Study of LFV in general flavor symmetric models

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1. motivation

Neutrino oscillation

→Large neutrino mixing angles

mixing angles	best fit	1σ range	3σ range
$\sin^2\theta_{12}$	0.304	0.292-0.317	0.270-0.344
$\sin^2\theta_{23}$ (NH)	0.452	0.424-0.504	0.382-0.643
$\sin^2\theta_{23}$ (IH)	0.579	0.542-0.604	0.389-0.644
$\sin^2\theta_{13}$ (NH)	0.0218	0.0208-0.0228	0.0186-0.0250
$\sin^2\theta_{13}$ (IH)	0.0219	0.0209-0.0230	0.0188-0.0251

These values of mixing angles may be approximated by simple rational numbers.

1． motivation

As well-known examples, there are…

$\sin^2\theta_{12} \sim 1/3$、$\sin^2\theta_{23} \sim 1/2$、$\sin^2\theta_{13} \sim 0$ ・・・ Tri-Bi maximal mixing

$\sin^2\theta_{12} \sim 1/2$、$\sin^2\theta_{23} \sim 1/2$、$\sin^2\theta_{13} \sim 0$ ・・・ Bi maximal mixing

$\sin^2\theta_{12} \sim 1/2$、$\sin^2\theta_{23} \sim 1/2$、$\sin^2\theta_{13} \sim 1/3$、$\delta \sim \pi/2$ ・・・ Tri maximal mixing

etc.

*)Here, mixing angles are defined as,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},$$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{12}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

$s_{ij} = \sin\theta_{ij}$、$c_{ij} = \cos\theta_{ij}$

1. motivation

Simple mixing patterns are derived from non-Abelian discrete flavor symm.

ex.)Tri-Bi maximal mixing pattern from A_4 (E.Ma, 2004)

	(L_e, L_μ, L_τ)	e_R	μ_R	τ_R	(H_1, H_2, H_3)	ξ_0	(ξ_1, ξ_2, ξ_3)
$SU(2)$	2	1	1	1	2	3	3
A_4	3	1	1”	1’	3	1	3

Flavor symmetry A_4 fixes Yukawa coupling terms.

$$\mathcal{L}_{yukawa} = y_1(\bar{L}H)_1 e_R + y_2(\bar{L}H)_{1'}\mu_R + y_2(\bar{L}H)_{1''}\tau_R$$
$$+f_{ab}^i \left(\xi_i^0 \nu_a \nu_b + \frac{1}{\sqrt{2}}\xi_i^+(\nu_a l_b^- + \nu_b l_a^-) + \xi_i^{++} l_a^- l_b^-\right) + h.c.$$

1． motivation

When A_4 triplet Higgs bosons H_i, ξ_i get their VEVs like,

$$(\langle H_1^0\rangle, \langle H_2^0\rangle, \langle H_3^0\rangle) = (v, v, v),\ \langle \xi_0^0\rangle = v_0,\ (\langle \xi_1^0\rangle, \langle \xi_2^0\rangle, \langle \xi_3^0\rangle) = (v_1, 0, 0),$$

charged lepton's Dirac mass m_l and left-handed neutrino's Majorana mass m_ν are fixed as follows.

