2nd String Theory in Greater Tokyo
@riken, 2015/6/9

Comments on entanglement entropy in the dS/CFT correspondence

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based on PRD 91 (2015) 8, 086009 [arXiv:1501.04903]

Plan of my talk

1. Introduction
2. Proposal for holographic entanglement entropy in Einstein gravity on dS
3. Comparison with a toy model
4. Conclusion & Discussion

1. Introduction

Motivation

- AdS/CFT relates gravitational theories on AdS with non-gravitational theories.

  ➡ We can analyze quantum gravitational theories using non-gravitational theories.

- Toward a quantum description of our Universe, AdS/CFT might be useful.

  ➡ We need dS/CFT instead of AdS/CFT
  since our Universe is approximately dS.

- The holographic entanglement entropy (HEE) is a useful quantity to analyze gravitational theories.

  In fact, Einstein's equation and a radial component of AdS metric are reproduced from HEE, for instance.

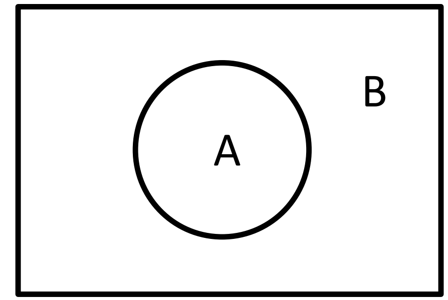
  [Nozaki-Ryu-Takayanagi, PRD 88 (2013) 2, 026012]
  [Lashkari-McDermott-Raamsdonk, JHEP 1404 (2014) 195]

Entanglement entropy

- Definition

  (i) divide the total system into A and B

$$\mathcal{H}_{\rm tot} = \mathcal{H}_A \times \mathcal{H}_B$$



  (ii) introduce a reduced density matrix

$$\rho_A = {\rm tr}_B \rho_{\rm tot} \quad \text{where} \quad \rho_{\rm tot} = |\Psi\rangle\langle\Psi|$$

wave function

  (iii) entanglement entropy (EE) is defined as

$$S_A = -{\rm tr}_A \rho_A \log \rho_A = -\lim_{n\to 1} \frac{\partial}{\partial n} {\rm tr}_A \rho_A^n = -\lim_{n\to 1} \frac{\partial}{\partial n} \log {\rm tr}_A \rho_A^n$$

definition   analytic continuation

- Entanglement entropy measures how subsystems A & B correlate each other.

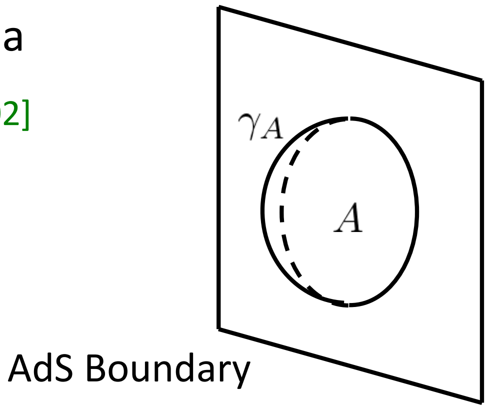
Ryu-Takayanagi formula

- Holographic dual of EE is given by Ryu-Takayanagi formula

[Ryu-Takayanagi, PRL 96 (2006) 181602]

$$S_A = \frac{\text{Area of } \gamma_A}{4G_{\text{N}}}$$

← extremal surfaces



- HEE is a generalised quantity of the black hole entropy.

