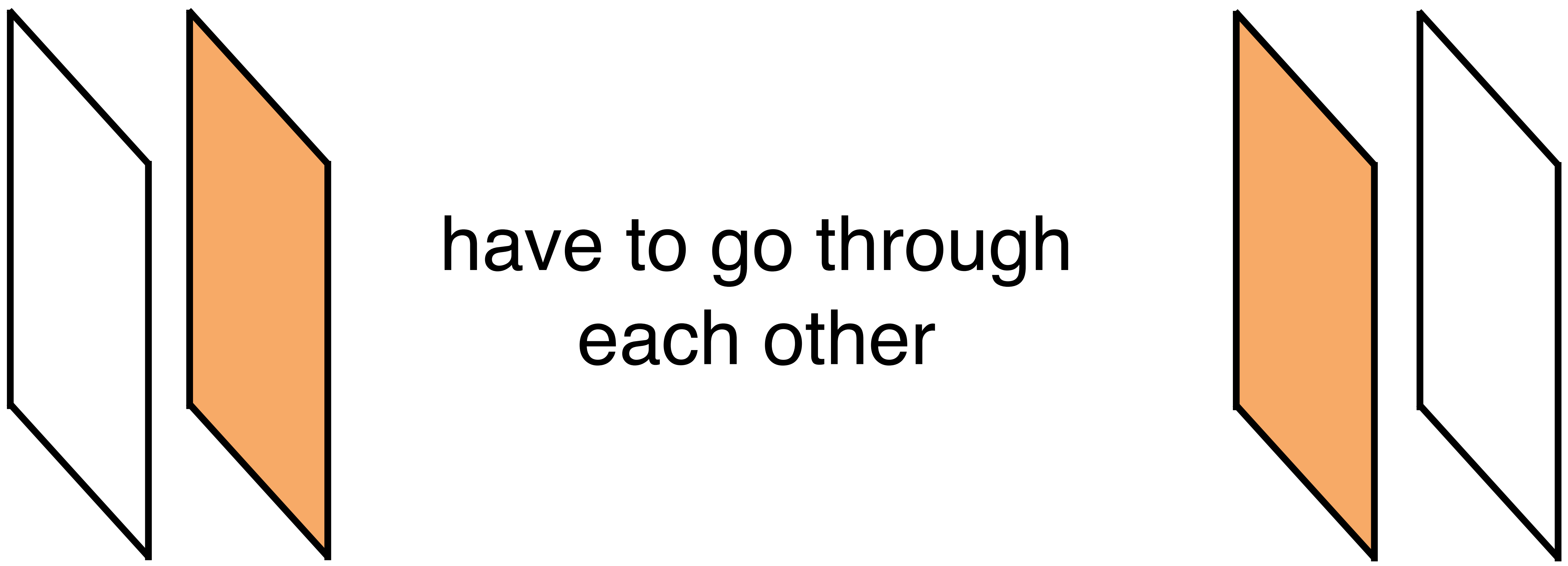
Abelianization

[McNamara ’21], [Debray, MD, Heckman, Montero ’23], see also [Yonekura ’20]

Bordisms are a **generalized homology theory**, in particular:

$$\Omega_k^{\xi}(BG)$$ **are Abelian groups**

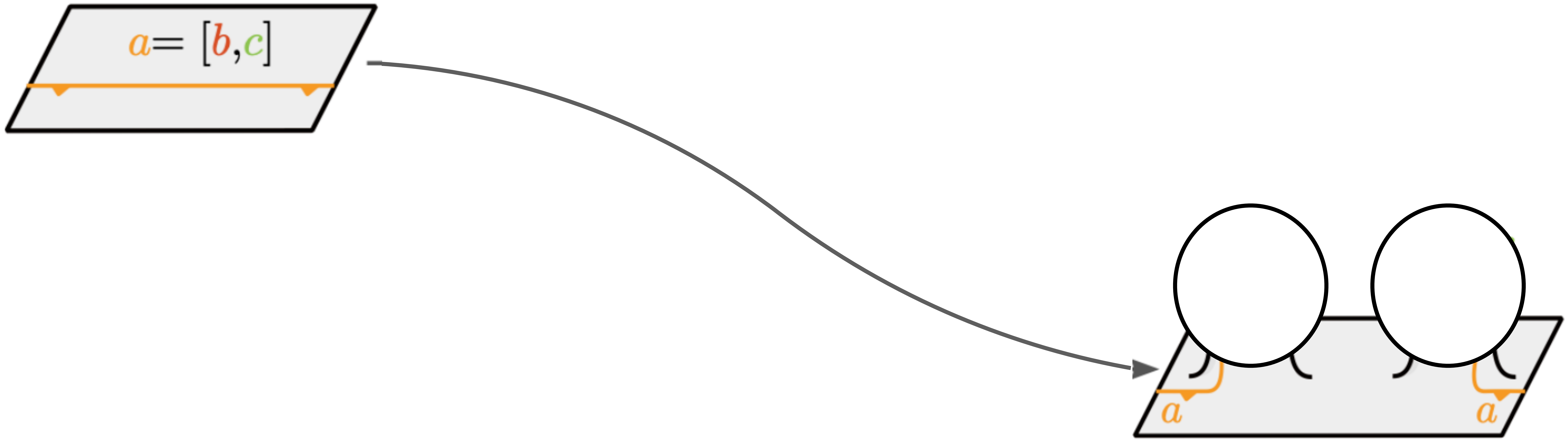
Clearly in QFT one can have at least **non-Abelian 0-form symmetries**



Abelianization

[McNamara ’21], [Debray, MD, Heckman, Montero ’23]

Even before the addition of defects **gravity has some work to do**



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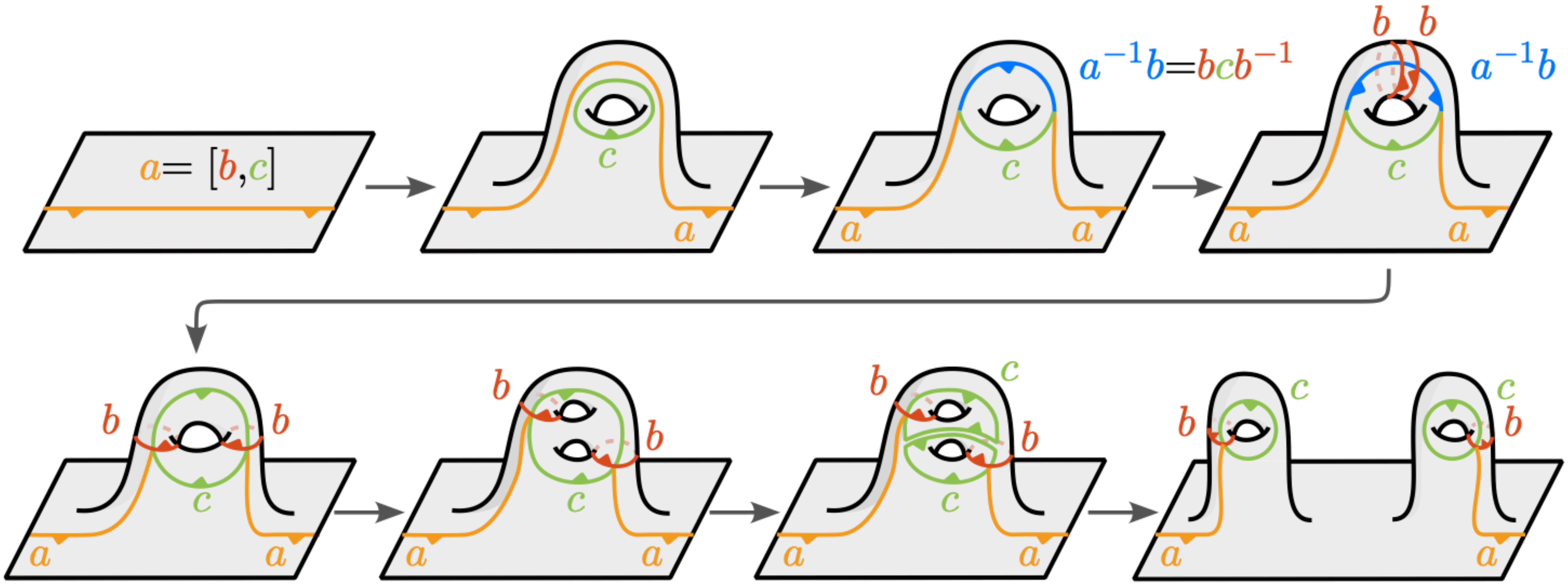


Abelianization

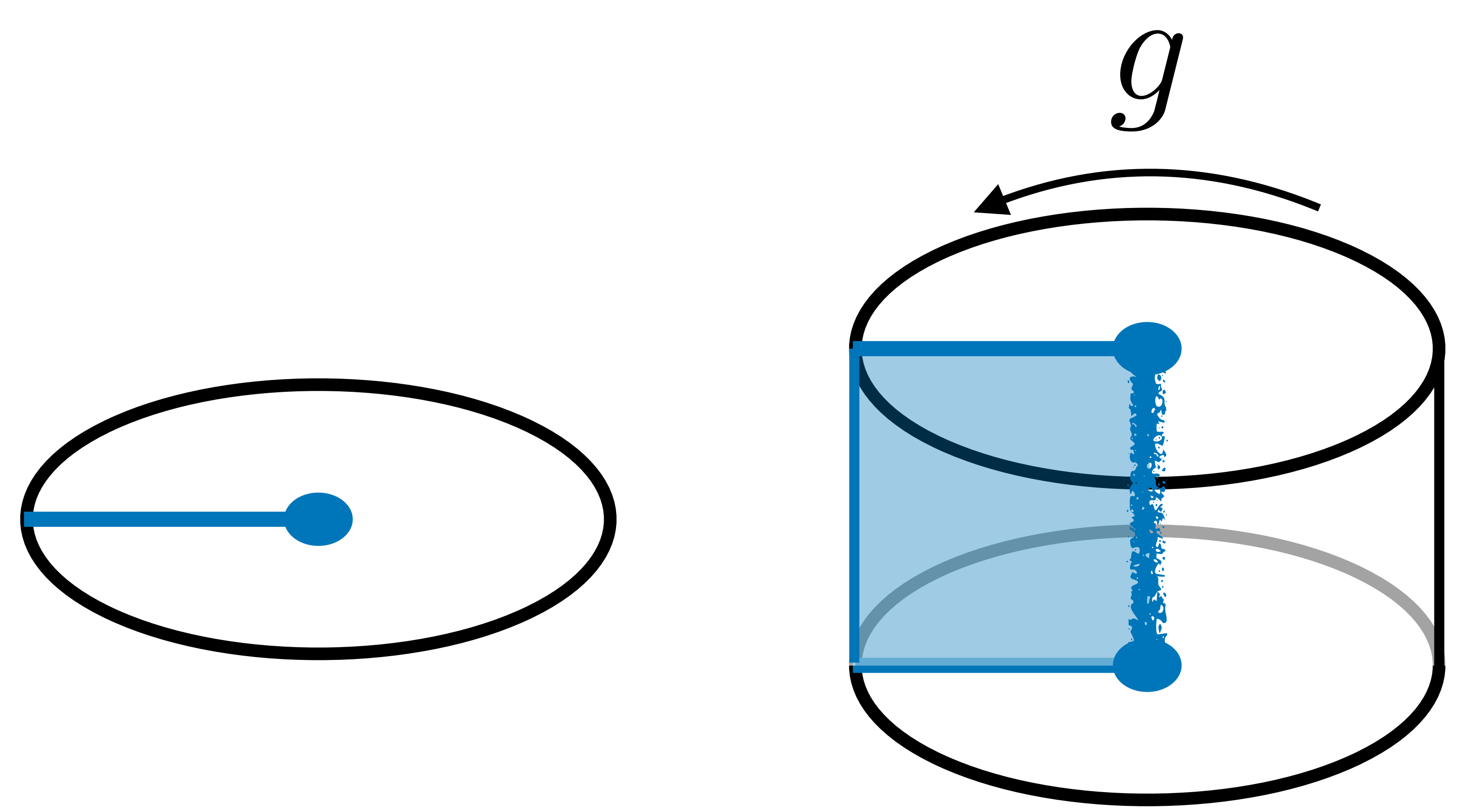
[McNamara ’21], [Debray, MD, Heckman, Montero ’23], see also [Yonekura ’20]

Some **topological operators break via gravitational solitons**

see also [Ruiz ’24], [Las Heras, Ruiz ’26]

$$\mathrm{Ab}(G) = \frac{G}{[G,G]}$$

➜ **Price to pay: topological complexity of spacetime**



$$\widetilde{\Omega}_1^{\xi}(BG) = \mathrm{Ab}(G)$$

(codimension-two defect)

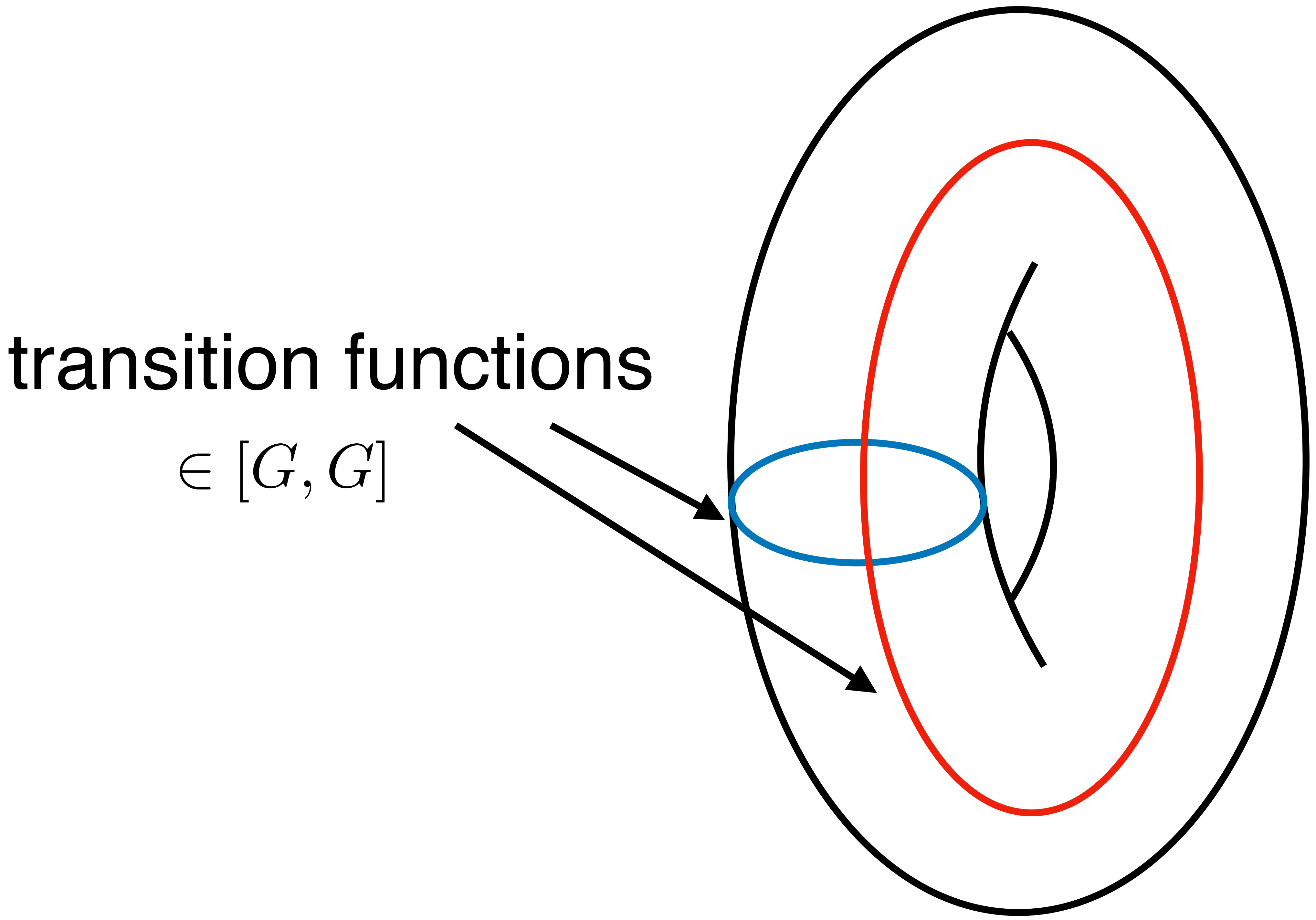
labeled by $g \in \mathrm{Ab}(G)$

Similar for non-trivial mixing [Chakrabhavi, Debray, MD, Heckman ’25]

