The Black Hole Microstate Geometry Program

– Past, Present, and Future

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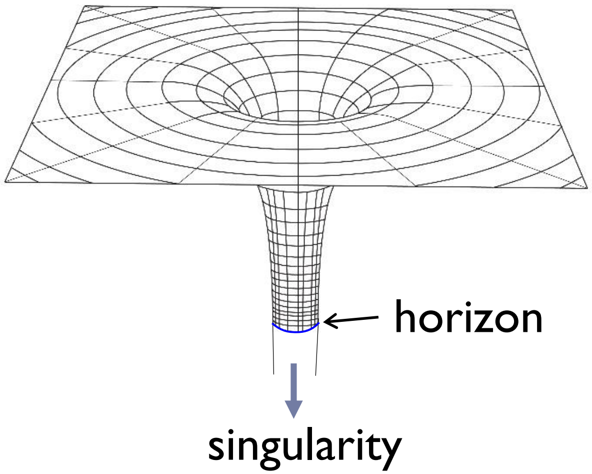
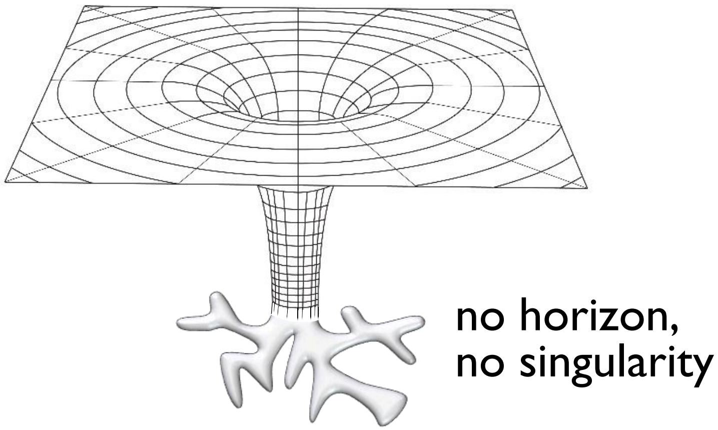
2nd String Theory in Greater Tokyo @ RIKEN

The Question:

How much of black hole entropy can be accounted for by smooth, horizonless solutions of classical gravity?





Why BH microphysics?

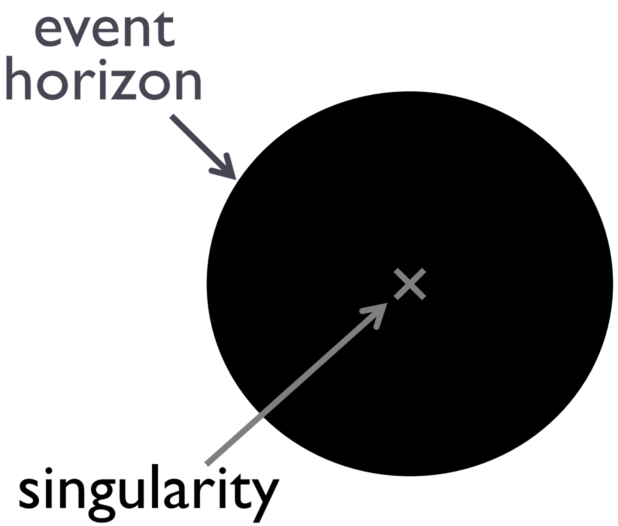
- Now nobody is sure about what's happening in BH
  - Conventional picture in doubt
- Observational consequences?
- Test of string theory as QG
- Related to various areas
  - Quantum information
  - Opening black box of AdS/CFT

Plan

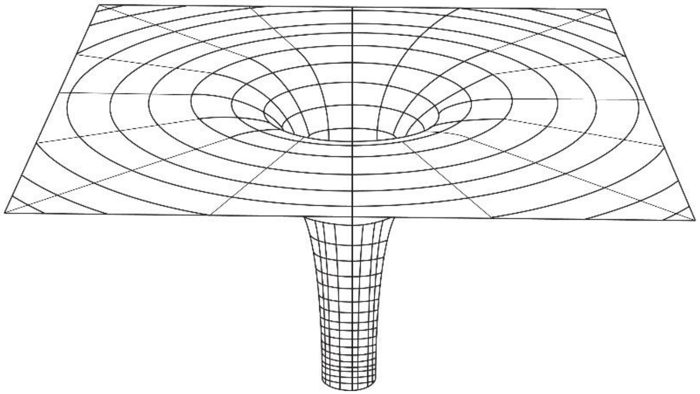
- BH microstates
- Microstate geom
- Fuzzball conjecture
  & microstate geom program
- Microstate geom in 5D
- Double bubbling
- Superstratum

Black hole microstates

Black holes



- Solution to Einstein equations
- Boundary of no return: event horizon
- Spacetime breaks down at spacetime singularity



BH entropy puzzle

- BH entropy:

$$S_{\mathrm{BH}} = \frac{A}{4G_{\mathrm{N}}}$$

⇨ Stat mech: $N_{\mathrm{micro}} = e^{S_{\mathrm{BH}}}$

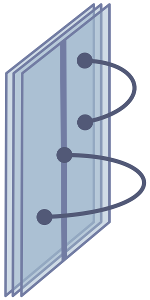
— ***Where are the microstates?***

  - Uniqueness theorems
  - Need quantum gravity?

AdS/CFT correspondence

string theory / gravity in AdS space ⟷ quantum field theory (CFT)

black hole → thermodyn. ensemble

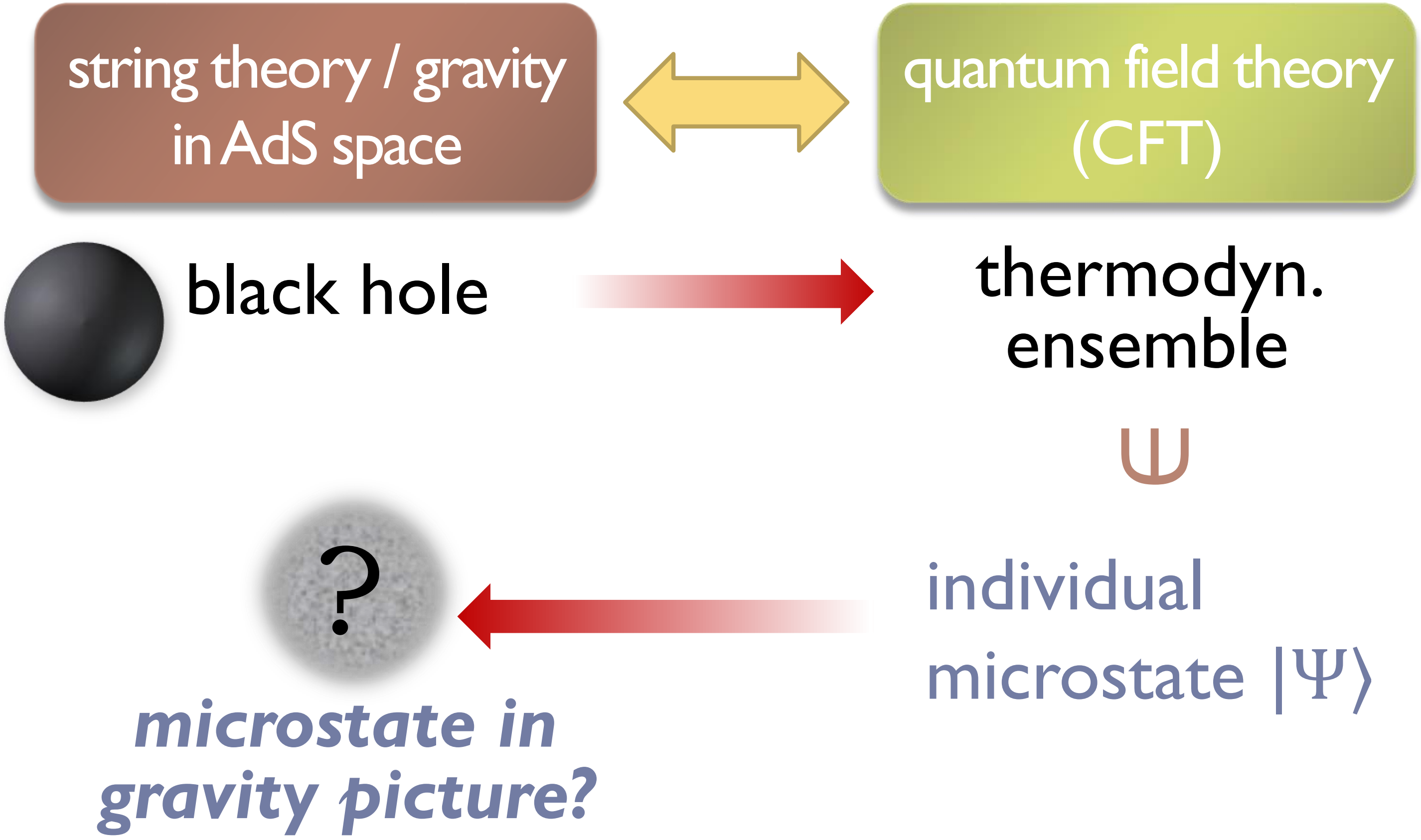


[Strominger-Vafa ’96]

$$S_{\mathrm{BH}} = \frac{A}{4G_{\mathrm{N}}} \overset{!}{=} S_{\mathrm{CFT}} = \log N_{\mathrm{micro}}$$

→ Stat mech interpretation of BH put on firm ground

BH microstates



string theory / gravity in AdS space

quantum field theory (CFT)

black hole

thermodyn. ensemble

$\rotatebox{90}{$\in$}$

individual microstate $|\Psi\rangle$

?

microstate in gravity picture?

- Must be a state of
  quantum gravity / string theory in general

Summary:

We want gravity picture
of BH microstates!

Microstate geometries

Are examples of gravity microstates known?

– Yes!

We know examples of microstates called *microstate geometries*.

