Deep inelastic scattering from nucleons and nuclei

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What is Deep Inelastic Scattering?

Lepton-nucleon scattering was born in 1956, when Hofstadter et al. performed the first elastic $e-p$ scattering experiment and found a finite radius of the proton.
About 10 years later it became 'deep inelastic', when experiments at 20 times higher energies, at SLAC, established the partonic nature of the proton.

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General process for the deep inelastic scattering is

$$l(k) + N(p) \longrightarrow l'(k') + X(p'), \quad l, l' = e^{\pm}, \mu^{\pm}, \nu_l, \bar{\nu}_l, \; N = n, p$$

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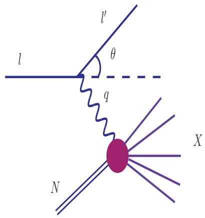
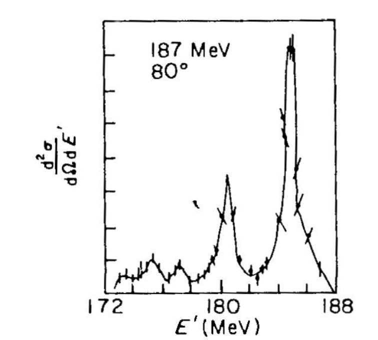
Kinematics(Nucleon in the rest frame)

$$Q^2=-q^2=-(k-k')^2=4EE'\sin^2\frac{\theta}{2}$$
$$M^2=p^2$$
$$\nu=p.q=M(E-E')$$
$$x=\frac{Q^2}{2M\nu}=\frac{Q^2}{2p.q}=\frac{Q^2}{2MEy}$$
$$y=\frac{p.q}{p.k}=1-\frac{E'}{E}$$
$$W^2=M^2+2p.q-Q^2$$



Electron-Carbon Scattering

$\theta = 80^\circ,\ Q^2 \to 0.06\ \ GeV^2$



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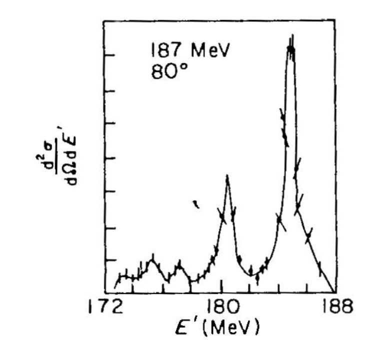
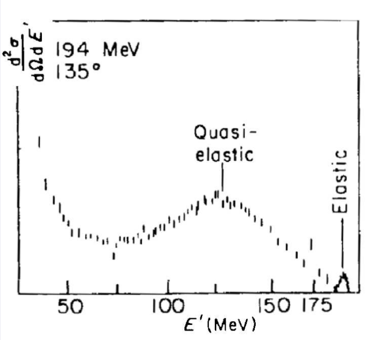


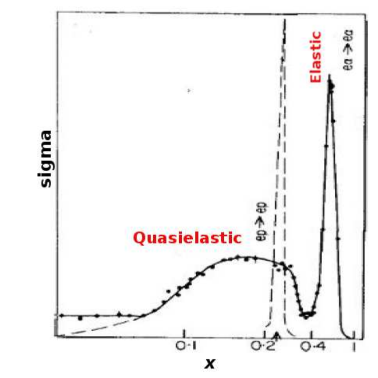
$\theta = 135^\circ,\ Q^2 \to 0.1\ GeV^2$



$$E' = \frac{E}{1 + \frac{E}{M_A}(1 - \cos\theta)}$$

Electron-Helium Scattering

$\theta = 45°, \ Q^2 \to 0.08 \ GeV^2$



Electron-Helium Scattering

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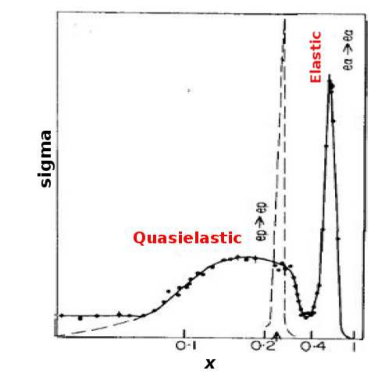
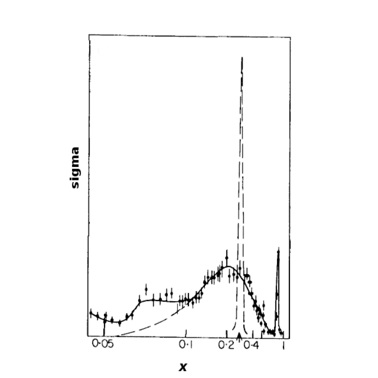
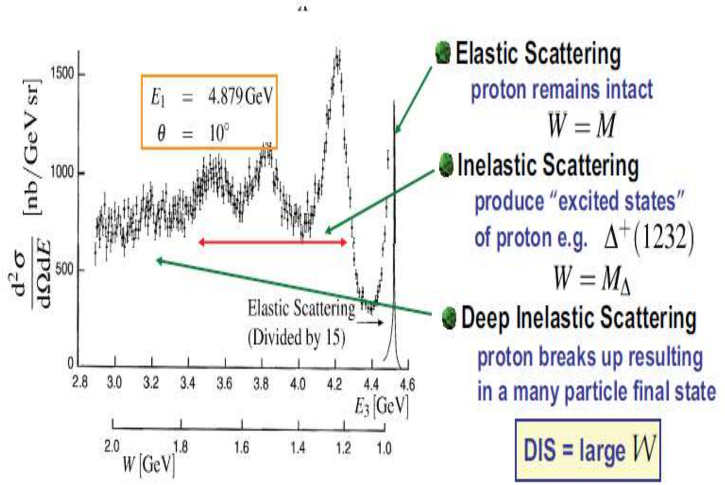
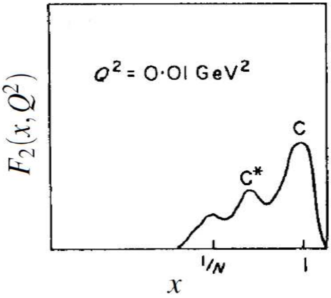


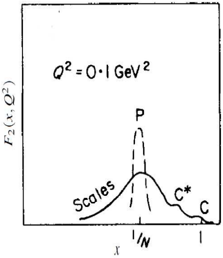
$\theta = 60°,\ Q^2 \to 0.1\ GeV^2$

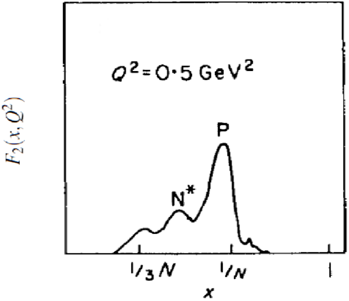


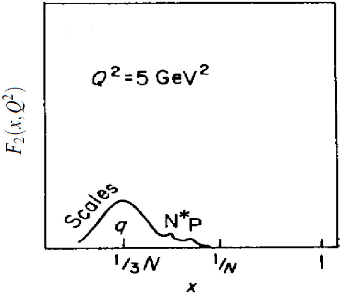
Scattering of 4.879 GeV electrons from protons at rest.



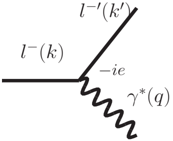

$Q^2 = 0{\cdot}01$ GeV^2
$F_2(x,Q^2)$
C
C*
1/N
1
x

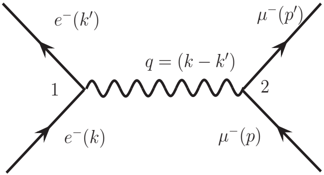

$Q^2 = 0{\cdot}1$ GeV^2
$F_2(x,Q^2)$
P
Scales
C*
C
1/N
1
x


$Q^2 = 0{\cdot}5$ GeV^2
$F_2(x,Q^2)$
P
N*
1/3 N
1/N
1
x


$Q^2 = 5$ GeV^2
$F_2(x,Q^2)$
Scales
q
N*P
1/3 N
1/N
1
x

$e^- - \mu^-$ *scattering*



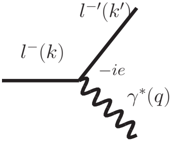


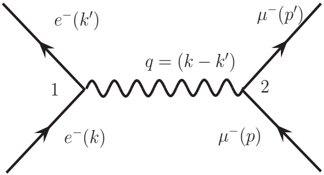
The interaction is described by the Lagrangian

$$\mathcal{L}_I \quad = \quad -ie\bar{\psi}(k')\gamma^\mu A_\mu \psi(k)$$

The transition amplitude M is given as:
$-iM \;=\;$ current at vertex 1 $\times$ propagator $\times$ current at vertex 2

$e^- - \mu^-$ scattering





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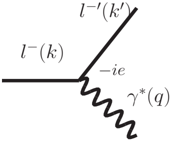

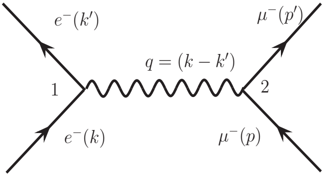

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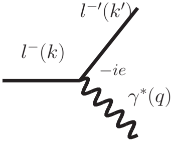

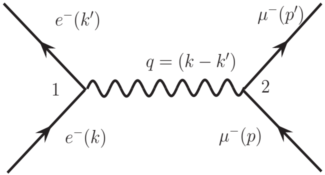

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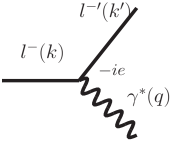

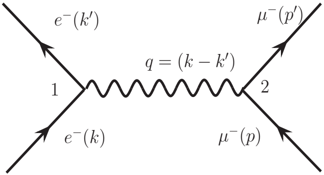

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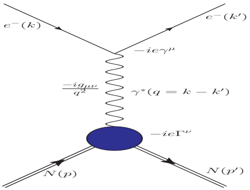
$e^- - p$ *elastic scattering*



Invariant amplitude is written as

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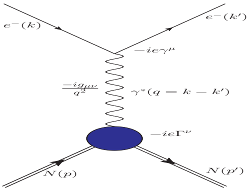

