Flat-Gauging Lines and Boundaries

Based on a WIP with Ho Tat Lam, Brandon C. Rayhaun

Kavli IPMU, Tokyo
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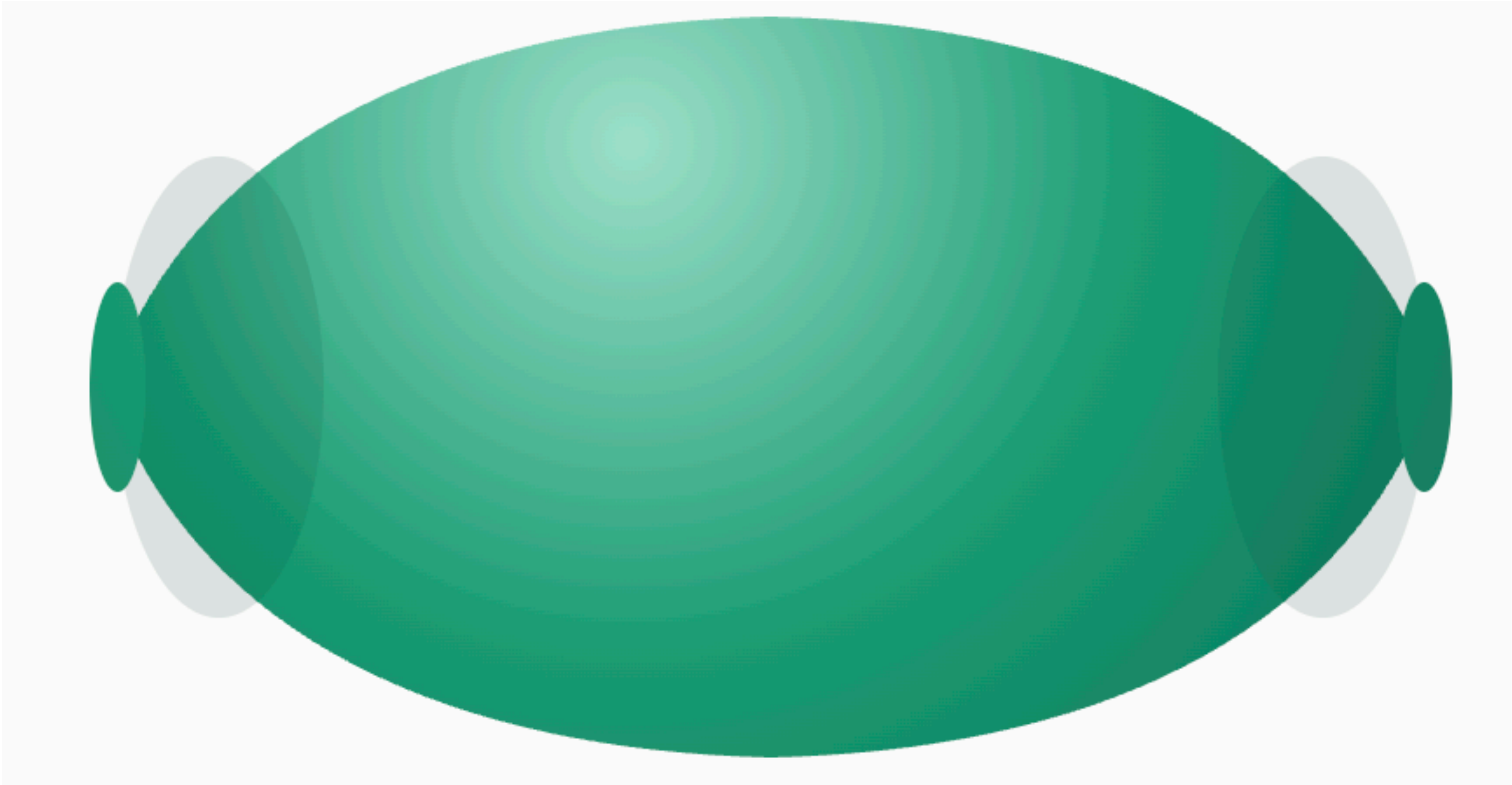
Yichul Choi (IAS)



Orbifold and Discrete Gauging

- In 1+1D CFTs, orbifold is a powerful construction that generates a new CFT from a known one.

- One takes a CFT Q with a discrete global symmetry G, and by gauging (a subgroup of) G, obtains a new CFT Q'.

- If the seed theory Q is solvable, then the gauged theory Q' is also (in principle) solvable.

Flat Gauging

- Our goal is to study a generalization of this procedure.

- We will take the global symmetry G to be a continuous Lie group. We gauge G while restricting the gauge field to be flat, $F = DA = 0$.

- The corresponding operation has been studied by various authors, and goes under various different names. […; Gaberdiel-Suchanek ’11;…; Dubovsky-Negro-Porrati ’23; Jia-Luo-Tian-Wang-Zhang ’25; Stockall-Yu ’25; Jia-Zhang ’26; …]

- Flat gauging, topological gauging, continuous orbifold, non-abelian T-duality, …

Flat Gauging

- We will see that flat-gauging is a well-defined operation in 1+1D CFTs.

- It provides a powerful tool to generate non-compact, irrational CFTs.

- We will propose a way to compute the spectrum of topological line defects (i.e. generalized global symmetries) and conformal boundary conditions of a flat-gauged theory, from those of the seed theory.

Main Results

- We classify all the generalized symmetries and conformal boundary conditions of a free, non-compact scalar field as well as those of the Runkel-Watts CFT.

- This is achieved by applying flat gauging to the $SU(2)_1$ Wess-Zumino-Witten (WZW) model.

- Along the way, we revisit certain aspects of the usual orbifold, and provide a physical interpretation of the Ostrik's formula in the study of category theory, which in physics parlance, computes the dual symmetries of the orbifold theory.

Outline

1. Revisiting lines and boundaries under discrete gauging

2. Flat-gauging

3. Runkel-Watts CFT

4. Non-compact boson

Lines and Boundaries Under Discrete Gauging

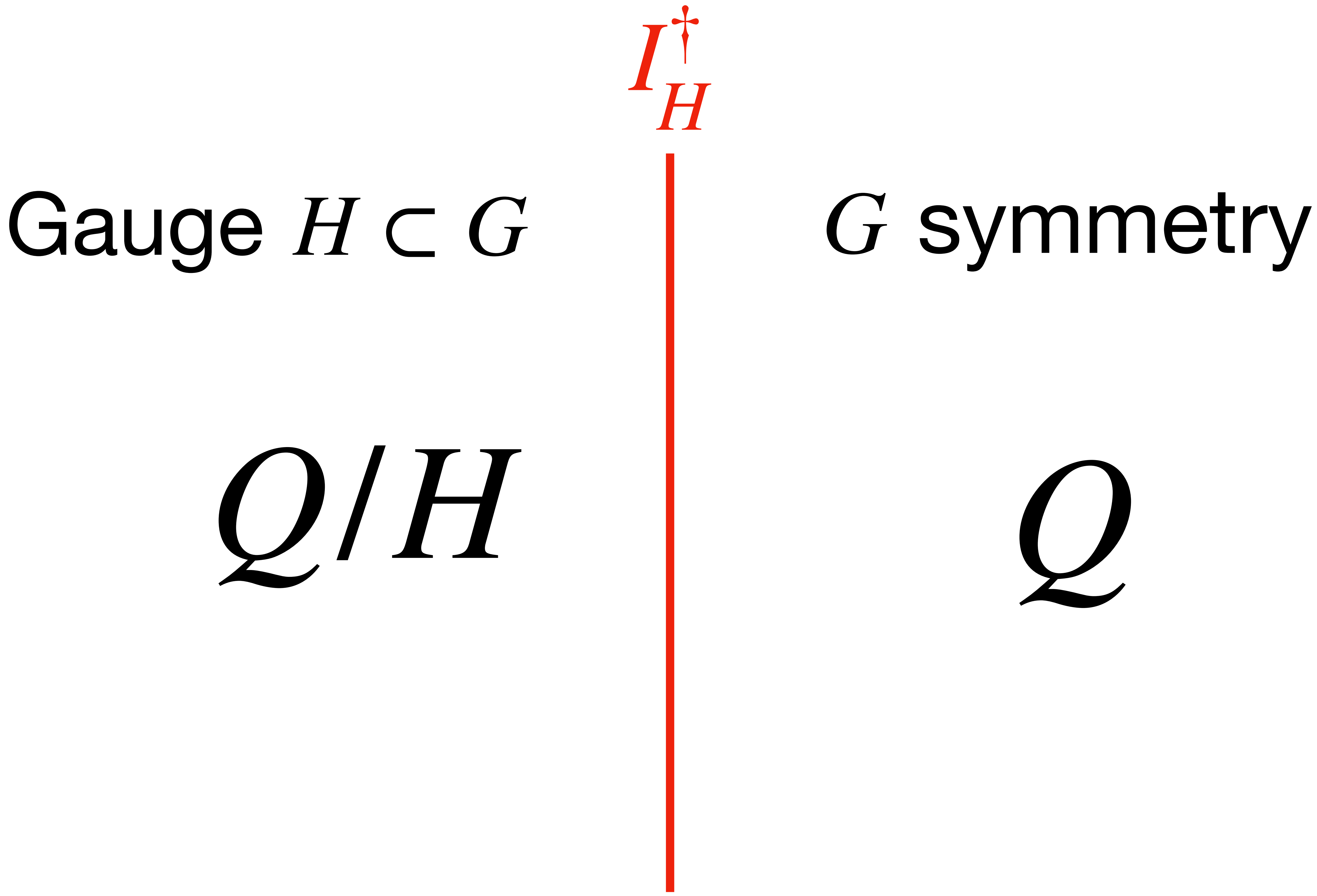
A useful tool, both conceptually and practically, will be the gauging interface:

