F-theory and 6D (1,0) theories

Michele Del Zotto

Harvard University

Based on joint works with:
Dave Morrison, Jonathan Heckman, Daniel Park, Tom Rudelius, Alessandro Tomasiello, Cumrun Vafa, and Dan Xie

2nd String Theory in Greater Tokyo - 9 June 2015

The classification of superconformal groups is a result by Nahm (77) building on the classification of super Lie algebras by himself, Rittenberg and Scheunert (77) and Kac (77).

The classification of superconformal groups is a result by Nahm (77) building on the classification of super Lie algebras by himself, Rittenberg and Scheunert (77) and Kac (77). The existence of superconformal groups relies on exceptional isomorphisms in between Lie groups of low rank.

The classification of superconformal groups is a result by Nahm (77) building on the classification of super Lie algebras by himself, Rittenberg and Scheunert (77) and Kac (77). The existence of superconformal groups relies on exceptional isomorphisms in between Lie groups of low rank. Essentially for this reason, superconformal algebras exist only in lower spacetime dimensions,

$$D \leq 6$$

The classification of superconformal groups is a result by Nahm (77) building on the classification of super Lie algebras by himself, Rittenberg and Scheunert (77) and Kac (77). The existence of superconformal groups relies on exceptional isomorphisms in between Lie groups of low rank. Essentially for this reason, superconformal algebras exist only in lower spacetime dimensions,

$$D \leq 6$$

A natural question is whether there exist corresponding QFTs with the desired superconformal symmetry.

For $D \leq 4$ it is possible to construct such models directly.

For $D \leq 4$ it is possible to construct such models directly. For $D = 5, 6$ the construction is indirect.

For $D \leq 4$ it is possible to construct such models directly. For $D = 5, 6$ the construction is indirect. From a pure QFT viewpoint there are interesting conjectures by Seiberg (95-96) see also Seiberg-Witten (96), which give necessary conditions for the existence of superconformal points in $D = 5, 6$.

For $D \leq 4$ it is possible to construct such models directly. For $D = 5, 6$ the construction is indirect. From a pure QFT viewpoint there are interesting conjectures by Seiberg (95-96) see also Seiberg-Witten (96), which give necessary conditions for the existence of superconformal points in $D = 5, 6$. On the other hand, String/M/F theory predicts their existence (Witten 95).

For $D \leq 4$ it is possible to construct such models directly. For $D = 5, 6$ the construction is indirect. From a pure QFT viewpoint there are interesting conjectures by Seiberg (95-96) see also Seiberg-Witten (96), which give necessary conditions for the existence of superconformal points in $D = 5, 6$. On the other hand, String/M/F theory predicts their existence (Witten 95). The logic for both constructions is very similar.

For $D \leq 4$ it is possible to construct such models directly. For $D = 5, 6$ the construction is indirect. From a pure QFT viewpoint there are interesting conjectures by Seiberg (95-96) see also Seiberg-Witten (96), which give necessary conditions for the existence of superconformal points in $D = 5, 6$. On the other hand, String/M/F theory predicts their existence (Witten 95). The logic for both constructions is very similar. One starts from a phase with broken conformal symmetry, and argues for the existence of a superconformal point.

This has the advantage that one can use 6D (1,0) susy multiplets, namely, as $SO(4)_{\text{spin}} \times USp(2)_R$

- $\frac{1}{2}$ hypers: $(1,1\,;\,2) \oplus (2,1\,;\,1)$
- vectors: $(2,2\,;\,1) \oplus (1,2\,;\,2)$
- tensors: $(3,1\,;\,1) \oplus (1,1\,;\,1) \oplus (2,1\,;\,2)$

Notice that vectors in 6D do not have scalars, therefore there is not a Coulomb branch. However, whenever a 6D model contain full hypers, Higgs branches arises, and whenever it contains tensor multiplets, giving vevs to the real scalars give rise to Coulomb like phase, the tensor branch.

This has the advantage that one can use 6D (1,0) susy multiplets, namely, as $SO(4)_{\text{spin}} \times USp(2)_R$

- $\frac{1}{2}$ hypers: $(1,1\,;\,2) \oplus (2,1\,;\,1)$
- vectors: $(2,2\,;\,1) \oplus (1,2\,;\,2)$
- tensors: $(3,1\,;\,1) \oplus (1,1\,;\,1) \oplus (2,1\,;\,2)$

Notice that vectors in 6D do not have scalars, therefore there is not a Coulomb branch. However, whenever a 6D model contain full hypers, Higgs branches arises, and whenever it contains tensor multiplets, giving vevs to the real scalars give rise to Coulomb like phase, the tensor branch. Along tensor branches, these models have BPS strings, which are non-critical, their tension is governed by the tensor branch vevs and by tuning them these can become massless.

This has the advantage that one can use 6D (1,0) susy multiplets, namely, as $SO(4)_{\text{spin}} \times USp(2)_R$

- $\frac{1}{2}$ hypers: $(1,1\,;\,2) \oplus (2,1\,;\,1)$
- vectors: $(2,2\,;\,1) \oplus (1,2\,;\,2)$
- tensors: $(3,1\,;\,1) \oplus (1,1\,;\,1) \oplus (2,1\,;\,2)$

Notice that vectors in 6D do not have scalars, therefore there is not a Coulomb branch. However, whenever a 6D model contain full hypers, Higgs branches arises, and whenever it contains tensor multiplets, giving vevs to the real scalars give rise to Coulomb like phase, the tensor branch. Along tensor branches, these models have BPS strings, which are non-critical, their tension is governed by the tensor branch vevs and by tuning them these can become massless. This is the hallmark for having a 6D superconformal point.

Some well known examples are:

Some well known examples are:

- $(2,0)$ theory of type A_{N-1}, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes

Some well known examples are:

- $(2,0)$ theory of type A_{N-1}, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes
- $(1,0)$ theory of heterotic E_8 instantons, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes in presence of an end-of-the-world M9 brane

Some well known examples are:

- $(2,0)$ theory of type A_{N-1}, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes
- $(1,0)$ theory of heterotic E_8 instantons, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes in presence of an end-of-the-world M9 brane
- $(1,0)$ superconformal matter, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes probing a singularity $\mathbb{C}^2/\Gamma_G$

Some well known examples are:

- $(2,0)$ theory of type A_{N-1}, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes
- $(1,0)$ theory of heterotic E_8 instantons, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes in presence of an end-of-the-world M9 brane
- $(1,0)$ superconformal matter, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes probing a singularity $\mathbb{C}^2/\Gamma_G$
- $(1,0)$ heterotic E_8 instantons probing a singularity $\mathbb{C}^2/\Gamma_G$

Some well known examples are:

- $(2,0)$ theory of type A_{N-1}, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes
- $(1,0)$ theory of heterotic E_8 instantons, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes in presence of an end-of-the-world M9 brane
- $(1,0)$ superconformal matter, engineered in M-theory as the theory on the worldvolume of a stack of N M5 branes probing a singularity $\mathbb{C}^2/\Gamma_G$
- $(1,0)$ heterotic E_8 instantons probing a singularity $\mathbb{C}^2/\Gamma_G$

From these examples it is evident that the study of such systems is deeply interconnected with the dynamics of extended objects in String and M theory, which is one motivation to study them.

Another motivation to understand these systems is to shed some light on the dynamics of lower dimensional systems obtained by compactifications to $D < 6$.

Another motivation to understand these systems is to shed some light on the dynamics of lower dimensional systems obtained by compactifications to $D < 6$. From compactification of the $(2, 0)$ theories on T^2, we have a beautiful geometrical explanation of the Montonen-Olive $SL(2, \mathbb{Z})$ duality for 4D $\mathcal{N} = 4$ SYM theories, Witten (95).

Another motivation to understand these systems is to shed some light on the dynamics of lower dimensional systems obtained by compactifications to $D < 6$. From compactification of the $(2,0)$ theories on T^2, we have a beautiful geometrical explanation of the Montonen-Olive $SL(2,\mathbb{Z})$ duality for 4D $\mathcal{N} = 4$ SYM theories, Witten (95). More generally, we have the class $\mathcal{S}$ construction (Gaiotto 08, and Gaiotto-Moore-Neitzke 09):

Another motivation to understand these systems is to shed some light on the dynamics of lower dimensional systems obtained by compactifications to $D < 6$. From compactification of the $(2,0)$ theories on T^2, we have a beautiful geometrical explanation of the Montonen-Olive $SL(2,\mathbb{Z})$ duality for 4D $\mathcal{N} = 4$ SYM theories, Witten (95). More generally, we have the class $\mathcal{S}$ construction (Gaiotto 08, and Gaiotto-Moore-Neitzke 09): compactification of $(2,0)$ theory on Riemann surfaces Σ leads to 4D $\mathcal{N} = 2$ theories whose S-duality group gets identified with the mapping class group of Σ.

Another motivation to understand these systems is to shed some light on the dynamics of lower dimensional systems obtained by compactifications to $D < 6$. From compactification of the $(2,0)$ theories on T^2, we have a beautiful geometrical explanation of the Montonen-Olive $SL(2,\mathbb{Z})$ duality for 4D $\mathcal{N} = 4$ SYM theories, Witten (95). More generally, we have the class $\mathcal{S}$ construction (Gaiotto 08, and Gaiotto-Moore-Neitzke 09): compactification of $(2,0)$ theory on Riemann surfaces Σ leads to 4D $\mathcal{N} = 2$ theories whose S-duality group gets identified with the mapping class group of Σ. Starting from $(1,0)$ and proceeding analogously one can obtain 4D $\mathcal{N} = 2$ or $\mathcal{N} = 1$ theories and try to explain aspects of their dynamics from this higher dimensional perspective.

The purpose of this lecture is to illustrate how to engineer (1,0) theories within F-theory, learn about some of their properties, and then exploit string dualities to study their compactification down to 4D.

The purpose of this lecture is to illustrate how to engineer (1,0) theories within F-theory, learn about some of their properties, and then exploit string dualities to study their compactification down to 4D. Much probably I am going to be able to explain just the necessary material to be able to start reading my papers on the subject, which I am going to review very briefly towards the end of the talk.

The purpose of this lecture is to illustrate how to engineer (1,0) theories within F-theory, learn about some of their properties, and then exploit string dualities to study their compactification down to 4D. Much probably I am going to be able to explain just the necessary material to be able to start reading my papers on the subject, which I am going to review very briefly towards the end of the talk. The main results are the following:

Studying 6D (1,0) theories in an F-theory framework, in joint work with Heckman, Tomasiello and Vafa, we have understood fractionalization of M-theory M5 and M9 branes probing $\mathbb{C}^2/\Gamma$ singularities.