- Solution of *classical* gravity
- Has same mass & charge as the BH
- Smooth & horizonless

Example 1:
LLM geometries

[Lin-Lunin-Maldacena 2004]

LLM geometries (1)

Sugra in $AdS_5 x S^5$ ⟷ D=4, $\mathcal{N}$=4 SYM

½ BPS BH: “superstar”
(singular, $A = 0$)

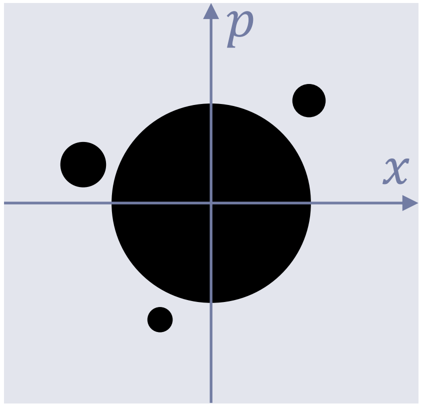
→

½ BPS states

||

fermion gas in
harmonic potential

$H = \frac{1}{2}(p^2 + x^2)$



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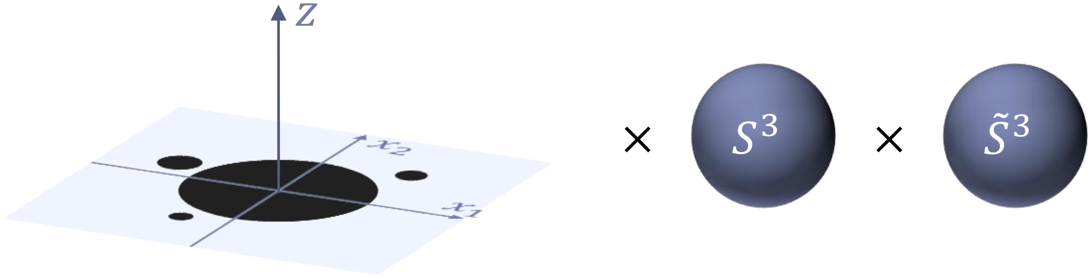
LLM (bubbling) geometries

LLM geometries (2)

$$ds^2 = -h^{-2}(dt+V)^2 + h^2\left(dy^2 + dx_1^2 + dx_2^2\right) + ye^G d\Omega_3^3 + ye^{-G} d\widetilde{\Omega}_3^2$$

$$h^{-2} = 2y \cosh G \qquad e^{2G} = \frac{1/2+z}{1/2-z}$$

$$[\partial_1^2 + \partial_2^2 + y\partial_y(y^{-1}\partial_y)]z(x_1,x_2,y) = 0$$



- LLM diagram encodes how S^3's shrink
- Smooth horizonless geometries
- Non-trivial topology supported by flux
- 1-to-1 correspondence with coherent states in CFT

$\}$ no uniqueness thm in 10D

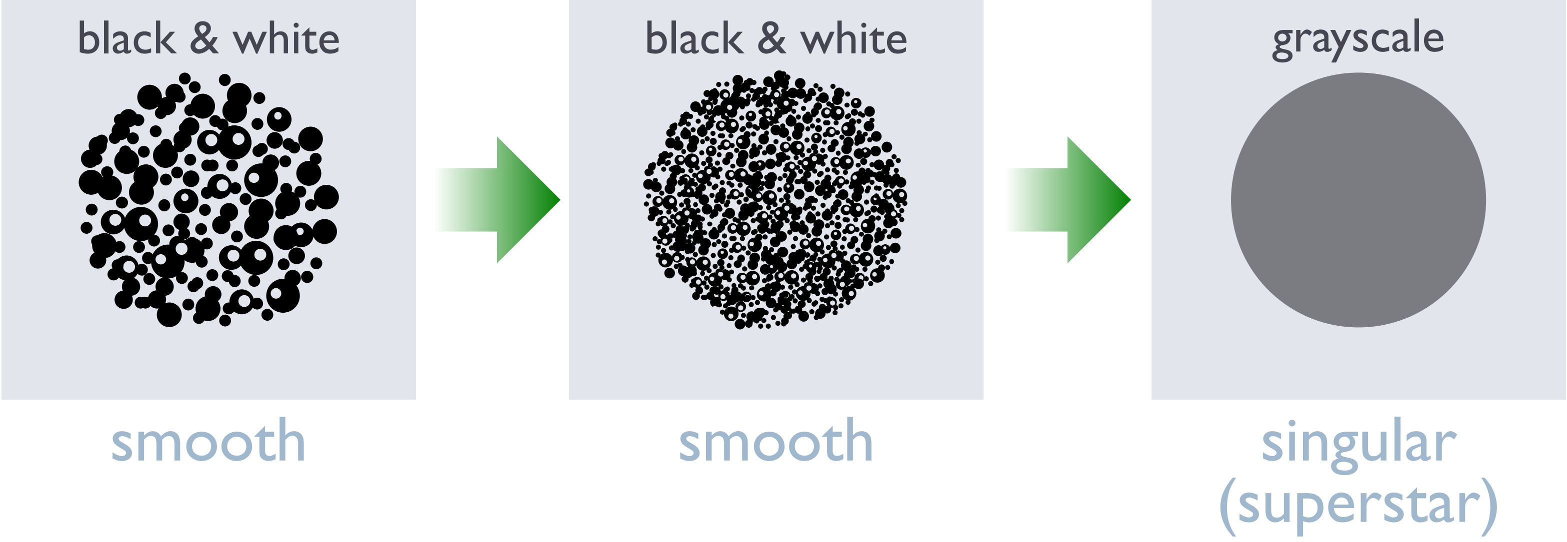
Classical limit

How is naive singular geometry (superstar) recovered?

- Bubble area quantized

$$(\text{area}) = 4\pi^2 l_p^4 N, \qquad h = 4\pi^2 l_p^4$$

- Classical limit: $l_p \to 0,\ N \to \infty$



Example 2:
LM geometries

[Lunin-Mathur 2001]

[Lunin-Maldacena-Maoz 2002]

LM geometries (1)

Sugra in $AdS_3 x S^3$ ⟺ D=2, $\mathcal{N}$=(4,4) CFT

2-charge BH
(singular, $A = 0$)

N_1 D1-branes
N_2 D5-branes

→

½ BPS states

||

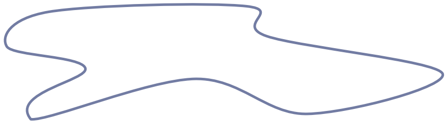
free bosons in 2D

Parametrized by integers
$n_1, n_2, n_3, \ldots$

$$\sum_k k n_k = N_1 N_2$$

←

"LM geometries"



LM geometries (2)

$$ds^2 = -\frac{2}{\sqrt{Z_1 Z_2}}(dv+\beta)(du+\omega) + \sqrt{Z_1 Z_2}\,dx_{1234}^2 + \sqrt{Z_1/Z_2}\,dx_{6789}^2$$

$$Z_1(\vec{x}) = 1 + \frac{Q_2}{L}\int_0^L \frac{|\dot{\vec{F}}|^2 d\lambda}{|\vec{x}-\vec{F}(\lambda)|^2}, \qquad Z_2(\vec{x}) = 1 + \frac{Q_2}{L}\int_0^L \frac{d\lambda}{|\vec{x}-\vec{F}(\lambda)|^2} \qquad \ldots$$

arbitrary curve $\vec{x} = \vec{F}(\lambda) \in \mathbb{R}^4_{1234}$

Fourier coeffs of $\vec{F}(\lambda)$ ⟺ $\{n_k\}$

- LM curve encodes how S^1 shrinks
- Smooth horizonless geometries supported by flux
- 1-to-1 correspondence with CFT states: $\vec{F}(\lambda) \leftrightarrow \{n_k\}$
- Entropy reproduced geometrically: $S \sim \sqrt{N_1 N_2}$

Classical limit

How is naive singular geometry recovered?

smooth

classical limit

singular

$$\mathcal{R} \sim g_s^{1/3} l_s^{1/3} (Q_1 Q_2)^{1/6}$$

$$\sim g_s^{2/3} l_s (N_1 N_2)^{1/6}$$

$g_s \to 0, l_s \to 0,$

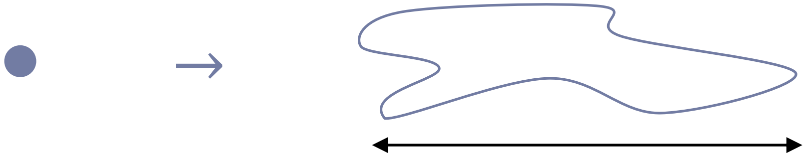
$N_{1,2} \to \infty$

Fix $Q_{1,2} \sim g_s l_s^2 N_{1,2}$

Summary:

Some BH microstates *are* represented by *microstate geometries.*

— Naive BH solutions are replaced by bubbling geometries with *finite spread.*



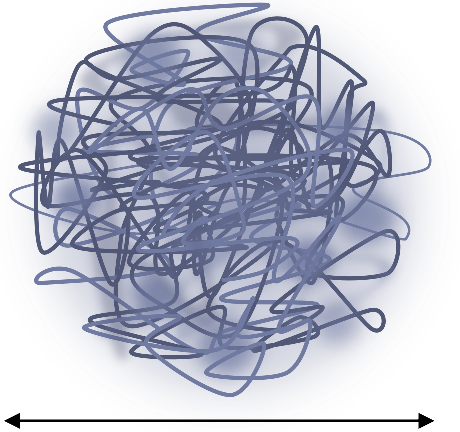
$\mathcal{R} \sim l_{\rm P} N^{\alpha}$

(but recall $A = 0$ so far)

Fuzzball conjecture & microstate geometry program

Maybe the same is true for genuine black holes?

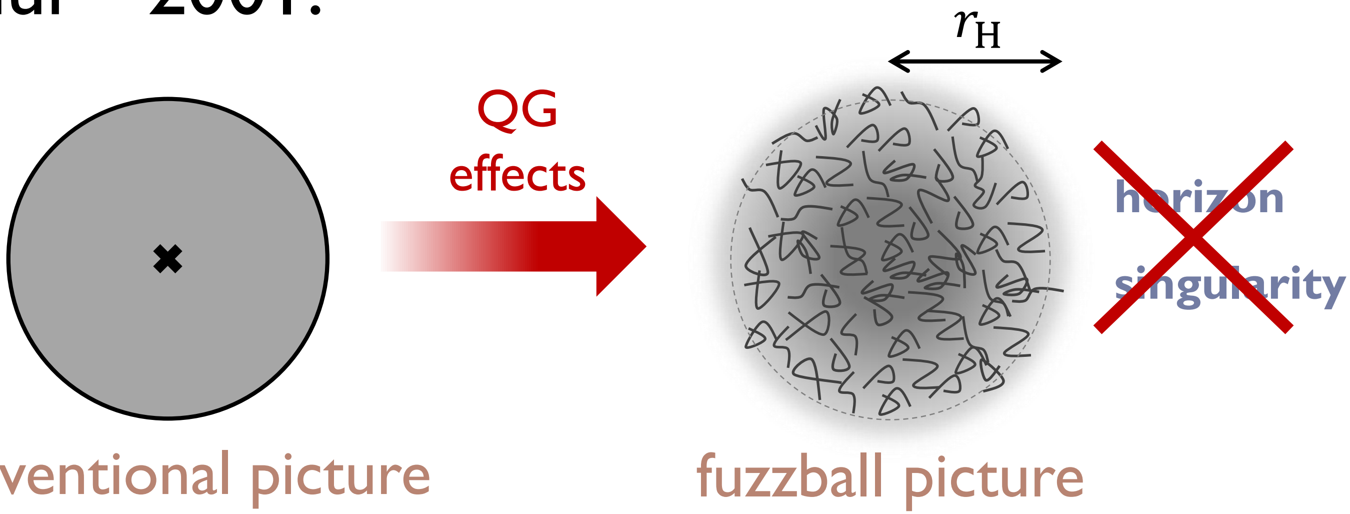
— BH microstates are some stringy configurations *spreading over a wide distance?*



$\mathcal{R} \sim l_{\mathrm{P}} N^{\alpha} \sim r_H$??

