Abelian 3d mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$

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2015/06/09, String Theory in Greater Tokyo @ RIKEN

“3d mirror symmetry”

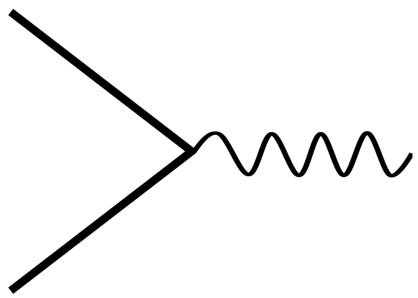
SQED

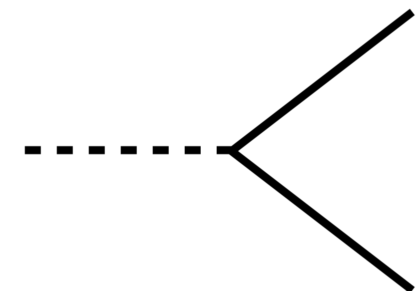
XYZ model

$$\mathcal{L}_{\boxed{\text{SQED}}} = \mathcal{L}_{\text{SYM}} + \mathcal{L}_{Q}^{(\boldsymbol{q}=+1)} + \mathcal{L}_{\tilde{Q}}^{(\boldsymbol{q}=-1)}$$

$$\mathcal{L}_{\text{XYZ}} = \mathcal{L}_X + \mathcal{L}_Y + \mathcal{L}_Z + XYZ + (\text{c.c.})$$

Gauge coupling





Yukawa coupling

IR: same fixed point (superconformal)

$$\mathcal{I}_{\boxed{\text{SQED}}} = \mathcal{I}_{\text{XYZ}}$$

Superconformal index is invariant along RG flow

Plan

- Superconformal index
- Mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$
- Summary & Outlook

☐ Superconformal index

#(BPS) having quantum numbers commuting with $\{Q, Q^\dagger\}$

Refinement of Witten index

$$\mathrm{Tr}_{\mathcal{H}}(-1)^{\hat{F}}$$

Superconformal index

#(BPS) having quantum numbers commuting with $\{Q, Q^\dagger\}$

$$\mathcal{I}(x,\alpha) = \mathrm{Tr}_{\mathcal{H}}(-1)^{\hat{F}} x^{\hat{H}+\hat{j}_3} \alpha^{\hat{f}}$$

Ex) $$= \cdots + 8x^2\alpha^4 + \cdots$$

(fermions if –)

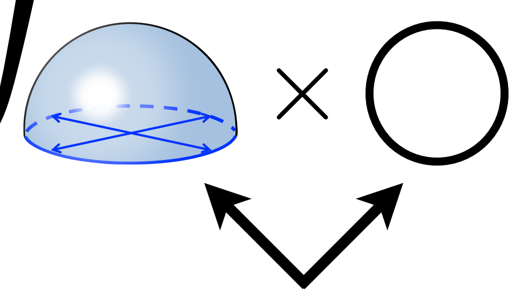
state	8 bosons
$\hat{H}+\hat{j}_3$	2
$\hat{f}$	4

Superconformal index

#(BPS) having quantum numbers commuting with $\{Q, Q^\dagger\}$

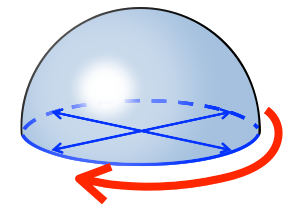
$$\mathcal{I}(x,\alpha) = \mathrm{Tr}_{\mathcal{H}}(-1)^{\hat{F}} x^{\hat{H}+\hat{j}_3} \alpha^{\hat{f}}$$

"Localization" $$= \int \mathcal{D}\Phi e^{-S[\Phi]-t\delta V}$$



Boundary conditions

Superconformal index

Boundary conditions on 

- Matter multiplet ($\longrightarrow$:vector with N_f)

$$\overrightarrow{\phi} \to \mathbf{M}\,\overrightarrow{\phi} \qquad \overrightarrow{\bar{\phi}} \to \mathbf{N}\,\overrightarrow{\bar{\phi}}$$

$$\overrightarrow{\psi} \to -i\gamma_1 \mathbf{M}\,\overrightarrow{\psi} \qquad \overrightarrow{\bar{\psi}} \to i\gamma_1 \mathbf{N}\,\overrightarrow{\bar{\psi}}$$