2

6D (1,0) theories are relative field theories: as their (2,0) cousins there are obstructions to define their partition functions on curved spaces; such an obstruction is measured by the defect group Λ^*/Λ where Λ is the charge lattice of BPS strings of the model while Λ^* is their lattice of codimension 4 defects.

2

6D (1,0) theories are relative field theories: as their (2,0) cousins there are obstructions to define their partition functions on curved spaces; such an obstruction is measured by the defect group Λ^*/Λ where Λ is the charge lattice of BPS strings of the model while Λ^* is their lattice of codimension 4 defects. This is joint with Heckman, Park, and Rudelius.

3

Compactification of 6D (1,0) theories on T^2 explains the appearance of the moduli spaces of flat G connections on T^2 as conformal manifolds of affine $\hat{G}$ quiver 4D $\mathcal{N} = 2$ SCFTs observed by Klemm, Mayr and Vafa (97), and predicts the existence of four infinite novel families of systems which enjoy an exact $SL(2, \mathbb{Z})$ duality and typically have strongly interacting superconformal subsystems.

Compactification of 6D (1,0) theories on T^2 explains the appearance of the moduli spaces of flat G connections on T^2 as conformal manifolds of affine $\hat{G}$ quiver 4D $\mathcal{N} = 2$ SCFTs observed by Klemm, Mayr and Vafa (97), and predicts the existence of four infinite novel families of systems which enjoy an exact $SL(2, \mathbb{Z})$ duality and typically have strongly interacting superconformal subsystems. We also extend the findings of Ganor, Morrison, and Seiberg (96) about the toroidal compactification of the theory of one E_8 heterotic instanton to a wide variety of 6D (1,0) SCFTs. This is joint with Vafa and Xie (also Ohomori, Shimizu, Tachikawa, and Yonekura (15)).

A lightening review of 6D F-theory backgrounds

This is based on Vafa (96) and Morrison-Vafa (96). Let X be an elliptically fibered 3-CY.

A lightening review of 6D F-theory backgrounds

This is based on Vafa (96) and Morrison-Vafa (96). Let X be an elliptically fibered 3-CY. Consider a compactification of M-theory on X.

A lightening review of 6D F-theory backgrounds

This is based on Vafa (96) and Morrison-Vafa (96). Let X be an elliptically fibered 3-CY. Consider a compactification of M-theory on X. As X is elliptically fibered we can use one S^1 in the elliptic fiber to reduce to IIA, the other S^1 to T-dualize to IIB.

A lightening review of 6D F-theory backgrounds

This is based on Vafa (96) and Morrison-Vafa (96). Let X be an elliptically fibered 3-CY. Consider a compactification of M-theory on X. As X is elliptically fibered we can use one S^1 in the elliptic fiber to reduce to IIA, the other S^1 to T-dualize to IIB. In the limit in which the elliptic fiber collapses to zero size, the winding modes of IIA becomes light and a new direction opens up in IIB.

A lightening review of 6D F-theory backgrounds

This is based on Vafa (96) and Morrison-Vafa (96). Let X be an elliptically fibered 3-CY. Consider a compactification of M-theory on X. As X is elliptically fibered we can use one S^1 in the elliptic fiber to reduce to IIA, the other S^1 to T-dualize to IIB. In the limit in which the elliptic fiber collapses to zero size, the winding modes of IIA becomes light and a new direction opens up in IIB. We end up with a compactification of IIB on $\mathbb{R}^{1,5} \times B$, where B is the base of the elliptically fibered 3-fold.

A lightening review of 6D F-theory backgrounds

This is based on Vafa (96) and Morrison-Vafa (96). Let X be an elliptically fibered 3-CY. Consider a compactification of M-theory on X. As X is elliptically fibered we can use one S^1 in the elliptic fiber to reduce to IIA, the other S^1 to T-dualize to IIB. In the limit in which the elliptic fiber collapses to zero size, the winding modes of IIA becomes light and a new direction opens up in IIB. We end up with a compactification of IIB on $\mathbb{R}^{1,5} \times B$, where B is the base of the elliptically fibered 3-fold. The IIB axio-dilaton field $\tau = C_0 + ie^{-\phi}$ is traded for the complex structure modulus of the shrunk elliptic fiber.

A lightening review of 6D F-theory backgrounds

This is based on Vafa (96) and Morrison-Vafa (96). Let X be an elliptically fibered 3-CY. Consider a compactification of M-theory on X. As X is elliptically fibered we can use one S^1 in the elliptic fiber to reduce to IIA, the other S^1 to T-dualize to IIB. In the limit in which the elliptic fiber collapses to zero size, the winding modes of IIA becomes light and a new direction opens up in IIB. We end up with a compactification of IIB on $\mathbb{R}^{1,5} \times B$, where B is the base of the elliptically fibered 3-fold. The IIB axio-dilaton field $\tau = C_0 + ie^{-\phi}$ is traded for the complex structure modulus of the shrunk elliptic fiber. If the fibration is trivial $X = B \times T^2$, τ is constant, the base itself is a 2-CY, and the background preserves 16 supercharges.

A lightening review of 6D F-theory backgrounds

This is based on Vafa (96) and Morrison-Vafa (96). Let X be an elliptically fibered 3-CY. Consider a compactification of M-theory on X. As X is elliptically fibered we can use one S^1 in the elliptic fiber to reduce to IIA, the other S^1 to T-dualize to IIB. In the limit in which the elliptic fiber collapses to zero size, the winding modes of IIA becomes light and a new direction opens up in IIB. We end up with a compactification of IIB on $\mathbb{R}^{1,5} \times B$, where B is the base of the elliptically fibered 3-fold. The IIB axio-dilaton field $\tau = C_0 + ie^{-\phi}$ is traded for the complex structure modulus of the shrunk elliptic fiber. If the fibration is trivial $X = B \times T^2$, τ is constant, the base itself is a 2-CY, and the background preserves 16 supercharges. If the fibration is non trivial, τ undergoes $SL(2, \mathbb{Z})$ monodromies, the base is a Kähler surface, and the background preserves 8 supercharges.

The only possible source for the $SL(2,\mathbb{Z})$ monodromies of τ must be objects of real codimension 2.

The only possible source for the $SL(2,\mathbb{Z})$ monodromies of τ must be objects of real codimension 2. Indeed, the monodromy in question is given by a representation of

$$\pi_1\Big(\text{spacetime} \setminus \text{sources}\Big) \to SL(2,\mathbb{Z})$$

The only possible source for the $SL(2,\mathbb{Z})$ monodromies of τ must be objects of real codimension 2. Indeed, the monodromy in question is given by a representation of

$$\pi_1\Big(\text{spacetime} \setminus \text{sources}\Big) \to SL(2,\mathbb{Z})$$

Examples of such sources are IIB D7-branes, but there are more general types of sources whose (rather unsatisfactory) definition we now turn.

X, being elliptic, has a canonical presentation in Weierstrass form:

$$X\colon y^2 = z^3 + f \cdot z + g$$

where $f \in H^0(B, -4K)$ and $g \in H^0(B, -6K)$, $K = \det\ T^*B$.

X, being elliptic, has a canonical presentation in Weierstrass form:

$$X\colon y^2 = z^3 + f \cdot z + g$$

where $f \in H^0(B, -4K)$ and $g \in H^0(B, -6K)$, $K = \det\ T^*B$. The locus where the elliptic fiber degenerates is a complex curve in the base B, the discriminant of the elliptic fibration

$$\Delta \equiv 4f^3 + 27g^2 = 0$$

X, being elliptic, has a canonical presentation in Weierstrass form:

$$X\colon\, y^2 = z^3 + f \cdot z + g$$

where $f \in H^0(B, -4K)$ and $g \in H^0(B, -6K)$, $K = \det\, T^*B$. The locus where the elliptic fiber degenerates is a complex curve in the base B, the discriminant of the elliptic fibration

$$\Delta \equiv 4f^3 + 27g^2 = 0$$

By definition $\Delta \in H^0(B, -12K)$.

X, being elliptic, has a canonical presentation in Weierstrass form:

$$X\colon y^2 = z^3 + f \cdot z + g$$

where $f \in H^0(B, -4K)$ and $g \in H^0(B, -6K)$, $K = \det\ T^*B$. The locus where the elliptic fiber degenerates is a complex curve in the base B, the discriminant of the elliptic fibration

$$\Delta \equiv 4f^3 + 27g^2 = 0$$

By definition $\Delta \in H^0(B, -12K)$. The discriminant has typically several irreducible components Δ_i.

X, being elliptic, has a canonical presentation in Weierstrass form:

$$X\colon y^2 = z^3 + f \cdot z + g$$

where $f \in H^0(B, -4K)$ and $g \in H^0(B, -6K)$, $K = \det\ T^*B$. The locus where the elliptic fiber degenerates is a complex curve in the base B, the discriminant of the elliptic fibration

$$\Delta \equiv 4f^3 + 27g^2 = 0$$

By definition $\Delta \in H^0(B, -12K)$. The discriminant has typically several irreducible components Δ_i. These are the loci around which τ undergoes nontrivial monodromies, which are dictated by Kodaira classification.

X, being elliptic, has a canonical presentation in Weierstrass form:

$$X\colon y^2 = z^3 + f \cdot z + g$$

where $f \in H^0(B, -4K)$ and $g \in H^0(B, -6K)$, $K = \det\ T^*B$. The locus where the elliptic fiber degenerates is a complex curve in the base B, the discriminant of the elliptic fibration

$$\Delta \equiv 4f^3 + 27g^2 = 0$$

By definition $\Delta \in H^0(B, -12K)$. The discriminant has typically several irreducible components Δ_i. These are the loci around which τ undergoes nontrivial monodromies, which are dictated by Kodaira classification. The real codimension 2 sources of τ-monodromy are interpreted as (exotic) 7-branes wrapping Δ_i.

X, being elliptic, has a canonical presentation in Weierstrass form:

$$X\colon\, y^2 = z^3 + f \cdot z + g$$

where $f \in H^0(B, -4K)$ and $g \in H^0(B, -6K)$, $K = \det\, T^*B$. The locus where the elliptic fiber degenerates is a complex curve in the base B, the discriminant of the elliptic fibration

$$\Delta \equiv 4f^3 + 27g^2 = 0$$

By definition $\Delta \in H^0(B, -12K)$. The discriminant has typically several irreducible components Δ_i. These are the loci around which τ undergoes nontrivial monodromies, which are dictated by Kodaira classification. The real codimension 2 sources of τ-monodromy are interpreted as (exotic) 7-branes wrapping Δ_i. The structure of the Kodaira elliptic singularity over Δ_i dictates the supersymmetric gauge theory living on the worldvolume of the corresponding 7-brane, with coupling $1/(g_i)^2 \sim \mathrm{vol}(\Delta_i)$.