I_H

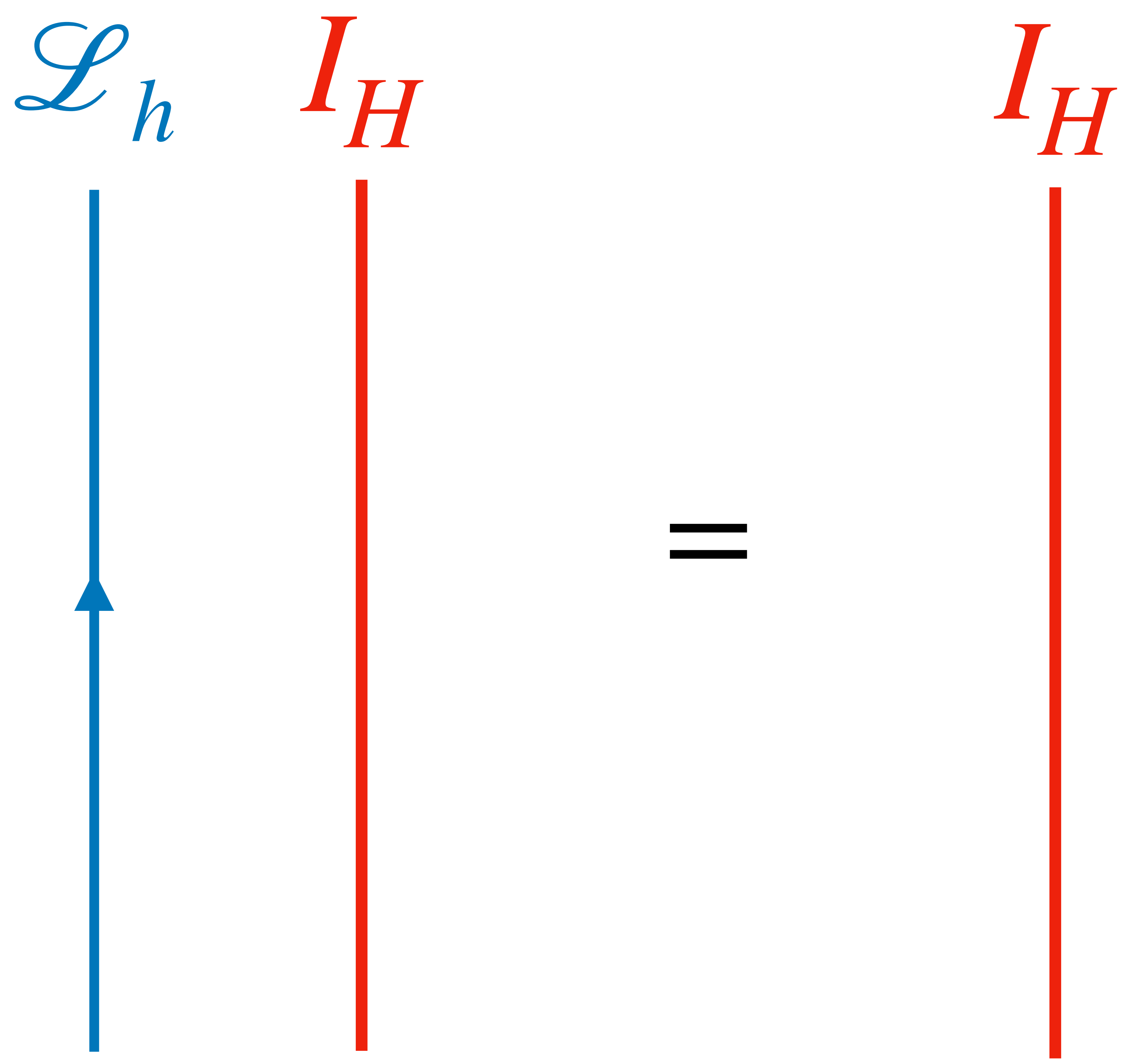
G symmetry | Gauge $H \subset G$

Q | Q/H

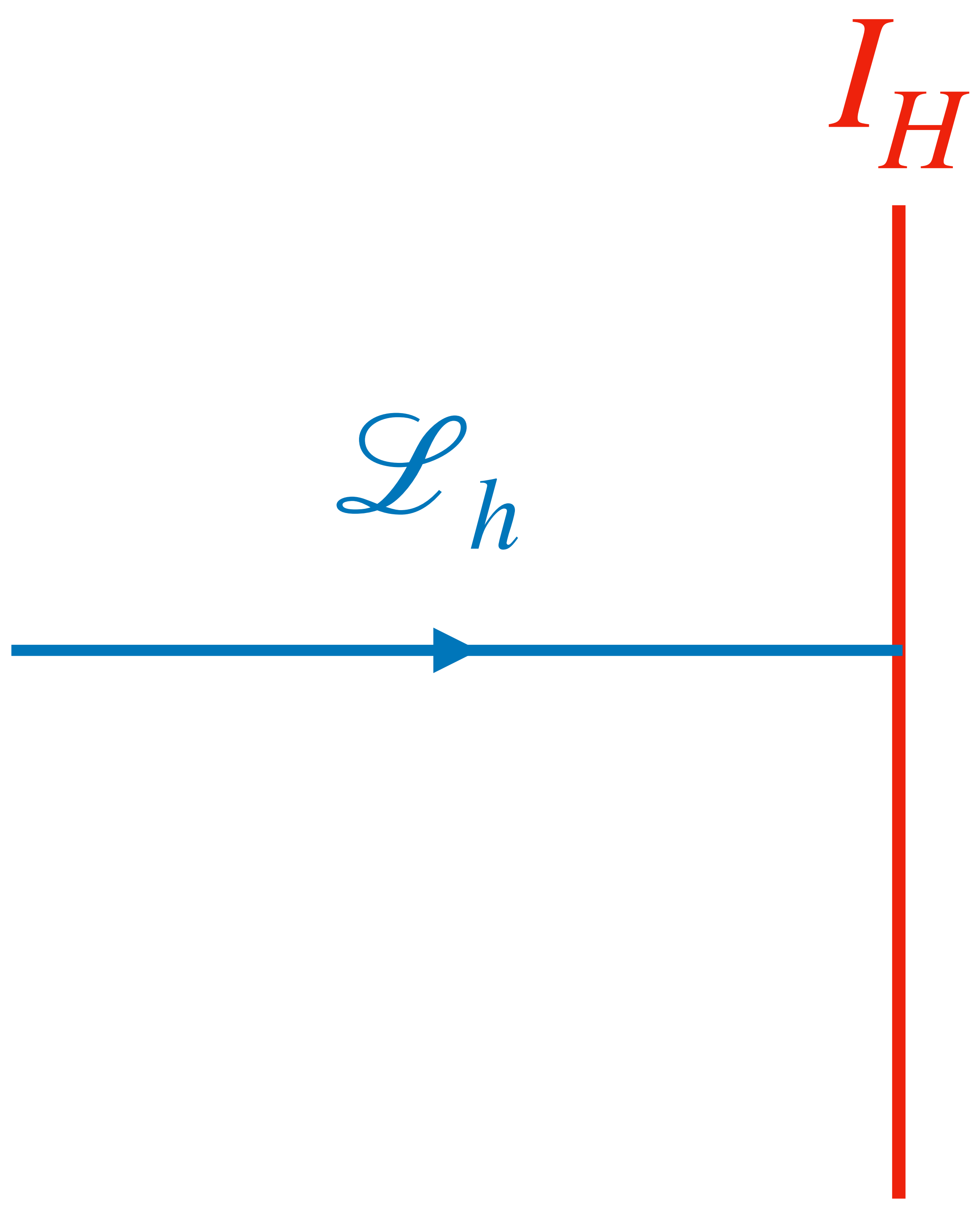
The interface in opposite orientation will be denoted by $I_H^\dagger$:



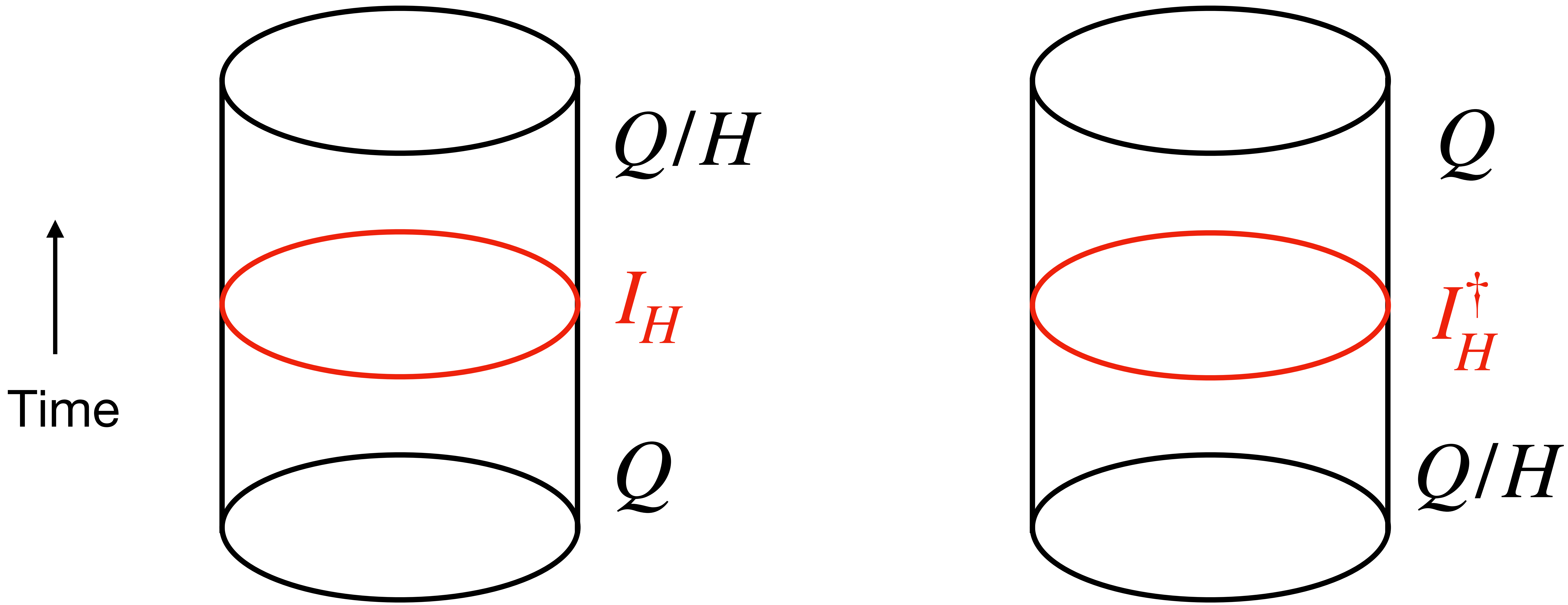
An important property is that the gauging interface absorbs topological lines that generate the H symmetry. For any $h \in H$,