Gravitational solitons can fail

Try to use the soliton

$$\Omega_2^{\xi}(BG)$$



Gravitational solitons can fail

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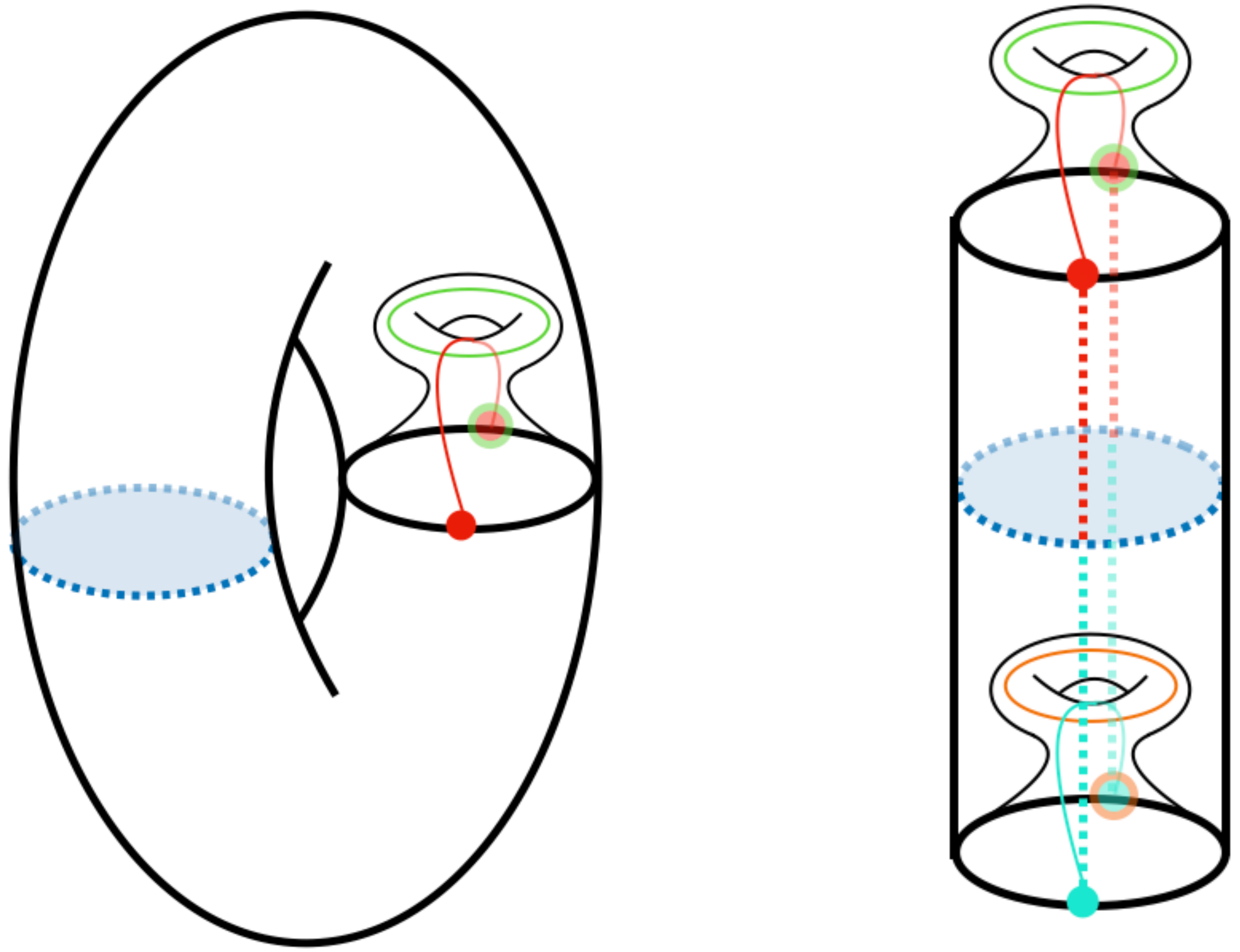
$$\Omega_2^\xi(BG)$$



action **incompatible** with transition function

➛ **does not lead to well-defined background**

can ‘resurrect’ codimension-two objects not seen in Abelianization

Brane networks

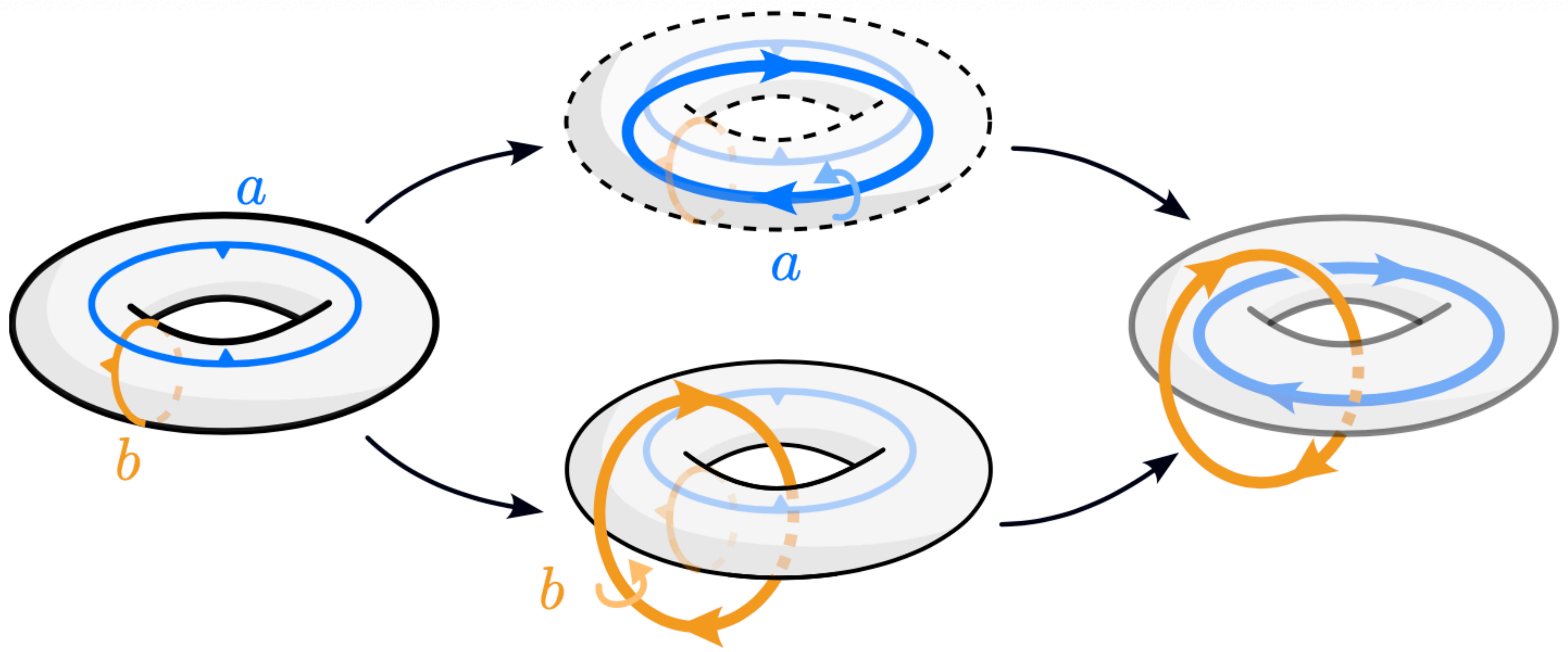
[MD, Ruiz '26]

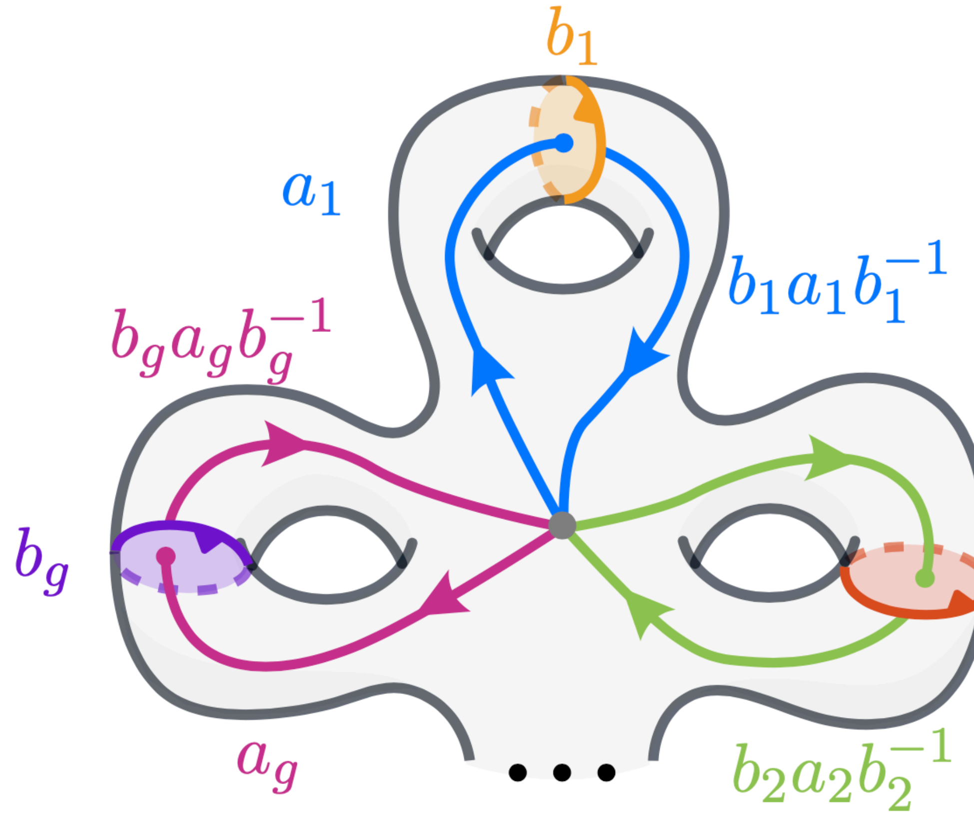
Can be related to non-trivial brane configurations and networks

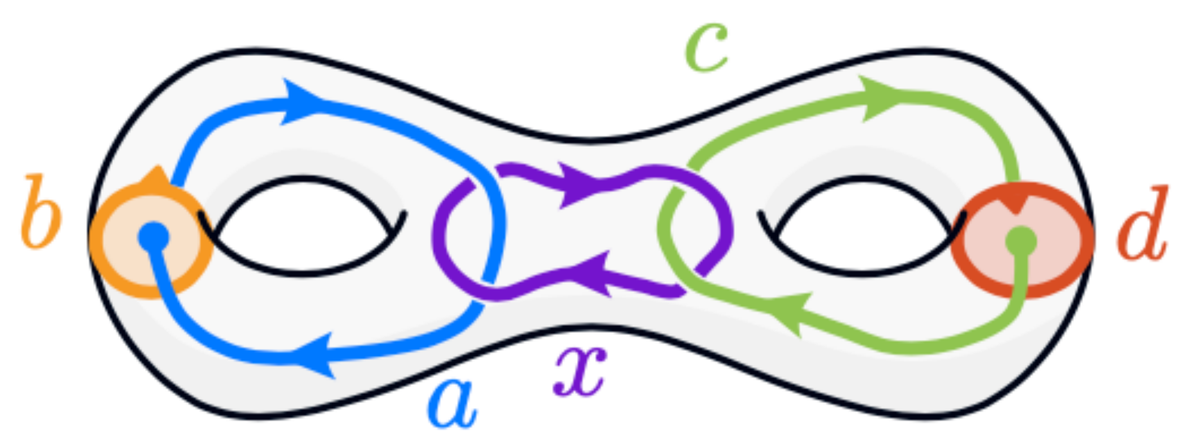
discrete groups $G = \frac{F}{R}$ $H_2(BG;\mathbb{Z}) = \frac{R \cap [F,F]}{[R,F]}$ **Hopf's theorem**

appears in bordism group $\Omega_2^{\xi}(BG)$

➛ **minimal genus** of **manifold carrying charge** (bordism generator)







Group (Co)Homology

Group (co)homology is important part of bordism groups

$$E^2_{p,q} = H_p\big(BG; \Omega^\xi_q(\mathrm{pt})\big) \Rightarrow \Omega^\xi_{p+q}\big(BG\big)$$

Atiyah-Hirzebruch spectral sequence

$$\Omega^{\mathrm{Spin}}_k\big(B\mathrm{U}(1)\big) \qquad \begin{array}{|ccccc} \mathbb{Z} & 0 & \mathbb{Z} & 0 & \mathbb{Z} \\ 0 & 0 & 0 & 0 & 0 \\ \mathbb{Z}_2 & 0 & \mathbb{Z}_2 & 0 & \mathbb{Z}_2 \\ \mathbb{Z}_2 & 0 & \mathbb{Z}_2 & 0 & \mathbb{Z}_2 \\ \mathbb{Z} & 0 & \mathbb{Z} & 0 & \mathbb{Z} \\ \hline \end{array}$$

can be affected by **differentials**

(p>2)

(imprint of geometrical/gravitational data)

Let's apply this strategy to supergravity theories

Maximal supergravity in ten dimensions

Type IIB supergravity:

- properties of **spacetime** (tangential structure)
- **S-duality**: a gauged discrete strong-weak coupling duality

$$\Omega_d^{\text{Spin-Mp}(2,\mathbb{Z})}(\mathrm{pt}) \qquad \Omega_d^{\text{Spin-GL}^+(2,\mathbb{Z})}(\mathrm{pt})$$

$$\begin{tikzcd}
\mathrm{SL}(2,\mathbb{Z}) \arrow[r] \arrow[d] & \mathrm{GL}(2,\mathbb{Z}) \arrow[d] \\
\mathrm{Mp}(2,\mathbb{Z}) \arrow[r] & \mathrm{GL}^+(2,\mathbb{Z})
\end{tikzcd}$$

