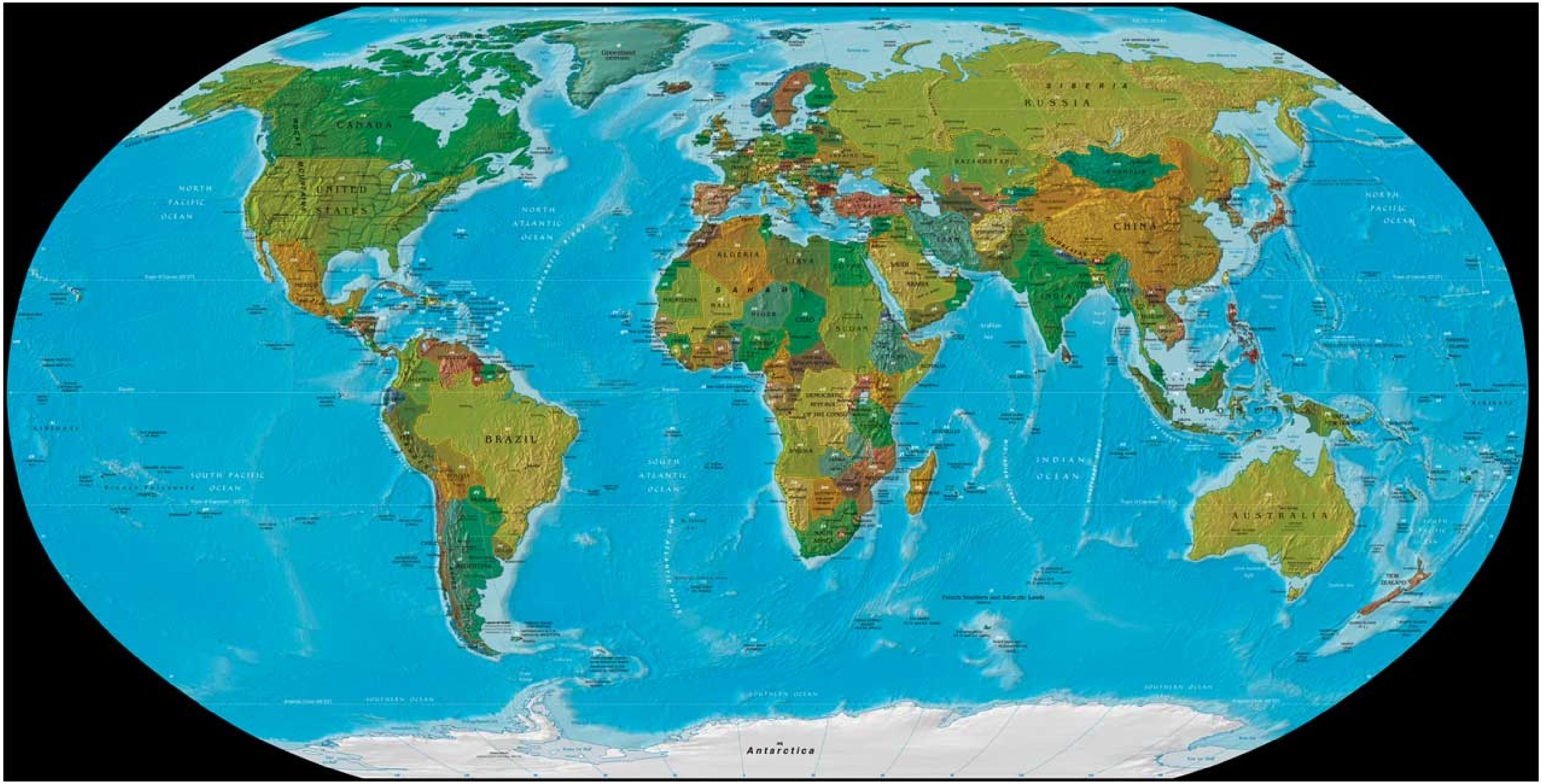
岡山市

Electroweak interactions on the nucleon

Luis Alvarez Ruso



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Outline

- General Introduction
- Electroweak interactions in the Standard Model
- Strong interactions in the Standard Model
- Inclusive neutrino-nucleon(nucleus) cross section
- Derivation of the nucleon EW current. Flavor structure. Form factors.
- Analysis of the CCQE and NCE cross sections
- Electroweak excitation of baryon resonances

Introduction

- Neutrino interactions with matter are at the heart many interesting and relevant physical processes
  - Astrophysics
    - Dynamics of the core-collapse in supernovae
    - r-process nucleosynthesis
  - Hadronic physics
    - Nucleon and Nucleon-Resonance (N-Δ, N-N*) axial form factors
    - Strangeness content of the nucleon spin
  - Nuclear physics
    - Information about: nuclear correlations, MEC, spectral functions
    - Complement electron scattering studies
  - Beyond Standard Model
    - Non-standard ν interactions

Introduction

- Neutrino interactions with matter are at the heart of all experiments seeking to unravel its nature.

- Oscillation experiments (with accelerator ν in the few-GeV region)
  - Good understanding of neutrino interactions are important for:
    - ν detection, E_ν reconstruction, ν flux calibration
    - determination of (irreducible) backgrounds
    - reduction of systematic errors
    - needed in the quest for CP violation and ν mass hierarchy
  - Near detectors help to reduce systematic errors but:
  - ND vs FD:
    - exposed to different fluxes with different flavor composition
    - different targets
  - All modern experiments are performed with nuclear targets
    - nuclear effects: essential for the interpretation of the data

Relevance for oscillation experiments

- (Kinematic) E_ν reconstruction:

$$E_\nu = \frac{2 m_n E_\mu - m_\mu^2 - m_n^2 + m_p^2}{2(m_n - E_\mu + p_\mu \cos\theta_\mu)}$$

  - Important for oscillations: $P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta_{23} \, \sin^2 \frac{\Delta m_{23}^2 L}{4 E_\nu}$

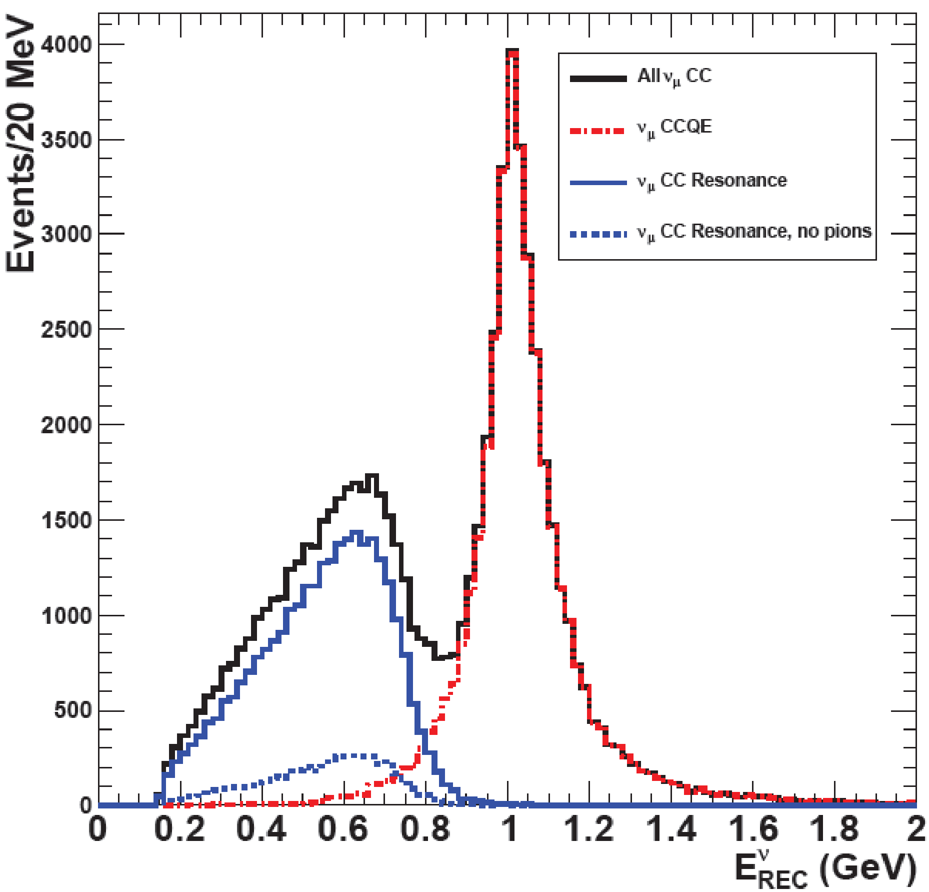
Relevance for oscillation experiments

- (Kinematic) E_ν reconstruction:

$$E_\nu = \frac{2m_n E_\mu - m_\mu^2 - m_n^2 + m_p^2}{2(m_n - E_\mu + p_\mu \cos\theta_\mu)}$$

  - Not exact on nuclear targets
  - CCQE-like events from
    - absorbed pions
    - 2p2h
    - ...

GENIE E_ν = 1 GeV



Relevance for oscillation experiments

- (Calorimetric) E_ν reconstruction (e.g. MINOS)
  - $E_\nu = E_{lep} + E_{had}$

but

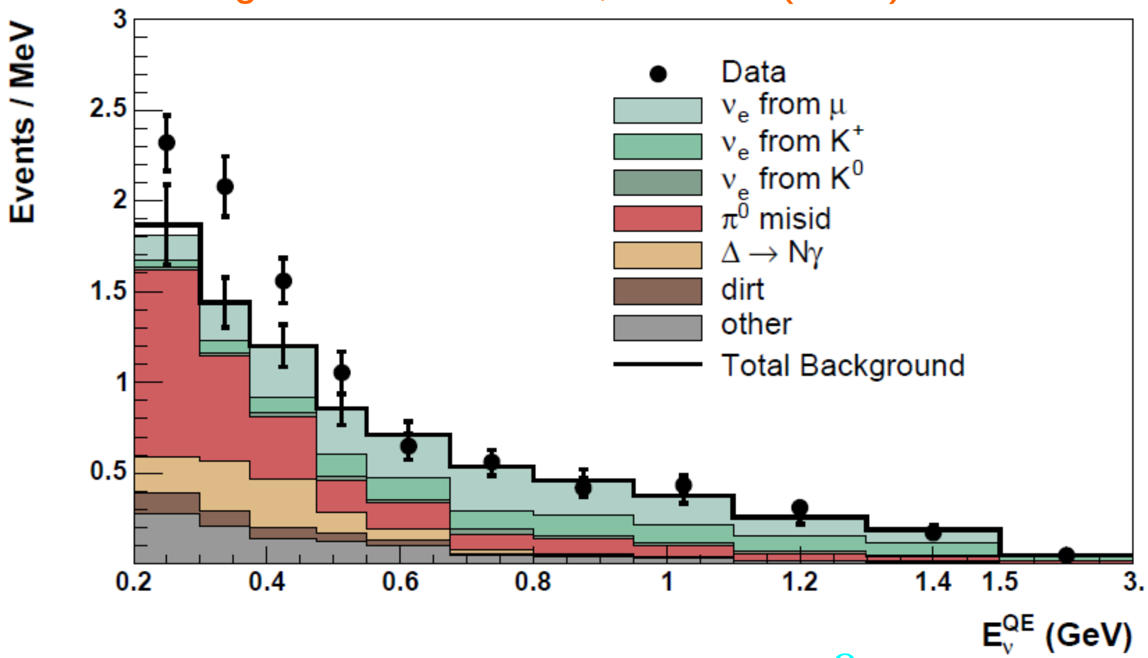
- There are invisible heavy fragments, neutrons or other undetected particles: $E_{vis} < E_{had}$

