New method for precise determination of top quark mass at LHC

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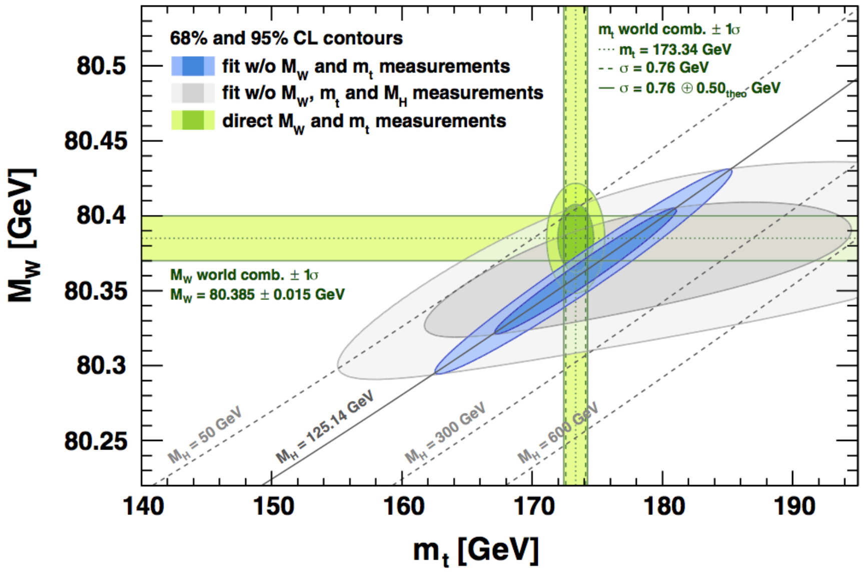
Phys. Lett. B 741 (2015) 232-238

Introduction

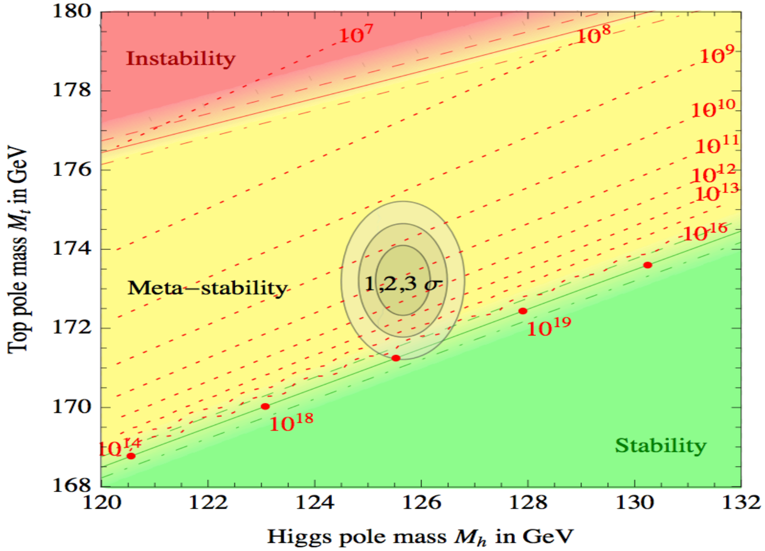
Top mass is an important input to various physics

★ EW precision tests for SM



Gfitter Group '14

★ SM vacuum stability



Buttazzo et al. '13

★ Beyond SM

Problem in m_t measurements

Tevatron+LHC m_t combination arXiv:1403.4427

$m_t = 173.34 \pm 0.76$ GeV 0.4 % precision !

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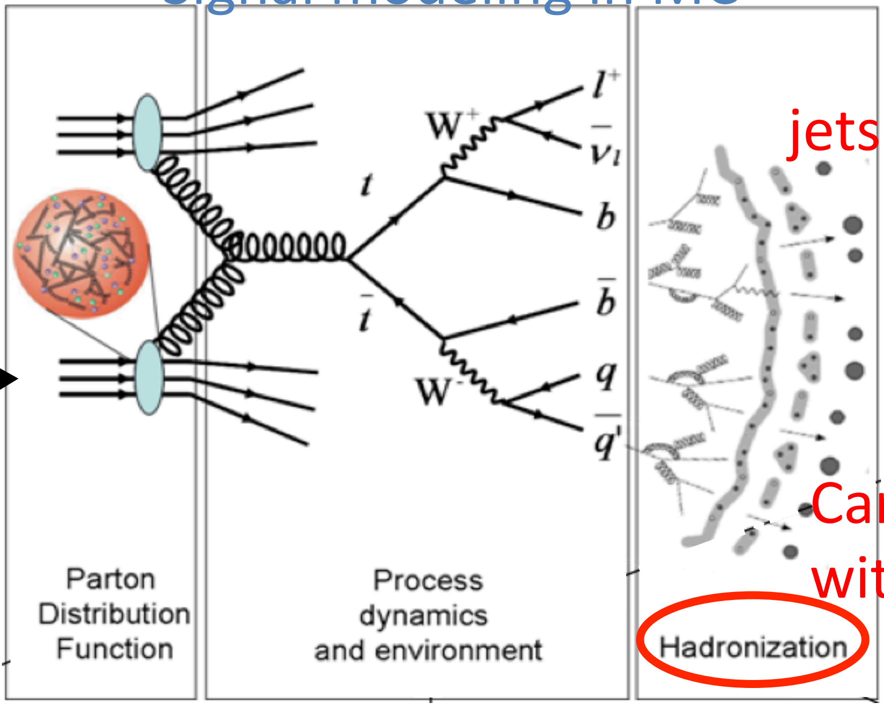
What kind of mass?

Experiment

↕ m_t measurement

Theory（MC） - - - ->

Signal modeling in MC



jets

Cannot be treated within pert. QCD

Aim of this study

Determine the top quark pole mass accurately at the LHC

We propose a new method
which uses lepton energy distribution

"Weight function method"

By a simulation analysis,
we show that this method works well.

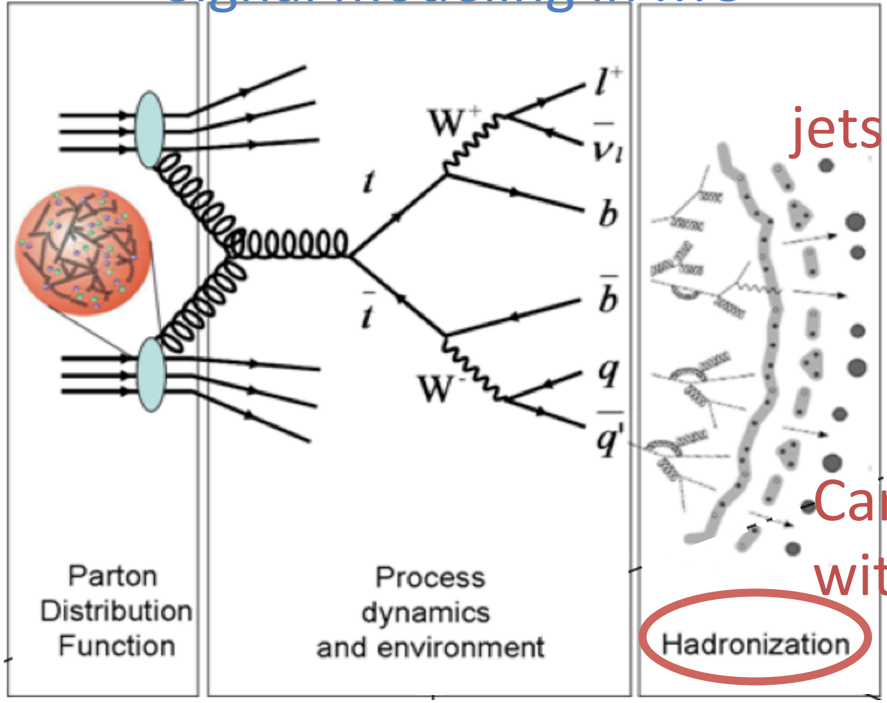
Weight function method

SK, Y.Shimizu, Y.Sumino, H.Yokoya, PLB 710, 658 (2012)
SK, Y.Shimizu, Y.Sumino, H.Yokoya, JHEP 08, 129 (2013)

New method for parent particle's mass reconstruction

- Only lepton energy distribution is needed
- Independent of top-quark velocity distribution

Signal modeling in MC



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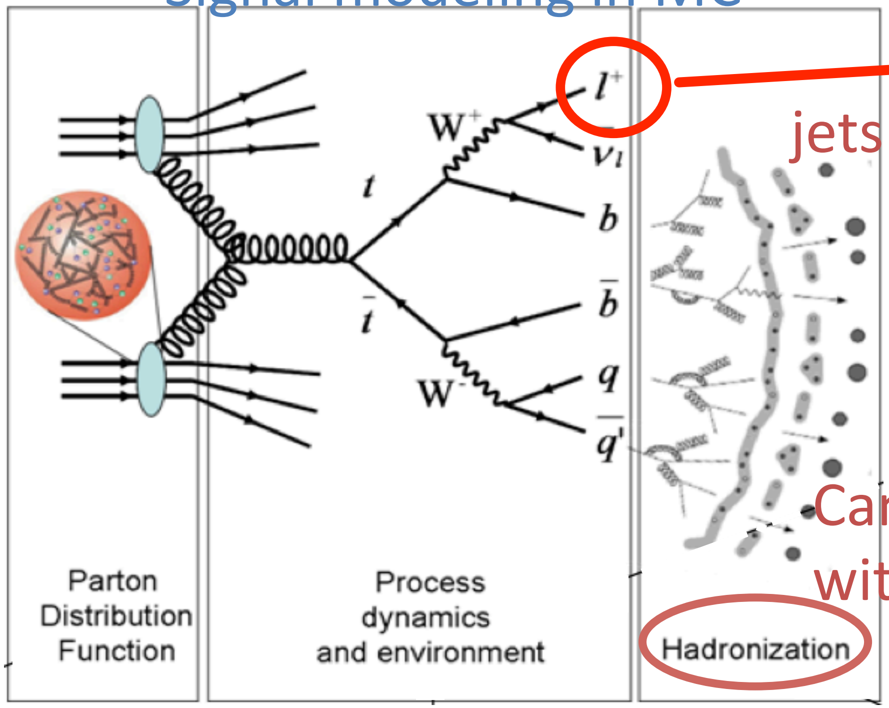


jets

Free from ambiguity of hadronization model

We can determine a theoretically well-defined m_t

Cannot be treated within pert. QCD

Weight functions and the weighted integrals

Weight function

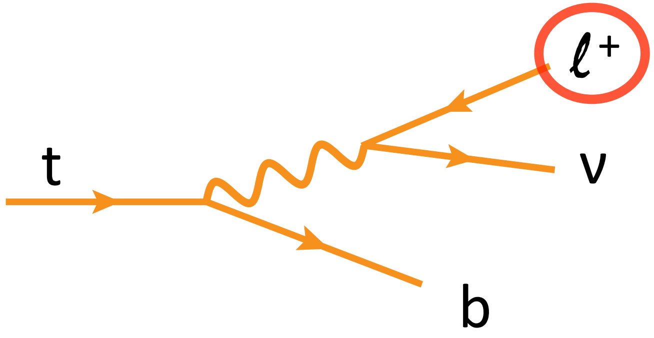
$$I(m) \equiv \int dE_l\, D(E_l)\, \boxed{W(E_l, m)}$$

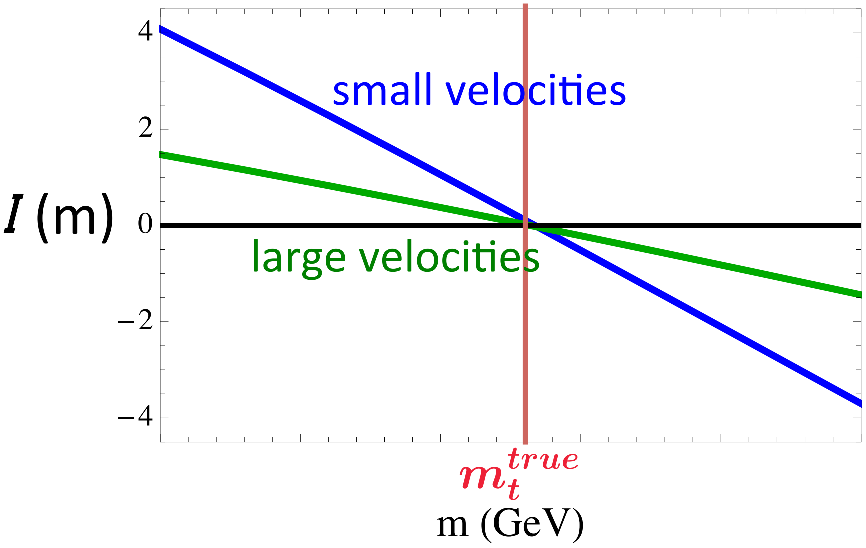
$\uparrow$

Lepton energy distribution in the lab. frame

There exist an infinite number of weight functions which satisfy

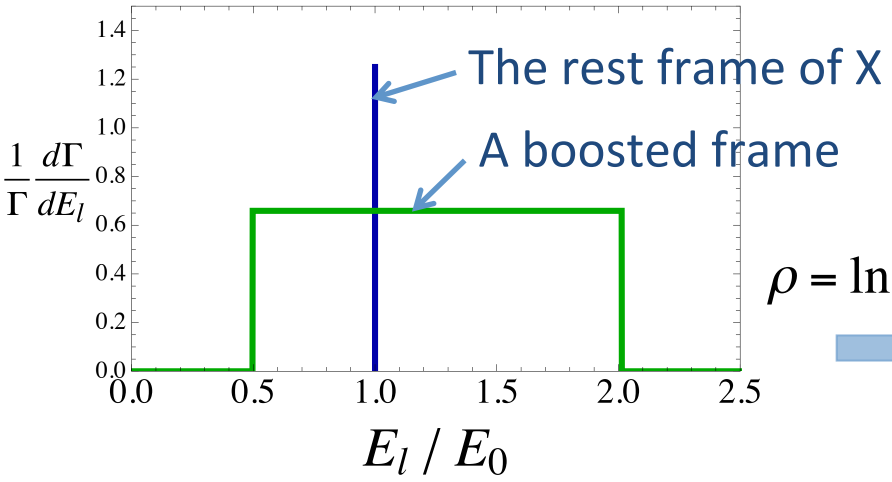
$I(m = m_t^{true}) = 0$ for an arbitrary velocity distribution of top quarks