The relation in between the singularities of the elliptic fibration and the order of vanishing of (f, g, Δ) is summarized in the following table:

ord (f)	ord (g)	ord (Δ)	singularity	nonabelian symmetry algebra
≥ 0	≥ 0	0	none	none
0	0	$n \geq 2$	A_{n-1}	$\mathfrak{su}(n)$ or $\mathfrak{sp}(\lfloor n/2 \rfloor)$
≥ 1	1	2	none	none
1	≥ 2	3	A_1	$\mathfrak{su}(2)$
≥ 2	2	4	A_2	$\mathfrak{su}(3)$ or $\mathfrak{su}(2)$
≥ 2	≥ 3	6	D_4	$\mathfrak{so}(8)$ or $\mathfrak{so}(7)$ or $\mathfrak{g}_2$
2	3	$n \geq 7$	D_{n-2}	$\mathfrak{so}(2n-4)$ or $\mathfrak{so}(2n-5)$
≥ 3	4	8	$\mathfrak{e}_6$	$\mathfrak{e}_6$ or $\mathfrak{f}_4$
3	≥ 5	9	$\mathfrak{e}_7$	$\mathfrak{e}_7$
≥ 4	5	10	$\mathfrak{e}_8$	$\mathfrak{e}_8$

Points with order of vanishing $(4, 6, 12)$ signal the presence of tensionless strings, curves with order of vanishing $(4, 6, 12)$ spoil the CY condition and hence are forbidden.

Useful fact about intersection theory on complex surfaces

Let D be an irreducible divisor of the base B such that $D \cdot D < 0$. Consider another divisor D' of B such that $D' \cdot D < 0$. Then D is an irreducible component of D', meaning that there is another divisor X of B such that

$$D' = D + X$$

This fact becomes very powerful when combined with the adjunction formula, which states that

$$(K + D) \cdot D = 2g - 2$$

where g is the genus of D. In particular, if $D \cdot D < 0$ and $g > 0$ this entails that along D we have $\mathrm{ord}(f, g, \Delta) \geq (4, 6, 12)$.
Proof: Adjunction $\Rightarrow K \cdot D \geq -D \cdot D \Rightarrow -nK = dD + X$ for some $d > 0 \Rightarrow X \cdot D = -n\, K \cdot D - d\, D \cdot D < 0$ unless $d \geq n$. Plug in $n = (4, 6, 12)$.

This last remark entails that $g(\Delta_i) = 0$ for all i. All irreducible components of the discriminant are topologically $\mathbb{P}^1$'s. D3 branes wrapping the 1-cycles Δ_i gives rise to strings in $\mathbb{R}^{1,5}$ with tension $\sim \mathrm{vol}(\Delta_i)$ which is the only (real) scalar mode arising quantizing the $\mathbb{P}^1$. Schematically:

	0	1	2	3	4	5	6	7	8	9
$\mathbb{R}^{1,5}$	X	X	X	X	X	X				
B							X	X	X	X
Δ							X	X		
D7_e	X	X	X	X	X	X	X	X		
D3	X	X					X	X		

This last remark entails that $g(\Delta_i) = 0$ for all i. All irreducible components of the discriminant are topologically $\mathbb{P}^1$'s. D3 branes wrapping the 1-cycles Δ_i gives rise to strings in $\mathbb{R}^{1,5}$ with tension $\sim \text{vol}(\Delta_i)$ which is the only (real) scalar mode arising quantizing the $\mathbb{P}^1$. Schematically:

	0	1	2	3	4	5	6	7	8	9
$\mathbb{R}^{1,5}$	X	X	X	X	X	X				
B							X	X	X	X
Δ							X	X		
$D7_e$	X	X	X	X	X	X	X	X		
D3	X	X					X	X		

Notice that the price for τ-monodromies is a superselection rule on the Hilbert space of IIB projecting onto monodromy-invariant states.

This last remark entails that $g(\Delta_i) = 0$ for all i. All irreducible components of the discriminant are topologically $\mathbb{P}^1$'s. D3 branes wrapping the 1-cycles Δ_i gives rise to strings in $\mathbb{R}^{1,5}$ with tension $\sim \mathrm{vol}(\Delta_i)$ which is the only (real) scalar mode arising quantizing the $\mathbb{P}^1$. Schematically:

	0	1	2	3	4	5	6	7	8	9
$\mathbb{R}^{1,5}$	X	X	X	X	X	X				
B							X	X	X	X
Δ							X	X		
$D7_e$	X	X	X	X	X	X	X	X		
D3	X	X					X	X		

Notice that the price for τ-monodromies is a superselection rule on the Hilbert space of IIB projecting onto monodromy-invariant states. In particular, this has the effect of projecting out all configurations with F1s, D1s, D5s, and NS5s.

6D SCFTs in F-theory

To engineer a 6D SCFTs one consider X local which entails that gravity is decoupled.

6D SCFTs in F-theory

To engineer a 6D SCFTs one consider X local which entails that gravity is decoupled. The hallmark of 6D SCFTs are tensionless strings, therefore one requires that it is possible to shrink Δ to zero size at finite distance in moduli space.

6D SCFTs in F-theory

To engineer a 6D SCFTs one consider X local which entails that gravity is decoupled. The hallmark of 6D SCFTs are tensionless strings, therefore one requires that it is possible to shrink Δ to zero size at finite distance in moduli space. By Grauert criterion, a necessary condition is that the intersection matrix

$$\Delta_i \cdot \Delta_j$$

is negative definite.

6D SCFTs in F-theory

To engineer a 6D SCFTs one consider X local which entails that gravity is decoupled. The hallmark of 6D SCFTs are tensionless strings, therefore one requires that it is possible to shrink Δ to zero size at finite distance in moduli space. By Grauert criterion, a necessary condition is that the intersection matrix

$$\Delta_i \cdot \Delta_j$$

is negative definite. If B is 2-CY, singularities must be crepant, and hence Du Val: these are in 1-to-1 correspondence with discrete subgroups of $SU(2)$, which are ADE classified (McKay).

6D SCFTs in F-theory

To engineer a 6D SCFTs one consider X local which entails that gravity is decoupled. The hallmark of 6D SCFTs are tensionless strings, therefore one requires that it is possible to shrink Δ to zero size at finite distance in moduli space. By Grauert criterion, a necessary condition is that the intersection matrix

$$\Delta_i \cdot \Delta_j$$

is negative definite. If B is 2-CY, singularities must be crepant, and hence Du Val: these are in 1-to-1 correspondence with discrete subgroups of $SU(2)$, which are ADE classified (McKay). In this case $-\Delta_i \cdot \Delta_j = (C_G)_{ij}$, the G type Cartan matrix.

6D SCFTs in F-theory

To engineer a 6D SCFTs one consider X local which entails that gravity is decoupled. The hallmark of 6D SCFTs are tensionless strings, therefore one requires that it is possible to shrink Δ to zero size at finite distance in moduli space. By Grauert criterion, a necessary condition is that the intersection matrix

$$\Delta_i \cdot \Delta_j$$

is negative definite. If B is 2-CY, singularities must be crepant, and hence Du Val: these are in 1-to-1 correspondence with discrete subgroups of $SU(2)$, which are ADE classified (McKay). In this case $-\Delta_i \cdot \Delta_j = (C_G)_{ij}$, the G type Cartan matrix. These are the (2,0) SCFTs of Shimizu's talk of this morning (Witten 95).

6D SCFTs in F-theory

To engineer a 6D SCFTs one consider X local which entails that gravity is decoupled. The hallmark of 6D SCFTs are tensionless strings, therefore one requires that it is possible to shrink Δ to zero size at finite distance in moduli space. By Grauert criterion, a necessary condition is that the intersection matrix

$$\Delta_i \cdot \Delta_j$$

is negative definite. If B is 2-CY, singularities must be crepant, and hence Du Val: these are in 1-to-1 correspondence with discrete subgroups of $SU(2)$, which are ADE classified (McKay). In this case $-\Delta_i \cdot \Delta_j = (C_G)_{ij}$, the G type Cartan matrix. These are the (2,0) SCFTs of Shimizu's talk of this morning (Witten 95). If B is Kähler, one obtains singularities which are in correspondence with discrete subgroups $\Gamma \subset U(2)$ giving rise to (1,0) SCFTs.

6D SCFTs in F-theory

To engineer a 6D SCFTs one consider X local which entails that gravity is decoupled. The hallmark of 6D SCFTs are tensionless strings, therefore one requires that it is possible to shrink Δ to zero size at finite distance in moduli space. By Grauert criterion, a necessary condition is that the intersection matrix

$$\Delta_i \cdot \Delta_j$$

is negative definite. If B is 2-CY, singularities must be crepant, and hence Du Val: these are in 1-to-1 correspondence with discrete subgroups of $SU(2)$, which are ADE classified (McKay). In this case $-\Delta_i \cdot \Delta_j = (C_G)_{ij}$, the G type Cartan matrix. These are the (2,0) SCFTs of Shimizu's talk of this morning (Witten 95). If B is Kähler, one obtains singularities which are in correspondence with discrete subgroups $\Gamma \subset U(2)$ giving rise to (1,0) SCFTs. In this case, however, several Γ's are such that the singularity $\mathbb{C}^2/\Gamma$ is not at finite distance in CY moduli space,

6D SCFTs in F-theory

To engineer a 6D SCFTs one consider X local which entails that gravity is decoupled. The hallmark of 6D SCFTs are tensionless strings, therefore one requires that it is possible to shrink Δ to zero size at finite distance in moduli space. By Grauert criterion, a necessary condition is that the intersection matrix

$$\Delta_i \cdot \Delta_j$$

is negative definite. If B is 2-CY, singularities must be crepant, and hence Du Val: these are in 1-to-1 correspondence with discrete subgroups of $SU(2)$, which are ADE classified (McKay). In this case $-\Delta_i \cdot \Delta_j = (C_G)_{ij}$, the G type Cartan matrix. These are the (2,0) SCFTs of Shimizu's talk of this morning (Witten 95). If B is Kähler, one obtains singularities which are in correspondence with discrete subgroups $\Gamma \subset U(2)$ giving rise to (1,0) SCFTs. In this case, however, several Γ's are such that the singularity $\mathbb{C}^2/\Gamma$ is not at finite distance in CY moduli space, e.g. resolving it one obtains curves with $\mathrm{ord}(f, g, \Delta) \geq (4, 6, 12)$.

