Developing fast and accurate CMB lensing analysis

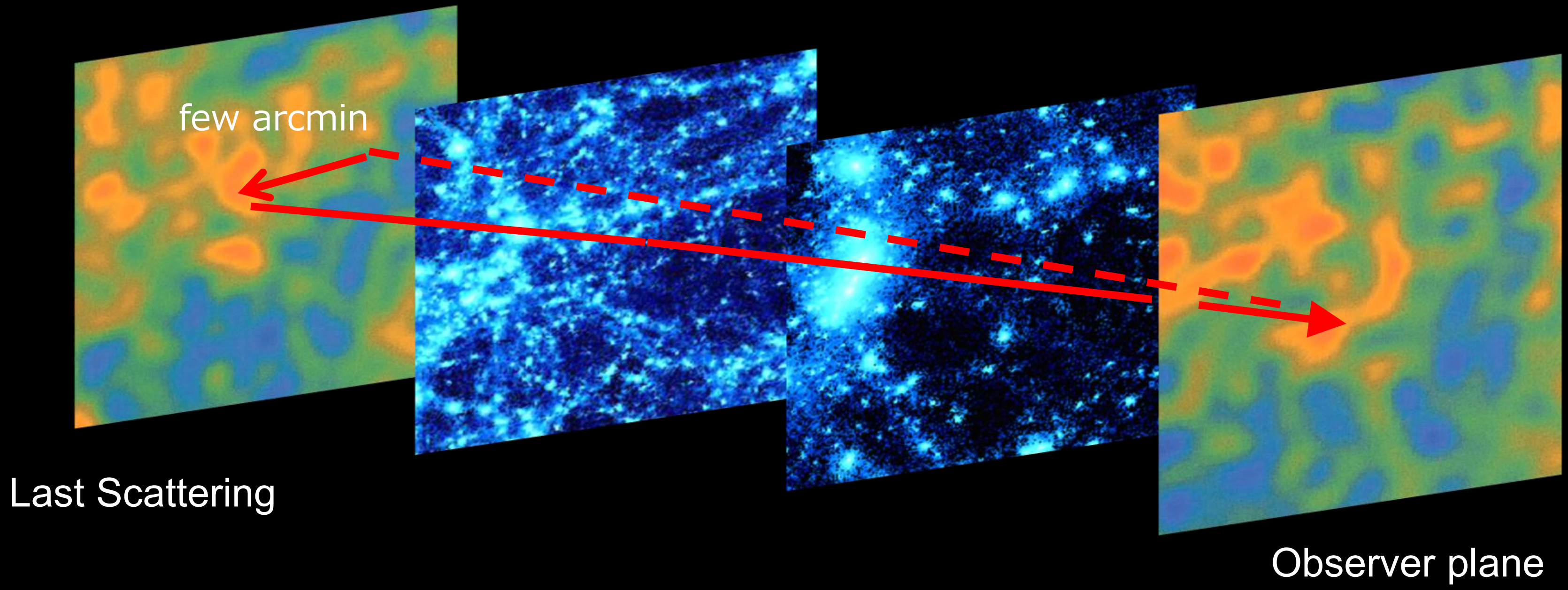
Toshiya Namikawa

KAVLI IPMU
東京大学 国際高等研究所 カブリ数物連携宇宙研究機構
KAVLI INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

What is the physical mechanism of the origin of the Universe? - challenges over transcending scales through the integration of theory, observations, and experiments, 8 July 2026

What is CMB lensing

- Path of CMB photons are deflected by the gravitational potential of the large-scale structure (gravitational lensing)



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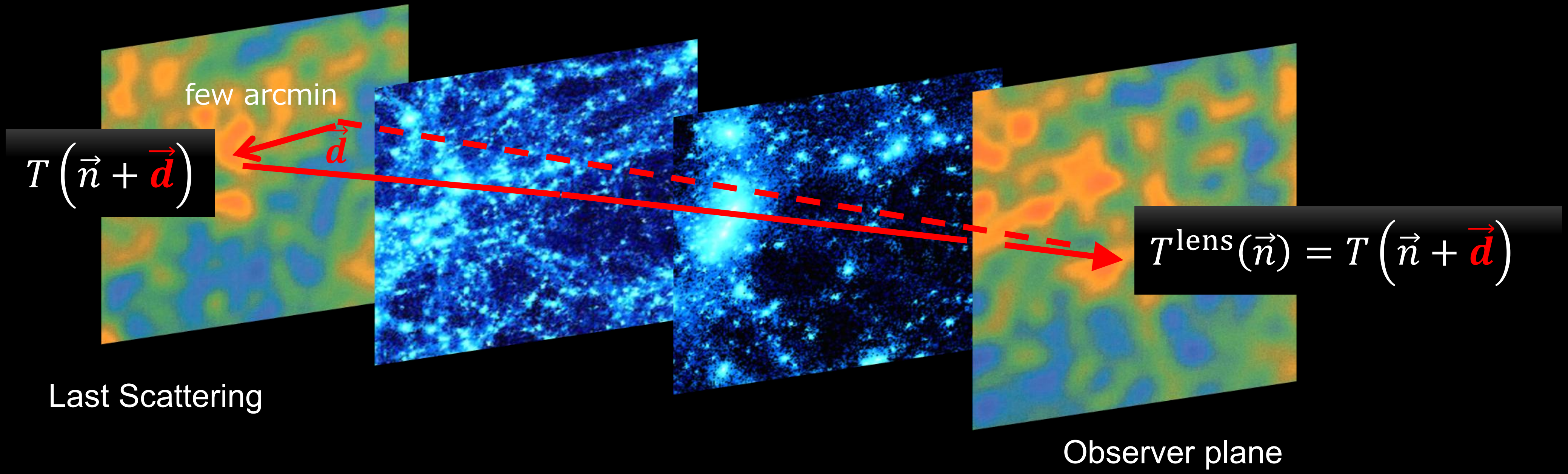


The lensing effect on CMB anisotropies = remapping the CMB anisotropies by $\vec{d}$

Why does CMB lensing important in cosmology

- We are interested in the lensing potential: $\nabla\phi = \vec{d}$

$$\phi(\vec{n}) \sim \int_0^{\chi_s} d\chi \frac{\chi_s - \chi}{\chi\chi_s} \Psi(\chi\vec{n}, \eta_0 - \chi)$$