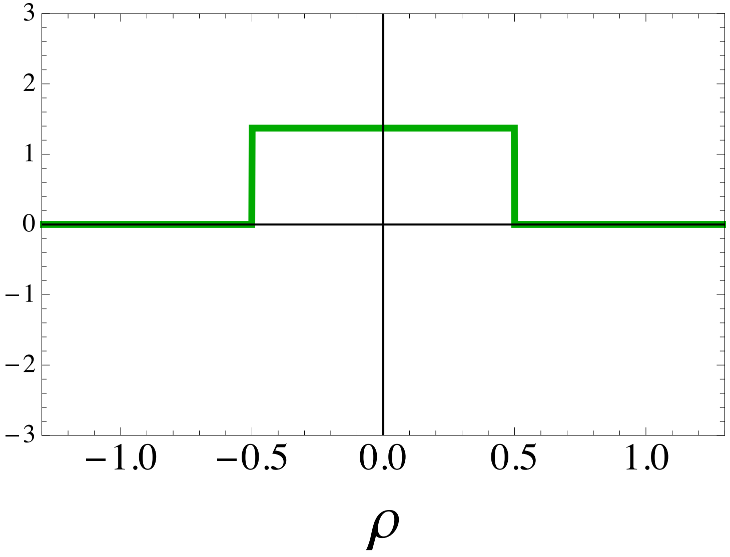
Construction of weight functions

For a two-body decay : X ⟶ ℓ + Y （X is scalar or unpolarized）



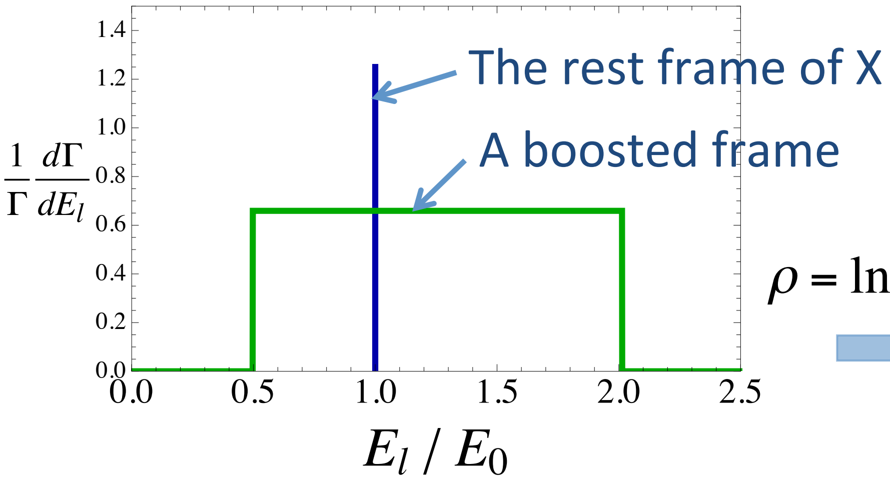
Lepton energy distribution

$$\rho = \ln(E_l / E_0)$$



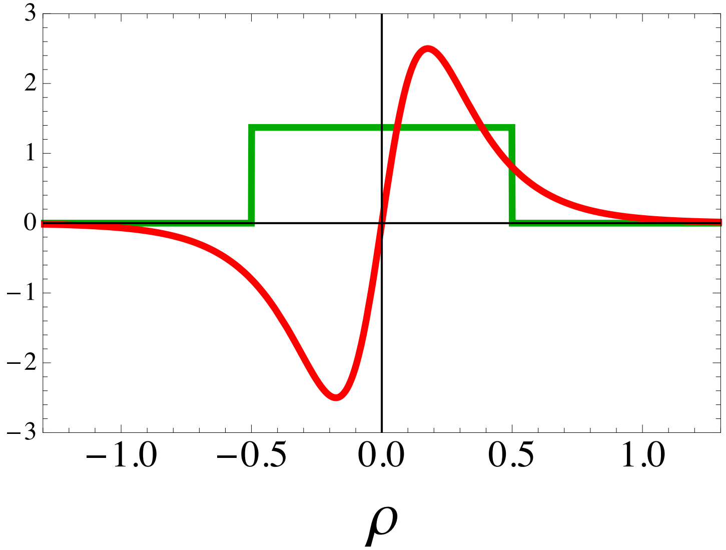
Construction of weight functions

For a two-body decay : $X \longrightarrow \ell + Y$ (X is scalar or unpolarized)



Lepton energy distribution

$\rho = \ln\,(E_l \,/\, E_0)$



$$\int dE_l\, D(E_l)\, W(E_l, m_X^{true}) \;=\; 0 \quad \Longleftrightarrow \quad \int d\rho(\text{even func. of } \rho)(\text{odd func. of } \rho) = 0$$

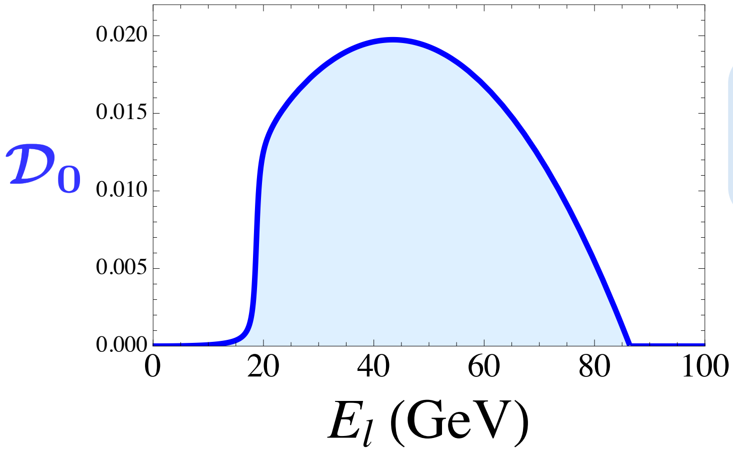
$$d\rho \propto e^{-\rho} dE_l$$

$$W(E_l, m_X^{true}) \;=\; \mathrm{e}^{-\rho}(\text{odd func. of } \rho)\Big|_{\mathrm{e}^{\rho} = E_l/E_0}$$

Construction of weight functions

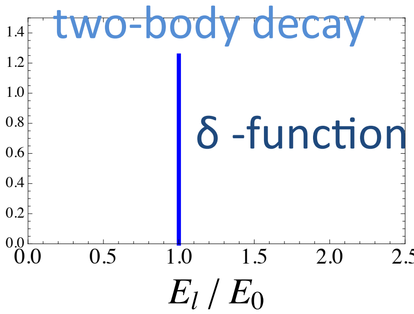
For a many-body decay : X $\longrightarrow \ell +$ anything (X is scalar or unpolarized)

Lepton energy distribution in the rest frame of X



Can be expressed as a superposition of lepton distribution for a two-body decay

$$\mathcal{D}_0(E_l) = \int dE\, \mathcal{D}_0(E)\, \delta(E_l - E)$$



A weight function would be also a superposition of that for a two-body decay

$$W(E_l, m) = \int dE\ \mathcal{D}_0(E\,; m) \frac{1}{E E_l} (\text{odd func. of } \rho) \Big|_{e^{\rho} = E_l / E}$$

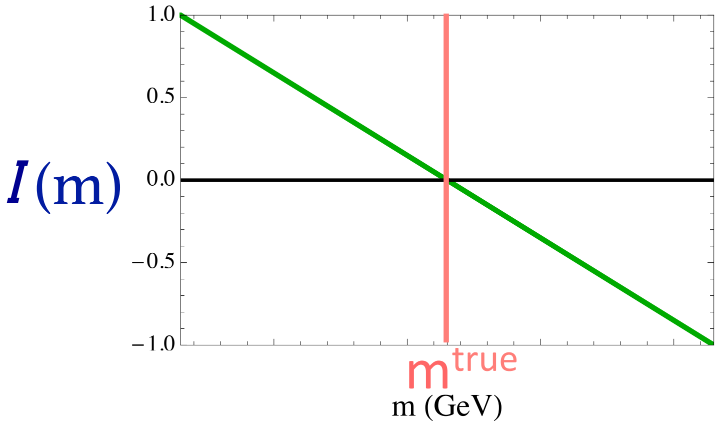
Summary of weight function method

1. Construct weight functions for the process

$$W(E_l, m) = \int dE\, \boxed{\mathcal{D}_0(E\,; m)}\, \frac{1}{EE_l}(\text{odd func. of } \rho)\bigg|_{e^{\rho}=E_l/E}$$

Lepton energy dist. in the rest frame of parent particle, which can be calculated in pert. QCD

2. Use lepton energy distribution measured by experiment as $D(E_\ell)$



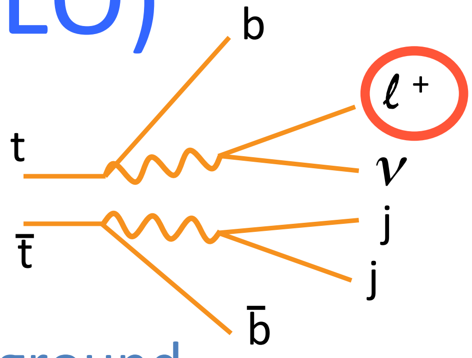
$$I(m) \equiv \int dE_l D(E_l) W(E_l, m)$$

3. Obtain the zero of I(m) as m^{true}

$$I(\, m = m^{\text{true}}\,) = 0$$

Simulation analysis (LO)

- LHC $\sqrt{s}$ = 14 TeV
- $t\bar{t}$ events, Lepton(μ)+jets channel



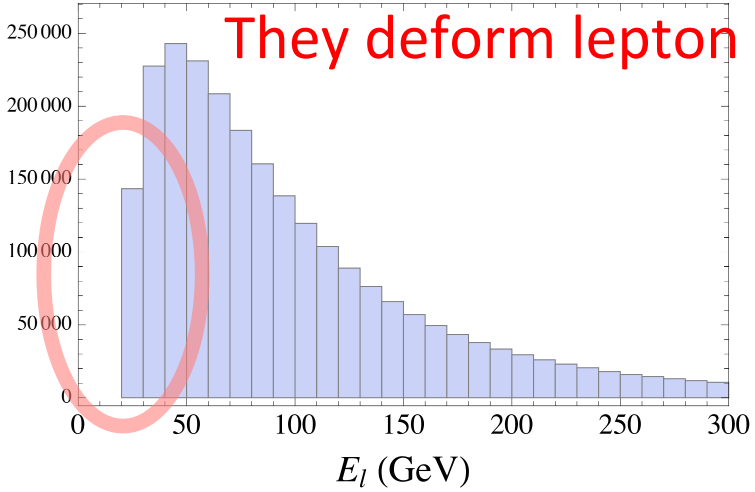
Event selection cuts

- 1 muon with $p_T > 20$GeV, $|\eta| < 2.4$
- At least 4 jets
- At least 1 b-tag
- $p_T(j_1) > 55$, $p_T(j_2) > 25$, $p_T(j_3) > 15$, $p_T(j_4) > 8$ GeV

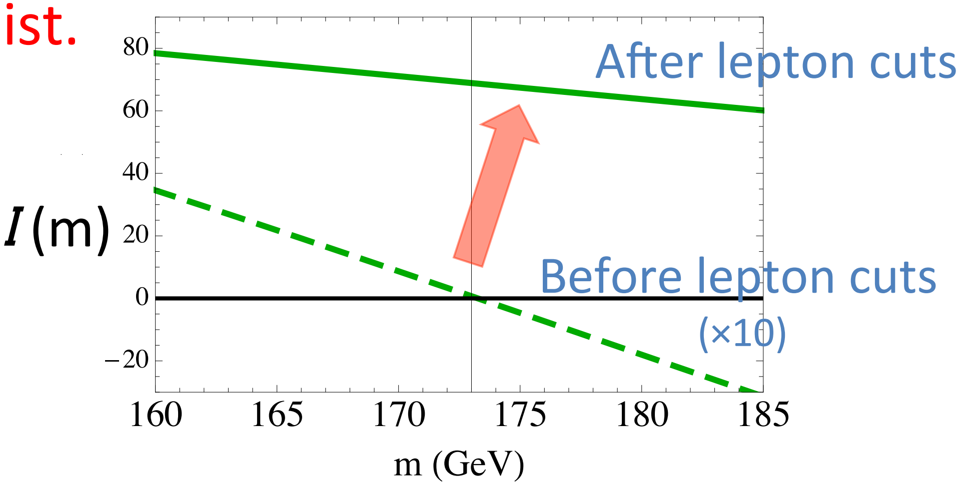
lepton cuts

Background

- Other $t\bar{t}$ events
- W+jets
- $Wb\bar{b}$+jets
- Single top



They deform lepton dist.