$\mathcal{Z}_{\text{1-loop}}$ (depending on $\mathbf{M}, \mathbf{N}$)

- Vector multiplet (U(1) gauge)

$V^{(\mathcal{P})}$

$$A_\vartheta \to -A_\vartheta \qquad A_{\varphi,y} \to +A_{\varphi,y}$$

$$\sigma \to -\sigma$$

$$\lambda \to +i\gamma_1\lambda \qquad \bar{\lambda} \to -i\gamma_1\bar{\lambda}$$

$$D \to +D$$

$V^{(\mathcal{CP})}$

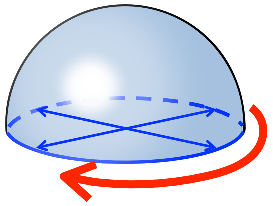
$$A_\vartheta \to +A_\vartheta \qquad A_{\varphi,y} \to -A_{\varphi,y}$$

$$\sigma \to +\sigma$$

$$\lambda \to -i\gamma_1\lambda \qquad \bar{\lambda} \to +i\gamma_1\bar{\lambda}$$

$$D \to -D$$

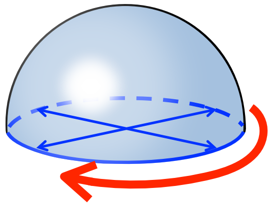
Superconformal index

Boundary conditions on 

- Vector multiplet (U(1) gauge)

$V^{(\mathcal{P})}$	$V^{(\mathcal{CP})}$
$A_\vartheta \to -A_\vartheta \quad A_{\varphi,y} \to +A_{\varphi,y}$	$A_\vartheta \to +A_\vartheta \quad A_{\varphi,y} \to -A_{\varphi,y}$
$\sigma \to -\sigma$	$\sigma \to +\sigma$
$\lambda \to +i\gamma_1\lambda \quad \bar{\lambda} \to -i\gamma_1\bar{\lambda}$	$\lambda \to -i\gamma_1\lambda \quad \bar{\lambda} \to +i\gamma_1\bar{\lambda}$
$D \to +D$	$D \to -D$

▢ Superconformal index

Boundary conditions on 

- Vector multiplet (U(1) gauge)

$V^{(\mathcal{P})}$ "Parity"

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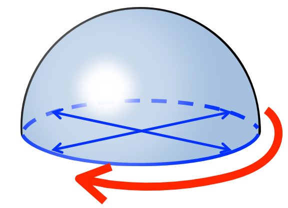
$D \rightarrow +D$

[redacted]

$$\text{Locus: } A = \left(\begin{array}{c} Z_2\text{–holonomy} \\ e^{i\oint_\gamma A_{\text{flat}}} = \pm 1 \end{array} \right) + \left(\begin{array}{c} \text{Wilson line} \\ \theta \pmod{2\pi} \end{array} \right)$$

$$\sum_{\{\text{holonomy } = \pm 1\}} \qquad \int_0^{2\pi} \frac{d\theta}{2\pi}$$

☐ Superconformal index

Boundary conditions on 

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$D \to +D$	$D \to -D$

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$$\Rightarrow \mathcal{I} = \sum_{\{\text{holonomy} = \pm 1\}} \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \mathcal{Z}_{\text{1-loop}}$$

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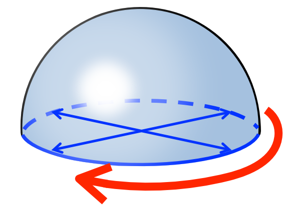
Boundary conditions on 

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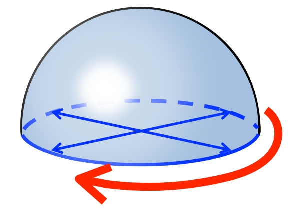
Superconformal index