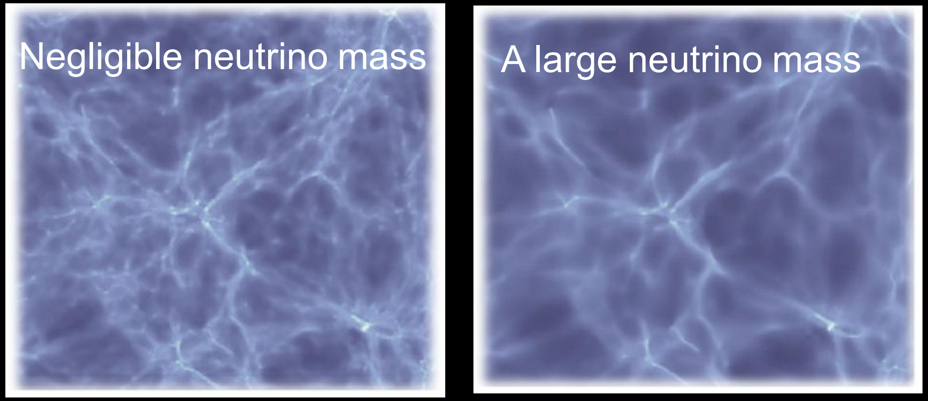
Gravitational potential

Sometimes we use convergence:

$$\kappa = -\frac{1}{2}\nabla^2\phi \sim \delta_m$$

- We can probe neutrino mass, dark energy, dark matter, etc, by measuring ϕ

✓ Small-scale structure with/without mass of neutrinos



Images: Viel et al. (2013)

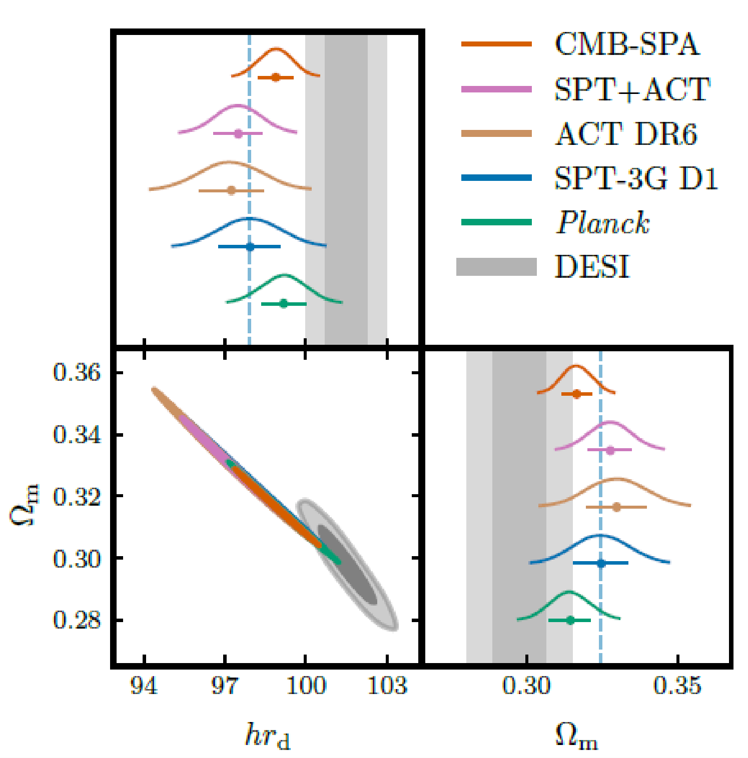
✓ Dark energy modifies the growth of structure formation

Why does CMB lensing important in cosmology

- The ΛCDM model faces a mild tension (CMB-BAO tension):

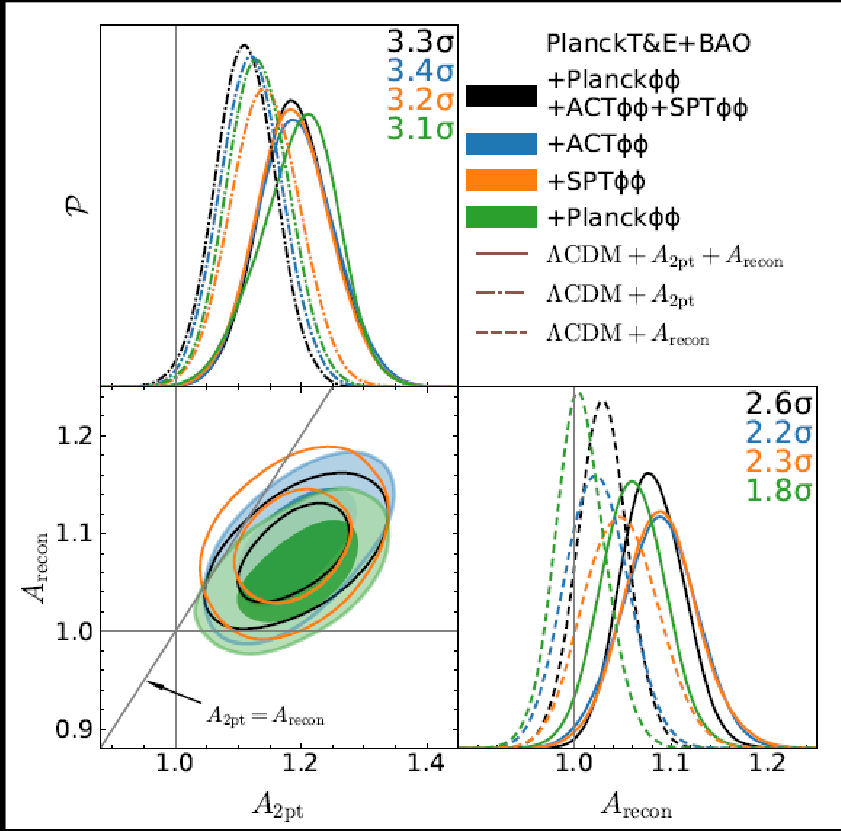
$$C_L^{\phi\phi} \to A C_L^{\phi\phi}$$

Tension in Ω_m constraints CMB (primary + lensing) / BAO



SPT-3G collaboration (2025)

Suggesting dynamical DE?