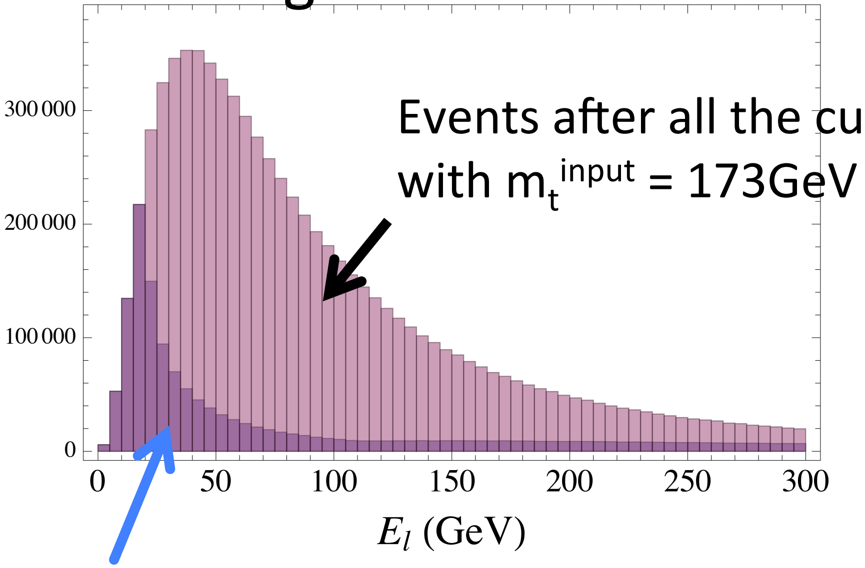


After lepton cuts

Before lepton cuts (×10)

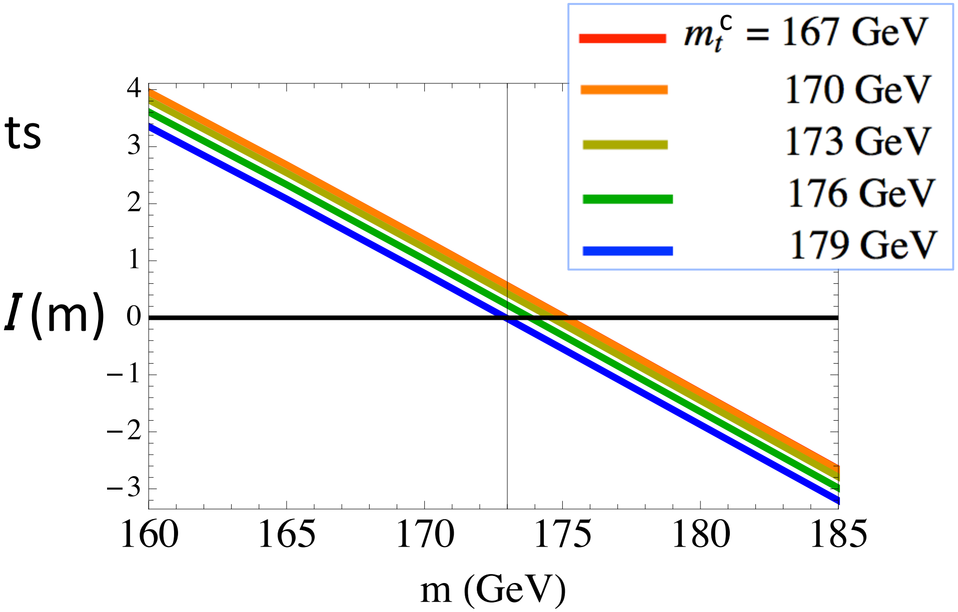
Solution to the problem of lepton cuts

We compensate for the loss caused by lepton cuts using MC events.



Events after all the cuts with m_t^{input} = 173GeV

Compensated MC events with m_t^c



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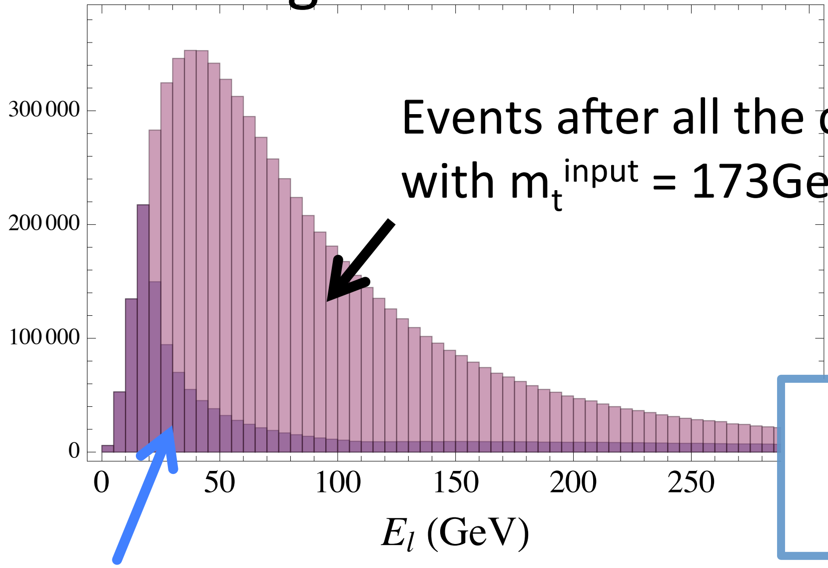

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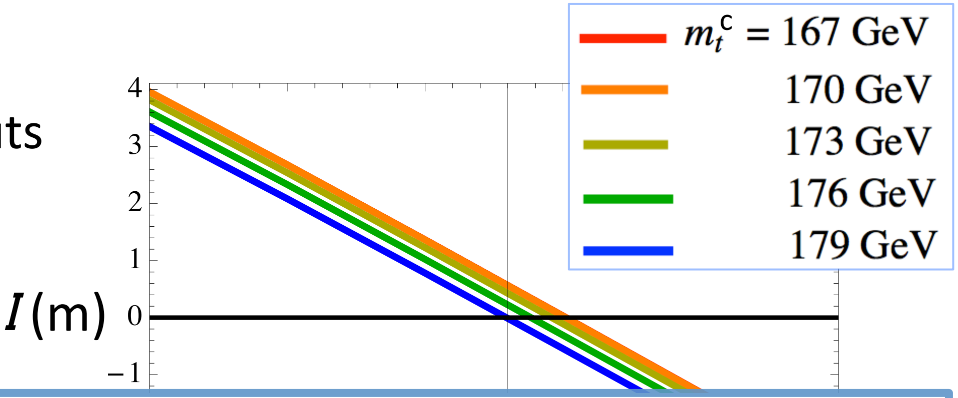
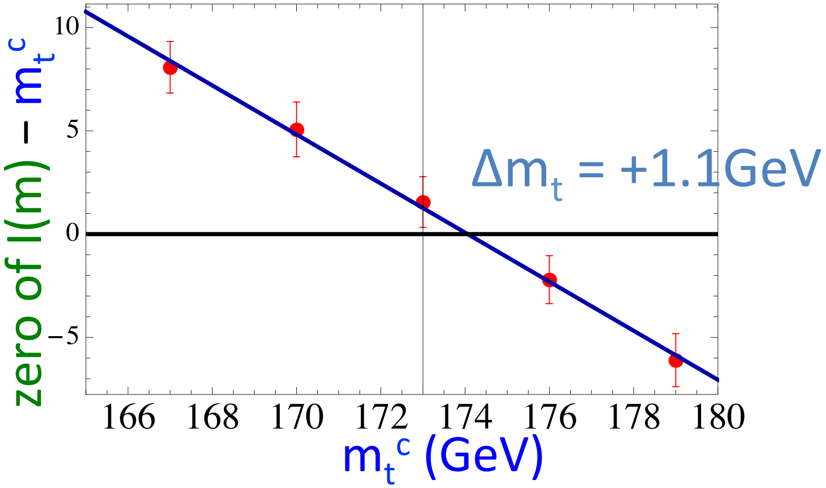


Effect of Γ_t : +0.34GeV
MC stat. error : ~1GeV
⇨ Consistent

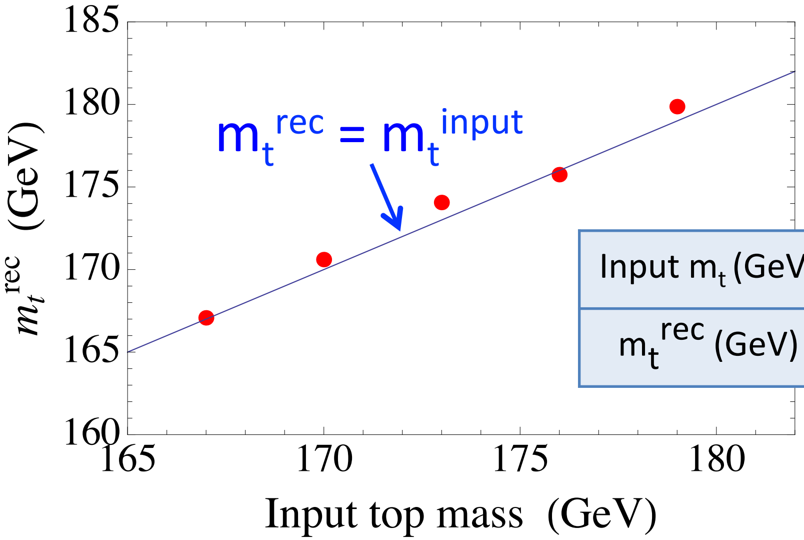


Δm_t = +1.1GeV

$m_t^c = m_t^{input}$ ⇒ zero of I(m) = m_t^c

$m_t^c \neq m_t^{input}$ ⇒ zero of I(m) ≠ m_t^c (guess)

Sensitivity of m_t determination (LO)



Input m_t (GeV)	167	170	173	176	179
m_t^{rec} (GeV)	167.1	170.6	174.1	175.7	179.9

Uncertainties [GeV]

Signal stat. error	0.4
μ_F scale	+1.5/-1.4
PDF	0.6
Jet energy scale	+0.5/-0.0
BG stat. error	0.4

← At 100 fb^{-1}, Lepton+jets channel

← Can be improved by including NLO

We aim for $\Delta m_t^{pole} < 1$ GeV

Summary and future works

- We proposed a new method to measure a theoretically well-defined top quark mass at LHC.
- We performed a simulation analysis of top mass reconstruction with lepton+jets channel at LO.
- The estimated stat. error is about 0.4GeV with 100fb^{-1}. Major systematic uncertainties are under good control.

Ongoing & future works

★ NLO, NNLO

Include NLO, NNLO corrections to the top decay process in weight functions. ⇨ m_t^{pole}

★ Effects of top off-shellness

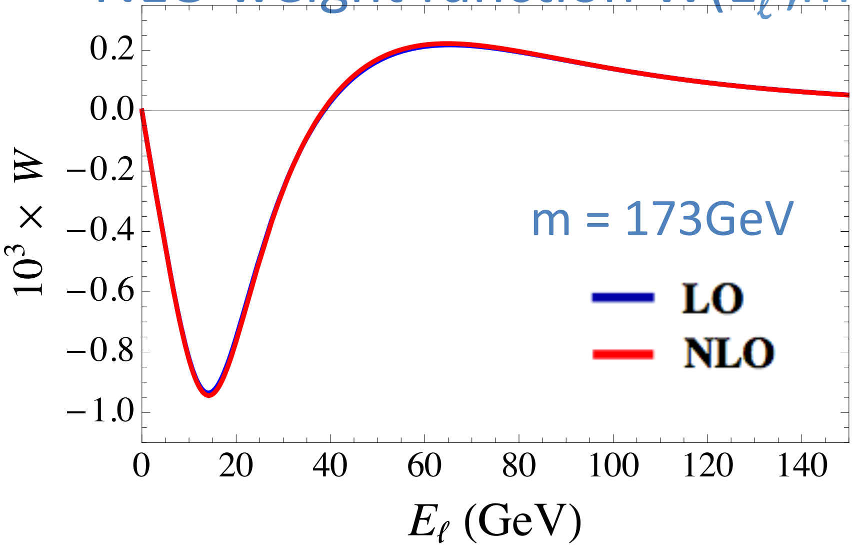
Backup

NLO analysis

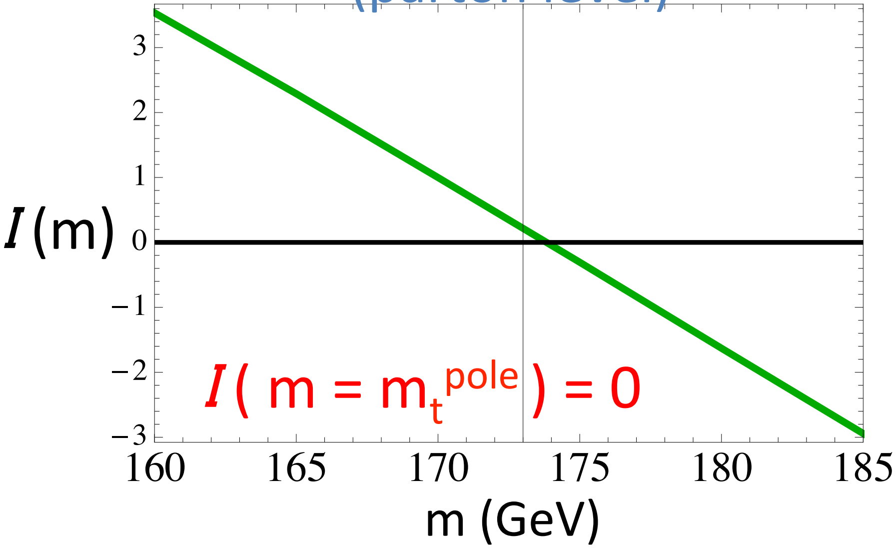
Required NLO correction

- involving top production → MC simulator for compensation
- involving top decay → MC for compensation + weight fn.
- involving both production and decay → Estimate

NLO weight function $W(E_\ell, m)$



簡易的simulation analysis (parton level)