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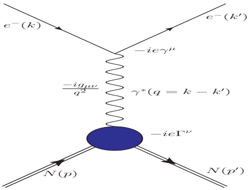

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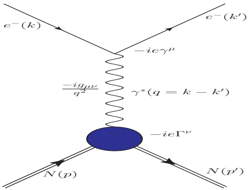

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Γ^ν is written in terms of p, p', q and γ-matrices:

$$\begin{aligned}\Gamma^\mu &= A(Q^2)\gamma^\mu + B(Q^2)(p'-p)^\mu + C(Q^2)(p'+p)^\mu \\ &+ D(Q^2)i\sigma^{\mu\nu}(p'-p)_\nu + E(Q^2)i\sigma^{\mu\nu}(p'+p)_\nu\end{aligned}$$

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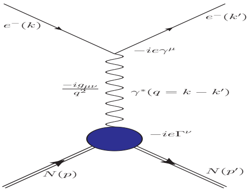

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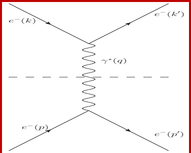
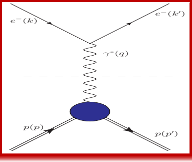
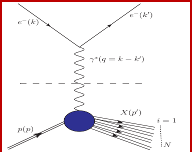
$$\Gamma^\mu = F_1(q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2M}q_\nu F_2(q^2) \quad \text{OR}$$

$$\Gamma^\mu = (F_1(q^2) + F_2(q^2))\gamma^\mu - \frac{F_2(q^2)}{2M}(p+p\prime)^\mu$$

$e^- - p$ *deep inelastic scattering*





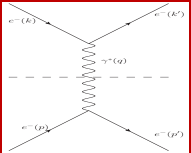


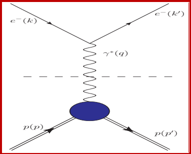
For the two body exclusive process $1+2 \to 3+4+\ldots+n$ *the differential cross section is*

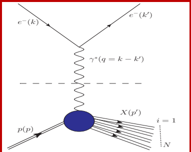
$$\sigma = \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - \ldots) \times$$

$$\Pi_{j=3}^{n} 2\pi \quad \delta(p_j^2 - m_j^2) \quad \theta(p_j^0) \frac{d^4 p_j}{(2\pi)^4}$$

$e^- - p$ *deep inelastic scattering*







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Dynamics of the scattering is contained in

$$|\mathcal{M}|^2 \propto L_{\mu\nu} W^{\mu\nu}$$

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We write a general parameterization of the hadronic tensor

$$\begin{aligned} W^{\mu\nu} &= -g_{\mu\nu}\ W_1 + \frac{p_\mu p_\nu}{M^2}\ W_2 - i\epsilon_{\mu\nu\lambda\sigma}\frac{p^\lambda q^\sigma}{2M^2}\ W_3 + \frac{q_\mu q_\nu}{M^2}\ W_4 \\ &+ \frac{(p_\mu q_\nu + p_\nu q_\mu)}{2M^2}\ W_5 + \frac{i(p_\mu q_\nu - p_\nu q_\mu)}{2M^2}\ W_6 \end{aligned}$$

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By contraction of hadronic tensor with $L_{\mu\nu}$

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Applying CVC: $q_\mu W^{\mu\nu} = 0$

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Hadronic tensor can be written as:

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W_1 and W_2 can be the functions of any two Lorentz-invariant scalars: q^2, $\nu = \frac{p.q}{M}$, $x = \frac{-q^2}{2p.q}$, $y = \frac{p.q}{p.k}$

$$L_{\mu\nu}W^{\mu\nu} = 4W_1(k.k') + \frac{2W_2}{M^2}[2(p.k)(p.k') - M^2(k.k')]$$

$$L_{\mu\nu}\, W^{\mu\nu} \quad = \quad 4\, W_1 (k.k') + \frac{2\, W_2}{M^2} [2(p.k)(p.k') - M^2 (k.k')]$$

In the Lab frame, we have

$$k.k' = 2EE' \sin\frac{\theta}{2}, \qquad p.k = EM, \qquad p.k' = E' M$$

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$L_{\mu\nu}\, W^{\mu\nu}$ is obtained as:

$$L_{\mu\nu}\, W^{\mu\nu} \;\; = \;\; 4EE' \left[\cos^2\frac{\theta}{2}\, W_2(\nu, q^2) + \sin^2\frac{\theta}{2} 2\, W_1(\nu, q^2)\right]$$

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$$\frac{d\sigma}{dE'\,d\Omega} = \frac{\alpha^2}{4E^2 sin^4(\frac{\theta}{2})}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\theta}{2}\right]\delta\left(\nu + \frac{q^2}{2m}\right)$$

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$ep \longrightarrow ep$

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$$\frac{d\sigma}{dE'\,d\Omega} = \frac{\alpha^2}{4E^2 sin^4(\frac{\theta}{2})}\left[\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\theta}{2}\right]\delta\left(\nu + \frac{q^2}{2m}\right)$$

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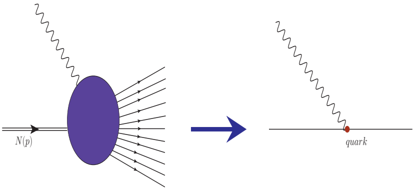
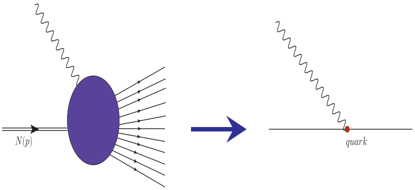
$$\frac{d\sigma}{dE'\,d\Omega} = \frac{\alpha^2}{4E^2 sin^4(\frac{\theta}{2})}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\right]\delta\left(\nu + \frac{q^2}{2M}\right)$$

$ep \longrightarrow eX$

$$\frac{d\sigma}{dE'\,d\Omega} = \frac{\alpha^2}{4E^2 sin^4(\frac{\theta}{2})}\left[W_2(\nu, Q^2)\cos^2\frac{\theta}{2} + 2\,W_1(\nu, Q^2)\sin^2\frac{\theta}{2}\right]$$


N(p)
quark



Proton structure functions:

$$2mW_1^{point}(\nu, Q^2) = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

$$\nu W_2^{point}(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

Structure function is independent of ν and Q^2 and depends on the ratio $\frac{Q^2}{2m\nu}$.

In contrast one may observe the behaviour of elastic scattering by setting $G_E = G_M \equiv G$

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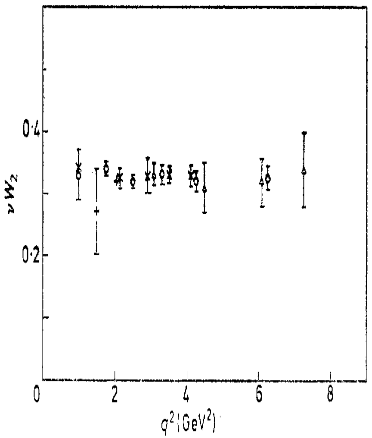
$$

The structure functions contain a form factor $G^2(Q^2)$ and so cannot be the functions of a single dimensionless variable.



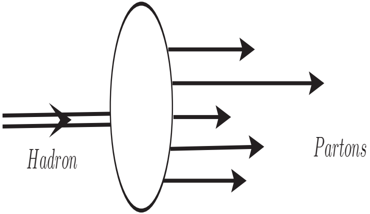
The naive parton model predicts that structure functions are independent of Q^2. This scale invariance is Bjorken scaling.

$$M W_1(\nu, Q^2) \longrightarrow F_1(\omega)$$
$$\nu W_2(\nu, Q^2) \longrightarrow F_2(\omega)$$

$\omega = \frac{2M\nu}{Q^2}$ (fixed)

Basic assumptions of parton model

1. A rapidly moving hadron appears as a jet of partons all of which travel in more or less same direction as the parent hadron



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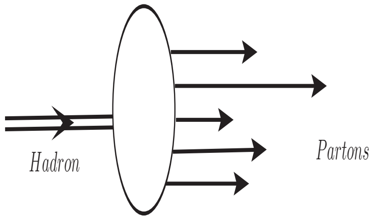
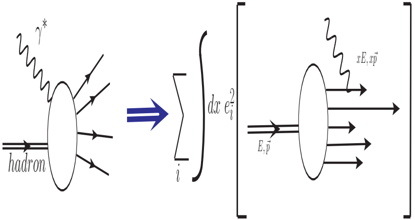
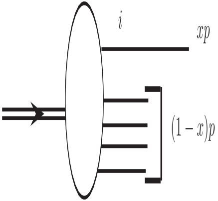


2. The rule for calculating reaction rate for hadron: the reaction rate for the basic process with free partons is calculated and summed incoherently over the contributions of partons in the hadron.



3. The three momentum of the hadron is shared out among the partons. One defines the parton momentum distribution

$$f_i(x) = \frac{dP_i}{dx} =$$



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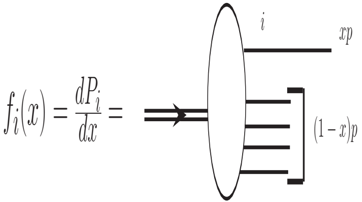

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4. They carry a different fraction x of the hadron's momentum and energy. All the fractions x add up to 1

$$\sum_{i'} \int dx \; x \; f_{i'}(x) = 1$$

	Hadron	Parton
Energy	E	x E
Momentum	p_L	xp_L
	$p_T = 0$	$p_T = 0$
Mass	M	m=xM

The dimensionless structure functions are given by:

$$F_1(\omega) = \frac{Q^2}{4m\nu x}\delta\left(1 - \frac{Q^2}{2m\nu}\right), \quad F_2(\omega) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

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For a proton, we may write F_1 and F_2 as

$$F_1(\omega) = \sum_i \int dx \;\; e_i^2 \;\; f_i(x) \;\; x \;\; \delta\left(x - \frac{1}{\omega}\right), \quad F_1(\omega) = \frac{\omega}{2}F_2(\omega)$$

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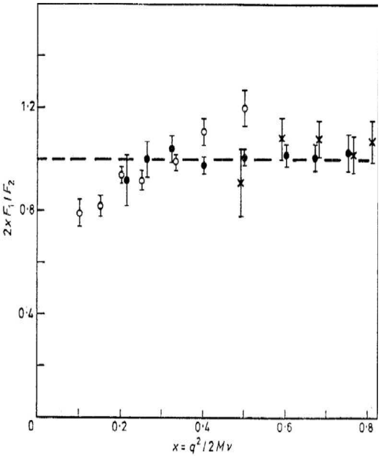
Structure functions are obtained as:

$$\nu W_2(\nu, Q^2) \longrightarrow F_2(x) = \sum e_i^2 \ x \ f_i(x)$$

$$MW_1(\nu, Q^2) \longrightarrow F_1(x) = \frac{1}{2x}F_2(x)$$

$F_{1,2}$ corresponds to the total momentum fraction carried by all the quarks and antiquarks in the nucleon, weighted by the squares of the quark charges.