$\downarrow$ Spin/Pin lifts due to action on fermions

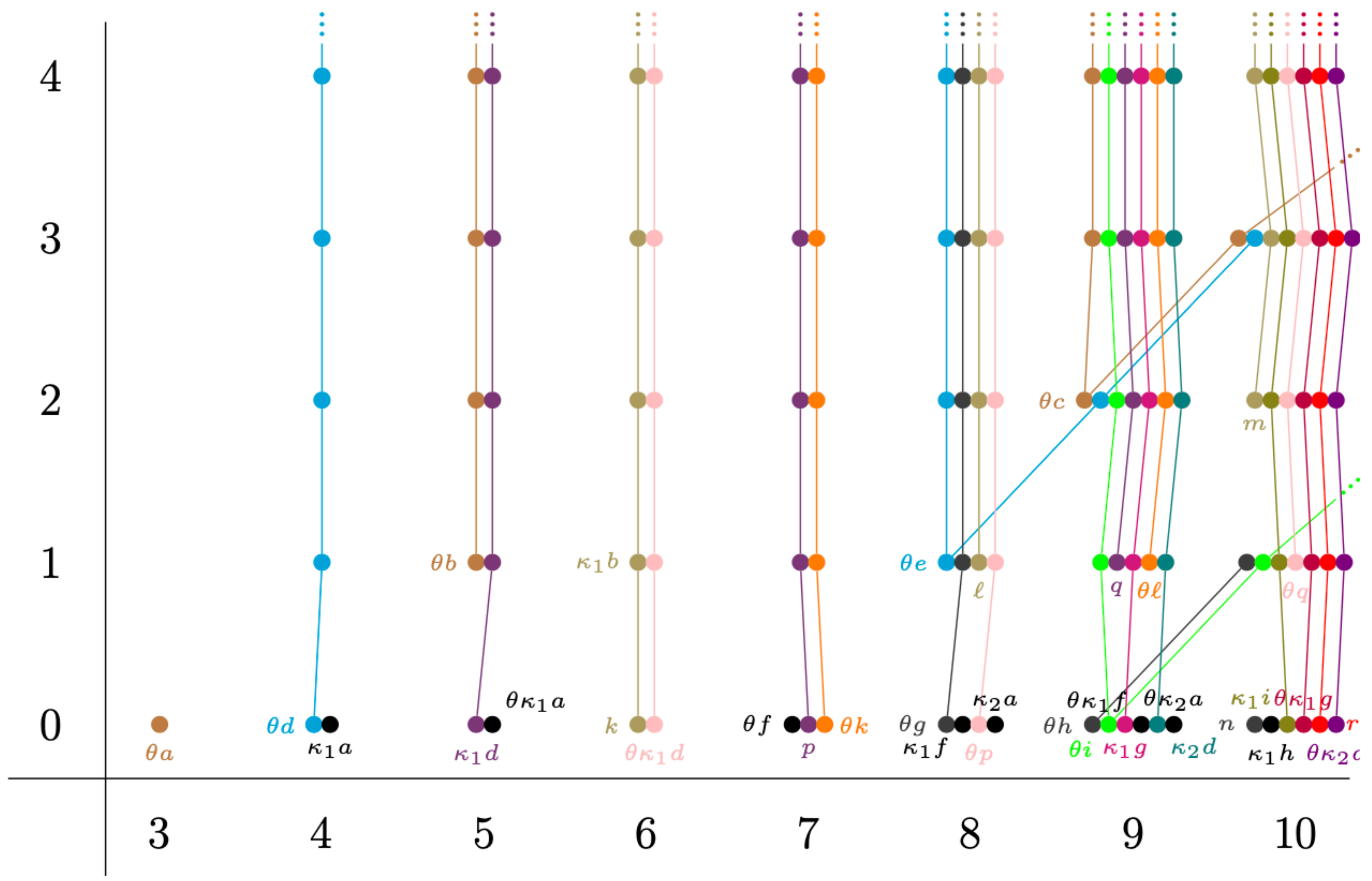
$\rightarrow$ orientation reversal of worldsheet

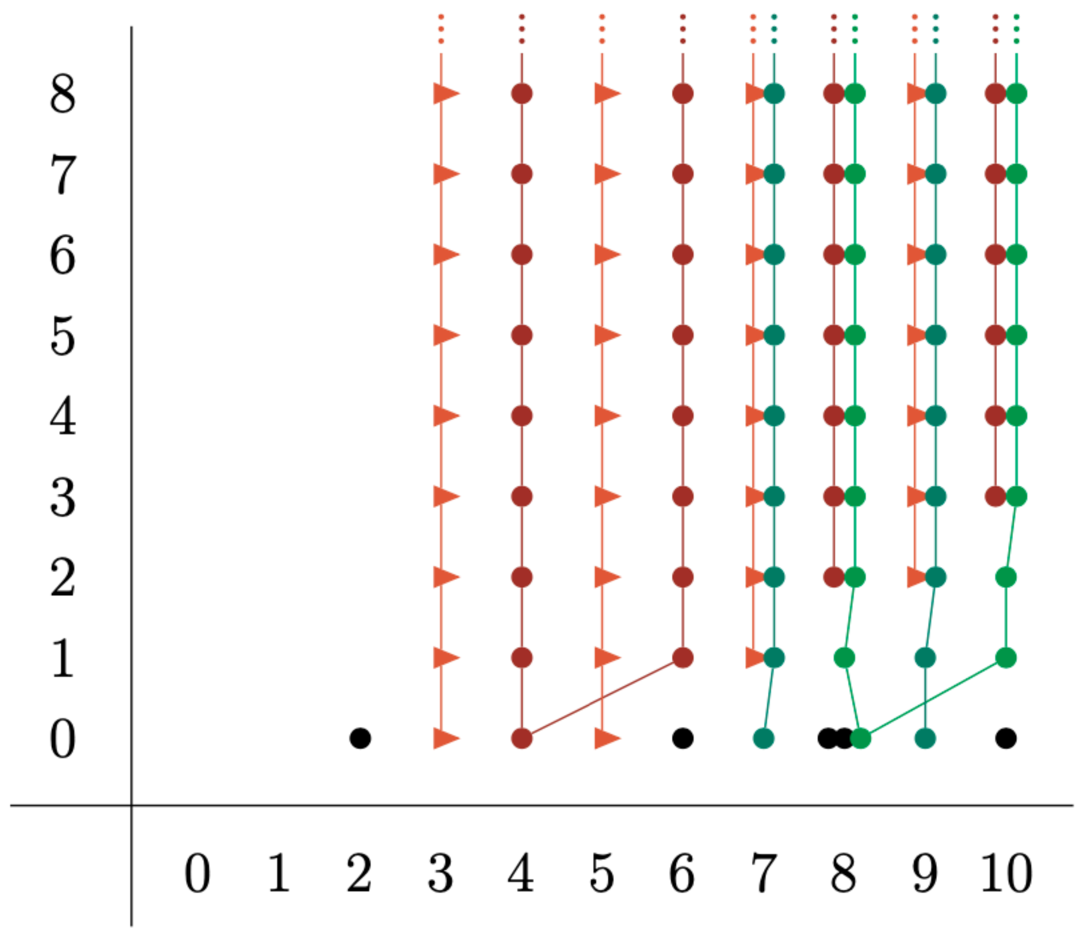
[Pantev, Sharpe '16], [Tachikawa, Yonekura '18]

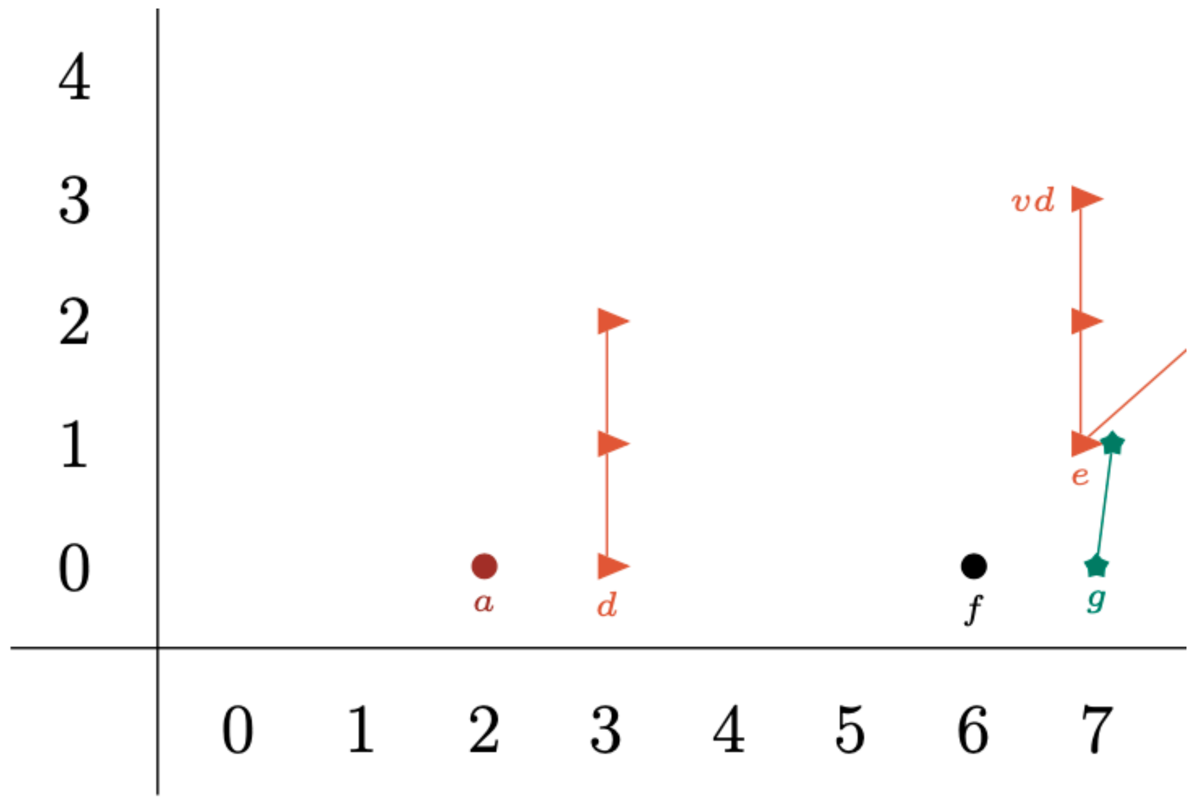
Calculation using spectral sequences

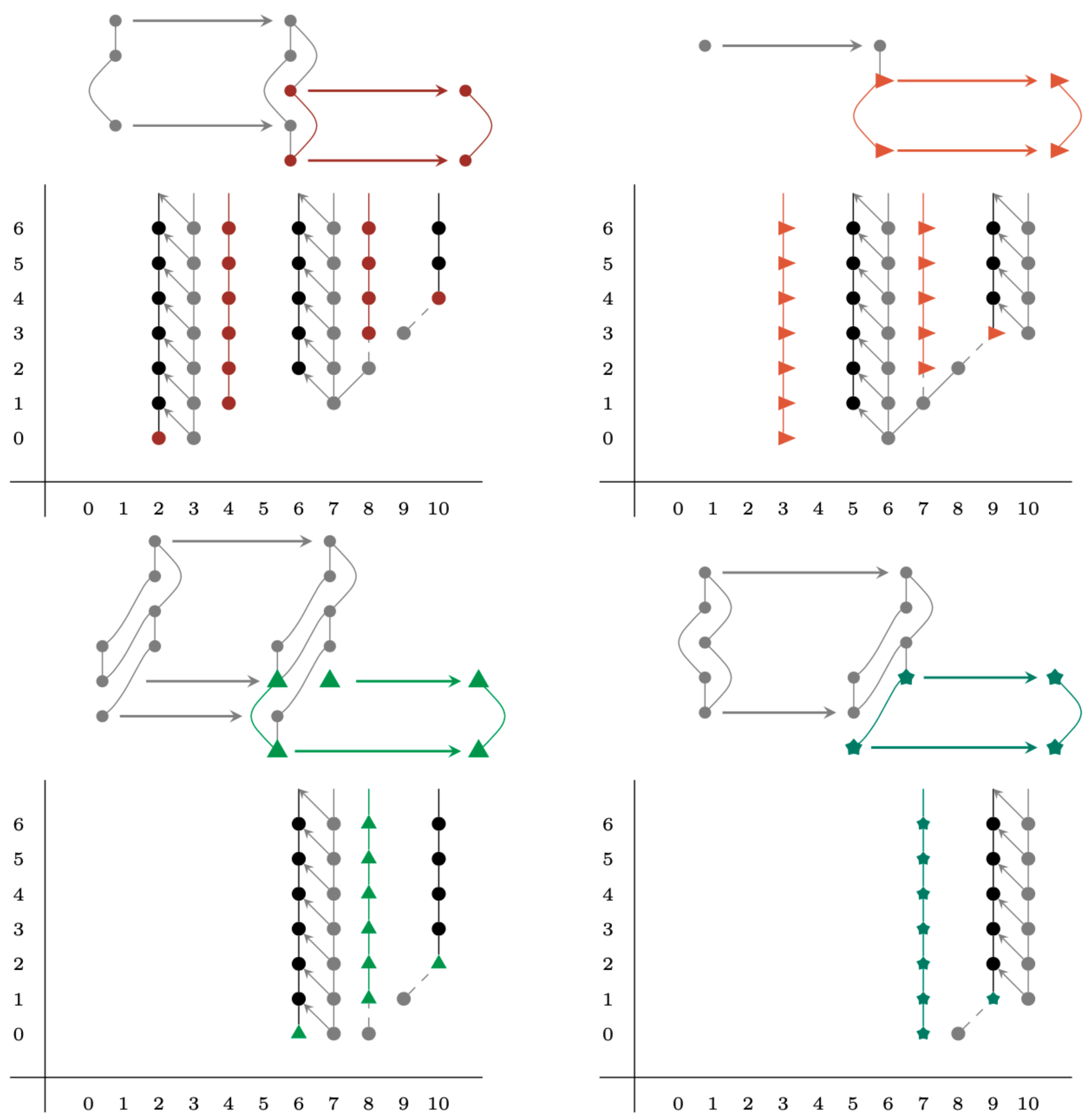
Mainly Adams (at prime 2),

(Also Atiyah Hirzebruch and homotopy equivalences to other spectra)









we have calculated the bordism groups

d	$\Omega_d^{\text{Spin}}(BSL(2,\mathbb{Z}))$ §4	$\Omega_d^{\text{Spin-Mp}(2,\mathbb{Z})}$ §5	$\Omega_d^{\text{Spin-GL}^+(2,\mathbb{Z})}$ §6
0	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
1	$\mathbb{Z}_2 \oplus \mathbb{Z}_{12}$	$\mathbb{Z}_3 \oplus \mathbb{Z}_8$	$2\mathbb{Z}_2$
2	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	0	$\mathbb{Z}_2$
3	$\mathbb{Z}_2 \oplus \mathbb{Z}_{24}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_3$	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3$
4	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}$
5	$\mathbb{Z}_{36}$	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$2\mathbb{Z}_2$
6	0	0	0
7	$\mathbb{Z}_2 \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_{32}$	$\mathbb{Z}_4 \oplus \mathbb{Z}_9$	$\mathbb{Z}_4 \oplus 3\mathbb{Z}_2 \oplus \mathbb{Z}_9$
8	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}_2$
9	$3\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_{128} \oplus \mathbb{Z}_4 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_8 \oplus \mathbb{Z}_{27}$	$8\mathbb{Z}_2$
10	$4\mathbb{Z}_2$	$\mathbb{Z}_2$	$4\mathbb{Z}_2$
11	$2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus 2\mathbb{Z}_8 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_{128}$	$\mathbb{Z}_8 \oplus 2\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_{27}$	$\mathbb{Z}_8 \oplus 9\mathbb{Z}_2 \oplus \mathbb{Z}_{27} \oplus \mathbb{Z}_3$

many potential global symmetries

many potential anomalies

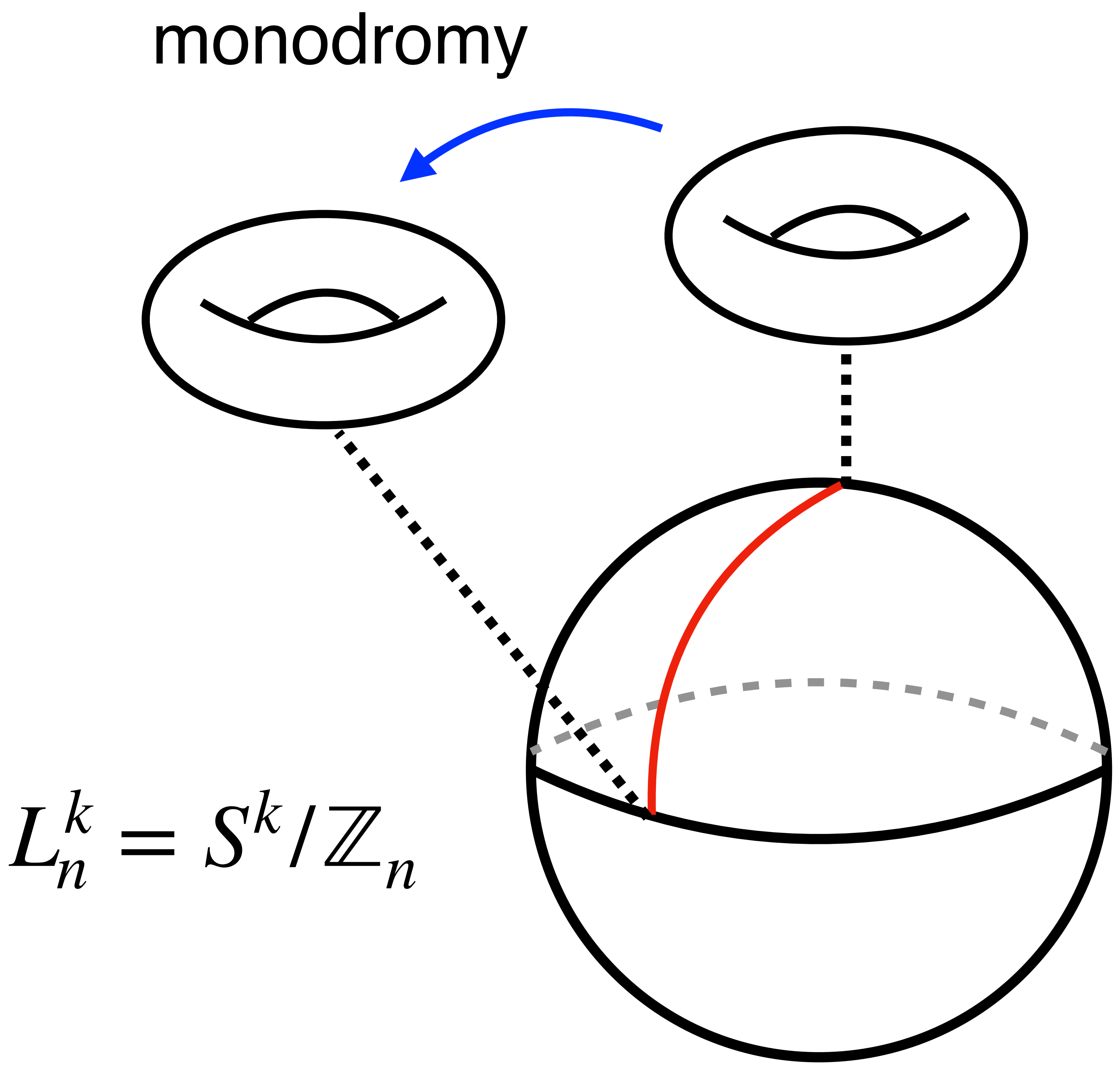
[Debray, MD, Heckman, Montero ’21 & ’23]