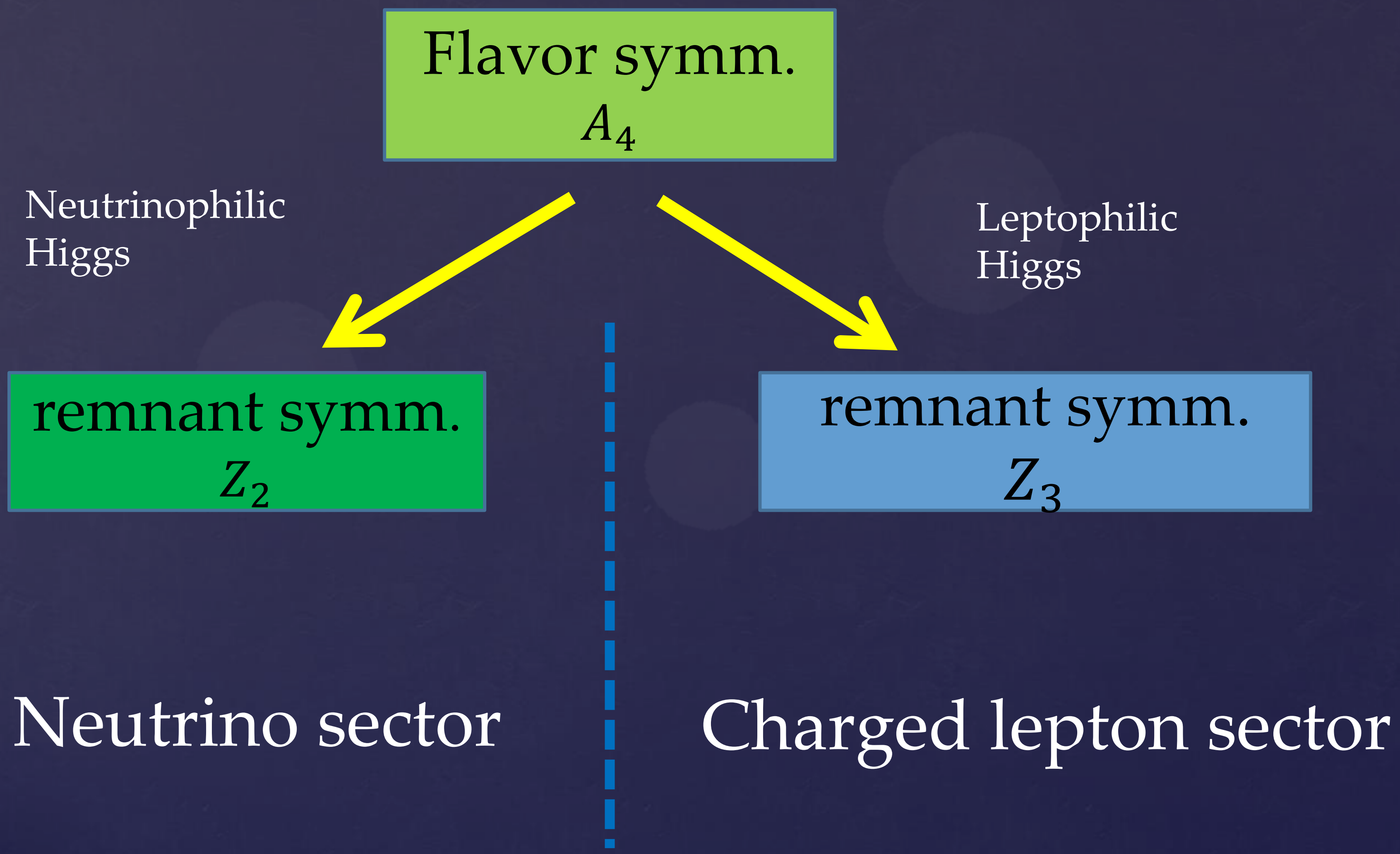
$$m_l = \begin{pmatrix} y_1 v & y_2 v & y_3 v \\ y_1 v & y_2 \omega v & y_2 \omega^2 v \\ y_1 v & y_2 \omega^2 v & y_2 \omega v \end{pmatrix},\ m_\nu = \begin{pmatrix} f^0 v_0 & 0 & 0 \\ 0 & f^0 v_0 & f^1 v_1 \\ 0 & f^1 v_1 & f^0 v_0 \end{pmatrix}.$$

From their diagonalizing matrices,

$$U_L^\dagger = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}、\ U_\nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}、$$

TBM mixing matrix $\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 1 & 0 & -i \\ 1 & 0 & i \end{pmatrix}$ is derived.

1. motivation



1. motivation

Under flavor symmetry G ,
leptophilic Higgs bosons H_i and neutrinophilic Higgs bosons Φ_i
are assumed to be triplet.

Yukawa coupling terms in the lepton sector are written as,

$$\mathcal{L}_{Yukawa} = Y_{ij}^{k}\overline{L_i}H_jE_R^k(+\tilde{Y}_{ij}^{k}\overline{L_i}\widetilde{\Phi}_jN_R^k + M_{ij}^{N}\overline{N_i^c}N_j) + h.c.$$

in general.

The VEVs of H_i, Φ_i break G, but they can leave subgroups, T, S ,
of the original flavor symmetry G
in the charged lepton, and the neutrino sector respectively.

Diagonalizing matrices of charged leptons and neutrinos
are fixed by these residual symmetry T, S in each sector .