Pole mass measurements

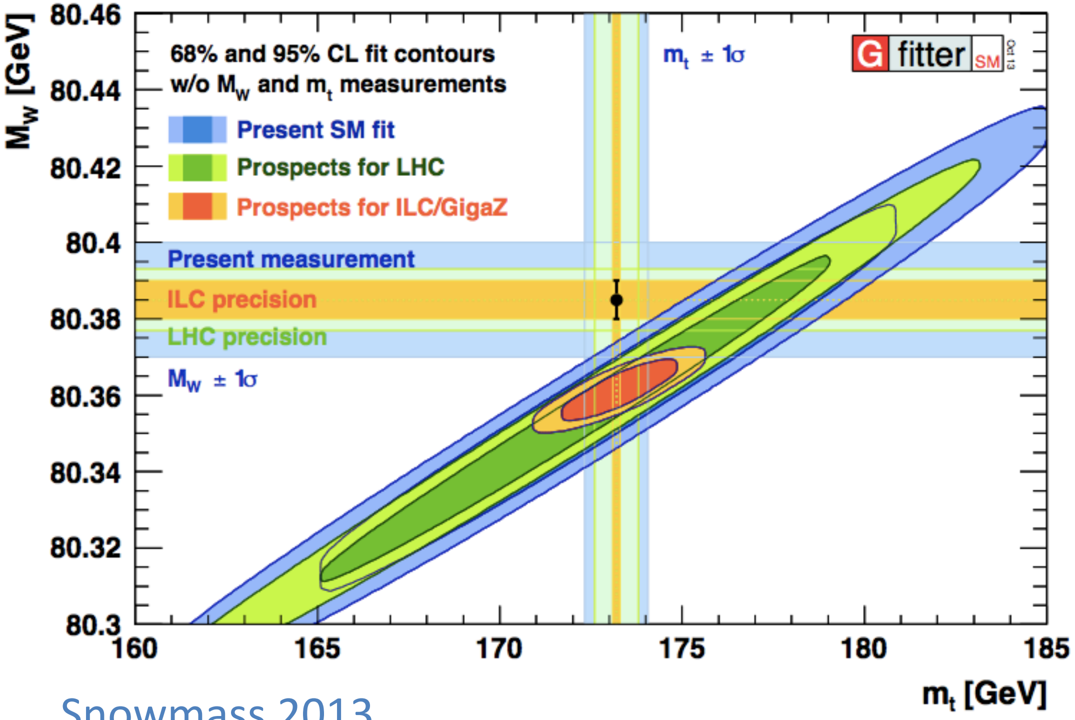
From tt cross section

- $m_t^{pole} = 172.9\,^{+2.5}_{-2.6}$ GeV ATLAS, Eur.Phys.J. C74, 3109 (2014)
- $m_t^{pole} = 176.7\,^{+3.8}_{-3.4}$ GeV CMS, PLB 728, 496 (2014)

Using $t\bar{t}$+1-jet events

- $m_t^{pole} = 173.7\,^{+2.3}_{-2.1}$ GeV ATLAS-CONF-2014-053

EW precision tests for SM

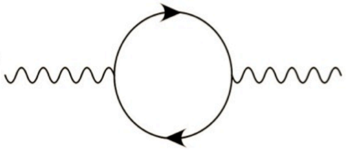


Snowmass 2013

Radiative corrections are often sensitive to the top quark mass

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2\theta_W}(1+\Delta r)$$

Radiative corrections



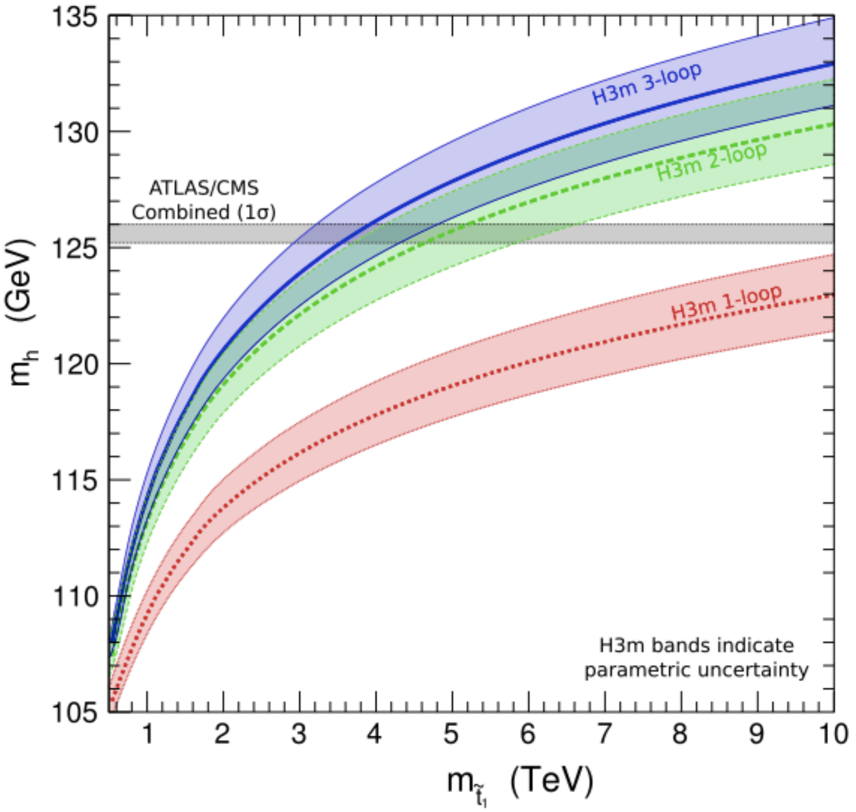
$$\Delta r = \frac{\alpha}{\pi s^2}\left(-\frac{3}{16}\boxed{\frac{m_t^2}{M_W^2}}\frac{c^2}{s^2} + \frac{11}{48}\ln\boxed{\frac{m_H^2}{m_Z^2}}\right) + \cdots$$

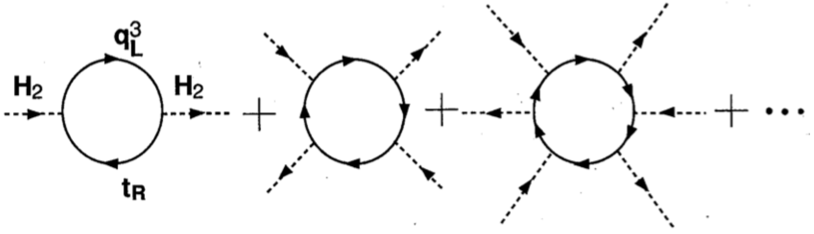
$$s^2 = \sin^2\theta_W,\ c^2 = \cos^2\theta_W$$

It is important to test for deviations from SM predictions in the EW sector

Beyond SM

For example, SUSY





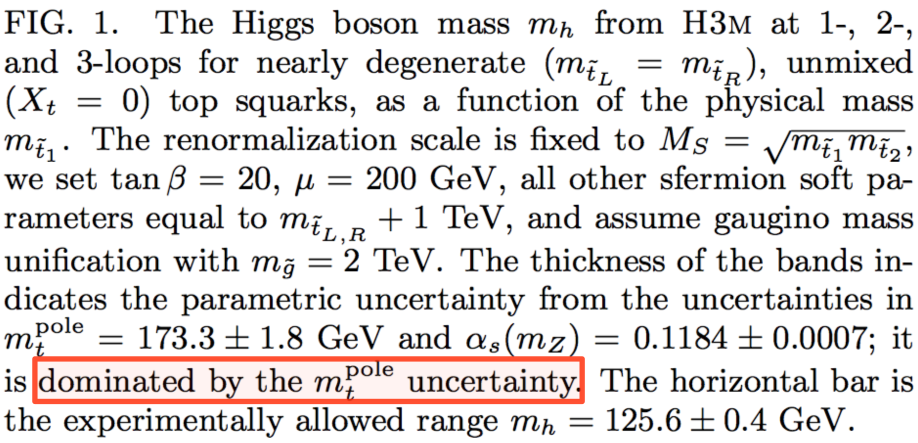


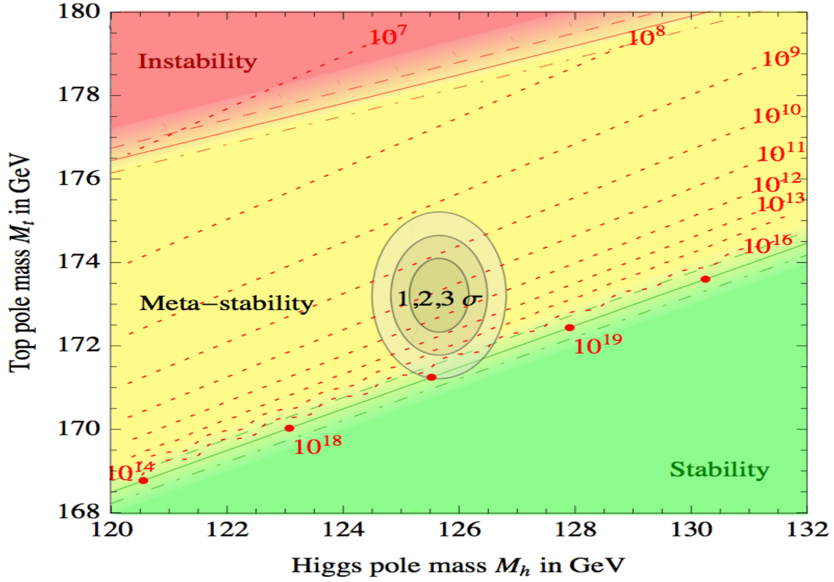
FIG. 1. The Higgs boson mass m_h from H3M at 1-, 2-, and 3-loops for nearly degenerate ($m_{\tilde{t}_L} = m_{\tilde{t}_R}$), unmixed ($X_t = 0$) top squarks, as a function of the physical mass $m_{\tilde{t}_1}$. The renormalization scale is fixed to $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, we set $\tan\beta = 20$, $\mu = 200$ GeV, all other sfermion soft parameters equal to $m_{\tilde{t}_{L,R}} + 1$ TeV, and assume gaugino mass unification with $m_{\tilde{g}} = 2$ TeV. The thickness of the bands indicates the parametric uncertainty from the uncertainties in $m_t^{\text{pole}} = 173.3 \pm 1.8$ GeV and $\alpha_s(m_Z) = 0.1184 \pm 0.0007$; it is dominated by the m_t^{pole} uncertainty. The horizontal bar is the experimentally allowed range $m_h = 125.6 \pm 0.4$ GeV.

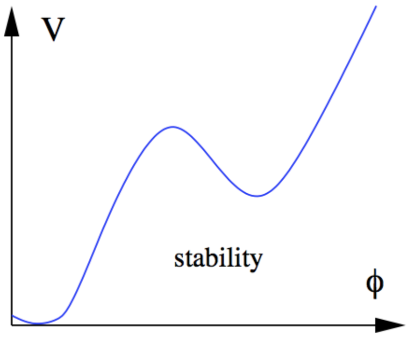
Feng et al., PRL 111,131802(2013)

The predictions beyond the SM often depend strongly on the value of the top quark mass.

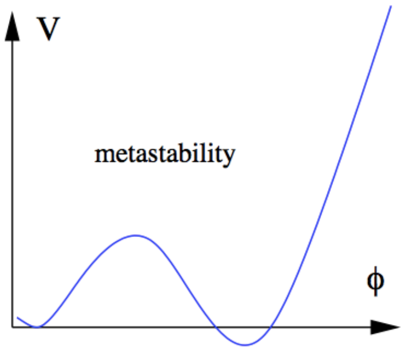
SM vacuum stability

Is the SM vacuum stable or metastable ?





Stable



Meta-stable

Degrassi et al. ’12

Buttazzo et al. ’13

Type of error	Estimate of the error	Impact on M_h
M_t	Experimental uncertainty in M_t	±1.4 GeV
α_s	Experimental uncertainty in α_s	±0.5 GeV
Experiment	Total combined in quadrature	±1.5 GeV
λ	Scale variation in λ	±0.7 GeV
y_t	$\mathcal{O}(\Lambda_{\rm QCD})$ correction to M_t	±0.6 GeV
y_t	QCD threshold at 4 loops	±0.3 GeV
RGE	EW at 3 loops + QCD at 4 loops	±0.2 GeV
Theory	Total combined in quadrature	±1.0 GeV

Precision measurements of m_t , α_s , m_H are needed.

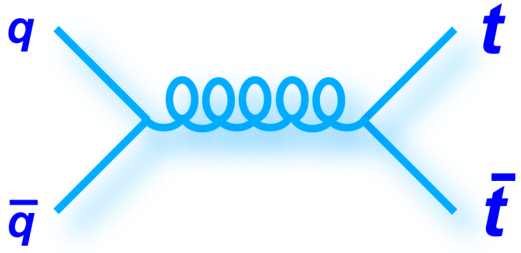
Top quarks at Tevatron and LHC

Tevatron

- $p\bar{p}$ collider
- discovered the top quark in 1995
- shut down in 2011

$\sigma(t\bar{t}) \sim 8$pb

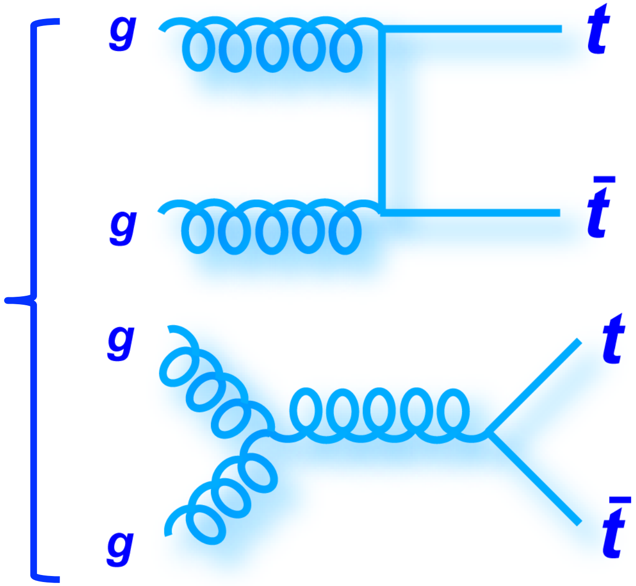
Main production process
: $t\bar{t}$ pair production