and

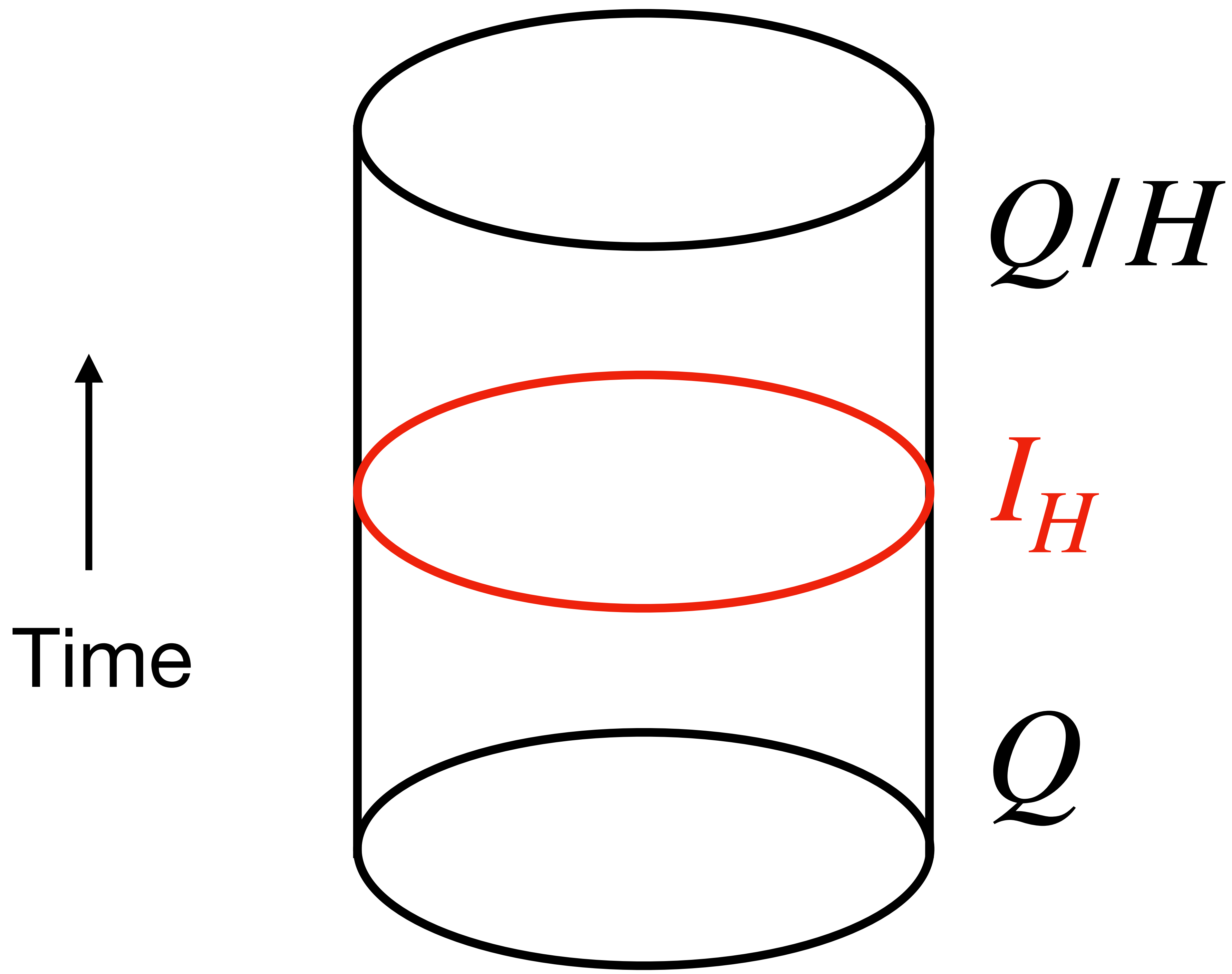


If we quantize the theory on a cylinder, the gauging interface defines a map between the Hilbert spaces of the two theory:



$I_H : \mathscr{H}_Q \to \mathscr{H}_{Q/H}$ $\qquad$ $I_H^\dagger : \mathscr{H}_{Q/H} \to \mathscr{H}_Q$

The map $I_H : \mathscr{H}_Q \to \mathscr{H}_{Q/H}$ projects out states that are charged under H.



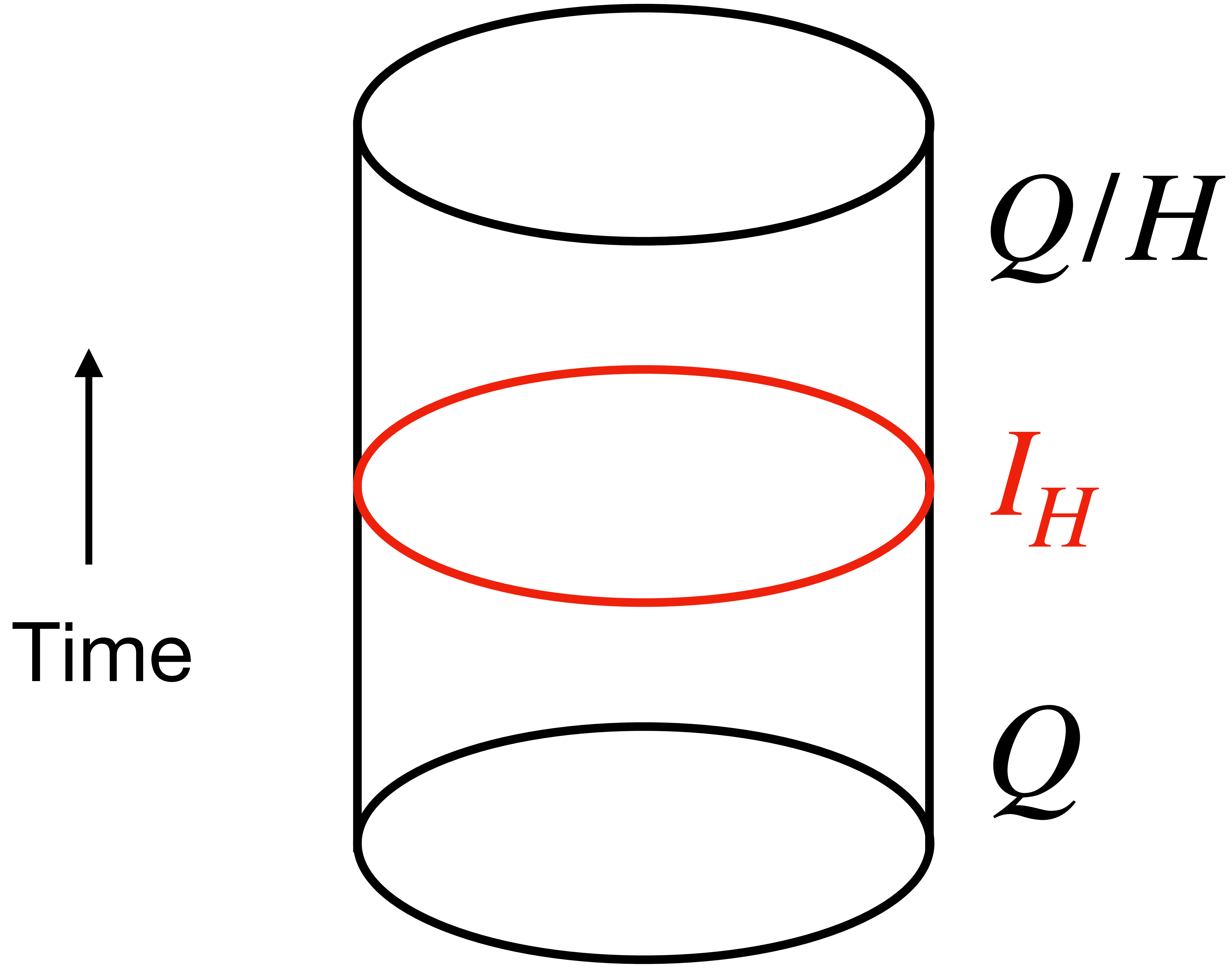
$I_H : \mathscr{H}_Q \to \mathscr{H}_{Q/H}$

$$I_H | \mathcal{O} \rangle_Q = 0$$

if

$| \mathcal{O} \rangle_Q$ is charged under H.

Invariant states are not affected:



$$I_H : \mathscr{H}_Q \to \mathscr{H}_{Q/H}$$

$$I_H |\mathcal{O}\rangle_Q = \sqrt{|H|}\, |\mathcal{O}\rangle_{Q/H}$$

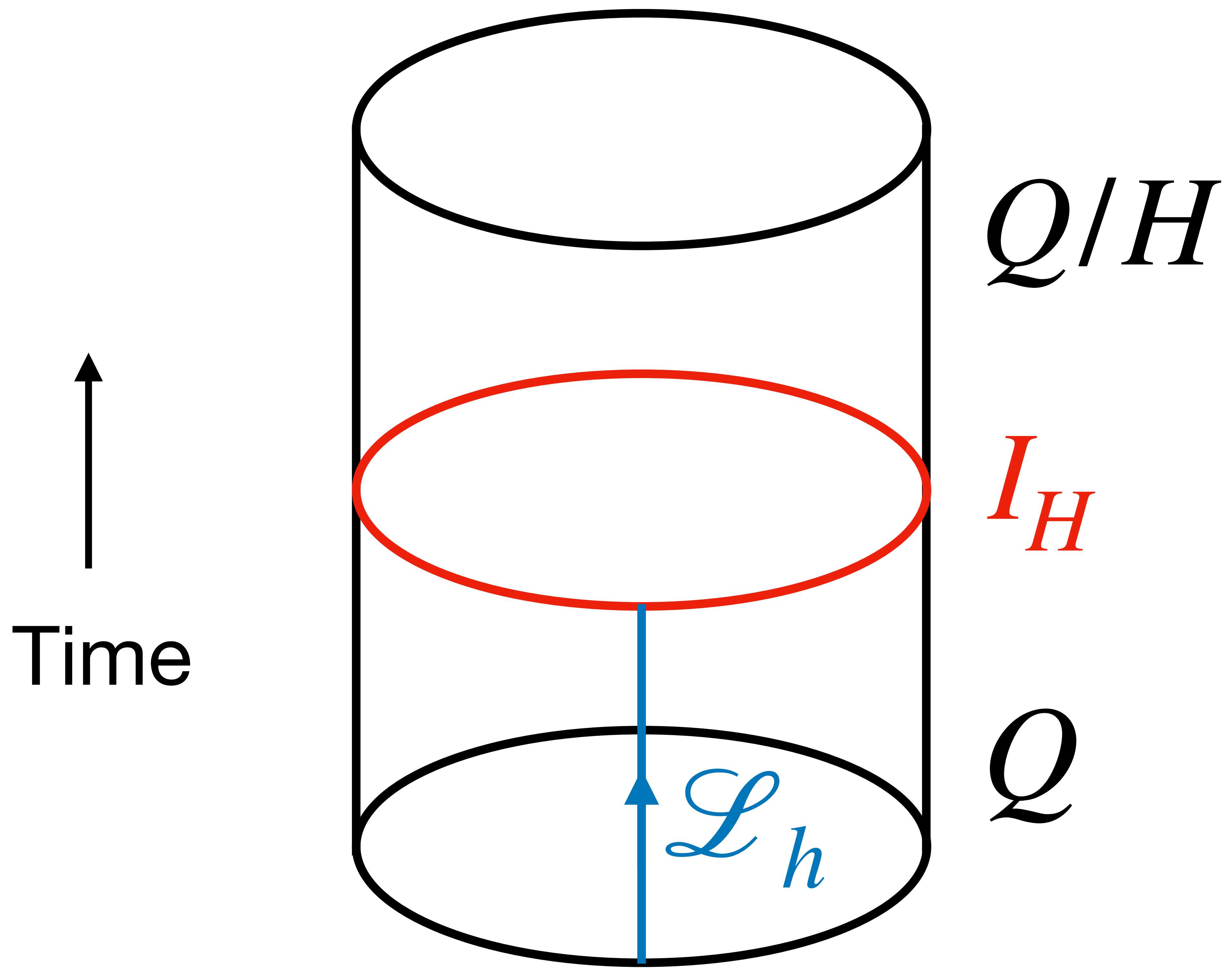
if

$|\mathcal{O}\rangle_Q$ is invariant under H.

