De Sitter Holography: Problems, Progress, Prospects

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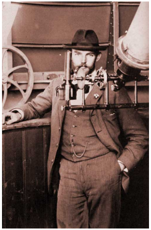
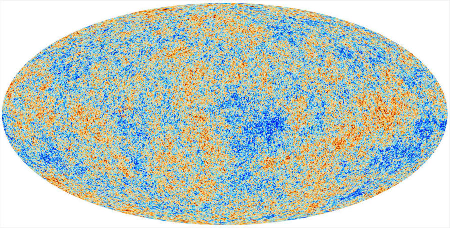


Outline

- Problems
- Progress
- Prospects

The prospect of an inflationary epoch and our current universe, with $\Lambda > 0$, provoke us to ask about de Sitter space.

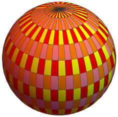


Problems

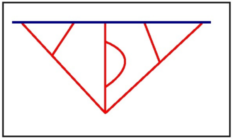
Sharp observables?

Accessible space is finite $\implies$ usual QG observables are absent. No asymptotic S-matrix, no boundary correlation functions

Meaningful sharp "local" quantities = dS entropy, ratio of dS entropy to maximal dS Nariai black hole



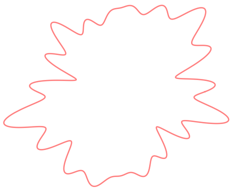
Meaningful sharp "global" quantities = Wavefunctional on late time slice



2d sigma model with S^N (Euclidean dS) target:

$$\boxed{S = \int d^2\sigma \sqrt{h} h^{ab}\, G_{IJ}(X^I)\, \partial_a X^I \partial_b X^J}$$

has NO fixed point: discrete spectrum, mass gap...



No go theorems $\Longrightarrow$ NO dS from compactifications of 10-dimensional SUGRA (Maldacena,Nunez...)

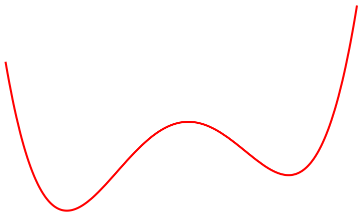
Are weakly coupled fundamental strings compatible with a long lived dS space?

SUSY, Stability?

dS breaks SUSY (thermal state, positive vac. energy...)

Cannot exploit SUSY toolkit (plus side: other useful symmetries)

dS Stability: YES classically, likely for certain quantum states perturbatively, unknown non-perturbatively



Progress

To proceed in any way we might have to find a different starting point in thinking about dS.

If holography is a general feature of QG, there should be a sense in which it applies to dS also.

Even though we cannot exploit SUSY, there are other highly symmetric theories admitting dS vacua.

Holography $\sim$ obtaining a gravity answer from a qm/statistical calculation:

- e.g. microstate counting of entropy (computed by area in gr)
- e.g. solution to Wheeler de Witt equation (gravitational path integral)

We will focus on the latter in what follows.

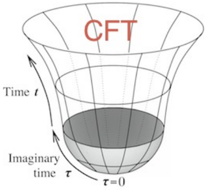
WdW equation: $$\boxed{\left[\frac{G_{ijkl}}{2\sqrt{h}}\frac{\delta}{\delta h_{ij}}\frac{\delta}{\delta h_{kl}}+\sqrt{h}\,(R[h_{ij}]-2\Lambda)\right]\Psi[h_{ij}]=0}$$

Large vol., $h_{ij}=a\hat{h}_{ij}$ with $a\to\infty$ (Papadimitrou;Pimentel) WdW implies (at tree level):

$$\boxed{\Psi[h_{ij}]=\Psi[e^{\omega(x^i)}\,h_{ij}]}$$

Hartle-Hawking solution: $$\boxed{\Psi_{HH}[h_{ij}]=\int_{\mathcal{M}}\mathcal{D}g_{\mu\nu}\,e^{-S_E}}$$

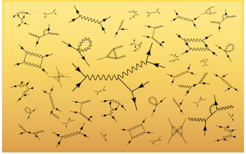
CONJECTURE: There exists Euclidean CFT s.t. $\boxed{\Psi_{HH} = Z_{CFT}}$
(Strominger,Witten,Maldacena)



Dictionary like Euclidean AdS/CFT: bulk fields $\sim$ single trace operators,
bulk masses $\sim$ conformal weights, Witten diagrams (not in-in) $\sim$ CFT correlators

Interesting connection between statistical (non-unitary) CFT and bulk QM.