- E_{vis} -> E_{had} relies on the simulation

Relevance for oscillation experiments

- Backgrounds
  - E.g. in the MiniBooNE $\nu_\mu \to \nu_e$ search

Aguilar-Arevalo et al., PRL102 (2009) 101802



  - NC backgrounds: $\nu_l \, N \to \nu_l \, \pi^0 \, N'$

$$\nu_l \, N \to \nu_l \, \gamma \, N'$$

  - Also important for $\nu_\mu \to \nu_e$ measurements at T2K

Electroweak interactions in the SM

- Spontaneously broken SU(2) x U(1) gauge symmetry

$$\mathcal{L}_{\mathrm{EW}} = -eJ^{\mu}_{em}A_{\mu} - \frac{g}{2\cos\theta_W}J^{\mu}_{nc}Z_{\mu} - \frac{g}{2\sqrt{2}}J^{\mu}_{cc}W^{\dagger}_{\mu} + h.c.$$

$$\sin\theta_W = \frac{e}{g} \qquad \cos\theta_W = \frac{M_W}{M_Z} \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

in the leptonic sector:

$$\begin{aligned} J^{\mu}_{em} &= \bar{l}_i\gamma^{\mu}l_i \qquad i = e,\mu,\tau \\ J^{\mu}_{cc} &= \bar{\nu}_i\gamma^{\mu}(1-\gamma_5)l_i \\ J^{\mu}_{nc} &= \frac{1}{2}\bar{l}_i\gamma^{\mu}(g_V - g_A\gamma_5)l_i + \frac{1}{2}\bar{\nu}_i\gamma^{\mu}(1-\gamma_5)\nu_i \end{aligned}$$

$$g_V = -1 + 4\sin^2\theta_W,\; g_A = -1$$

$$|g_V| \approx 0.04 \ll |g_A|$$

Electroweak interactions in the SM

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in the quark sector:

$$J^\mu_{em} = Q_i\bar{q}_i\gamma^\mu q_i = \frac{2}{3}\bar{q}_u\gamma^\mu q_u - \frac{1}{3}(\bar{q}_d\gamma^\mu q_d + \bar{q}_s\gamma^\mu q_s) + \ldots$$

$$\begin{aligned} J^\mu_{nc} &= \bar{q}_u\gamma^\mu\left[\frac{1}{2} - \left(\frac{2}{3}\right)2\sin^2\theta_W - \frac{1}{2}\gamma_5\right]q_u + (u\to c) + (u\to t) \\ &+ \bar{q}_d\gamma^\mu\left[-\frac{1}{2} - \left(-\frac{1}{3}\right)2\sin^2\theta_W + \frac{1}{2}\gamma_5\right]q_d + (d\to s) + (d\to b) \end{aligned}$$

Electroweak interactions in the SM

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$$\mathcal{L}_{\rm EW} = -eJ^\mu_{em}A_\mu - \frac{g}{2\cos\theta_W}J^\mu_{nc}Z_\mu - \frac{g}{2\sqrt{2}}J^\mu_{cc}W^\dagger_\mu + h.c.$$

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in the quark sector:

$$J^\mu_{cc} = (\bar{q}_u \bar{q}_c \bar{q}_t)\,\gamma^\mu\,(1-\gamma_5)\,U \begin{pmatrix} q_d \\ q_s \\ q_b \end{pmatrix}$$

$U \leftarrow$ CKM matrix

$$U \approx \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

$\theta_C \approx$ 13 deg $\leftarrow$ Cabibbo angle

$$W^- p(uud) \rightarrow n(udd) \qquad \sim \cos^2\theta_C$$

$$W^- p(uud) \rightarrow \Lambda(usd)$$

$$W^- p(uud) \rightarrow p(uud)\,K^-(\bar{u}s)$$

$\Big\} \sim \sin^2\theta_C$

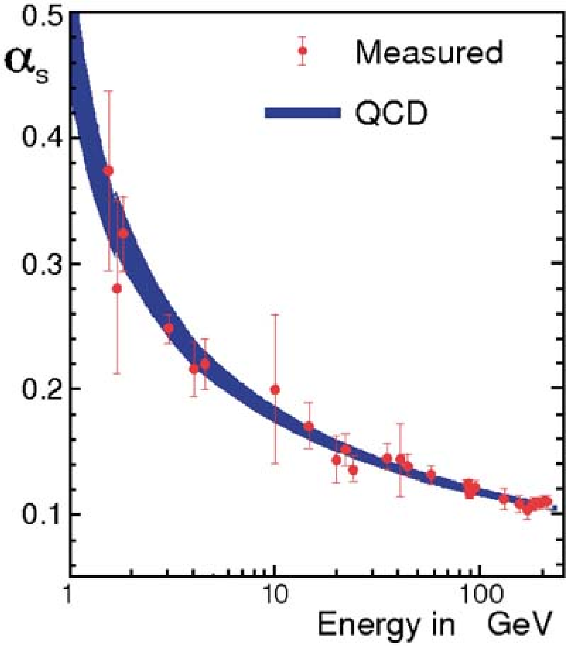
Strong interactions in the SM

- SU(3) (color) gauge symmetry: QCD

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_q \left(i\gamma^\mu D_\mu - m_q \right) \psi_q - \frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} \qquad q = u, d, s, \ldots \quad a = 1 - 8$$

$$D_\mu \psi = \left(\partial_\mu - ig \frac{\lambda_a}{2} A_\mu^a \right) \psi \qquad G_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu$$

- Asymptotically free $\Rightarrow$ perturbative at high energies
- Nonperturbative at low energies
- Confining



$$\alpha_s = \frac{g^2}{4\pi}$$

Strong interactions in the SM

- Approximate symmetries of N_f =3 QCD
  - $m_u = m_d = m_s$ ⇔ Global SU(3)$_{flavor}$ symmetry

$$V_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q \Leftrightarrow \partial_\mu V_a^\mu = 0 \qquad a = 1-8 \qquad \bar{q} = (\bar{q}_u \, \bar{q}_d \, \bar{q}_s) \qquad q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}$$

m_u (1 GeV) = 4 ± 2 MeV

m_d (1 GeV) = 8 ± 4 MeV

m_s (1 GeV) = 164 ± 33 MeV

$$\partial_\mu V_a^\mu = \bar{q} \left[m, \frac{\lambda_a}{2} \right] q \qquad m = \mathrm{diag}(m_u, m_d, m_s)$$

  - $m_u = m_d$ ⇔ Global SU(2)$_{flavor}$ isospin symmetry

$$V_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q = \bar{q}'\gamma^\mu \frac{\tau_a}{2} q' \Leftrightarrow \partial_\mu V_a^\mu = 0 \qquad a = 1-3 \qquad q' = \begin{pmatrix} q_u \\ q_d \end{pmatrix}$$

Strong interactions in the SM

Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ +i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ +i & 0 & 0 \end{pmatrix},$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & +i & 0 \end{pmatrix}, \lambda_8 = \tfrac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Strong interactions in the SM

- Approximate symmetries of N_f =3 QCD
  - $m_u = m_d = m_s$ ⇔ Global SU(3)$_{\text{flavor}}$ symmetry

$$V_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q \Leftrightarrow \partial_\mu V_a^\mu = 0 \qquad a = 1-8 \qquad \bar{q} = (\bar{q}_u\, \bar{q}_d\, \bar{q}_s) \quad q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}$$

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$$V_a^\mu = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q = \bar{q}'\gamma^\mu \frac{\tau_a}{2} q' \Leftrightarrow \partial_\mu V_a^\mu = 0 \qquad a = 1-3 \qquad q' = \begin{pmatrix} q_u \\ q_d \end{pmatrix}$$

Strong interactions in the SM

- Flavor structure of the EW quark currents:

$$\mathcal{L}_{\rm EW} = -eJ^\mu_{em}A_\mu - \frac{g}{2\cos\theta_W}J^\mu_{nc}Z_\mu - \frac{g}{2\sqrt{2}}J^\mu_{cc}W^\dagger_\mu + h.c.$$

$$\begin{aligned} J^\mu_{em} &= \frac{2}{3}\bar{q}_u\gamma^\mu q_u - \frac{1}{3}(\bar{q}_d\gamma^\mu q_d + \bar{q}_s\gamma^\mu q_s) = \bar{q}Q\gamma^\mu q \qquad Q = \mathrm{diag}\left(\tfrac{2}{3} \; -\tfrac{1}{3} \; -\tfrac{1}{3}\right) \\ &= \frac{1}{2}\bar{q}\frac{\lambda_8}{\sqrt{3}}\gamma^\mu q + \bar{q}\frac{\lambda_3}{2}\gamma^\mu q = \frac{1}{2}V^\mu_Y + V^\mu_3 \end{aligned}$$

$$J^\mu_{cc} = \bar{q}_u\gamma^\mu(1-\gamma_5)(q_d\cos\theta_C + q_s\sin\theta_C)$$

$$V^\mu_+ = \bar{q}_u\gamma^\mu q_d = \bar{q}\gamma^\mu\frac{\lambda_1 + i\lambda_2}{2}q = V^\mu_1 + iV^\mu_2$$

$V_{1,2,3}$: components of the same conserved flavor vector current

$$J^\mu_{nc} = \bar{q}_u\gamma^\mu\left[\frac{1}{2} - \left(\frac{2}{3}\right)2\sin^2\theta_W - \frac{1}{2}\gamma_5\right]q_u + \bar{q}_d\gamma^\mu\left[-\frac{1}{2} - \left(-\frac{1}{3}\right)2\sin^2\theta_W + \frac{1}{2}\gamma_5\right]q_d + (d\to s)$$

$$V^\mu_{nc} = (1-2\sin^2\theta_W)V^\mu_3 - 2\sin^2\theta_W\frac{1}{2}V^\mu_Y - \frac{1}{2}\bar{q}_s\gamma^\mu q_s$$

Strong interactions in the SM

- Approximate symmetries of N_f =3 QCD
  - $m_u = m_d = m_s = 0 \Leftrightarrow$ Chiral $SU(3)_L \times SU(3)_R$ symmetry

m_u (1 GeV) = 4 ± 2 MeV
m_d (1 GeV) = 8 ± 4 MeV $\ll$ rho meson, nucleon mass
m_s (1 GeV) = 164 ± 33 MeV

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_{qL} i\gamma^\mu D_\mu \psi_{qL} + \bar{\psi}_{qR} i\gamma^\mu D_\mu \psi_{qR} - m_q(\bar{\psi}_{qL}\psi_{qR} + \bar{\psi}_{qL}\psi_{qR}) + \ldots$$

- Conserved currents:

$$R^\mu_a = \bar{q}_R \gamma^\mu \frac{\lambda_a}{2} q_R \qquad V^\mu_a = R^\mu_a + L^\mu_a = \bar{q}\gamma^\mu \frac{\lambda_a}{2} q$$

$$\Leftrightarrow$$

$$L^\mu_a = \bar{q}_L \gamma^\mu \frac{\lambda_a}{2} q_L \qquad A^\mu_a = R^\mu_a - L^\mu_a = \bar{q}\gamma^\mu \gamma_5 \frac{\lambda_a}{2} q$$

$$q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix} \qquad \bar{q} = (\bar{q}_u \, \bar{q}_d \, \bar{q}_s)$$

- Explicit chiral symmetry breaking:

$$\partial_\mu V^\mu_a = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q \qquad \partial_\mu A^\mu_a = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\}\gamma_5 q \quad \leftarrow \text{PCAC}$$