The normalization $\sqrt{|H|}$ is chosen

such that $I_H^\dagger \times I_H = \sum_{h\in H} \mathscr{L}_h$.

More generally, we have a family of maps $I_H^h : \mathscr{H}_Q^h \to \mathscr{H}_{Q/H}$, $h \in H$:

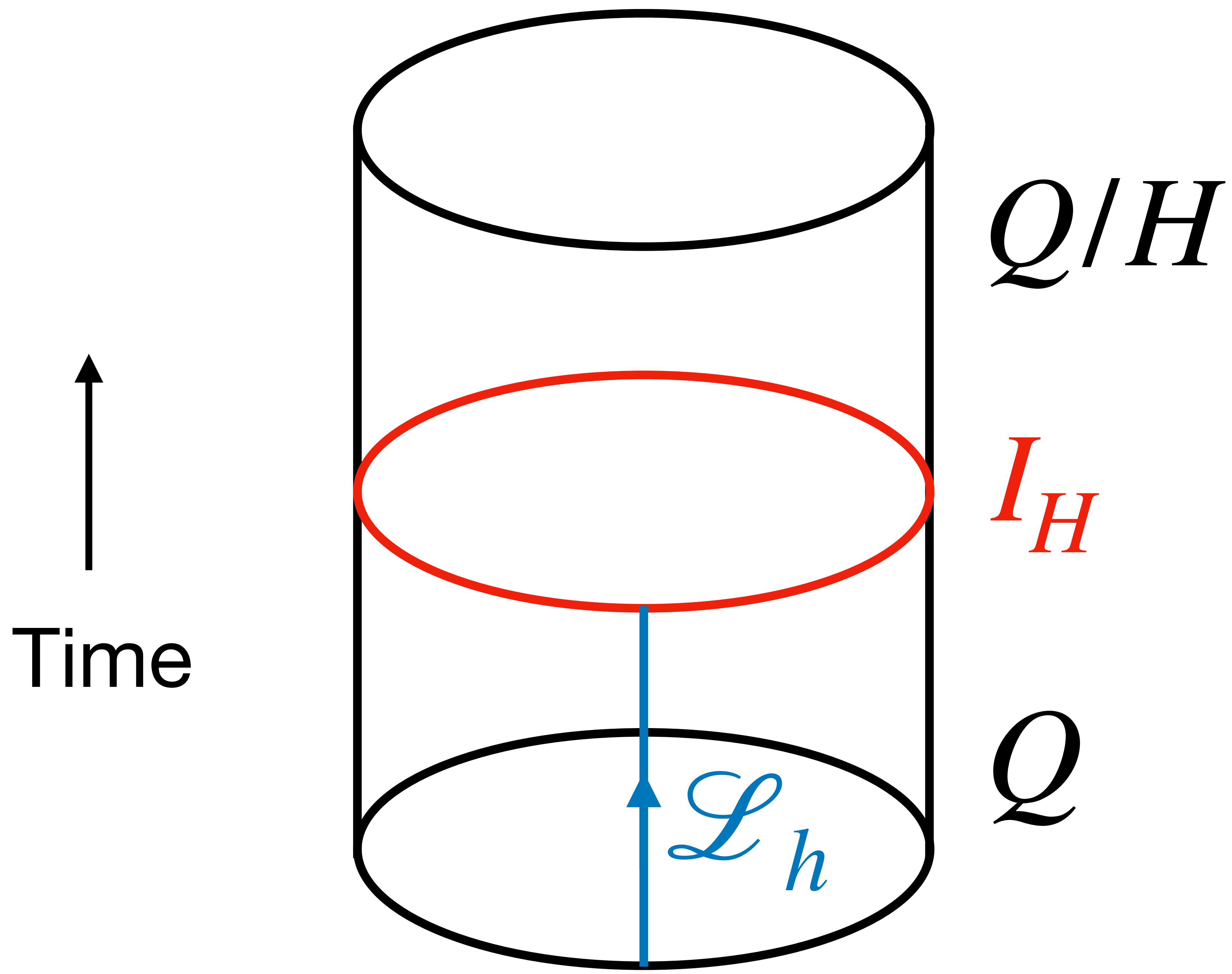


$I_H^h : \mathscr{H}_Q^h \to \mathscr{H}_{Q/H}$

$\mathscr{L}_h$, $h \in H$ are topological line defects generating the H symmetry of Q.

$\mathscr{H}_Q^h$ is a twisted Hilbert space obtained by quantizing the theory Q with a twisted boundary condition by $h \in H$.

The map $I_H^h : \mathscr{H}_Q^h \to \mathscr{H}_{Q/H}$ acts on states in a similar way as before:



$$I_H^h : \mathscr{H}_Q^h \to \mathscr{H}_{Q/H}$$

The twisted Hilbert space $\mathscr{H}_Q^h$ is acted by the centralizer subgroup $C_H(h) \subset H$.

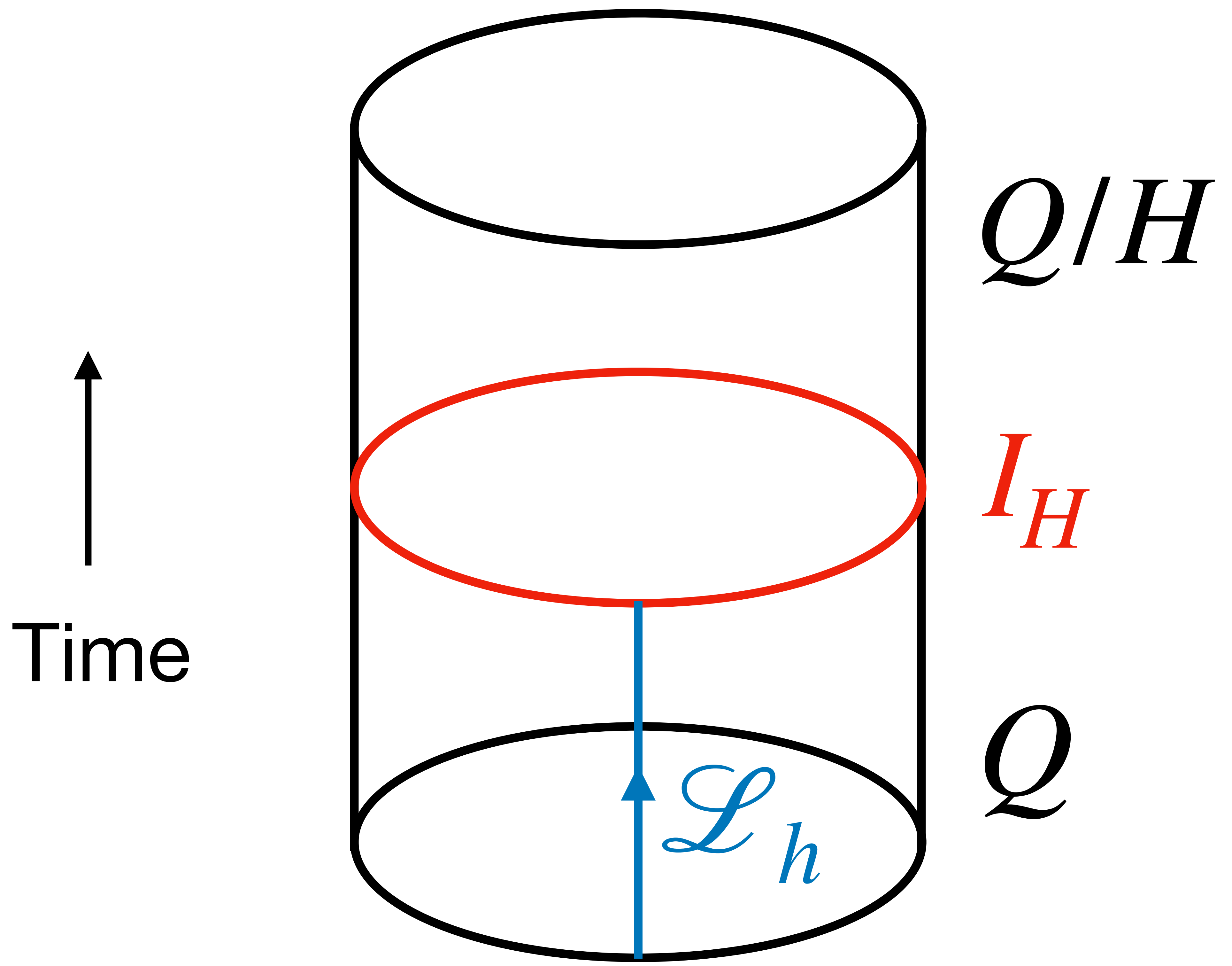
$$I_H^h | \mathcal{O}^h \rangle_Q = 0$$

if

$| \mathcal{O}^h \rangle_Q$ is charged under $C_H(h)$.

For simplicity, we assume no discrete torsion.

The map $I_H^h : \mathscr{H}_Q^h \to \mathscr{H}_{Q/H}$ acts on states in a similar way as before:



$I_H^h : \mathscr{H}_Q^h \to \mathscr{H}_{Q/H}$

The twisted Hilbert space $\mathscr{H}_Q^h$ is acted by the centralizer subgroup $C_H(h) \subset H$.

$$I_H^h |\mathcal{O}^h\rangle_Q = \sqrt{|C_H(h)|}\, |\mathcal{O}^{[h]}\rangle_{Q/H}$$

if

$|\mathcal{O}^h\rangle_Q$ is invariant under $C_H(h)$.

For simplicity, we assume no discrete torsion.