The list of allowed $\Gamma \subset U(2)$ has been worked out by Heckman, Morrison, and Vafa (13) building on Morrison and Taylor (12).

The list of allowed $\Gamma \subset U(2)$ has been worked out by Heckman, Morrison, and Vafa (13) building on Morrison and Taylor (12). The key ingredient in this story are non-Higgasable clusters, to which we now turn.

The list of allowed $\Gamma \subset U(2)$ has been worked out by Heckman, Morrison, and Vafa (13) building on Morrison and Taylor (12). The key ingredient in this story are non-Higgasable clusters, to which we now turn. A crucial difference in between 2-Kähler singularities and 2-CY ones is that, while in the latter case the self-intersection of the blow-up exceptional divisors can have only one value, $\Delta_i^2 = -2$, in the resolution of the former Δ_i^2 can have several values.

The list of allowed $\Gamma \subset U(2)$ has been worked out by Heckman, Morrison, and Vafa (13) building on Morrison and Taylor (12). The key ingredient in this story are non-Higgasable clusters, to which we now turn. A crucial difference in between 2-Kähler singularities and 2-CY ones is that, while in the latter case the self-intersection of the blow-up exceptional divisors can have only one value, $\Delta_i^2 = -2$, in the resolution of the former Δ_i^2 can have several values. This fact has a clear physical interpretation.

The list of allowed $\Gamma \subset U(2)$ has been worked out by Heckman, Morrison, and Vafa (13) building on Morrison and Taylor (12). The key ingredient in this story are non-Higgasable clusters, to which we now turn. A crucial difference in between 2-Kähler singularities and 2-CY ones is that, while in the latter case the self-intersection of the blow-up exceptional divisors can have only one value, $\Delta_i^2 = -2$, in the resolution of the former Δ_i^2 can have several values. This fact has a clear physical interpretation. In 6D the Dirac pairing among non-critical strings is symmetric.

The list of allowed $\Gamma \subset U(2)$ has been worked out by Heckman, Morrison, and Vafa (13) building on Morrison and Taylor (12). The key ingredient in this story are non-Higgasable clusters, to which we now turn. A crucial difference in between 2-Kähler singularities and 2-CY ones is that, while in the latter case the self-intersection of the blow-up exceptional divisors can have only one value, $\Delta_i^2 = -2$, in the resolution of the former Δ_i^2 can have several values. This fact has a clear physical interpretation. In 6D the Dirac pairing among non-critical strings is symmetric. In F-theory engineering, the intersection matrix equals minus the Dirac pairing in between the elementary non-critical strings of a given system.

The list of allowed $\Gamma \subset U(2)$ has been worked out by Heckman, Morrison, and Vafa (13) building on Morrison and Taylor (12). The key ingredient in this story are non-Higgasable clusters, to which we now turn. A crucial difference in between 2-Kähler singularities and 2-CY ones is that, while in the latter case the self-intersection of the blow-up exceptional divisors can have only one value, $\Delta_i^2 = -2$, in the resolution of the former Δ_i^2 can have several values. This fact has a clear physical interpretation. In 6D the Dirac pairing among non-critical strings is symmetric. In F-theory engineering, the intersection matrix equals minus the Dirac pairing in between the elementary non-critical strings of a given system. For (2,0) SCFTs there is only one type of non-critical string with self-Dirac pairing 2.

The list of allowed $\Gamma \subset U(2)$ has been worked out by Heckman, Morrison, and Vafa (13) building on Morrison and Taylor (12). The key ingredient in this story are non-Higgasable clusters, to which we now turn. A crucial difference in between 2-Kähler singularities and 2-CY ones is that, while in the latter case the self-intersection of the blow-up exceptional divisors can have only one value, $\Delta_i^2 = -2$, in the resolution of the former Δ_i^2 can have several values. This fact has a clear physical interpretation. In 6D the Dirac pairing among non-critical strings is symmetric. In F-theory engineering, the intersection matrix equals minus the Dirac pairing in between the elementary non-critical strings of a given system. For (2,0) SCFTs there is only one type of non-critical string with self-Dirac pairing 2. For (1,0) theories there are several different types of non-critical strings, each distinct from another by its self-Dirac pairing.

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model.

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model. Blowing-up the singularity we flow along the tensor branch of the 6D theory.

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model. Blowing-up the singularity we flow along the tensor branch of the 6D theory. While for (2,0) theories, the tensor branch is an abelian theory of (2,0) tensor multiplets, for (1,0) theories there are non abelian gauge fields as well.

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model. Blowing-up the singularity we flow along the tensor branch of the 6D theory. While for (2,0) theories, the tensor branch is an abelian theory of (2,0) tensor multiplets, for (1,0) theories there are non abelian gauge fields as well. Geometry teaches us that for $\Delta_i^2 < -2$, the tensor branches must be characterized by tensor-vector systems.

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model. Blowing-up the singularity we flow along the tensor branch of the 6D theory. While for (2,0) theories, the tensor branch is an abelian theory of (2,0) tensor multiplets, for (1,0) theories there are non abelian gauge fields as well. Geometry teaches us that for $\Delta_i^2 < -2$, the tensor branches must be characterized by tensor-vector systems. Let me discuss an example.

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model. Blowing-up the singularity we flow along the tensor branch of the 6D theory. While for (2,0) theories, the tensor branch is an abelian theory of (2,0) tensor multiplets, for (1,0) theories there are non abelian gauge fields as well. Geometry teaches us that for $\Delta_i^2 < -2$, the tensor branches must be characterized by tensor-vector systems. Let me discuss an example. Consider a component of Δ with $\Delta_i^2 = -3$. Recall that $g = 0$, therefore the adjunction formula gives $K \cdot \Delta_i = -\Delta_i^2 - 2 = 1$

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model. Blowing-up the singularity we flow along the tensor branch of the 6D theory. While for (2,0) theories, the tensor branch is an abelian theory of (2,0) tensor multiplets, for (1,0) theories there are non abelian gauge fields as well. Geometry teaches us that for $\Delta_i^2 < -2$, the tensor branches must be characterized by tensor-vector systems. Let me discuss an example. Consider a component of Δ with $\Delta_i^2 = -3$. Recall that $g = 0$, therefore the adjunction formula gives $K \cdot \Delta_i = -\Delta_i^2 - 2 = 1 \Rightarrow -nK \cdot \Delta_i < 0$

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model. Blowing-up the singularity we flow along the tensor branch of the 6D theory. While for (2,0) theories, the tensor branch is an abelian theory of (2,0) tensor multiplets, for (1,0) theories there are non abelian gauge fields as well. Geometry teaches us that for $\Delta_i^2 < -2$, the tensor branches must be characterized by tensor-vector systems. Let me discuss an example. Consider a component of Δ with $\Delta_i^2 = -3$. Recall that $g = 0$, therefore the adjunction formula gives $K \cdot \Delta_i = -\Delta_i^2 - 2 = 1 \Rightarrow -nK \cdot \Delta_i < 0$ $\Rightarrow -nK = d\Delta_i + X$

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model. Blowing-up the singularity we flow along the tensor branch of the 6D theory. While for (2,0) theories, the tensor branch is an abelian theory of (2,0) tensor multiplets, for (1,0) theories there are non abelian gauge fields as well. Geometry teaches us that for $\Delta_i^2 < -2$, the tensor branches must be characterized by tensor-vector systems. Let me discuss an example. Consider a component of Δ with $\Delta_i^2 = -3$. Recall that $g = 0$, therefore the adjunction formula gives $K \cdot \Delta_i = -\Delta_i^2 - 2 = 1 \Rightarrow -nK \cdot \Delta_i < 0$ $\Rightarrow -nK = d\Delta_i + X \Rightarrow X \cdot \Delta_i = -nK \cdot \Delta_i - d\Delta_i^2 = 3d - n \geq 0$

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model. Blowing-up the singularity we flow along the tensor branch of the 6D theory. While for (2,0) theories, the tensor branch is an abelian theory of (2,0) tensor multiplets, for (1,0) theories there are non abelian gauge fields as well. Geometry teaches us that for $\Delta_i^2 < -2$, the tensor branches must be characterized by tensor-vector systems. Let me discuss an example. Consider a component of Δ with $\Delta_i^2 = -3$. Recall that $g = 0$, therefore the adjunction formula gives $K \cdot \Delta_i = -\Delta_i^2 - 2 = 1 \Rightarrow -nK \cdot \Delta_i < 0$ $\Rightarrow -nK = d\Delta_i + X \Rightarrow X \cdot \Delta_i = -nK \cdot \Delta_i - d\Delta_i^2 = 3d - n \geq 0$ $\Leftrightarrow d \geq n/3$.

The sizes of the blow-up exceptional divisors are naturally interpreted as vevs of the scalars in the tensor multiplets of the model. Blowing-up the singularity we flow along the tensor branch of the 6D theory. While for (2,0) theories, the tensor branch is an abelian theory of (2,0) tensor multiplets, for (1,0) theories there are non abelian gauge fields as well. Geometry teaches us that for $\Delta_i^2 < -2$, the tensor branches must be characterized by tensor-vector systems. Let me discuss an example. Consider a component of Δ with $\Delta_i^2 = -3$. Recall that $g = 0$, therefore the adjunction formula gives $K \cdot \Delta_i = -\Delta_i^2 - 2 = 1 \Rightarrow -nK \cdot \Delta_i < 0$ $\Rightarrow -nK = d\Delta_i + X \Rightarrow X \cdot \Delta_i = -nK \cdot \Delta_i - d\Delta_i^2 = 3d - n \geq 0$ $\Leftrightarrow d \geq n/3$. Now let's plug in $n = (4, 6, 12)$ and we obtain that

$$d \geq (2, 2, 4)$$

.

Lookin at the table we find that $d \geq (2, 2, 4)$ are precisely the order of vanishing corresponding to the 5th line below

ord (f)	ord (g)	ord (Δ)	singularity	nonabelian symmetry algebra
≥ 0	≥ 0	0	none	none
0	0	$n \geq 2$	A_{n-1}	$\mathfrak{su}(n)$ or $\mathfrak{sp}(\lfloor n/2 \rfloor)$
≥ 1	1	2	none	none
1	≥ 2	3	A_1	$\mathfrak{su}(2)$
≥ 2	2	4	A_2	$\mathfrak{su}(3)$ or $\mathfrak{su}(2)$
≥ 2	≥ 3	6	D_4	$\mathfrak{so}(8)$ or $\mathfrak{so}(7)$ or $\mathfrak{g}_2$
2	3	$n \geq 7$	D_{n-2}	$\mathfrak{so}(2n-4)$ or $\mathfrak{so}(2n-5)$
≥ 3	4	8	$\mathfrak{e}_6$	$\mathfrak{e}_6$ or $\mathfrak{f}_4$
3	≥ 5	9	$\mathfrak{e}_7$	$\mathfrak{e}_7$
≥ 4	5	10	$\mathfrak{e}_8$	$\mathfrak{e}_8$

Hence there is a nonabelian gauge symmetry which is forced on us.