Fuzzball conjecture

- Mathur ~2001:



  - BH microstates = QG/stringy “fuzzballs”
  - No horizon, no singularity
  - Spread over horizon scale

Sugra fuzzballs (1)

Are fuzzballs describable in sugra?

- Unlikely in general
  - General fuzzballs must involve all string modes
  - Massive string modes are not in sugra

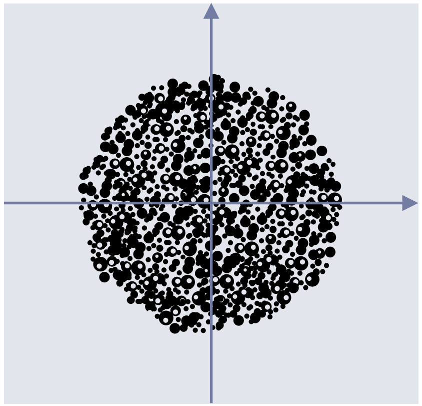
- Hope for BPS states
  - Massive strings break susy
    - → Only massless (sugra) modes allowed?
  - “Example”: MSW (wiggling M5)

[Maldacena+Strominger+Witten 1997]

Sugra fuzzballs (2)

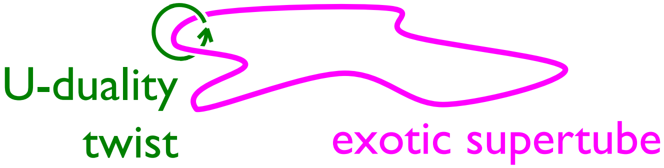
Caveats:

- Generic states have large curvature
  - Higher derivative corrections nonnegligible
  - But should not change qualitative picture; DoF must be the same



smooth, but curvature large

- Non-geometries
  - Non-geometric microstates possible [Park+MS 2015]
  - Need to extend framework (DFT, EFT)



Microstate geometry program:

What portion of the BH entropy
of supersymmetric BHs is accounted for
by smooth, horizonless solutions of classical sugra?

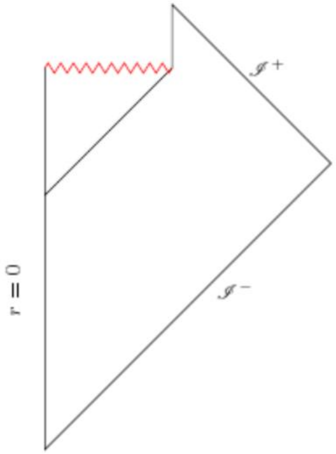
Comment: bottom up approach

[Mathur '09] $O(1)$ deviation from flat space is needed for Hawking radiation to carry information

- Based on Q info (strong subadditivity)

[AMPS '12] "Firewall"

- Same result, same Q info (monogamy etc.)



These arguments are "bottom-up"

→ Mechanism to support finite size not explained

Microstate geometry program is "top-down"

→ Finite size supported by topology with fluxes

Microstate geometries in 5D

Let’s review a class of BH microstate geometries, including their pros & cons.

5D microstate geometries:
circa 2004–09

Setup

- $D = 5, \mathcal{N} = 1$ sugra with 2 vector multiplets

gauge fields: $A^I_\mu\,,\ I = 1,2,3.\quad F^I \equiv dA^I.$

scalars: $X^I,\ \ X^1X^2X^3 = 1$

- Action

$$S_{\text{bos}} = \int \Big(*_5 R - Q_{IJ} dX^I \wedge *_5\, dX^I - Q_{IJ} F^I \wedge *_5\, F^J \\ - \tfrac{1}{6} C_{IJK} \underbrace{F^I \wedge F^J \wedge A^K}_{\text{Chern-Simons interaction}} \Big)$$

$$C_{IJK} = |\epsilon_{IJK}|\,, \qquad Q_{IJ} = \tfrac{1}{2}\text{diag}(1/X^1, 1/X^2, 1/X^3)$$

11D interpretation

- M-theory on $T^6_{56789\mathrm{A}}$

$\mathrm{A} = 10$

$$ds_{11}^2 = ds_5^2 + X^1\left(dx_5^2 + dx_6^2\right) + X^2\left(dx_7^2 + dx_8^2\right) + X^3\left(dx_9^2 + dx_\mathrm{A}^2\right)$$

$$\mathcal{A}_3 = \underbrace{A^1}\, dx_5 \wedge dx_6 + \underbrace{A^2}\, dx_7 \wedge dx_8 + \underbrace{A^3}\, dx_9 \wedge dx_\mathrm{A}$$

$$\begin{tikzcd} \text{M2(56)} \arrow[d, leftrightarrow] & \text{M2(78)} \arrow[d, leftrightarrow] & \text{M2(9A)} \arrow[d, leftrightarrow] \\ \text{M5}(\lambda 789\text{A}) & \text{M5}(\lambda 569\text{A}) & \text{M5}(\lambda 5678) \end{tikzcd}$$

BPS solutions [Gutowski-Reall ’04] [Bena-Warner ’04]

- Require susy

$$ds_5^2 = -Z^{-2}(dt+k)^2 + Z\,\overbrace{ds_4^2}^{\text{4D base } \mathcal{B}^4 \text{ (hyperkähler)}}$$

$$A^I = \underbrace{-Z_I^{-1}(dt+k)}_{\text{elec}} + \underbrace{B^I}_{\text{mag}}, \qquad dB^I = \Theta^I$$

$$Z = (Z_1 Z_2 Z_3)^{1/3}; \quad X^1 = \left(\frac{Z_2 Z_3}{Z_1^2}\right)^{1/3} \text{ and cyclic}$$

All depends only on B_4 coordinates

- Linear system

$$\Theta^I = *_4\,\Theta^I,$$

$$\nabla^2 Z_I = C_{IJK}\ *_4\left(\Theta^J \wedge \Theta^K\right)$$

$$(1 + *_4)dk = Z_I \Theta^I$$

Sol'ns with U(1) sym

[Gutowski-Gauntlett '04]

Solving eqs in general is difficult.
Assume $U(1)$ symmetry in $\mathcal{B}^4$

$$ds_4^2 = V^{-1}(d\psi + A)^2 + V\left(\overbrace{dy_1^2 + dy_2^2 + dy_3^2}^{\text{flat } \mathbb{R}^3}\right),$$

(Gibbons-Hawking space)

V is harmonic in $\mathbb{R}^3$:

$$V = v_0 + \sum_p \frac{v_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}$$

Multi-center KK monopole / Taub-NUT

Complete solution

All eqs solved in terms of harmonic functions in $\mathbb{R}^3$:

$$\boxed{H = (V, K^I, L_I, M), \qquad H = h + \sum_p \frac{Q_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}}$$

$$\Theta^I = d\left(\frac{K^I}{V}\right) \wedge (d\psi + A) - V *_3 d\left(\frac{K^I}{V}\right)$$

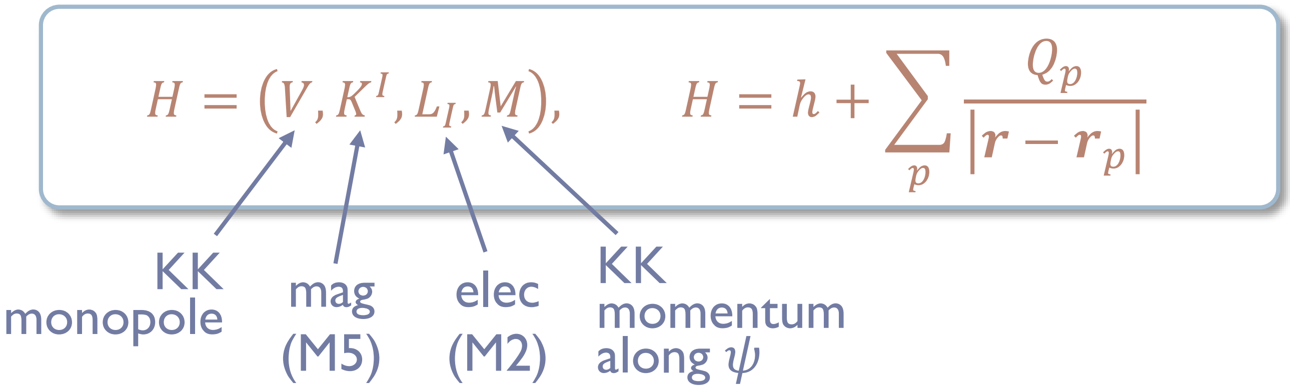
$$Z_I = L_I + \frac{1}{2V} C_{IJK} K^J K^K$$

$$k = \mu(d\psi + A) + \omega$$

$$\mu = M + \frac{1}{2V} K^I L_I + \frac{1}{6V^2} C_{IJK} K^I K^J K^K$$

$$*_3\, d\omega = V dM - M dV + \frac{1}{2}\left(K^I dL_I - L_I dK^I\right)$$

Multi-center solution



$$H = (V, K^I, L_I, M), \qquad H = h + \sum_p \frac{Q_p}{|\boldsymbol{r} - \boldsymbol{r}_p|}$$

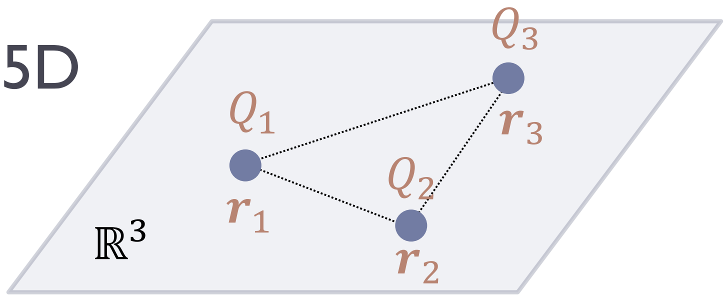
KK monopole

mag (M5)

elec (M2)