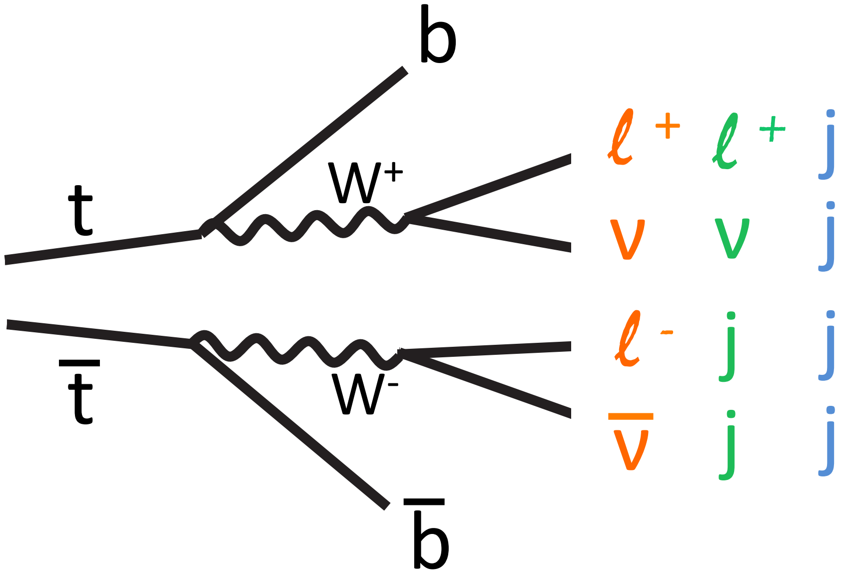
LHC

$\sigma(t\bar{t}) \sim 900$pb
($\sqrt{s}$ = 14 TeV)

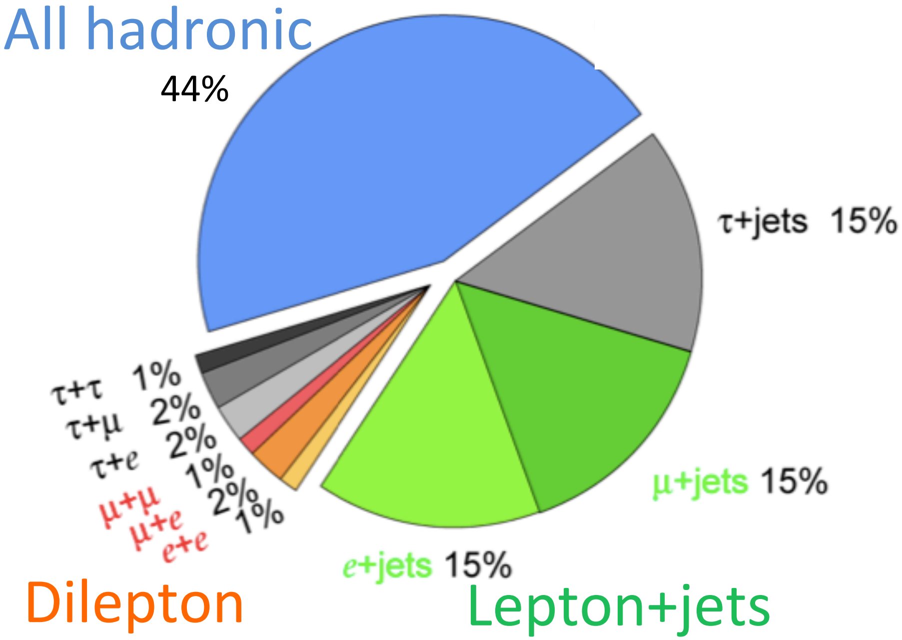
- pp collider
- ended data taking at $\sqrt{s}$ = 7, 8 TeV
- restart in 2015 at $\sqrt{s}$ = 13/14 TeV



$t\bar{t}$ decay channel



$t\bar{t}$ branching ratio

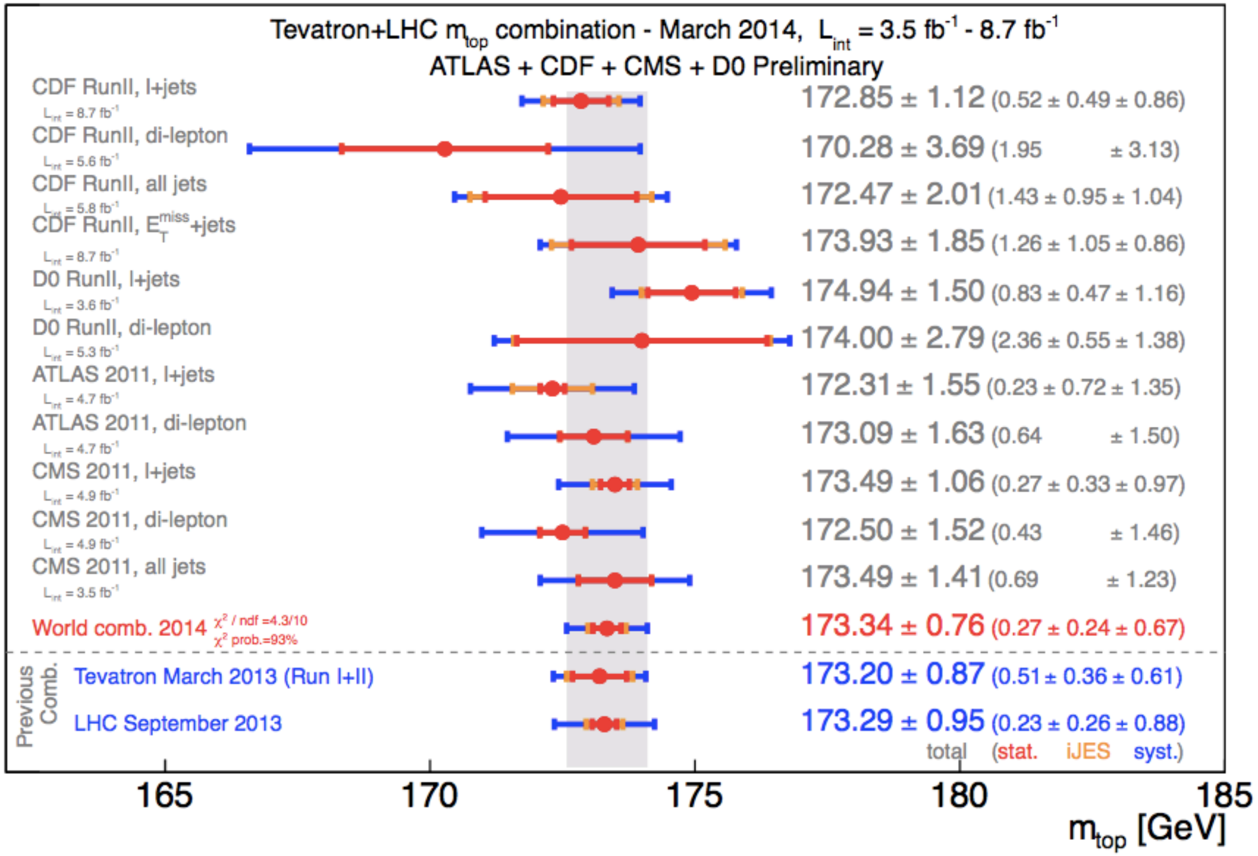


	Cross section	S / N
Dilepton	Small	Very good
Lepton+jets	Medium	Good
All hadronic	Large	Not good

Current status of top mass measurements

(direct measurement)

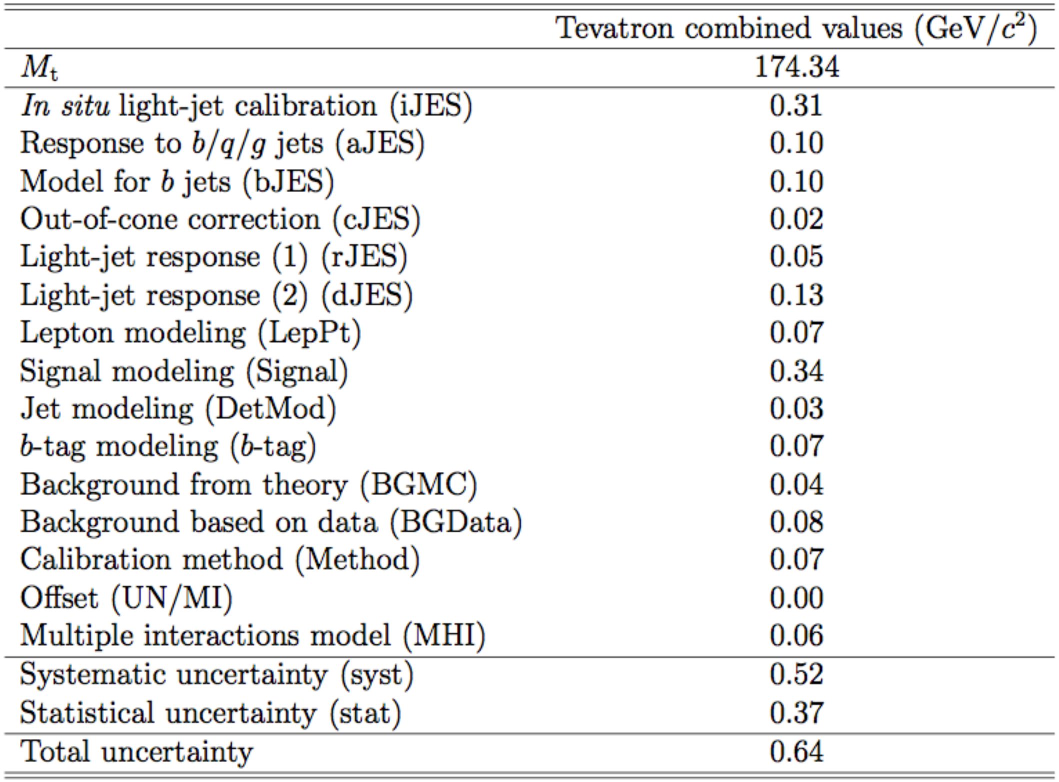
- Tevatron+LHC m_t combination (arXiv:1403.4427)



0.4% precision!

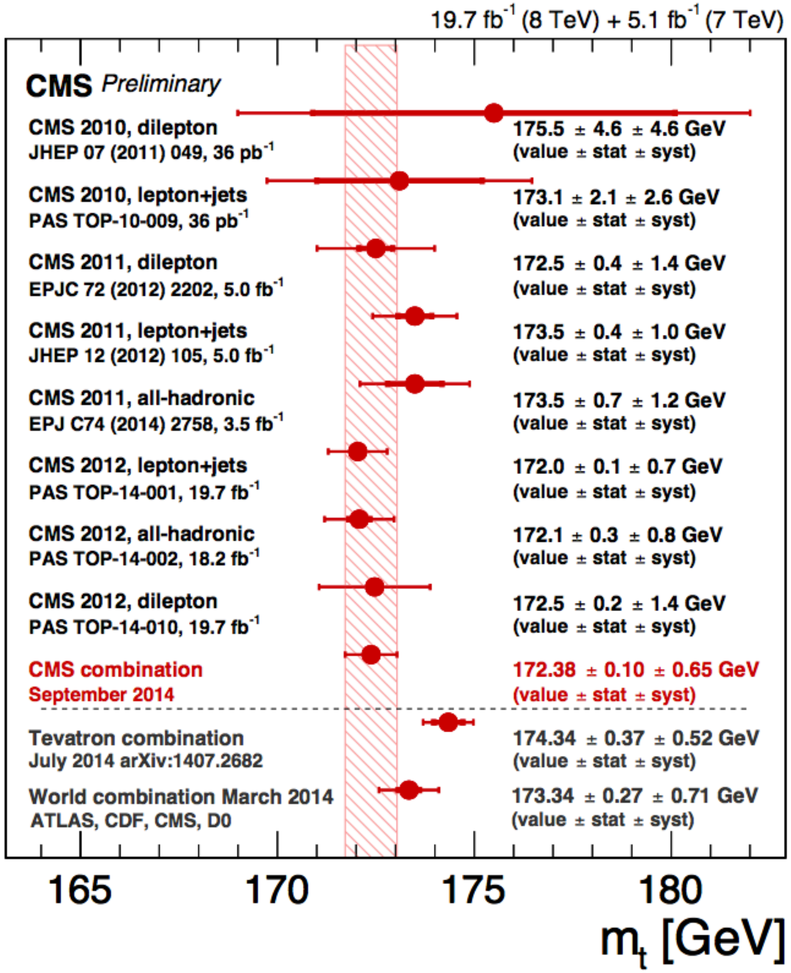
- Tevatron combination (arXiv:1407.2682) m_t = 174.34 ± 0.64 GeV
- CMS combination (Sep. 2014) m_t = 172.38 ± 0.65 GeV

Tevatron combination



	Tevatron combined values (GeV/c^2)
M_t	174.34
In situ light-jet calibration (iJES)	0.31
Response to $b/q/g$ jets (aJES)	0.10
Model for b jets (bJES)	0.10
Out-of-cone correction (cJES)	0.02
Light-jet response (1) (rJES)	0.05
Light-jet response (2) (dJES)	0.13
Lepton modeling (LepPt)	0.07
Signal modeling (Signal)	0.34
Jet modeling (DetMod)	0.03
b-tag modeling (b-tag)	0.07
Background from theory (BGMC)	0.04
Background based on data (BGData)	0.08
Calibration method (Method)	0.07
Offset (UN/MI)	0.00
Multiple interactions model (MHI)	0.06
Systematic uncertainty (syst)	0.52
Statistical uncertainty (stat)	0.37
Total uncertainty	0.64

CMS combination



Top quark mass?

