Anomaly polynomial of general 6d SCFTs

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Based on arXiv:1408.5572 with Ohmori, Tachikawa, Yonekura

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Review: What are 't Hooft anomalies?

- 't Hooft anomaly: obstruction to gauging global symmetries.

  Couple the theory to background gauge fields A_μ, $g_{\mu\nu}$

  $\rightarrow$ effective action fails to be gauge invariant:

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- Descent equation and anomaly polynomial:

$$I_{d+2} = dI_{d+1}^{(0)},\ \delta I_{d+1}^{(0)} = dI_d^{(1)}.$$

  Anomaly polynomial I_{d+2}: polynomial of characteristic classes

$p_1(T) = -\frac{1}{8\pi^2}\,\mathrm{tr}\,R^2$, $p_2(T) = \frac{1}{128\pi^2}\left((\mathrm{tr}\,R^2)^2 - 2\,\mathrm{tr}\,R^4\right)$ etc...

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- 6d SCFT: self-dual 2-form gauge fields and tensionless strings.

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- 't Hooft anomalies for global symmetries:
  - N^3 scaling law of d.o.fs
  - Central charges of compactified theory [Benini, Tachikawa, Wecht '09]
  - RG flow between 6d SCFTs [Heckman Morrison Rudelius Vafa '15]

etc...

How to calculate 't Hooft anomalies of 6d SCFTs?

- Gravitational calculcation:

  embed 6d SCFT into M-theory and use anomaly inflow.

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In this short talk, I will explain the field theoretical calculation of anomaly polynomial of 6d $\mathcal{N} = (2, 0)$ theory.

- G-type $\mathcal{N}=(2,0)$ theory: IIB on ADE orbifold $\mathbb{C}^2/\Gamma_G$.

  Anomaly polynomial conjecture: [Intriligator '00]

$$I_G^{\mathcal{N}=(2,0)} = \frac{h_G^\vee d_G}{24} p_2(N) + r_G I^{\mathcal{N}=(2,0)\ \text{tensor}},$$

$$I^{\mathcal{N}=(2,0)\ \text{tensor}} = \frac{1}{48}\left(p_2(N) - p_2(T) + \frac{1}{4}(p_1(T) - p_1(N))^2\right).$$

$$h^\vee_{\mathrm{SU}(k)} d_{\mathrm{SU}(k)} = k^3 - k \qquad N\text{: } \mathrm{SO}(5) \text{ R-symmetry}$$

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- When $G = A_{k-1}, D_k$, gravitational calculation is available:

  $G = A_{n-1} \to$ coincident k M5-branes, [(Freed,) Harvey, Minasian, Moore '98]

  $G = D_n \to$ coincident k M5-branes + orientifold. [Yi '01]

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- Field theory calculation is valid for any G. [Ohmori, HS, Tachikawa, Yonekura]

- Tensor branch RG flow: giving tension to self-dual strings.

  G-type $\mathcal{N}=(2,0)$ theory $\to$ r_G free $\mathcal{N}=(2,0)$ tensor multiplets.

  UV IR (B_2, $\phi^{i=1\cdots 5}$, fermions)

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- Anomaly matching on tensor branch:

$$I_G^{\mathcal{N}=(2,0)} = (\text{anomaly matching term}) + r_G I^{\mathcal{N}=(2,0)\ \text{tensor}}.$$

  Origin of anomaly matching term?

Green-Schwarz mechanism for anomaly matching

- Integrate out massive strings: induce electric/magnetic coupling for B_2

$$\Delta L = \int_{X_6} \Omega^{ij} B_i I_j \quad \text{and} \quad \underbrace{dH_i = I_i}_{\text{Bianchi identity}} \quad i = 1 \cdots r_G.$$

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- Additional contribution from Green-Schwarz mechanism:

$$I^{\mathsf{GS}} = \frac{1}{2} \Omega^{ij} I_i I_j$$

$$I_G^{\mathcal{N}=(2,0)} = I^{\mathsf{GS}} + r_G I^{\mathcal{N}=(2,0)\ \text{tensor}} .$$

How to determine I_i ?

S^1 compactification of 6d $\mathcal{N}=(2,0)$ theory

- G-type 6d $\mathcal{N}=(2,0)$ theory on $S^1 \to$ 5d G $\mathcal{N}=2$ SYM.

  6d tensor branch $\to$ 5d Coulomb branch:$G \to \mathrm{U}(1)^{r_G}$.

  $\langle \phi^a_{i=1\cdots 4} \rangle = 0$ and $\langle \phi^a_5 \rangle = v^a$, then $\mathrm{SO}(5)_R \to \mathrm{SO}(4)_R$.

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- Massive strings $\to$ massive $\mathcal{N}=2$ vectors Φ_α (α: roots of G) w/ mass $v \cdot \alpha$.

  Integrating out Φ_α: induced Chern-Simons terms

  $$S^{CS} = \Omega^{ij} A_i I_j, \qquad A_i : \mathrm{U}(1)_i \text{ gauge field.}$$

  Reduce to ordinary 1-loop caluculation!

Caluculation

- Induced Chern-Simons term for A_i $i = 1 \cdots r_G$:

$$\frac{1}{2}\sum_{\alpha>0}(\alpha \cdot A)\Big[\underbrace{(c_2(L) + \frac{2}{24}p_1(T))}_{\text{pos real mass fermions}} - \underbrace{(c_2(R) + \frac{2}{24}p_1(T))}_{\text{neg real mass fermions}}\Big]$$

$$= \rho \cdot A(c_2(L) - c_2(R)),$$

$\rho = \frac{1}{2}\sum_{\alpha>0}\alpha$: Weyl vector, $\mathrm{SO}(4)_R \sim \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$.

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- Green-Schwarz contribution:

$$\frac{1}{2}\langle\rho,\rho\rangle(c_2(L) - c_2(R))^2 = \frac{h_G^\vee d_G}{24}(c_2(L) - c_2(R))^2.$$

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- These polynomials may be used to prove a-theorem for 6d SCFTs/ investigate compactification etc……