Electroweak nucleon current

- Flavor structure of the quark currents:

$$\mathcal{L}_{\rm EW} = -eJ^\mu_{em}A_\mu - \frac{g}{2\cos\theta_W}J^\mu_{nc}Z_\mu - \frac{g}{2\sqrt{2}}J^\mu_{cc}W^\dagger_\mu + h.c.$$

$$J^\mu_{cc} = \bar{q}_u\gamma^\mu(1-\gamma_5)(q_d\cos\theta_C + q_s\sin\theta_C)$$

$$A^\mu_+ = \bar{q}_u\gamma^\mu\gamma_5 q_d = \bar{q}_u\gamma^\mu\gamma_5\frac{\lambda_1 + i\lambda_2}{2}q_d = A^\mu_1 + iA^\mu_2$$

$$J^\mu_{nc} = \bar{q}_u\gamma^\mu\left[\frac{1}{2} - \left(\frac{2}{3}\right)2\sin^2\theta_W - \frac{1}{2}\gamma_5\right]q_u + \bar{q}_d\gamma^\mu\left[-\frac{1}{2} - \left(-\frac{1}{3}\right)2\sin^2\theta_W + \frac{1}{2}\gamma_5\right]q_d + (d\to s)$$

$$A^\mu_{nc} = A^\mu_3 + \frac{1}{2}\bar{q}_s\gamma^\mu\gamma_5 q_s$$

$A_{1,2,3}$: components of the same partially conserved flavor axial current

Strong interactions in the SM

- Approximate symmetries of $N_f = 3$ QCD
  - $m_u = m_d = m_s = 0 \Leftrightarrow$ Chiral $SU(3)_L \times SU(3)_R$ symmetry
  - Explicit chiral symmetry breaking:

$$\partial_\mu V_a^\mu = \bar{q}\left[m, \frac{\lambda_a}{2}\right] q \qquad \partial_\mu A_a^\mu = i\bar{q}\left\{m, \frac{\lambda_a}{2}\right\}\gamma_5 q \qquad \leftarrow \text{PCAC}$$

  - Spontaneous chiral symmetry breaking:
    - the ground state does not have the chiral symmetry of the Lagrangian
    - m_ρ = 770 MeV (1^-) $\neq$ m_{a_1} = 1230 MeV (1^+)
    - $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$
    - Goldstone bosons: π, K, η

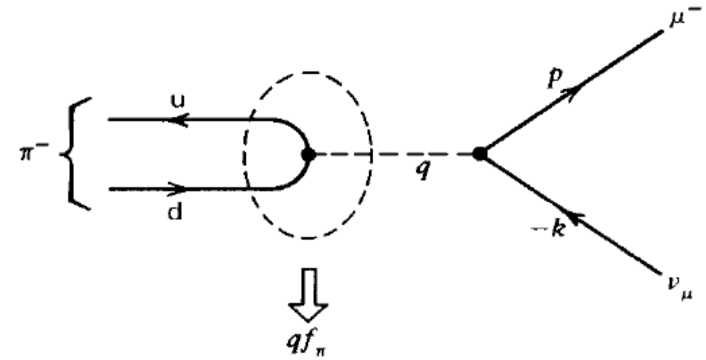
$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{effective}}(\pi, K, \eta \ldots)$$

    - In terms of hadronic degrees of freedom: $A_a^\mu = -f_\pi \partial^\mu \pi_a + \ldots$

$$\langle 0 | A_+^\mu | \pi^- \rangle = \sqrt{2} f_\pi i q^\mu$$

Electroweak interactions in the SM

- Example: $\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$



$$\mathcal{L}_{\rm EW} = -\frac{g}{2\sqrt{2}} J^\mu_{cc} W^+_\mu + h.c.$$

$$(-i)\mathcal{M} = (-i)\left\langle \mu^- \bar{\nu}_\mu \right| \left(-\frac{g}{2\sqrt{2}}\right) J^\mu_{cc} \left|0\right\rangle (-i) D_{\mu\nu}(q) (-i) \left\langle 0 \right| \left(-\frac{g}{2\sqrt{2}}\right) J^\nu_{cc} \left|\pi^-\right\rangle$$

$$D_{\mu\nu} = \frac{1}{q^2 - M_W^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right) \approx -\frac{g_{\mu\nu}}{M_W^2} \qquad q^2 = m_\pi^2 \ll M_W^2$$

$$\left\langle \mu^- \bar{\nu}_\mu \right| J^\mu_{cc} \left|0\right\rangle = \bar{u}(p)\gamma_\mu (1-\gamma_5) v(k)$$

$$\left\langle 0 \right| J^\nu_{cc} \left|\pi^-\right\rangle = \bar{v}_u \gamma^\nu (1-\gamma_5) u_{d'} = \sqrt{2} f_\pi i q^\nu$$

$$\left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} = \frac{G_F}{\sqrt{2}}$$

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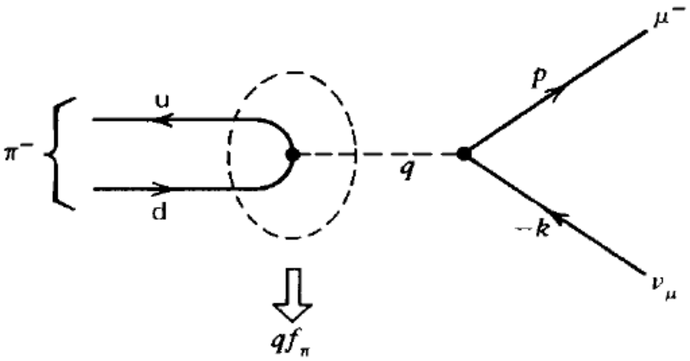


$$\overline{|\mathcal{M}|^2} = \overline{\sum_{\text{polar.}}} |\mathcal{M}|^2 = 4 G_F^2 L_{\mu\nu} H^{\mu\nu}$$

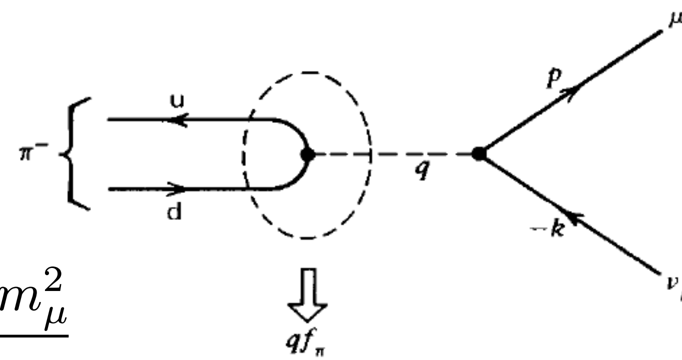
$$\mathrm{Tr}\left[(\not{p} + m_\mu)\gamma_\mu(1-\gamma_5)\not{k}\gamma_\nu(1-\gamma_5)\right] = 8 L_{\mu\nu}$$

$$L_{\mu\nu} = p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu} k\cdot p + i\epsilon_{\mu\nu\alpha\beta} p^\alpha k^\beta$$

$$H^{\mu\nu} = 2 f_\pi^2 q^\mu q^\nu$$

Electroweak interactions in the SM

- Example: $\pi^-(q) \to \mu^-(p) + \bar{\nu}_\mu(k)$



$$\overline{|\mathcal{M}|^2} = 8 G_F^2 f_\pi^2 m_\mu^2 (p \cdot k)$$

$$q^2 = (p+k)^2 = m_\mu^2 + 2(p \cdot k) \Rightarrow (p \cdot k) = \frac{m_\pi^2 - m_\mu^2}{2}$$

Decay width, in the π rest frame:

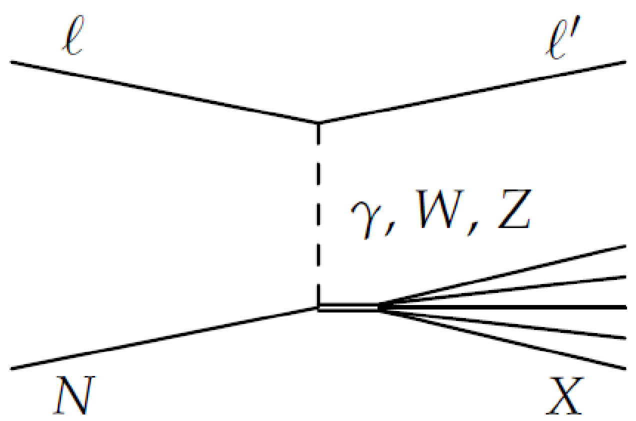
$$\Gamma = \frac{1}{2m_\pi} \int \frac{d^3p}{2p^0(2\pi)^3} \frac{d^3k}{2k^0(2\pi)^3} (2\pi)^4 \delta^4(k+p-q) \overline{|\mathcal{M}|^2}$$

$$\Gamma = \frac{G_F^2}{4\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

$$\tau = \frac{1}{\Gamma} = 2.6\, 10^{-8}\, s \Rightarrow f_\pi = 92.4\, \mathrm{MeV}$$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



$$k = (k_0, \vec{k}) \qquad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}) \qquad p' = (E', \vec{p'})$$

$$q = k - k' = p' - p = (\omega, \vec{q}) \qquad q^2 = -Q^2 < 0$$

In Lab: $p = (M, \vec{0})$

For CC: $$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} L_{\mu\nu} W^{\mu\nu}$$

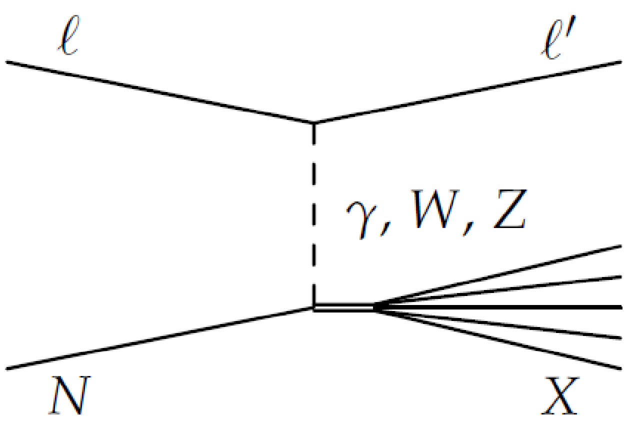
$$L_{\mu\nu} = k'_\mu k_\nu + k'_\nu k_\mu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\alpha\beta} k'^\alpha k^\beta$$

$$W^{\mu\nu} = \frac{1}{2M} \overline{\sum_{\text{polar.}}} \sum_i \left(\int \frac{d^3 p_i}{2E'_i (2\pi)^3} \right) (2\pi)^3 \delta^4(k' + p' - k - p) \langle X| J^\mu |N\rangle \langle X| J^\nu |N\rangle^*$$

For EM: $L_{\mu\nu} \to L_{\mu\nu}^{(\text{sym})} \qquad \frac{G_F^2}{(2\pi)^2} \to \frac{\alpha^2}{q^4}$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



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$$q = k - k' = p' - p = (\omega, \vec{q}) \qquad q^2 = -Q^2 < 0$$

General structure of the hadronic tensor:

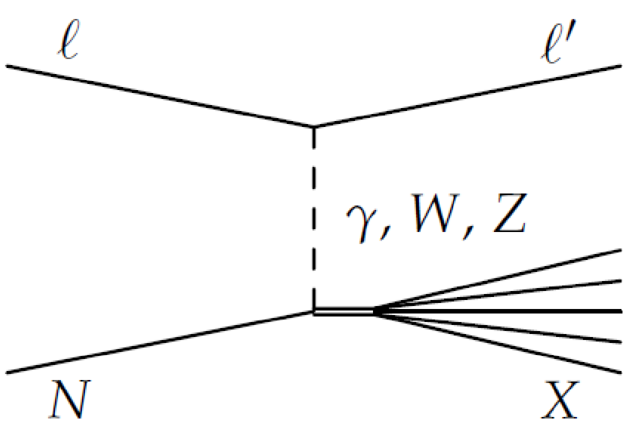
Ingredients: $g^{\mu\nu}$, q^μ, p^μ, $\epsilon^{\alpha\beta\mu\nu}$ $\quad p' = p + q \leftarrow$ not independent