1. motivation

Q. How far can we discuss
about the phenomenology of this kind of models
without restricting the flavor symmetry group?

2． Set up

- L_i、H_i、Φ_i $(i = 1,2,3)$ are triplets under G.
  E_R^i are (non-)trivial singlets。
- VEVs of H_i break G into T, and VEVs of Φ_i break G into S.
- Another Higgs boson which couples to quark is needed to give almost diagonal quark mass matrix.
  $\Rightarrow H_q$ (G-singlet)

Lepton Yukawa coupling terms：

$$\mathcal{L}_{Yukawa} = Y_{ij}^k \overline{L_i} H_j E_R^k + \tilde{Y}_{ij}^k \overline{L_i} \widetilde{\Phi_j} N_R^k + h.c.$$

2． Set up

Higgs bosons we are interested in are,

H_q、H_1、H_2、H_3

that couple to SM particle.
SM limit (Φ_i decouple) ⇒ We can use T-conservation.

$$H_q = \begin{pmatrix} H_q^+ \\ \frac{1}{\sqrt{2}}(v \sin\beta + H_q^0 + i\chi_q) \end{pmatrix}、 H_1 = \begin{pmatrix} H_1^+ \\ \frac{1}{\sqrt{2}}(v \cos\beta + H_1^0 + i\chi_1) \end{pmatrix}$$ ・・・ T-trivial

(Type-X 2HDM)

H_2、H_3 ・・・ T-charged

Especially, T-charged fields $H_{2,3}$ have only small mixing with other scalars thanks to T-charge conservation.

3． Phenomenology

T-conserving contribution

H_2, H_3 carry T-charge, and cause FCNC which doesn't occur in SM

$$
\begin{aligned}
\mathcal{L}_T^{(4)} = \frac{1}{v^2\cos^2\beta}\Big\{ & \frac{|b_3|^2}{m_{\phi_2}^2}(\overline{\tau_R}e_L)(\overline{e_L}\tau_R) + \frac{|b_2|^2}{m_{\phi_2}^2}(\overline{\mu_R}\tau_L)(\overline{\tau_L}\mu_R) + \frac{|b_1|^2}{m_{\phi_2}^2}(\overline{e_R}\mu_L)(\overline{\mu_L}e_R) \\
& + \frac{|c_3|^2}{m_{\phi_3}^2}(\overline{\tau_R}\mu_L)(\overline{\mu_L}\tau_R) + \frac{|c_2|^2}{m_{\phi_3}^2}(\overline{\mu_R}e_L)(\overline{e_L}\mu_R) + \frac{|c_1|^2}{m_{\phi_3}^2}(\overline{e_R}\tau_L)(\overline{\tau_L}e_R) \\
& + \frac{b_2^*b_3}{m_{\phi_2}^2}(\overline{\mu_R}\tau_L)(\overline{e_L}\tau_R) + \frac{b_1^*b_2}{m_{\phi_2}^2}(\overline{e_R}\mu_L)(\overline{\tau_L}\mu_R) + \frac{b_3^*b_1}{m_{\phi_2}^2}(\overline{\tau_R}e_L)(\overline{\mu_L}e_R) + h.c. \\
& + \frac{c_2^*c_3}{m_{\phi_3}^2}(\overline{\mu_R}e_L)(\overline{\mu_L}\tau_R) + \frac{c_1^*c_2^2}{m_{\phi_3}^2}(\overline{e_R}\tau_L)(\overline{e_L}\mu_R) + \frac{c_3^*c_1^2}{m_{\phi_3}^2}(\overline{\tau_R}\mu_L)(\overline{\tau_L}e_R) + h.c.\Big\}.
\end{aligned}
$$

Effective 4-point interaction terms caused by T-charged neutral scalar bosons exchange.

Ex.) $\tau^- \longrightarrow e^-\mu^-\mu^+$

$$\mathrm{Br}(\tau^- \longrightarrow e^-\mu^-\mu^+) = \frac{m_\tau^5}{3(8\pi)^3\Gamma_\tau}\left|\frac{m_\tau m_\mu}{{m_2}^2(v\cos\beta)^2}\right|^2$$