KK momentum along ψ

- Multi-center config of BHs & BRs in 5D
- Positions $\boldsymbol{r}_p$ satisfy "bubbling eq"
  (force balance)
- Reducing on ψ gives 4D BHs
  (same as Bates-Denef 2003)



Microstate geometries (1)

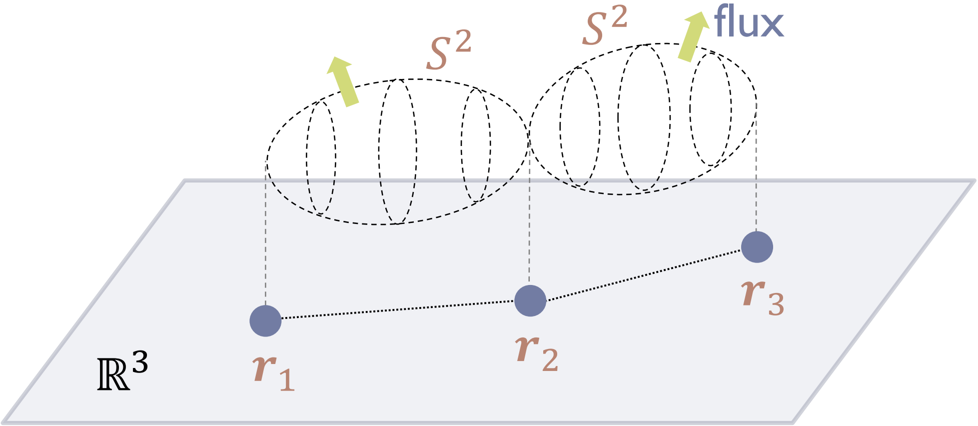
Tune charges:

$$l_p^I = -\frac{C_{IJK}}{2}\frac{k_p^J k_p^K}{v_p}$$

$$m_p = \frac{C_{IJK}}{12}\frac{k_p^I k_p^J k_p^K}{v_p^2}$$

Smooth horizonless solutions

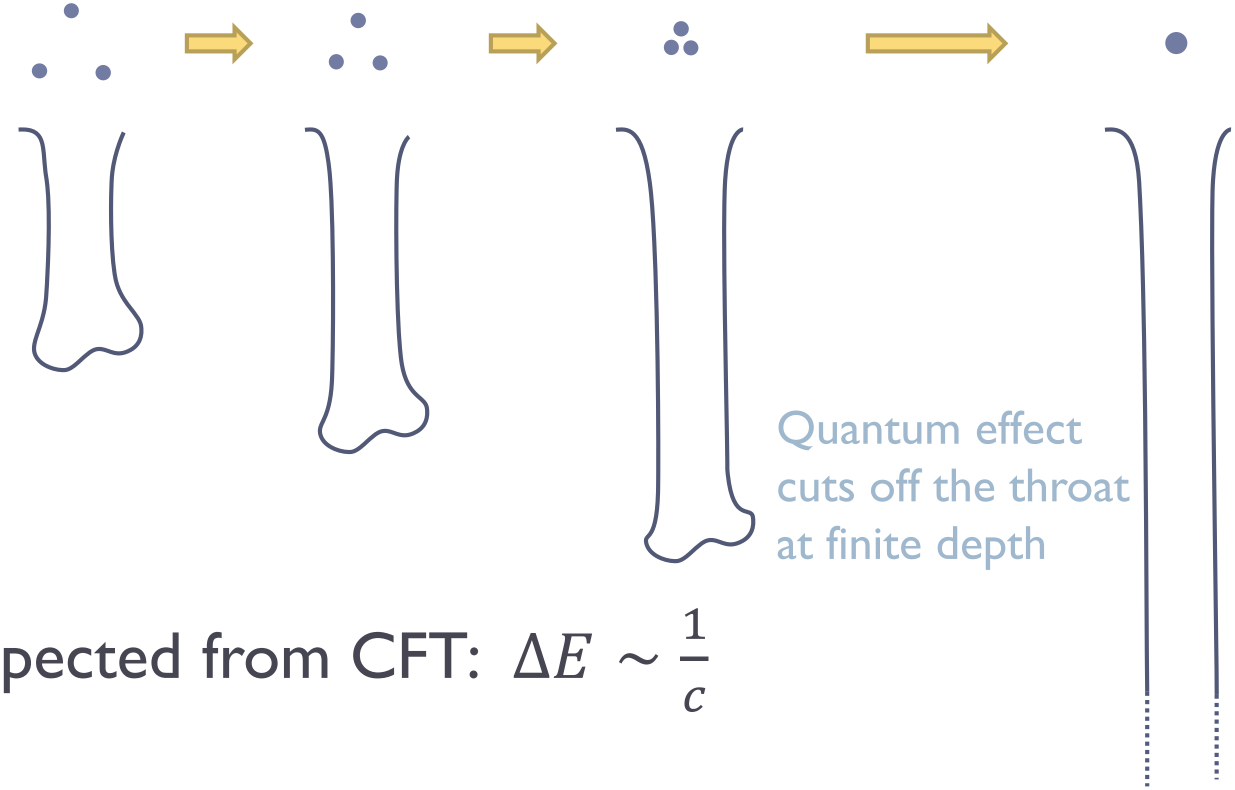
[Bena-Warner 2006] [Berglund-Gimon-Levi 2006]



- Microstate geometries for 5D (and 4D) BHs ☺
  - □ Same asymptotic charges as BHs
- Topology & fluxes support the soliton
- Mechanism to support horizon-sized structure!

Microstate geometries (2)

- Various nice properties ☺
  - Scaling solutions [BW et al., 2006, 2007]



Quantum effect cuts off the throat at finite depth

  - Gap expected from CFT: $\Delta E \sim \frac{1}{c}$

The real question:

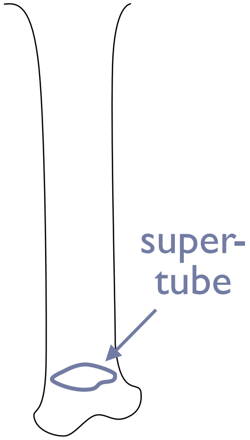
Are there enough?

- 3-chage sys (+ fluctuating supertube)
  - Entropy enhancement mechanism [BW et al., 2008]

    → Much more entropy?
  - An estimate [BW et al., 2010]

$$S \sim Q^{\frac{5}{4}} \ll Q^{\frac{3}{2}}$$

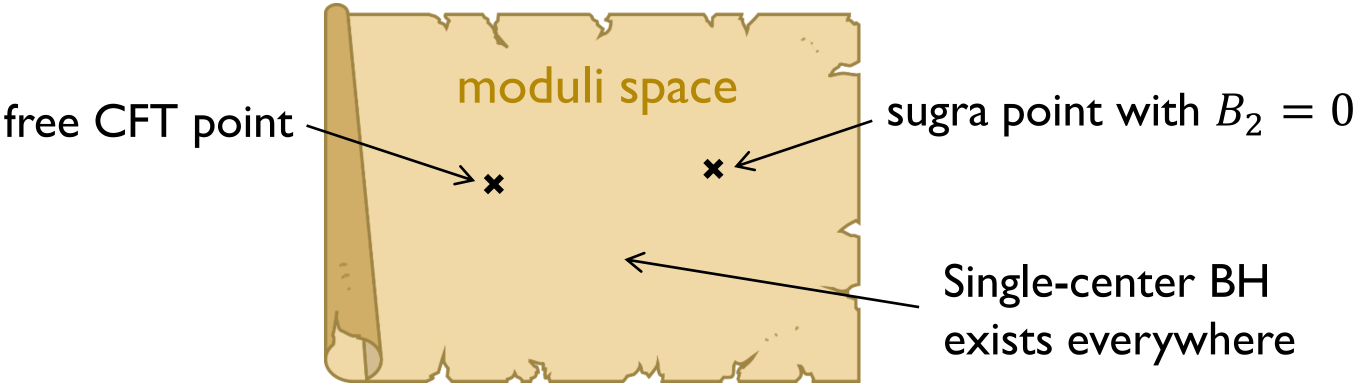
Parametrically smaller ☹



- 4-chage sys [de Boer et al., 2008-09]
  - Quantization of D6-$\overline{\text{D6}}$-D0 config → *much less entropy* ☹

Further issues (1)

- Lifting [Dabholkar, Giuca, Murthy, Nampuri '09]



  - Single-ctr BH exists everywhere and contributes to index (elliptic genus).
  - Microstates must also exist everywhere and contribute to index.
  - But >2 center solns do not contribute to index!

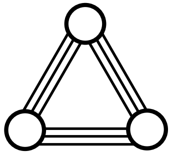
    → They disappear when generic moduli are turned on?

    → They are irrelevant for microstates?

  - Cf. Moulting BH [Bena, Chowdhury, de Boer, El-Showk, MS 2011]

Further issues (2)

- Pure Higgs branch [Bena, Berkooz, de Boer, El-Showk, Van den Bleeken '12]
  - Vacua of Quiver QM (scaling regime)



Coulomb branch

- Corresponds to multi-center solutions
- Small entropy
- Generally $J \neq 0$

Pure Higgs branch

- Corresponding sugra solution unclear
- Large entropy
- $J = 0$

Summary:

We found microstate geometries
for genuine BHs,
but they are *too few*.

Possibilities:

A) Sugra is not enough

B) Need more general ansatz ⇦ this talk

Double bubbling

2010–

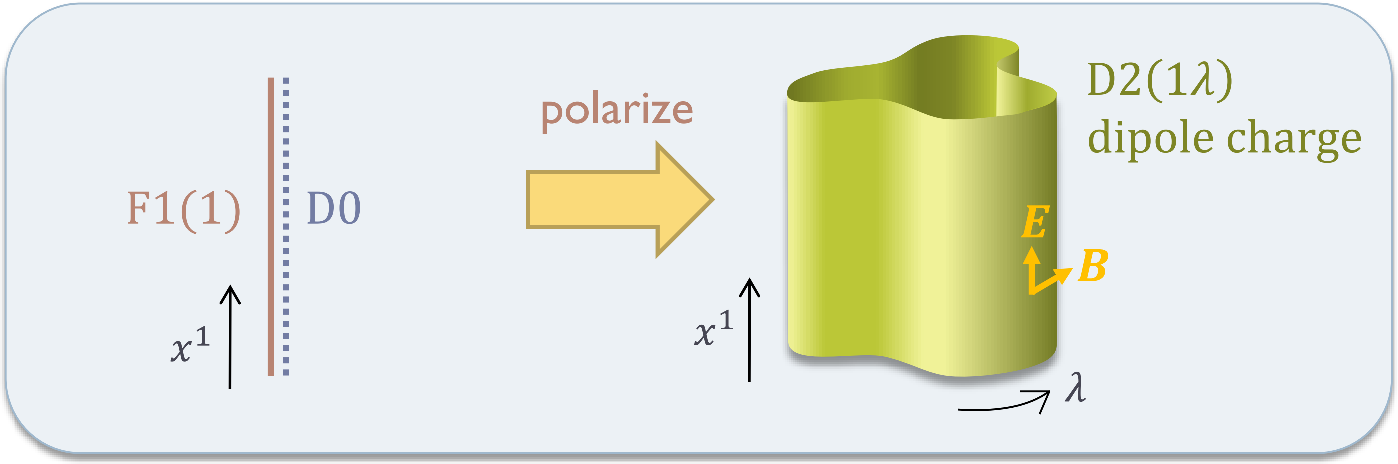
What are we missing?