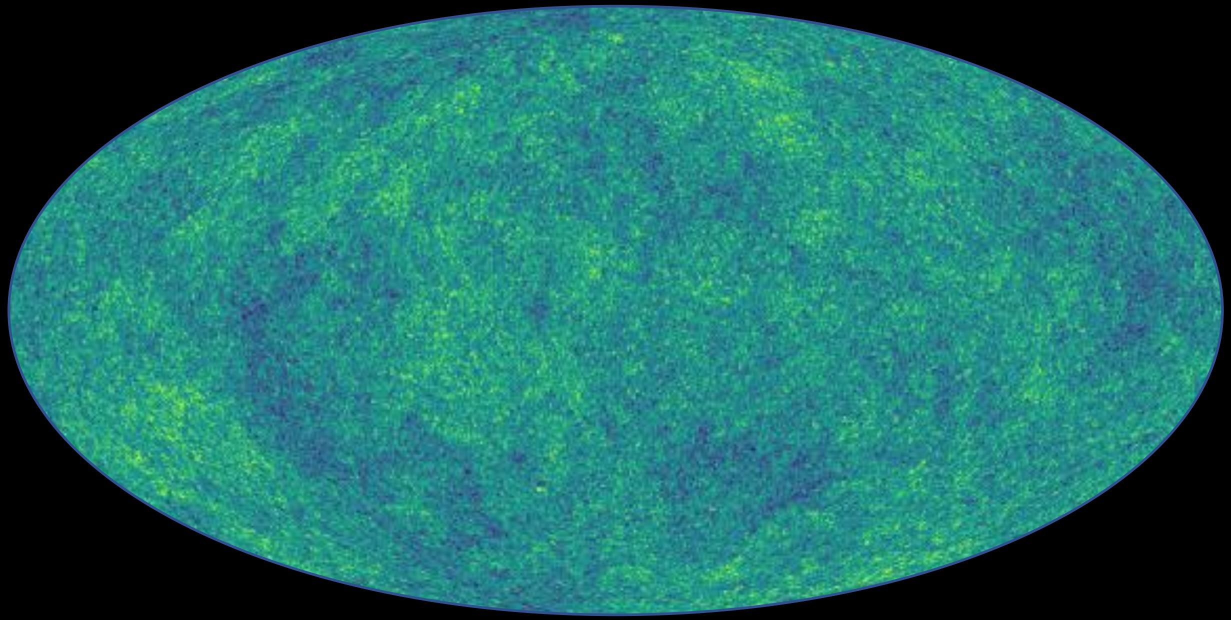
Amplitude of lensing signal not consistent with ΛCDM?

We can address these problems by measuring CMB lensing more precisely

How to reconstruct lensing signal from CMB map

Unlensed CMB map (at the last scattering)

Statistically isotropic – CMB fluctuations are distributed with the same variance at each direction



(at the CMB last scattering)

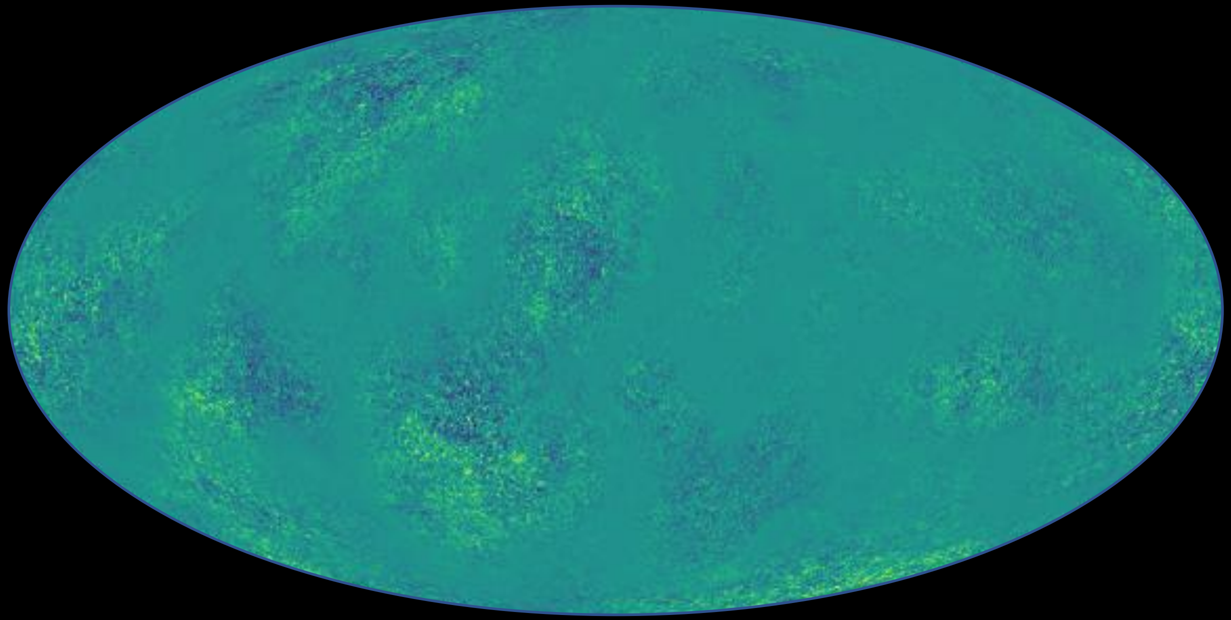
Statistical isotropy = CMB fluctuations at different angular scales are ***not*** correlated

$$\boxed{T_{\vec{l}-\vec{L}}T_{\vec{l}} \sim 0}$$

Lensed CMB map

Statistically "anisotropic" – Lensing modifies the variance "locally", introducing a "spatially-varying" variance

(Zaldarriaga & Seljack'99)



Distortion by lensing (at observer)