Boundary conditions on 

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$V^{(\mathcal{CP})}$ "Charge conjugation"+"Parity"

$A_\vartheta \rightarrow +A_\vartheta \qquad A_{\varphi,y} \rightarrow -A_{\varphi,y}$

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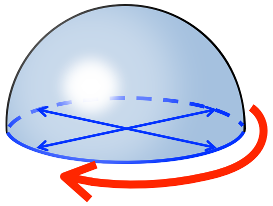
$D \rightarrow -D$

Locus: $A = \left(\begin{array}{c} \text{monopole} \\ B \in 2\mathbb{Z} \end{array} \right) + \left(\begin{array}{c} \text{Wilson line} \\ \theta_\pm \pmod{2\pi} \end{array} \right)$

▢ Superconformal index

Boundary conditions on 

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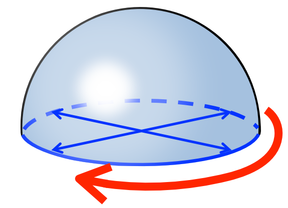
$\sum_{B \in 2\mathbb{Z}}$ $\qquad \sum_{\theta_\pm = 0,\pi}$

☐ Superconformal index

Boundary conditions on 

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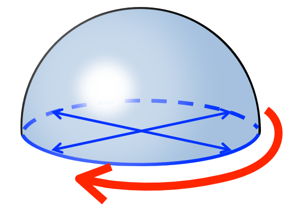
Locus: $A = \left(\begin{matrix} \text{monopole} \\ B \in 2\mathbb{Z} \end{matrix} \right) + \left(\begin{matrix} \text{Wilson line} \\ \theta_\pm \pmod{2\pi} \end{matrix} \right)$

$$\Rightarrow \mathcal{I} = \boxed{\sum_{B \in 2\mathbb{Z}} \sum_{\theta_\pm = 0, \pi} \mathcal{Z}_{\text{1-loop}}}$$

Superconformal index

Boundary conditions on 

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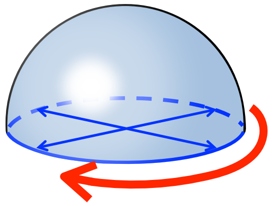
Boundary conditions on 

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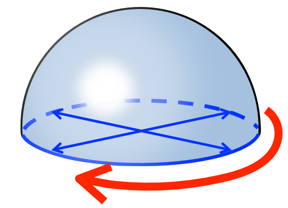
Boundary conditions on 

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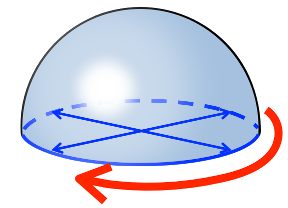
Plan

- [x] Superconformal index
- [ ] Mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$
- [ ] Summary & Outlook

Mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$

Boundary conditions on 

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$\mathcal{Z}_{\text{1-loop}}$ (depending on $\mathbf{M}, \mathbf{N}$)

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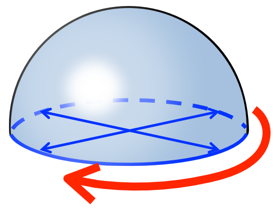
☐ Mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$

$V^{(\mathcal{P})}$ $\boxed{\text{SQED}}$ XYZ model

- To decide M, N: correspondence of moduli



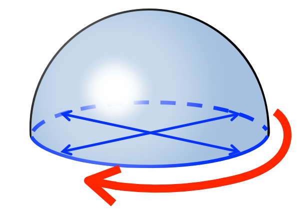
$$e^{+\sigma} \iff X$$

$$e^{-\sigma} \iff Y$$

$$Q\tilde{Q} \iff Z$$

☐ Mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$

$V^{(\mathcal{P})}$ SQED | XYZ model

- To decide $\mathbf{M}, \mathbf{N}$: correspondence of moduli 

$$e^{+\sigma} \to e^{-\sigma} \quad \Longleftrightarrow \quad X \to Y$$

$$e^{-\sigma} \to e^{+\sigma} \quad \Longleftrightarrow \quad Y \to X$$

$$Q\tilde{Q} \longrightarrow Q\tilde{Q} \quad \Longleftrightarrow \quad Z \longrightarrow Z$$