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{M^2} + W_4 \frac{q^\mu q^\nu}{M^2} + W_5 \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} + W_3 i\epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2M^2} + W_6 \frac{p^\mu q^\nu - q^\mu p^\nu}{M^2}$$

Structure functions: $W_i = W_i(p^2 = M^2, q \cdot p = \omega M, q^2) = W_i(\omega, q^2)$

Inclusive cross section

$$l(k) + N(p) \to l'(k') + X(p')$$



$$k = (k_0, \vec{k}) \qquad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}) \qquad p' = (E', \vec{p'})$$

$$q = k - k' = p' - p = (\omega, \vec{q}) \qquad q^2 = -Q^2 < 0$$

General structure of the hadronic tensor:

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{M^2} + W_4 \frac{q^\mu q^\nu}{M^2} + W_5 \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} + W_3 i\epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2M^2} + W_6 \frac{p^\mu q^\nu - q^\mu p^\nu}{M^2}$$

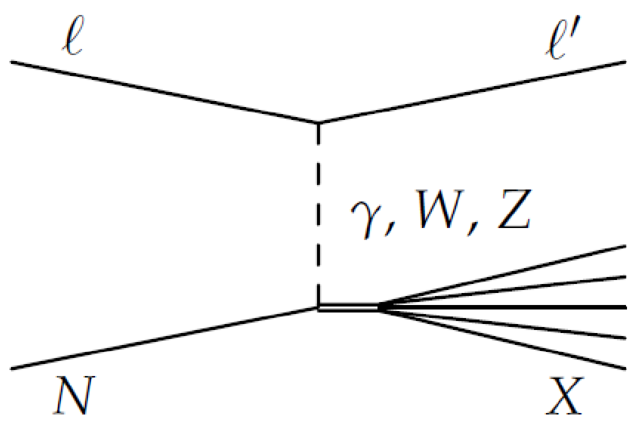
Structure functions: $W_i = W_i(\omega, q^2)$

For EM interactions: $q_\mu J^\mu = 0 \Rightarrow q_\mu W^{\mu\nu}_{em} = W^{\mu\nu}_{em} q_\nu = 0$

$$W^{\mu\nu}_{em} = W_1 \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) + \frac{W_2}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right)$$

Inclusive cross section

$$l(k) + N(p) \rightarrow l'(k') + X(p')$$



$$k = (k_0, \vec{k}) \qquad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}) \qquad p' = (E', \vec{p'})$$

$$q = k - k' = p' - p = (\omega, \vec{q}) \qquad q^2 = -Q^2 < 0$$

In Lab: $p = (M, \vec{0})$

$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} \{W_1 2k \cdot k' + W_2 (2k'_0 k_0 - k \cdot k') + 2\frac{m_l^2}{M^2}[W_4 k \cdot k' - W_5 M k_0] + \frac{W_3}{M}[(k_0 + k'_0) k \cdot k' - k_0 m_l^2]\}$$

$m_l \rightarrow 0$

$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{2\pi^2}(k'_0)^2 \left[W_1 2\sin^2\frac{\theta'}{2} + W_2 \cos^2\frac{\theta'}{2} \pm W_3 \frac{(k_0 + k'_0)}{M} \sin^2\frac{\theta'}{2}\right]$$

Inclusive cross section

- Example: EM scattering on a point-like spin ½ particle

$$\langle X | J^\mu | N \rangle \rightarrow \bar{u}(p')\gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3p'}{2E'} \delta^4(k' + p' - k - p) 4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p \cdot p' - M^2)$$

Using that:

$$\delta(E' + k'_0 - M - k_0) = \frac{E'}{M} \delta\left(k'_0 - k_0 - \frac{q^2}{2M}\right)$$

$$q^2 = (p' - p)^2 = 2M^2 - 2p \cdot p' \;\Rightarrow\; p \cdot p' - M^2 = -\frac{q^2}{2}$$

$$p'^2 = M^2 = (q + p)^2 = q^2 + 2(p \cdot q) + M^2 \Rightarrow \frac{p \cdot q}{q^2} = \frac{M\omega}{q^2} = -\frac{1}{2}$$

Inclusive cross section

- Example: EM scattering on a point-like spin ½ particle

$$\langle X|\, J^\mu \,|N\rangle \to \bar{u}(p')\gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M}\int \frac{d^3p'}{2E'}\delta^4(k'+p'-k-p)4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p\cdot p' - M^2)$$

one finds:

$$W_1 = -\frac{q^2}{4M^2\omega}\delta\left(1+\frac{q^2}{2M\omega}\right)$$
$$W_2 = \frac{1}{\omega}\delta\left(1+\frac{q^2}{2M\omega}\right)$$

$\Leftrightarrow$

$$2MW_1 \equiv F_1 = x\delta(1-x) = F_1(x)$$
$$\omega W_2 \equiv \delta(1-x) = F_2(x)$$
$$x = -\frac{q^2}{2M\omega}$$

Inclusive cross section

- Example: EM scattering on a point-like spin ½ particle

$$\langle X|\, J^\mu \,|N\rangle \rightarrow \bar{u}(p')\gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M}\int \frac{d^3p'}{2E'}\delta^4(k'+p'-k-p)4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p\cdot p' - M^2)$$

for real nucleons, at low $\vec{q}^{\,2}$

$$W_1 = -\frac{q^2}{4M^2\omega}\delta\left(1+\frac{q^2}{2M\omega}\right) \rightarrow -\frac{q^2}{4M^2\omega}G^2(q^2)\delta\left(1+\frac{q^2}{2M\omega}\right)$$

$$W_2 = \frac{1}{\omega}\delta\left(1+\frac{q^2}{2M\omega}\right) \rightarrow \frac{1}{\omega}G^2(q^2)\delta\left(1+\frac{q^2}{2M\omega}\right)$$

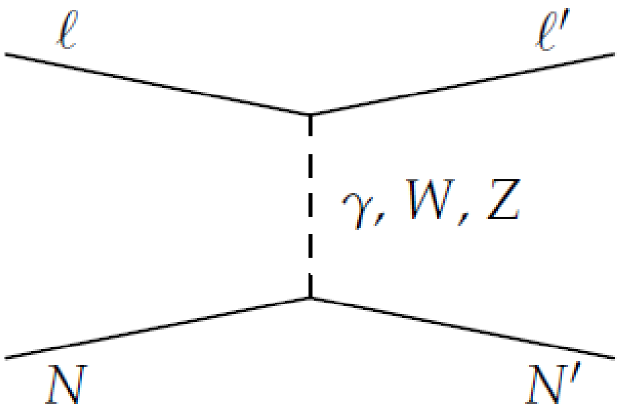
Form factors

$\rho(r)$	$\lvert F(q^2) \rvert$	Example
pointlike	constant	Electron
exponential	dipole	Proton
gauss	gauss	^{6}Li
homogeneous sphere	oscillating	–
sphere with a diffuse surface	oscillating	^{40}Ca

r → $\lvert q \rvert$ →

QE scattering on the nucleon

$$\begin{aligned}
\text{EM}: l^{\pm}(k) + N(p) &\rightarrow l^{\pm}(k') + N(p') \\
\text{CC}: \nu(k) + n(p) &\rightarrow l^{-}(k') + p(p') \\
\bar{\nu}(k) + p(p) &\rightarrow l^{+}(k') + n(p') \\
\text{NC}: \nu(k) + N(p) &\rightarrow \nu(k') + N(p') \\
\bar{\nu}(k) + N(p) &\rightarrow \bar{\nu}(k') + N(p')
\end{aligned}$$

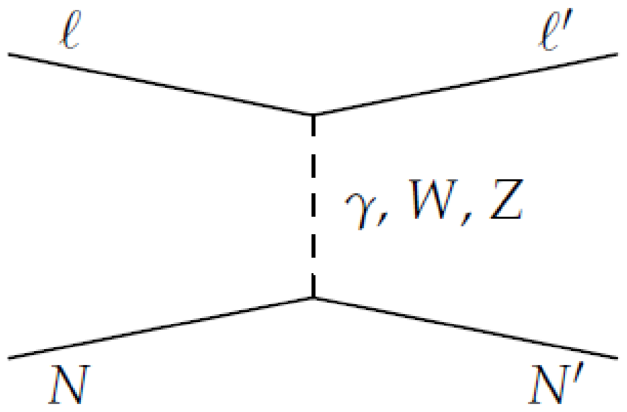


- QE kinematics:

$$\begin{aligned}
&(q+p)^2 = (p')^2 \\
&q^2 + 2M\omega + M^2 = M^2 \\
&\omega = -\frac{q^2}{2M} \\
&x = 1
\end{aligned}$$

QE scattering on the nucleon

$$
\begin{aligned}
\text{EM}: l^\pm(k) + N(p) &\rightarrow l^\pm(k') + N(p') \\
\text{CC}: \nu(k) + n(p) &\rightarrow l^-(k') + p(p') \\
\bar\nu(k) + p(p) &\rightarrow l^+(k') + n(p') \\
\text{NC}: \nu(k) + N(p) &\rightarrow \nu(k') + N(p') \\
\bar\nu(k) + N(p) &\rightarrow \bar\nu(k') + N(p')
\end{aligned}
$$



- Cross section:

$$\frac{d\sigma}{dk_0' d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} L_{\mu\nu} W^{\mu\nu}$$

For EM: $L_{\mu\nu} \rightarrow L_{\mu\nu}^{(\mathrm{sym})}$ $\quad \frac{G_F^2}{(2\pi)^2} \rightarrow \frac{\alpha^2}{q^4}$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3p'}{2E'} \delta^4(k' + p' - k - p) H^{\nu\mu}$$

$$H^{\alpha\beta} = \mathrm{Tr}\left[(\not{p} + M)\, \gamma^0\, (\Gamma^\alpha)^\dagger\, \gamma^0\, (\not{p}' + M)\, \Gamma^\beta \right]$$

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

Electroweak nucleon current

$$\langle N'|\, J^\mu \,|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\begin{aligned} \mathcal{V}^\mu &= \bar{u}(p')\left[\gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2\right]u(p) \\ \mathcal{A}^\mu &= \bar{u}(p')\left[\gamma^\mu\gamma_5 F_A + \frac{q^\mu}{M}\gamma_5 F_P\right]u(p) \end{aligned}$$

- $F_1 \leftarrow$ Dirac form factor
- $F_2 \leftarrow$ Pauli form factor
- $F_A \leftarrow$ axial form factor
- $F_P \leftarrow$ pseudoscalar form factor

Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

- $\Gamma^\mu \leftarrow$ 4-vector constructed from:

$$(1)\ p^\mu\ ,\ p'^\mu$$

$$(2)\ \epsilon_{\alpha\beta\mu\nu}\ ,\ g^{\mu\nu}$$

$$(3)\ \left\{\gamma_\mu\ ,\ \gamma_5\ ,\ \gamma_\mu\gamma_5\ ,\ \sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]\right\}$$

Electroweak nucleon current

$$\langle N' | \, J^\mu \, | N \rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

- $\Gamma^\mu \leftarrow$ 4-vector constructed from:

$$(1) \; p^\mu \, , \; p'^\mu$$

$$(2) \; \epsilon_{\alpha\beta\mu\nu} \, , \; g^{\mu\nu}$$

$$(3) \; \left\{ \gamma_\mu \, , \, \gamma_5 \, , \, \gamma_\mu\gamma_5 \, , \, \sigma_{\mu\nu} = \tfrac{i}{2}\left[\gamma_\mu, \gamma_\nu\right] \right\}$$

Any other combination of γ matrices can be reduced to (3).