It is observed that $F_1(x)$ and $F_2(x)$ are not independent:

Callan-Gross relation for spin $\frac{1}{2}$ constituents: $F_1(x) = \frac{1}{2x} F_2(x)$

In the 'naive' parton model:

$$
\begin{aligned}
u(x) &= u_v(x) + u_s(x) \\
d(x) &= d_v(x) + d_s(x) \\
u_v(x) &= 2 d_v(x) \\
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Using these one may obtain

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F_2(x) &= \sum e_i^2 \; x \; f_i(x) \\
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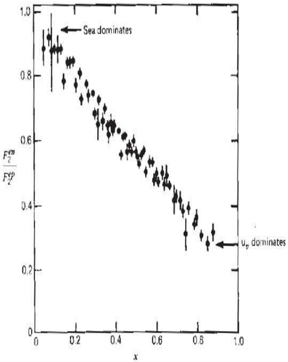

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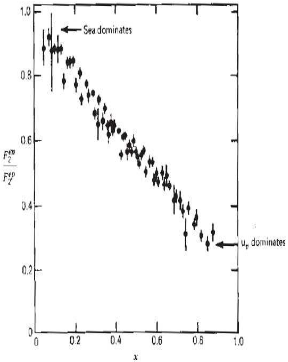


Valence quarks are dominant at large x and sea quarks at low x.

Missing momentum

Summing the measured momenta of partons should give the proton momentum

$$\int_x^1 dx\ x(u+\bar{u}+d+\bar{d}+s+\bar{s}) = 1-\epsilon_g$$

where $\epsilon_g = \int_x^1 dx\ x(q+\bar{q})$

Missing momentum

Experimental observation(neglecting strange quarks)

$$\int_0^1 dx\ F_2^{ep} = \frac{4}{9}\epsilon_u + \frac{1}{9}\epsilon_d = 0.18$$

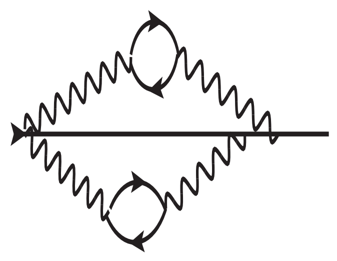
$$\int_0^1 dx\ F_2^{en} = \frac{1}{9}\epsilon_u + \frac{4}{9}\epsilon_u = 0.12$$

$$\epsilon_u = 0.36, \quad \epsilon_d = 0.12$$

Fraction of momentum not carried by quarks

$\epsilon_g = 1 - \epsilon_u - \epsilon_d = 0.46$

1. *In QED electrons are dressed.*
2. *Propagator is photon* $\frac{-ig^{\mu\nu}}{q^2}$



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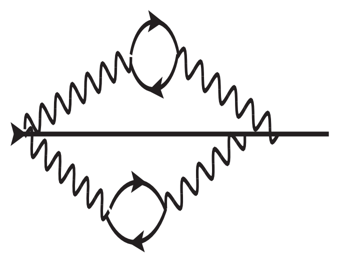
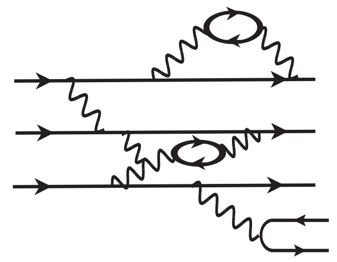
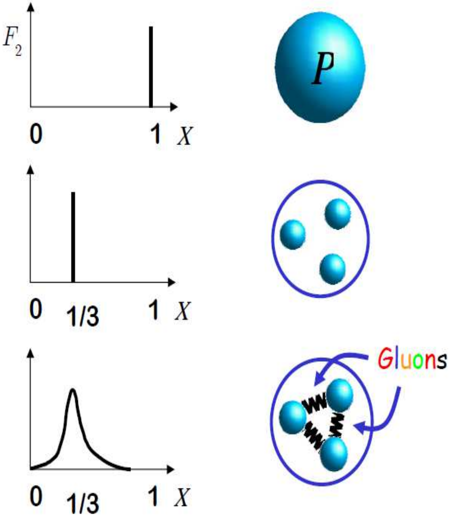


1. *In QCD gluons replace photons*
2. *Gluons bind the quarks inside hadron*





- *(a)* if proton is a point particle
- *(b)* 3 free valence quarks have discrete spectrum at $x = \frac{1}{3}$
- *(c)* Valence quarks bound with gluons have continuous spectrum peaked at $x = \frac{1}{3}$

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The structure function scales i.e. $F(x, Q^2) \longrightarrow F(x)$ *in the asymptotic (Bjorken) limit:* $Q^2 \longrightarrow \infty$

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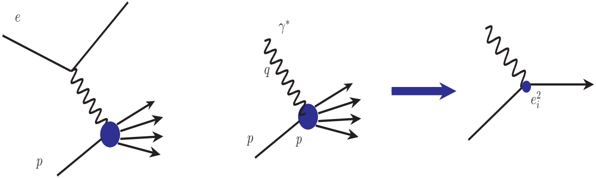

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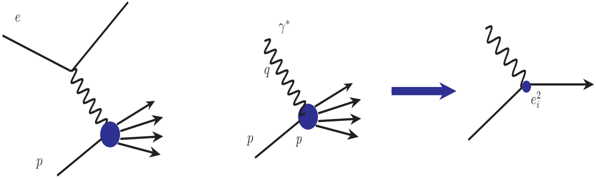
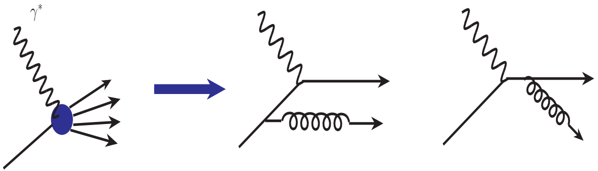


QCD extends the naive quark parton model by allowing interactions between the partons via the exchange of gluons.

Two possibilities may arise:

• Quark can radiate a gluon before or after being struck by the virtual photon i.e. $\gamma^* q \longrightarrow qg$.



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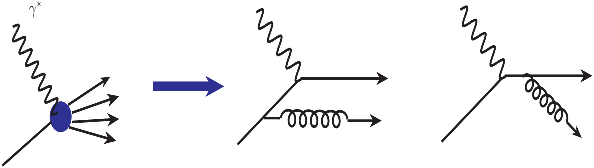
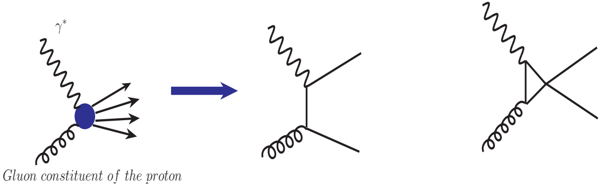


- Gluon constituents can contribute to DIS via $\gamma^* g \longrightarrow q\bar{q}$



Effect of gluons dynamics on structure functions:

Violation of scaling property of structure functions.

Outgoing quark will no longer be collinear with the virtual photon($p_T \neq 0$).

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Violation of scaling property of structure functions.

Outgoing quark will no longer be collinear with the virtual photon($p_T \neq 0$).

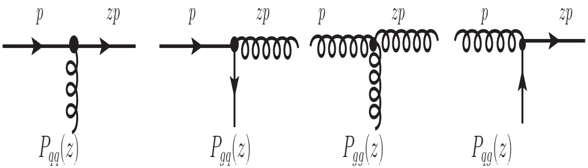
Considering $\gamma^* q \longrightarrow qg$, F_2 modifies to:

$$\frac{F_2(x)}{x} = \sum_q e_q^2 \int_x^1 \frac{dy}{y} q(y) \left[\delta\left(1 - \frac{x}{y}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{y}\right) log\left(\frac{Q^2}{\mu^2}\right) \right]$$

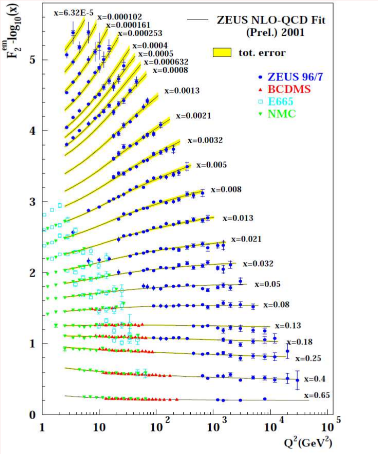
which introduces a logarithmic Q^2 dependence due to the gluon emission, violating the scaling behavior.

Splitting Functions P_{qq}

1. The effect of all interactions is described by splitting functions P_{qq}.
2. P_{qq} is the splitting function which represent the probability of a quark of momentum p to emit a gluon and become a quark with momentum $\frac{x}{y}.p(= zp)$
3. Splitting functions have been calculated from perturbative QCD.



Logarithmic variation of Q^2 in F_2



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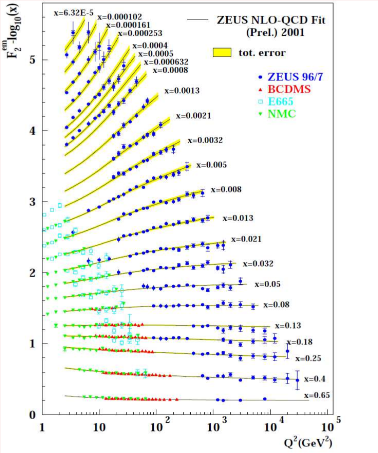

At large x, $F_2^{e.m.}$ decreases with Q^2, at small x, $F_2^{e.m.}$ increases with Q^2.