- m_t^{MC} : Not a parameter defined in perturbative theory

- m_t^{pole} : Top quark has a color, so the physical on-shell quark cannot exist

⇨ Far from a fundamental param.

$$\frac{1}{\not p - m_0 - \Sigma(p, m_0)} = \frac{c}{\not p - m}$$

$$m = m(\mu)\Big(1 + \alpha_s(\mu)d^1 + \alpha_s^2(\mu)d^2 + \ldots\Big)$$

- $m_t^{\overline{MS}}$: Short-distance mass
  Free from IR contamination

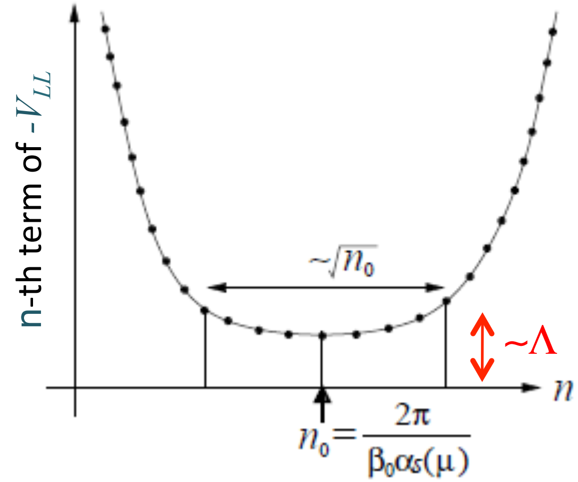
$$m_0 = m(\mu)\Big(1 + \frac{\alpha_s}{\pi}\Big[\frac{1}{\epsilon}\Big]\Big)$$

⇨ Known as a good parameter in pert. QCD

Important to determine $m_t^{\overline{MS}}$ accurately

Pole mass and $\overline{\text{MS}}$ mass

From Sumino-san's slide

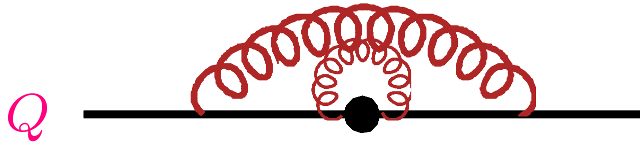


$$m_{\text{pole}} \simeq m_{\overline{\text{MS}}}(\mu) + \frac{1}{2}\int_{q<\mu} \frac{d^3\vec{q}}{(2\pi)^3} C_F \frac{4\pi\alpha_{1L}(q)}{q^2}$$

Pole mass m_{pole}

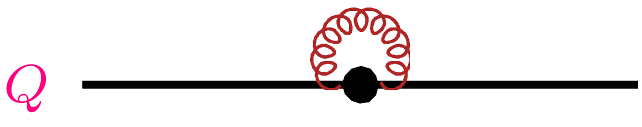
$\overline{\text{MS}}$ mass $\overline{m} \equiv m_{\overline{\text{MS}}}(m_{\overline{\text{MS}}})$

$0 < \lambda_g < \infty$



m_{bare}

$0 < \lambda_g < 1/\overline{m}$

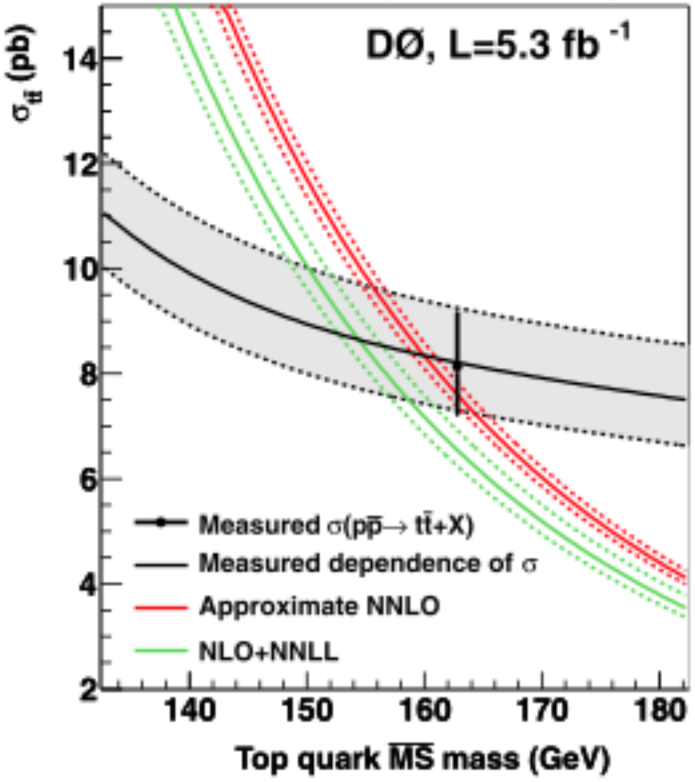


m_{bare}

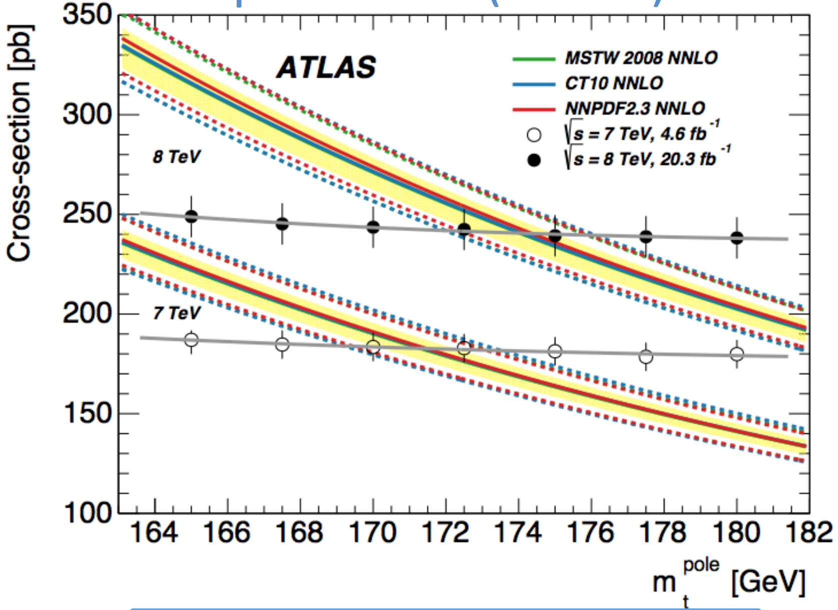
Measurement of $\overline{\text{MS}}$ mass (and pole mass)

(from $t\bar{t}$ cross section)

$\overline{\text{MS}}$ mass (Tevatron)



pole mass (ATLAS)



$$m_t^{\text{pole}} = 172.9^{+2.5}_{-2.6}\ \text{GeV}$$

$$m_t^{\overline{\text{MS}}} = 160.0^{+5.1}_{-4.5}\ \text{GeV}$$

$$m_t^{\overline{\text{MS}}} = 154.5^{+5.2}_{-4.5}\ \text{GeV}$$

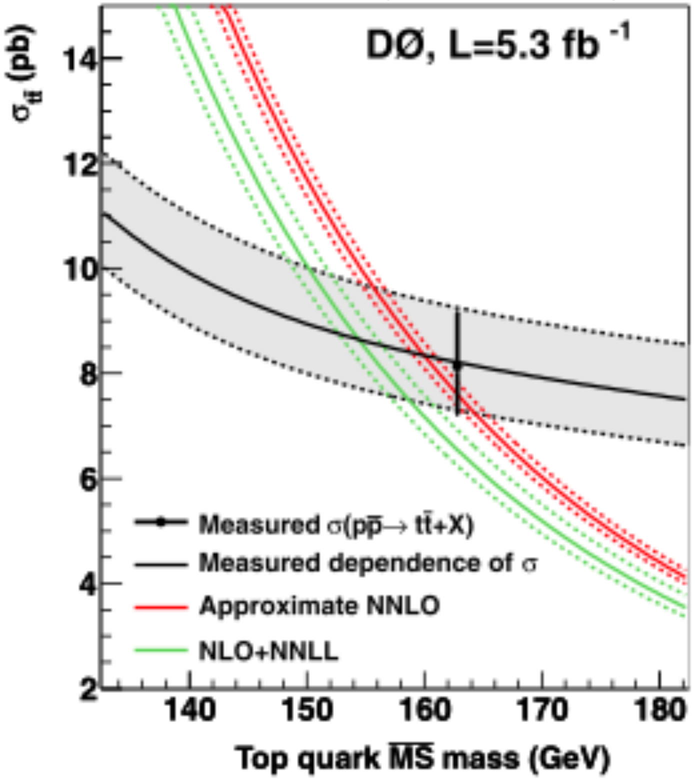
The errors are still large.

- The sensitivity of $\sigma_{t\bar{t}}$ to m_t is not so strong
- Theoretical uncertainties ∼ 1.5 - 2 GeV

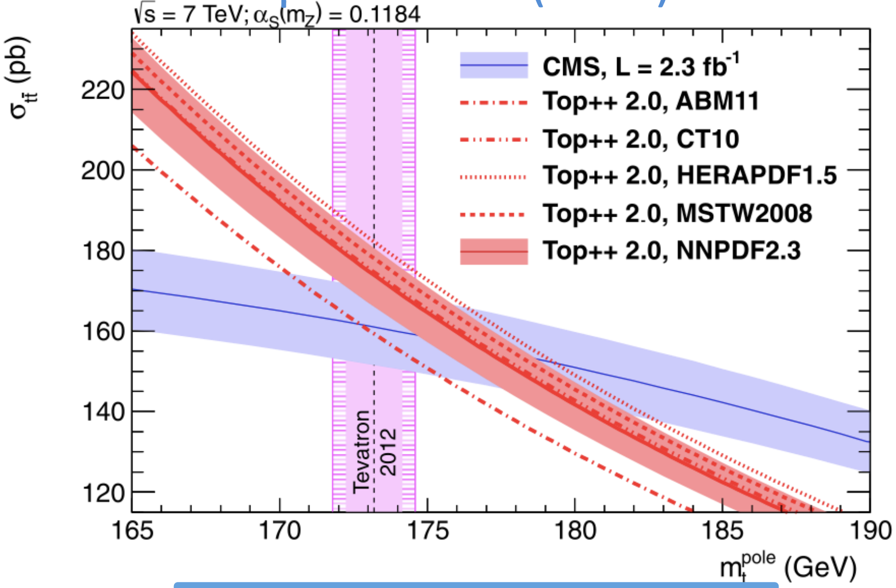
Measurement of $\overline{\text{MS}}$ mass (and pole mass)

(from $t\bar{t}$ cross section)

$\overline{\text{MS}}$ mass (Tevatron)



pole mass (CMS)



$$m_{\mathrm{t}}^{\mathrm{pole}} = 176.7^{+3.8}_{-3.4}\ \mathrm{GeV}$$

$$m_t^{\overline{\mathrm{MS}}} = 160.0^{+5.1}_{-4.5}\ \mathrm{GeV}$$

$$m_t^{\overline{\mathrm{MS}}} = 154.5^{+5.2}_{-4.5}\ \mathrm{GeV}$$

The measured $\overline{\text{MS}}$ mass still has a large error

- The sensitivity of $\sigma_{t\bar{t}}$ to m_t is not so strong
- Theoretical uncertainties ∼ 1.5 - 2 GeV

Other methods for measurement of m_t

- $t\bar{t}$ cross section D0, PLB 703, 422 (2011)
  CMS, PLB 728, 496 (2014)
- Kinematic endpoints CMS, Eur.Phys.J. C73, 2494 (2013)
- J/ψ method A. Kharchilava, PLB 476, 73 (2000)
- B-hadron lifetime (L_{xy}) C. S. Hill, J. R. Incandela, J. M. Lamb, PRD 71, 054029 (2005)
- $t\bar{t}$+1-jet differential distribution
  S. Alioli, P. Fernandez, J. Fuster, A. Irles, S.O. Moch, P. Uwer, M. Vos, Eur.Phys.J. C73, 2438 (2013)

⋮

We propose a totally different method

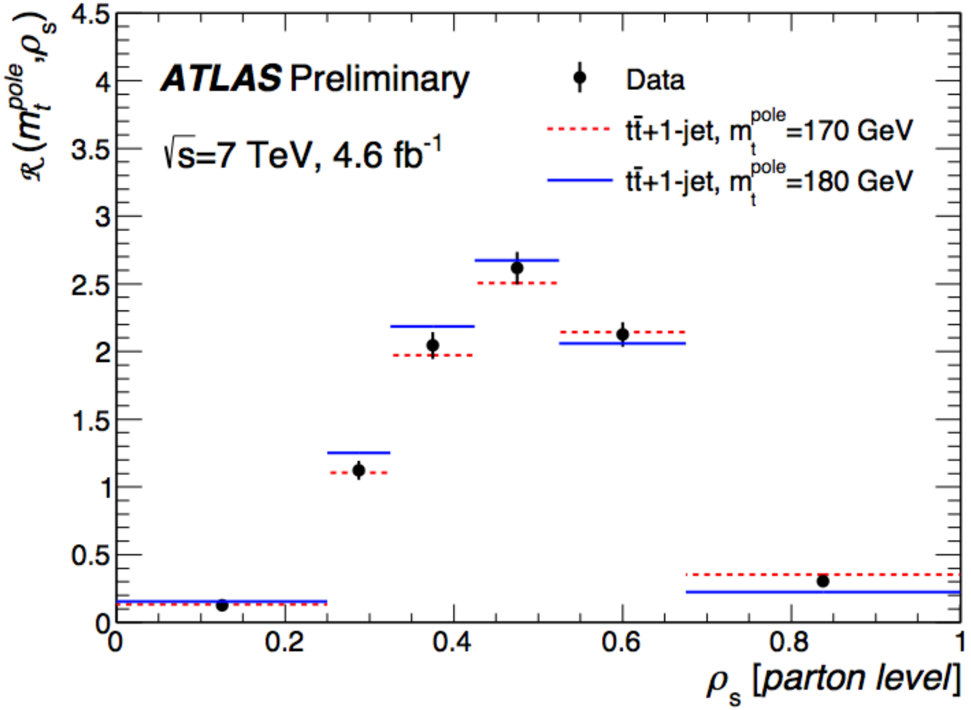
- to measure a theoretically well-defined top mass
- using lepton energy distribution