$\Rightarrow$ constraints on scalar mass and Higgs mixing angle β

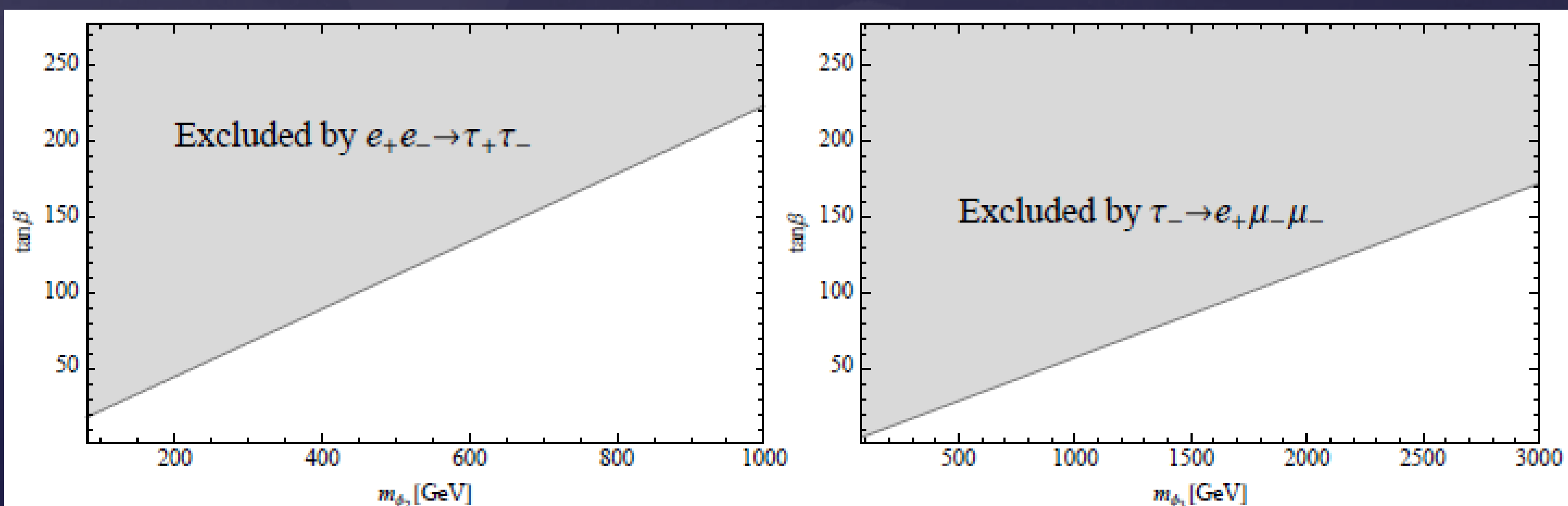
3．Phenomenology

T-conserving contribution

T-conserving process like $e^- e^+ \longrightarrow \mu^- \mu^+, \tau^- \tau^+$

can also be enhanced by T channel H_2, H_3 exchange.

From these, $m_2 \gtrsim 0.62 \times \frac{m_\tau}{v \cos\beta}$ TeV , $m_3 \gtrsim 2.23 \times \frac{m_\mu}{v \cos\beta}$ TeV.



3． Phenomenology

T-breaking contribution

Through interactions of H_i and Φ_i , differnt T-charges can mix each other.

There is no mixing between
T-trivial fields (H_q, H_1) and T-charged fields (H_2, H_3)
for the vacuum stability.

⇒ T-breaking in the charged lepton sector causes mixing among T-charged scalars.

$$(U^H)_{a\alpha} {m_{H\alpha}}^2 (U^H)_{b\alpha} = \begin{pmatrix} {m_{H2}}^2 + (\delta {m_H}^2)_{22} & (\delta {m_H}^2)_{23} \\ (\delta {m_H}^2)_{23} & {m_{H3}}^2 + (\delta {m_H}^2)_{33} \end{pmatrix}$$

$$(U^A)_{a\alpha} {m_{A\alpha}}^2 (U^A)_{b\alpha} = \begin{pmatrix} {m_{A2}}^2 + (\delta {m_A}^2)_{22} & (\delta {m_A}^2)_{23} \\ (\delta {m_A}^2)_{23} & {m_{A3}}^2 + (\delta {m_A}^2)_{33} \end{pmatrix}$$

3． Phenomenology

T-breaking contribution

$\mu \longrightarrow e\gamma$

$$\mathcal{L}_{\mu\longrightarrow e\gamma} = \mathrm{e}C_\mu \bar{e_L}\sigma_{\mu\nu}\mu_R F^{\mu\nu}$$

$$C_\mu = \frac{m_\tau Y^\tau_{e2} Y^\mu_{\tau 2}}{64\pi^2}\left\{\frac{U^H_{2\alpha}U^H_{2\alpha}}{m^2_{H\alpha}}\left(\ln\frac{m^2_{H\alpha}}{m^2_\tau} - \frac{3}{2}\right) - \frac{U^A_{2\alpha}U^A_{2\alpha}}{m^2_{A\alpha}}\left(\ln\frac{m^2_{A\alpha}}{m^2_\tau} - \frac{3}{2}\right)\right\}$$