[Lewkowycz-Maldacena, JHEP 1308 (2013) 090]

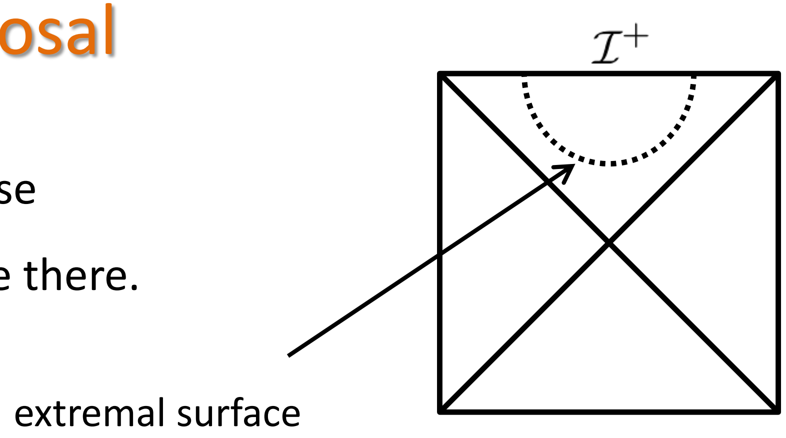
Black hole entropy formula holds even in dS and flat spacetime.

➡ HEE formula should hold in dS!!

I investigate HEE in dS/CFT.

2. Proposal for HEE in Einstein gravity on dS

Proposal

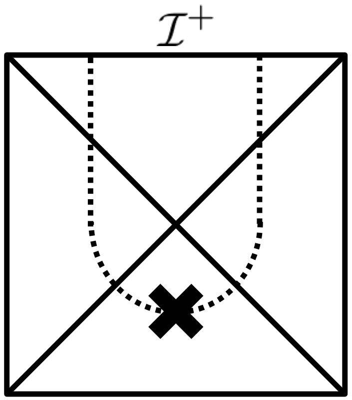


- We need to find extremal surfaces whose boundaries sit in $\mathcal{I}^+$ since dual CFTs live there.

- We propose that "extremal surfaces" in dS are given by analytic continuations of that in Euclidean AdS.

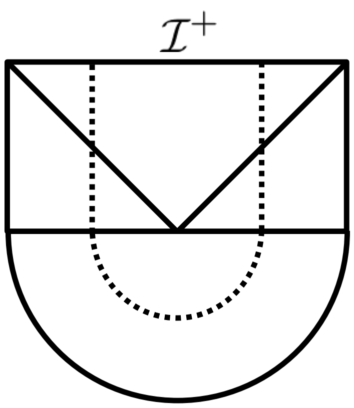
- Comments on other possibilities

① time-like surface



In general, time-like surfaces are not closed.

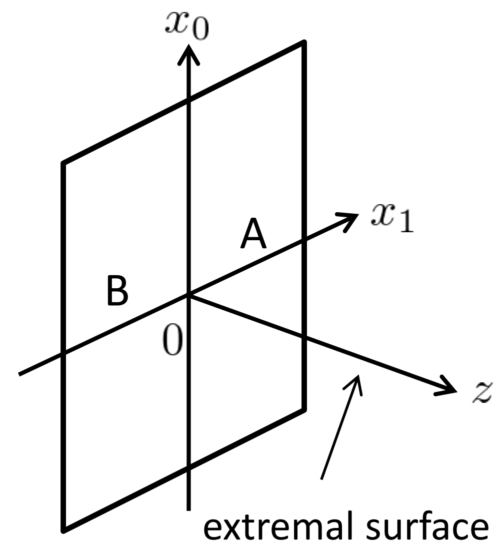
② Hartle-Hawking vacuum



HEE becomes a sum of a pure real part and a pure imaginary part.

This result largely disagrees with our results in Sp(N) model.

Example: half plane



- Take the region A as a half plane.

Metric: $$ds^2 = \ell_{\rm AdS}^2 \frac{dz^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

Extremal surface: $0 \le z < \infty$

$$\Rightarrow \quad S_A = \frac{V_{d-2}}{4G_{\rm N}} \int_{\varepsilon}^{\infty} dz \left(\frac{\ell_{\rm AdS}}{z}\right)^{d-1} = \frac{V_{d-2}\ell_{\rm AdS}^{d-1}}{4G_{\rm N}(d-2)} \cdot \frac{1}{\varepsilon^{d-2}}$$

- Double analytic continuation: $z \to -i\eta\,, \quad \ell_{\rm AdS} \to -i\ell_{\rm dS}$

Metric: $$ds^2 = \ell_{\rm dS}^2 \frac{-d\eta^2 + \sum_{i=1}^{d} dx_i^2}{\eta^2}$$

Extremal surface: $0 \le \eta < i\infty$

Entanglement entropy: $$S_A = (-i)^{d-1} \frac{V_{d-2}\ell_{\rm dS}^{d-1}}{4G_{\rm N}(d-2)} \cdot \frac{1}{\varepsilon^{d-2}}$$

In general, extremal surfaces extend in complex-valued coordinates.