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad\qquad \mathbf{M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

☐ Mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$

$V^{(\mathcal{P})}$ SQED | XYZ model

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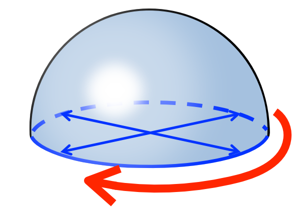
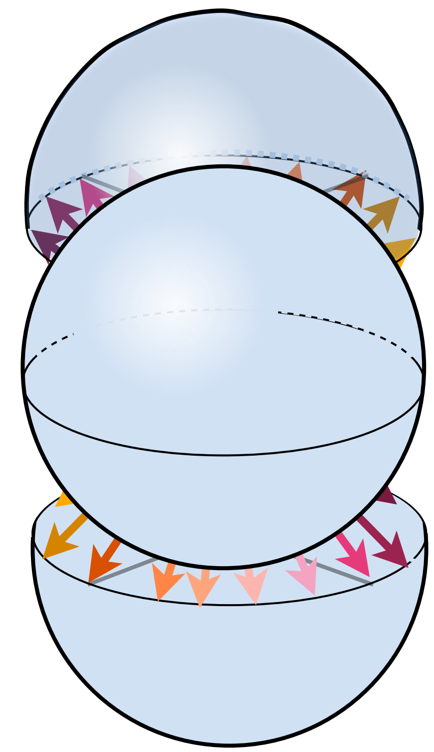


$$e^{+\sigma} \to e^{-\sigma} \quad \Longleftrightarrow \quad X$$

$$e^{-\sigma} \to e^{+\sigma} \quad \Longleftrightarrow \quad Y$$

$$Q\tilde{Q} \to Q\tilde{Q} \quad \Longleftrightarrow \quad Z \to Z$$



single matter on $\mathbb{S}^2$

$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathbf{M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

☐ Mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$

$V^{(\mathcal{P})}$ SQED XYZ model

$$\mathcal{I}_{\text{SQED}} \qquad\qquad \mathcal{I}_{\text{XYZ}}$$

$$\begin{aligned} &q^{\frac{1}{8}} \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \\ &\times \Big\{ a^{-\frac{1}{4}} \left(a^{-\frac{1}{2}} q^{\frac{1}{2}}\right)^{|s|} \frac{(a,q;q^2)_\infty}{(a^{-1}q,q^2;q^2)_\infty} \, {}_2\varphi_1(a^{-1}q^2, a^{-1}q; q; q^2, aq^{2|s|}) \\ &\quad + a^{\frac{1}{4}} \left(a^{-\frac{1}{2}} q^{\frac{3}{2}}\right)^{|s|} \frac{(a,q^3;q^2)_\infty}{(a^{-1}q^3,q^2;q^2)_\infty} \, {}_2\varphi_1(a^{-1}q^2, a^{-1}q^3; q^3; q^2, aq^{2|s|}) \Big\} \end{aligned}$$

$$= q^{\frac{1}{8}} \tilde{a}^{-\frac{1}{4}} \left(\tilde{a}^{-\frac{1}{2}} q^{\frac{1}{2}}\right)^{|\tilde{s}|} \frac{(\tilde{a}^{-\frac{1}{2}} q^{1+|\tilde{s}|};q)_\infty}{(\tilde{a}^{\frac{1}{2}} q^{|\tilde{s}|};q)_\infty} \frac{(\tilde{a};q^2)_\infty}{(\tilde{a}^{-1}q;q^2)_\infty}$$

${}_2\varphi_1$: basic hypergeometric series

New!!

$$(z;q)_\infty = \prod_{k=0}^{\infty} (1 - zq^k)$$

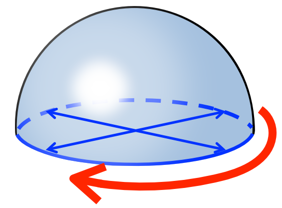
$$\sum_{n\geq 0} \frac{(\lambda;q)_n}{(q;q)_n} z^n = \frac{(\lambda z;q)_\infty}{(z;q)_\infty}$$