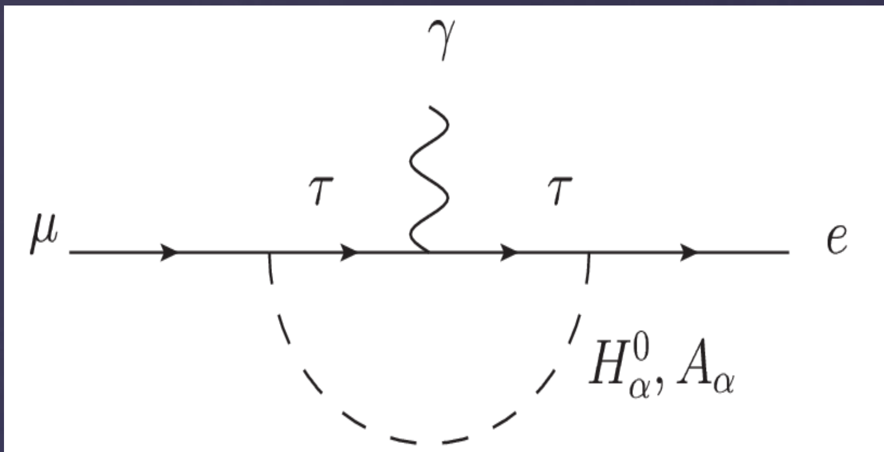
$\tau \longrightarrow e\gamma$

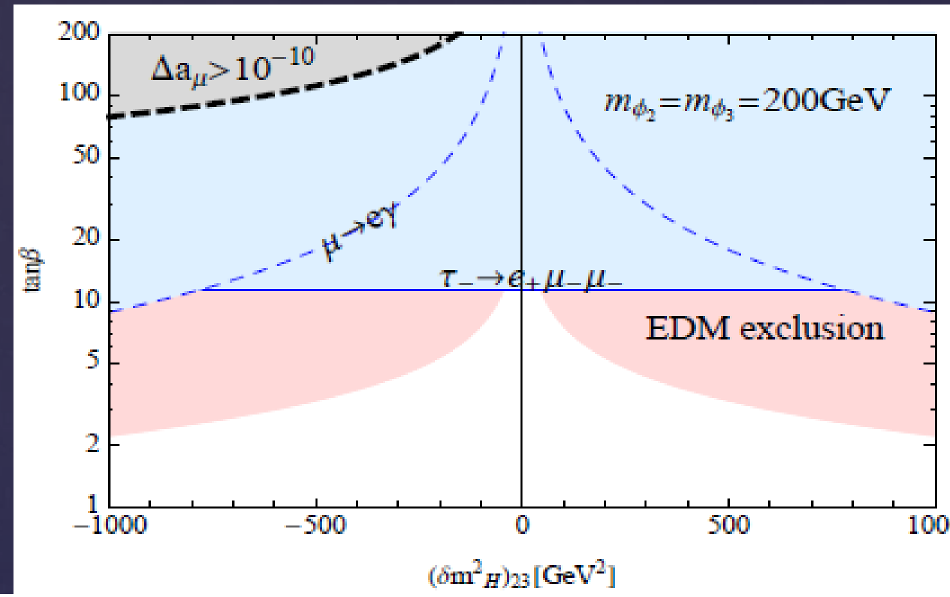
$$\mathcal{L}_{\tau\longrightarrow e\gamma} = \mathrm{e}C_\tau \bar{e_L}\sigma_{\mu\nu}\tau_R F^{\mu\nu}$$