— A guiding principle for constructing microstate geometries.

Revisit better understood example:
2-charge system (LM geometries)

Supertube transition [Mateos+Townsend 2001]

$$\mathrm{D0} + \mathrm{F1}(1) \longrightarrow \mathrm{D2}(1\lambda)$$



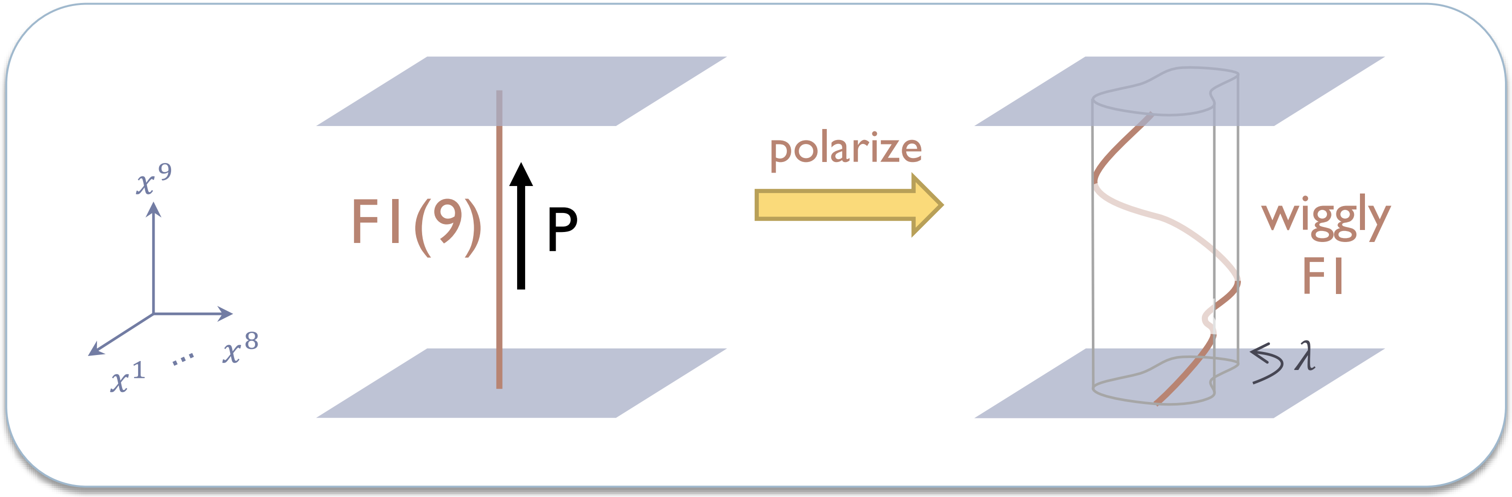
- Spontaneous polarization phenomenon

(cf. Myers effect)

- Produces new dipole charge
- Represents *genuine bound state*
- Cross section = *arbitrary* curve

F1-P frame

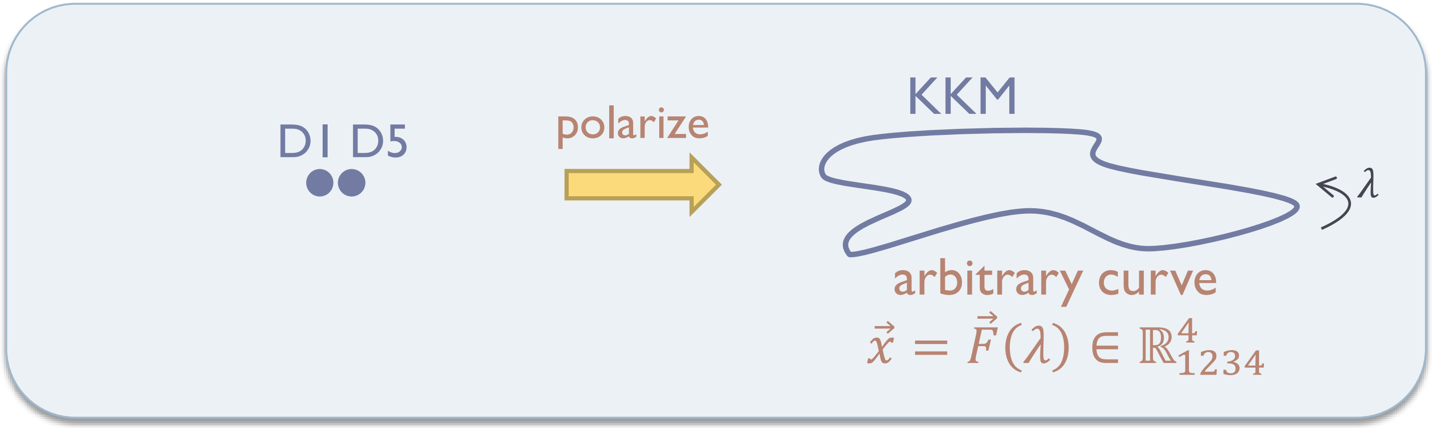
$\mathrm{F1}(9) + \mathrm{P}(9) \Longrightarrow \mathrm{F1}(\lambda)$



- To carry momentum,
  F1 must wiggle in transverse $\mathbb{R}^8$
- Projection onto transverse $\mathbb{R}^8$ is an arbitrary curve

D1-D5 frame

$$\mathrm{D1}(5) + \mathrm{D5}(56789) \rightarrow \mathrm{KKM}(\lambda 6789{,}5)$$



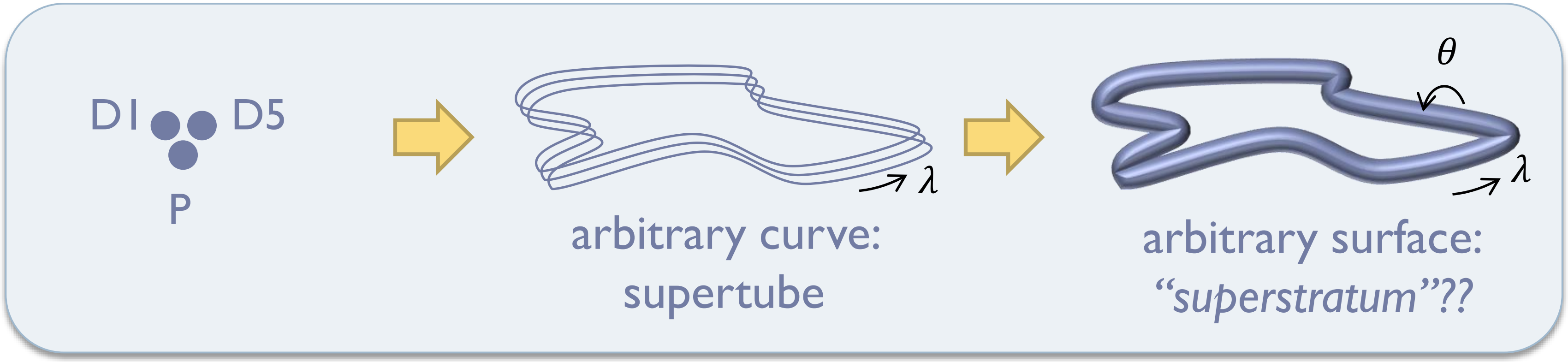
$$\vec{x} = \vec{F}(\lambda) \in \mathbb{R}^4_{1234}$$

- This is LM geometry
- Arbitrary curve → large entropy $S \sim \sqrt{N_1 N_2}$
- Explains origin of 2-charge microstate geometries

3-charge case

“Double bubbling”

$$\begin{array}{lcl c lcl} \mathrm{D1}(5) & & \mathrm{KKM}(\lambda 6789,5) & & 1_3^6(\theta,\lambda 56789) \\ \mathrm{D5}(56789) & \Rightarrow & \mathrm{D5}(\lambda 6789) & \Rightarrow & \mathrm{KKM}(\lambda 6789,\theta) \\ \mathrm{P}(5) & & \mathrm{D1}(\lambda) & & 5_3^2(\theta 6789,\lambda) \end{array}$$



arbitrary curve:
supertube

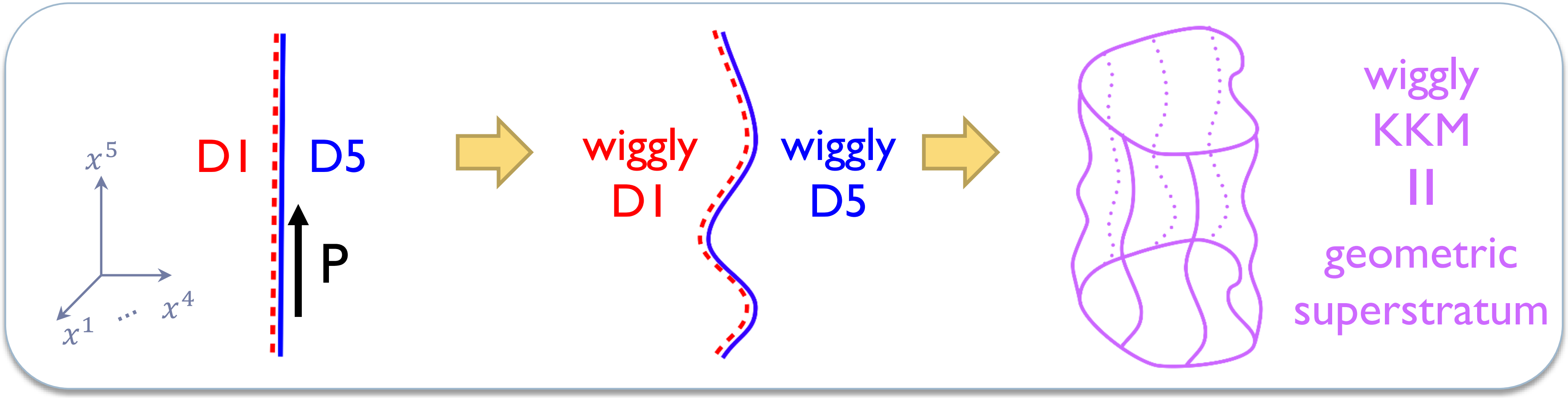
arbitrary surface:
“superstratum”??

