# Nuclear effects in neutrino and electron interactions

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Blind monks examining an elephant Hanabusa Itchō

# Outline

#### 1) Introduction

- Impulse approximation
- Off-shell effects

#### 2) Nuclear models

- Fermi gas model
- Shell model
- Spectral function approach

#### 3) Kinematic energy reconstruction

- Simplest case
- Realistic case

#### 4) Summary



*Assumption*: the dominant process of lepton-nucleus interaction is **scattering off a single nucleon**, with the remaining nucleons acting as a spectator system.



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It is valid when the momentum transfer  $|\mathbf{q}|$  is high enough, as the probe's spatial resolution is  $\sim 1/|\mathbf{q}|$ .





$$\frac{d\sigma_{\ell A}^{\mathrm{IA}}}{d\omega d\Omega} = \sum_{N} \int d^{3}p \, dE \, P_{\mathrm{hole}}^{N}(\mathbf{p}, E) \, \frac{M}{E_{\mathbf{p}}} \frac{d\sigma_{\ell N}^{\mathrm{elem}}}{d\omega d\Omega} \, P_{\mathrm{part}}^{N}(\mathbf{p}', \mathcal{T}')$$

The (hole) spectral function describes the ground-state properties of the target nucleus.





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Ensures the energy conservation and Pauli blocking





For scattering in a given angle, neutrinos and electrons differ only due to **the elementary cross section**.

In neutrino scattering, uncertainties come from (i) interaction dynamics and (ii) **nuclear effects**.

It is **highly improbable** that theoretical approaches unable to reproduce *(e,e')* data would describe nuclear effects in neutrino interactions at similar kinematics.

To be **reliable**, a description of nuclear effects has to be validated by **systematic comparisons** to *(e,e')* data, allowing its uncertainties to be estimated.

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Consider a nucleus stable against emission of nucleons.

As in its ground state,  $E_A = M_A$ , the energy cannot be decreased by emission of a nucleon

$$E_A = E_{A-1} + E_p < E_{A-1} + M$$



so the energy of a nucleon in the nucleus is lower than M.

V.R. Pandharipande, Nucl. Phys. B (Proc. Suppl.) 112, 51 (2002)

In a nuclear model, the initial nucleon's energy may

differ from the on-shell energy by a constant

$$E_p = \sqrt{M^2 + |\mathbf{p}|^2} - \epsilon$$

sophistication

ncreasing

be a function of the momentum

$$E_p = \sqrt{M^2 + |\mathbf{p}|^2} - \varepsilon(|\mathbf{p}|)$$

• only be correlated with the momentum

The elementary cross section,

$$\frac{d\sigma_{\ell N}^{\rm elem}}{dE_{\bf k'}d\Omega dE_{\bf p'}d\Omega} \propto L_{\mu\nu}H^{\mu\nu}$$

contains two tensors

$$L_{\mu\nu} \propto j_{\mu}^{\text{lept}} j_{\nu}^{\text{lept*}}$$
 and  $H^{\mu\nu} \propto j_{\text{hadr}}^{\mu} j_{\text{hadr}}^{\nu*}$ 

#### with only the hadron one affected by off-shell effects.

The current appearing in the hadron tensor is known on the mass shell,

$$j_{\text{hadr}}^{\mu} = \overline{u}(\mathbf{p}', s') \left( \gamma^{\mu} F_1 + i \sigma^{\mu\kappa} \frac{q_{\kappa}}{2M} F_2 + \dots \right) u(\mathbf{p}, s)$$

#### or equivalently

$$j_{\text{hadr}}^{\mu} = \overline{u}(\mathbf{p}', s') \left( \gamma^{\mu}(F_1 + F_2) - \frac{(p+p')^{\mu}}{2M} F_2 + \dots \right) u(\mathbf{p}, s)$$

The prescription of de Forest [NPA 392, 232 (1983)]:

to approximate the off-shell hadron tensor, one can use the on-shell expression with the same momentum transfer and a modified energy transfer,

$$\begin{split} H^{\mu\nu}_{\text{off-shell}}(p,q) &\to H^{\mu\nu}_{\text{off-shell}}(\tilde{p},\tilde{q}) \\ \\ \tilde{p} &= (\sqrt{M^2 + \mathbf{p}^2}, \mathbf{p}) \quad \text{and} \quad \tilde{q} = (\tilde{\omega}, \mathbf{q}) \end{split}$$

with

The prescription of de Forest [NPA 392, 232 (1983)]:

as the initial nucleon's energy is now  $E_p = \sqrt{M^2 + p^2}$ in our calculations, and the final energy is an observable, the energy transfer has to be

$$\tilde{\omega} = \sqrt{M^2 + (\mathbf{p} + \mathbf{q})^2} - \sqrt{M^2 + \mathbf{p}^2}$$

the difference between the "lepton"  $\omega$  and "hadron"  $\widetilde{\omega}$  is transferred to the spectator system of (A-1) nucleons.