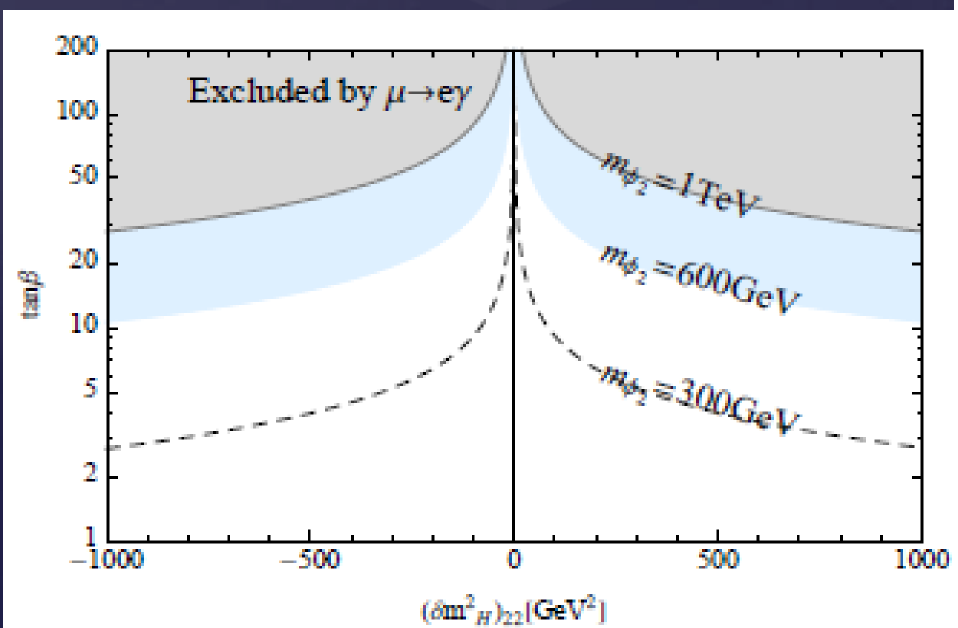
$$C_\tau = \frac{m_\mu Y^\mu_{e3} Y^\tau_{\mu 3}}{64\pi^2}\left\{\frac{U^H_{3\alpha}U^H_{3\alpha}}{m^2_{H\alpha}}\left(\ln\frac{m^2_{H\alpha}}{m^2_\mu} - \frac{m_\tau}{6m_\mu}\right) - \frac{U^A_{3\alpha}U^A_{3\alpha}}{m^2_{A\alpha}}\left(\ln\frac{m^2_{A\alpha}}{m^2_\mu} - \frac{m_\tau}{6m_\mu}\right)\right\}$$

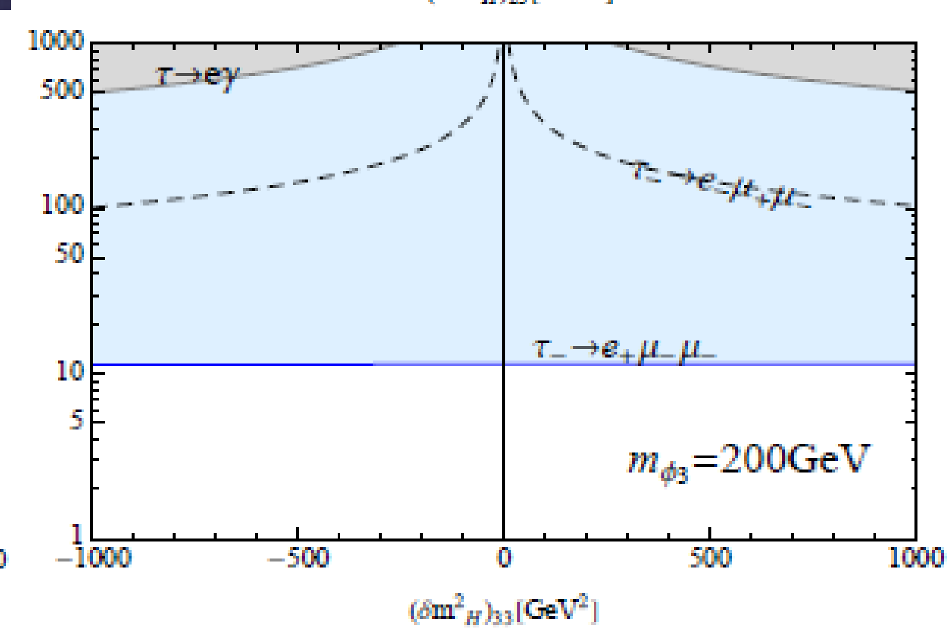
3． Phenomenology

T-breaking contribution









3. Phenomenology

$$\Delta q_L^i = \sum_{a=2,3}\sum_{k=1}^{3} \frac{|Y_{ia}^k|^2}{16\pi^2}\frac{M_Z^2}{m_{\phi_a}^2}\left(-\frac{1}{36}-\frac{1}{3}\sin^2\theta_W\right),$$

$$\Delta q_R^i = \sum_{a=2,3}\sum_{k=1}^{3} \frac{|Y_{ka}^i|^2}{16\pi^2}\frac{M_Z^2}{m_{\phi_a}^2}\left(\frac{7}{36}-\frac{1}{3}\sin^2\theta_W\right),$$

$$\Delta a_\mu = \frac{m_\mu m_\tau Y_{\tau 2}^{\mu} Y_{\mu 3}^{\tau}}{(4\pi)^2}\left\{\frac{U_{2\alpha}^h U_{3\alpha}^h}{m_{h_\alpha}^2} F\left(m_{h_\alpha}^2/m_\tau^2\right) - \frac{U_{2\alpha}^A U_{3\alpha}^A}{m_{A_\alpha}^2} F\left(m_{A_\alpha}^2/m_\tau^2\right)\right\}.$$