- Multiple transitions can happen in principle
- Arbitrary surface → larger entropy?
- Non-geometric in general

[de Boer+MS 2010, 2012]
[Bena+de Boer +Warner+MS 2011]

A geometric channel

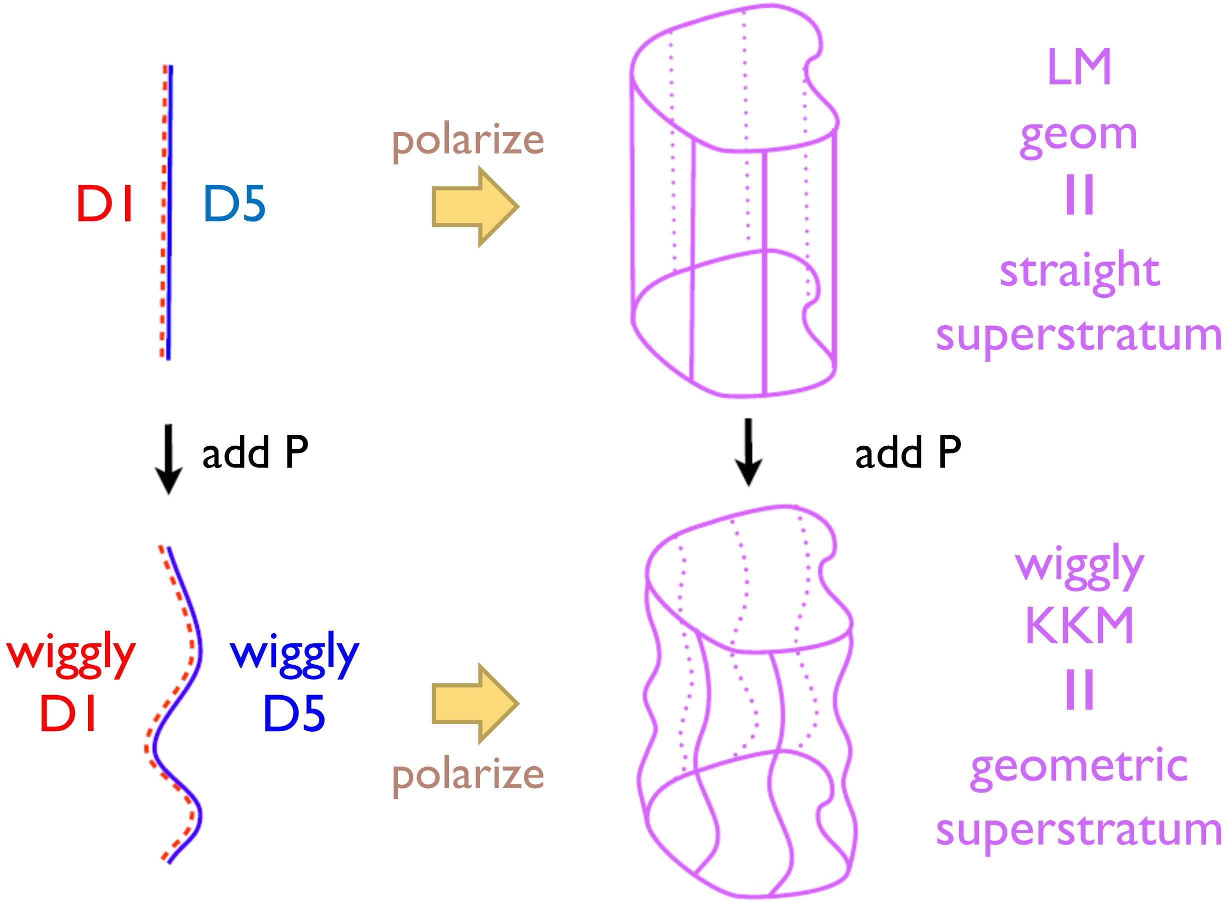
$\mathrm{D1}(5)$
$\mathrm{D5}(56789)$
$\mathrm{P}(5)$
→
$\mathrm{D5}(\lambda 6789)$
$\mathrm{D1}(\lambda)$
→
$\mathrm{KKM}(\lambda 6789, \theta)$



- Dependence on x^5 is crucial
- Must live in *6D*
- Possibility to recover $S \sim \sqrt{N_1 N_2 N_3}$

[Bena+de Boer
+Warner+MS 2014]

Two routes to superstratum



D1

D5

polarize

LM
geom
||
straight
superstratum

add P

add P

wiggly
D1

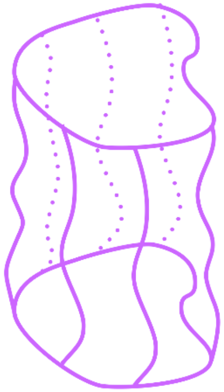
wiggly
D5

polarize

wiggly
KKM
||
geometric
superstratum

Summary:

Existence of superstrata
depending on functions of two variables
is a necessary condition for
$S_{\mathrm{BH}} \sim S_{\mathrm{geom}}$



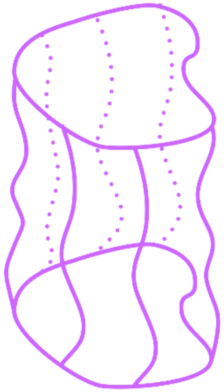
Microstate geometries in 6D (sugra superstratum)

2011–

Goal:

Explicitly construct "superstrata" or wiggly KKM in 6D

They must depend on functions of two variables: $F(v, w)$



Susy solutions in 6D

- IIB sugra on T^4_{6789}
- No dependence on T^4 coordinates
- Require same susy as preserved by D1-D5-P
- Expected charges / dipole charges:

D1(v) D1(λ) KKM(λ6789, v)
D5(v6789) D5(λ6789)
P(v)

$$u = \frac{t-x^5}{\sqrt{2}}, \quad v = \frac{t+x^5}{\sqrt{2}}$$

x^5: compact

[Gutowski+Martelli+Reall 2003]
[Cariglia+Mac Conamhna 2004]
[Bena+Giusto+MS+Warner 2011]
[Giusto+Martucci+Petrini+Russo 2013]

The sol'n is characterized by…

scalars

$Z_1 \leftrightarrow \mathrm{D1}(v)$

$Z_2 \leftrightarrow \mathrm{D5}(v6789)$

$\mathcal{F} \leftrightarrow \mathrm{P}(v)$

$Z_4 \leftrightarrow \mathrm{NS5}(v6789)+\mathrm{F1}(v)$

2-forms

$\Theta_1 \leftrightarrow \mathrm{D1}(\lambda)$

$\Theta_2 \leftrightarrow \mathrm{D5}(\lambda6789)$

$\Theta_4 \leftrightarrow \mathrm{NS5}(\lambda6789)+\mathrm{F1}(\lambda)$

1-forms

$\beta \leftrightarrow \mathrm{KKM}(\lambda6789, v)$

$\omega \leftrightarrow \mathrm{P}(\lambda)$

Explicit form of solution

$$ds_{10}^2 = -\frac{2\alpha}{\sqrt{Z_1 Z_2}}(dv+\beta)\left(du+\omega+\tfrac{1}{2}\mathcal{F}(dv+\beta)\right) - \sqrt{Z_1 Z_2}\,ds^2(\mathcal{B}^4) + \sqrt{\frac{Z_1}{Z_2}}\,ds^2(T^4)$$

$$e^{2\Phi} = \frac{\alpha Z_1}{Z_2} \qquad \alpha \equiv \frac{Z_1 Z_2}{Z_1 Z_2 - Z_4^2} \qquad \mathcal{D} \equiv d_4 - \beta \wedge \partial_v \qquad \dot{} \equiv \partial_v$$

$$H_3 = -(du+\omega)\wedge(dv+\beta)\wedge\left(\mathcal{D}\left(\frac{\alpha Z_4}{Z_1 Z_2}\right) - \frac{\alpha Z_4}{Z_1 Z_2}\dot{\beta}\right)$$

$$+(dv+\beta)\wedge\left(\Theta_4 - \frac{\alpha Z_4}{Z_1 Z_2}\mathcal{D}\omega\right) + \frac{\alpha Z_4}{Z_1 Z_2}(du+\beta)\wedge\mathcal{D}\beta + *_4\,(\mathcal{D}Z_4 + Z_4\dot{\beta})$$

$$F_1 = \mathcal{D}\left(\frac{Z_4}{Z_1}\right) + (dv+\beta)\wedge\partial_v\left(\frac{Z_4}{Z_1}\right)$$

$$F_3 = -(du+\omega)\wedge(dv+\beta)\wedge\left(\mathcal{D}\left(\frac{1}{Z_1}\right) - \frac{1}{Z_1}\dot{\beta} + \frac{\alpha Z_4}{Z_1 Z_2}\mathcal{D}\left(\frac{Z_4}{Z_1}\right)\right)$$

$$+(dv+\beta)\wedge\left(\Theta_1 - \frac{Z_4}{Z_1}\Theta_4 - \frac{1}{Z_1}\mathcal{D}\omega\right) + \frac{1}{Z_1}(du+\beta)\wedge\mathcal{D}\beta + *_4\,(\mathcal{D}Z_2 + Z_2\dot{\beta}) - \frac{Z_4}{Z_1} *_4\,(\mathcal{D}Z_4 + Z_4\dot{\beta})$$

… … … …

0^{th} layer: 4D base

6D spacetime: (u, v, x^m)

u: isometry
$v \sim x^5$ (compact)
x^m: 4D base

- 4D base $\mathcal{B}^4(v)$: almost hyper-Kähler

$$ds^2(\mathcal{B}^4) = h_{mn}(x, v) dx^m dx^n, \quad m, n = 1,2,3,4$$

$\beta(x, v)$: 1-form (↔ KKM)

$J^{(A)}(x, v),\ A = 1,2,3$: almost HK 2-forms

$$J^{(A)m}{}_n J^{(B)n}{}_p = \epsilon^{ABC} J^{(C)m}{}_p - \delta^{AB} \delta^m_p$$

$$d_4 J^{(A)} = \partial_v \left(\beta \wedge J^{(A)}\right), \qquad D \equiv d_4 - \beta \wedge \partial_v$$