For example:

$$\gamma_\mu\gamma_\nu = g_{\mu\nu} - i\sigma_{\mu\nu}$$

$$\gamma^\alpha\sigma^{\mu\nu} = i\left(g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\nu}g^{\beta\mu}\right)\gamma_\beta + \epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta$$

Electroweak nucleon current

$$\langle N'|\, J^\mu \,|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

- $\Gamma^\mu \leftarrow$ 4-vector constructed from:

$$(1)\ p^\mu \,,\ p'^\mu$$

$$(2)\ \epsilon_{\alpha\beta\mu\nu} \,,\ g^{\mu\nu}$$

$$(3)\ \left\{\gamma_\mu \,,\ \gamma_5 \,,\ \gamma_\mu\gamma_5 \,,\ \sigma_{\mu\nu} = \tfrac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]\right\}$$

Using:

- Dirac algebra
- Dirac equation: $(\not{p} - M)u(p) = \bar{u}(p')(\not{p}' - M) = 0$

one finds:

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2 + \frac{q^\mu}{M}F_S - \gamma^\mu\gamma_5 F_A - \frac{i}{2M}\sigma^{\mu\nu}q_\nu\gamma_5 F_T - \frac{q^\mu}{M}\gamma_5 F_P$$

Electroweak nucleon current

$$\langle N'| J^\mu |N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2 + \frac{q^\mu}{M}F_S - \gamma^\mu\gamma_5 F_A - \frac{i}{2M}\sigma^{\mu\nu}q_\nu\gamma_5 F_T - \frac{q^\mu}{M}\gamma_5 F_P$$

- $F_i = F_i\,(q^2) \Leftrightarrow 2\,p\cdot q + q^2 = 0$

Electroweak nucleon current

$$\langle N'|\, J^\mu \,|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2 + \frac{q^\mu}{M}F_S - \gamma^\mu\gamma_5 F_A - \frac{i}{2M}\sigma^{\mu\nu}q_\nu\gamma_5 F_T - \frac{q^\mu}{M}\gamma_5 F_P$$

- Time reversal (T) transformation:

$$T\left(\bar{u}\Gamma^\mu u\right)T^\dagger = (-1)^\mu \sum_i F_i^* \bar{u}\mathcal{O}_i^\mu u \qquad T l_\mu T^\dagger = (-1)^\mu l_\mu$$

the interaction is proportional to:

$$l_\mu \bar{u}\Gamma^\mu u = \sum_i F_i l_\mu \bar{u}\mathcal{O}_i^\mu u$$

$$T\left(l_\mu \bar{u}\Gamma^\mu u\right)T^\dagger = (-1)^{2\mu} \sum_i F_i^* l_\mu \bar{u}\mathcal{O}_i^\mu u$$

- T-inv $\Rightarrow$ $F_i = F^*_i$

Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2 + \frac{q^\mu}{M}F_S - \gamma^\mu\gamma_5 F_A - \frac{i}{2M}\sigma^{\mu\nu}q_\nu\gamma_5 F_T - \frac{q^\mu}{M}\gamma_5 F_P$$

- Parity (P) transformation:

$$P\bar{u}(p_0', \vec{p}\,')\Gamma^\mu(q_0, \vec{q})u(p_0, \vec{p})P^\dagger = \bar{u}(p_0', -\vec{p}\,')\gamma_0\Gamma^\mu(q_0, -\vec{q})\gamma_0 u(p_0, -\vec{p})$$

γ^μ , $\sigma^{\mu\nu}q_\nu$, q^μ $\leftarrow$ transform as vectors

$\gamma^\mu\gamma_5$, $\sigma^{\mu\nu}\gamma_5 q_\nu$, $q^\mu\gamma_5$ $\leftarrow$ transform as axial-vectors

Electroweak nucleon current

$$\langle N'|\, J^\mu \,|N\rangle = \bar{u}(p')\Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\mathcal{V}^\mu = \bar{u}(p')\left[\gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2 + \frac{q^\mu}{M}F_S\right]u(p)$$

$$\mathcal{A}^\mu = \bar{u}(p')\left[\gamma^\mu\gamma_5 F_A + \frac{i}{2M}\sigma^{\mu\nu}q_\nu\gamma_5 F_T + \frac{q^\mu}{M}\gamma_5 F_P\right]u(p)$$

- Current conservation + Dirac eq. :

$$q_\mu \mathcal{V}^\mu = \bar{u}(p')\left[(\not{p}' - \not{p})F_1 + \frac{q^2}{M}F_S\right]u(p) = 0 \Rightarrow F_S = 0$$

Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\mathcal{V}^\mu = \bar{u}(p') \left[\gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 + \frac{q^\mu}{M} F_S \right] u(p)$$

$$\mathcal{A}^\mu = \bar{u}(p') \left[\gamma^\mu \gamma_5 F_A + \frac{i}{2M} \sigma^{\mu\nu} q_\nu \gamma_5 F_T + \frac{q^\mu}{M} \gamma_5 F_P \right] u(p)$$

- G-parity (G) transformation: $G = C e^{i\pi \frac{\tau_2}{2}}$
  - Isospin rotation: $e^{i\pi \frac{\tau_2}{2}} \begin{pmatrix} q_u \\ q_d \end{pmatrix} = i\tau_2 \begin{pmatrix} q_u \\ q_d \end{pmatrix} = \begin{pmatrix} q_d \\ -q_u \end{pmatrix}$

$$G \mathcal{V}^\mu G^\dagger = \mathcal{V}^\mu$$ $\leftarrow$ except for the F_S term

$$G \mathcal{A}^\mu G^\dagger = -\mathcal{A}^\mu$$ $\leftarrow$ except for the F_T term

  - $\Rightarrow F_S = F_T = 0 \Leftrightarrow$ absence of 2nd class currents

Electroweak nucleon current

$$\mathcal{V}^\mu = \bar{u}(p')\left[\gamma^\mu F_1 + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2\right]u(p)$$

$$\mathcal{A}^\mu = \bar{u}(p')\left[\gamma^\mu\gamma_5 F_A + \frac{q^\mu}{M}\gamma_5 F_P\right]u(p)$$

- Sachs form factors: $G_E = F_1 + \frac{q^2}{2m_N}F_2$

  $G_M = F_1 + F_2$

- In the Breit frame: $p = (E, -\vec{q}/2)\,,\ p' = (E, -\vec{q}/2)\,,\ q = p' - p = (0, \vec{q})\,,\ q^2 = -\vec{q}^2$

$$\langle N'_{s'}|\,\mathcal{V}^0\,|N_s\rangle = G_E(\vec{q}^2)\delta_{ss'}$$

$$\langle N'_{s'}|\,\vec{\mathcal{V}}\,|N_s\rangle = G_M(\vec{q}^2)\frac{i}{2M}\chi_{s'}(\vec{\sigma}\times\vec{q})\chi_s$$

$$\langle N'_{s'}|\,\mathcal{A}^0\,|N_s\rangle = 0$$

$$\langle N'_{s'}|\,\vec{\mathcal{A}}\,|N_s\rangle = \chi_{s'}\left[\frac{E}{M}F_A(\vec{q}^2)\vec{\sigma}_T + \left[F_A(\vec{q}^2) - \frac{\vec{q}^2}{2M}F_P(\vec{q}^2)\right]\vec{\sigma}_L\right]\chi_s$$

$$\vec{\sigma}_T = \vec{\sigma} - \hat{\vec{q}}(\vec{\sigma}\cdot\hat{\vec{q}})\,,\quad \vec{\sigma}_L = \hat{\vec{q}}(\vec{\sigma}\cdot\hat{\vec{q}})$$

Electroweak nucleon current

- Vector and EM form factors:

$V_a^\alpha = \mathcal{V}^\alpha \frac{\tau_a}{2}$ ← isovector current $\quad V_Y^\alpha = \mathcal{V}_Y^\alpha I$ ← hypercharge (isoscalar) current

$$\langle p | V_{\mathrm{EM}}^\alpha | p \rangle = \langle p | V_3^\alpha + \frac{1}{2} V_Y^\alpha | p \rangle = \frac{\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_p^\alpha$$

$$\langle n | V_{\mathrm{EM}}^\alpha | n \rangle = \langle n | V_3^\alpha + \frac{1}{2} V_Y^\alpha | n \rangle = \frac{-\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_n^\alpha$$

Then: $\langle p | V_{\mathrm{CC}}^\alpha | n \rangle = \langle p | V_1^\alpha + i V_2^\alpha | n \rangle = \mathcal{V}^\alpha = \mathcal{V}_p^\alpha - \mathcal{V}_n^\alpha$

$$\begin{aligned} \langle p | V_{\mathrm{NC}}^\alpha | p \rangle &= \langle p | (1 - 2\sin^2\theta_W) V_3^\alpha - \sin^2\theta_W V_Y^\alpha | p \rangle \\ &= \left(\frac{1}{2} - \sin^2\theta_W \right) \mathcal{V}^\alpha + \sin^2\theta_W \mathcal{V}_Y^\alpha \\ &= \left(\frac{1}{2} - 2\sin^2\theta_W \right) \mathcal{V}_p^\alpha - \mathcal{V}_n^\alpha \end{aligned}$$

- Vector CC and NC form factors can be expressed in terms of EM ones

Electroweak nucleon current

- Consequences of PCAC and pion pole dominance

$$\langle n| (-i)\mathcal{V}_\pi^\mu |p\rangle = \langle n| (-i)\mathcal{L}_{N\pi\pi} \left|p\pi^-\right\rangle (-i)D_\pi(q)(-i) \left\langle \pi^-\right| \mathcal{A}_-^\mu |0\rangle$$

$$\mathcal{L}_{NN\pi} = -\frac{g_{NN\pi}}{f_\pi}\bar{N}\gamma_\mu\gamma_5(\partial^\mu\vec{\pi})\vec{\tau}N \qquad D_\pi = \frac{1}{q^2 - m_\pi^2} \qquad \left\langle \pi^-\right| A_-^\mu |0\rangle = -\sqrt{2}f_\pi i q^\mu$$

$$\begin{aligned} \langle n| \mathcal{V}_\pi^\mu |p\rangle &= -2g_{N\pi\pi}F_{N\pi\pi}(q^2)\frac{1}{q^2 - m_\pi^2}\bar{u}\not{q}\gamma_5 q^\mu u \qquad F_{N\pi\pi}(0) = 1 \\ &= \underbrace{-2g_{N\pi\pi}F_{N\pi\pi}(q^2)\frac{2M}{q^2 - m_\pi^2}}_{=\frac{F_P}{M}}\bar{u}\gamma_5 q^\mu u \end{aligned}$$

Electroweak nucleon current

- Consequences of PCAC and pion pole dominance

$$\langle n | \mathcal{V}_\pi^\mu | p \rangle = -2 g_{N\pi\pi} F_{N\pi\pi}(q^2) \frac{1}{q^2 - m_\pi^2} \bar{u} \not{q} \gamma_5 q^\mu u$$

$$= \underbrace{-2 g_{N\pi\pi} F_{N\pi\pi}(q^2) \frac{2M}{q^2 - m_\pi^2}}_{} \bar{u} \gamma_5 q^\mu u \quad = \frac{F_P}{M}$$