"q-binomial theorem"

☐ Mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$

$V^{(\mathcal{CP})}$ SQED — XYZ model

- To decide $\mathbf{M}, \mathbf{N}$: correspondence of moduli



$$e^{+\sigma} \longrightarrow e^{+\sigma} \quad \Longleftrightarrow \quad X \longrightarrow X$$

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$$Q\tilde{Q} \longrightarrow \tilde{Q}Q \quad \Longleftrightarrow \quad Z \longrightarrow Z$$

single matter on $\mathbb{S}^2$

$$\mathbf{M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

▢ Mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$

$V^{(\mathcal{CP})}$ SQED XYZ model

$$\mathcal{I}_{\text{SQED}} \qquad \mathcal{I}_{\text{XYZ}}$$

$$\frac{1}{2}q^{-\frac{1}{8}}\frac{(q;q^2)_\infty}{(q^2;q^2)_\infty}\left\{\frac{(a^{-\frac{1}{2}}q;q)_\infty}{(a^{\frac{1}{2}};q)_\infty}{}_1\psi_1(a^{\frac{1}{2}};a^{-\frac{1}{2}}q;q,q^{\frac{1}{2}}a^{-\frac{1}{2}}w) + \frac{(-a^{-\frac{1}{2}}q;q)_\infty}{(-a^{\frac{1}{2}};q)_\infty}{}_1\psi_1(-a^{\frac{1}{2}};-a^{-\frac{1}{2}}q;q,q^{\frac{1}{2}}a^{-\frac{1}{2}}w)\right\} = q^{-\frac{1}{8}}\frac{(\tilde{a}^{\frac{1}{2}}\tilde{w}^{-1}q^{\frac{1}{2}},\tilde{a}^{\frac{1}{2}}\tilde{w}q^{\frac{1}{2}},\tilde{a}^{-1}q;q^2)_\infty}{(\tilde{a}^{-\frac{1}{2}}\tilde{w}q^{\frac{1}{2}},\tilde{a}^{-\frac{1}{2}}\tilde{w}^{-1}q^{\frac{1}{2}},\tilde{a};q^2)_\infty}$$

${}_1\psi_1$: bilateral basic hypergeometric series

New!!

$$(z;q)_\infty = \prod_{k=0}^{\infty}(1-zq^k)$$

$${}_1\psi_1(a;b;q,z) = \frac{(q,b/a,az,q/az;q)_\infty}{(b,q/a,z,b/az;q)_\infty}$$

"Ramanujan's sum"

+

"Product-to-sum identity of theta functions"

Plan

- [x] Superconformal index
- [x] Mirror symmetry on $\mathbb{RP}^2 \times \mathbb{S}^1$
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☐ Summary & Outlook

(1)Classify **parity conditions** and compute superconformal indices on $\mathbb{RP}^2 \times \mathbb{S}^1$

$$V^{(\mathcal{P})} \Rightarrow \boxed{\sum_{\{\text{ holonomy } =\pm 1\}} \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \mathcal{Z}_{\text{1-loop}}}$$

$$V^{(\mathcal{CP})} \Rightarrow \boxed{\sum_{B \in 2\mathbb{Z}} \sum_{\theta_\pm = 0,\pi} \mathcal{Z}_{\text{1-loop}}}$$

(2)Give exact proof of $\boxed{\text{SQED}} = \text{XYZ model}$ on $\mathbb{RP}^2 \times \mathbb{S}^1$ as **new** mathematical identities

$V^{(\mathcal{P})} \Rightarrow$ "q–binomial theorem"

$V^{(\mathcal{CP})} \Rightarrow$ "Ramanujan's sum"

▢ Summary & Outlook

1. Extend to N_f flavors/non-Abelian group
2. Insertion of Wilson/Votex loops
3. Find "Holomophic blocks”
4. Apply to 3d-3d correspondence
5. From brane construction in string theory

$$\vdots$$