Symmetry-breaking defects

String theory takes care of it ➞ **interesting corners**

F-theory

Typically:



boundary of $(T^2 \times \mathbb{C}^j)/\mathbb{Z}_n$

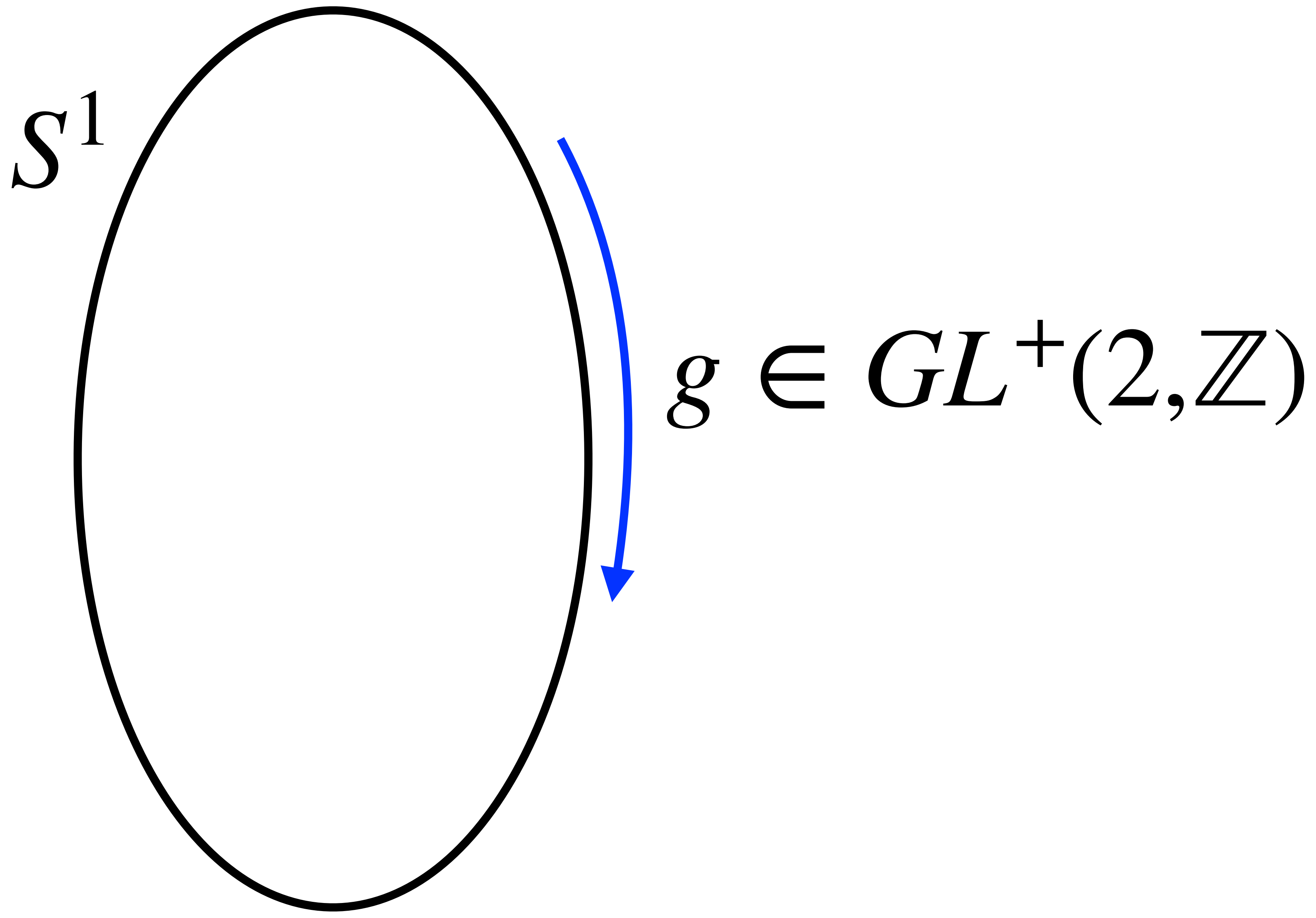
- **Non-Higgsable clusters**
  [Morrison, Taylor '12]
- $\mathcal{N} = 3$ **S-folds**
  [Garcia-Etxebarria, Regalado '15]
- **7-branes**
- **Topologically twisted theories**

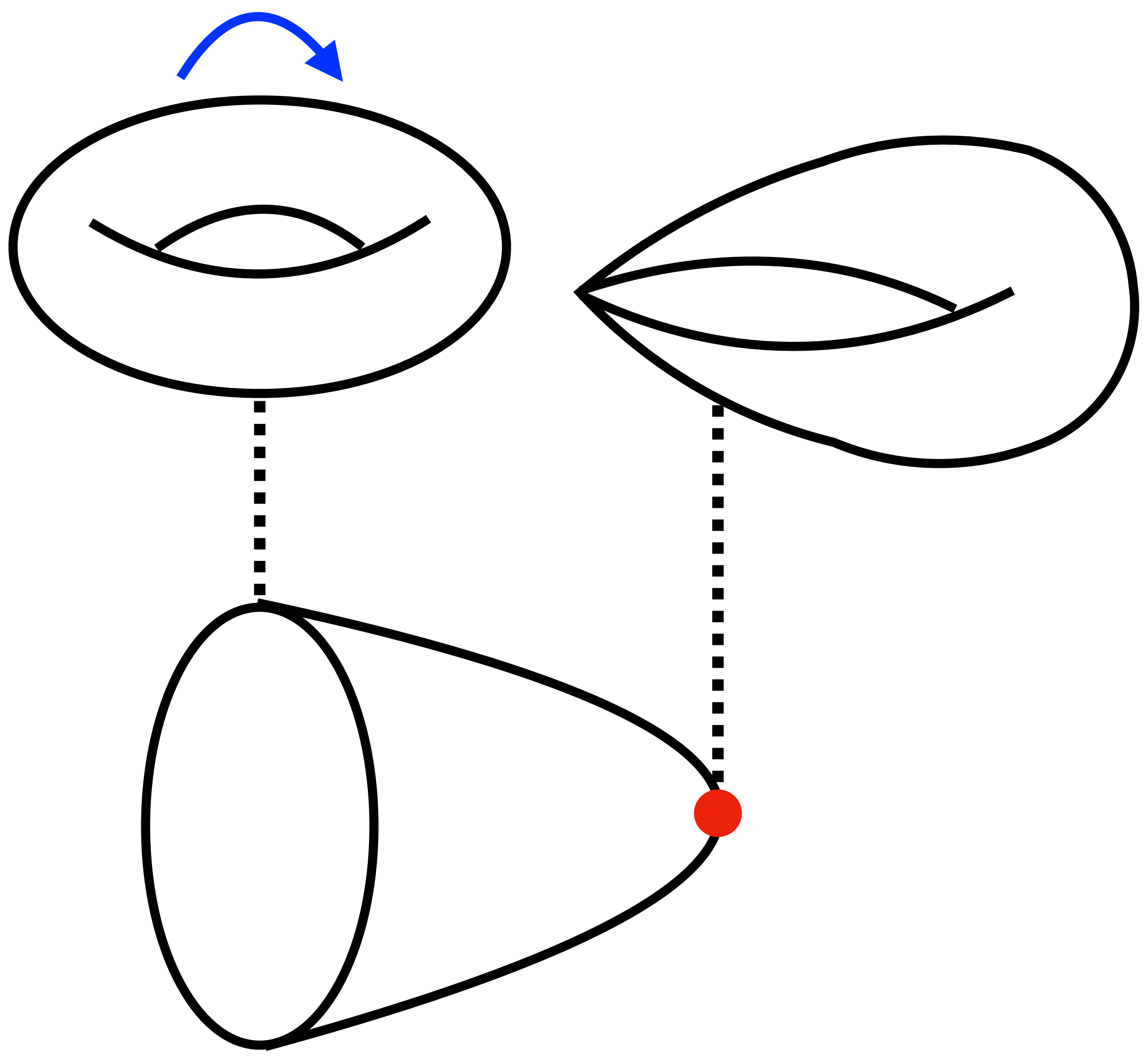
k = 1: 7-branes

$$\boxed{\Omega_1^{\text{Spin-GL}^+(2,\mathbb{Z})}(\text{pt}) = \mathbb{Z}_2 \oplus \mathbb{Z}_2}$$

taken care of by [p,q]-7-branes

(F-theory) see also [MD, Heckman '20]



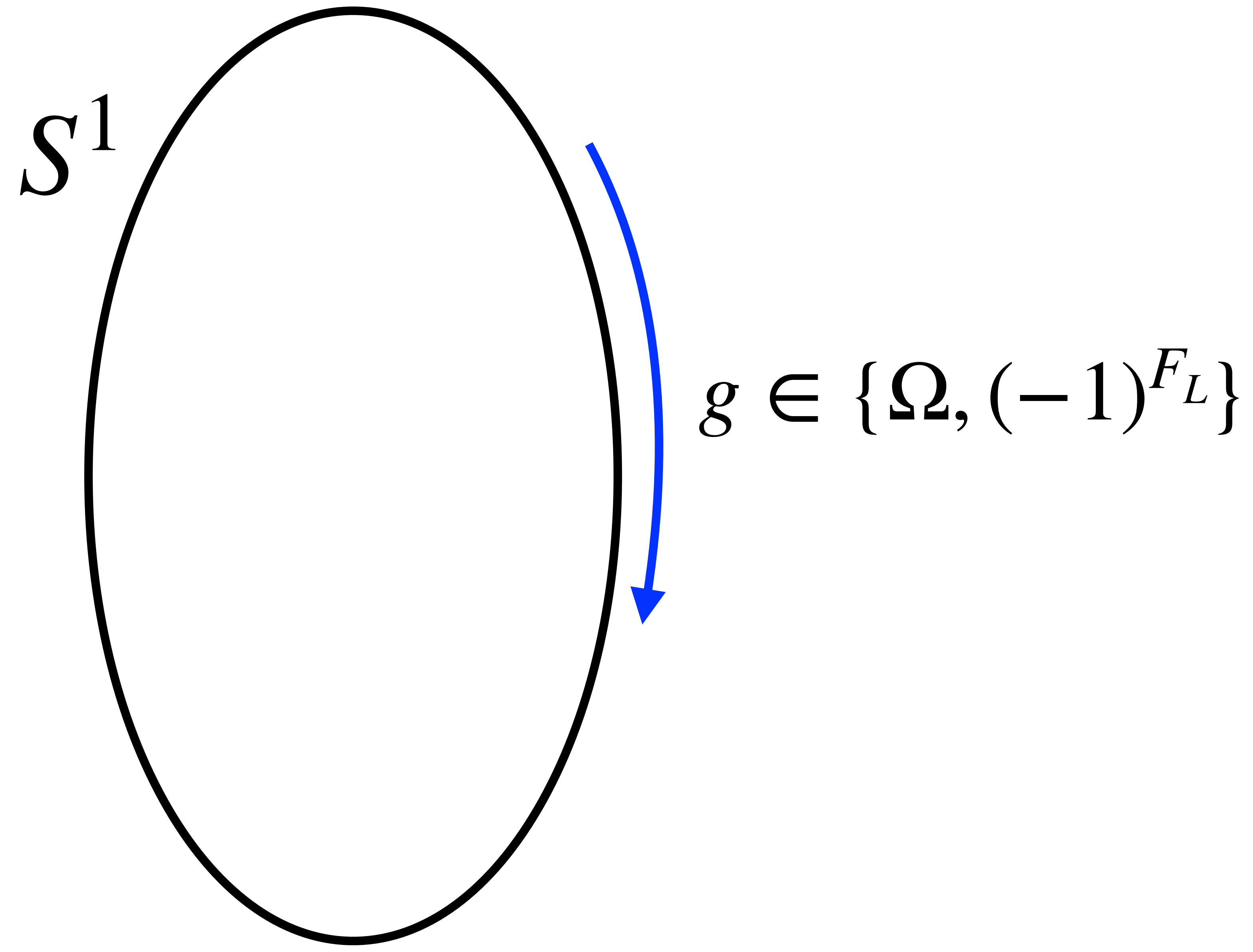


k = 1: 7-branes

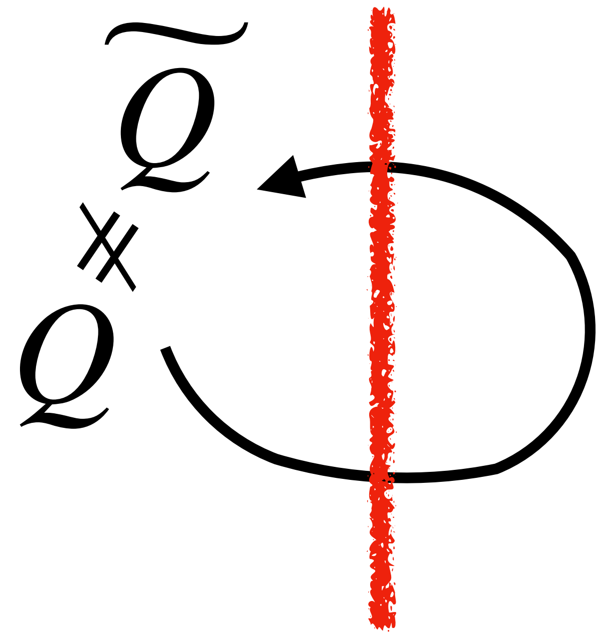
$$\boxed{\Omega_1^{\text{Spin-GL}^+(2,\mathbb{Z})}(\text{pt}) = \mathbb{Z}_2 \oplus \mathbb{Z}_2}$$

new ‘reflection’ 7-brane

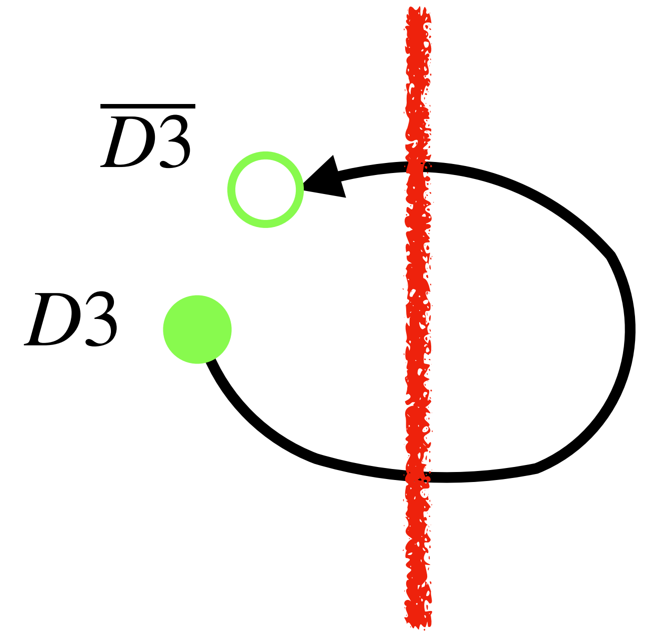
hinted at in [Distler, Freed, Moore ’09]