#### Examples of an oversimplified treatment:





Imagine an infinite space filled uniformly with nucleons



Due to the translational invariance, the eigenstates can be labeled using momentum,  $\psi(x) = C e^{-ipx}$ .



Due to the boundary conditions,  $p_i \frac{L}{2} = \frac{\pi}{2} + n\pi$ every state occupies  $(2\pi/L)^3$  in the momentum space





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Momentum space



Coordinate space

The corresponding hole and particle spectral functions are

$$P_{\text{hole}}^{\text{FG}}(\mathbf{p}, E) = \frac{3}{4\pi p_F^3} \,\theta(p_F - |\mathbf{p}|) \,\delta(E_p - \varepsilon - M + E),$$
$$P_{\text{part}}^{\text{FG}}(\mathbf{p}', \mathcal{T}') = [1 - \theta(p_F - |\mathbf{p}'|)] \,\delta(E_{p'} - M - \mathcal{T}'),$$

putting them to the cross section (below), you can recover the standard formula for the Fermi gas.

$$\frac{d\sigma_{\ell A}^{\mathrm{IA}}}{d\omega d\Omega} = \sum_{N} \int d^{3}p \, dE \, P_{\mathrm{hole}}^{N}(\mathbf{p}, E) \, \frac{M}{E_{\mathbf{p}}} \frac{d\sigma_{\ell N}^{\mathrm{elem}}}{d\omega d\Omega} \, P_{\mathrm{part}}^{N}(\mathbf{p}', \mathcal{T}')$$

#### Electron scattering off carbon, 500 MeV, 60 deg



Moniz et al., PRL 26, 445 (1971)

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Moniz et al., PRL 26, 445 (1971)



#### What happens at a kinematics other than 500 MeV, 60deg?



### **Charge-density in nuclei**



# Local Fermi gas model

A spherically symmetric nucleus can be approximated by concentric spheres of a constant density.





#### Shell model

#### **Example: oxygen nucleus**

In a spherically symmetric potential, the eigenstates can be labeled using the total angular momentum.



#### **Example: oxygen nucleus**



Leuschner et al., PRC 49, 955 (1994)

#### **Example: oxygen spectral function**

3

2

 $P(\mathbf{p}, E) \ (10^{-8} \ \mathrm{MeV^{-4}})$ 


#### **Depletion of the shell-model states**



De Witt Huberts, JPG 16, 507 (1990)

## **Depletion of the shell-model states**

The observed depletion is  $\sim$ 35% for the valence shells

and ~20% overall, when higher missing energy is probed.



D. Rohe, NuInt05



# **Spectral function approach**

The main source of the depletion of the shell-model states are **short-range nucleon-nucleon correlations**.

Yielding NN pairs (typically pn pairs) with high relative momentum, they move ~20% of nucleons to the states of high removal energies.





Acciari et al. (ArgoNeuT), PRD 90, 012008 (2014)

The hole spectral function can be expressed as





Benhar&Pandharipande, RMP 65, 817 (1993)



Benhar&Pandharipande, RMP 65, 817 (1993)

## **Local-density approximation**

The correlation component in nuclei can be obtained combining the results for infinite nuclear matter obtained at different densities:



$$P_{\mathrm{corr}}^{N}(\mathbf{p}, E) = \int dR \rho(R) P_{\mathrm{corr}}^{NM,N}(\rho, \mathbf{p}, E).$$

Benhar et al., NPA 579 493, (1994)



## **Final-state interactions**

Their effect on the cross section is easy to understand in terms of the complex optical potential:

- the real part modifies the struck nucleon's energy spectrum: it differes from  $\sqrt{M^2 + p'^2}$
- the imaginary part reduces the single-nucleon final states and produces multinucleon final states

$$e^{i(E+U)t} = e^{i(E+U_V)t}e^{-U_Wt}$$

Horikawa et al., PRC 22, 1680 (1980)

$$E_{\mathbf{k}} + M_A = E_{\mathbf{k}'} + E_{A-1} + E_{\mathbf{p}'}$$













#### **Final-state interactions**

In the convolution approach,

$$\frac{d\sigma^{\rm FSI}}{d\omega d\Omega} = \int d\omega' f_{\bf q} (\omega - \omega') \frac{d\sigma^{\rm IA}}{d\omega' d\Omega},$$

with the folding function

$$f_{\mathbf{q}}(\omega) = \delta(\omega)\sqrt{T_A} + \left(1 - \sqrt{T_A}\right)F_{\mathbf{q}}(\omega),$$
  
Nucl. transparency

#### **Nuclear transparency**



#### **Nuclear transparency**





Benhar et al., PRC 44, 2328 (1991)