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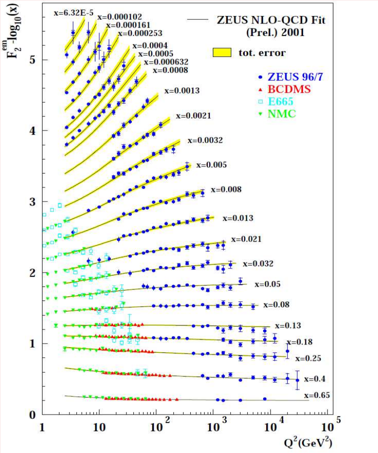

At large x, $F_2^{e.m.}$ decreases with Q^2, at small x, $F_2^{e.m.}$ increases with Q^2.

At Larger Q^2

- The probability of finding a quark at small x increases.
- The probability of finding a quark at large x decreases since high momentum quarks lose momentum by radiating gluons.

Re-expressing $\frac{F_2(x)}{x}$ as:

$$\frac{F_2(x)}{x} = \sum_q e_q^2 \int_x^1 \frac{dy}{y} \left[q(y) + \Delta q(y, Q^2)\right] \delta \left(1 - \frac{x}{y}\right)$$

where

$$\Delta q(y, Q^2) = \left[\frac{\alpha_s}{2\pi} \left(\frac{x}{y}\right) log \left(\frac{Q^2}{\mu^2}\right)\right] \int_x^1 \frac{dy}{y} q(y) P_{qq}(\frac{x}{y})$$

$$\frac{dq(x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} q(y, Q^2) P_{qq}(\frac{x}{y})$$

This is 'Altarelli-Parisi Evolution equation' which gives the Q^2 evolution of the quark.

Q^2 evolution of parton densities:

$$\frac{dq(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[q(y,Q^2)P_{qq}(\frac{x}{y}) + g(y,Q^2)P_{qg}(\frac{x}{y})\right]$$

Similarly, Q^2 evolution of gluon densities:

$$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[q(y,Q^2)P_{gq}(\frac{x}{y}) + g(y,Q^2)P_{gg}(\frac{x}{y})\right]$$

In general the splitting function can be expressed as a power series in α_s: $P_{ab} = P_{ab}^{LO} + \alpha_s P_{ab}^{NLO} + \alpha_s^2 P_{ab}^{NNLO} + ...$

Proton contains both quarks and gluons, so coupled DGLAP:

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$$\frac{d}{d\log Q^2}\begin{pmatrix} q \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q \\ g \end{pmatrix}$$

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which allows us to write

$$\frac{dq(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}(P_{ab}\otimes q)(x,Q^2)$$

Differential scattering cross section for the reaction $ep \longrightarrow eX$ in terms of dimensionless structure functions:

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$$\frac{d\sigma}{dE'\,d\Omega} = \frac{\alpha^2}{4E^2 sin^4(\frac{\theta}{2})}\left[F_2(x)\cos^2\frac{\theta}{2} + 2F_1(x)\sin^2\frac{\theta}{2}\right]$$

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Recall

$$x = \frac{Q^2}{2M\nu}, \quad y = \frac{\nu}{E}, \quad \nu = q_0 = E - E', \quad Q^2 \;=\; 4EE'\sin^2\frac{\theta}{2}$$

Using Jacobian identity

Differential scattering cross section may be expressed as:

$$\frac{d\sigma}{dxdy} = \frac{8\pi\alpha^2 ME}{Q^4}\left[\left(1 - y - \frac{Mxy}{2E}\right)F_2(x) + \frac{y^2}{2}2xF_1(x)\right]$$

Electromagnetic structure functions

$$
\begin{aligned}
F_2^{ep} &= \frac{4\ x}{9}\left(u_v + u_s + \bar{u}_s + c + \bar{c} + ...\right) + \frac{x}{9}\left(d_v + d_s + \bar{d}_s + s + \bar{s} + ...\right) \\
F_2^{en} &= \frac{x}{9}\left(u_v + u_s + \bar{u}_s + s + \bar{s} + ...\right) + \frac{4\ x}{9}\left(d_v + d_s + \bar{d}_s + c + \bar{c} + ...\right)
\end{aligned}
$$

Electromagnetic structure functions

$$
\begin{aligned}
F_2^{ep} &= \frac{4\ x}{9}\left(u_v + u_s + \bar{u}_s + c + \bar{c} + ...\right) + \frac{x}{9}\left(d_v + d_s + \bar{d}_s + s + \bar{s} + ...\right) \\
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\end{aligned}
$$

where $v \equiv$ valence quark and $s \equiv$ sea quark

These distribution functions are generally determined by using global QCD analysis with the inputs from various sets of experimental data specially obtained from DIS experiments revealing proton structure.

Data set	$N_{\text{pts.}}$
H1 MB 99 e^+p NC	8
H1 MB 97 e^+p NC	64
H1 low Q^2 96–97 e^+p NC	80
H1 high Q^2 98–99 e^-p NC	126
H1 high Q^2 99–00 e^+p NC	147
ZEUS SVX 95 e^+p NC	30
ZEUS 96–97 e^+p NC	144
ZEUS 98–99 e^-p NC	92
ZEUS 99–00 e^+p NC	90
H1 99–00 e^+p CC	28
ZEUS 99–00 e^+p CC	30
H1/ZEUS $e^\pm p$ F_2^{charm}	83
H1 99–00 e^+p incl. jets	24
ZEUS 96–97 e^+p incl. jets	30
ZEUS 98–00 $e^\pm p$ incl. jets	30
DØ II $p\bar{p}$ incl. jets	110
CDF II $p\bar{p}$ incl. jets	76
CDF II $W \to l\nu$ asym.	22
DØ II $W \to l\nu$ asym.	10
DØ II Z rap.	28
CDF II Z rap.	29

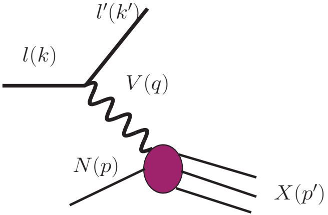
Data set	$N_{\text{pts.}}$
BCDMS μp F_2	163
BCDMS μd F_2	151
NMC μp F_2	123
NMC μd F_2	123
NMC $\mu n/\mu p$	148
E665 μp F_2	53
E665 μd F_2	53
SLAC ep F_2	37
SLAC ed F_2	38
NMC/BCDMS/SLAC F_L	31
E866/NuSea pp DY	184
E866/NuSea pd/pp DY	15
NuTeV νN F_2	53
CHORUS νN F_2	42
NuTeV νN xF_3	45
CHORUS νN xF_3	33
CCFR $\nu N \to \mu\mu X$	86
NuTeV $\nu N \to \mu\mu X$	84
All data sets	**2743**

- Red = New w.r.t. MRST 2006 fit.

Presently, parton distribution functions have been parametrized by many groups like:

pdfs	authors	arXiv
ABKM	S. Alekhin, J. Blümlein, S. Klein, S. Moch, and others	0908.3128, 0908.2766, ...
CTEQ	H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. Nadolsky, J. Pumplin, C.-P. Yuan, and others	1007.2241, 1004.4624, 0910.4183, 0904.2424, 0802.0007, ...
GJR	M. Glück, P. Jimenez-Delgado, E. Reya, and others	0909.1711, 0810.4274, ...
HERAPDF	H1 and ZEUS collaborations	1006.4471, 0906.1108, ...
MSTW	A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt	1006.2753, 0905.3531, 0901.0002, ...
NNPDF	R. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. Latorre, J. Rojo, M. Ubiali, and others	1005.0397, 1002.4407, 0912.2276, 0906.1958, ...

Neutrino nucleon scattering



$$\nu_l(k) + N(p) \to l^-(k') + X(p'),\ \ l = \ \ e,\ \mu,$$

Anti(neutrino)-nucleon double differential scattering cross section:

ν-N DCX

$$\frac{d^2\sigma^N_{\nu,\bar{\nu}}}{d\Omega' \, dE'} = \frac{{G_F}^2}{(2\pi)^2} \, \frac{|\vec{k}'|}{|\vec{k}|} \, \left(\frac{m_W^2}{q^2 - m_W^2} \right)^2 L^{\alpha\beta}_{\nu,\bar{\nu}} \ W^N_{\alpha\beta}\,,$$

Leptonic Tensor

$$L^{\alpha\beta} = k^{\alpha} k'^{\beta} + k^{\beta} k'^{\alpha} - k.k' g^{\alpha\beta} \pm i\epsilon^{\alpha\beta\rho\sigma} k_{\rho} k'_{\sigma}$$

The differential cross section:

$$\begin{aligned}\frac{d^2\sigma^{\nu(\bar\nu)}}{dx\ dy} &= \frac{G_F^2 M E_\nu}{\pi(1+Q^2/M_W^2)^2}\left(\left[y^2 x + \frac{m_l^2 y}{2E_\nu M}\right] F_1(x,Q^2)\right.\\ &+ \left[(1-\frac{m_l^2}{4E_\nu^2}) - (1+\frac{Mx}{2E_\nu})y\right] F_2(x,Q^2)\\ &\left.\pm \left[xy(1-\frac{y}{2}) - \frac{m_l^2 y}{4E_\nu M}\right] F_3(x,Q^2)\right)\end{aligned}$$

where

$$MW_1^N(\nu,Q^2) = F_1^N(x,Q^2);\ \ \nu W_2^N(\nu,Q^2) = F_2^N(x,Q^2);\ \ \nu W_3^N(\nu,Q^2) = F_3^N(x,Q^2)$$

Hadronic tensor

$$W_{\alpha\beta}^{N} = \left(\frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta}\right) W_1^{\nu(\bar\nu)} + \frac{1}{M^2}\left(p_\alpha - \frac{p.q}{q^2}\, q_\alpha\right)\left(p_\beta - \frac{p.q}{q^2}\, q_\beta\right) W_2^{\nu(\bar\nu)} - \frac{i}{2M^2}\epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma W_3^{\nu(\bar\nu)}$$