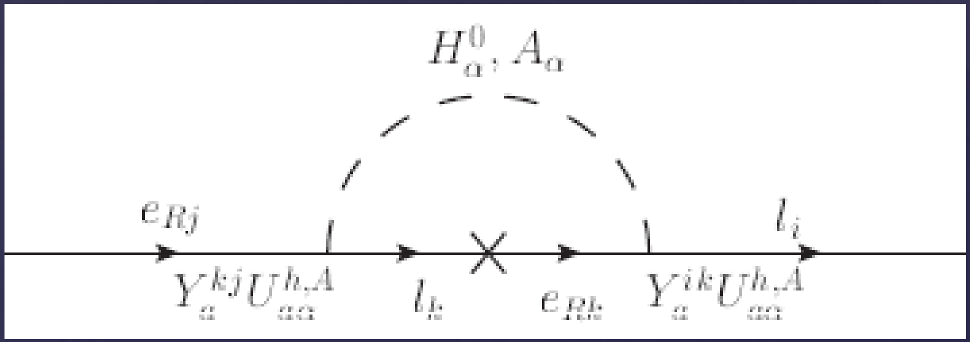
$$d_e = \frac{e}{32\pi^2}\mathrm{Im}(Y_{e2}^{\tau} Y_{\tau 3}^{e})\left\{U_{3\alpha}^h U_{2\alpha}^h \frac{m_\tau}{m_{h\alpha}^2} F\left(m_{h\alpha}^2/m_\tau^2\right) - U_{3\alpha}^A U_{2\alpha}^A \frac{m_\tau}{m_{A\alpha}^2} F\left(m_{A\alpha}^2/m_\tau^2\right)\right\},$$

$$d_\mu = \frac{e}{32\pi^2}\mathrm{Im}(Y_{\mu 3}^{\tau} Y_{\tau 2}^{\mu})\left\{U_{3\alpha}^h U_{2\alpha}^h \frac{m_\tau}{m_{h\alpha}^2} F\left(m_{h\alpha}^2/m_\tau^2\right) - U_{3\alpha}^A U_{2\alpha}^A \frac{m_\tau}{m_{A\alpha}^2} F\left(m_{A\alpha}^2/m_\tau^2\right)\right\}.$$

3． Phenomenology

Non zero θ_{13}

An important effect of T-breaking is shifting mass basis.

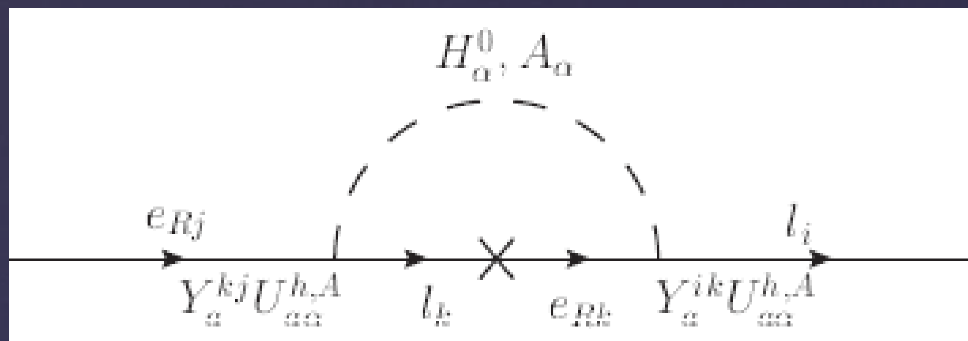


T-breaking effect in the neutrino sector caused by Φ_i propagate to the charged lepton sector through loop processes .

Simple patterns of neutrino mixing angles are altered by this effect.