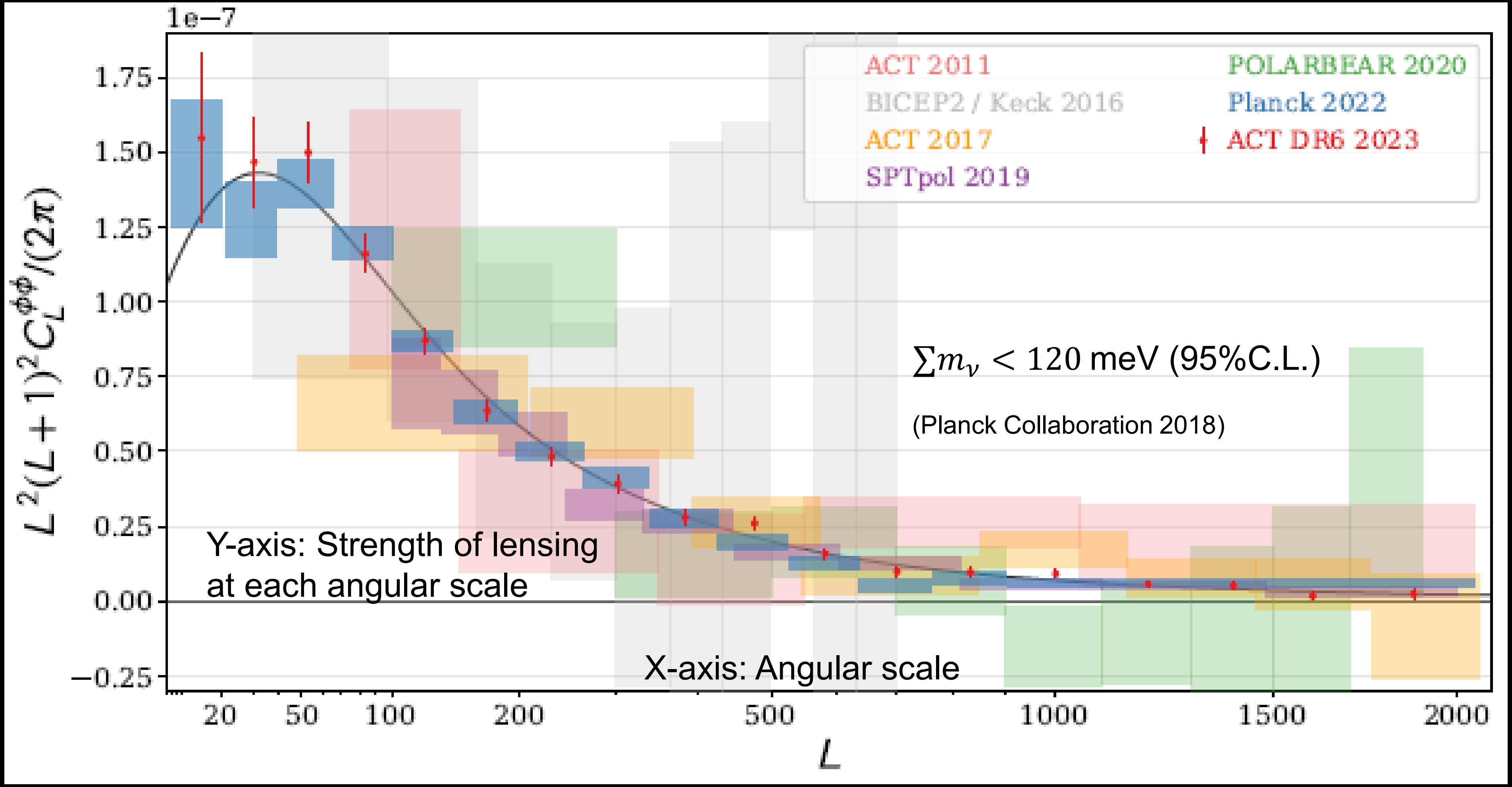
$$\kappa = -\frac{\nabla \cdot \vec{d}}{2}$$

Statistical anisotropy = CMB fluctuations at different angular scales are correlated