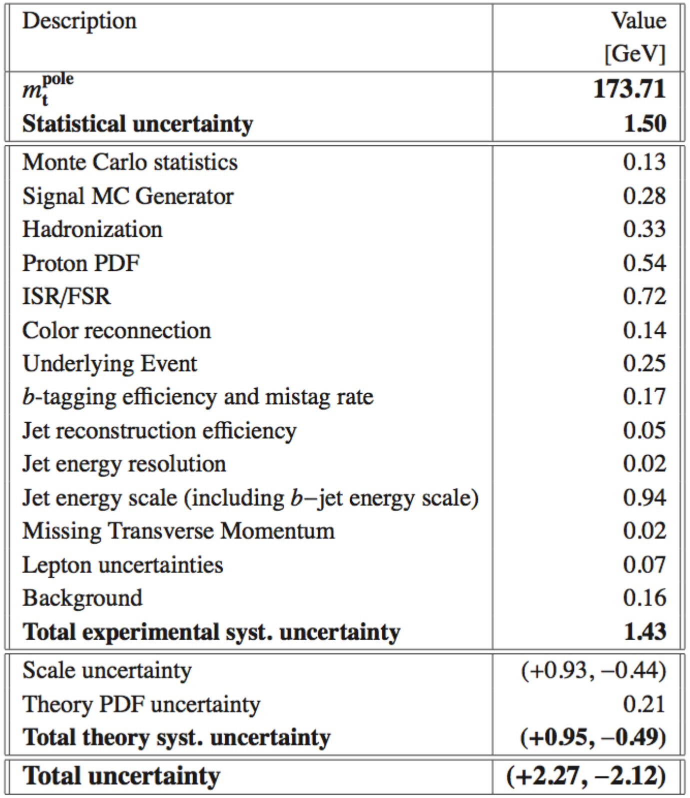
Using tt+1-jet differential distribution

ATLAS-CONF-2014-053

$$\mathcal{R}(m_t^{\text{pole}}, \rho_s) = \frac{1}{\sigma_{tt+1\text{-jet}}} \frac{d\sigma_{tt+1\text{-jet}}}{d\rho_s}(m_t^{\text{pole}}, \rho_s),$$

$$\rho_s = \frac{2m_0}{\sqrt{s_{ttj}}},$$





Description	Value [GeV]
m_t^{pole}	**173.71**
Statistical uncertainty	**1.50**
Monte Carlo statistics	0.13
Signal MC Generator	0.28
Hadronization	0.33
Proton PDF	0.54
ISR/FSR	0.72
Color reconnection	0.14
Underlying Event	0.25
b-tagging efficiency and mistag rate	0.17
Jet reconstruction efficiency	0.05
Jet energy resolution	0.02
Jet energy scale (including b−jet energy scale)	0.94
Missing Transverse Momentum	0.02
Lepton uncertainties	0.07
Background	0.16
Total experimental syst. uncertainty	**1.43**
Scale uncertainty	(+0.93, −0.44)
Theory PDF uncertainty	0.21
Total theory syst. uncertainty	**(+0.95, −0.49)**
Total uncertainty	**(+2.27, −2.12)**

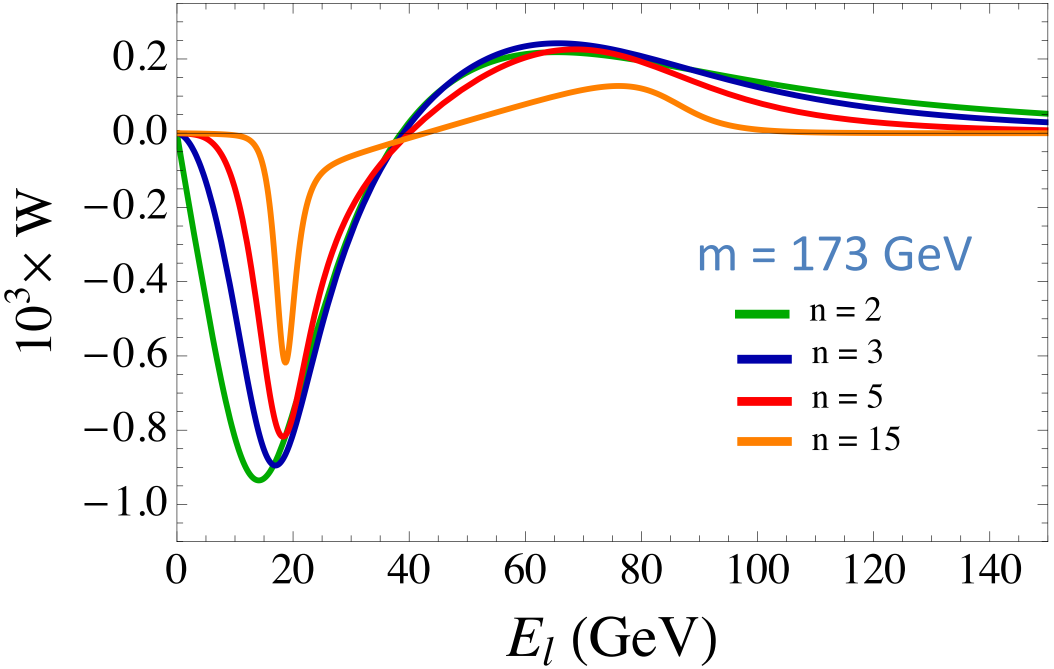
Table 3: Value of the inferred top-quark pole mass and of its uncertainties.

Examples of weight functions

For a top quark decay : t → Wb → ℓ νb

$$W(E_l, m) = \int dE\, \mathcal{D}_0(E\,; m) \frac{1}{E E_l} \boxed{(\text{odd func. of } \rho)} \Bigg|_{e^{\rho} = E_l / E}$$

$$\boxed{(\text{odd func. of } \rho) = \frac{n \tanh(n\rho)}{\cosh(n\rho)}}$$



$$W(E_l, m) = \int dE\, \mathcal{D}_0(E\,; m) \frac{2n E_l^{n-1} E^{n-1} (E_l^{2n} - E^{2n})}{(E_l^{2n} + E^{2n})^2}$$

Polarization of top quarks at LHC

The polarization of top quarks at LHC ?

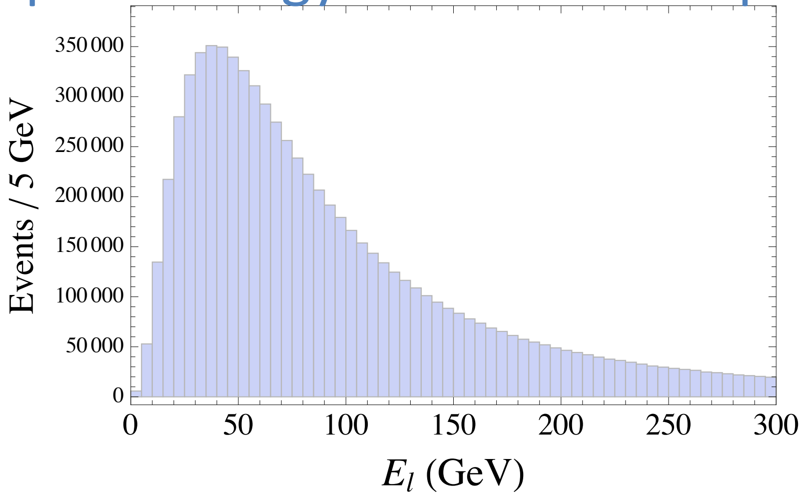
$\longrightarrow$ Almost unpolarized

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{lt}} = \frac{1}{2}(1 + B\cos\theta_{lt})$$

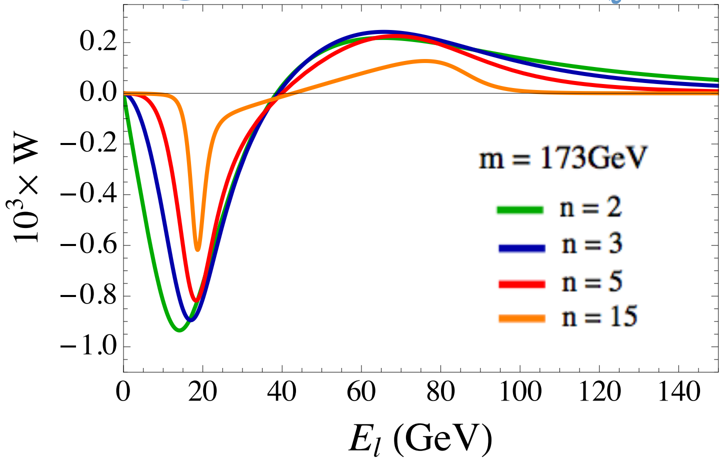
SM: B $\sim$ 0.006,   ATLAS: compatible with zero

Parton level analysis

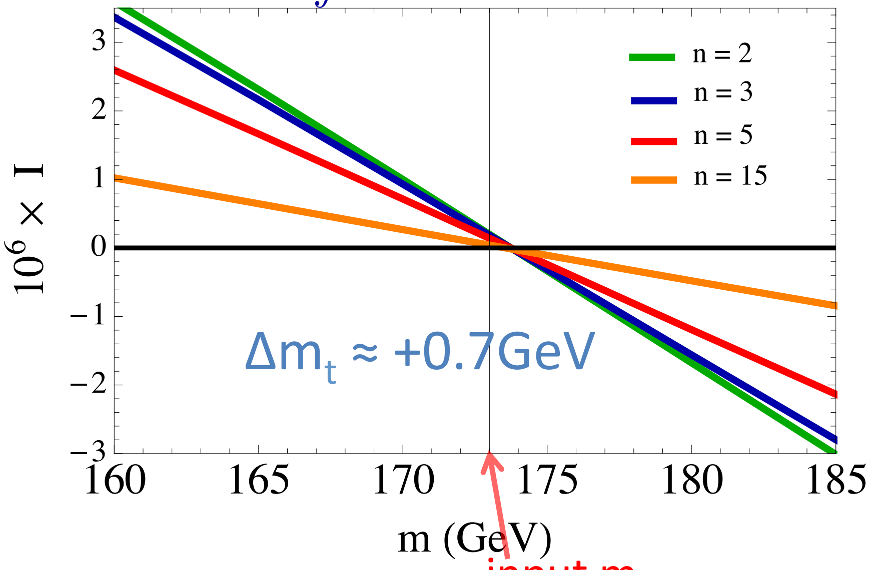
Lepton energy distribution at parton level (signal)



Weight function $W(E_\ell, m)$



$$I(m) \equiv \int dE_l D(E_l) W(E_l, m)$$



$\Delta m_t \approx +0.7$GeV

input m_t

- Effect of Γ_t : +0.34GeV
- MC stat. error : 0.4GeV

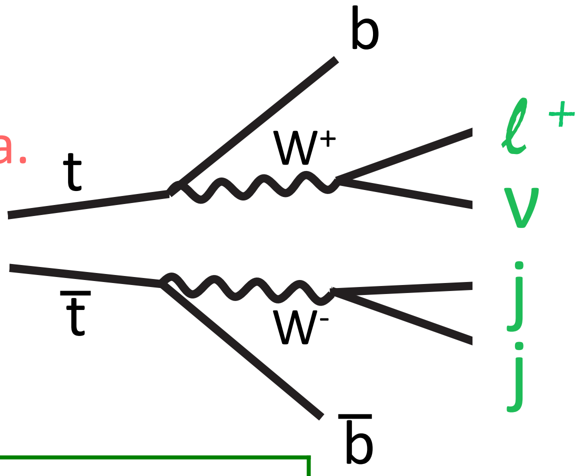
➡ Consistent with expectation

In principle, our method works

Event selection cuts

- 1 muon with $p_T > 20\text{GeV}$, $|\eta| < 2.4$ (lepton cuts)
- At least 4 jets
- At least 1 b-tag with the b-tag efficiency 0.4 independent of p_T and η
- $p_T(j_1) > 55$, $p_T(j_2) > 25$, $p_T(j_3) > 15$, $p_T(j_4) > 8\text{GeV}$

We do not use cuts concerning missing momenta.



Cross section after all cuts

Signal (m_t=173GeV)	Other $t\bar{t}$ BG	W+jets BG	$Wb\bar{b}$+jets BG	Single top BG
22.4 pb	5.7 pb	1.8 pb	1.8 pb	1.3 pb

Effects of lepton isolation and photon emissions

These effects are well understood, it should be possible in principle to estimate them and restore the parton-level lepton distributions.

⇨ We assumed that they can be estimated and restored completely for signal events in this analysis.