3． Phenomenology

Non zero θ_{13}



$$M_l = \begin{pmatrix} m_e & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{\mu e} & m_\mu & \epsilon_{\mu\tau} \\ \epsilon_{\tau e} & \epsilon_{\tau\mu} & m_\tau \end{pmatrix}, \qquad \epsilon_{ij} = \sum_{a,b,k,\alpha,\beta} \frac{Y^k_{ia} m_k Y^j_{kb}}{32\pi^2} \left\{ U^H_{a\alpha} U^H_{b\alpha} \ln \frac{m_{H\alpha}{}^2}{\Lambda^2} - U^A_{a\alpha} U^A_{b\alpha} \ln \frac{m_{A\alpha}{}^2}{\Lambda^2} \right\}$$

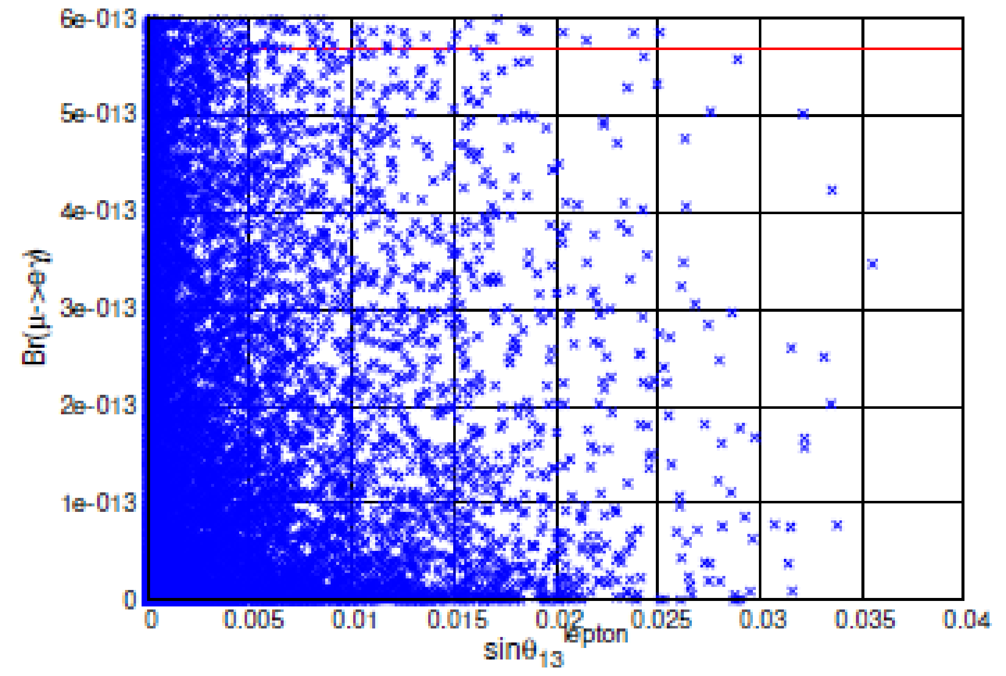
$$U'_{PMNS} = U_L^\dagger U_{PMNS} = \begin{pmatrix} 1 & -\frac{\epsilon_{e\mu}}{m_\mu} & -\frac{\epsilon_{e\tau}}{m_\tau} \\ \frac{\epsilon_{e\mu}}{m_\mu} & 1 & -\frac{\epsilon_{\mu\tau}}{m_\tau} \\ \frac{\epsilon_{e\tau}}{m_\tau} & \frac{\epsilon_{\mu\tau}}{m_\tau} & 1 \end{pmatrix} U_{PMNS}$$

When U_{PMNS} is TBM

$$\sin\theta_{13} = \frac{\epsilon_{e\mu}}{\sqrt{2} m_\mu} - \frac{\epsilon_{e\tau}}{\sqrt{2} m_\tau} \simeq \frac{Y^\tau_{e2} Y^\mu_{\tau 2}}{\sqrt{2}(4\pi)^2} \frac{m_\tau}{m_\mu} \left(U^H_{2\alpha} U^H_{2\alpha} \ln \frac{m_{H\alpha}{}^2}{m_\tau^2} - U^A_{2\alpha} U^A_{2\alpha} \ln \frac{m_{A\alpha}{}^2}{m_\tau^2} \right)$$

3． Phenomenology

Non zero θ_{13}



Size of T-breaking which satisfies above constraints can only derive O(0.01) as a value of θ_{13}.

⇒flavor breaking effect in neutrino sector have to be considered.

4. Summary

Non-Abelian discrete flavor symmetry is often used to explain peculiar pattern of neutrino mixing angles.

In such models, original flavor symmetry(G) is broken so that different subgroups(S, T) is left in the charged lepton, and neutrino sectors.

Charged leptons is considered almost T-conserving. So phenomenology of LFV can be studied through T-charge conservation and its breaking effect.

Because experimental constraints on LFV restrict T-breaking effect, loop corrections can not give large value to θ_{13}. Physics in the neutrino sector or additional content is needed to give correct value of neutrino mixing angles.