$$\boxed{T_{\vec{l}-\vec{L}} T_{\vec{l}} \sim \kappa_{\vec{L}}}$$

Measurements of CMB lensing power spectrum

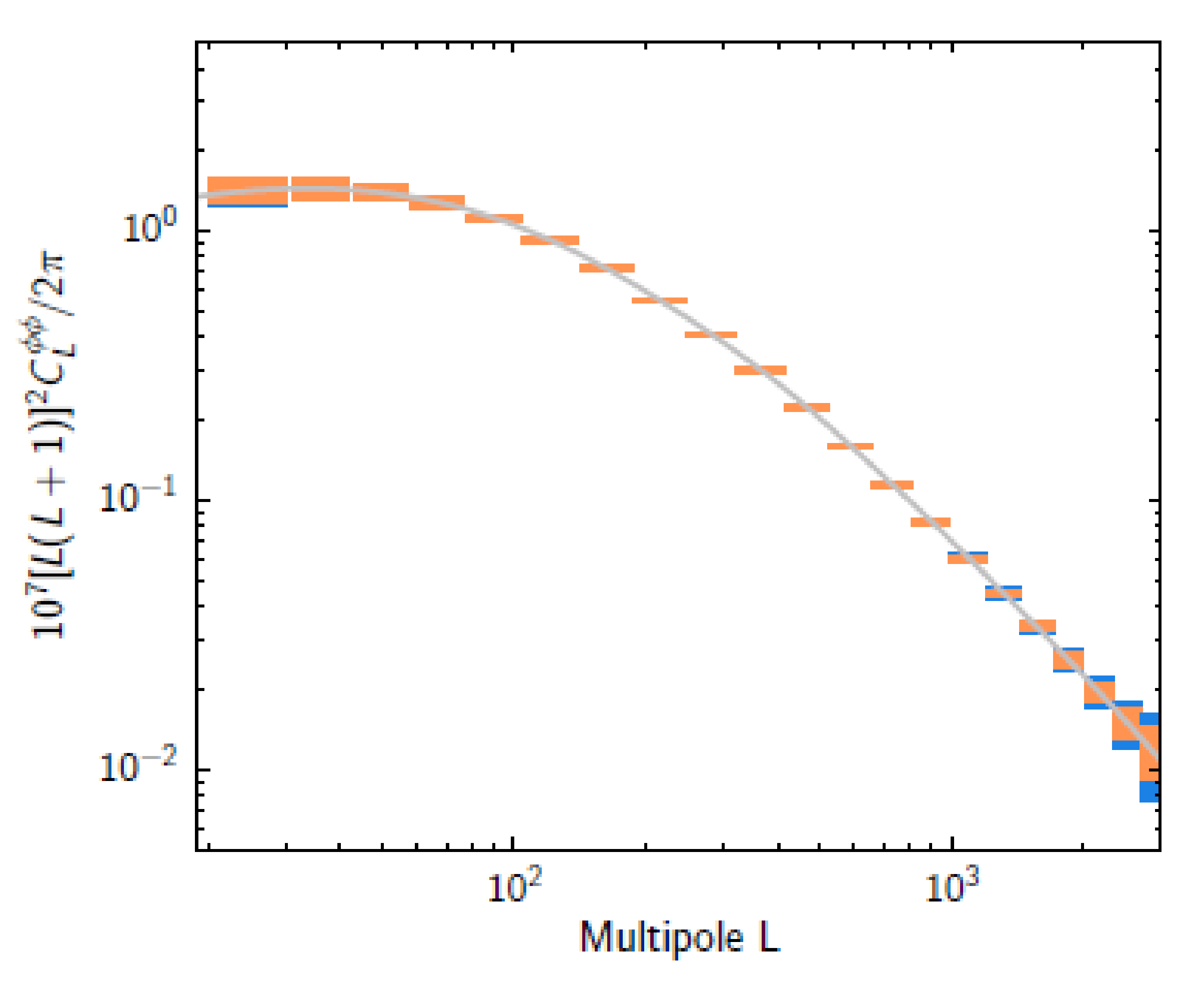
- Planck detects lensing at ~40 sigma



(ACT Collaboration 2023)

Measurements of CMB lensing power spectrum

- SO will detect lensing at ~160 sigma



(SO Collaboration 2019)

Tightly constrain cosmology

$\sigma(A_{\rm lens}) < 0.007$

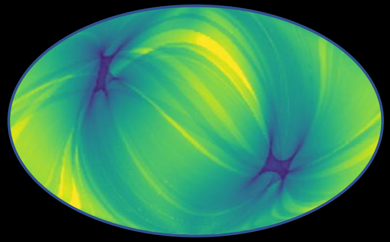
$\sum m_\nu < 31$ meV (1σ)
w/ BAO

Practical issues of reconstructing lensing map and a goal of B02

However, in practice, there are several challenges:

- Bias from other sources of a statistical anisotropy (foregrounds/sources, noise, beam, etc)

  Usually important at low-L, a problem for fNL with cross-correlation



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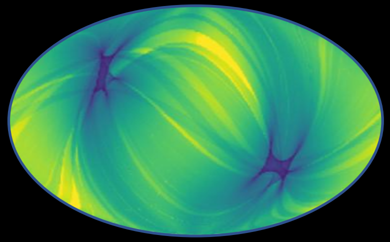
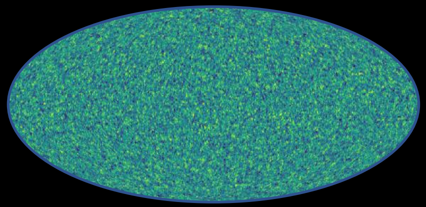


- Accurate subtraction of large noise



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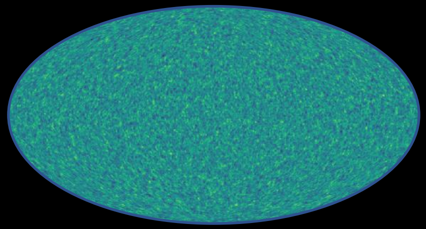

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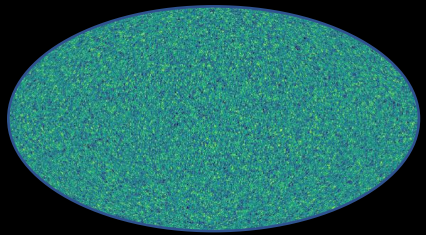


Reconstructed map ϕ | Signal ϕ | Noise

- Computational cost for optimal analysis

One of the goals of the B02 group is to realize an accurate and fast computation of CMB lensing signals

Current status of our work: A fast computation of noise bias

- Lensing estimator utilizes the correlation of temperature at different angular scales

$$\kappa_{\vec{L}} \sim T_{\vec{l}-\vec{L}} T_{\vec{l}}$$

- Lensing power spectrum can be estimated by squaring the lensing estimator

$$\left|\kappa_{\vec{L}}\right|^2 \sim \left|T^4\right| \sim C_L^{\kappa\kappa} + N_L^{\kappa\kappa}$$

(a four-point correlation of the CMB fluctuations)

Problem:
$N_L^{\kappa\kappa}$ is usually estimated using two set of Monte Carlo simulations, and evaluated as an average of many Monte Carlo realizations, but its computational cost is very high

We develop the control variate, a method that accelerates convergence by correcting a large Monte Carlo average with a similar but cheaper-to-compute quantity

Basic idea: Use control variate for faster computation

- A control variate (Kahn & Marshall 1953, Lavenberg & Welch 1981)

Expectation of $\tilde{\theta}^i$

$$\hat{\theta} = \langle \theta^i \rangle \quad \Rightarrow \quad \hat{\theta} = \langle \theta^i - \alpha(\tilde{\theta}^i - \tilde{\theta}) \rangle$$

Original estimator

Control variate

$$\alpha = \frac{\mathrm{Cov}(\theta^i, \tilde{\theta}^i)}{\sigma^2(\tilde{\theta}^i)} = \rho(\theta^i, \tilde{\theta}^i)\frac{\sigma(\theta^i)}{\sigma(\tilde{\theta}^i)}$$

- Two requirements
  - θ^i and $\tilde{\theta}^i$ are highly correlated ($\gg 90\%$)

    Reduce the MC variance as $\mathrm{Var}(\hat{\theta}) = 1 - \rho^2$
  - $\tilde{\theta}$ should be computed much faster than the MC average