The differential cross section:

$$\frac{d^2\sigma^{\nu(\bar\nu)}}{dx\,dy} = \frac{G_F^2 M E_\nu}{\pi(1+Q^2/M_W^2)^2}\left(\left[y^2 x + \frac{m_l^2 y}{2E_\nu M}\right] F_1(x,Q^2) + \left[(1-\frac{m_l^2}{4E_\nu^2}) - (1+\frac{Mx}{2E_\nu})y\right] F_2(x,Q^2) \pm \left[xy(1-\frac{y}{2}) - \frac{m_l^2 y}{4E_\nu M}\right] F_3(x,Q^2)\right)$$

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The nucleon structure functions in terms of PDFs:

$$
\begin{aligned}
F_2^{\nu p} &= 2x[d(x)+s(x)+\bar{u}(x)+\bar{c}(x)], \\
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xF_3^{\nu p} &= 2x[d(x)+s(x)-\bar{u}(x)-\bar{c}(x)], \\
xF_3^{\nu n} &= 2x[u(x)+s(x)-\bar{d}(x)-\bar{c}(x)].
\end{aligned}
$$

Similarly for the antineutrino-nucleon scattering case:

$$
\begin{aligned}
F_2^{\bar{\nu} p} &= 2x[u(x)+c(x)+\bar{d}(x)+\bar{s}(x)], \\
F_2^{\bar{\nu} n} &= 2x[d(x)+c(x)+\bar{u}(x)+\bar{s}(x)], \\
xF_3^{\bar{\nu} p} &= 2x[u(x)+c(x)-\bar{d}(x)-\bar{s}(x)], \\
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\end{aligned}
$$

If we look inside the nucleus

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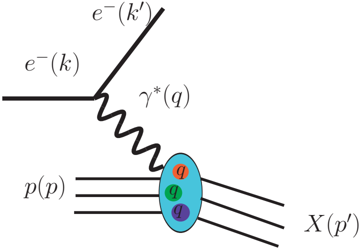

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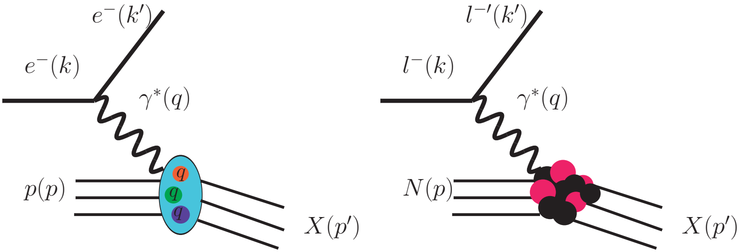

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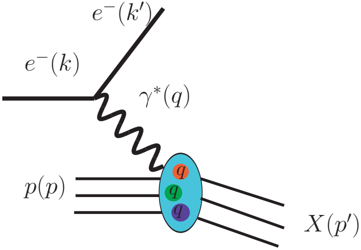

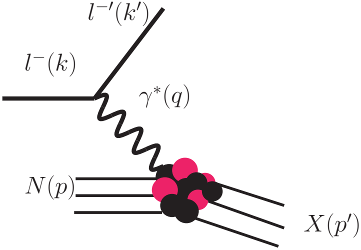

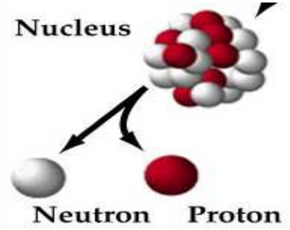

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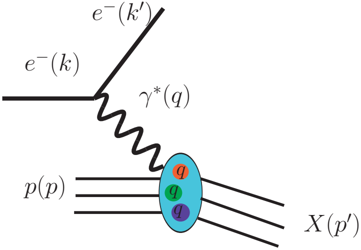

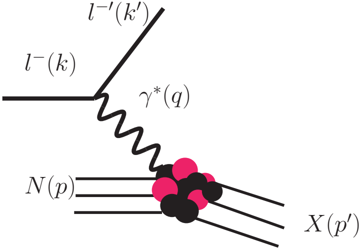

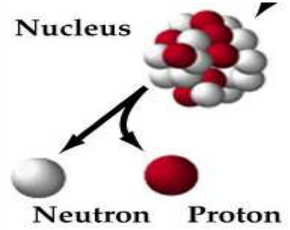

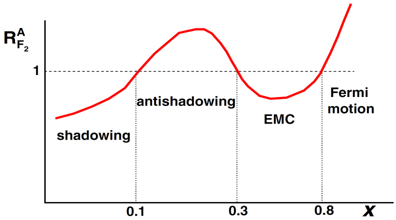

Phenomenological Efforts

The present understanding of the nuclear medium effects in DIS is mainly based on charged lepton-nucleus DIS data.

In some theoretical analysis, NME have been phenomenologically described in terms of a few parameters which are determined by using $l^{\pm}$-A, $l^{\pm}$-A and DY or $l^{\pm}$-A, DY and $\nu/\bar{\nu}$-A scattering data.

Paukkunen and Salgado:JHEP2010: "find no apparent disagreement with the nuclear effects in neutrino DIS and those in charged lepton DIS."

CTEQ-Grenoble-Karlsruhe collaboration "observed that the nuclear corrections in ν-A DIS are indeed incompatible with the predictions derived from $l^{\pm}$-A DIS and DY data"

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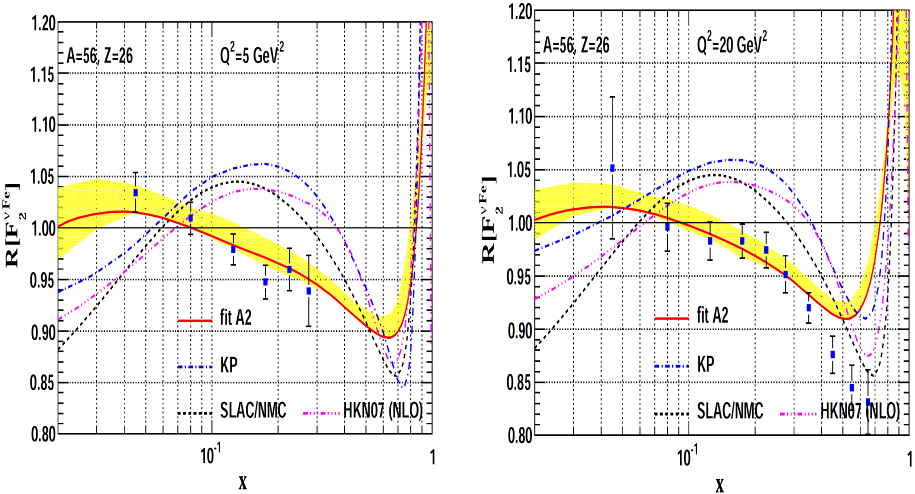
Phenomenological group	data types used
EKS98	l+A DIS, p+A DY
HKM	l+A DIS
HKN04	l+A DIS, p+A DY
nDS	l+A DIS, p+A DY
EKPS	l+A DIS, p+A DY
HKN07	l+A DIS, p+A DY
EPS08	l+A DIS, p+A DY, $h^{\pm}$,π^0,$\pi^{\pm}$ in d+Au
EPS09	l+A DIS, p+A DY, π^0 in d+Au
nCTEQ	l+A DIS, p+A DY
nCTEQ	l+A and ν+A DIS, p+A DY
DSSZ	l+A and ν+A DIS, p+A DY, π^0,$\pi^{\pm}$ in d+Au



J G Morfin J. of Physics: Conf. Ser. 408 (2013) 012054;
Kovarik et al. Phys.Rev.Lett. 106 (2011) 122301

Eskola

- Expression used by Eskola et al. analysis:

$$f_i^A(x, Q^2) = R_i^A(x, Q^2) f_i(x, Q^2)$$

$R_i^A(x, Q^2)$ is the nuclear modification to the free proton PDF.
$f_i(x, Q^2$ parton distribution function in nucleon.

Hirai

- In their analysis the NPDFs are expressed as

$$f_i^A(x, Q_0^2) = w_i(x, A, Z) f_i(x, Q_0^2)$$

$f_i^A(x, Q_0^2)$ and $f_i(x, Q_0^2)$ are the type-i NPDF and nucleonic PDF. w_i is the weight function that shows the nuclear modification for the type i parton distribution.

Assumed $s = \bar{s}$ and $\bar{u}_v^A = \bar{d}_v^A = \bar{s}_v^A$

Tzanov

In NuTeV analysis Tzanov et al. obtained the nuclear correction factor of the form

$$f(x) = 1.10 - 0.36x - 0.28e^{-21.9x} + 2.77x^{14.4}$$

from a fit to charged-lepton scattering on nuclear targets. The correction is independent of Q^2 and is small at intermediate x but is large at low and high x.

We have incorporated the following NME in the present calculation

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5. Shadowing and antishadowing

Medium effects in lepton-A scattering

- Kinematic effect which arises as the struck nucleon is not at rest but is moving with a Fermi momentum in the rest frame of the nucleus.
- Dynamic effect which arises due to the strong interaction of the initial nucleon in the nuclear medium.