Breaks supersymmetry



Alice string for D3 branes



Maximal supergravity

U-duality in (11-k) dimensions (rather its **bosonic version**):

See e.g. [Obers, Pioline '98]

$$\mathrm{SL}(k;\mathbb{Z}) \bowtie \mathrm{SO}(k-1,k-1;\mathbb{Z})$$

D	$G_U^{D\mathrm{d}}$
10	1
9	$\mathrm{SL}(2,\mathbb{Z})$
8	$\mathrm{SL}(3,\mathbb{Z}) \times \mathrm{SL}(2,\mathbb{Z})$
7	$\mathrm{SL}(5,\mathbb{Z})$
6	$\mathrm{SO}(5,5,\mathbb{Z})$
5	$\mathrm{E}_{6(6)}(\mathbb{Z})$
4	$\mathrm{E}_{7(7)}(\mathbb{Z})$
3	$\mathrm{E}_{8(8)}(\mathbb{Z})$

comes from:

M-theory on T^k

$\wr|$

type IIB on T^{k-1}

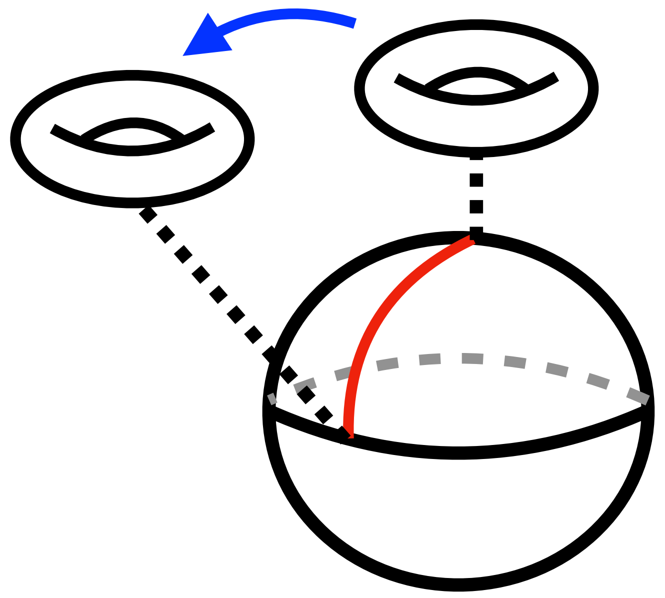
d	$\Omega_d^{\mathrm{Spin}}(BSL(2,\mathbb{Z}) \times BSL(3,\mathbb{Z}))$
1	$(\mathbb{Z}_2 \oplus)\mathbb{Z}_3 \oplus \mathbb{Z}_4$
2	$(\mathbb{Z}_2 \oplus)\mathbb{Z}_2^{\oplus 3}$
3	$\mathbb{Z}_3^{\oplus 3} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_8^{\oplus 3}$
4	$(\mathbb{Z} \oplus)\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_4^{\oplus 2}$
5	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_9 \oplus \mathbb{Z}_2^{\oplus 5} \oplus \mathbb{Z}_4$
6	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_2^{\oplus 3} \oplus \mathbb{Z}_4^{\oplus 2}$
7	$\mathbb{Z}_3^{\oplus 2} \oplus \mathbb{Z}_9^{\oplus 3} \oplus \mathbb{Z}_2^{\oplus 6} \oplus \mathbb{Z}_8^{\oplus 2} \oplus \mathbb{Z}_{16}^{\oplus 2} \oplus \mathbb{Z}_{32}$

a LOT of topological charges

What are the objects?

What we find (examples)

Type IIB:



$\mathbb{Z}_3 \in \Omega_3^{\mathrm{Spin}}(BG_U)$

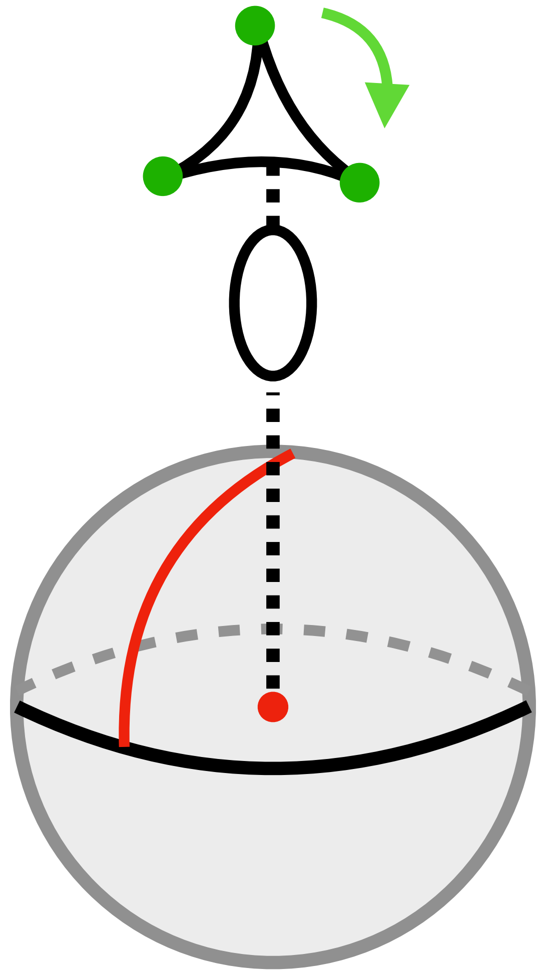
non-Higgsable cluster compactified on torus

(a descendant of IIB in 10d)

➡ **Argyres-Douglas theory** $D_3(\mathrm{SU}(2))$

[del Zotto, Vafa, Xie ’15], [Carta, Giacomelli, Mekareeya, Mininno ’23]

M-theory:



$\mathbb{Z}_3 \in \Omega_3^{\mathrm{Spin}}(BG_U)$

three sector of E_0 theory (M-theory on $\mathbb{C}^3/\mathbb{Z}_3$)

[Seiberg ’96], [Morrison, Seiberg ’97]

$$(\mathbb{C}^3/\mathbb{Z}_3) \oplus (\mathbb{C}^3/\mathbb{Z}_3) \oplus (\mathbb{C}^3/\mathbb{Z}_3)$$

See also U-folds, e.g., [Kumar, Vafa ’96], [Liu, Minasian ’97], …

Including fermions and reflections

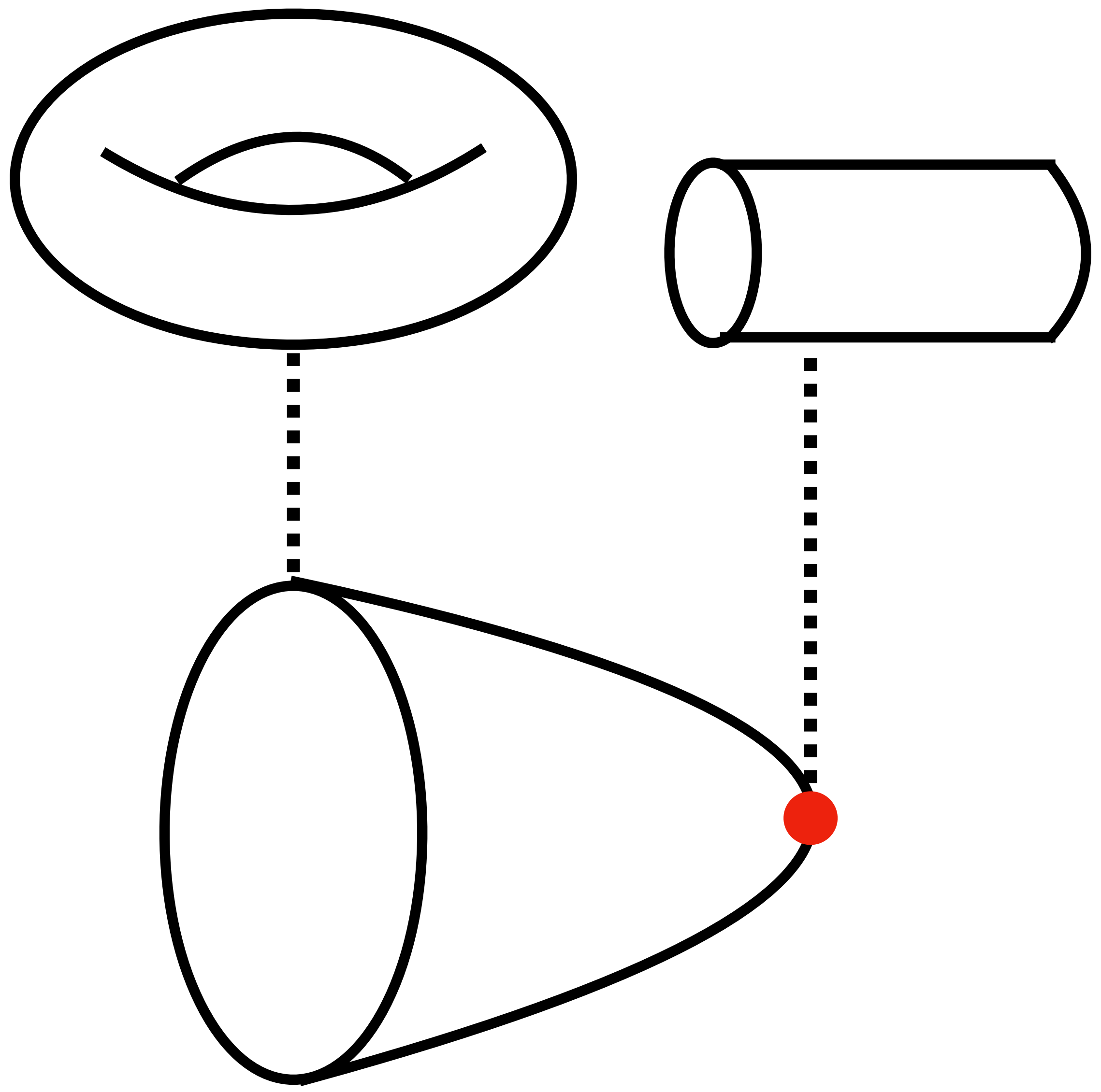
[Chakrabhavi, Debray, MD, Heckman '25]

But there is also a **reflection**:

$$\widetilde{G}_U \rtimes \mathbb{Z}_2 \qquad\qquad \Omega_1^{\mathrm{Spin}\text{-}(\widetilde{G}_U \rtimes \mathbb{Z}_2)}(\mathrm{pt}) = \mathbb{Z}_2$$

always **survives**

M-theory makes sense **on un-oriented** spaces



need to include **new reflection branes**

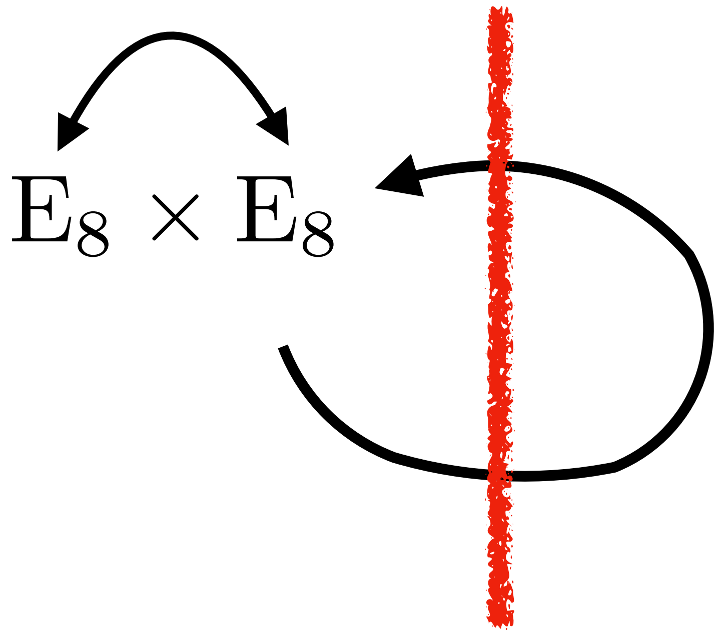
Heterotic supergravity

Gauge degrees of freedom $(\mathrm{E}_8 \times \mathrm{E}_8) \rtimes \mathbb{Z}_2$ or $\mathrm{Spin}(32)/\mathbb{Z}_2$

Bianchi identity: $dH_3 \sim \frac{1}{2}p_1 - c_2$

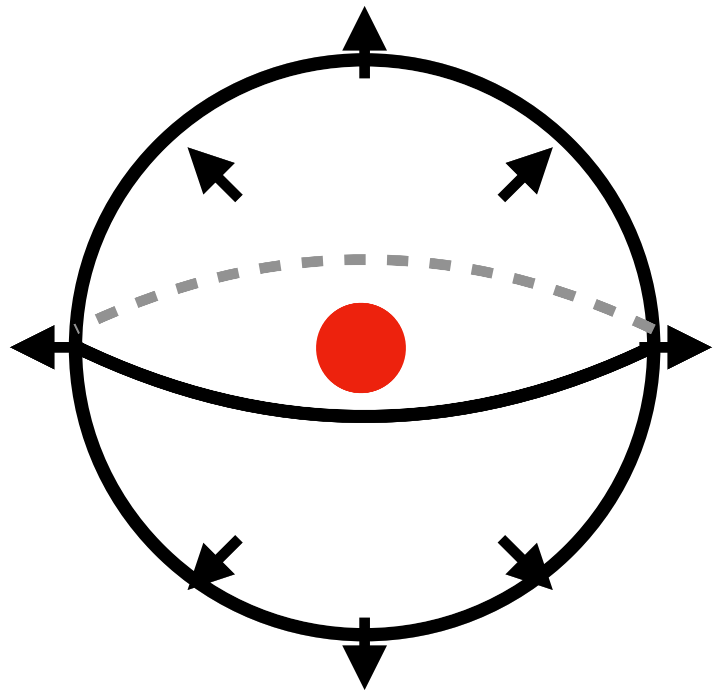
➡ **twisted String structure**

$$\Omega_k^{\text{String-}G}(\mathrm{pt})$$



7-brane

6-brane



lift to $\mathrm{Spin}(32)$ obstructed

...

e.g. see [Basile, Debray, Delgado, Montero '23], [Debray '23]