3. Comparison with a toy model

Sp(N) model

- The CFT holographic dual to Einstein gravity on dS is not known yet.

  ➡ analyze the free Sp(N) model, which is the holographic dual of Vasiliev's higher spin theory

- Action (d-dim)

$$I = \int \mathrm{d}^d x \, \Omega_{ab} \partial \chi^a \cdot \partial \chi^b \quad \text{where} \quad \Omega_{ab} = \begin{pmatrix} 0 & 1_{N/2\times N/2} \\ -1_{N/2\times N/2} & 0 \end{pmatrix}$$

Introducing $\eta^a = \chi^a + i\chi^{a+\frac{N}{2}} \, , \quad \bar{\eta}^a = -i\chi^a - \chi^{a+\frac{N}{2}} \qquad (a = 1, \cdots, N/2)$

➡ $$I = \int \mathrm{d}^d x \, \partial \bar{\eta}^a \cdot \partial \eta^a$$

- χ^a , η^a and $\bar{\eta}^a$ are anti-commuting scalar fields.

  ➡ violates the spin-statics theorem ➡ non-unitary

Replica trick

- Entanglement entropy

$$S_A = -\mathrm{tr}_A \rho_A \log \rho_A = -\lim_{n\to 1} \frac{\partial}{\partial n} \mathrm{tr}_A \rho_A^n = -\lim_{n\to 1} \frac{\partial}{\partial n} \log \mathrm{tr}_A \rho_A^n$$

This calculation is difficult. Instead, we calculate $\mathrm{tr}_A \rho_A^n$.

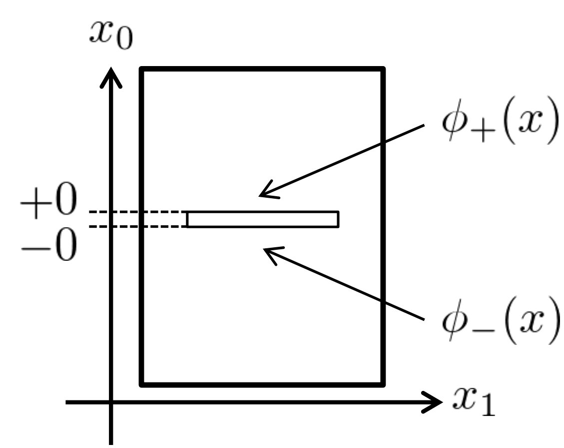
- Wave function for a ground state

$$\Psi[\phi(\boldsymbol{x}, x_0 = 0)] = \frac{1}{\sqrt{Z}} \int \prod_{-\infty < x_0 < 0} \prod_{\boldsymbol{x}} \mathrm{d}\phi \, \mathrm{e}^{-S[\phi]} \delta[\phi(0, \boldsymbol{x}) - \phi(\boldsymbol{x})]$$

$$\Psi^*[\phi(\boldsymbol{x}, x_0 = 0)] = \frac{1}{\sqrt{Z}} \int \prod_{0 < x_0 < \infty} \prod_{\boldsymbol{x}} \mathrm{d}\phi \, \mathrm{e}^{-S[\phi]} \delta[\phi(0, \boldsymbol{x}) - \phi(\boldsymbol{x})]$$