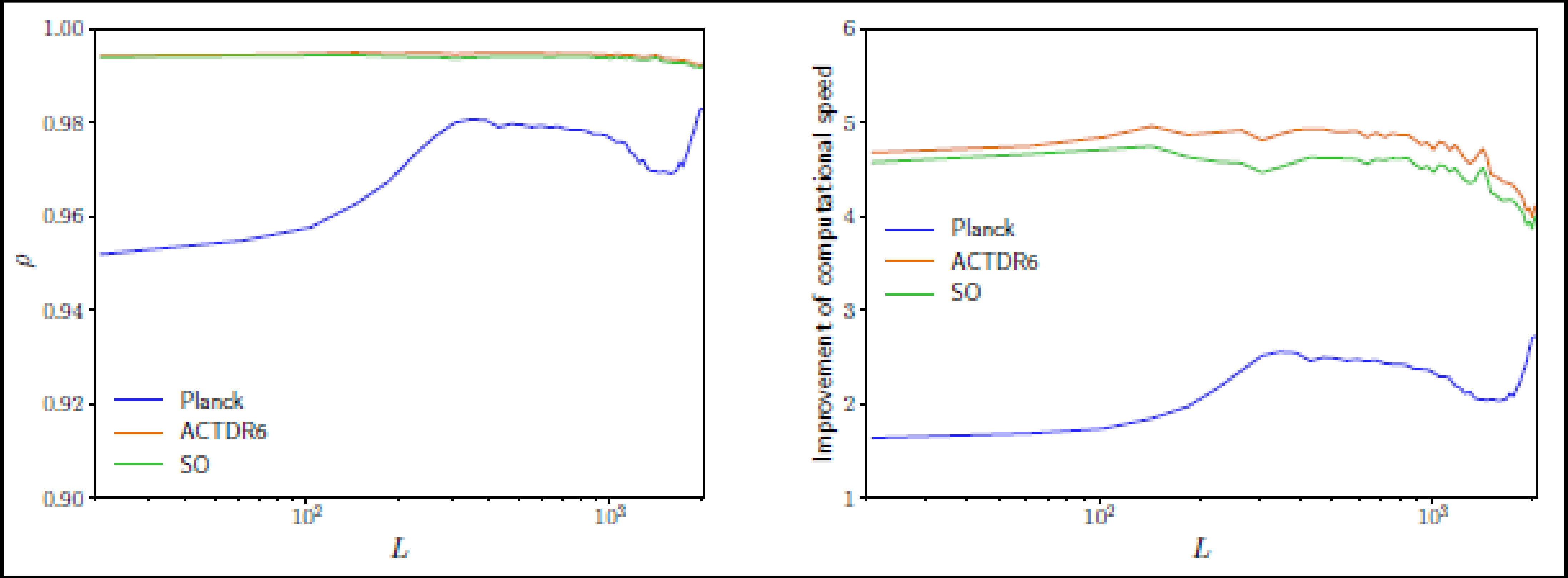
- Recently applied for computing LSS statistics from simulations

  (Chartier et al. 2021, Kokron et al. 2022, Hadzhiyska et al; 2023, Doytcheva et al. 2024)

Check correlations

- We find a control variate that correlates with the original noise bias by 99% that reduces the computational time by a factor of ~ 5 for ACT/SO-like experiment

(Namikawa+26)



$$= \frac{0.5}{\sqrt{1-\rho^2}}$$

Summary

- CMB lensing becomes one of the important cosmological probes
  - E.g., CMB-BAO tension, dark energy, primordial non-Gaussianity, $A_{\rm lens}$
  - SO will provide very precise lensing measurements (SNR ~ 160)
- One of the goals of the B02 group is to realize an accurate and fast computation of SO lensing signals
  - Our recent work found a control variate that can reduce computational time of the noise bias calculation by a factor of 5 for ACT/SO-like experiments
- Future plans
  - Polarization
  - Reducing impact of the Fourier-mode filtering
  - Implement this algorithm in the SO lensing pipeline
  - Other 4pt correlations

Backup slides

Diagonal approximation for RDN0?

- An approximation to ignore mode coupling between different multipoles

$$\langle \Theta_{\ell m} \Theta^*_{\ell' m'} \rangle = C_\ell^{\Theta\Theta} \delta_{\ell\ell'} \delta_{mm'}$$

→ RDN0 can be expressed using only C_ℓ

(Hanson et al. 2012)

This “diagonal”-RDN0 does not well correlate with the full-RDN0 because the off-diagonal terms are equally an important source of fluctuations of RDN0

Introduction: precise CMB lensing needs accurate bias subtraction

- Bias term is given by

$$N_L^{\mathrm{MC}} = \langle N_L^{\mathrm{MC},S_iS_i'} \rangle_i \,,$$

where

$$N_L^{\mathrm{MC},S_iS_i'} = \frac{1}{2L+1}\sum_M \left[\left| \hat{\kappa}_{LM}[\Theta^{S_i},\Theta^{S_i'}] \right|^2 + \hat{\kappa}_{LM}[\Theta^{S_i},\Theta^{S_i'}]\hat{\kappa}^*_{LM}[\Theta^{S_i'},\Theta^{S_i}] \right] .$$

- Difficult to evaluate analytically, and we use MC simulation in practice
- We use two independent MC realizations, S_i, S_i'

- A more accurate/optimal way to evaluate the bias term is to mix data and simulation

$$N_L^{\mathrm{RD}} = \langle N_L^{\mathrm{DS},S_i} \rangle_i - N_L^{\mathrm{MC}}$$

$$N_L^{\mathrm{DS},S_i} \equiv \frac{1}{2L+1}\sum_M \Big[\hat{\kappa}_{LM}[\Theta^{S_i},\Theta]\hat{\kappa}^*_{LM}[\Theta^{S_i},\Theta] + \hat{\kappa}_{LM}[\Theta^{S_i},\Theta]\hat{\kappa}^*_{LM}[\Theta,\Theta^{S_i}] + \hat{\kappa}_{LM}[\Theta,\Theta^{S_i}]\hat{\kappa}^*_{LM}[\Theta^{S_i},\Theta] + \hat{\kappa}_{LM}[\Theta,\Theta^{S_i}]\hat{\kappa}^*_{LM}[\Theta,\Theta^{S_i}] \Big] .$$