- Bulk late time $\sim$ CFT UV cutoff $\implies$ CFT interpretation of late time (bulk IR) divergences.



e.g. 3d CFT has no Weyl anomaly $\implies$ no log divergences of graviton contributions to Ψ.

massless scalar $\sim$ marginal operator with $\Delta = 3$. $1/N$ contributions to Δ lead to (resumable) logs.

‘Practicality’ of DS/CFT II

- Properly defines the Hartle-Hawking path integral (as in EAdS/CFT)
- New language for CMB quantities (as opposed to features of inflationary potential, no need for semiclassical picture...)
- Selects a PARTICULAR solution to WdW equation

AdS useful picture: low energy limit of worldvolume theory on stack of branes. Typically gauge theories, adjoint matter...

Dual is NOT unitary, e.g. $\boxed{\Delta = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - m^2\ell^2} \in \mathbb{C}}$

Instead of adjoint matter, we might consider vector matter.

dS_4 is consistent vacuum solution in theories of interacting massless higher spin fields (s=0,1,2,...)

Has infinite dimensional higher symmetry group (with $SO(4,1)$ subgroup).

Perturbation theory works as usual in the bulk. Bulk scalar perturbatively stable $\boxed{V(\phi) \sim +2\phi^2/\ell^2}$. No ghosts at quadratic level.

Inspired by AdS_4 case (Klebanov-Polyakov,Sezgin-Sundel,Giombi-Yin...)

Postulate CFT dual to higher spin de Sitter is theory of GHOSTS ($N \to -N$) in fundamental representation of $U(N)$.

Simplest CFT is free:

$$\boxed{S_{CFT} = \int d^3x \, \partial_i \phi^I \partial^i \bar{\phi}_I \ , \qquad I = 1, 2, \ldots, N}$$

(More generally can add CS gauge field, quartic interactions, switch to commuting spinors. Imposing $U(N)$ constraint leads has interesting topological consequences (Banerjee,Hellerman,Maltz,Shenker))

Traceless and conserved currents $\boxed{J^{(s)} = \phi^I \partial_{i_1} \dots \partial_{i_s} \bar{\phi}_I}$ with $(\Delta, s) = (s+1, s)$

Includes stress tensor T_{ij} with $(\Delta, s) = (3, 2)$ dual to bulk graviton h_{ij}

Also scalar $J^{(0)} = \phi^I \bar{\phi}_I$ with $(\Delta, s) = (1, 0)$

(Interesting that light bulk scalar is necessary for consistency of theory)

Single trace operators $\phi^I(x)\bar{\phi}_I(y)$ are sourced by complex matrices $B(x, y)$ (Das,Jevicki;Doulas,Mazzucato,Razamat;...)

$$\boxed{\delta S_{CFT} = \int d^3x \int d^3y \, \phi^I(x) B(x, y) \bar{\phi}_I(y)}$$

Generally B may contain many higher spin sources:

$$\boxed{B(x, y) = \sum_{s=0}^{\infty} (-i)^s \, h^{i_1 \ldots i_s}(x) \, \partial_{i_1} \ldots \partial_{i_s} \, \delta(x - y)}$$

Recall Z_{CFT} computes the wavefunction. For free theory this yield a remarkably simple formula:

$$\boxed{\Psi[B(x,y)] = Z_{CFT}[B(x,y)] = [\det(B(x,y))]^N}$$

Far beyond any minisuperspace approximation.

Relevant deformations:

$$\boxed{\Psi[g_{ij}, m] = \left[\det{}_{\zeta}\left(-\nabla^2_{(g)} + \frac{R[g]}{8} + m(x)\right)\right]^N}$$

ζ-function regularization implemented. Maximum (global?) about dS vacuum.

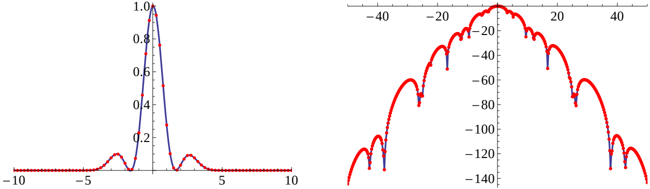


Figure : Examples of Z_{CFT} (and $\log Z_{CFT}$)) for an $SO(3)$ preserving deformation (in this case S^3 harmonics).

