

# Mass Minimization without Prejudice

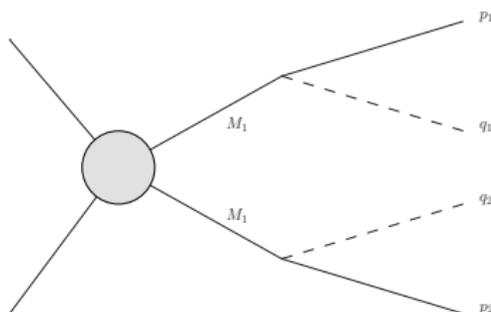
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to be appeared on arXiv soon

# Pair Production of Semi-invisibly Decaying Particles and Cambridge- $M_{T2}$



$M_{T2}$  is a mass variable designed to detect masses of identical pair of particles decaying semi-invisibly.

$$M_{T2} \equiv \min_{q_{1,T}, q_{2,T}} \max[M_T(p_1, q_1), M_T(p_2, q_2)]$$

$$q_{1,T} + q_{2,T} = E_T$$

$M_{T2}$  distribution is bounded above by mother particle's true mass, like  $M_T$ 's case.

$$M_{T2} \leq \text{parent mass}$$

However, when we do an analysis at the beginning, we don't need to assume the parents are identical.

# $M_{T2}$ in Non-identical Pair Production

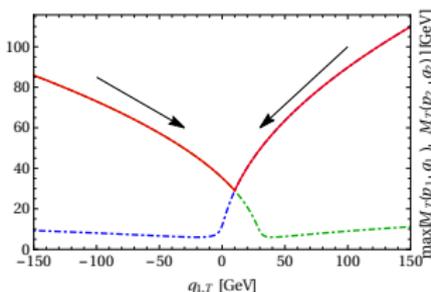
If we consider non-identical pair production,  $M_{T2}$  falls into several difficulties:

- The endpoint is only sensitive to the heavy one.

$$M_{T2} \leq \max(M_1, M_2)$$

- $M_{T2}$  does not saturate to the expected endpoint.

These weakness of  $M_{T2}$  is originated from the fact that significant portion of the  $M_{T2}$  solutions are balanced in two transverse mass.



In the case of balanced configuration of transverse momenta,  $M_{T2}$  has an intrinsic constraint

$$M_T(p_1, q_1) = M_T(p_2, q_2)$$

Since the balancedness is encoded in the maximum function, we can try alternative objective functions to get away from symmetric intrinsic constraint.

# Minimization of Generalized Mean: Power Mean

As a continuous and smooth generalization of maximum, we considered a power mean  $\mu_p$ .

$$\hat{\mu}_p(f_1, \dots, f_n) = \left( \frac{1}{n} \sum_{i=1}^n f_i^p \right)^{\frac{1}{p}}$$

p	function
$\infty$	Maximum
2	Root mean square
1	Mean
0	Geometric Mean
-1	Harmonic Mean
$-\infty$	Minimum

- One can define a *minimized power mean*, to construct mass-bounding variables.

$$\mu_p[f] = \min \hat{\mu}_p(f(1), \dots, f(n))$$

- As  $p \rightarrow \infty$ , they converges to their maximum variant such as

$$\lim_{p \rightarrow \infty} \mu_p[M_T] = M_{T2} \equiv \min_{q_1, T+q_2, T \in \mathbb{E}_T} \max[M_T(p_1, q_1), M_T(p_2, q_2)]$$

$$\lim_{p \rightarrow \infty} \mu_p[M] = M_2 \equiv \min_{q_1, q_2, T \in \mathbb{E}_T} \max[M(p_1, q_1), M(p_2, q_2)]$$