PCAC: $\langle n | q_\mu \mathcal{A}^\mu | p \rangle = 0 \qquad m_\pi \to 0$

$$\bar{u} \left[\not{q} \gamma_5 F_A - 2 g_{N\pi\pi} F_{N\pi\pi}(q^2) \frac{2M}{q^2 - m_\pi^2} q^2 \gamma_5 \right] u = 0 \qquad m_\pi \to 0$$

$$F_A(q^2) = 2 g_{N\pi\pi} F_{N\pi\pi}(q^2)$$

$$F_A(0) \equiv g_A = 2 g_{NN\pi}$$ $\leftarrow$ Goldberger-Treiman relation

Electroweak nucleon current

- Consequences of PCAC and pion pole dominance

$$\mathcal{A}^\mu = \bar{u}(p')\left[\gamma^\mu\gamma_5 F_A(q^2) + \frac{q^\mu}{M}\gamma_5 F_A(q^2)\frac{2M}{Q^2+m_\pi^2}\right]u(p)$$

- F_P has a small impact on CCQE cross sections (except for ν_τ !)
  - appear in terms proportional to $(m_l/M)^4$
- F_P does not contribute to NCE cross sections
- F_p is studied in muon capture $\mu^- + p \to \nu_\mu + n$
  - $F_P(0)/g_A$ consistent with the PCAC+pion-pole model
- F_p is also studied in radiative muon capture $\mu^- + p \to \nu_\mu + n + \gamma$
  - more difficult to measure and to model

Electron scattering on the nucleon

$$\text{EM}: l^{\pm}(k) + N(p) \to l^{\pm}(k') + N(p')$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4k_0^2 \sin^4\frac{\theta}{2}} \frac{k_0'}{k_0} \cos^2\frac{\theta}{2}$$

$\leftarrow$ Scattering on a point-like spinless target in Lab

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[1 - \frac{q^2}{2M^2}\tan^2\frac{\theta}{2}\right]$$

$\leftarrow$ Scattering on a point-like spin 1/2 target in Lab

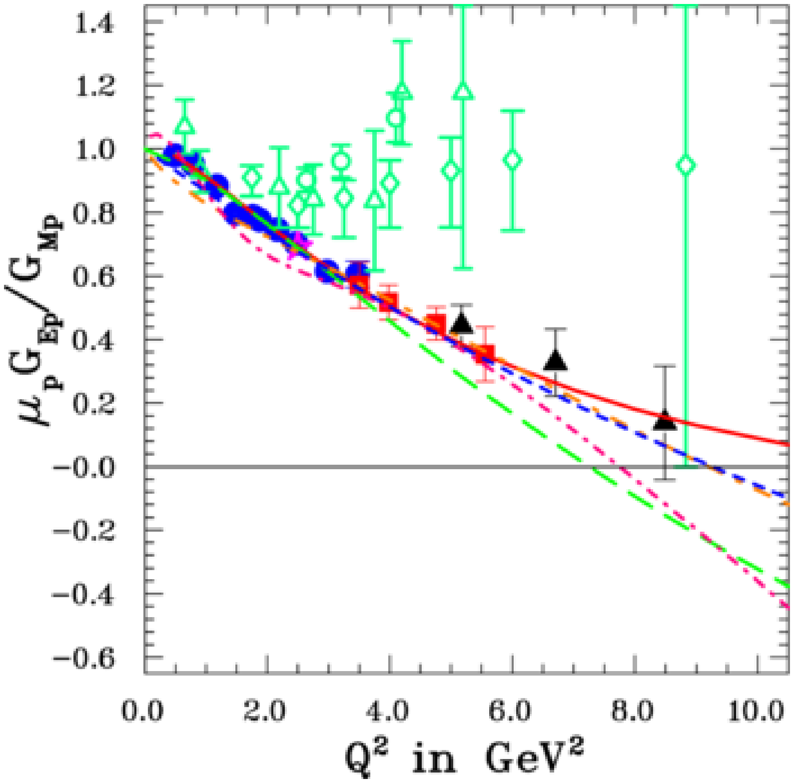
$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\frac{G_E^2 - \frac{q^2}{4M^2}G_M^2}{1 - \frac{q^2}{4M^2}} - \frac{q^2}{2M^2}G_M^2 \tan^2\frac{\theta}{2}\right]$$

$$q^2 = -4k_0 k_0' \sin^2\frac{\theta}{2}$$

- Rosenbluth separation $\Rightarrow G_E$, G_M

Electron scattering on the nucleon

- More precision, particularly at high Q^2, with polarization techniques (Jlab)
  - Polarization transfer: $\vec{e} + p \to e + \vec{p}$
  - Double polarization: $\vec{e} + \vec{p} \to e + p$



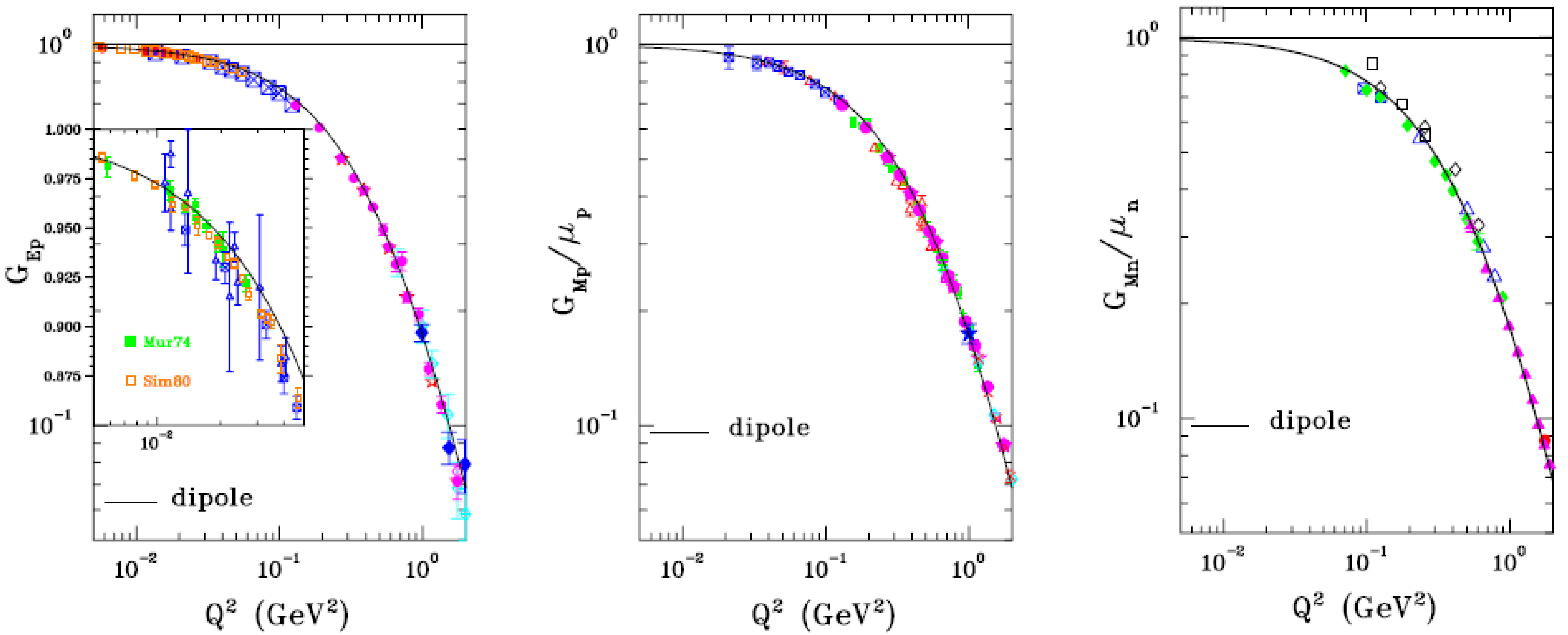
Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764, Scolarpedia

Nucleon EM form factors

- $Q^2 \lesssim 1$ GeV^2

$$G_E^p(q^2) = G_D(q^2)\,,\ G_M^p = \mu_p G_D(q^2)\,,\ G_M^p = \mu_p G_D(q^2)\,,\ G_D = \left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

$$\mu_p = 2.793\,,\ \mu_n = -1.913\,,\ M_V^2 = 0.71\ \mathrm{GeV}^2$$



Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764

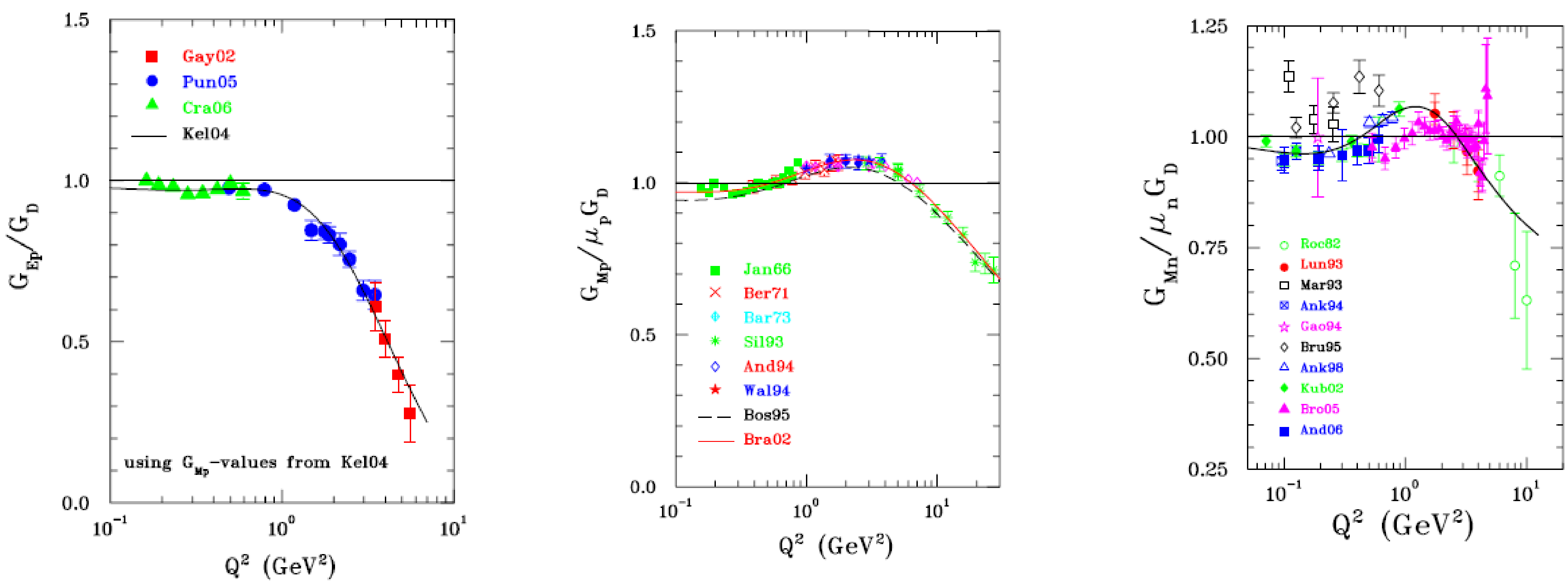
Nucleon EM form factors

- $Q^2 \lesssim 1$ GeV^2

$$G_E^p(q^2) = G_D(q^2)\,,\; G_M^p = \mu_p G_D(q^2)\,,\; G_M^p = \mu_p G_D(q^2)\,,\; G_D = \left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