BPS equations

- First layer (Z, Θ)

$$\mathcal{D} *_4 \left(\mathcal{D}Z_1 + \dot{\beta} Z_1\right) = -\mathcal{D}\beta \wedge \Theta_2$$

$$\mathcal{D}\Theta_2 - \dot{\beta} \wedge \Theta_2 = \partial_v\left[*_4 \left(\mathcal{D}Z_1 + \dot{\beta} Z_1\right)\right]$$

$$\Theta_2 - Z_1\psi = *_4 \left(\Theta_2 - Z_1\psi\right)$$

…

$$\psi = \frac{1}{8}\epsilon^{ABC} J^{(A)mn} \dot{J}^{(B)}_{mn} J^{(C)}$$

- Second layer $(\omega, \mathcal{F})$

$$(1 + *_4)\mathcal{D}\omega + \mathcal{F}\mathcal{D}\beta = Z_1 *_4 \Theta_1 + Z_2\Theta_2 - Z_4(1 + *_4)\Theta_4$$

$$\begin{aligned} *_4 \mathcal{D} *_4 L + 2\dot{\beta}_i L^i = {} & \dot{Z}_1\dot{Z}_2 + \ddot{Z}_1 Z_2 + Z_1\ddot{Z}_2 - \dot{Z}_4^2 - 2Z_4\ddot{Z}_4 + \frac{1}{2}\partial_v\left(Z_1Z_2 - Z_4^2\right)h^{mn}\dot{h}_{mn} \\ & + \tfrac{1}{2}\left(Z_1Z_2 - Z_4^2\right)\left(h^{mn}\ddot{h}_{mn} - \tfrac{1}{2}h^{mn}\dot{h}_{np}h^{pq}\dot{h}_{qm}\right) \\ & - \tfrac{1}{2} *_4 \left((\Theta_1 - Z_2\psi) \wedge (\Theta_2 - Z_1\psi) - (\Theta_4 - Z_4\psi) \wedge (\Theta_4 - Z_4\psi) + \tfrac{Z_1Z_2}{\alpha}\psi \wedge \psi - 2\psi \wedge \mathcal{D}\omega\right) \end{aligned}$$

$$L \equiv \dot{\omega} + \mathcal{F}\dot{\beta} - \mathcal{D}\mathcal{F}$$

— Linear if solved in the right order

— *Very* complicated!
Hard to find general superstrata

Strategy:

To prove concept,
construct simple superstrata
depending on functions of two variables

[Bena-Giusto-Russo-MS-Warner ’15]

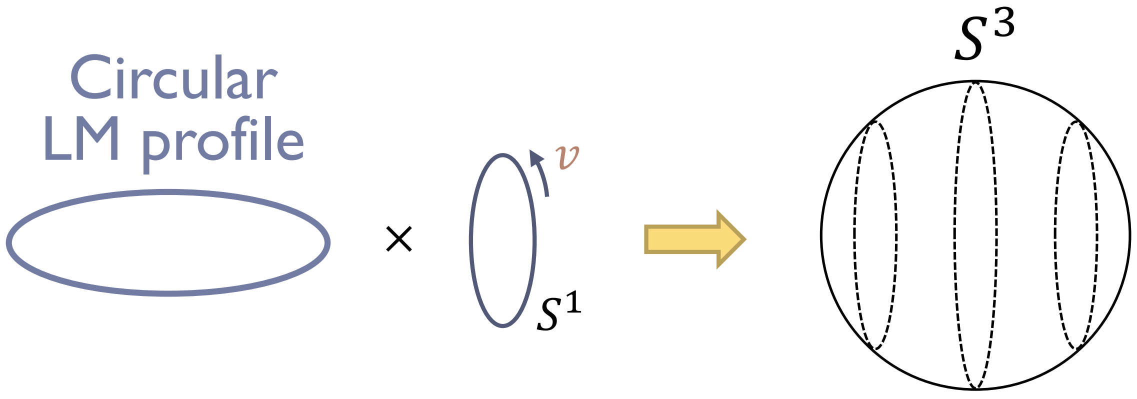
Background (1)

Starting point: simplest D1-D5 configuration (no P yet):

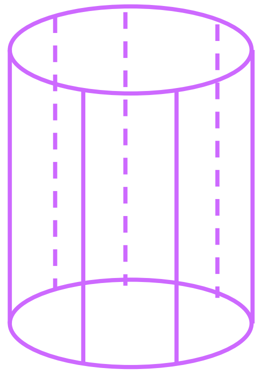
circular LM geom = pure $\mathrm{AdS}_3 \times S^3$

= "round" superstratum with no wiggle (yet)

Circular LM profile



at the center of AdS_3

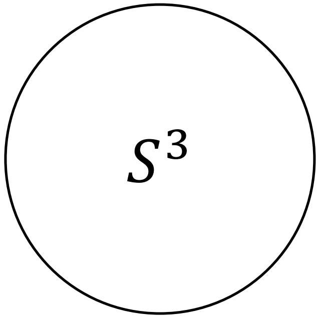


(previous naive picture)

Background (2)

Circular profile:

$$F_1 + iF_2 = a\,\exp(2\pi i\lambda/L)$$



Explicit solution:

Flat base ($\mathcal{B}^4 = \mathbb{R}^4$)

$$ds^2(\mathbb{R}^4) = \Sigma\left(\frac{dr^2}{r^2+a^2} + d\theta^2\right) + (r^2+a^2)\sin^2\theta\,d\phi^2 + r^2\cos^2\theta\,d\psi^2$$

$$\Sigma \equiv r^2 + a^2\cos^2\theta \qquad \beta = \frac{R_5 a^2}{\sqrt{2}\Sigma}(\sin^2\theta d\phi - \cos^2\theta d\psi)$$

Other data:

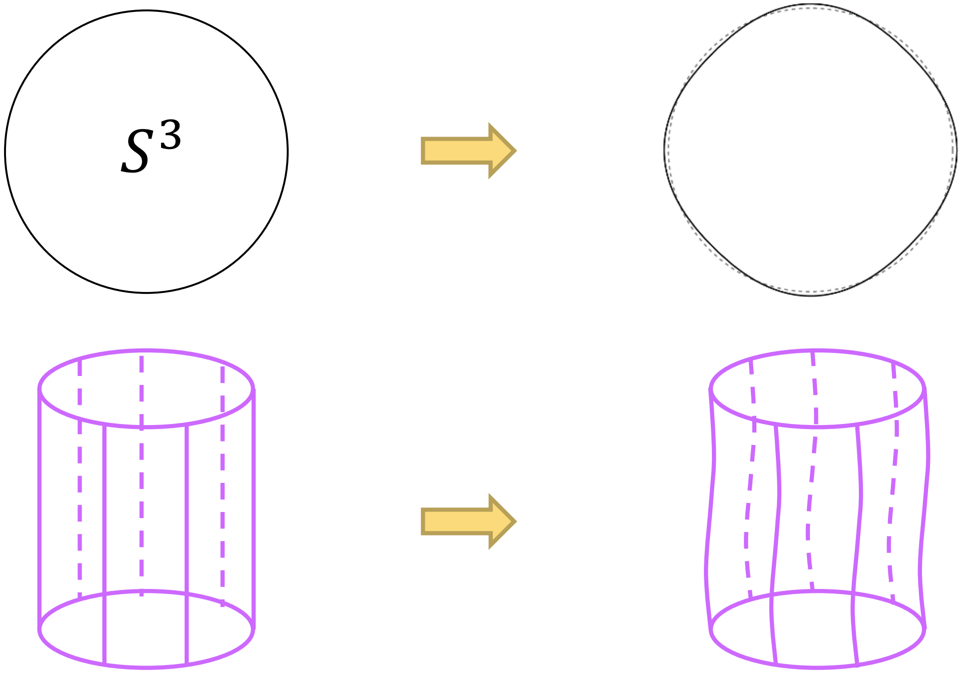
$$Z_1 = 1 + \frac{Q_1}{\Sigma} \qquad Z_2 = 1 + \frac{Q_2}{\Sigma} \qquad \omega = \frac{R_5 a^2}{\sqrt{2}\Sigma}(\sin^2\theta d\phi + \cos^2\theta d\psi)$$

$$Z_4 = \mathcal{F} = \Theta_1 = \Theta_2 = \Theta_4 = 0$$

Putting momentum

Now we want to add P

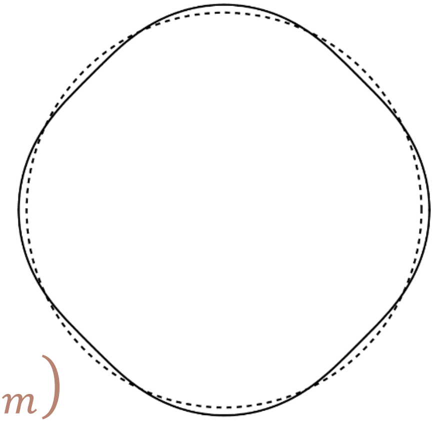
Putting momentum deforms
the round superstratum = S^3
by putting wiggles on it



Linear fluctuation

Certain *linear* solutions can be found by solution generating technique

[Mathur+Saxena+Srivastava 2003]



$$Z_4 = b\frac{R_5\Delta_{km}}{\Sigma}\cos\hat{v}_{km}$$

$$\Theta_4 = -\sqrt{2}bm\Delta_{km}\left(r\sin\theta\,\Omega^{(1)}\sin\hat{v}_{km} + \Omega^{(2)}\cos\hat{v}_{km}\right)$$

$$\Delta_{km} \equiv \left(\frac{a}{\sqrt{r^2+a^2}}\right)^k \sin^{k-m}\theta\,\cos^m\theta \qquad \hat{v}_{km} \equiv \frac{m\sqrt{2}}{R_5}v + (k-m)\phi - m\psi$$

v dependence (P)

$ds^2(\mathcal{B}^4),\ Z_{1,2,},\ \beta,\ \omega,\ \Theta_{1,2}$: unchanged at $\mathcal{O}(b)$

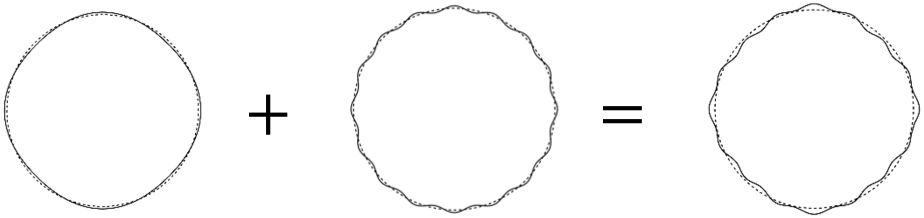
- Depends on two params (k,m)
- CFT dual: descendants of *chiral primary*

How to get function of two variables

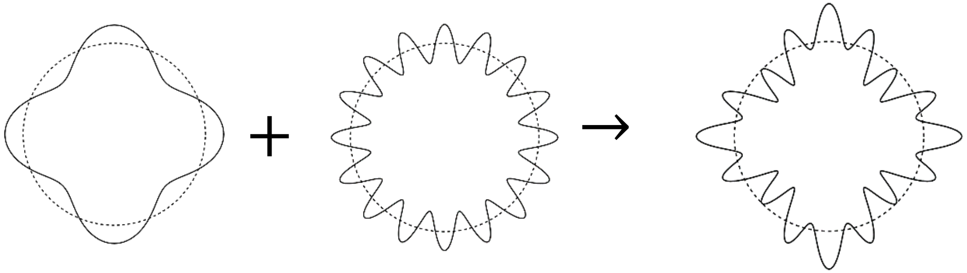
- Regard solution with (k, m) as Fourier modes on S^3

$$f(S^3) = \sum_{k,m} b_{k,m} Y_{k,m} \qquad S^3 : \underbrace{SU(2)_L}_{\text{BPS}} \times SU(2)_R$$

b_{km} independent ⇔ function of two variables!