Lookin at the table we find that $d \geq (2, 2, 4)$ are precisely the order of vanishing corresponding to the 5th line below

ord (f)	ord (g)	ord (Δ)	singularity	nonabelian symmetry algebra
≥ 0	≥ 0	0	none	none
0	0	$n \geq 2$	A_{n-1}	$\mathfrak{su}(n)$ or $\mathfrak{sp}(\lfloor n/2 \rfloor)$
≥ 1	1	2	none	none
1	≥ 2	3	A_1	$\mathfrak{su}(2)$
≥ 2	2	4	A_2	$\mathfrak{su}(3)$ or $\mathfrak{su}(2)$
≥ 2	≥ 3	6	D_4	$\mathfrak{so}(8)$ or $\mathfrak{so}(7)$ or $\mathfrak{g}_2$
2	3	$n \geq 7$	D_{n-2}	$\mathfrak{so}(2n-4)$ or $\mathfrak{so}(2n-5)$
≥ 3	4	8	$\mathfrak{e}_6$	$\mathfrak{e}_6$ or $\mathfrak{f}_4$
3	≥ 5	9	$\mathfrak{e}_7$	$\mathfrak{e}_7$
≥ 4	5	10	$\mathfrak{e}_8$	$\mathfrak{e}_8$

Hence there is a nonabelian gauge symmetry which is forced on us. This is an example of non-Higgsable cluster: for generic values of the complex structure, the gauge group is $SU(3)$.

The same reasoning for $\Delta_i^2 = -k$ gives $K \cdot \Delta_i = k - 2$ hence we obtain $d \geq n(k-2)/k$.

The same reasoning for $\Delta_i^2 = -k$ gives $K \cdot \Delta_i = k - 2$ hence we obtain $d \geq n(k-2)/k$. It is easy to see that $k > 12$ leads to $d \geq (4, 6, 12)$, and hence it is forbidden.

The same reasoning for $\Delta_i^2 = -k$ gives $K \cdot \Delta_i = k - 2$ hence we obtain $d \geq n(k-2)/k$. It is easy to see that $k > 12$ leads to $d \geq (4, 6, 12)$, and hence it is forbidden. Analogous techniques allows one to classify all possible such configurations, The same reasoning for $\Delta_i^2 = -k$ gives $K \cdot \Delta_i = k - 2$ hence we obtain $d \geq n(k-2)/k$. It is easy to see that $k > 12$ leads to $d \geq (4, 6, 12)$, and hence it is forbidden. Analogous techniques allows one to classify all possible such configurations, the result is:

-3	$\mathfrak{su}(3)$	0
-4	$\mathfrak{so}(8)$	0
-5	$\mathfrak{f}_4$	0
-6	$\mathfrak{e}_6$	0
-7	$\mathfrak{e}_7$	$\frac{1}{2}\mathbf{56}$
-8	$\mathfrak{e}_7$	0
-12	$\mathfrak{e}_8$	0
$-3, -2$	$\mathfrak{g}_2 \oplus \mathfrak{su}(2)$	$(\mathbf{7}+\mathbf{1}, \frac{1}{2}\mathbf{2})$
$-3, -2, -2$	$\mathfrak{g}_2 \oplus \mathfrak{su}(2)$	$(\mathbf{7}+\mathbf{1}, \frac{1}{2}\mathbf{2})$
$-2, -3, -2$	$\mathfrak{su}(2) \oplus \mathfrak{so}(7) \oplus \mathfrak{su}(2)$	$(\mathbf{1}, \mathbf{8}, \frac{1}{2}\mathbf{2})$ $+(\frac{1}{2}\mathbf{2}, \mathbf{8}, \mathbf{1})$

More general non-Higgsable basis are obtained from the non-Higgsable clusters using the following glueing rule:

$$\ldots, \overset{\mathfrak{g}_1}{n_1}, 1, \overset{\mathfrak{g}_2}{n_2}, \ldots$$

$$\Longleftrightarrow$$

$$\mathfrak{g}_1 \oplus \mathfrak{g}_2 \subset \mathfrak{e}_8 \qquad \text{maximal subalgebra}$$

Blowing down such configurations one obtains $\Gamma \subset U(2)$ singularities which are at finite distance.

More general non-Higgsable basis are obtained from the non-Higgsable clusters using the following glueing rule:

$$\ldots, \overset{\mathfrak{g}_1}{n_1}, 1, \overset{\mathfrak{g}_2}{n_2}, \ldots$$

$$\Longleftrightarrow$$

$$\mathfrak{g}_1 \oplus \mathfrak{g}_2 \subset \mathfrak{e}_8 \qquad \text{maximal subalgebra}$$

Blowing down such configurations one obtains $\Gamma \subset U(2)$ singularities which are at finite distance. Let me discuss one example of blow down.

12, 1, 2, 2, 3, 1, 5

$$12, 1, 2, 2, 3, 1, 5$$

$$11, 1, 2, 3, 1, 5$$

12, 1, 2, 2, 3, 1, 5

11, 1, 2, 3, 1, 5

10, 1, 3, 1, 5

$$12, 1, 2, 2, 3, 1, 5$$

$$11, 1, 2, 3, 1, 5$$

$$10, 1, 3, 1, 5$$

$$9, 2, 1, 5$$

$$12, 1, 2, 2, 3, 1, 5$$

$$11, 1, 2, 3, 1, 5$$

$$10, 1, 3, 1, 5$$

$$9, 2, 1, 5$$

$$9, 1, 4$$

$$12, 1, 2, 2, 3, 1, 5$$

$$11, 1, 2, 3, 1, 5$$

$$10, 1, 3, 1, 5$$

$$9, 2, 1, 5$$

$$9, 1, 4$$

$$8, 3$$

$$12, 1, 2, 2, 3, 1, 5$$

$$11, 1, 2, 3, 1, 5$$

$$10, 1, 3, 1, 5$$

$$9, 2, 1, 5$$

$$9, 1, 4$$

$$8, 3$$

This is an example of Hirzebruch-Jung singularity of type $A_{p,q}$ where $p/q = 8 - 1/3 = 23/3$ meaning that it corresponds to an orbifold action

$$(z_1, z_2) \to (\omega z_1, \omega^q z_2) \qquad \omega \in \mathbb{Z}_p$$

Many infinite families of singularities of type $A_{p,q}$ are realized in this way, as well as singularities of type $D_{p,q}$ and several exceptional configurations.

Many infinite families of singularities of type $A_{p,q}$ are realized in this way, as well as singularities of type $D_{p,q}$ and several exceptional configurations. For the complete list I refer you to the paper by Heckman, Morrison and Vafa (13).

Many infinite families of singularities of type $A_{p,q}$ are realized in this way, as well as singularities of type $D_{p,q}$ and several exceptional configurations. For the complete list I refer you to the paper by Heckman, Morrison and Vafa (13). It is easy to see that the glueing rule is very constraining.

Many infinite families of singularities of type $A_{p,q}$ are realized in this way, as well as singularities of type $D_{p,q}$ and several exceptional configurations. For the complete list I refer you to the paper by Heckman, Morrison and Vafa (13). It is easy to see that the glueing rule is very constraining. For example there are essentially 5 sequences of generalized A–type, namely

$$SU : \cdots, 2, 2, 2, 2, 2, \cdots$$

$$SO : \cdots, 4, 1, 4, 1, 4, 1, \cdots$$

$$E_6 : \cdots, 6, 1, 3, 1, 6, 1, 3, 1, 6, 1, 3, 1, \cdots$$

$$E_7 : \cdots, 8, 1, 2, 3, 2, 1, 8, 1, 2, 3, 2, 1, \cdots$$

$$E_8 : \cdots, 12, 1, 2, 2, 3, 1, 5, 1, 3, 2, 2, 1, 12, 1, 2, 2, 3, 1, 5, 1, 3, 2, 2, 1, \cdots$$

which can be truncated on the left and on the right at arbitrary postions.

It is remarkable that the same patterns arise when colliding non-compact singular fibers — see Bershadsky and Johansen (96) and Aspinwall and Morrison (97), for example:

$$SO(8) \times SO(8) \to [SO(8)], 1, [SO(8)]$$

$$E_6 \times E_6 \to [E_6], 1, 3, 1, [E_6]$$

$$E_7 \times E_7 \to [E_7], 1, 2, 3, 2, 1, [E_7]$$

$$E_8 \times E_8 \to [E_8], 1, 2, 2, 3, 1, 5, 1, 3, 2, 2, 1, [E_8]$$

The interpretation of this fact was found in a joint collaboration with Heckman, Tomasiello and Vafa (14).

The interpretation of this fact was found in a joint collaboration with Heckman, Tomasiello and Vafa (14). Since $1/(g_i)^2 \sim \mathrm{vol}(\Delta_i)$ we have that

$$g_i \to 0 \qquad \Longleftrightarrow \qquad \mathrm{vol}(\Delta_i) \to \infty$$

The interpretation of this fact was found in a joint collaboration with Heckman, Tomasiello and Vafa (14). Since $1/(g_i)^2 \sim \mathrm{vol}(\Delta_i)$ we have that

$$g_i \to 0 \qquad \Longleftrightarrow \qquad \mathrm{vol}(\Delta_i) \to \infty$$

In other words, opening up a cycle gives rise to a flavor symmetry:

The interpretation of this fact was found in a joint collaboration with Heckman, Tomasiello and Vafa (14). Since $1/(g_i)^2 \sim \text{vol}(\Delta_i)$ we have that

$$g_i \to 0 \qquad \Longleftrightarrow \qquad \text{vol}(\Delta_i) \to \infty$$

In other words, opening up a cycle gives rise to a flavor symmetry: non-compact components of Δ can be interpreted as flavor divisors.