Gauging Interface

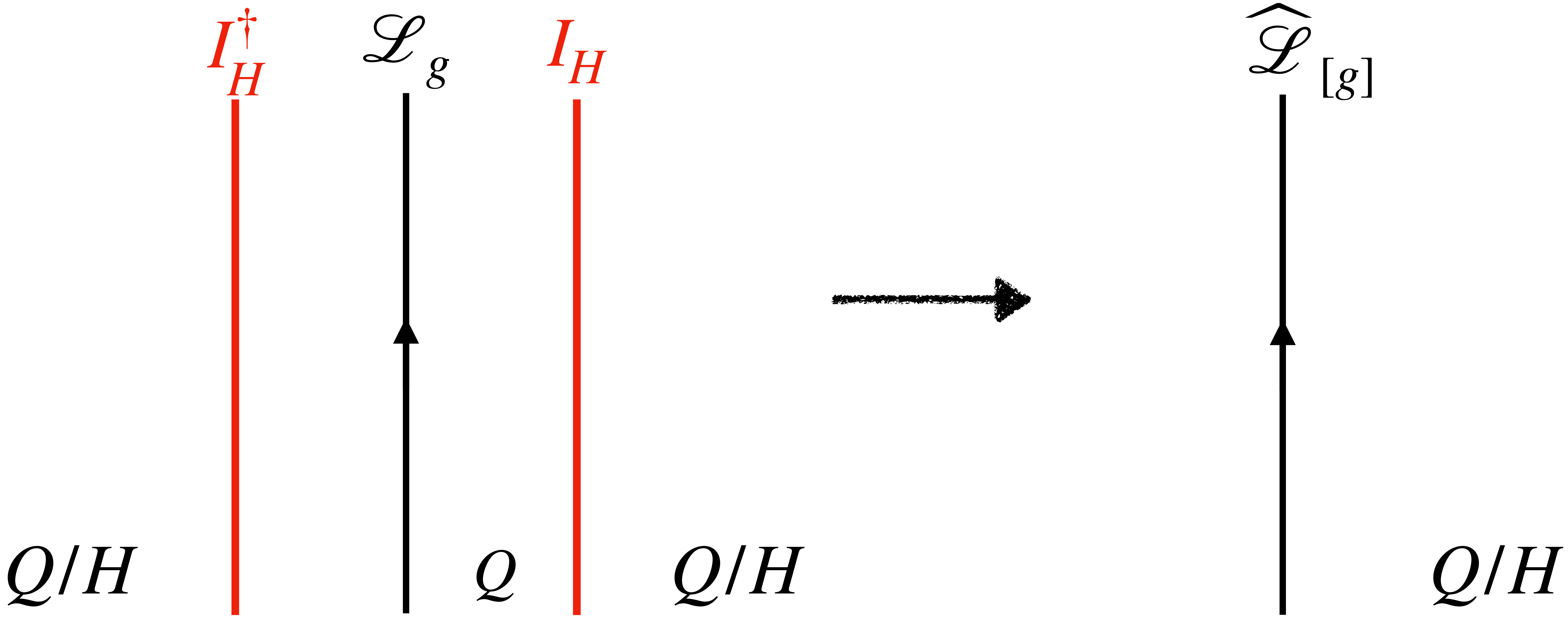
- The Hilbert space $\mathscr{H}_{Q/H}$ of the gauged theory is the image of the family of gauging interface maps:

$$\bigoplus_{h \in H} I_H^h : \bigoplus_{h \in H} \mathscr{H}_Q^h \to \mathscr{H}_{Q/H} \,.$$

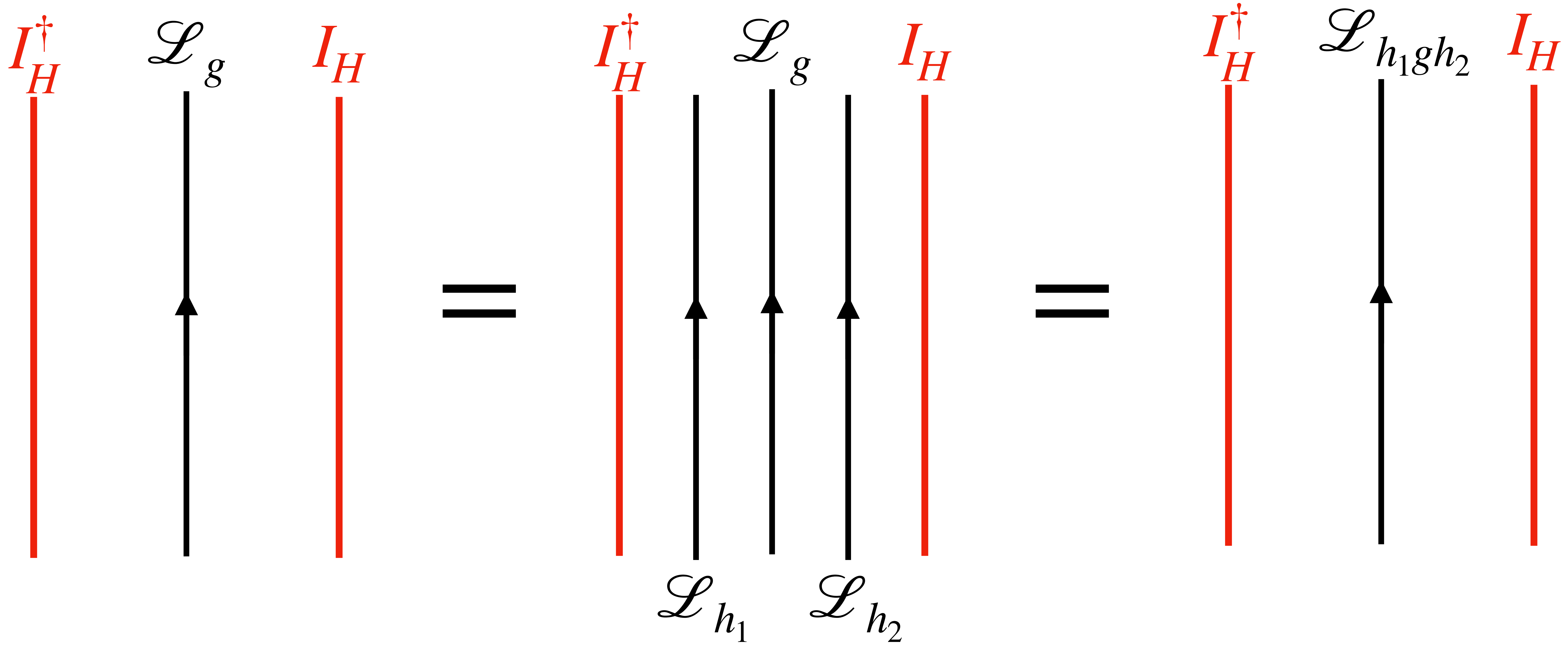
- We have rephrased the familiar two step procedure of the usual orbifold.
- (1) One extends the Hilbert space to include twisted sectors, and then (2) project down to gauge-invariant states.

- These gauging interfaces are useful in computing the (generalized) global symmetries of the gauged theory Q/H from those of the original theory Q.

- The "sandwich construction" [Hsin-Kobayashi-Zhang '24, '25]:



The topological line defect $\widehat{\mathscr{L}}_{[g]}$ of the theory Q/H depends on $g \in G$ only through the double coset $[g] \in H\backslash G/H$. For any $h_1, h_2 \in H$,



Ostrik's Theorem

- Abstractly, it is known that [Ostrik '02; Bhardwaj-Tachikawa '17], after gauging $H \subset G$, the gauged theory Q/H enjoys a dual symmetry described by a fusion category, whose simple objects are labeled by a pair $([g], \rho)$,

$$\widehat{\mathscr{L}}_{[g],\rho} \,.$$

- Here, $[g] \in H \backslash G / H$, and ρ is an irreducible representation of a group

$$H^g \equiv H \cap gHg^{-1} \subset G \,.$$

Ostrik's Theorem

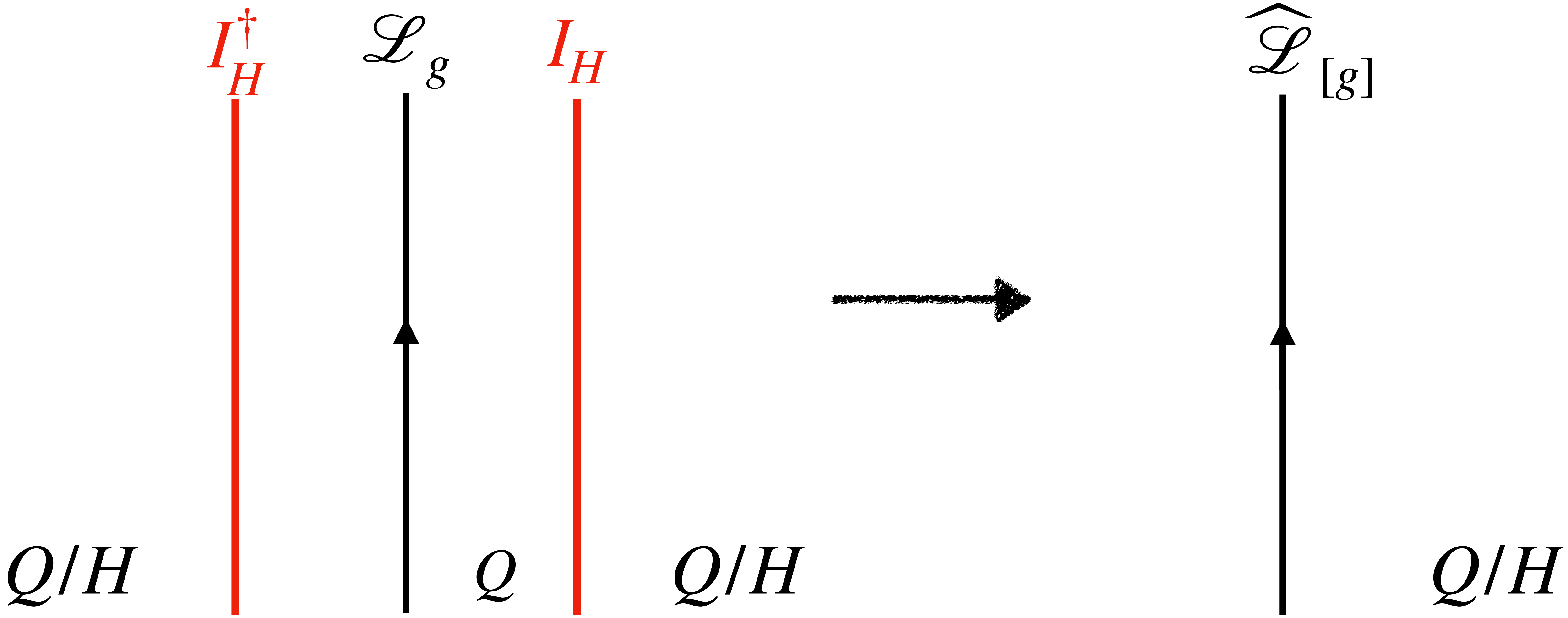
$$\widehat{\mathscr{L}}_{[g],\rho}\,, \quad [g] \in H\backslash G/H\,, \quad \rho \in \mathrm{Rep}(H \cap gHg^{-1})\,.$$

- The physical origin of the double coset is clear from the sandwich picture.

- Where did the group $H^g \equiv H \cap gHg^{-1}$ and the representation ρ come from?