# Endpoint of $\mu_p$ and the Intersection

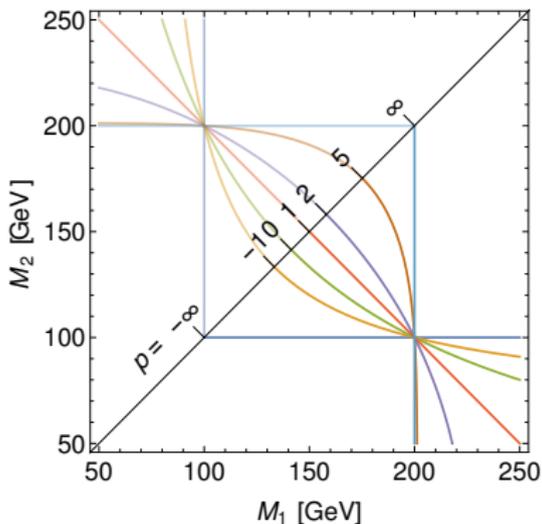
- $\mu_p$  also has upper bound originated from its functional form.

$$\mu_p[M_T] \leq \hat{\mu}_p(M_1, M_2), \quad \mu_p[M] \leq \hat{\mu}_p(M_1, M_2)$$

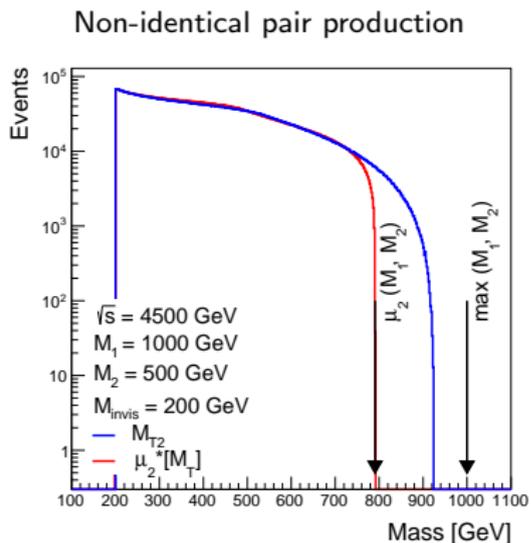
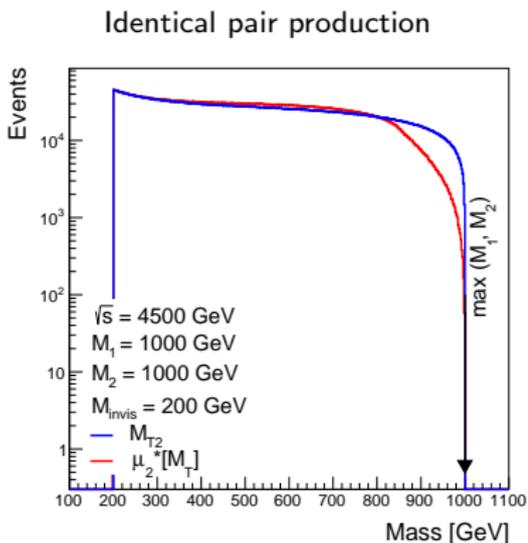
- Each endpoints of power means  $\mu_p$  distribution constrains distinct region of mass spectrum.

$$\hat{\mu}_p(M_1, M_2) = \mu_p[M_T]^{\max}$$

- The constrained regions eventually intersect around the true parents masses.
- We can pin down the mass spectrum by the endpoint analysis of  $\mu_p$ 's.



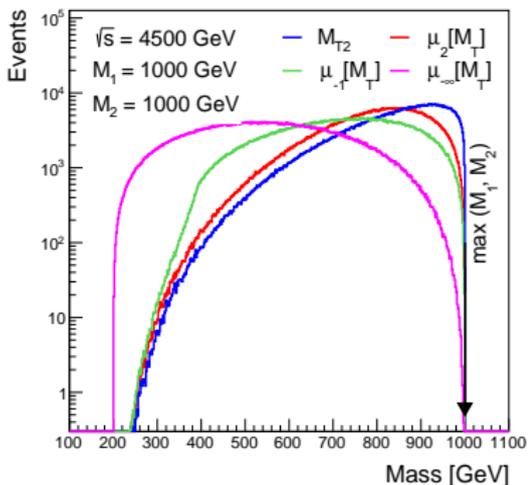
# Distribution of $\mu_p$ : Two-body Decay