Invariance under h.s. 'diffeomorphisms' (leading to momentum constraint):

$$\boxed{\Psi[B_{xy}] = \Psi[B'_{xy}] \ , \qquad B'_{xy} = U_{xp} B_{pq} U^\dagger_{qy} \ , \quad U_{xy} \in U(\mathbb{R}^3) \ .}$$

If UV part of B_{xy}'s spectrum is that of 3d Laplacian, invariant under local Weyl transformations (leading to Hamiltonian constraint):

$$\boxed{\Psi[B_{xy}] = \Psi[e^{\omega_x} B_{xy} e^{\omega_y}] \ .}$$

Hyper-Weyl transformations $B'_{xy} = e^{\omega_{xz}} B_{zw} e^{\omega_{wy}}$ (with $\omega_{xy} = \omega^\dagger_{xy}$) transform non-trivially:

$$\boxed{\delta \log \Psi[B_{xy}] = N \delta \omega_{xy} \ .}$$

$\{B_{xy}, \Pi_{xy}\}$ overparameterizatize the (non-gauge fixed) phase space?

B_{xy} sources bilinear $\phi^I_x \bar{\phi}^I_y$ which has $\sim N \times V$ d.o.f. $(N < V)$

B_{xy} and $\langle \phi^I_x \bar{\phi}^I_y \rangle_{B_{xy}}$ are different pieces (falloffs) of the same fluctuating bulk fields

POSTULATE:

$$\boxed{B_{xy} = Q^I_x \bar{Q}^I_y}$$

(unlike AdS/CFT, sources also fluctuate in dS/CFT)

If Q_x^I bosonic $Q_x^I \bar{Q}_y^I$ has reduced rank (for $N < V$) $\implies$ det $Q_x^I \bar{Q}_y^I = 0$

If Q_x^I Grassman determinant non-vanishing...

$$\boxed{\Psi = \Psi[Q_x^I, \bar{Q}_x^I] = \left(\det\, Q_x^I \bar{Q}_y^I\right)^N}$$

Bosonic representation (M is $N \times N$ Hermitean matrix):

$$\boxed{\int dQ\, \Psi(Q_x^I)\Psi^*(Q_x^I) = \int dM\, e^{-\mathrm{tr} M^2 + V \mathrm{tr} \log M}}$$

Classical potential has minimum, diagonalizing M leads to N d.o.f. with some eigenvalue distribution.

Interestingly: $\boxed{N \sim S_{dS}}$

Prospects

Degrees of freedom in general DS?

If our picture is general, it means that inflation does not generate new degrees of freedom as time proceeds in the naive way seen in perturbation theory.

Once N degrees of freedom are produced no more are produced. Many relations between CMB correlations?

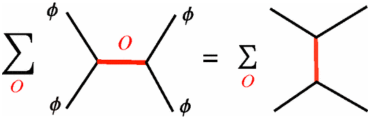
Toward Einstein duals?

Deformations of hs models to obtain Einstein-like de Sitter?

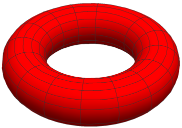
HS particles with small finite mass have a negative norm mode (Higuchi;Deser,Waldron). This is UNLIKE AdS.

Also, avenue from free $U(N)$ model to ABJM model (Chang,Minwalla,Sharma,Yin) leads to tachyons in dS...

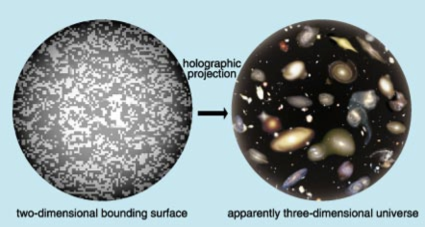
Bulk Hermitean Hamiltonian $\implies$ reality conditions between CFT correlators. Input into bootstrap equations instead of unitarity?



dS_3/CFT_2 also exploit modular invariance. Does a $Z[\tau] = Z[-1/\tau]$ exist with dS_3 properties (i.e. imaginary c, complex weights...)?



Holographic formulation of static patch from the get go?



Static patch conformal to $\mathrm{AdS}_2 \times S^2$, worldline maps to boundary of AdS_2, horizon-to-horizon. Starting point conformal gravity?

THANK YOU VERY MUCH FOR YOUR TIME!