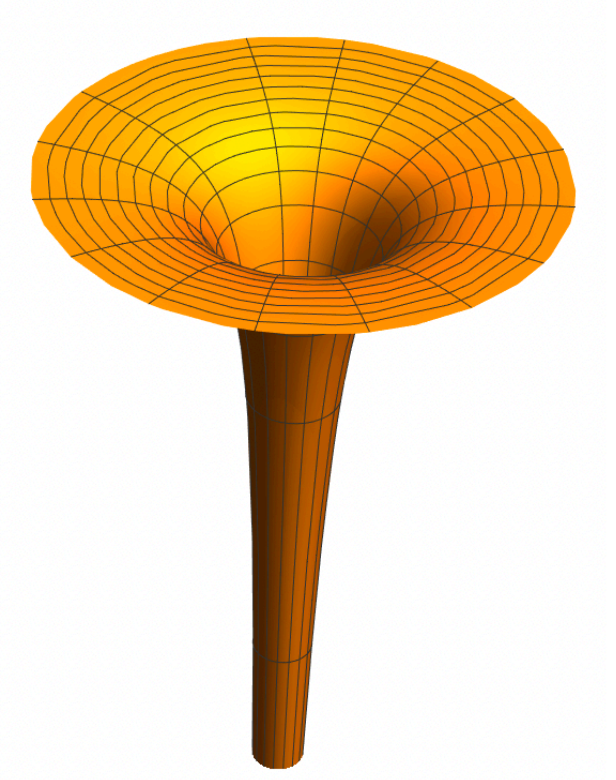
Heterotic string theory

[Kaidi, Ohmori, Tachikawa, Yonekura ’23], [Kaidi, Tachikawa, Yonekura ’24]

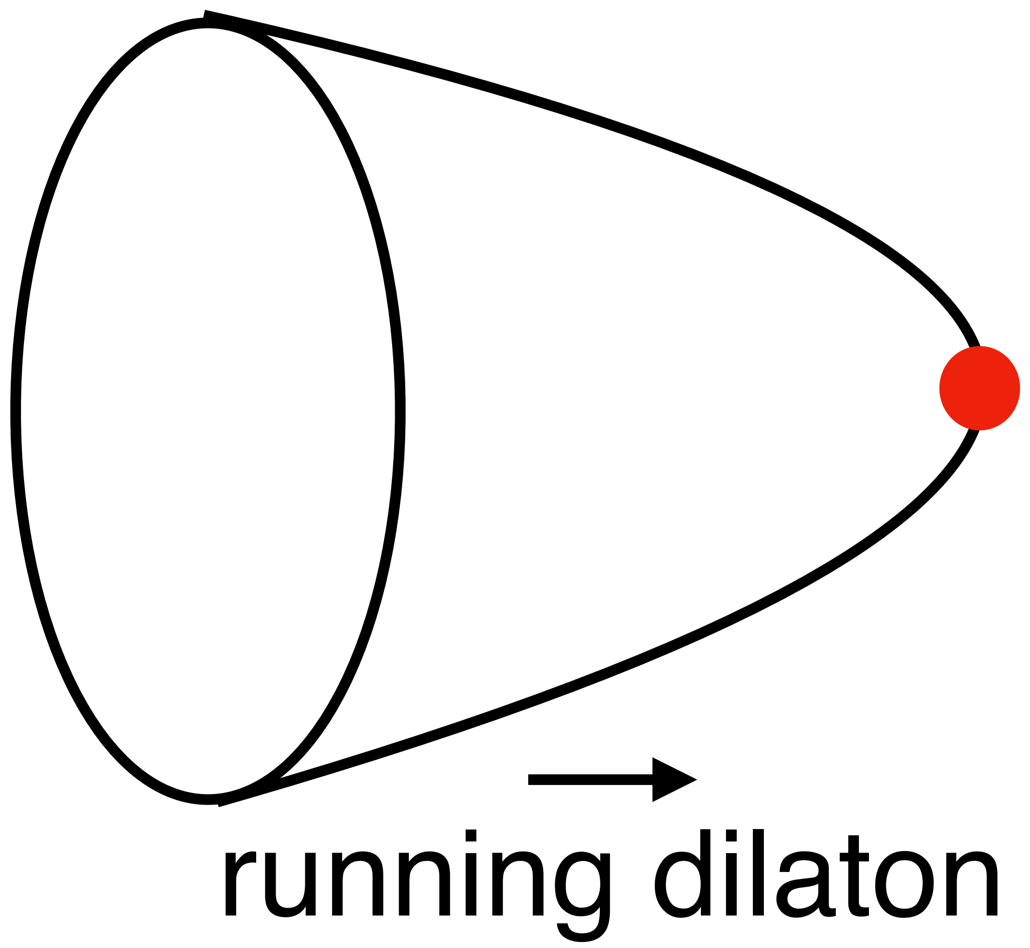
Have been **found before from a worldsheet perspective**

$$G \leftrightarrow (H \times \mathrm{SO}(n))/(-1)^{F_L}$$

equivalence of **left-moving current algebra**



remaining gauge
degrees of freedom
H



interesting **relation to non-SUSY heterotic**

Properties of the defect

[WIP with Novicic, Ruiz]

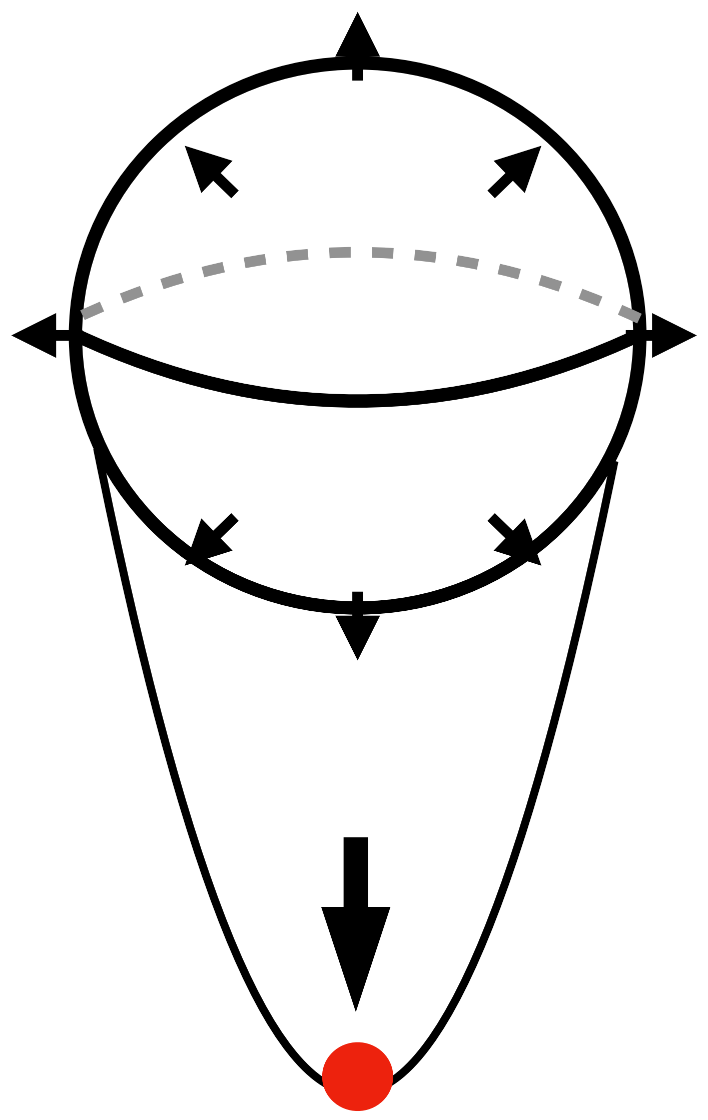
Hard to study because typically the core region is at **strong coupling**

→ use **asymptotic region** and **robust properties**

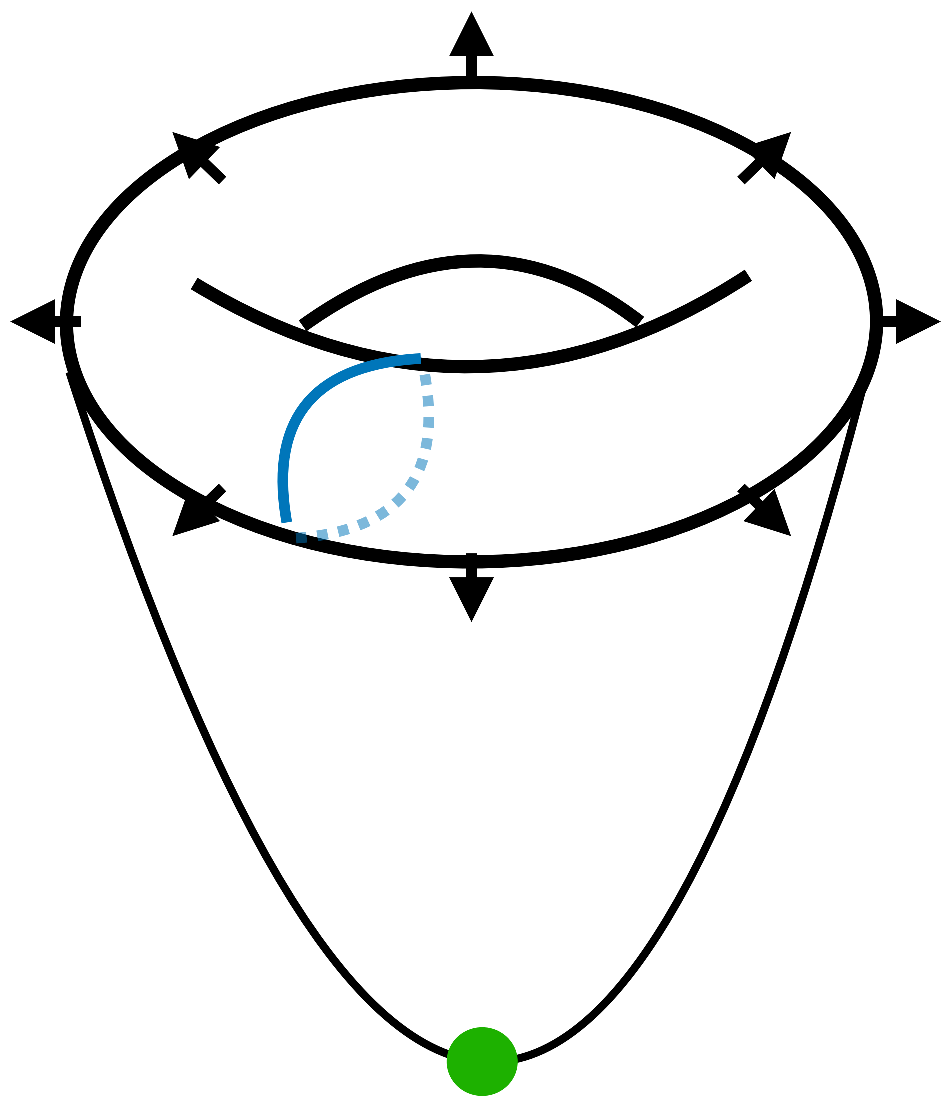
Properties of the defect

[WIP with Novicic, Ruiz]

Asymptotics dictated by **a representative of deformation class**



ambiguity



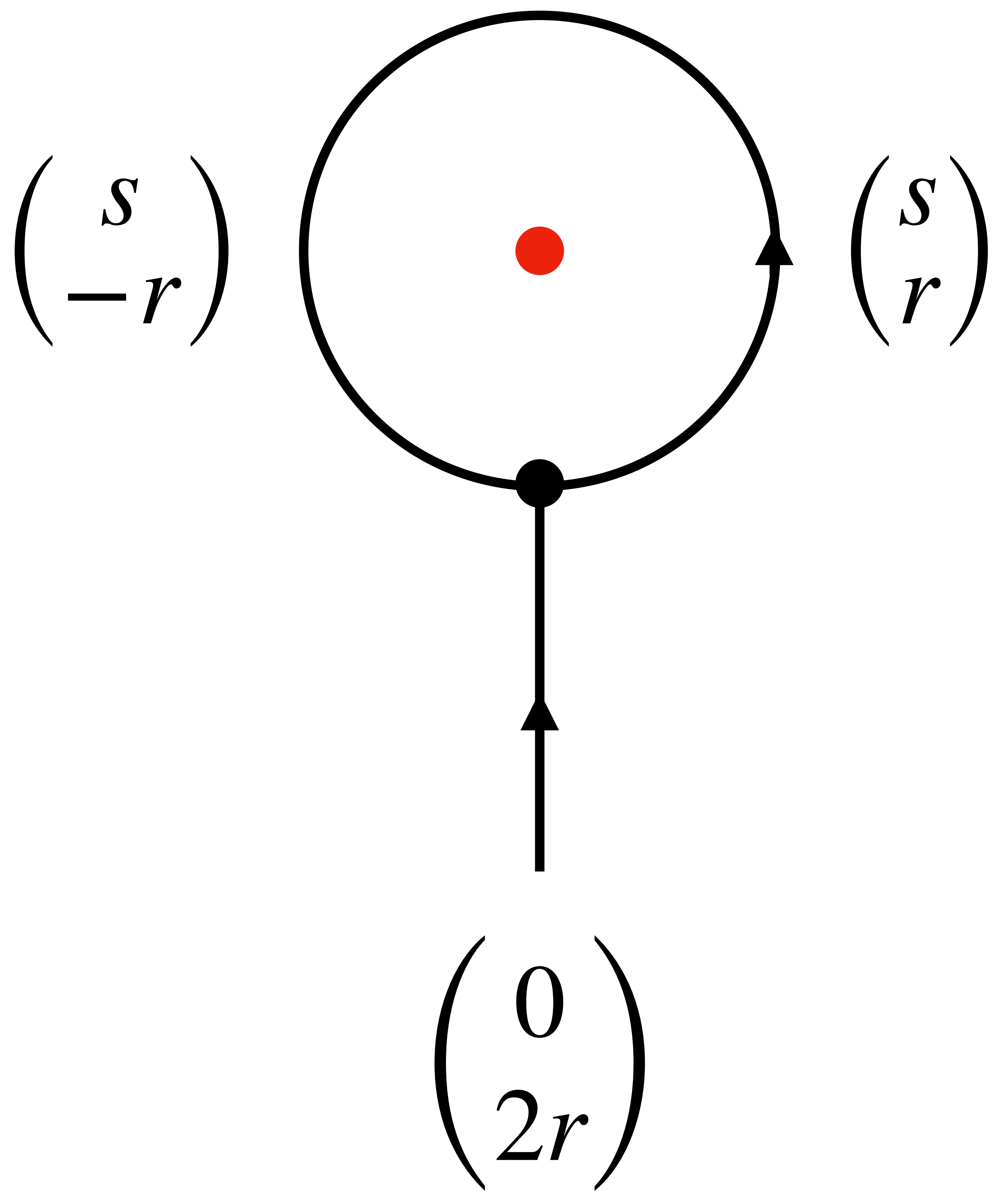
study **anomaly inflow**

study **probe branes**

→ **properties of the symmetry-breaking defects**

Strings can end on R7-brane

[MD, Heckman, Montero, Torres ’22]