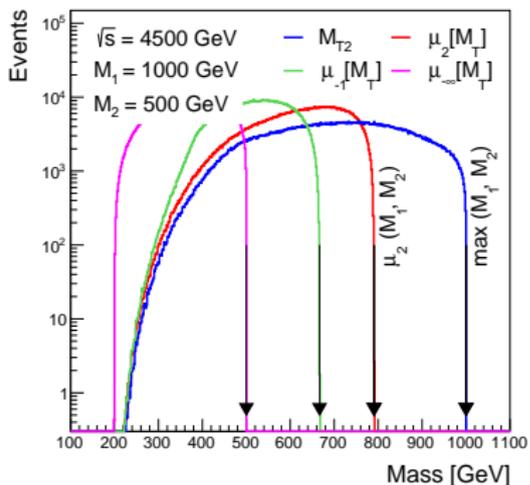
- Sample events are generated from Monte Carlo simulation assuming constant cross section. ISR is not considered.
- Balanced  $\mu$  configurations are dominates.

# Distribution of $\mu_p^*$ : Three-body Decay

## Identical pair production



## Non-identical pair production

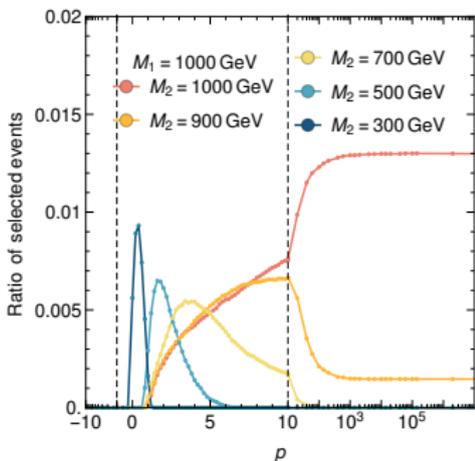


- Sample events are generated from Monte Carlo simulation assuming constant cross section. ISR is not considered.
- Unbalanced configurations contributes to endpoint region because of rich invariant mass spectrum of net visible momenta in one decay chain.

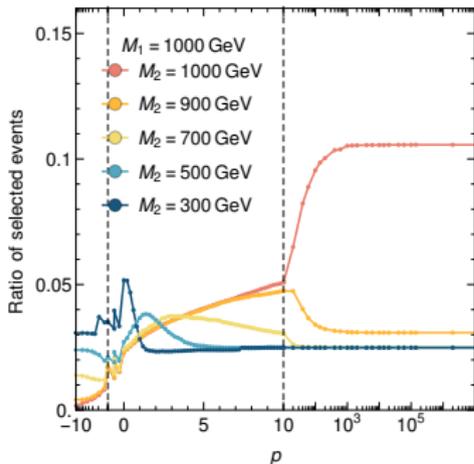
# Endpoint Saturation of $\mu_p$

- The fingerprint for distinguishing true mass spectrum is the endpoint region of Histogram.
- The saturation depends on type of configuration:
  - Balanced configuration sensitive to compatibility between intrinsic constraint and true mass spectrum
  - Unbalanced configuration sensitive to the invariant mass of net visible momenta on each decay chain