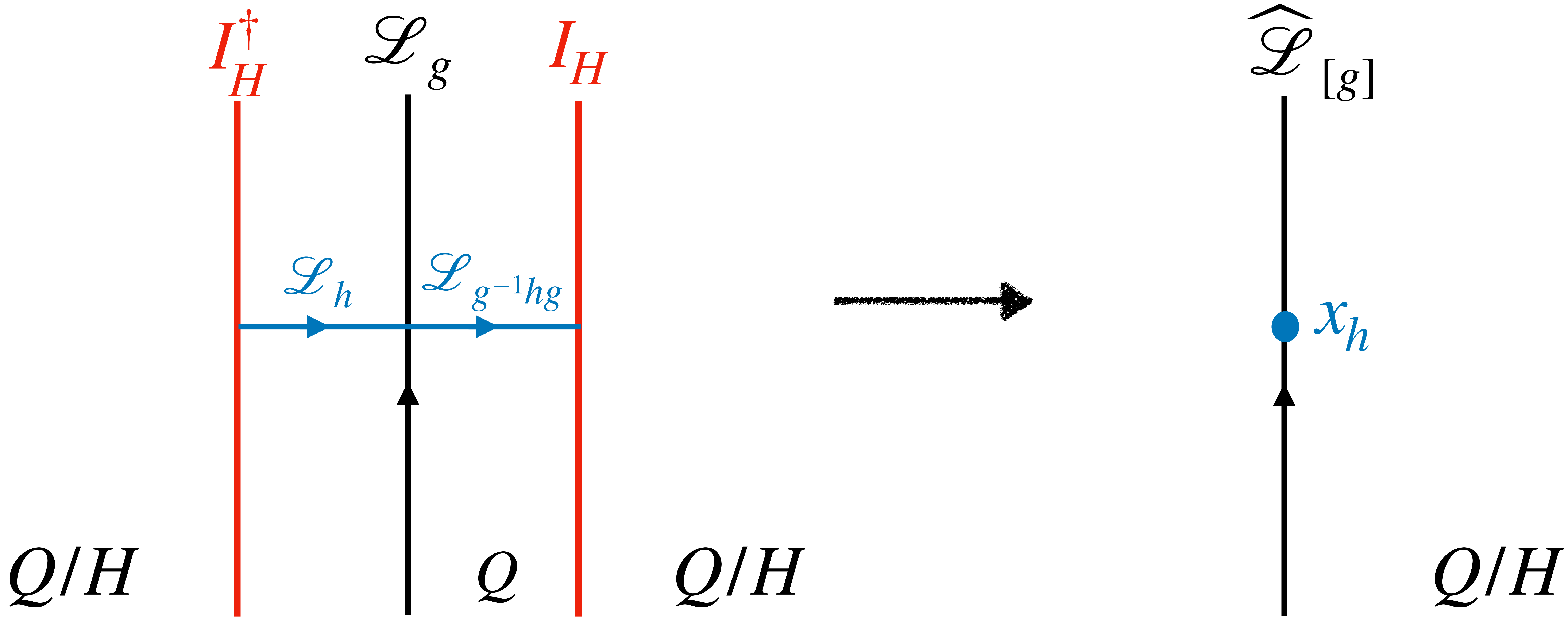
Improving the Sandwiches

- The issue is that, in general, the line $\widehat{\mathscr{L}}_{[g]}$ is not a simple line, and may decompose into direct sum of several simple topological line defects.

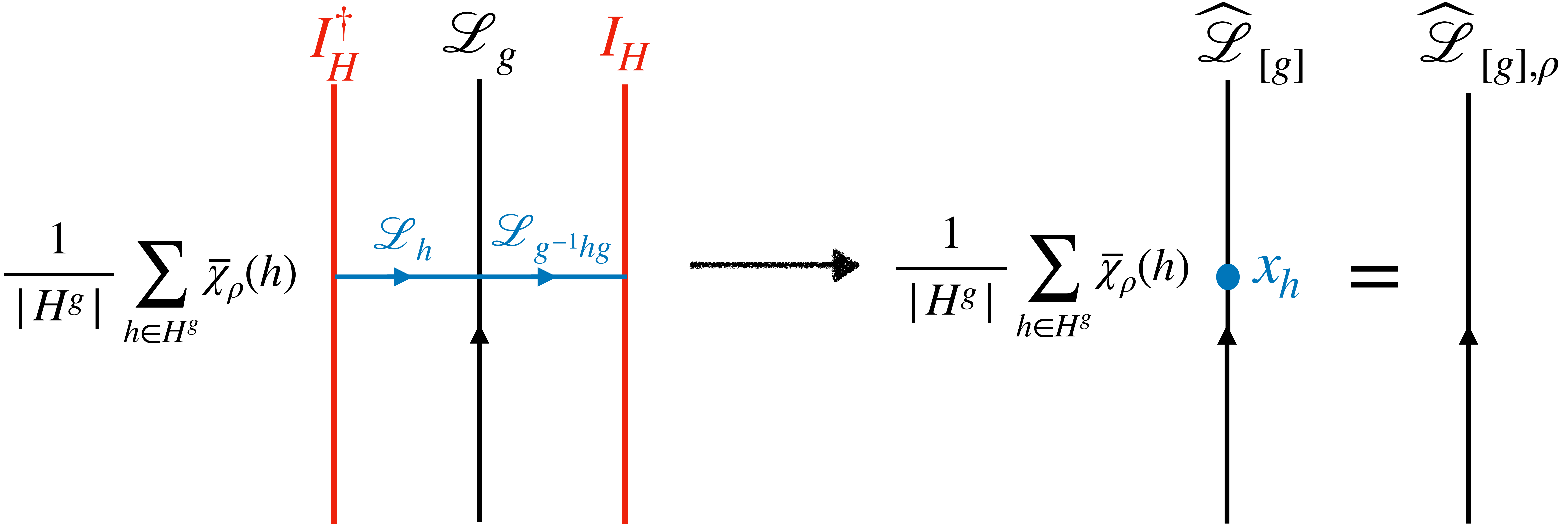


- A topological line defect is simple iff there is no nontrivial topological point operator on the line defect.

- However, for each $h \in H^g \equiv H \cap gHg^{-1}$, we have



Motivated by the Ostrik's theorem, we propose that each simple summand inside $\widehat{\mathscr{L}}_{[g]}$ is obtained by performing a projection:

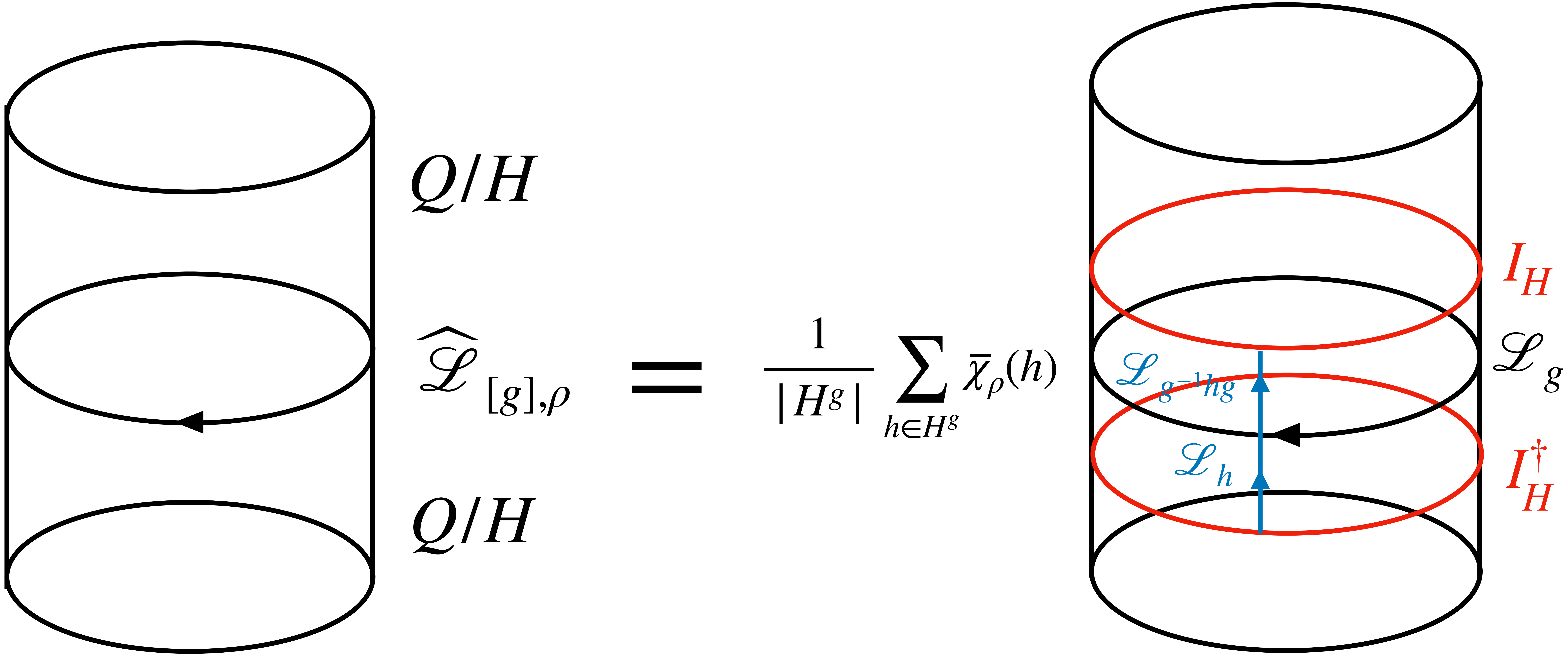


In particular, one can show $\widehat{\mathscr{L}}_{[g]} = \sum_\rho \dim(\rho)\widehat{\mathscr{L}}_{[g],\rho}$.

Dual Symmetries

- We have provided a pictorial understanding of Ostrik's theorem.

- Moreover, they are practically useful in computing the action of the topological line defects $\widehat{\mathscr{L}}_{[g],\rho}$ on the Hilbert space $\mathscr{H}_{Q/H}$ of the gauged theory.