- Non-linearly complete to get genuine geometric superstratum



Non-linear completion

Use linear structure of BPS eqs to nonlinearly complete

- Assume 0th data $\mathcal{B}^4, \beta$ are unchanged
- Regard Z_4, Θ_4 as non-linear sol'n of 1st layer

$$\mathcal{D} *_4 \mathcal{D}Z_4 = -\mathcal{D}\beta \wedge \Theta_4 \qquad \mathcal{D}\Theta_4 = \partial_v *_4 \mathcal{D}Z_4$$

- Find $\omega, \mathcal{F}$ as non-linear sol'n of 2nd layer

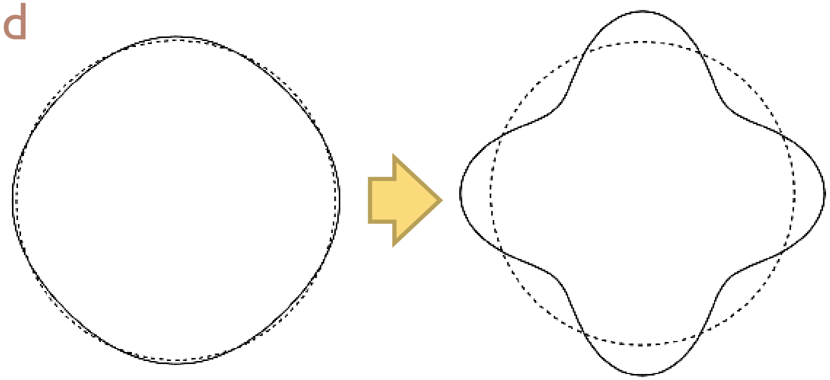
$$(1 +*_4)d\omega + \mathcal{F}d\beta = Z_1\Theta_1 + Z_2\Theta_2 - 2Z_4\Theta_4$$

$$*_4\, \mathcal{D} *_4 \left(\dot{\omega} - \tfrac{1}{2}d\mathcal{F}\right) = \dot{Z}_1\dot{Z}_2 + \ddot{Z}_1 Z_2 + Z_1\ddot{Z}_2 - \dot{Z}_4^2 - 2Z_4\ddot{Z}_4$$

  - ☐ Enough to do it for each pair of modes
- Regularity determines solution
  - ☐ It also determines $Z_{1,2}, \Theta_{1,2}$

Ex 1: $(k_1, m_1) = (k_2, m_2)$

$Z_4 \sim b \frac{\Delta_{k_1 m_1}}{\Sigma} \cos \hat{v}_{k_1 m_1}$, Z_2: unchanged



$Z_1 \supset b^2 \Delta_{2k_1,2m_1} \cos \hat{v}_{2k_1,2m_1}$

: needed to make ω regular

$\mathcal{F} = (2m_1)^2 F^{(0,0)}_{2k_1,2m_1}$

$\omega = \mu(d\psi + d\phi) + \zeta(d\psi - d\phi)$

undetermined

$$\mu = \frac{R_5}{4\sqrt{2}} \left(-\frac{\Delta_{2k_1,2m_1}}{\Sigma} + (2k_1 - 2m_1)^2 F^{(0,0)}_{2k_1,2m_1+2} - (2m_1)^2 \frac{r^2 + a^2 \sin^2\theta}{\Sigma} F^{(0,0)}_{2k_1,2m_1} \right) + \frac{x}{\Sigma}$$

Regularity ⇨ $\omega = 0$ at $r = \theta = 0$ ⇨ $x = \frac{R_5}{4\sqrt{2}} \left(\frac{k_1}{m_1} \right)^{-1}$

$\zeta = \cdots$

→ NL completed, with coeff fixed by regularity

Ex 2: (k_1, m_1): any, $(k_2, m_2) = (1,0)$

$Z_4 \sim b_1 \frac{\Delta_{k_1 m_1}}{\Sigma} \cos \hat{v}_{k_1 m_1} + b_2 \frac{\Delta_{10}}{\Sigma} \cos \hat{v}_{10}$, Z_2: unchanged

$$Z_1 \supset b_1 b_2 \left(\frac{\Delta_{k_1+1,m_1}}{\Sigma} \cos \hat{v}_{k_1+1,m_1} + \underset{\uparrow}{c} \frac{\Delta_{k_1-1,m_1}}{\Sigma} \cos \hat{v}_{k_1-1,m_1} \right),$$

undetermined

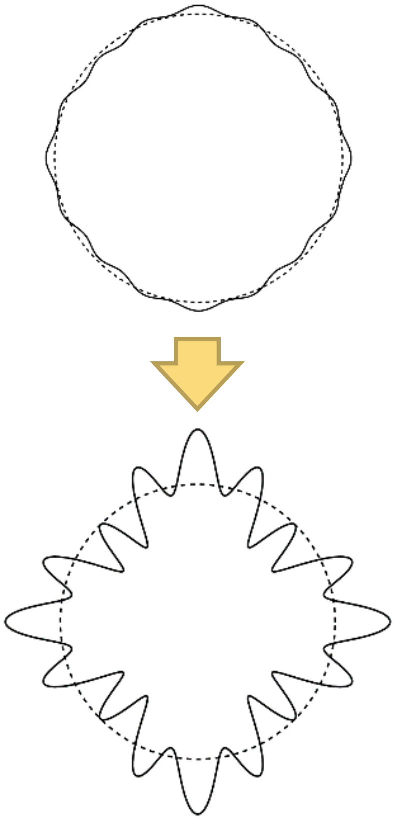
$\mathcal{F} = 0$

$\omega = c\omega^{(1)} + \omega^{(2)}$

$$\omega^{(1)} = \frac{R_5}{\sqrt{2}} \Delta_{k_1-1,m_1} \left(-\frac{dr}{r(r^2+a^2)} \sin \hat{v}_{k_1-1,m_1} + \frac{\text{¥}\sin^2\theta d\phi + \cos^2\theta d\psi}{\Sigma} \cos \hat{v}_{k_1-1,m_1} \right)$$

$$\omega^{(2)} = -\frac{R_5}{\sqrt{2}} \frac{\Delta_{k_1-1,m_1}}{r^2+a^2} \left[\left(\frac{m_1-k_1}{k_1} \frac{dr}{r} - \frac{m_1}{k_1} \tan\theta d\theta \right) \sin \hat{v}_{k_1-1,m_1} + \left(\frac{r^2+a^2}{\Sigma} \sin^2\theta d\phi + \left(\frac{r^2+a^2}{\Sigma} \cos^2\theta - \frac{m_1}{k_1} \right) d\psi \right) \cos \hat{v}_{k_1-1,m_1} \right]$$

Regularity ⇨ $\omega = 0$ at $r = \theta = 0$ ⇨ $c = \frac{k_1 - m_1}{k_1}$



→ NL completed, with coeff fixed by regularity

Summary

∃ Superstratum depending of two variables

⟺ Having modes with different (k, m)

⟺ NL completion for pair of modes

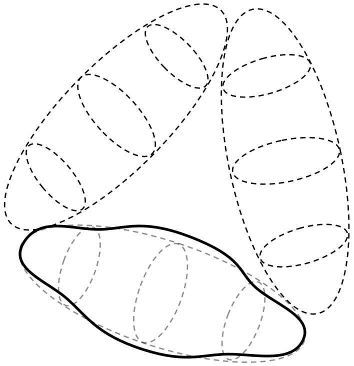
- Succeeded in NL completion for various pairs of modes
  - → *Constructive proof of existence of superstrata!*
  - → Big step toward general 3-charge microstate geometries
- Correspond to *non-chiral* primaries in CFT
  - → Most general microstate geom with known CFT dual

Toward more general superstrata

- Does this class of superstrata reproduce S_{BH}?
  - → No. These correspond to coherent states of graviton gas. Entropy is parametrically smaller.
- Need more general superstrata
  - → In CFT language, we only considered rigid generators of $SU(1,1|2)_L \times SU(1,1|2)_R$ e.g. L_0, L_1, L_{-1}, J_0^-
  - → Need higher and fractional modes e.g. $J_{-\frac{1}{k}}^-$
  - → They probably correspond to multiple superstrata

Multiple superstrata

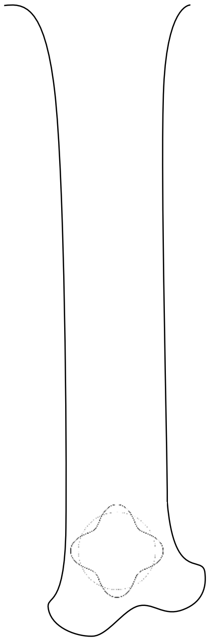
- More generally, one has multiple S^3's
- Can fluctuate each S^3 — multi-superstratum



- Can use $AdS_3 \times S^3$ as local model
- Large redshift in scaling geometries

→ entropy enhancement?

→ $S \sim Q^{3/2}$?



Comment on “issues”

- Lifting
  - Not directly applicable to 6D configuration
- Pure Higgs branch
  - Superstratum reminiscent of Higgs branch

Maybe only states that have $J = 0$
survive when moduli are turned on?

Conclusions

Conclusions

- Microstate geometry program
  - Interesting enterprise elucidating micro nature of BHs, whether answer turns out to be yes or no
- Microstate geometries in 5D sugra
  - Have properties expected from CFT, but too few
- Superstratum
  - A new class of microstate geometries
  - CFT duals precisely understood
  - More general superstrata are crucial to reproduce S_{BH}

Future directions

- Superstratum
  - More general solution, multi-strata
  - Clarify issues (lifting, pure Higgs)
  - Count states, reproduce entropy (or not)
- Non-geometric microstates
  - Exotic branes, DFT
  - Novel ways to store information
- More
  - Non-extremal BHs
  - Information paradox
  - Observational consequences?
  - Early universe
  - …

Thanks!