The interpretation of this fact was found in a joint collaboration with Heckman, Tomasiello and Vafa (14). Since $1/(g_i)^2 \sim \mathrm{vol}(\Delta_i)$ we have that

$$g_i \to 0 \qquad \Longleftrightarrow \qquad \mathrm{vol}(\Delta_i) \to \infty$$

In other words, opening up a cycle gives rise to a flavor symmetry: non-compact components of Δ can be interpreted as flavor divisors. In the limit in which all compact irreducible components shrunk to zero size, flavor divisors remains non-compact and do not introduce additional scales in the F-theory geometry.

The interpretation of this fact was found in a joint collaboration with Heckman, Tomasiello and Vafa (14). Since $1/(g_i)^2 \sim \mathrm{vol}(\Delta_i)$ we have that

$$g_i \to 0 \qquad \Longleftrightarrow \qquad \mathrm{vol}(\Delta_i) \to \infty$$

In other words, opening up a cycle gives rise to a flavor symmetry: non-compact components of Δ can be interpreted as flavor divisors. In the limit in which all compact irreducible components shrunk to zero size, flavor divisors remains non-compact and do not introduce additional scales in the F-theory geometry. The particular 6D SCFTs which arise at a collision of two flavor divisors are the so called 'superconformal matter'.

The interpretation of this fact was found in a joint collaboration with Heckman, Tomasiello and Vafa (14). Since $1/(g_i)^2 \sim \text{vol}(\Delta_i)$ we have that

$$g_i \to 0 \qquad \Longleftrightarrow \qquad \text{vol}(\Delta_i) \to \infty$$

In other words, opening up a cycle gives rise to a flavor symmetry: non-compact components of Δ can be interpreted as flavor divisors. In the limit in which all compact irreducible components shrunk to zero size, flavor divisors remains non-compact and do not introduce additional scales in the F-theory geometry. The particular 6D SCFTs which arise at a collision of two flavor divisors are the so called 'superconformal matter'. These are labeled by the flavor symmetries on the superconformal divisors and by the order of the singularity where they meet.

The interpretation of this fact was found in a joint collaboration with Heckman, Tomasiello and Vafa (14). Since $1/(g_i)^2 \sim \mathrm{vol}(\Delta_i)$ we have that

$$g_i \to 0 \qquad \Longleftrightarrow \qquad \mathrm{vol}(\Delta_i) \to \infty$$

In other words, opening up a cycle gives rise to a flavor symmetry: non-compact components of Δ can be interpreted as flavor divisors. In the limit in which all compact irreducible components shrunk to zero size, flavor divisors remains non-compact and do not introduce additional scales in the F-theory geometry. The particular 6D SCFTs which arise at a collision of two flavor divisors are the so called 'superconformal matter'. These are labeled by the flavor symmetries on the superconformal divisors and by the order of the singularity where they meet. Together with Heckman, Morrison and Park we have recently examined the problem of re-coupling such systems to gravity — it is a very interesting story: I refer you to the paper for details.

To obtain more general gauge groups, one has to consider non generic complex structures that allow for fiber singularities of higher rank, which are still highly constrained by the self-intersection numbers of the corresponding divisors.

To obtain more general gauge groups, one has to consider non generic complex structures that allow for fiber singularities of higher rank, which are still highly constrained by the self-intersection numbers of the corresponding divisors. The glueing rule for such enhanced fibers is obtained by requiring 6D gauge anomaly cancellation.

To obtain more general gauge groups, one has to consider non generic complex structures that allow for fiber singularities of higher rank, which are still highly constrained by the self-intersection numbers of the corresponding divisors. The glueing rule for such enhanced fibers is obtained by requiring 6D gauge anomaly cancellation. Again superconformal matter arises at the collision of non-compact flavor divisors with enhanced gauge symmetries.

To obtain more general gauge groups, one has to consider non generic complex structures that allow for fiber singularities of higher rank, which are still highly constrained by the self-intersection numbers of the corresponding divisors. The glueing rule for such enhanced fibers is obtained by requiring 6D gauge anomaly cancellation. Again superconformal matter arises at the collision of non-compact flavor divisors with enhanced gauge symmetries. It is also interesting to consider T-brane flavor divisors.

To obtain more general gauge groups, one has to consider non generic complex structures that allow for fiber singularities of higher rank, which are still highly constrained by the self-intersection numbers of the corresponding divisors. The glueing rule for such enhanced fibers is obtained by requiring 6D gauge anomaly cancellation. Again superconformal matter arises at the collision of non-compact flavor divisors with enhanced gauge symmetries. It is also interesting to consider T-brane flavor divisors. The rules for glueing together superconformal matter building blocks are again determined using 6D gauge anomaly cancellation.

To obtain more general gauge groups, one has to consider non generic complex structures that allow for fiber singularities of higher rank, which are still highly constrained by the self-intersection numbers of the corresponding divisors. The glueing rule for such enhanced fibers is obtained by requiring 6D gauge anomaly cancellation. Again superconformal matter arises at the collision of non-compact flavor divisors with enhanced gauge symmetries. It is also interesting to consider T-brane flavor divisors. The rules for glueing together superconformal matter building blocks are again determined using 6D gauge anomaly cancellation. This analysis has been carried over systematically by Heckman, Morrison, Rudelius and Vafa (15), from which a classification of all allowed possibilities follows.

Let me discuss the example of (E_6, E_6) conformal matter meeting at a $\mathbb{C}^2/\mathbb{Z}_2$ singularity.

Let me discuss the example of (E_6, E_6) conformal matter meeting at a $\mathbb{C}^2/\mathbb{Z}_2$ singularity. Schematically the collision of the two noncompact divisors look like:

$$[E_6]\, A_1\, [E_6]$$

Let me discuss the example of (E_6, E_6) conformal matter meeting at a $\mathbb{C}^2/\mathbb{Z}_2$ singularity. Schematically the collision of the two noncompact divisors look like:

$$[E_6]\, A_1\, [E_6]$$

$$[E_6]\, \overset{\mathfrak{e}_6}{2}\, [E_6]$$

Let me discuss the example of (E_6, E_6) conformal matter meeting at a $\mathbb{C}^2/\mathbb{Z}_2$ singularity. Schematically the collision of the two noncompact divisors look like:

$$[E_6]\, A_1\, [E_6]$$

$$[E_6]\, \overset{\mathfrak{e}_6}{2}\, [E_6]$$

$$[E_6]\, 1, \overset{\mathfrak{e}_6}{4}, 1\, [E_6]$$

Let me discuss the example of (E_6, E_6) conformal matter meeting at a $\mathbb{C}^2/\mathbb{Z}_2$ singularity. Schematically the collision of the two noncompact divisors look like:

$$[E_6]\, A_1\, [E_6]$$

$$[E_6]\, \overset{\mathfrak{e}_6}{2}\, [E_6]$$

$$[E_6]\, 1, \overset{\mathfrak{e}_6}{4}, 1\, [E_6]$$

$$[E_6]\, 1, 2, \overset{\mathfrak{e}_6}{4}, 2, 1\, [E_6]$$

Let me discuss the example of (E_6, E_6) conformal matter meeting at a $\mathbb{C}^2/\mathbb{Z}_2$ singularity. Schematically the collision of the two noncompact divisors look like:

$$[E_6]\, A_1\, [E_6]$$

$$[E_6]\, \overset{\mathfrak{e}_6}{2}\, [E_6]$$

$$[E_6]\, 1, \overset{\mathfrak{e}_6}{4}, 1\, [E_6]$$

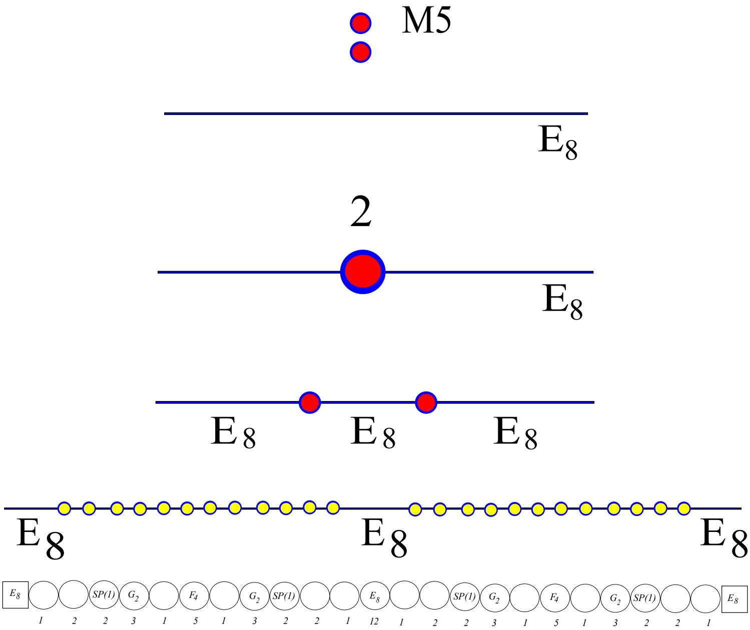
$$[E_6]\, 1, 2, \overset{\mathfrak{e}_6}{4}, 2, 1\, [E_6]$$

$$[E_6]\, 1, \overset{\mathfrak{su}_3}{3}, 1, \overset{\mathfrak{e}_6}{6}, 1, \overset{\mathfrak{su}_3}{3}, 1 [E_6]$$

Several examples of conformal matter are realized explicitly in M-theory.

Several examples of conformal matter are realized explicitly in M-theory. Either as M5s or as E_8 heterotic instantons probing $\mathbb{C}^2/\Gamma_{ADE}$ singularities.

Several examples of conformal matter are realized explicitly in M-theory. Either as M5s or as E_8 heterotic instantons probing $\mathbb{C}^2/\Gamma_{ADE}$ singularities. In both cases we predict, using F-theory, the structure of fractional M5 and M9 branes.



M5
E8
2
E8
E8 E8 E8
E8 E8 E8
E8 SP(1) G2 F4 G2 SP(1) E8 SP(1) G2 F4 G2 SP(1) E8
1 2 2 3 1 5 1 3 2 2 1 12 1 2 2 3 1 5 1 3 2 2 1

The defect group

From the F-theory engineering of these systems it follows that the lattice of BPS string charges is identified with the mid-dimensional homology group of the base B

$$\Lambda \equiv H_2(B, \mathbb{Z})$$

The defect group

From the F-theory engineering of these systems it follows that the lattice of BPS string charges is identified with the mid-dimensional homology group of the base B

$$\Lambda \equiv H_2(B, \mathbb{Z})$$

The 6D symmetric Dirac pairing in between charged BPS strings is captured by the intersection pairing on B.