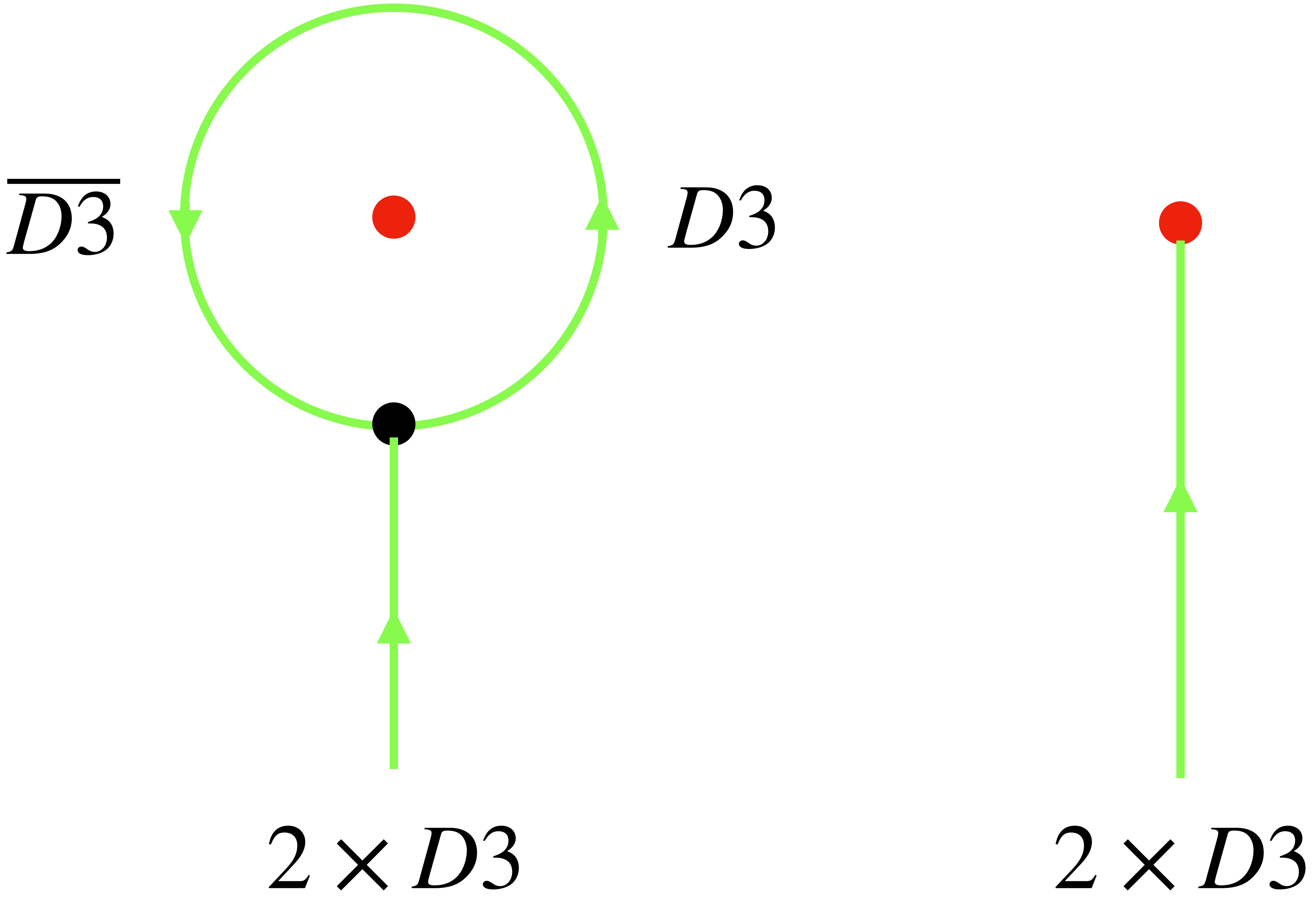
F1-strings end on Ω brane

D1-strings end on $(-1)^{F_L}$ brane

➡ something, e.g., gauge fields, should absorb charge

See also [Cvetic, MD, Lin, Zhang ’21, ’22] for [p,q]-7-branes

D3-branes can end on R7-brane

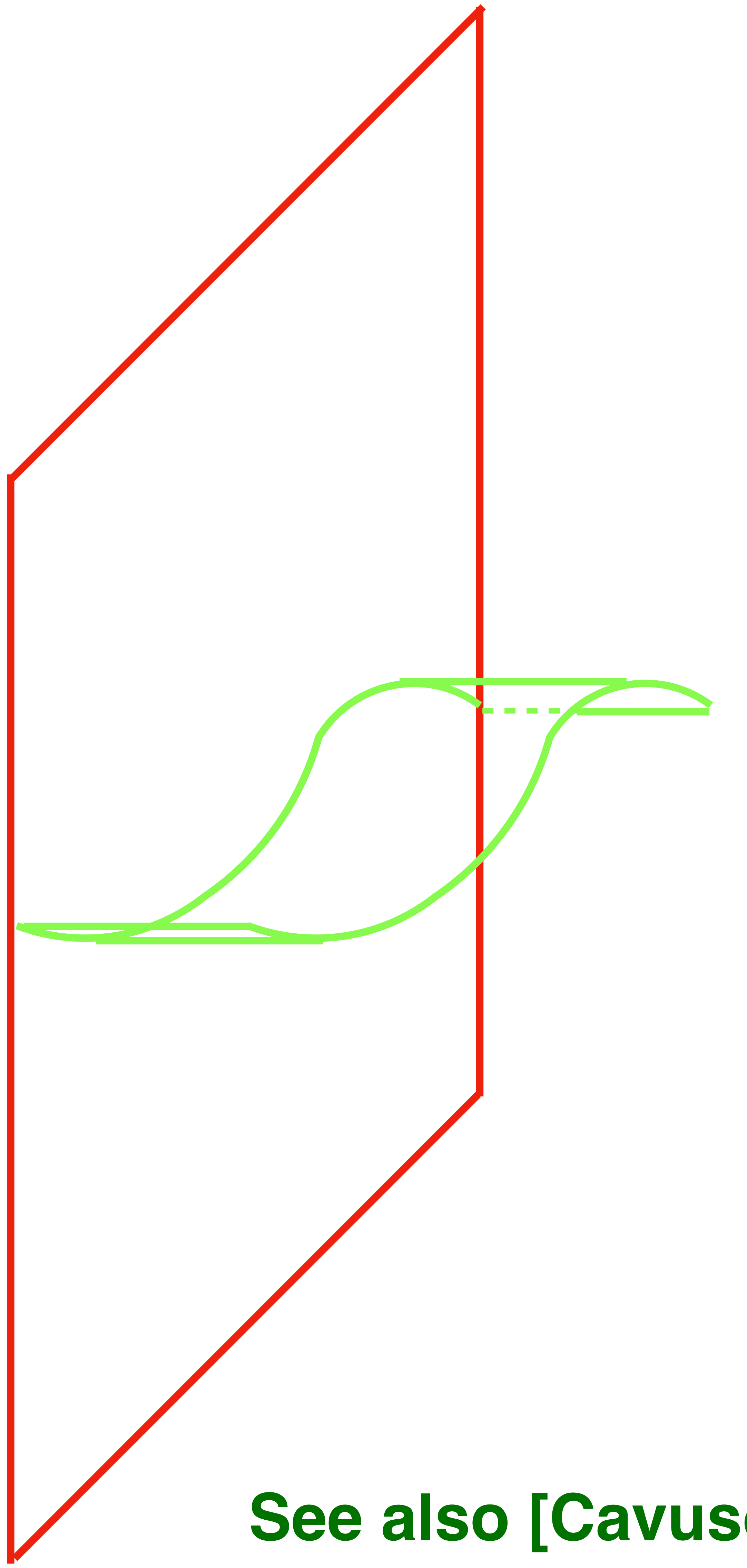


D3-branes end because of
$C_4 \to -C_4$ transformation

➔ something should absorb charge

See also [Chakrabhavi, Debray, MD, Heckman '25], [Heckman, McNamara, Parra-Martinez, Torres '26]

3-form fields



D3-brane creates 3d worldvolume in R7-brane

➡ flux on transverse S^4

suggests $F_4 = dC_3$ (odd under reflections)

➡ **massless 3-form on R7-brane**

(potentially interesting behavior under S-duality;
interacting non-supersymmetric CFT in 8d???)

anomaly inflow doesn't help

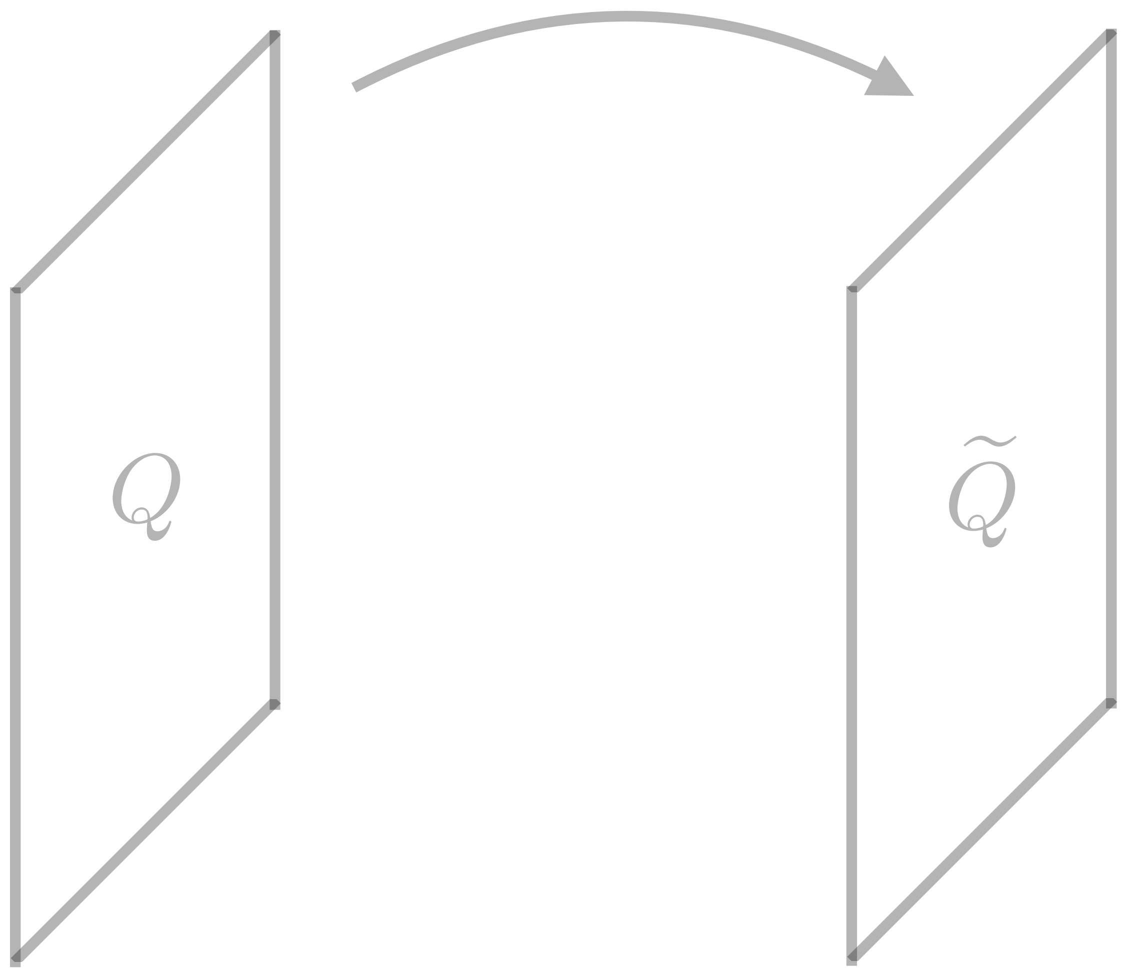
See also [Cavusoglu, Cvetic, Heckman, Kuntz, Murdia '26], [Dibitetto, Petri (Gibson, Sterckx) '26]

Alternatives?

No global symmetries

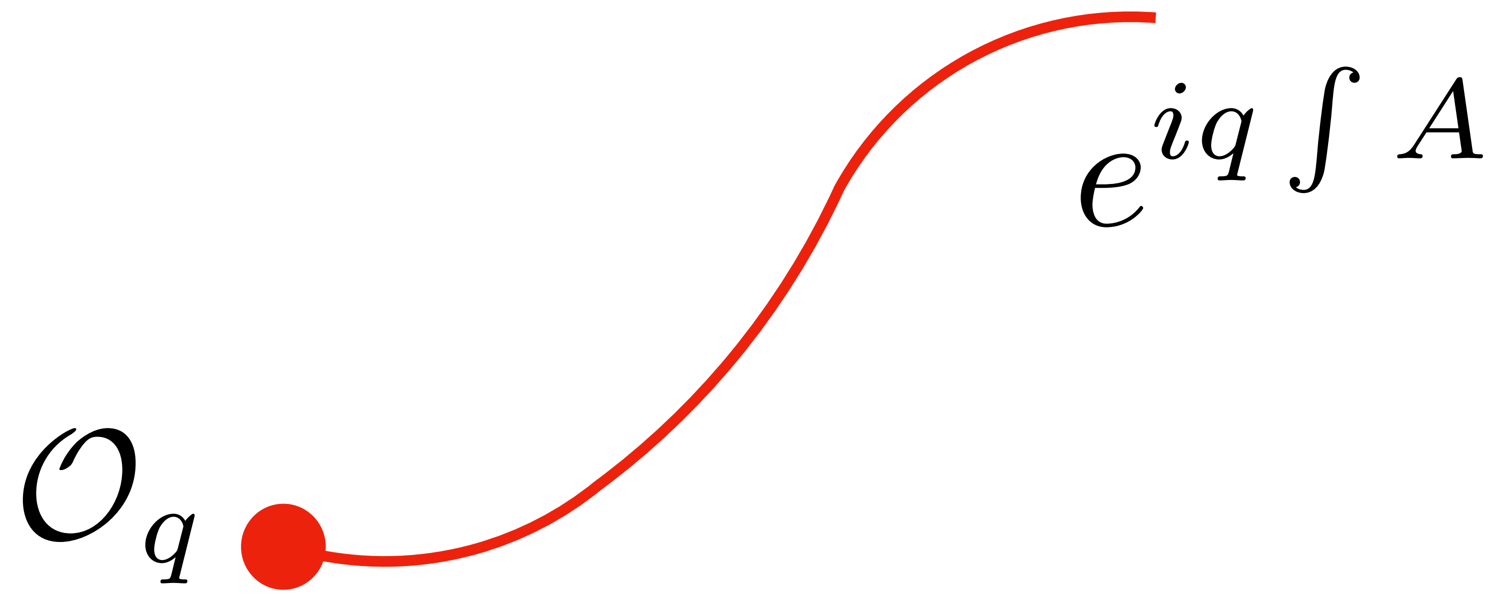
If no global symmetries they **need to be:**

broken



charge not conserved

gauged



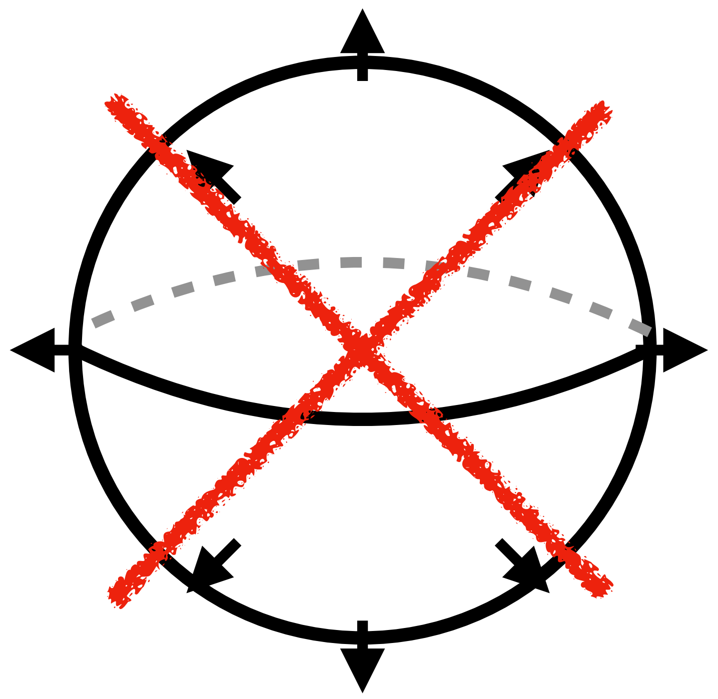
no anomalies (incl. gravity)

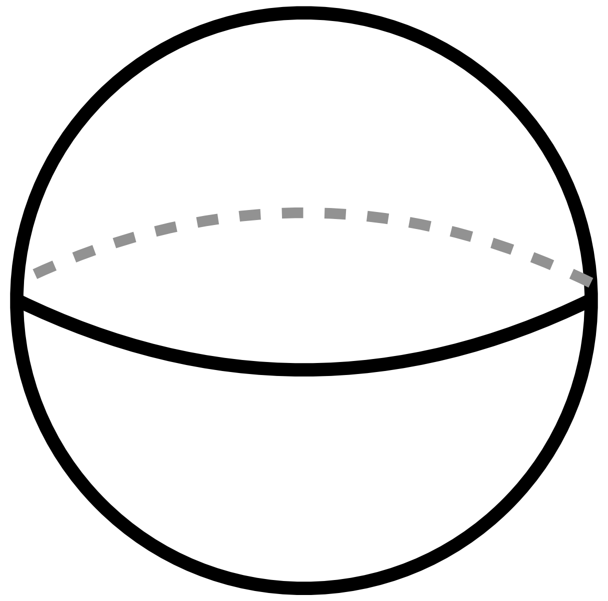
(breaking by quantum corrections)