$$\mu_p = 2.793\,,\; \mu_n = -1.913\,,\; M_V^2 = 0.71 \text{ GeV}^2$$

- Q^2 $\gtrsim$ GeV^2

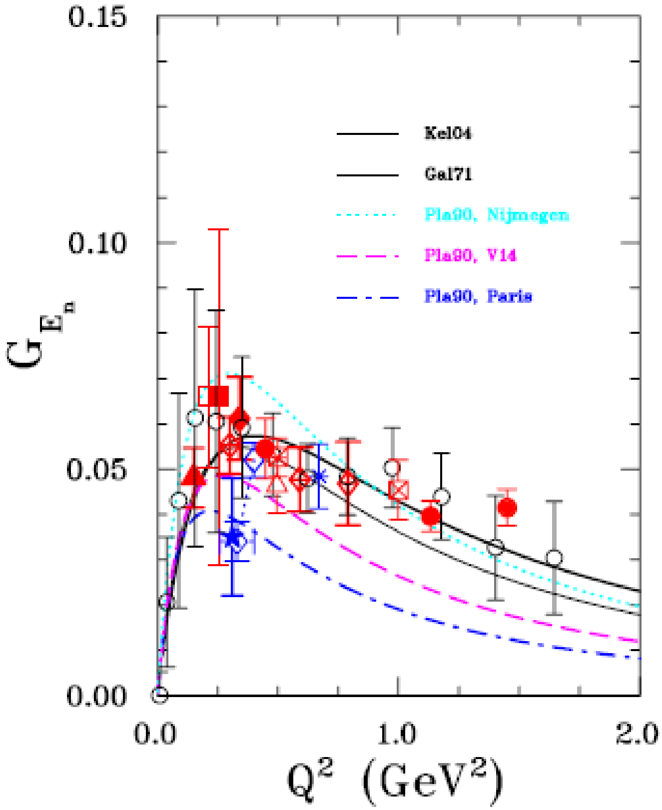


Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764

Nucleon EM form factors

- Neutron electric form factor

$$G_E^n(0) = 0$$



Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764, Scolarpedia

ν QE scattering on the nucleon

- Cross section:
  - As an expansion in small variables $q^2, m_l^2 \ll M^2, E_\nu^2$
  - Mean square radii

$$
\begin{aligned}
G_E^p(Q^2) &= 1 - \frac{1}{6}\langle r_p^2\rangle Q^2 + \ldots \\
G_E^n(Q^2) &= -\frac{1}{6}\langle r_n^2\rangle Q^2 + \ldots \\
F_A(Q^2) &= g_A - \frac{1}{6}\langle r_A^2\rangle Q^2 + \ldots
\end{aligned}
$$

CCQE scattering on the nucleon

- Cross section:
  - As an expansion in small variables $q^2, m_l^2 \ll M^2, E_\nu^2$

$$\frac{d\sigma}{dq^2} = \frac{1}{2\pi} G^2 c_{\rm EW}^2 \left[R - \frac{m_l^2}{4E_\nu^2} S + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

  - CC: $c_{\rm CC} = \cos\theta_C$

$$\begin{aligned} R_{\rm CC} &= 1 + g_A^2 \\ S_{\rm CC} &= \frac{2E_\nu + M}{M} + g_A^2 \frac{2E_\nu - M}{M} \\ T_{\rm CC} &= 1 - g_A^2 + 2\frac{E_\nu}{M} (1 \mp g_A)^2 \mp 4\frac{E_\nu}{M} g_A \kappa^{\rm V} - \left(\frac{E_\nu}{M}\kappa^{\rm V}\right)^2 \\ &+ 4E_\nu^2 \left[\frac{1}{3} \left(\langle r_p^2 \rangle - \langle r_n^2 \rangle + g_A^2 \langle r_A^2 \rangle \right) - \frac{1}{2M^2} \kappa^{\rm V} \right] \end{aligned}$$

$$\kappa^{\rm V} = \mu_p - \mu_n - 1$$

CCQE scattering on the nucleon

- Cross section:
  - As an expansion in small variables $q^2, m_l^2 \ll M^2, E_\nu^2$

$$\frac{d\sigma}{dq^2} = \frac{1}{2\pi} G^2 c^2_{\mathrm{EW}} \left[R - \frac{m_l^2}{4E_\nu^2} S + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

  - CC: $c_{\mathrm{CC}} = \cos\theta_C$

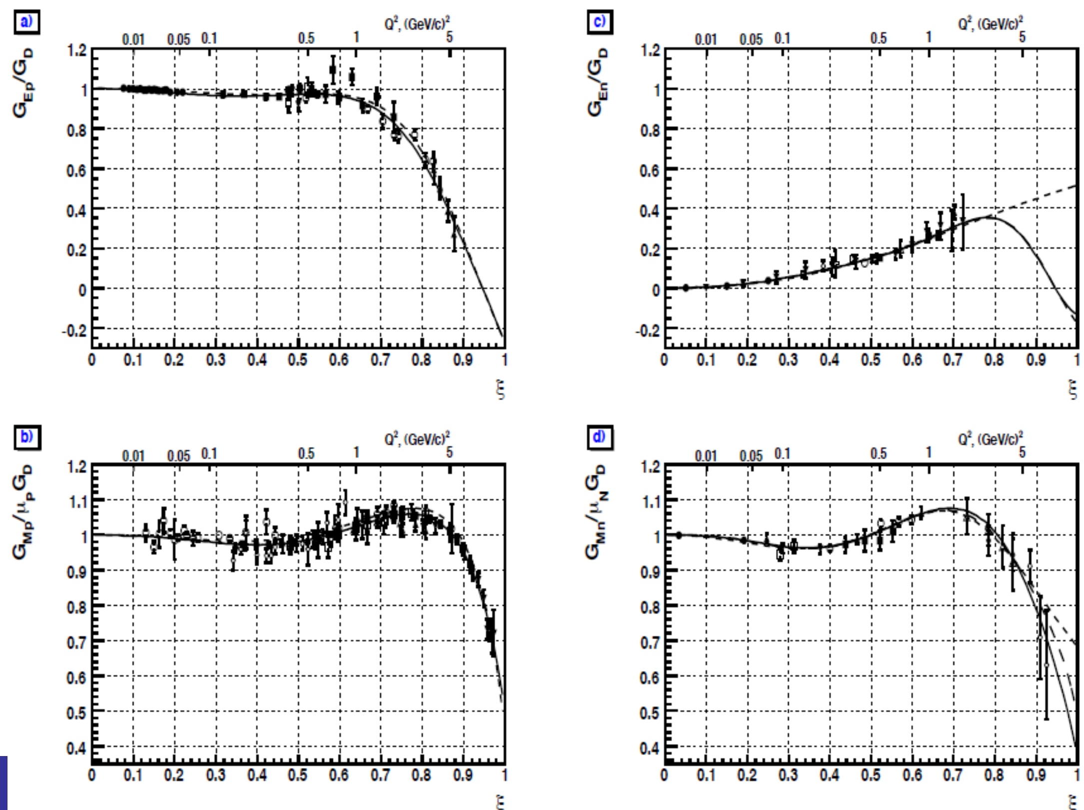
  - Large fraction of the CCQE cross section depends on a small number of nucleon properties:
    - Charges, magnetic moments, charge mean squared radii
    - axial coupling and axial radius

CCQE scattering on the nucleon

- Measurement of the axial radius:
  - CCQE on H and D (BNL, ANL)

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{M_A^2}\right)^{-2} \qquad \langle r_A^2 \rangle = \frac{12}{{M_A}^2}$$

  - M_A = 1.016 ± 0.026 GeV   Bodek et al., EPJC 53 (2008)
  - Using:



CCQE scattering on the nucleon

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  - CCQE on H and D (BNL, ANL)

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{M_A^2}\right)^{-2} \qquad \langle r_A^2 \rangle = \frac{12}{{M_A}^2}$$

    - $M_A = 1.016 \pm 0.026$ GeV   Bodek et al., EPJC 53 (2008)

  - From π electroproduction on p:

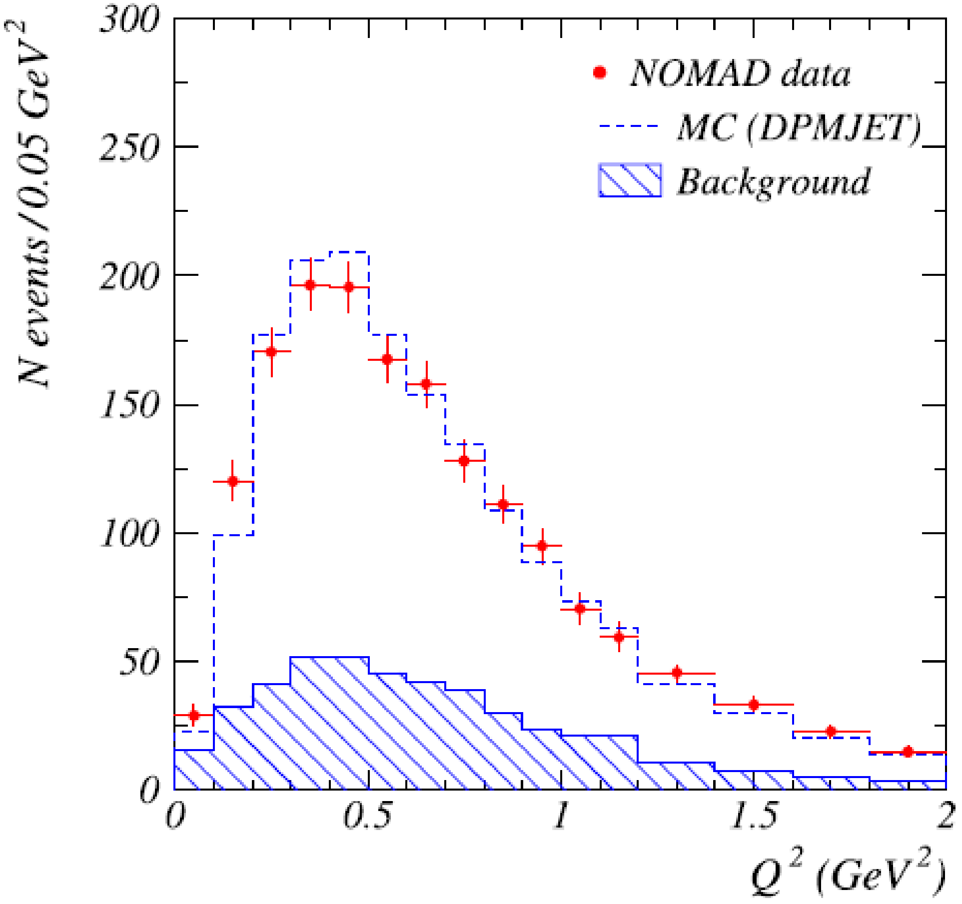
$$6 \left.\frac{dE_{0+}^{(-)}}{dq^2}\right|_{q^2=0} = \langle r_A^2 \rangle + \frac{3}{M}\left(\kappa^{\mathrm{V}} + \frac{1}{2}\right) + \frac{3}{64 f_\pi^2}\left(1 - \frac{12}{\pi^2}\right)$$

    - $M_A = 1.014 \pm 0.016$ GeV   Liesenfeld et al., PLB 468 (1999) 20

  - No indication of deviations from the dipole dep. in BNL, ANL data

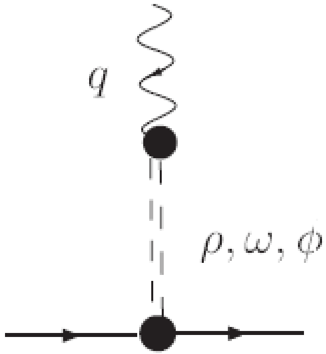
CCQE scattering on the nucleon

- No indication of deviations from the dipole dep. in NOMAD data (on ^{12}C)



CCQE scattering on the nucleon

- Why dipoles?