Problem: Computational cost of RDN0 is very high

Our idea: Isotropic simulation for RDN0

- We replace one of the simulation sets (S_i) to an isotropic full-sky signal simulation (I_i)

$$N_L^{\mathrm{RD}} = \langle N_L^{\mathrm{DS},S_i} \rangle - \langle N_L^{\mathrm{MC},S_i S_i'} \rangle$$

$$N_L^{\mathrm{DS},S_i} \Rightarrow N_L^{\mathrm{DS},I_i}$$

$$N_L^{\mathrm{MC},S_i S_i'} \Rightarrow N_L^{\mathrm{MC},S_i,I_i}$$

- We propose to use N_L^{DS,I_i} and $N_L^{\mathrm{MC},S_i,I_i}$ for control variates of each term:

$$N_L^{\mathrm{DS}} = \langle N_L^{\mathrm{DS},S_i} - \alpha^{\mathrm{DS}} \left(N_L^{\mathrm{DS},I_i} - \widetilde{N}_L^{\mathrm{DS}} \right) \rangle \qquad N_L^{\mathrm{MC}} = \langle N_L^{\mathrm{MC},S_i} - \alpha^{\mathrm{MC}} \left(N_L^{\mathrm{MC},I_i} - \widetilde{N}_L^{\mathrm{MC}} \right) \rangle$$

(this replaces one of the leg of the lensing estimator to isotropic simulation)

The use of the isotropic simulation simplifies the expectation values to the diagonal-RDN0

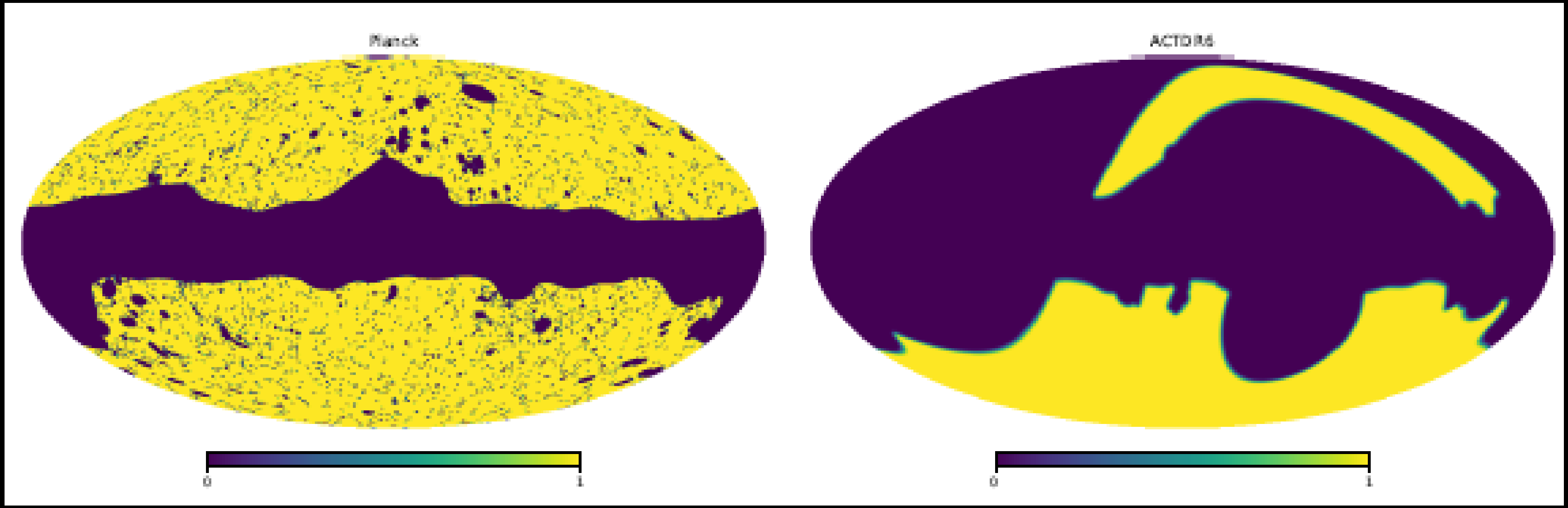
The locality of the lensing estimator leads to a high correlation

Check correlations

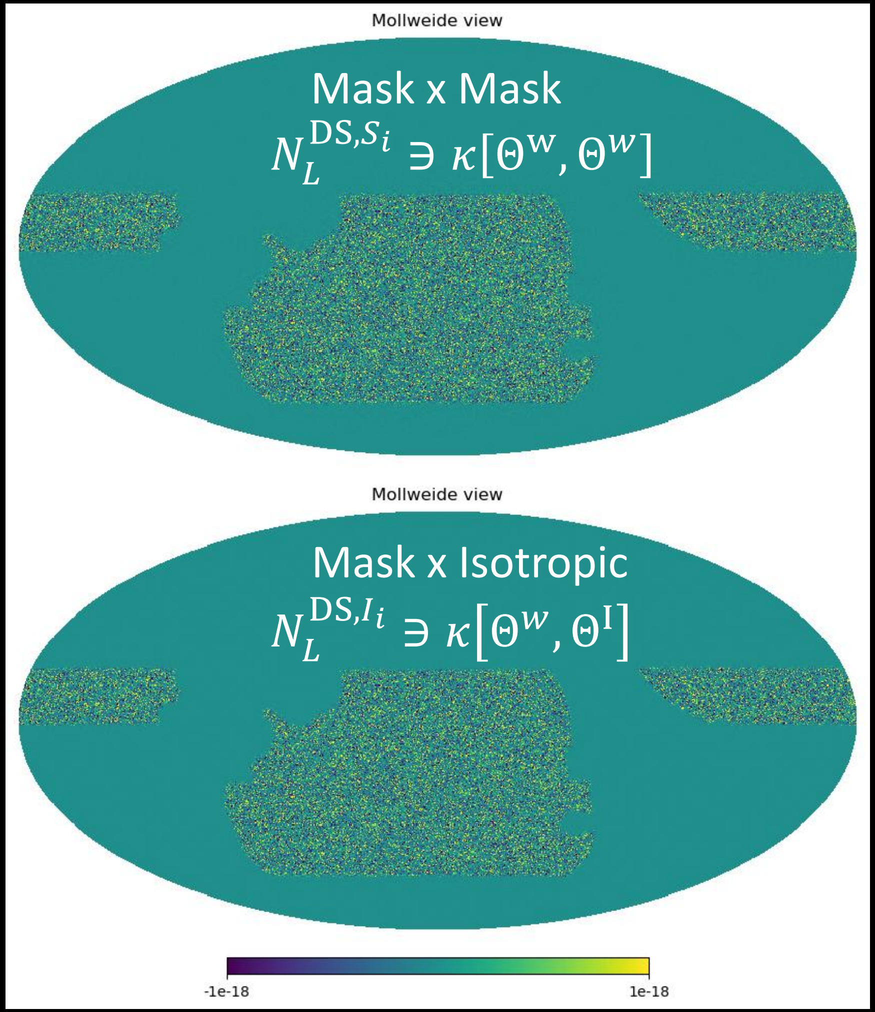
- We next check the correlation between the original term and control variate