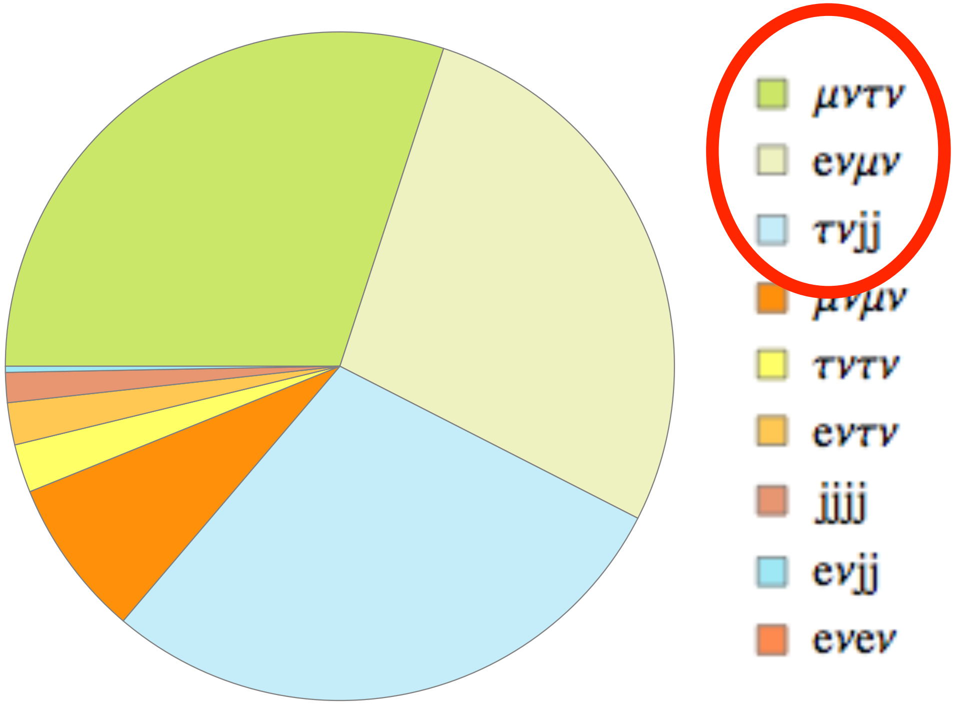
- Estimate of lepton isolation
  - Isolation cone angle → 0
  - Compensate for the loss caused by the lepton isolation
- Effect of photon emissions
  - Can include it into weight function by calculating the lepton distribution with the effect

Flat b-tagging efficiency

At least 1 b-tag with the b-tag efficiency 0.4 independent of p_T and η (in the region $p_T > 15\text{GeV}$ and $|\eta| < 2.5$)

- This flat efficiency would be attainable in experiments in principle.
- If a b-tagging efficiency $\varepsilon(p_T,\eta)$ can be estimated with a good accuracy, multiplying the lepton energy distribution by $\varepsilon(p_T,\eta)^{-1}$ can be an alternative way.

Other $t\bar{t}$ BG after cuts

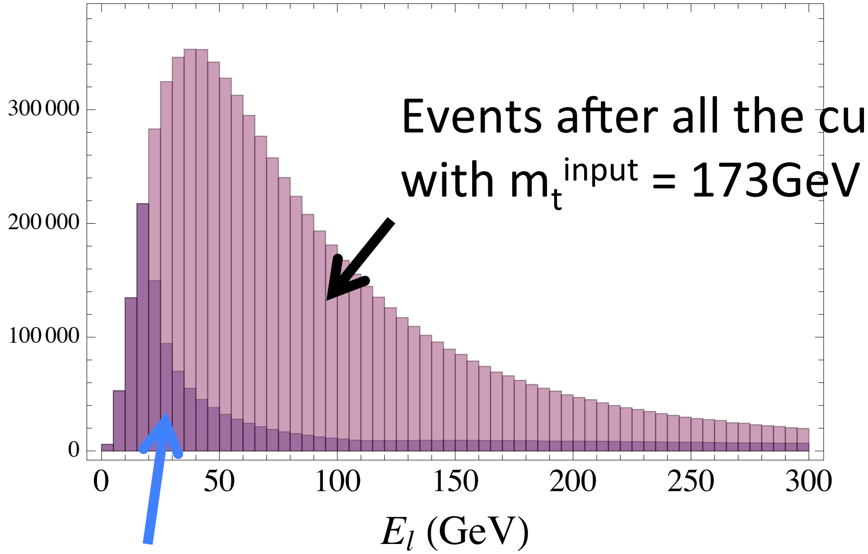


These decays can be predicted accurately

- μντν, eνμν : Can be included in the signal events in principle
- Muon from tau decay : Can be regarded as a signal process by including the contribution of the muon energy distribution into weight functions

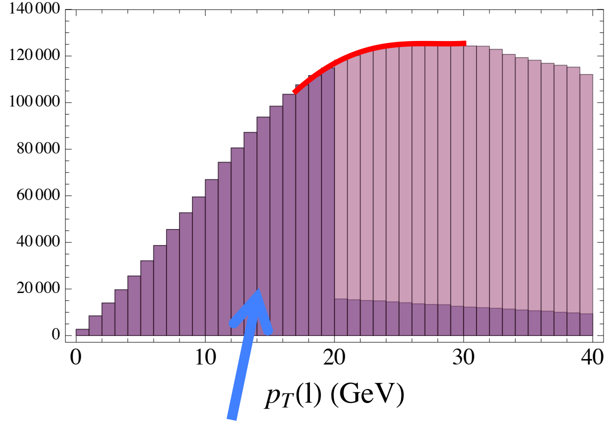
A solution to the problem of lepton cuts

We compensate for the loss using MC events.



Events after all the cuts with m_t^{input} = 173GeV

Compensated MC events with m_t^c



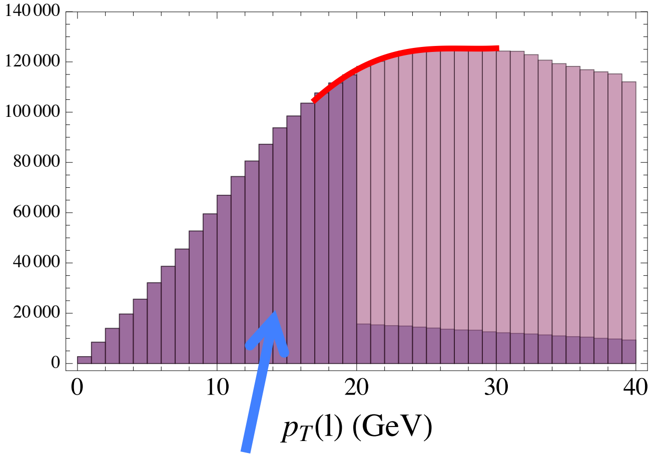
Compensated MC events

- We need to assume some value for m_t of the compensated MC events

- To fix the normalization of the added events, perform a χ^2-fit so that p_T(l) distributions are connected smoothly

Compensating method

We compensate for the loss ($p_T(l)<20$GeV or $|\eta(l)|>2.4$) using MC events.



Compensated MC events

To fix the normalization of the compensated events, perform a χ^2 fit so that $p_T(l)$ distribution are connected smoothly.

We do not use detailed knowledge on the global shape of the distribution

➡ As a result, we obtain a good feature that I(m) does not depend strongly on the top quark mass of the compensated events.

We expect that we can improve the quality of the fit.

(fitting function, correction to the p_T distribution, raise lepton p_T cut)

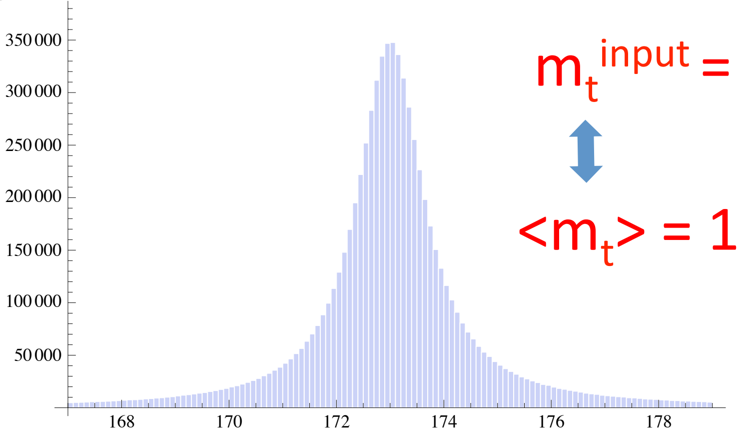
Effects of top width and MC stat. error

- Effects of top width

The weight function method assumes that the top quark is on-shell

⇨ We should include corrections for off-shellness of the top quark

In our analysis (BW dist. with cut-off at $m_t \pm 50*\Gamma_t$)



$m_t^{input} = 173\text{GeV}$

↕

$\langle m_t \rangle = 173.34\text{GeV}$

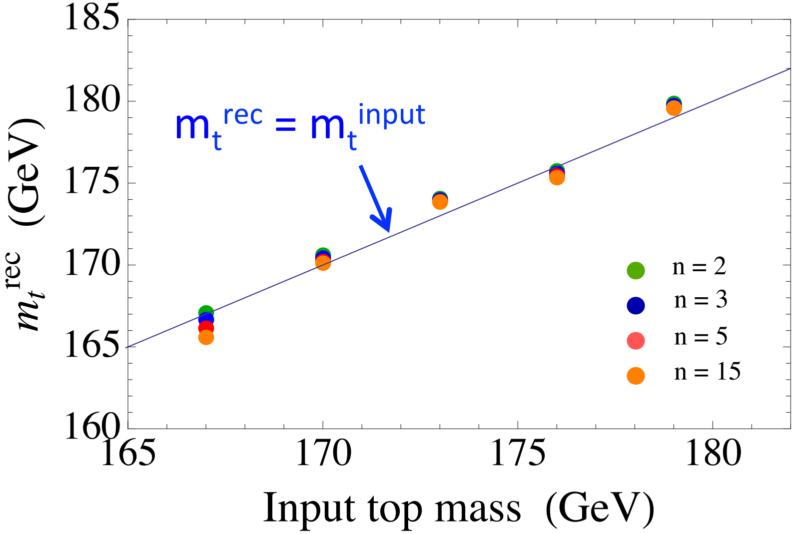
Top invariant mass distribution at parton level

- MC statistical errors after all cuts [GeV] ($m_t^{input} = 173\text{GeV}$)

n = 2	3	5	15
+1.0/-1.1	±1.1	±1.1	+1.2/-1.3

They include the MC statistical errors of the compensated part.

Sensitivity of mass determination



n=2

Input top mass(GeV)	167	170	173	176	179
m_t^{rec}(GeV)	167.1	170.6	174.1	175.7	179.9

- At 100 fb^{-1}
- Lepton(e,μ)+jets channel
- Assuming that the error of electron mode is the same as the muon mode

Uncertainties [GeV]

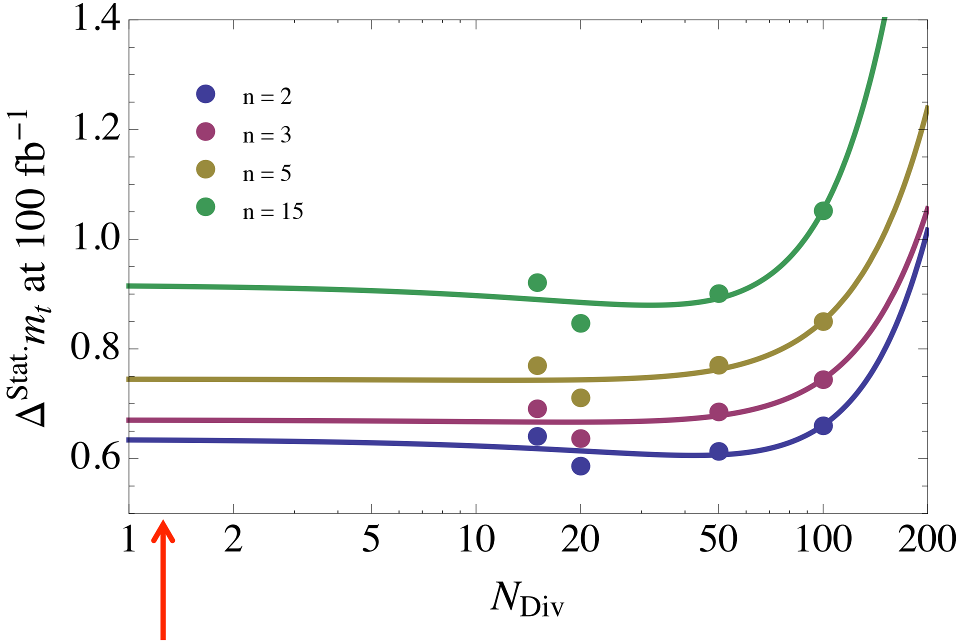
	Signal stat. error	μ_F scale	PDF	JES	BG stat. error
n = 2	0.4	+1.5/-1.4	0.6	+0.5/-0.0	0.4
3	0.5	+1.4/-1.3	0.8	+0.7/-0.1	0.4
5	0.5	+1.4/-1.2	1.1	+0.8/-0.2	0.4
15	0.6	+1.4/-1.2	1.4	+0.9/-0.3	0.4

Can be improved by including NLO

Signal statistical error

$m_t^{\text{input}} = 173\text{GeV}$

We divide the generated events into 15, 20, 50 and 100 subgroups of equal sizes and perform the top mass reconstruction.



Results of fits depend on the number of events

We extrapolate statistical errors at the number of events for 100fb^{-1}

N_{Div} corresponding to the number of events with 100 fb^{-1} = 1.2

	n=2	n=3	n=5	n=15
only μ mode	0.63	0.67	0.74	0.91
semi-leptonic channel	0.44	0.47	0.53	0.65

Future work: NLO and NNLO

- Lepton energy distribution in the rest frame of the top quark at NLO, NNLO

  ➡ Weight function at NLO, NNLO

- MC simulator with NLO top decay

  ➡ Compensated events at NLO

- MC simulator with NNLO top decay

  We can obtain parton level lepton distributions
  using the lepton dist. in the rest frame of top quark at NNLO
  and velocity dist. of top quarks from MC.

  ➡ Compensated events at NNLO

- MC simulator with NLO top production

  ➡ Reduce uncertainties due to μ_F scale dependence

Future work: top off-shellness

Incorporate the effects of off-shellness of the top quark into weight function:

- Contributions from diagrams with top quark

Use the superposition of the lepton distribution in the top rest frame with the weight of top invariant mass distribution

- Contributions from diagrams without top quark (Irreducible background)

Define "the rest frame of the top quark" as the center-of-mass frame of b and W boson, and incorporate this contribution into the lepton dist.

We do not know whether these work. At least, we can estimate corrections caused by top off-shellness effects.

Future work: $\overline{\mathrm{MS}}$ mass

- Determination of m_t^{pole} → $m_t^{\overline{\mathrm{MS}}}$

3-loop relation

- Direct determination of $m_t^{\overline{\mathrm{MS}}}$

Lepton energy dist. in the top rest frame with $m_t^{\overline{\mathrm{MS}}}$

α_s expansion is not a good approximation
in a part of phase space

→ But good for the weighted integral I(m) ?

Also, Including effects of

- final state radiation,
- top off-shellness,

the convergence will improve.