In nuclear medium for em interaction the expression for the cross section is written as:

$$\frac{d^2\sigma^A}{d\Omega' dE'} = \frac{\alpha^2}{q^4} \frac{|\vec{k}'|}{|\vec{k}|} L^{\mu\nu} W^A_{\mu\nu},$$

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Nuclear hadronic tensor:

$$
\begin{aligned}
W^{A}_{\mu\nu} \quad &= \quad \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\, W^A_1(\nu, Q^2) \\
&\quad + \frac{W^A_2(\nu, Q^2)}{M_A^2}\left(p_\mu - \frac{p.q}{q^2}\, q_\mu\right)\left(p_\nu - \frac{p.q}{q^2}\, q_\nu\right)
\end{aligned}
$$

$W^A_i(\nu, Q^2)$ are redefined as:

$$
\begin{aligned}
M_A\, W^A_1(\nu, Q^2) \quad &= \quad F^A_1(x) \\
\nu\, W^A_2(\nu, Q^2) \quad &= \quad F^A_2(x)
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Nuclear hadronic tensor:

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W_{\mu\nu}^{A} &= \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right) W_1^A(\nu, Q^2) \\
&\quad + \frac{W_2^A(\nu, Q^2)}{M_A^2}\left(p_\mu - \frac{p.q}{q^2}\, q_\mu\right)\left(p_\nu - \frac{p.q}{q^2}\, q_\nu\right)
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Theoretical efforts

- A relativistic nucleon spectral function is used to describe the momentum distribution of nucleons in nuclei.
- The spectral function has been calculated using Lehmann's representation for the relativistic nucleon propagator.
- Nuclear many body theory is used to calculate it for an interacting Fermi sea in nuclear matter.
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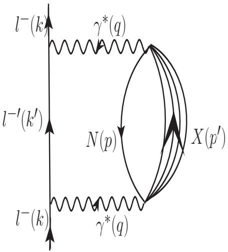
We obtain cross section as:

$$d\sigma = \frac{-2m}{E_l(\mathbf{k})} Im\Sigma(k) \frac{E_l(\mathbf{k})}{\mid \mathbf{k} \mid} d^3 r,$$

Thus to get $d\sigma$ for (l, l') scattering on the nucleus, we are required to evaluate imaginary part of lepton self energy $Im\Sigma(k)$.



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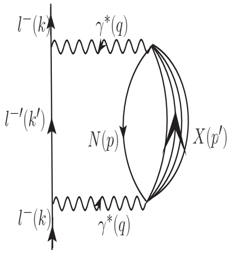

Lepton self energy $\Sigma(k)$ is written as:

$$-i\Sigma(k) \quad = \quad \int \frac{d^4q}{(2\pi)^4} \bar{u}_l(\vec{k})$$

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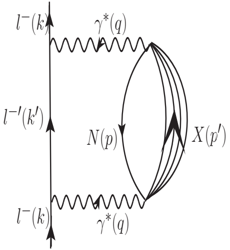

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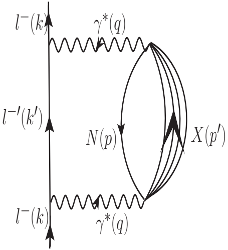

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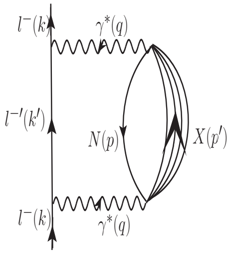
$$-i\Sigma(k) \quad = \quad \int \frac{d^4q}{(2\pi)^4} \bar{u}_l(\vec{k}) \; ie\gamma^\mu \; i\frac{\not{k}' + m}{k'^2 - m^2 + i\epsilon}$$

We obtain cross section as:

$$d\sigma = \frac{-2m}{E_l(\mathbf{k})} Im\Sigma(k) \frac{E_l(\mathbf{k})}{\mid \mathbf{k} \mid} d^3 r,$$

Thus to get $d\sigma$ for (l, l') scattering on the nucleus, we are required to evaluate imaginary part of lepton self energy $Im\Sigma(k)$.



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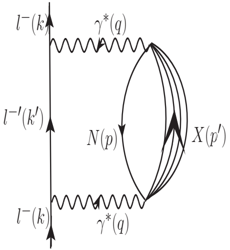

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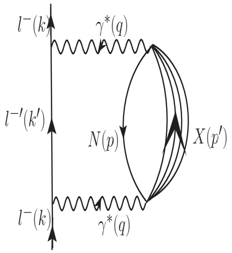

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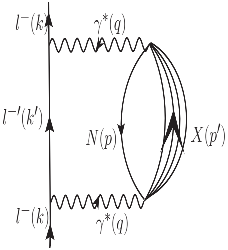

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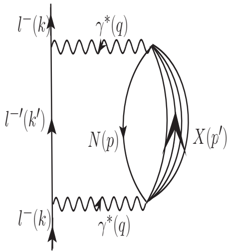

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Re-expressing Lepton self energy as:

$$\Sigma(k) = ie^2 \int \frac{d^4 q}{(2\pi)^4} \, \frac{1}{q^4} \, \frac{1}{2m} \, L_{\mu\nu} \, \frac{1}{k'^2 - m^2 + i\epsilon} \, \Pi^{\mu\nu}(q),$$

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Apply Cutkosky rules

$$\begin{array}{lcl}
\Sigma(k) & \rightarrow & 2i \; Im\Sigma(k) \\
D(k') & \rightarrow & 2i\theta(k'^0) \; ImD(k') \text{ (boson propagator)} \\
\Pi^{\mu\nu}(q) & \rightarrow & 2i\theta(q^0) \; Im\Pi^{\mu\nu}(q) \\
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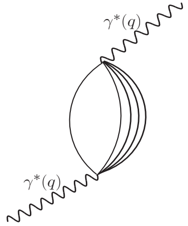
Thus we obtain:

$$Im\Sigma(k) = e^2 \int \frac{d^3 q}{(2\pi)^3} \, \frac{1}{2E_l} \theta(q^0) \, Im(\Pi^{\mu\nu}) \frac{1}{q^4} \frac{1}{2m} \, L_{\mu\nu}$$



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$$\Pi^{\mu\nu}(q) = e^2 \int \frac{d^4p}{(2\pi)^4} G(p) \sum_X \sum_{s_p, s_l} \prod_{i=1}^{N} \int \frac{d^4p'_i}{(2\pi)^4} \prod_l G_l(p'_l) \prod_j D_j(p'_j)$$

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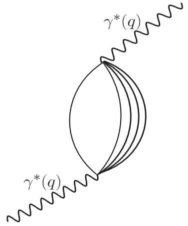

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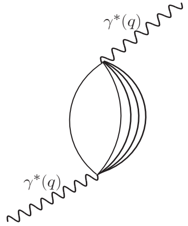

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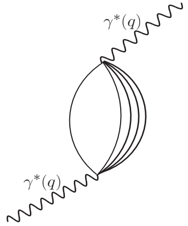

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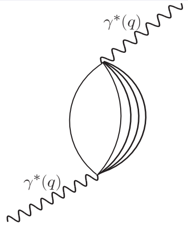

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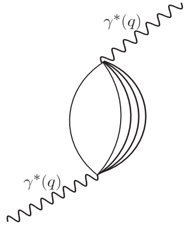

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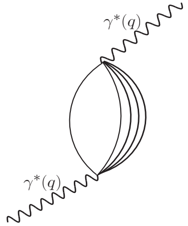

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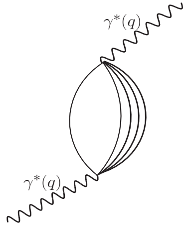

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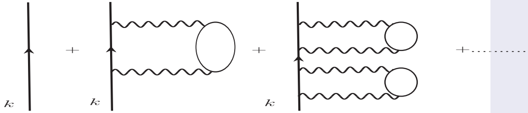
Relativistic Dirac propagator $G^0(p_0, \vec{p})$ for a free nucleon:

$$G^0(p_0, \vec{p}) = \frac{M}{E(\vec{p})} \left\{ \frac{\sum_r u_r(p) \bar{u}_r(p)}{p^0 - E(\vec{p}) + i\epsilon} + \frac{\sum_r v_r(-p) \bar{v}_r(-p)}{p^0 + E(\vec{p}) - i\epsilon} \right\}$$

The nucleon propagator in the interacting Fermi sea is obtained by making a perturbative expansion of $G(p^0, p)$ in terms of $G^0(p^0, p)$ by retaining the positive energy contributions only:



$$G(p_0, \vec{p}) = \frac{M}{E(\vec{p})} \frac{\sum_r u_r(p) \bar{u}_r(p)}{(p^0 - E(\vec{p}) + i\epsilon)} + \left(\frac{M}{E(\vec{p})} \right)^2 \frac{1}{(p^0 - E(\vec{p}) + i\epsilon)} \sum \frac{\sum_r u_r(p) \bar{u}_r(p)}{(p^0 - E(\vec{p}) + i\epsilon)} +$$

$$= \frac{M}{E(\vec{p})} \frac{\sum_r u_r(p) \bar{u}_r(p)}{\left(p^0 - E(\vec{p}) + i\epsilon \frac{M}{E(\vec{p})} \sum \right)}$$

This allows us to write the relativistic nucleon propagator in a nuclear medium in terms of the spectral functions of holes and particles as:

$$G(p^0,\vec{p}) = \frac{M}{E(\vec{p})} \sum_r u_r(\vec{p})\bar{u}_r(\vec{p}) \left[\int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,\vec{p})}{p^0-\omega-i\epsilon} + \int_{\mu}^{\infty} d\omega \frac{S_p(\omega,\vec{p})}{p^0-\omega+i\epsilon}\right]$$

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for $p^0 \leq \mu$

$$S_h(p^0,\vec{p}) = \frac{1}{\pi} \frac{\frac{M}{E(\vec{p})} Im\Sigma(p^0,\vec{p})}{(p^0 - E(\vec{p}) - \frac{M}{E(\vec{p})} Re\Sigma(p^0,\vec{p}))^2 + (\frac{M}{E(\vec{p})} Im\Sigma(p^0,\vec{p}))^2}$$

for $p^0 > \mu$

$$S_p(p^0,\vec{p}) = -\frac{1}{\pi} \frac{\frac{M}{E(\vec{p})} Im\Sigma(p^0,\vec{p})}{(p^0 - E(\vec{p}) - \frac{M}{E(\vec{p})} Re\Sigma(p^0,\vec{p}))^2 + (\frac{M}{E(\vec{p})} Im\Sigma(p^0,\vec{p}))^2}$$

Nuclear hadronic tensor:

In the LDA, the nuclear hadronic tensor can be written as a convolution of nucleonic hadronic tensor with the hole spectral function

$$W^A_{\alpha\beta} = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 \, \frac{M}{E(\vec{p})} S_h(p^0, \vec{p}, \rho(r)) \, W^N_{\alpha\beta}(p, q)$$

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$$W^N_{xx} = \left(\frac{q_x q_x}{q^2} - g_{xx}\right) \; W^N_1 + \frac{1}{M^2}\left(p_x - \frac{p.q}{q^2}\; q_x\right)\left(p_x - \frac{p.q}{q^2}\; q_x\right) \; W^N_2$$