  We consider a simulation situation to ACTDR6

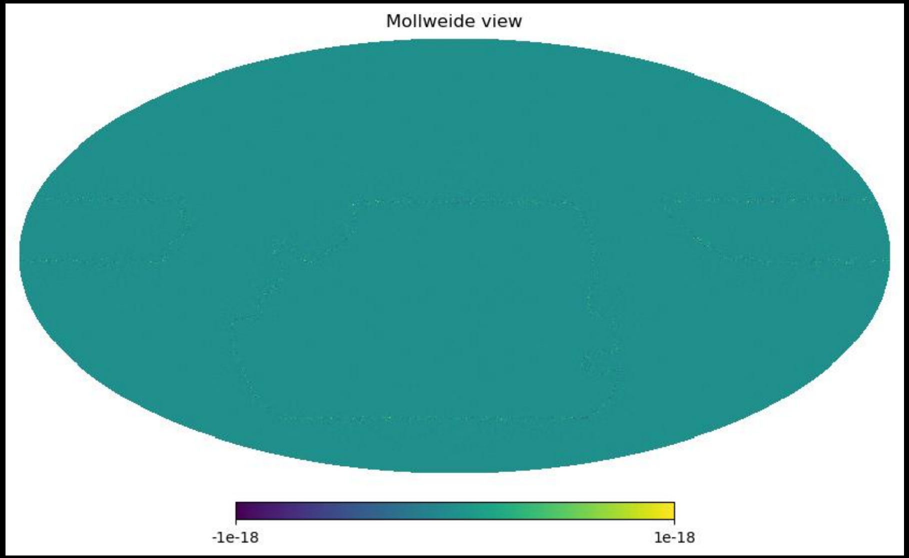
  - Signal-only simulation (because of the data split)
  - Mask (but we also consider Planck lensing mask)
  - w/ and w/o Fourier-filtering



- Reconstructed map (unnormalized)

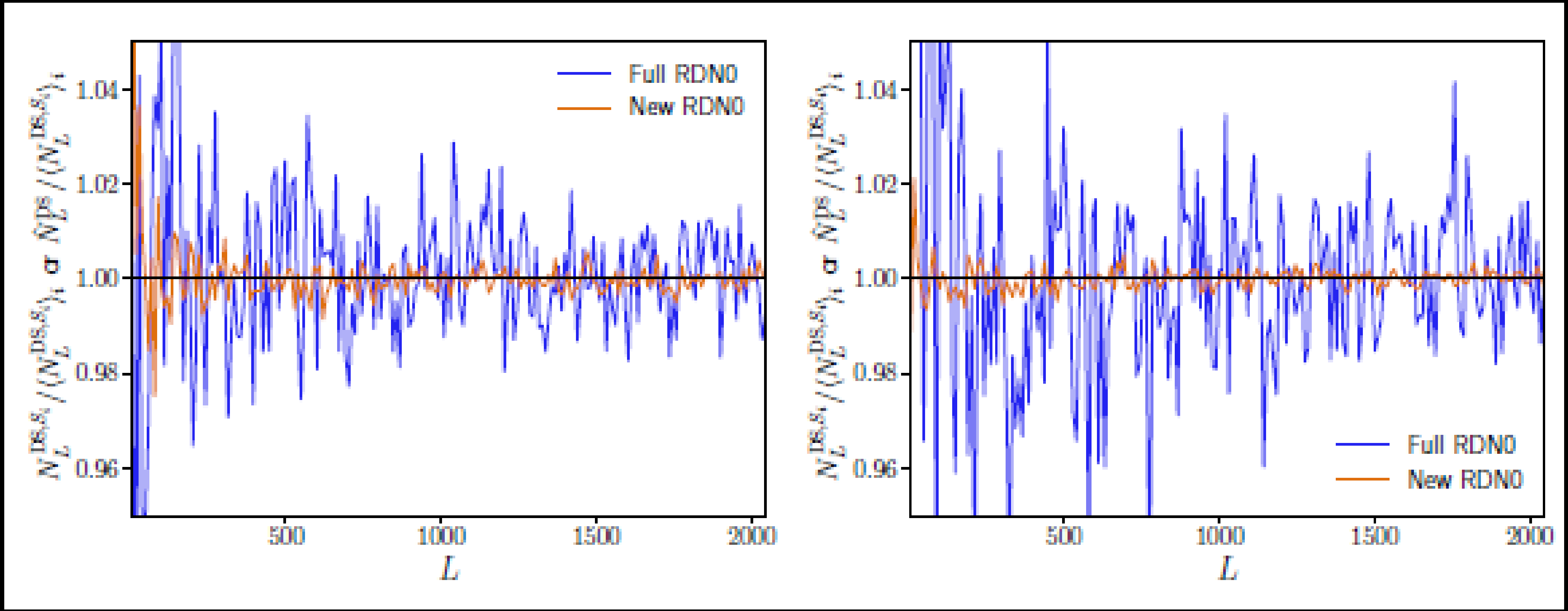


Difference



Check correlations

- The difference of $N_L^{\mathrm{DS},S_i} - \alpha^{\mathrm{DS}} N_L^{\mathrm{DS},I_i}$



$\alpha^{\mathrm{DS}} = \frac{W_6}{W_4} \simeq 1$ (for simplicity)