- Path integral representation of ρ_A

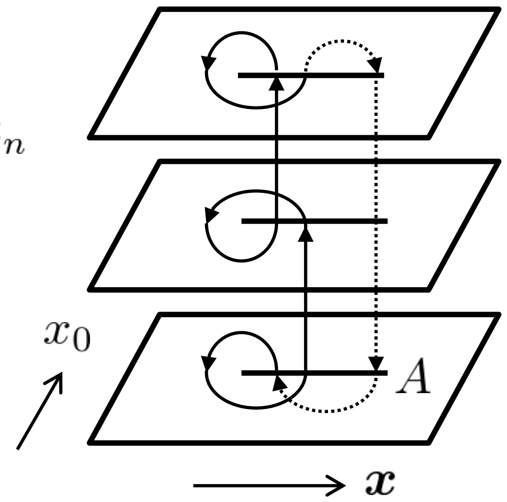
$$[\rho_A]_{\phi_- \phi_+} = \frac{1}{Z} \int \prod_{x} \mathrm{d}\phi \, \mathrm{e}^{-S[\phi]} \prod_{\boldsymbol{x} \in A} \delta[\phi(-0, \boldsymbol{x}) - \phi_-(\boldsymbol{x})] \times \delta[\phi(+0, \boldsymbol{x}) - \phi_+(\boldsymbol{x})]$$



Replica trick

- $\mathrm{tr}\rho_A^n$ is given by a partition function on Riemann surface Σ_n

$$\mathrm{tr}\rho_A^n = \frac{1}{Z^n}\int \prod_{x\in\Sigma_n} \mathrm{d}\phi\, \mathrm{e}^{-S[\phi]}$$

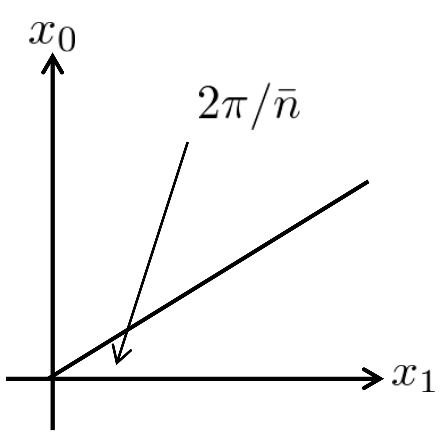


- EE for a free scalar field theory (the subsystem is a half plane)

We take $n \to 1/\bar{n}$ where $\bar{n}$ is integer.

➡ We need to evaluate the partition function on $\mathbb{R}^2/Z_{\bar{n}} \times \mathbb{R}^{d-2}$.

$$S_A = -\lim_{n\to 1}\frac{\partial}{\partial(1/n)}\left(\log Z_{\mathbb{R}^2/Z_n\times\mathbb{R}^{d-2}} - \frac{1}{n}\log Z_{\mathbb{R}^d}\right)$$
$$= \frac{V_{d-2}}{6(d-2)(4\pi)^{(d-2)/2}}\cdot\frac{1}{\varepsilon^{d-2}} + \mathcal{O}(\varepsilon^{-(d-4)})$$



Entanglement entropy in Sp(N) model

- Differences from the scalar field theory

  (i) anti-commuting scalars

  ➡ EE in Sp(N) model is minus that of the usual scalar field theory.

  (ii) N complex fields

  ➡ EE is proportional to N.

$$S_A = -\frac{NV_{d-2}}{6(d-2)(4\pi)^{\frac{d-2}{2}}} \cdot \frac{1}{\varepsilon^{d-2}}$$

- Comment

  EE is a positive definite quantity in usual.

  However, EE in Sp(N) model is negative.

  ➡ Hilbert space of Sp(N) model is not positive definite.

4. Conclusion & Discussion

Conclusion & Discussion

- We have obtained HEE & EE.

  HEE behaves as $S_A \propto (-i)^{d-1}$ in dS_d+1.

  EE behaves as $S_A \propto -S_A^{\text{standard field theories}}$.

- dS_d+1/CFT_d correspondence makes senses only when $d+1 \in 4\mathbb{Z}$

  ➡ The most interesting case, dS_4/CFT_3, is included.

  The most simple case, dS_3/CFT_2, is excluded.

  This is consistent with the result in subsection 5.2 in [Maldacena, JHEP 0305 (2003) 013].

- Our proposal has been checked only in the simple case, half plane.

  However, our proposal holds in any entanglement surfaces.

Thank you for
your attention!!