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Chosing $\vec{q}$ along the z-axis

$$W^N_{xx}(\nu_N, Q^2) = W^N_1(\nu_N, Q^2) + \frac{1}{M^2} p_x^2 \, W^N_2(\nu_N, Q^2)$$

Similarily taking xx component of nuclear hadronic tensor

$$W_{xx}^{A}(\nu_A, Q^2) = W_1^{A}(\nu_A, Q^2) = \frac{F_1^{A}(x_A)}{AM}$$

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$$W^A_{xx}(\nu_A, Q^2) = W^A_1(\nu_A, Q^2) = \frac{F^A_1(x_A)}{AM}$$

$F_1(x) = M \ W_1(\nu, Q^2), \quad F_2(x) = \nu \ W_2(\nu, Q^2)$

$$\frac{F^A_1(x_A)}{AM} = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\vec{p})} \int_{-\infty}^{\mu} dp^0 S_h(p^0, \vec{p}, \rho(\vec{r})) \times \left[\frac{F^N_1(x_N)}{M} + \frac{1}{M^2} {p_x}^2 \frac{F^N_2(x_N)}{\nu}\right]$$

To obtain $F_2^A(x, Q^2)$

$$W^N_{zz} = \left(\frac{q_z^2}{q^2} - g_{zz}\right) W_1^N + \frac{1}{M^2}\left(p_z - \frac{p.q}{q^2}\, q_z\right)^2 W_2^N$$
$$= \frac{q_0^2}{q^2} W_1^N + \frac{1}{M^2}\left(\frac{(p_z q^2 - p.q\; q_z)^2}{q^4}\right) W_2^N$$

$$W^A_{zz}(\nu_A, Q^2) = \left(\frac{q_z^2}{q^2} - g_{zz}\right) W_1^A + \frac{1}{M_A^2}\left(-\frac{p_A \cdot q}{q^2}\, q_z\right)^2 W_2^A$$

$$F_2(x) = \nu W_2(\nu, Q^2)$$

Finally, we obtain the expression for nuclear nuclear structure function $F_2^A(x_A)$

$$F_2^A(x_A) = 2\sum_{p,n}\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\vec{p})}\int_{-\infty}^{\mu} dp^0\, S_h^{p,n}(p^0, \vec{p}, \rho_{p,n}(\vec{r}))\, F_2^N(x_N)\, C$$

$$C = \left[\frac{Q^2}{q_z^2}\left(\frac{p^2 - p_z^2}{2M^2}\right) + \frac{(p.q)^2}{M^2\nu^2}\left(\frac{p_z\, Q^2}{p.qq_z} + 1\right)^2 \frac{q_0\, M}{p_0\, q_0 - p_z\, q_z}\right]$$

Taking xy component nucleonic hadronic tensor:

$$W_{xy}^N = \frac{p_x p_y}{M^2} W_2^N + \frac{i}{2M^2}\left(p_z q^0 - p^0 q_z\right) W_3^N$$

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Taking xy component nucleonic hadronic tensor:

$$W^N_{xy} = \frac{p_x p_y}{M^2} W^N_2 + \frac{i}{2M^2}\left(p_z q^0 - p^0 q_z\right) W^N_3$$

Taking xy component of nuclear hadronic tensor:

$$W^A_{xy} = -\frac{i}{2M_A^2}\epsilon_{xy\rho\sigma} p^\rho q^\sigma W^A_3 = -\frac{i}{2M_A} q_z W^A_3$$

One may write W^A_3 in term W^N_3:

$$-\frac{i}{2M_A} q_z W^A_3 = 4\int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 \frac{M}{E(\vec{p})} S_h(p^0, \vec{p}, \rho(r)) \left[\frac{p_x p_y}{M^2} W^N_2 + \frac{i}{2M^2}\left(p_z q^0 - p^0 q_z\right)\right] W^N_3$$

Using $\nu W_3^{\nu(\bar{\nu})}(\nu, Q^2) = F_3^{\nu(\bar{\nu})}(x, Q^2)$, we obtain

$$F_3^A(x_A, Q^2) = 4\int d^3r \int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\vec{p})}\int_{-\infty}^{\mu} dp^0 S_h(p^0, \mathbf{p}, \rho(\mathbf{r})) \times \frac{p^0\gamma - p_z}{(p^0 - p_z\gamma)\gamma} F_3^N(x_N, Q^2)$$

π and ρ mesons contributions

"Significant at low-x and mid-x"

1. There are virtual mesons associated with each nucleon bound inside the nucleus.
2. These meson clouds get strengthened by the strong attractive nature of nucleon-nucleon interactions.
3. This leads to an increase in the interaction probability of virtual photons with the meson cloud.
4. The effect of meson cloud is more pronounced in heavier nuclear targets and dominate in the intermediate region of x($0.2<x<0.6$).

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2. These meson clouds get strengthened by the strong attractive nature of nucleon-nucleon interactions.
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π and ρ mesons contributions

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For mesons cloud contribution

$$2\pi \frac{M}{E(\vec{p})} S_h(p_0, \vec{p})\, W_N^{\alpha\beta}(p,q) \to 2 Im D(p) \theta(p_0)\, W_\pi^{\alpha\beta}(p,q)$$

Pion propagator in the nuclear medium

$$D(p) = [{p_0}^2 - \vec{p}^{\,2} - m_\pi^2 - \Pi_\pi(p_0, \vec{p})]^{-1}$$

with

$$\Pi_\pi = \frac{f^2/m_\pi^2 F^2(p) \vec{p}^{\,2} \Pi^*}{1 - f^2/m_\pi^2\, V_L' \Pi^*}$$

πNN form factor

$$F(p) = (\Lambda^2 - m_\pi^2)/(\Lambda^2 + \vec{p}^{\,2})$$

Similar to nucleonic case

$$W^{\mu\nu}_{A,\pi} = 3 \int d^3r \int \frac{d^4p}{(2\pi)^4}\, \theta(p_0)(-2)\; ImD(p)\; 2m_\pi\, W^{\mu\nu}_{\pi}(p,q)$$

Factor 3 $\Rightarrow$ Three Charged states of pion

For pion excess in nuclear medium

$$ImD(p) \;\rightarrow\; \delta ImD(p) \equiv ImD(p) - \rho\, \frac{\partial ImD(p)}{\partial \rho}\,|_{\rho=0}$$

which leads to

$$\begin{aligned} F^{A}_{1,\pi}(x_\pi) \;&=\; -6AM \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\; \delta ImD(p)\; 2m_\pi \;\times \\ &\left[\frac{F_{1\pi}(x_\pi)}{m_\pi} + \frac{|\vec{p}|^2 - p_z^2}{2(p_0\, q_0 - p_z q_z)} \frac{F_{2\pi}(x_\pi)}{m_\pi} \right] \end{aligned}$$

Contribution from rho meson

Propagator for *rho* meson

$$D_\rho(p) = [{p_0}^2 - \vec{p}^{\,2} - m_\rho^2 - \Pi_\rho^*(p_0, \vec{p})]^{-1}$$

with irreducible ρ self-energy

$$\Pi_\rho^* = \frac{f^2/m_\rho^2\, C_\rho F_\rho^2(p)\vec{p}^{\,2}\Pi^*}{1 - f^2/m_\rho^2\, V_T'\Pi^*}$$

ρNN form factor

$$F_\rho(p) = (\Lambda_\rho^2 - m_\rho^2)/(\Lambda_\rho^2 + \vec{p}^{\,2})$$

Finally,

$$F_{1,\rho}^A(x_\rho) \;=\; -12AM\int d^3r \int \frac{d^4p}{(2\pi)^4}\theta(p_0)\;\delta ImD_\rho(p)\;2m_\rho \times \left[\frac{F_{1\rho}(x_\rho)}{m_\rho} + \frac{|\vec{p}|^2 - p_z^2}{2(p_0\;q_0 - p_z\;q_z)}\frac{F_{2\rho}(x_\rho)}{m_\rho}\right]$$

Structure functions for π and ρ mesons:

$$F^A_{2,\pi}(x_\pi) \quad = \quad -6 \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\ \delta ImD(p)\, 2m_\pi \ \frac{m_\pi}{p_0 \ - \ p_z\ \gamma} C_1 F_{2\pi}(x_\pi)$$

$$C_1 = \frac{Q^2}{q_z^2}\left(\frac{|\vec{p}|^2 \ - \ p_z^2}{2m_\pi^2}\right) + \frac{(p_0 \ - \ p_z\ \gamma)^2}{m_\pi^2}\left(\frac{p_z\ Q^2}{(p_0 \ - \ p_z\ \gamma)\, q_0\, q_z} \ + \ 1\right)^2$$

$$F^A_{2,\rho}(x_\rho) \quad = \quad -12 \int d^3r \int \frac{d^4p}{(2\pi)^4} \theta(p_0)\ \delta ImD_\rho(p)\, 2m_\rho \ \frac{m_\rho}{p_0 \ - \ p_z\ \gamma} C_2 F_{2\rho}(x_\rho)$$

$$C_2 = \frac{Q^2}{q_z^2}\left(\frac{|\vec{p}|^2 \ - \ p_z^2}{2m_\rho^2}\right) + \frac{(p_0 \ - \ p_z\ \gamma)^2}{m_\rho^2}\left(\frac{p_z\ Q^2}{(p_0 \ - \ p_z\ \gamma)\, q_0\, q_z} \ + \ 1\right)^2$$

Shadowing and antishadowing effects

"Significant at low-x and low-Q^2"

1. The shadowing suppression at small x occurs due to coherent multiple scattering inside the nucleus of a quark-anti quark pair coming from the virtual boson with destructive interference of the amplitudes.
2. The shadowing effect is important at low x and low Q^2 .
3. The anti-shadowing effect is due to constructive interference of the multiple scattering amplitudes.
4. This effect is also important at low x but greater than x region of shadowing.