Gauging

Trivialize obstruction → **only allowed sector has** $Q = 0$

$$\frac{1}{2\pi}\oint F \neq 0$$





$$\frac{1}{2\pi}\oint F = 0$$

similar for

$$\int F^n$$

Coupling the conserved current to a gauge field imposes

$$*J = dH$$

Charge vanishes since $*J$ needs to be integrated over closed manifold
(Gauss's law)

see also [Heidenreich, McNamara, Montero, Reece, Rudelius, Valenzuela '20], [Blumenhagen, Cribiori '21]

Modification of classifying space

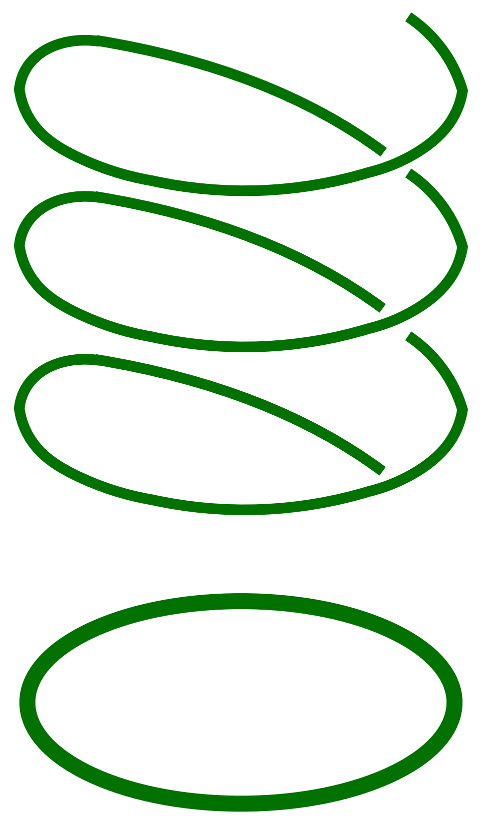
[Saito, Tachikawa ’24], [MD, Minasian, Novicic ‘26]

$$\begin{tikzcd} Q \arrow[r] & BU(1) \arrow[r, "\alpha"] & Y \\ & X \arrow[ul] \arrow[u, "f"'] & \end{tikzcd}$$

There is a space Q , the **homotopy fiber, implementing constraint**

➞ new **modified classifying space**

Itself **a fibration:** $$\Omega Y \longrightarrow Q \longrightarrow BU(1)$$



$\mathbb{Z} \to \mathbb{R} \to S^1$

$\Omega K(A,m) = K(A,m-1)$ trivialization of element in $H^n(BU(1);\mathbb{Z})$

New obstructions

[MD, Minasian, Novicic '26]

Also as input of the **bordism computation** (gravitational charges)

Example: $dH = nF \wedge F$

k	0	1	2	3	4	5	6
$\Omega_k^{\mathrm{SO}}(Q)$	$\mathbb{Z}$	0	$\mathbb{Z}$	$\mathbb{Z}_n$	$\mathbb{Z}$	$\mathbb{Z}_n \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$	$\mathbb{Z}$
$\Omega_k^{\mathrm{SO}}(B\mathrm{U}(1))$	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z} \oplus \mathbb{Z}$	$\mathbb{Z}_2$	$\mathbb{Z} \oplus \mathbb{Z}$

modifications generally persist

➡ **other global symmetries; might not solve the problem**

Gauging and the Whitehead tower

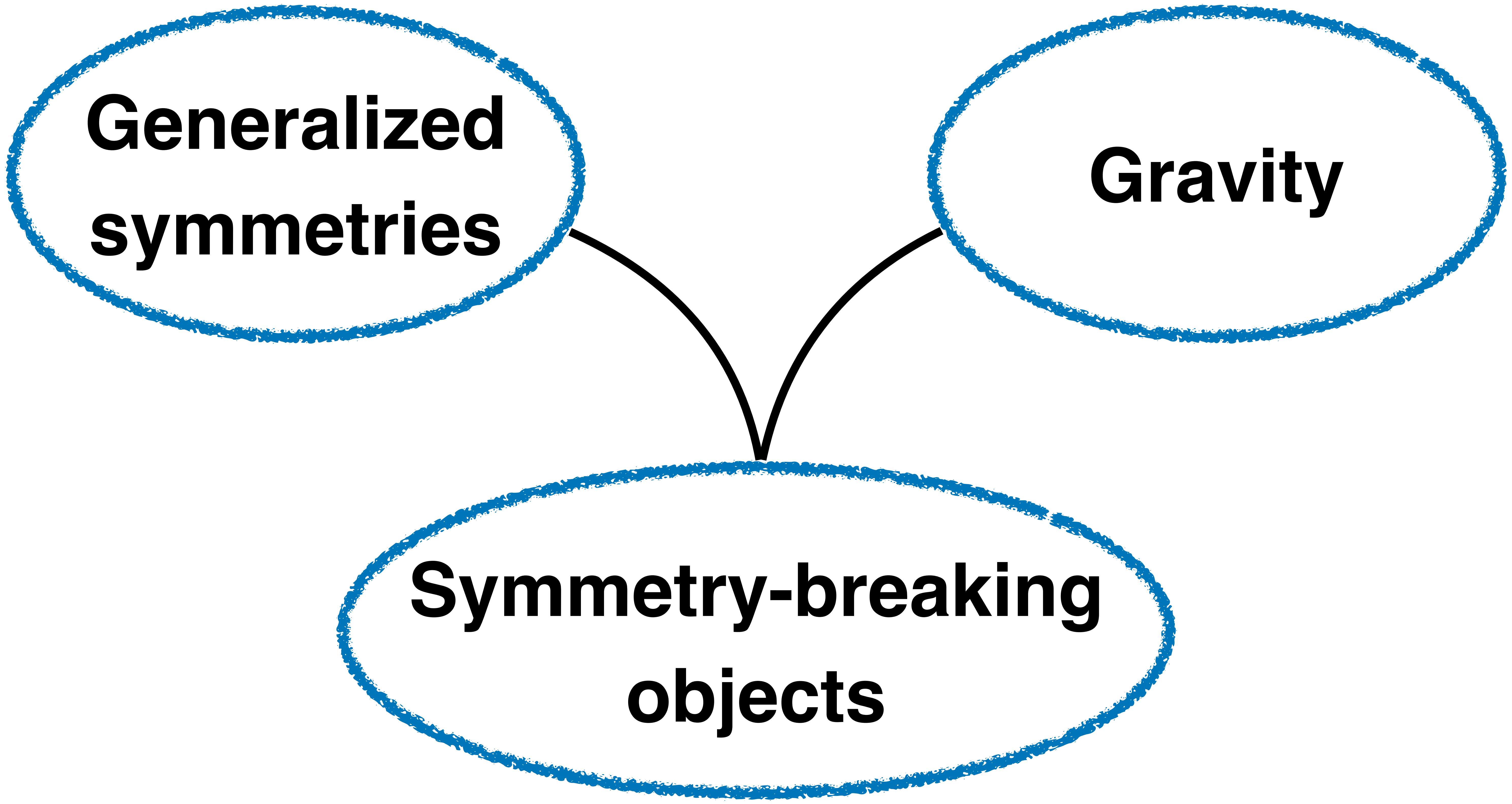
In gravity the **topological charges** come from the **tangent bundle**

➔ **iterative trivialization** or **gauging** leads to interesting sequence

$$\begin{tikzcd}
 & B\mathrm{String} \arrow[d, "\frac{1}{2}p_1 = 0"] & \text{(String structure)} \\
 & B\mathrm{Spin} \arrow[d, "w_2 = 0"] & \text{(Spin structure)} \\
 & B\mathrm{SO} \arrow[d, "w_1 = 0"] & \text{(orientation)} \\
X \arrow[r] \arrow[ur] \arrow[uur] \arrow[uuur] & B\mathrm{O} &
\end{tikzcd}$$

Summary

Surprises everywhere!



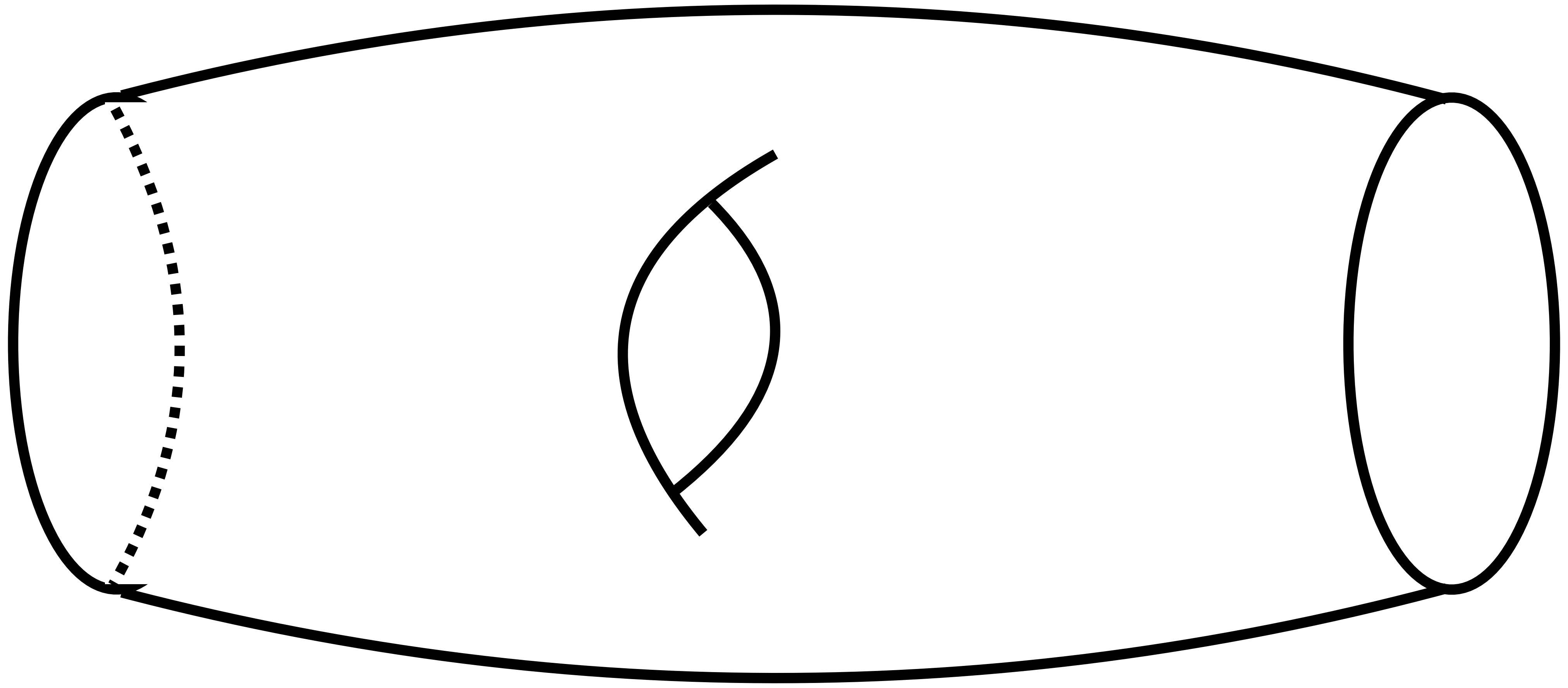
Much to explore!

Omission

[Dai, Freed '94], [Witten '15], [Freed, Hopkins '16], [Yonekura '16], [Yamashita, Yonekura '21], see also [Montero, Garcia-Etxebarria '18]

Bordism groups also **important for gauging ➛ anomalies**

$$Z[A^\lambda] \stackrel{?}{=} Z[A]$$

$Z(A, g)$  $Z(A^\lambda, g^\theta)$

Further **classify topological terms**, i.e., θ**-angles**

Outlook: Across phase transitions

[Davighi, MD: WIP]

UV

G

$H \subset G$

IR

If symmetry is **anomalous**: **must be matched**

['t Hooft '80]

Anomaly of G

Anomaly of H + topological terms G/H for Goldstone modes

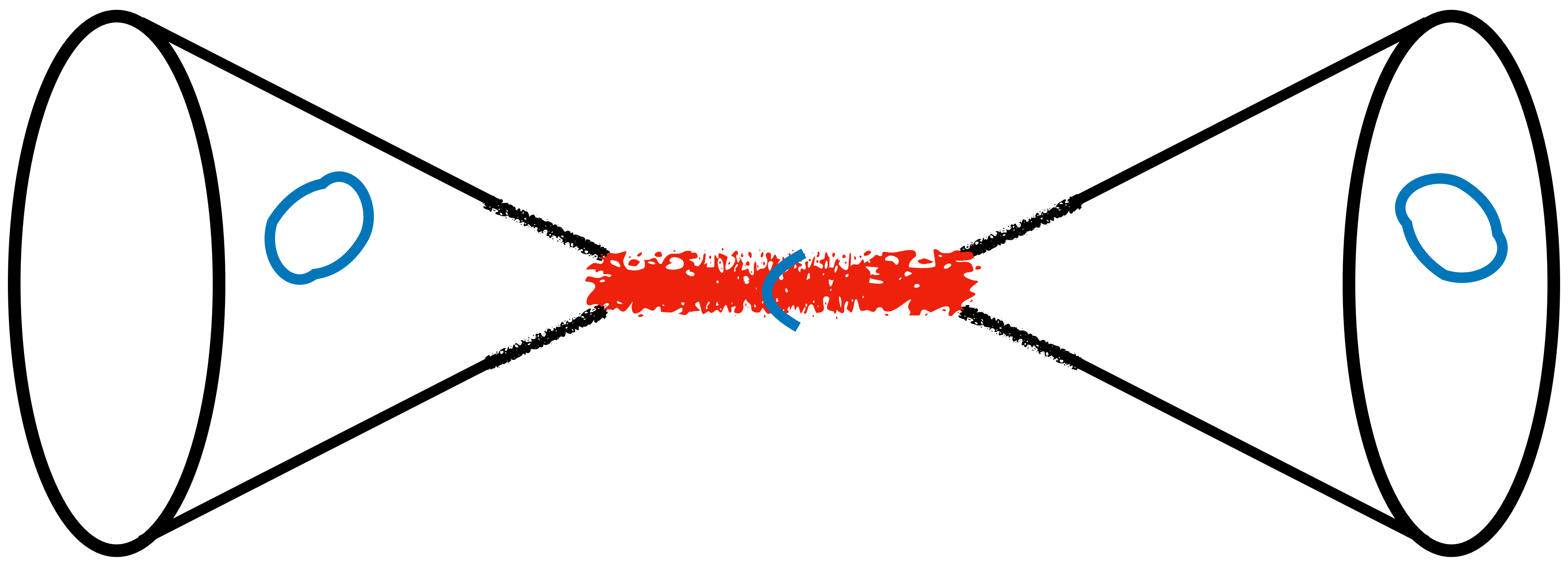
can **enforce mixing**

$$\ldots \to (I_{\mathbb{Z}}\Omega^{\xi})^{D+1}(G/H) \to (I_{\mathbb{Z}}\Omega^{\xi})^{D+2}(BG) \to (I_{\mathbb{Z}}\Omega^{\xi})^{D+2}(BH) \to \ldots$$

Outlook: If I was a string

Explore defects and exotic background with string worldsheet:

- **Strings see less:** $g_s < 1$ (problem with RR fluxes)
- **Strings see more:** SQFT_2 (no smooth spacetime needed)



→ **Deformations of supersymmetric QFT in 2d:** **topological modular forms**

See e.g. work by Gaiotto, Johnson-Freyd, Kaidi, Lin, Tachikawa, Tominaga, Yamashita, Yonekura, Zhang, …