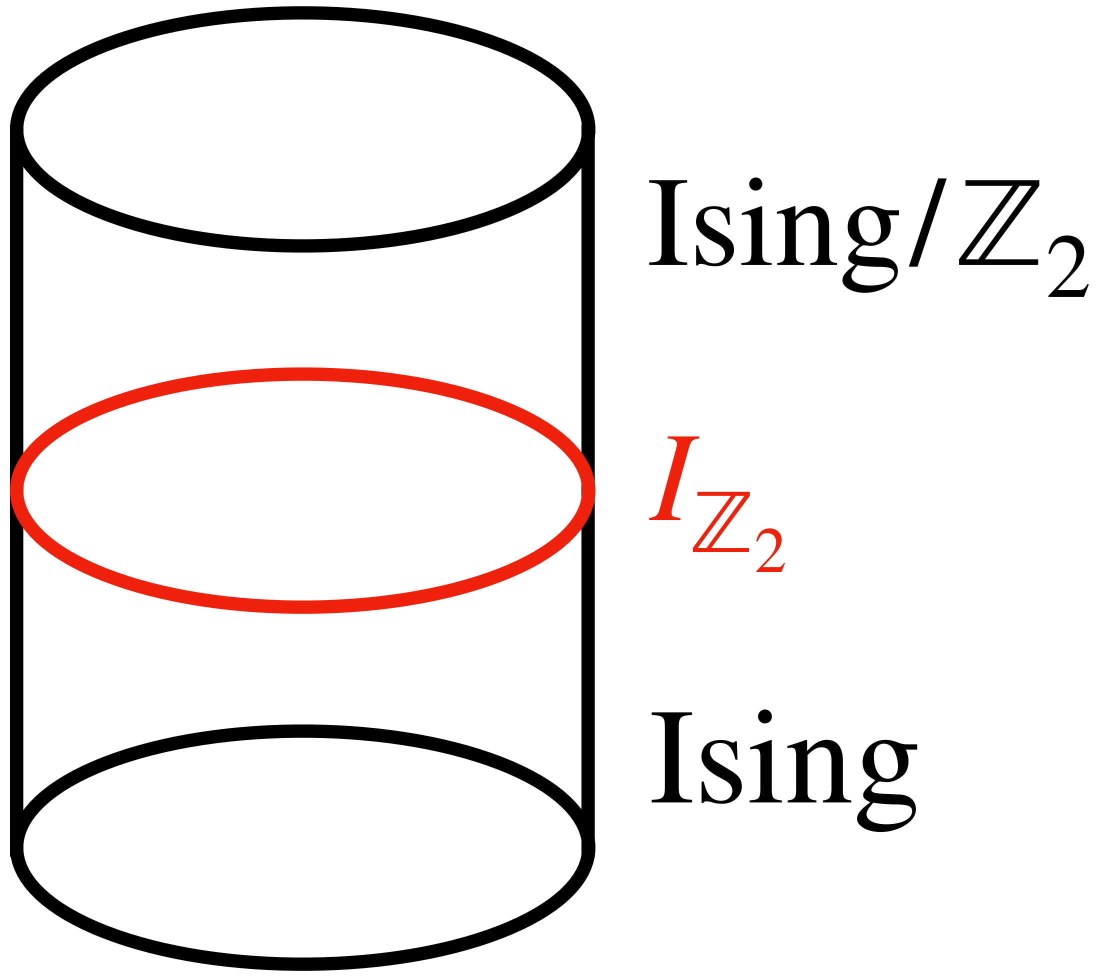
$$\boxed{\widehat{\mathscr{L}}_{[g],\rho}\,|\,\mathscr{O}\rangle_{Q/H} = \frac{1}{|H^g|}\sum_{h\in H^g}\bar{\chi}_\rho(h)\, I_H^h \circ \mathscr{L}_g^h \circ (I_H^h)^\dagger\,|\,\mathscr{O}\rangle_{Q/H}\,.}$$

Warm-up: Gauging $\mathbb{Z}_2$

- Let's test in a simple example: Ising and $\text{Ising}/\mathbb{Z}_2$.

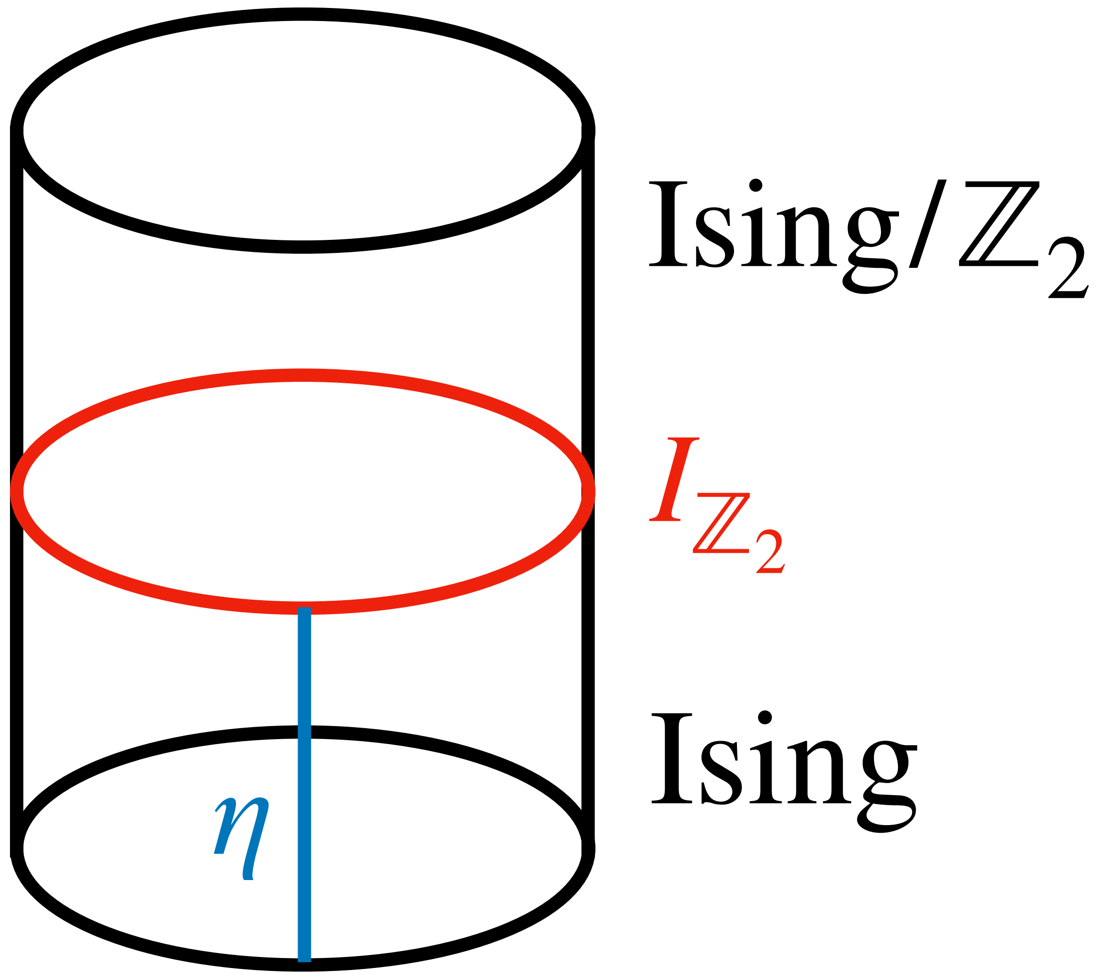
- Ising has a global symmetry $\mathbb{Z}_2 = \{1, \eta\}$, and $\text{Ising}/\mathbb{Z}_2$ has a dual symmetry $\mathbb{Z}_2 = \{1, \hat{\eta}\}$.

- Using the (improved) sandwich picture, let's reproduce the action of the dual $\mathbb{Z}_2$ symmetry in $\text{Ising}/\mathbb{Z}_2$.



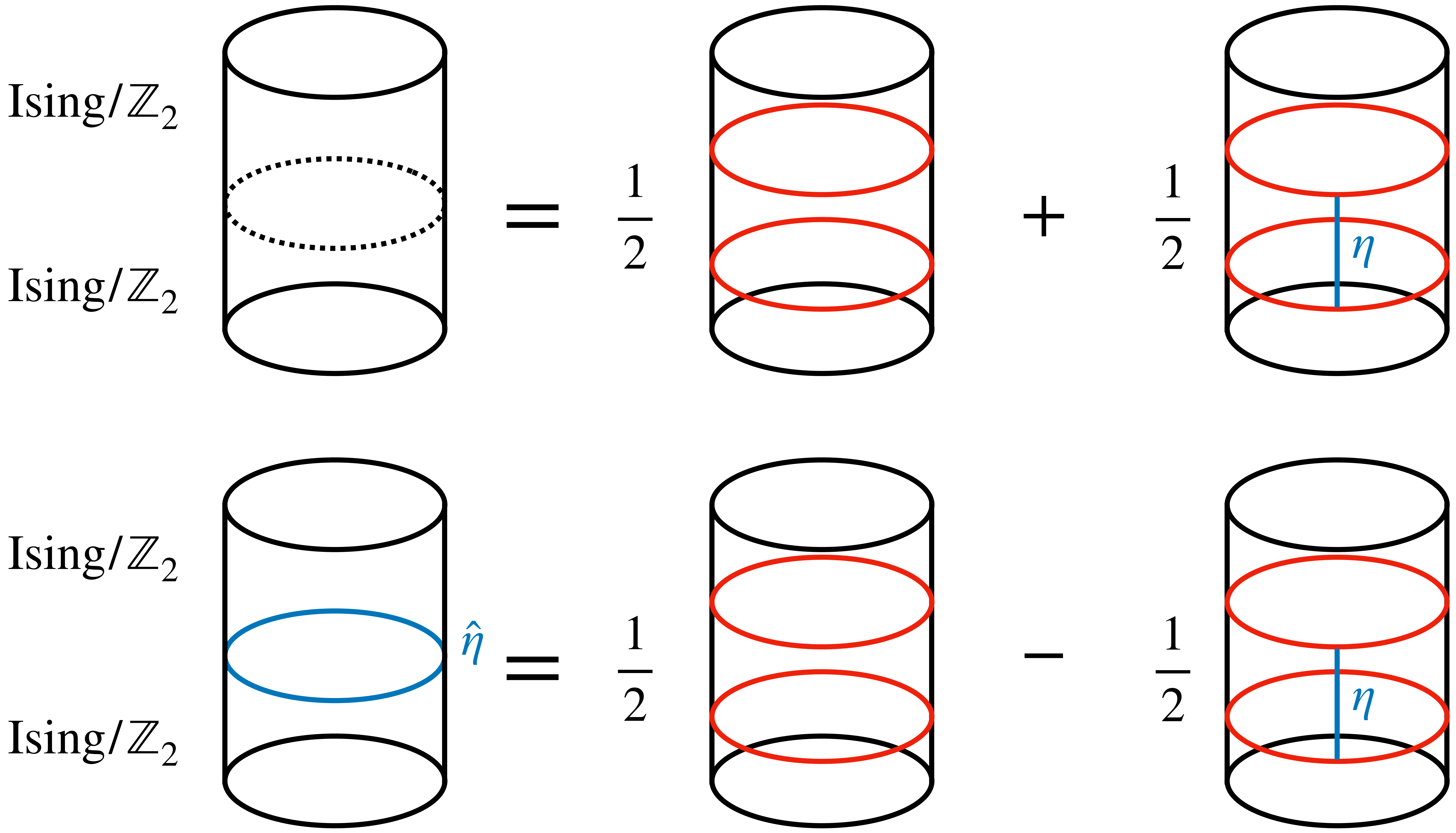
$$I_{\mathbb{Z}_2}|1\rangle_{\text{Ising}} = \sqrt{2}\,|1\rangle_{\text{Ising}/\mathbb{Z}_2}$$
$$I_{\mathbb{Z}_2}|\epsilon\rangle_{\text{Ising}} = \sqrt{2}\,|\epsilon\rangle_{\text{Ising}/\mathbb{Z}_2}$$
$$I_{\mathbb{Z}_2}|\sigma\rangle_{\text{Ising}} = 0$$