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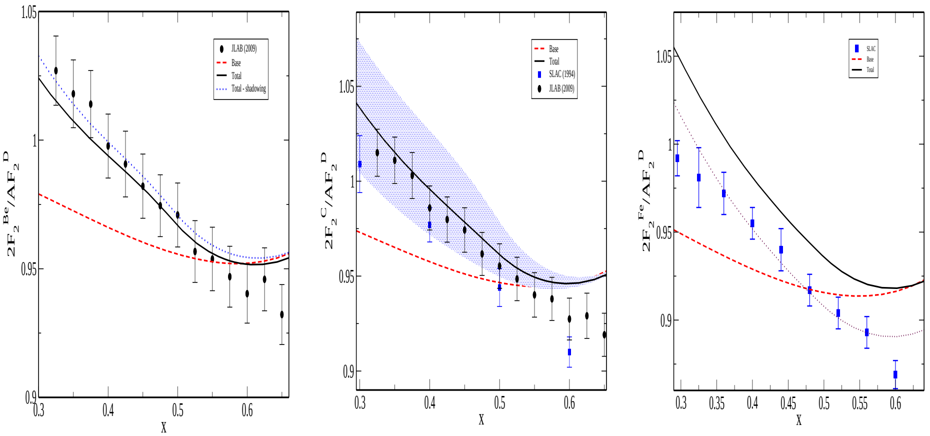
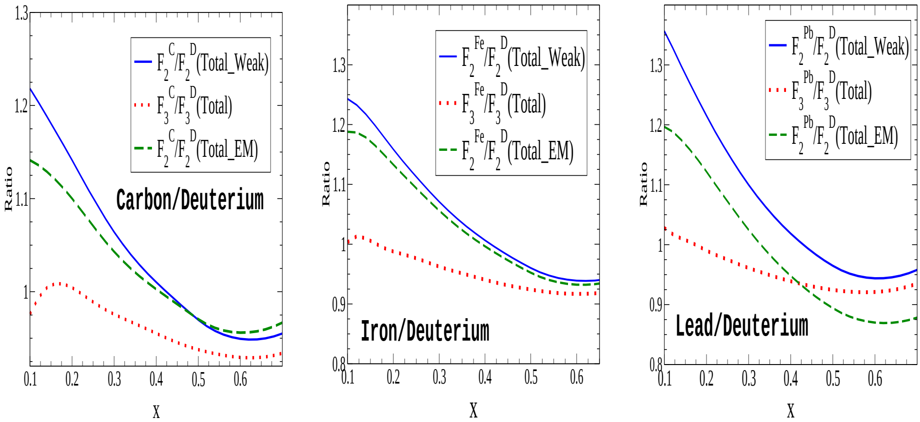
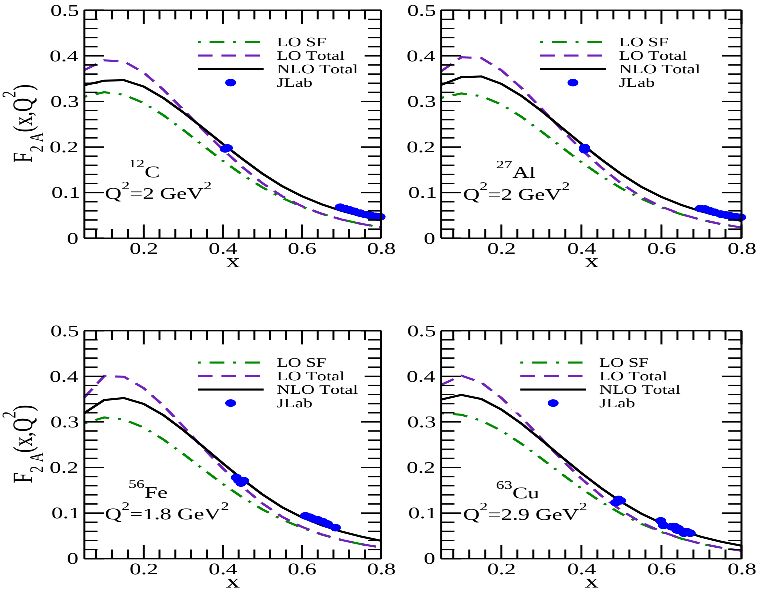
Electromagnetic Nuclear Structure Function

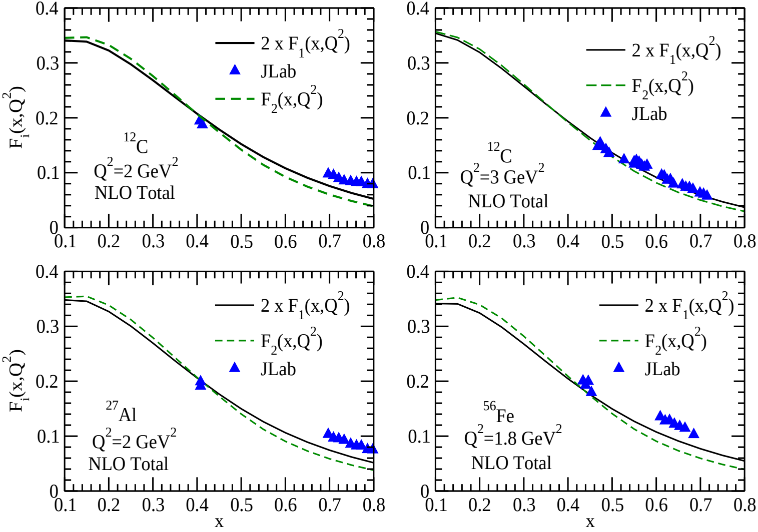
$\frac{2F_2^A}{AF_2^D}(A = Be, C, Fe)$ vs x



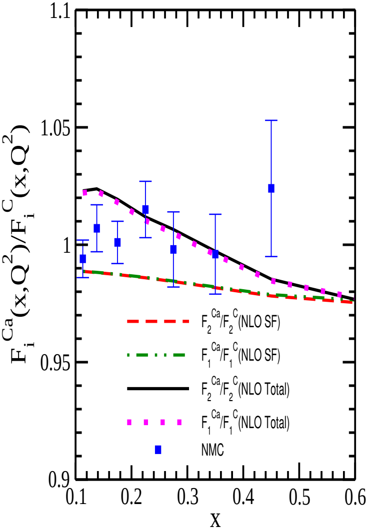
Ratio of Structure functions in Weak and E.M. cases

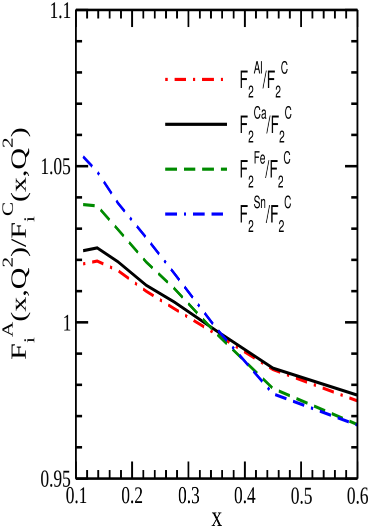



LO SF
LO Total
NLO Total
JLab
$F_{2A}(x,Q^2)$
x
^{12}C
Q^2=2 GeV^2
^{27}Al
Q^2=2 GeV^2
^{56}Fe
Q^2=1.8 GeV^2
^{63}Cu
Q^2=2.9 GeV^2

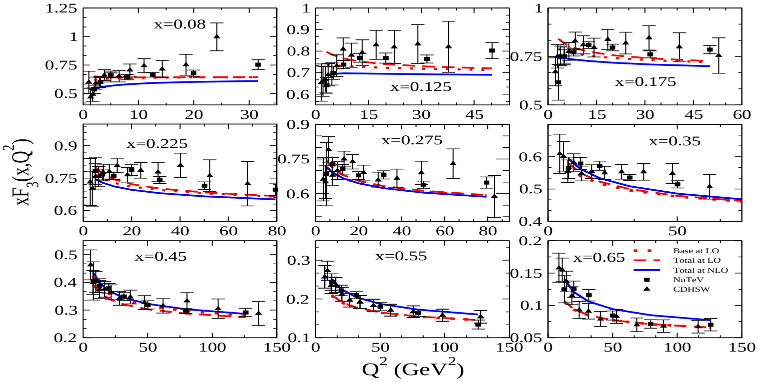

2 x F1(x,Q2)
JLab
F2(x,Q2)
12C
Q2=2 GeV2
NLO Total
12C
Q2=3 GeV2
NLO Total
27Al
Q2=2 GeV2
NLO Total
56Fe
Q2=1.8 GeV2
NLO Total
Fi(x,Q2)
x

Nuclear dependence in $\frac{F_i^A(x,Q^2)}{F_i^C(x,Q^2)}$





xF_3 vs Q^2 ^{56}Fe




DOMO ARIGATO
GOZAIMASU

Backup

The cross section for an element of volume dV in the nucleus is related to the probability per unit time (Γ) of the lepton interacting with the nucleons and is written as:

$$d\sigma \;=\; \Gamma\, dt dS = \Gamma \frac{dt}{dl} dS dl = \Gamma \frac{1}{v} dV = \Gamma \frac{E_l}{\mid \mathbf{k} \mid} dV = \Gamma \frac{E_l}{\mid \mathbf{k} \mid} d^3 r,$$

dl is the length of the interaction, $v(=\frac{dl}{dt})$ is the velocity of the incoming lepton and we have used $\mathbf{k} = \mathbf{v} E_l$.

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Also probability per unit time of the lepton interacting with the nucleons in the medium to give the final state is related to the imaginary part of the lepton self energy i.e.

$$-\frac{\Gamma}{2} = \frac{m}{E_l(\mathbf{k})} Im\Sigma$$

Local Density Approximation

In the local density approximation reaction takes place at a point r, lying inside a volume d^3r with local density $\rho_p(r)$ and $\rho_n(r)$ corresponding to the proton and neutron densities

$$\rho_p(r) = \frac{Z}{A}\rho(r), \quad \rho_n(r) = \frac{A-Z}{A}\rho(r)$$

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Fermi momentum of the nucleon is

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This leads to the spectral functions for the protons and neutrons to be the function of local Fermi momentum given by

$$2\int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega, \vec{p}, p_{F_{p,n}}(\vec{r}))\ d\omega = \rho_{p,n}(\vec{r})$$

$$4\int d^3r \int \frac{d^3p}{(2\pi)^3}\int_{-\infty}^{\mu} S_h(\omega, \vec{p}, \rho(r))\ d\omega = A$$