## **Real part of the optical potential**

We account for the spectrum modification by

$$f_{\mathbf{q}}(\omega - \omega') \to f_{\mathbf{q}}(\omega - \omega' - U_V).$$

This procedure is similar to that from the Fermi gas model to introduce the binding energy in the argument of  $\delta(...)$ .

$$U_V = U_V(t_{\rm kin})$$

$$t_{\rm kin} = \frac{E_{\bf k}^2(1 - \cos\theta)}{M + E_{\bf k}(1 - \cos\theta)}$$

#### **Optical potential by Cooper** *et al.*



Deb et al., PRC 72, 014608 (2005)

## **Optical potential by Cooper** *et al.*



## Simple comparison

#### Real part of the OP

- acts in the final state
- shifts the QE peak
  to low ω at low |q|
  (to high ω at high |q|)

#### Binding energy in RFG

- acts in the initial state
- shifts the QE peak to high  $\omega$







## **Compared calculations**



#### Low excitation-energy phenomena





Barreau *et al*., NPA 402, 515 (1983)



Barreau *et al.*, NPA 402, 515 (1983)



Barreau *et al.*, NPA 402, 515 (1983)

Baran *et al.*, PRL 61, 400 (1988) Whitney *et al.*, PRC 9, 2230 (1974)

- The supplemental material of PRD 91,033005 (2015)
- shows comparisons to the data sets collected
- at 54 kinematical setups
  - energies from ~160 MeV to ~4 GeV,
  - angles from 12 to 145 degrees,
  - at the QE peak, the values of momentum transfer from ~145 to ~1060 MeV/c and  $0.02 \le Q^2 \le 0.86$  (GeV/c)<sup>2</sup>.

#### **CCQE MINERvA data**



#### **CCQE MINERvA data**

| TABLE I. Fit results to the CC QE MINERvA data. |               |                                       |               |  |
|---|---------------|---------------------------------------|---------------|--|
|   | antineutrino  | neutrino                              | combined fit  |  |
|   | including     | including theoretical uncertainties:  |               |  |
| $M_A$ (GeV)                                     | $1.16\pm0.06$ | $1.17\pm0.06$                         | $1.16\pm0.06$ |  |
| $\chi^2/d.o.f.$                                 | 0.38          | 1.33                                  | 0.93          |  |
|   | neglectin     | neglecting theoretical uncertainties: |               |  |
| $M_A$ (GeV)                                     | $1.15\pm0.10$ | $1.15\pm0.07$                         | $1.13\pm0.06$ |  |
| $\chi^2/d.o.f.$                                 | 0.44          | 1.38                                  | 1.00          |  |
|   | neglectin     | neglecting FSI ( $M_A = 1.16$ GeV):   |               |  |
| $\chi^2/d.o.f.$                                 | 2.49          | 2.45                                  | 2.42          |  |
|   |               |                                       |               |  |





# Kinematic energy reconstruction: simplest (unrealistic) case
Consider the simplest (unrealistic) case:

the beam is **monochromatic** but its energy is **unknown** and has to be reconstructed





$$E' = 768 \text{ MeV}$$
  
 $\theta = 37.5 \text{ deg}$   
 $\Delta E' = 5 \text{ MeV}$ 



$$E' = 768 \text{ MeV}$$
  
 $\theta = 37.5 \text{ deg}$   
 $\varDelta E' = 5 \text{ MeV}$ 

for 
$$\epsilon = 25$$
 MeV  
 $E = 960$  MeV  
 $\Delta E = 7$  MeV



| $\theta$ (deg)              | 37.5                                      | 37.1  | 36                       | 36                                  |  |  |  |  |
|-----------------------------|---|---|--------------------------|-------------------------------------|--|--|--|--|
| E' (MeV)                    | 768.0                                     | 615.0   | 487.5                    | 287.5                               |  |  |  |  |
| $\Delta E'$ (MeV)           | 5   | 5   | 5                        | 2.5                                 |  |  |  |  |
| $\epsilon = 25 \text{ MeV}$ |   |   |                          |                                     |  |  |  |  |
| <b>rec.</b> <i>E</i>        | <b>960</b> ± 7                            | $741 \pm 7$   | $571 \pm 6$              | $333 \pm 3$                         |  |  |  |  |
| true E                      | 961                                       | 730   | 560                      | 320                                 |  |  |  |  |
|                             | Sealock et al.,<br>PRL 62, 1350<br>(1989) | O'Connell <i>et al.</i> ,<br>PRC 35, 1063<br>(1987) | Barreau<br>NPA 40<br>(19 | u <i>et al.</i> ,<br>02, 515<br>83) |  |  |  |  |

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|                   |               |        |        |            |
| true E            | 961           | 730    | 560    | 320        |
| <i>ϵ</i>          | <b>26 ± 5</b> | 16 ± 5 | 16 ± 3 | $13 \pm 3$ |
|                   |               |        |        |            |
|                   |               |        |        |            |