The defect group

From the F-theory engineering of these systems it follows that the lattice of BPS string charges is identified with the mid-dimensional homology group of the base B

$$\Lambda \equiv H_2(B, \mathbb{Z})$$

The 6D symmetric Dirac pairing in between charged BPS strings is captured by the intersection pairing on B. Such pairing being non-degenerate by construction, its dual lattice is uniquely defined as the set

$$\Lambda^* \equiv \{\ell \in \mathbb{Q}^{n_T} : \ell \cdot \lambda \in \mathbb{Z} \; \forall \, \lambda \in \Lambda\}$$

The defect group

From the F-theory engineering of these systems it follows that the lattice of BPS string charges is identified with the mid-dimensional homology group of the base B

$$\Lambda \equiv H_2(B, \mathbb{Z})$$

The 6D symmetric Dirac pairing in between charged BPS strings is captured by the intersection pairing on B. Such pairing being non-degenerate by construction, its dual lattice is uniquely defined as the set

$$\Lambda^* \equiv \{\ell \in \mathbb{Q}^{n_T} : \ell \cdot \lambda \in \mathbb{Z} \; \forall \, \lambda \in \Lambda\}$$

The dual lattice is naturally associated with the lattice of allowed charges for BPS defects.

The defect group

From the F-theory engineering of these systems it follows that the lattice of BPS string charges is identified with the mid-dimensional homology group of the base B

$$\Lambda \equiv H_2(B, \mathbb{Z})$$

The 6D symmetric Dirac pairing in between charged BPS strings is captured by the intersection pairing on B. Such pairing being non-degenerate by construction, its dual lattice is uniquely defined as the set

$$\Lambda^* \equiv \{\ell \in \mathbb{Q}^{n_T} : \ell \cdot \lambda \in \mathbb{Z} \; \forall \, \lambda \in \Lambda\}$$

The dual lattice is naturally associated with the lattice of allowed charges for BPS defects. A necessary condition for a given 6D SCFT to have a well defined partition function on curved spaces is that its lattice of string charges is self–dual or unimodular, $\Lambda^* = \Lambda$, which was argued by Seiberg and Taylor (11).

The defect group

From the F-theory engineering of these systems it follows that the lattice of BPS string charges is identified with the mid-dimensional homology group of the base B

$$\Lambda \equiv H_2(B, \mathbb{Z})$$

The 6D symmetric Dirac pairing in between charged BPS strings is captured by the intersection pairing on B. Such pairing being non-degenerate by construction, its dual lattice is uniquely defined as the set

$$\Lambda^* \equiv \{\ell \in \mathbb{Q}^{n_T} : \ell \cdot \lambda \in \mathbb{Z} \; \forall \, \lambda \in \Lambda\}$$

The dual lattice is naturally associated with the lattice of allowed charges for BPS defects. A necessary condition for a given 6D SCFT to have a well defined partition function on curved spaces is that its lattice of string charges is self–dual or unimodular, $\Lambda^* = \Lambda$, which was argued by Seiberg and Taylor (11). A measure of the discrepancy from modularity is given by the defect group Λ^*/Λ.

It is interesting to observe that

$$\Lambda^*/\Lambda \equiv \mathrm{Ab}\ \Gamma$$

where Γ is the discrete subgroup of $U(2)$ which characterizes the singular point,

It is interesting to observe that

$$\Lambda^*/\Lambda \equiv \mathsf{Ab}\ \Gamma$$

where Γ is the discrete subgroup of $U(2)$ which characterizes the singular point, from this fact it follows that the defect group is a property which, though computed using tensor branch data, is intrinsic of the SCFT.

It is interesting to observe that

$$\Lambda^*/\Lambda \equiv \text{Ab } \Gamma$$

where Γ is the discrete subgroup of $U(2)$ which characterizes the singular point, from this fact it follows that the defect group is a property which, though computed using tensor branch data, is intrinsic of the SCFT. Moreover, the class of models which have trivial defect group coincides by construction with the class of very-Higgsable 6D models introduced recently by Ohomori, Shimizu, Tachikawa and Yonekura (15).

It is interesting to observe that

$$\Lambda^*/\Lambda \equiv \text{Ab}\ \Gamma$$

where Γ is the discrete subgroup of $U(2)$ which characterizes the singular point, from this fact it follows that the defect group is a property which, though computed using tensor branch data, is intrinsic of the SCFT. Moreover, the class of models which have trivial defect group coincides by construction with the class of very-Higgsable 6D models introduced recently by Ohomori, Shimizu, Tachikawa and Yonekura (15). Indeed, it is easy to show that the defect group is conserved along blow ups and downs of the exceptional divisors in the F-theory base,

It is interesting to observe that

$$\Lambda^*/\Lambda \equiv \text{Ab } \Gamma$$

where Γ is the discrete subgroup of $U(2)$ which characterizes the singular point, from this fact it follows that the defect group is a property which, though computed using tensor branch data, is intrinsic of the SCFT. Moreover, the class of models which have trivial defect group coincides by construction with the class of very-Higgsable 6D models introduced recently by Ohomori, Shimizu, Tachikawa and Yonekura (15). Indeed, it is easy to show that the defect group is conserved along blow ups and downs of the exceptional divisors in the F-theory base, hence blowing down to nothing (which is very Higgsability) becomes equivalent to the requirement that Ab $\Gamma = 1$.

Toroidal Compactifications to 4D

The strategy that we have employed to study toroidal compactifications to 4D is to exploit string theory dualities.

Toroidal Compactifications to 4D

The strategy that we have employed to study toroidal compactifications to 4D is to exploit string theory dualities. The idea is very simple, and is based on the following fact:

$$F/X \times S^1 = M/X$$

Toroidal Compactifications to 4D

The strategy that we have employed to study toroidal compactifications to 4D is to exploit string theory dualities. The idea is very simple, and is based on the following fact:

$$F/X \times S^1 = M/X$$

$$M/X \times S^1 = IIA/X$$

Toroidal Compactifications to 4D

The strategy that we have employed to study toroidal compactifications to 4D is to exploit string theory dualities. The idea is very simple, and is based on the following fact:

$$F/X \times S^1 = M/X$$

$$M/X \times S^1 = IIA/X$$

The SW curves of the corresponding 4D $\mathcal{N} = 2$ theories are obtained by considering IIB on the Hori-Vafa mirror of X.

The models which are easily amenable to such an analysis are those which are elliptically fibered 3-CY which are simple enough.

The models which are easily amenable to such an analysis are those which are elliptically fibered 3-CY which are simple enough. We have identified a large class of such models, corresponding to simple orbifolds of $T^2 \times \mathbb{C}^2$.

The models which are easily amenable to such an analysis are those which are elliptically fibered 3-CY which are simple enough. We have identified a large class of such models, corresponding to simple orbifolds of $T^2 \times \mathbb{C}^2$. T^2 admits the following orbifold actions: $\mathbb{Z}_{2,3,4,6}$.

The models which are easily amenable to such an analysis are those which are elliptically fibered 3-CY which are simple enough. We have identified a large class of such models, corresponding to simple orbifolds of $T^2 \times \mathbb{C}^2$. T^2 admits the following orbifold actions: $\mathbb{Z}_{2,3,4,6}$. Therefore it is natural to consider actions of cyclic discrete groups generated by an element

$$g = \mathrm{diag}(\alpha^2, \alpha^{-1}, \alpha^{-1}) \qquad \alpha^2 \in \mathbb{Z}_{2,3,4,6}$$

The models which are easily amenable to such an analysis are those which are elliptically fibered 3-CY which are simple enough. We have identified a large class of such models, corresponding to simple orbifolds of $T^2 \times \mathbb{C}^2$. T^2 admits the following orbifold actions: $\mathbb{Z}_{2,3,4,6}$. Therefore it is natural to consider actions of cyclic discrete groups generated by an element

$$g = \mathrm{diag}(\alpha^2, \alpha^{-1}, \alpha^{-1}) \qquad \alpha^2 \in \mathbb{Z}_{2,3,4,6}$$

The corresponding F-theory models are precisely the nHc's with basis $4, 6, 8, 12$ (Witten 96).

The models which are easily amenable to such an analysis are those which are elliptically fibered 3-CY which are simple enough. We have identified a large class of such models, corresponding to simple orbifolds of $T^2 \times \mathbb{C}^2$. T^2 admits the following orbifold actions: $\mathbb{Z}_{2,3,4,6}$. Therefore it is natural to consider actions of cyclic discrete groups generated by an element

$$g = \mathrm{diag}(\alpha^2, \alpha^{-1}, \alpha^{-1}) \qquad \alpha^2 \in \mathbb{Z}_{2,3,4,6}$$

The corresponding F-theory models are precisely the nHc's with basis $4, 6, 8, 12$ (Witten 96). Of course, we can also take $\Gamma_{ADE} \subset SU(2)$ orbifolds of the $\mathbb{C}^2$ base without spoiling the CY condition.

The models which are easily amenable to such an analysis are those which are elliptically fibered 3-CY which are simple enough. We have identified a large class of such models, corresponding to simple orbifolds of $T^2 \times \mathbb{C}^2$. T^2 admits the following orbifold actions: $\mathbb{Z}_{2,3,4,6}$. Therefore it is natural to consider actions of cyclic discrete groups generated by an element

$$g = \mathrm{diag}(\alpha^2, \alpha^{-1}, \alpha^{-1}) \qquad \alpha^2 \in \mathbb{Z}_{2,3,4,6}$$

The corresponding F-theory models are precisely the nHc's with basis $4, 6, 8, 12$ (Witten 96). Of course, we can also take $\Gamma_{ADE} \subset SU(2)$ orbifolds of the $\mathbb{C}^2$ base without spoiling the CY condition. The corresponding F-theory models are orbifolds of $T^2 \times \mathbb{C}^2$ with respect to the group generated by $\mathbb{Z}_{2,3,4,6}$ and Γ_{ADE}.

Let $k = 2, 3, 4, 6$, notice that for $\Gamma_{ADE} = \mathbb{Z}_{2Nk}$ we have an action

$$h = \mathrm{diag}(1, \omega, \omega^{-1}) \qquad \omega \in \mathbb{Z}_{2Nk}$$

Let $k = 2, 3, 4, 6$, notice that for $\Gamma_{ADE} = \mathbb{Z}_{2Nk}$ we have an action

$$h = \mathrm{diag}(1, \omega, \omega^{-1}) \qquad \omega \in \mathbb{Z}_{2Nk}$$

In particular $\omega^N = \alpha$. Therefore $gh^N = \mathrm{diag}(\alpha^2, 1, \alpha^{-2})$ and $g^{-1}h^N = \mathrm{diag}(\alpha^{-2}, \alpha^2, 1)$ leave fixed respectively the loci $z_2 = 0$ and $z_1 = 0$.