### Two-body decay



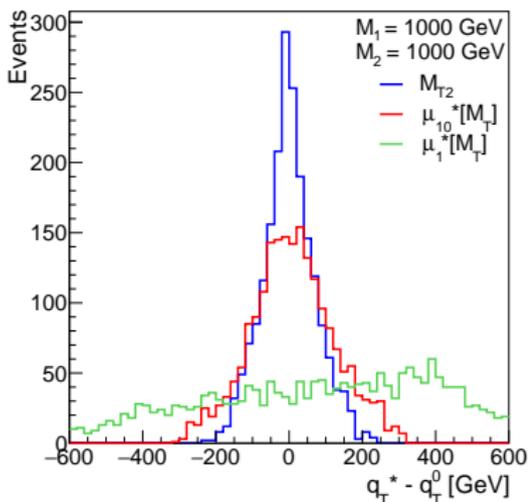
### Three-body decay



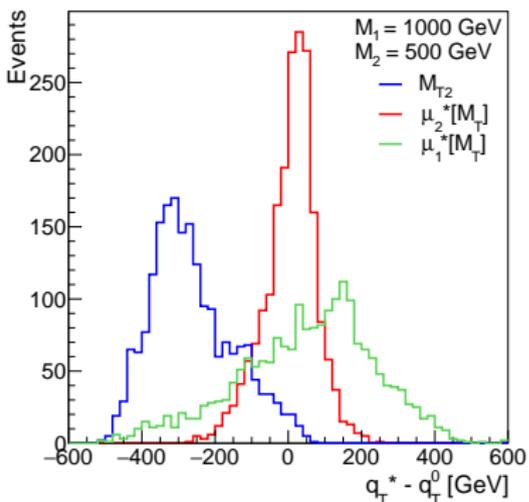
- Ratio of selected events having mass value at least 95% of expected endpoint mass

Event Reconstruction of  $\mu_p$ 

Identical pair production



Non-identical pair production

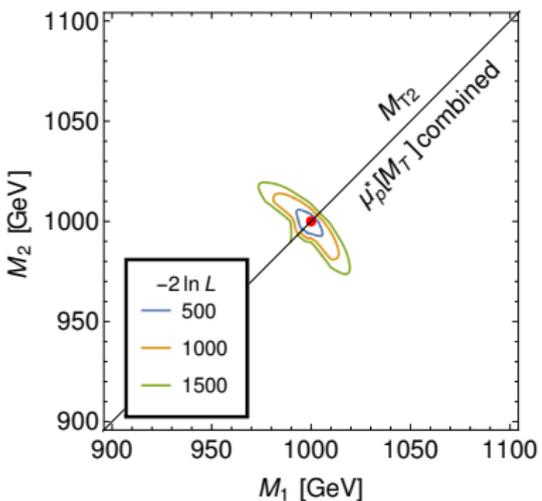


Difference of reconstructed transverse momentum  $q_T^*$  and exact transverse momentum  $q_T^0$  of missing particles. Events having the top 0.1%  $M_{T2}$  or  $\mu_p$  are selected.

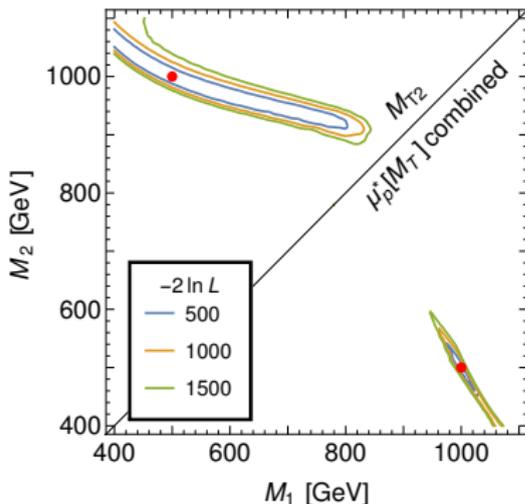
Likelihood Analysis of  $\mu_p$ 

- To estimate resolving power of each mass variable, we calculated Poisson log-likelihood between a reference sample and a template

Identical pair production



Non-identical pair production



- Red point is reference  $\mu_p$  mass spectrum used as a sample.
- From 90% of  $M_{T2}$  or  $\mu_p$  value to endpoint region on template is considered.
- $\mu_p$  with  $p = 1, 2, 5, 10, 100, 1000$  and  $M_{T2}$  is used for combined analysis

# Conclusion

- We developed a class of mass functions. which is based on power means, in multiple resonance decay system, which to be minimized over invisible missing momenta with minimal kinematic constraints.
- We show and emphasize that mass variables in the general class can provide significantly enhanced resolving power of measuring generally asymmetric resonance masses, which should be complementary to the  $M_{T2}$  where its expected endpoint becomes mass-sensitive only when symmetric and identical mother particle masses are assumed.