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|-------------------|----------------------------------|-------------------|---------------------|----------------------|
| E' (MeV)          | 768.0                            | 615.0             | 487.5               | 287.5                |
| $\Delta E' (MeV)$ | ) 5                              | 5                 | 5                   | 2.5                  |
|                   |                                  |                   |                     |                      |
|                   |                                  |                   |                     |                      |
| true E            | 961                              | 730               | 560                 | 320                  |
|                   | 26 + 5                           | $16 \pm 5$        | 16 + 3              | 13 + 3               |
| <i>ϵ</i>          | $20\pm 3$                        | $10\pm 3$         | $10\pm 3$           | $13\pm 3$            |
| $\epsilon$        | $20 \pm 5$<br>different <i>E</i> | $\equiv$ differen | $t Q^2 \equiv diff$ | $\frac{13 \pm 3}{2}$ |



## Kinematic energy reconstruction: realistic case

## **Polychromatic beam**

In modern experiments, the neutrino beams are not monochromatic, and the **energy must be reconstructed** from the observables, typically E' and  $\cos \theta$  under the CCQE event hypothesis.



#### **CCQE** events

In practice, CCQE event candidates are defined as containing **no pions observed**.

+ CCQE (1p1h and 2p2h)pion production and followed by absorption undetected pions

CCQE with pions from FSI

**CCQE-like events** 

### **Recall the monochromatic-beam case**



## **CCQE events of given** *l*<sup>±</sup> **kinematics**



## **CCQE events of given** *l*<sup>±</sup> **kinematics**

Very different processes and neutrino energies contribute to CCQE-like events of a given E' and  $\cos \theta$ .

An undetected pion typically lowers the reconstructed energy by ~300–350 MeV.

Note that in the reconstruction formula,  $M_{\Delta} = 1232 \text{ MeV}$ would be more suitable than M' = 939 MeV.

$$E_{v}^{\text{rec}} = \frac{2(M-\varepsilon)E_{\ell} + M'^{2} - (M-\varepsilon)^{2} - m_{\ell}^{2}}{2(M-\varepsilon-E_{\ell} + |\mathbf{k}_{\ell}|\cos\theta)}. \qquad \frac{M_{\Delta}^{2} - M'^{2}}{2M} \approx 340 \text{ MeV}$$

### **Absorbed or undetected pions**



## **Summary**

- An accurate description of nuclear effects, including finalstate interactions, is crucial for accurate reconstruction of neutrino energy.
- Theoretical models must be validated against (e,e') data to estimate their uncertainties.
- The spectral function formalism can be used in Monte Carlo simulations to improve the accuracy of description of nuclear effects.
- Final-state interactions can have an important effect on neutrino energy reconstruction, even at E ~ few GeV.

#### **Question 1**

Consider the process of quasielasic scattering on a free nucleon. Why is the antineutrino cross section lower than that for neutrino?

#### **Question 2**

At a given kinematics, the quasielastic cross sections  $d\sigma_{IA}/d\omega d\Omega$ for neutrino and electron scattering off a nucleus are similar, barring the normalization. What is the total cross section for electrons? Why do neutrino and electron interactions differ qualitatively?

#### **Question 3**

At high neutrino energies, high scattering angles are strongly suppressed in quasielastic scattering. Can you explain it?

#### **Problem 1**

Consider charged-current scattering off a nuclear target leading to excitation of a resonance of the invariant mass *W*. How to reconstruct the neutrino energy from the charged-lepton's kinematics?

#### **Problem 2**

Assume a general case of charged-current interaction, with *n* nucleon and *m* meson tracks reconstructed in the detector. How to approximate the neutrino energy using the momentum conservation?

#### Problem 3

How to reconstruct the neutrino energy when a pion is produced in a single-nucleon knockout from a nucleus? Assume that the pion kinematics is known. Hint: use the relation between the energy and momentum of the knocked-out nucleon.



# **Backup slides**

## **Energy reconstruction**

Consider the probability distribution that a muon of given energy and scattering angle is produced by a neutrino of energy  $E_{\nu}$ 

$$\mathcal{P}(E_{\nu})\big|_{E_{\mu},\,\cos\theta} = \frac{\frac{d\sigma(E_{\nu})}{dE_{\mu}d\cos\theta}}{\int dE_{\nu}\frac{d\sigma(E_{\nu})}{dE_{\mu}d\cos\theta}}$$



kinematics relevant to the T2K experiment

At  $\cos = 0.97$ the difference is ~16 MeV for RFG with  $\varepsilon = 25$  MeV



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To get the maxima right

 $\varepsilon$  = 9 MeV @ cos = 0.97  $\varepsilon$  =27 MeV @ cos = 0.92  $\varepsilon$  =29 MeV @ cos = 0.87