Let $k = 2, 3, 4, 6$, notice that for $\Gamma_{ADE} = \mathbb{Z}_{2Nk}$ we have an action

$$h = \text{diag}(1, \omega, \omega^{-1}) \qquad \omega \in \mathbb{Z}_{2Nk}$$

In particular $\omega^N = \alpha$. Therefore $gh^N = \text{diag}(\alpha^2, 1, \alpha^{-2})$ and $g^{-1}h^N = \text{diag}(\alpha^{-2}, \alpha^2, 1)$ leave fixed respectively the loci $z_2 = 0$ and $z_1 = 0$. These are non-compact divisors over which the elliptic fiber has e singularity of type $SO(8), E_6, E_7, E_8$ respectively for $k = 2, 3, 4, 6$.

Let $k = 2, 3, 4, 6$, notice that for $\Gamma_{ADE} = \mathbb{Z}_{2Nk}$ we have an action

$$h = \mathrm{diag}(1, \omega, \omega^{-1}) \qquad \omega \in \mathbb{Z}_{2Nk}$$

In particular $\omega^N = \alpha$. Therefore $gh^N = \mathrm{diag}(\alpha^2, 1, \alpha^{-2})$ and $g^{-1}h^N = \mathrm{diag}(\alpha^{-2}, \alpha^2, 1)$ leave fixed respectively the loci $z_2 = 0$ and $z_1 = 0$. These are non-compact divisors over which the elliptic fiber has e singularity of type $SO(8), E_6, E_7, E_8$ respectively for $k = 2, 3, 4, 6$. They meet at $z_1 = z_2 = 0$ at an A_{N-1} singularity.

Let $k = 2, 3, 4, 6$, notice that for $\Gamma_{ADE} = \mathbb{Z}_{2Nk}$ we have an action

$$h = \text{diag}(1, \omega, \omega^{-1}) \qquad \omega \in \mathbb{Z}_{2Nk}$$

In particular $\omega^N = \alpha$. Therefore $gh^N = \text{diag}(\alpha^2, 1, \alpha^{-2})$ and $g^{-1}h^N = \text{diag}(\alpha^{-2}, \alpha^2, 1)$ leave fixed respectively the loci $z_2 = 0$ and $z_1 = 0$. These are non-compact divisors over which the elliptic fiber has e singularity of type $SO(8), E_6, E_7, E_8$ respectively for $k = 2, 3, 4, 6$. They meet at $z_1 = z_2 = 0$ at an A_{N-1} singularity. These are precisely conformal matter systems!

The LG mirrors of these systems are very elegant and universal: they have the form

$$W_{T^2/\mathbb{Z}_k}(x_1, x_2, x_3) + W_G(y_1, y_2) + \text{ 2D exactly marginal deformations}$$

These have $\hat{c} = 3$.

The LG mirrors of these systems are very elegant and universal: they have the form

$W_{T^2/\mathbb{Z}_k}(x_1, x_2, x_3) + W_G(y_1, y_2) +$ 2D exactly marginal deformations

These have $\hat{c} = 3$. To find $\mathcal{N} = 2$ 4D SCFTs, by tuning the 2D exactly marginal deformations of the 2D LG mirrors we need to find special points where, by decoupling a Liouville subsector with $\hat{c} > 1$, we find a residual LG model with $\hat{c} < 2$ that has a scaling symmetry, which we identify with the $U(1)_R$ symmetry of the 4D SCFT.

The LG mirrors of these systems are very elegant and universal: they have the form

$W_{T^2/\mathbb{Z}_k}(x_1, x_2, x_3) + W_G(y_1, y_2) +$ 2D exactly marginal deformations

These have $\hat{c} = 3$. To find $\mathcal{N} = 2$ 4D SCFTs, by tuning the 2D exactly marginal deformations of the 2D LG mirrors we need to find special points where, by decoupling a Liouville subsector with $\hat{c} > 1$, we find a residual LG model with $\hat{c} < 2$ that has a scaling symmetry, which we identify with the $U(1)_R$ symmetry of the 4D SCFT. Doing so we find the SW curves of the models $(E_r^{(1,1)}, G)$, $r = 4, 6, 7, 8$, which I have constructed in 2012 with Cecotti and Giacomelli.

The LG mirrors of these systems are very elegant and universal: they have the form

$$W_{T^2/\mathbb{Z}_k}(x_1, x_2, x_3) + W_G(y_1, y_2) + \text{ 2D exactly marginal deformations}$$

These have $\hat{c} = 3$. To find $\mathcal{N} = 2$ 4D SCFTs, by tuning the 2D exactly marginal deformations of the 2D LG mirrors we need to find special points where, by decoupling a Liouville subsector with $\hat{c} > 1$, we find a residual LG model with $\hat{c} < 2$ that has a scaling symmetry, which we identify with the $U(1)_R$ symmetry of the 4D SCFT. Doing so we find the SW curves of the models $(E_r^{(1,1)}, G)$, $r = 4, 6, 7, 8$, which I have constructed in 2012 with Cecotti and Giacomelli. At such points, the marginal parameter which corresponds to the mirror T^2 is a 4D exactly marginal deformation, and therefore we predict that all these models enjoy an exact $SL(2, \mathbb{Z})$ action.

In particular, for the case of superconformal matter we obtain $(E^{(1,1)}_{4,6,7,8}, SU(kN))$ for $k = 2, 3, 4, 6$, which are just the lagrangian SCFTs corresponding to affine quivers of $\hat{D}_4(N), \hat{E}_{6,7,8}(N)$ type respectively.

In particular, for the case of superconformal matter we obtain $(E^{(1,1)}_{4,6,7,8}, SU(kN))$ for $k = 2, 3, 4, 6$, which are just the lagrangian SCFTs corresponding to affine quivers of $\hat{D}_4(N), \hat{E}_{6,7,8}(N)$ type respectively. This explains the reason why such systems have conformal manifolds which are given by moduli spaces of T^2 flat connections: we started with a flavor symmetry $E_8 \times E_8$ and we broke it completely by means of a diagonal Wilson line on T^2.

In particular, for the case of superconformal matter we obtain $(E^{(1,1)}_{4,6,7,8}, SU(kN))$ for $k = 2, 3, 4, 6$, which are just the lagrangian SCFTs corresponding to affine quivers of $\hat{D}_4(N), \hat{E}_{6,7,8}(N)$ type respectively. This explains the reason why such systems have conformal manifolds which are given by moduli spaces of T^2 flat connections: we started with a flavor symmetry $E_8 \times E_8$ and we broke it completely by means of a diagonal Wilson line on T^2. The moduli space of such Wilson line, is identified with the moduli space of the 4D theory.

Moreover, by tuning parameters in a different way we find other points in LG moduli space which admit similar decoupling limits.

Moreover, by tuning parameters in a different way we find other points in LG moduli space which admit similar decoupling limits. In particular, we find points where the $\hat{c} < 2$ scale invariant theory has the structure

$$f_{ADE}(w_1(x_i, y), w_2(x_i, y), w_3(x_i, y))$$

where $w_i = 0$ is a punctured Riemann surface. This is the IIB version of the class $\mathcal{S}$ construction!

Moreover, by tuning parameters in a different way we find other points in LG moduli space which admit similar decoupling limits. In particular, we find points where the $\hat{c} < 2$ scale invariant theory has the structure

$$f_{ADE}(w_1(x_i, y), w_2(x_i, y), w_3(x_i, y))$$

where $w_i = 0$ is a punctured Riemann surface. This is the IIB version of the class $\mathcal{S}$ construction! Using this method it is extremely easy to read off the structure of the punctures.

Moreover, by tuning parameters in a different way we find other points in LG moduli space which admit similar decoupling limits. In particular, we find points where the $\hat{c} < 2$ scale invariant theory has the structure

$$f_{ADE}(w_1(x_i, y), w_2(x_i, y), w_3(x_i, y))$$

where $w_i = 0$ is a punctured Riemann surface. This is the IIB version of the class $\mathcal{S}$ construction! Using this method it is extremely easy to read off the structure of the punctures. One interesting remark is that a single (1,0) SCFT can give rise to several class $\mathcal{S}[G]$ theories with different G and Σ.

Moreover, by tuning parameters in a different way we find other points in LG moduli space which admit similar decoupling limits. In particular, we find points where the $\hat{c} < 2$ scale invariant theory has the structure

$$f_{ADE}(w_1(x_i, y), w_2(x_i, y), w_3(x_i, y))$$

where $w_i = 0$ is a punctured Riemann surface. This is the IIB version of the class $\mathcal{S}$ construction! Using this method it is extremely easy to read off the structure of the punctures. One interesting remark is that a single (1,0) SCFT can give rise to several class $\mathcal{S}[G]$ theories with different G and Σ. For example, $\mathcal{T}(E_8, N)$ superconformal matter gives rise to both $\mathcal{S}[E_8]$ theory with Σ a sphere with $N+2$ punctures of which two are full and N are simple.

Moreover, by tuning parameters in a different way we find other points in LG moduli space which admit similar decoupling limits. In particular, we find points where the $\hat{c} < 2$ scale invariant theory has the structure

$$f_{ADE}(w_1(x_i, y), w_2(x_i, y), w_3(x_i, y))$$

where $w_i = 0$ is a punctured Riemann surface. This is the IIB version of the class $\mathcal{S}$ construction! Using this method it is extremely easy to read off the structure of the punctures. One interesting remark is that a single (1,0) SCFT can give rise to several class $\mathcal{S}[G]$ theories with different G and Σ. For example, $\mathcal{T}(E_8, N)$ superconformal matter gives rise to both $\mathcal{S}[E_8]$ theory with Σ a sphere with $N + 2$ punctures of which two are full and N are simple. But also to class $\mathcal{S}[D_4]$ theory with Σ a hyperelliptic curve of genus $g = N - 1$ with $4N + 4$ punctures.

The story continues...

For the nearest future several applications of all this machinery: $\mathcal{N} = 1$ theories by compactification on Σ (Gaiotto, Razamat (15), Aharony, Franco (15)), applications to study non-perturbative effects in String theory, applications to the classification of 5D SCFTs, ... and more! Stay tuned!

The story continues...

For the nearest future several applications of all this machinery: $\mathcal{N} = 1$ theories by compactification on Σ (Gaiotto, Razamat (15), Aharony, Franco (15)), applications to study non-perturbative effects in String theory, applications to the classification of 5D SCFTs, ... and more! Stay tuned!

Thanks!