Ising	$\mathbb{Z}_2$ even	$\mathbb{Z}_2$ odd
$\mathscr{H}_{\text{Ising}}$	$\|1\rangle_{\text{Ising}}\,,\|\epsilon\rangle_{\text{Ising}}$	$\|\sigma\rangle_{\text{Ising}}$
$\mathscr{H}^{\eta}_{\text{Ising}}$	$\|\mu\rangle_{\text{Ising}}$	$\|\psi\rangle_{\text{Ising}}\,,\|\bar{\psi}\rangle_{\text{Ising}}$



$$I_{\mathbb{Z}_2}|\mu\rangle_{\text{Ising}} = \sqrt{2}\,|\mu\rangle_{\text{Ising}/\mathbb{Z}_2}$$
$$I_{\mathbb{Z}_2}|\psi\rangle_{\text{Ising}} = 0$$
$$I_{\mathbb{Z}_2}|\bar{\psi}\rangle_{\text{Ising}} = 0$$

Ising/$\mathbb{Z}_2$	$\mathbb{Z}_2$ even	$\mathbb{Z}_2$ odd
$\mathscr{H}_{\text{Ising}/\mathbb{Z}_2}$	$\|1\rangle\,,\|\epsilon\rangle$	$\|\mu\rangle_{\text{Ising}/\mathbb{Z}_2}$
$\mathscr{H}^{\hat{\eta}}_{\text{Ising}/\mathbb{Z}_2}$	$\|\sigma\rangle_{\text{Ising}/\mathbb{Z}_2}$	$\|\psi\rangle\,,\|\bar{\psi}\rangle$



One can explicitly check that, for instance, $\hat{\eta}\,|\,\mu\rangle_{\mathrm{Ising}/\mathbb{Z}_2} = -\,|\,\mu\rangle_{\mathrm{Ising}/\mathbb{Z}_2}$

Conformal Boundaries

- A similar method allows us to obtain conformal boundary conditions of the gauged theory Q/H from those of the original theory Q.

- In short, a simple boundary condition B of Q splits into a family of simple boundary conditions $\widehat{B_\rho}$ where ρ is an irreducible representation of the subgroup of H which is preserved by B. [This is well-known in the literature.]

- Similar to before, using the gauging interface, one can explicitly compute the corresponding Cardy boundary states.

Flat Gauging

Flat Gauging

- We would like to ask a similar question in the case of flat gauging.
- Starting from a seed theory Q with a continuous global symmetry G, what is the dual symmetry we get after flat-gauging $H \subset G$ in Q/H?
- We will be cavalier and blindly extrapolate what we know from discrete gauging.
- We find that this works surprisingly well, and reproduce many known results.
- We achieve two things: (1) Gain confidence that flat-gauging is indeed a well-defined operation, and (2) turn it into a powerful machine to generate irrational, non-compact CFTs whose symmetries (and boundaries) we can understand explicitly.

Two Examples

$$SU(2)_1 \xrightarrow{\text{Flat-Gauge } U(1)} \text{Non-compact Free Boson}$$

$$SU(2)_1 \xrightarrow{\text{Flat-Gauge } SO(3)} \text{“Runkel-Watts” Theory}$$

See also […; Gaberdiel-Suchanek ’11;…; Dubovsky-Negro-Porrati ’23; Jia-Luo-Tian-Wang-Zhang ’25; Jia-Zhang ’26; …]

Why $SU(2)_1$?

- $SU(2)_1$ Wess-Zumino-Witten model is one of rare examples (other than Virasoro minimal models) where we are quite confident that we know all the global symmetries and (unitary) conformal boundaries of the theory. [Gaberdiel-Recknagel-Watts '01; Fuchs-Gaberdiel-Runkel-Schweigert '07]

- In particular, the theory only has an ordinary global symmetry

$$G = \frac{SU(2)_L \times SU(2)_R}{\mathbb{Z}_2} \cong SO(4)\,.$$

- By flat-gauging (non-anomalous) subgroups of $SO(4)$, we generate non-compact theories whose symmetries and boundaries we can again fully classify.

Example 1: Runkel-Watts Theory

- What is Runkel-Watts theory? [Runkel-Watts '01]
- The following 3 different presentations are very likely to be equivalent (modulo some subtleties associated with degenerate representations).

  1. $c \to 1^-$ limit of unitary Virasoro minimal models [Runkel-Watts '01]
  2. Flat $SO(3)$ gauging of $SU(2)_1$ [Gaberdiel-Suchanek '11]
  3. $c \to 1^+$ limit of Liouville CFT [Schomerus '03; Ribault-Santachiara '15]

- Presentation 1 is the most difficult to understand.
- We will compute various things using Presentation 2, and compare with 3. We reproduce known answers from 3. This provides further evidence between the equivalence $2 \Leftrightarrow 3$.

Interlude - Liouville CFT

- The Liouville conformal field theory is usually defined for arbitrary values of central charge $c \in \mathbb{C} \backslash (-\infty, 1]$.
- It is a prototypical example of a nontrivial, solvable, irrational CFT.
- The spectrum consists of a continuum of primary states $|P\rangle$, $P \in \mathbb{R}$, $P \sim -P$, with conformal weights $h = \bar{h} = \dfrac{c-1}{24} + P^2$.
- The exact 3-point structure constant $C(P_1, P_2, P_3)$ is known [Dorn-Otto; Zamolodchikov^2] and for $c \in \mathbb{C} \backslash (-\infty, 1]$, it is a meromorphic function of the momenta P's.

Interlude - Liouville CFT

- As $c \to 1^+$, we can make a few observations.
- $h = \bar{h} = \dfrac{c-1}{24} + P^2 \ \to P^2 \in \mathbb{R}_{\geq 0}$.
- The spectrum is diagonal, and every $c = 1$ Virasoro representation appears in the spectrum (modulo subtleties with degenerate representations).
- $C(P_1, P_2, P_3)$ becomes the RW structure constant (which is non-analytic in P's). [Schomerus '03; Ribault-Santachiara '15]

Interlude - Liouville CFT

- Topological line defects of the Liouville CFT were classified in [Drukker-Gaiotto-Gomis ’10].

- They come in two families.

- “FZZT” [Fateev-Zamolodchikov^2; Techner] topological lines: In 1-to-1 correspondence with generic irreducible representations of Virasoro algebra at central charge c.

- “ZZ” [Zamolodchikov^2] topological lines: In 1-to-1 correspondence with degenerate irreducible representations of Virasoro algebra at central charge c.

- Their action on the states of the Liouville CFT is explicitly known.

Example 1: Runkel-Watts Theory

- Extrapolating our knowledge from discrete gauging, the expectation is that, once we flat-gauge $SO(3)$ symmetry of $SU(2)_1$, the dual symmetry operators are given by

$$\widehat{\mathscr{L}}_{[g],\rho}\,, \quad [g] = SO(3)\backslash SO(4)/SO(3)\,, \quad \rho \in \mathrm{Rep}(SO(3) \cap gSO(3)g^{-1})$$

- By computing this explicitly, we recover the spectrum of all the FZZT and ZZ topological line defects!

- For instance, the subset of ZZ lines corresponding to $h = j^2, j \in \mathbb{Z}_{\geq 0}$ degenerate $c = 1$ Virasoro representations, physically correspond to the $SO(3)$ Wilson lines, and satisfy the $\mathrm{Rep}(SO(3))$ fusion rule.

Example 2: Non-Compact Scalar

$$\mathscr{L} = \partial_\mu \Phi \partial^\mu \Phi \,.$$

- We reproduce all the known conformal boundary conditions by flat gauging.
  [Callan-Klebanov ’94; Polchinski-Thorlacius ’94; Callan-Klebanov-Ludwig-Maldacena ’94]

- We find that the full set of simple topological line defects, describing all the generalized global symmetries of the 1+1D free scalar field, is given by

$$[\mathring{D}^2 \cup \mathbb{R}] \cup [S^3 \times (0,1)] \cup [\mathring{D}^2 \cup \mathbb{R}] \,.$$

Symmetries of Free Scalar

Invertible $\mathbb{R} \rtimes \mathbb{Z}_2$

$$\Phi \to \Phi + \alpha$$
$$\Phi \to -\Phi$$

$$[\mathring{D}^2 \cup \mathbb{R}] \cup [S^3 \times (0,1)] \cup [\mathring{D}^2 \cup \mathbb{R}]\,.$$

- Other than the obvious invertible symmetry $\mathbb{R} \rtimes \mathbb{Z}_2$, we find a continuous family of non-invertible topological line defects that were not known before.
- Their fusion, action on the Hilbert space, and defect Hilbert spaces are all explicitly computable.

Continuous Tambara-Yamagami

$$[\overset{\mathscr{D}}{\mathring{D}^2} \cup \overset{\mathscr{L}_\alpha}{\mathbb{R}}] \cup [S^3 \times (0,1)] \cup [\mathring{D}^2 \cup \mathbb{R}]\,.$$

- An interesting "subcategory" is generated by the $\mathbb{R}$ shift symmetry lines $\mathscr{L}_\alpha$, together with a single non-invertible line $\mathscr{D}$ coming from at the center of one the two disks $\mathring{D}^2$.

$$\mathscr{D} \times \mathscr{D} = \int_{\mathbb{R}} d\mu(\alpha)\, \mathscr{L}_\alpha\,.$$

- We thus found an "$\mathbb{R}$ Tambara-Yamagami symmetry" in free non-compact boson. […; Mar´inSalvador '26]

Conclusion

- We generalized the (improved) sandwich construction to flat-gauging, and proposed a way to explicitly compute dual symmetries (and boundaries).

- We reproduced various known results in Runkel-Watts theory and non-compact boson theory, giving us more confidence that flat-gauging is a well-defined operation in 1+1D CFTs.

- We proposed a full classification of all the generalized global symmetries of non-compact free boson. We find the continuous $\mathbb{R}$ Tambara-Yamagami symmetry as a subcategory.

Outlook

- Once we are confident enough about flat-gauging, both physically and mathematically, we can turn it into a very useful machinery.

- It appears that there maybe is a zoo of under-explored irrational, non-compact CFTs that we can generate from better-understood compact CFTs!

Thank you!