- Dipole behavior might arise from the contribution of two mesons with similar masses and opposite couplings

$$F_{1,2}(q^2) \sim \frac{a}{q^2 - m_{V1}^2} + \frac{(-a)}{q^2 - m_{V2}^2} = \frac{a\,(m_{V1}^2 - m_{V2}^2)}{(q^2 - m_{V1}^2)(q^2 - m_{V2}^2)}$$

QE scattering on the nucleon

- Cross section:
  - As an expansion in small variables $q^2, m_l^2 \ll M^2, E_\nu^2$

$$\frac{d\sigma}{dq^2} = \frac{1}{2\pi} G^2 c_{\mathrm{EW}}^2 \left[R - \frac{m_l^2}{4E_\nu^2} S + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

  - NC: $c_{\mathrm{NC}} = 1/4$

$$R_{\mathrm{NC}}^{(p)} = \alpha_{\mathrm{V}}^2 + (g_A - \Delta s)^2$$

$$T_{\mathrm{NC}}^{(p)} = \alpha_{\mathrm{V}}^2 - (g_A - \Delta s)^2 + 2\frac{E_\nu}{M}\left[\alpha_{\mathrm{V}} \mp (g_A - \Delta s)\right]^2 \mp 4\frac{E_\nu}{M}(g_A - \Delta s)\kappa_{\mathrm{NC}}^{(p)} - \left(\frac{E_\nu}{M}\kappa_{\mathrm{NC}}^{(p)}\right)^2$$
$$+ \quad 4E_\nu^2 \left\{ \alpha_{\mathrm{V}} \left[\frac{1}{3}\left(\alpha_{\mathrm{V}}\langle r_p^2\rangle - \langle r_n^2\rangle - \langle r_s^2\rangle\right) - \frac{1}{2M^2}\kappa_{\mathrm{NC}}^{(p)} \right] + \frac{1}{3}(g_A - \Delta s)\left(g_A\langle r_A^2\rangle - \Delta s\langle r_{As}^2\rangle\right) \right\}$$

$$R_{\mathrm{NC}}^{(n)} = 1 + (g_A + \Delta s)^2$$

$$T_{\mathrm{NC}}^{(n)} = 1 - (g_A + \Delta s)^2 + 2\frac{E_\nu}{M}\left[1 \mp (g_A + \Delta s)\right]^2 \pm 4\frac{E_\nu}{M}(g_A + \Delta s)\kappa_{\mathrm{NC}}^{(n)} - \left(\frac{E_\nu}{M}\kappa_{\mathrm{NC}}^{(n)}\right)^2$$
$$+ \quad 4E_\nu^2 \left\{ -\frac{1}{3}\left(\alpha_{\mathrm{V}}\langle r_n^2\rangle - \langle r_p^2\rangle - \langle r_s^2\rangle\right) + \frac{1}{2M^2}\kappa_{\mathrm{NC}}^{(n)} + \frac{1}{3}(g_A + \Delta s)\left(g_A\langle r_A^2\rangle + \Delta s\langle r_{As}^2\rangle\right) \right\}$$

$$\kappa_{\mathrm{NC}}^{(p)} = \alpha_{\mathrm{V}}(\mu_p - 1) - \mu_n - \mu_s \qquad \kappa_{\mathrm{NC}}^{(n)} = 1 - \mu_p + \alpha_{\mathrm{V}}\mu_n - \mu_s \qquad \alpha_{\mathrm{V}} = 1 - 4\sin^2\theta_W$$

QE scattering on the nucleon

- Strangeness content of the nucleon:
  - $\langle r_s^2 \rangle,\ \mu_s,\ \langle r_{As}^2 \rangle$ $\leftarrow$ insignificant
  - Δs strange axial coupling $\Leftrightarrow$ strange quark contribution to the spin

$$\left.\frac{d\sigma_{\mathrm{NC}}^{(p)}/dq^2}{d\sigma_{\mathrm{NC}}^{(n)}/dq^2}\right|_{q^2=0} = \frac{\alpha_{\mathrm{V}}^2 + (g_A - \Delta s)^2}{1 + (g_A + \Delta s)^2} \approx \frac{(g_A - \Delta s)^2}{1 + (g_A + \Delta s)^2} \approx \begin{cases} 0.62 & \text{if } \Delta s = 0 \\ 1.27 & \text{if } \Delta s = -0.3 \end{cases}$$

  - A recent global fit: Pate, Trujillo, arXiv:1308.5694

Weak Resonance excitation

- Resonances contribute to:
  - the inclusive $\nu_l \, N \to l \, X$ cross section
  - several exclusive channels: $\nu_l \, N \to l \, N' \, \pi$

$$\nu_l \, N \to l \, N' \, \gamma$$

$$\nu_l \, N \to l \, N' \, \eta$$

$$\nu_l \, N \to l \, \Lambda(\Sigma) \, \bar{K}$$

  - At $E_\nu \sim 1$ GeV (MiniBooNE, SciBooNE, T2K,...) $\Delta(1232)$ is dominant
  - At $E_\nu > 1$ GeV (MINERνA) N* become also important

Weak Resonance excitation

- CC R excitation: $\nu_l(k)\, N(p) \to l^-(k')\, R(p')$

$$\frac{d\sigma}{dk'_0 d\Omega'} = \frac{1}{32\pi^2}\frac{|\vec{k}'|}{k_0 M_N}\mathcal{A}(p')|\bar{\mathcal{M}}|^2$$ ← Inclusive cross section

$$\mathcal{A}(p') = \frac{M^*}{\pi}\frac{\Gamma(p')}{(p'^2 - M^{*2})^2 + M^{*2}\Gamma^2(p')}$$

$\Gamma(p')$ ← total momentum dependent width

$$\mathcal{M} = \frac{G_F \cos\theta_C}{\sqrt{2}} l^\alpha J_\alpha$$

$$l^\alpha = \bar{u}(k')\gamma^\alpha(1-\gamma_5)u(k)$$ ← leptonic current

$$J_\alpha = V_\alpha - A_\alpha$$ ← hadronic current

can be parametrized in terms of
N-R transition form factors

Weak Resonance excitation

- $\Delta(1232)$ $J^P=3/2^+$

$$J_\alpha = \bar{u}^\mu(p') \left[\left(\frac{C_3^V}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^V}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\alpha p'_\mu) + \frac{C_5^V}{M_N^2}(g_{\alpha\mu}q\cdot p - q_\alpha p_\mu) \right) \gamma_5 + \frac{C_3^A}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^A}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\beta p'_\mu) + C_5^A g_{\alpha\mu} + \frac{C_6^A}{M_N^2} q_\alpha q_\mu \right] u(p)$$

C^V_{3-5} , C^A_{3-6} $\leftarrow$ N-Δ transition form factors

- Rarita-Schwinger fields: spin 3/2

$$u_\mu(p, s_\Delta) = \sum_{\lambda,s} \left(1\lambda \frac{1}{2} s \middle| \frac{3}{2} s_\Delta \right) \epsilon_\mu(p,\lambda) u(p,s)$$

- Eq. of motion: $(\not{p} - M_\Delta)\, u_\mu = 0$

- with constrains: $\gamma^\mu u_\mu = p^\mu u_\mu = 0$

Weak Resonance excitation

- Second resonance peak: N*(1440), N*(1520), N*(1535)
  - N*(1440) $J^P=1/2^+$

$$J_\alpha = \bar{u}(p')\left[\frac{F_1^V}{(2M_N)^2}(\not{q}q_\alpha - q^2\gamma_\alpha) + i\frac{F_2^V}{2M_N}\sigma_{\alpha\beta}q^\beta - F_A\gamma_\alpha\gamma_5 - \frac{F_P}{M_N}\gamma_5 q_\alpha\right]u(p)$$

  - N*(1535) $J^P=1/2^-$

$$J_\alpha = \bar{u}(p')\left[\frac{F_1^V}{(2M_N)^2}(\not{q}q_\alpha - q^2\gamma_\alpha)\gamma_5 + i\frac{F_2^V}{2M_N}\sigma_{\alpha\beta}q^\beta\gamma_5 - F_A\gamma_\alpha - \frac{F_P}{M_N}q_\alpha\right]u(p)$$

  - N*(1520) $J^P=3/2^-$

$$J_\alpha = \bar{u}^\mu(p')\left[\frac{C_3^V}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^V}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\alpha p'_\mu) + \frac{C_5^V}{M_N^2}(g_{\alpha\mu}q\cdot p - q_\alpha p_\mu)\right.$$
$$\left. + \left(\frac{C_3^A}{M_N}(g_{\alpha\mu}\not{q} - q_\alpha\gamma_\mu) + \frac{C_4^A}{M_N^2}(g_{\alpha\mu}q\cdot p' - q_\beta p'_\mu) + C_5^A g_{\alpha\mu} + \frac{C_6^A}{M_N^2}q_\alpha q_\mu\right)\gamma_5\right]u(p)$$

Weak Resonance excitation

- Vector CC and NC form factors can be expressed in terms of EM ones
  - CC: $F^V_{1,2} = F^p_{1,2} - F^n_{1,2}$
  - NC: $\tilde{F}^{p(n)}_{1,2} = \left(\frac{1}{2} - 2\sin^2\theta_W\right) F^{p(n)}_{1,2} - F^{n(p)}_{1,2}$
  - The same applies for $C^V_{1,2,3}$
- Helicity amplitudes from π photo- and electro-production data

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 1/2 \middle| \epsilon^+_\mu J^\mu_{\mathrm{EM}} \middle| N, J_z = -1/2 \right\rangle \zeta$$

$$A_{3/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 3/2 \middle| \epsilon^+_\mu J^\mu_{\mathrm{EM}} \middle| N, J_z = 1/2 \right\rangle \zeta$$

$$S_{1/2} = -\sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\mathbf{q}|}{\sqrt{Q^2}} \left\langle R, J_z = 1/2 \middle| \epsilon^0_\mu J^\mu_{\mathrm{EM}} \middle| N, J_z = 1/2 \right\rangle \zeta$$

- Helicity amplitudes $\Rightarrow$ EM form factors

Weak Resonance excitation

- Axial transition form factors
  - Poorly known (if at all...)
  - PCAC: $q^\alpha A_\alpha \approx 0$
  - π-pole dominance of the pseudoscalar form factor: C_6^A

- $\Delta(1232)$ $J^P=3/2^+$

PCAC $\Rightarrow$ $C_6^A = -\frac{M_N^2}{q^2} C_5^A$

Using $\mathcal{L}_{\Delta N\pi} = -\frac{g_{\Delta N\pi}}{f_\pi} \bar{\Delta}_\mu (\partial^\mu \vec{\pi}) \vec{T}^\dagger N$ $\quad g_{\Delta N\pi} \Leftrightarrow \Gamma(N^* \to N\pi)$

π-pole dominance $\Rightarrow$ $C_6^A = -\sqrt{\frac{2}{3}} g_{N^* N\pi} F(q^2) \frac{M_N^2}{q^2 - m_\pi^2}$ $\quad F(0) = 1$

Therefore $C_5^A(0) = \sqrt{\frac{2}{3}} g_{\Delta N\pi} \leftarrow$ Goldberger-Treiman relation

$C_4^A = -\frac{1}{4} C_5^A \quad C_3^A = 0 \leftarrow$ Adler model

MAID

- Helicity amplitudes

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \left\langle R, J_z = 1/2 \left| \epsilon^{+}_{\mu} J^{\mu}_{\mathrm{EM}} \right| N, J_z = -1/2 \right\rangle \zeta$$

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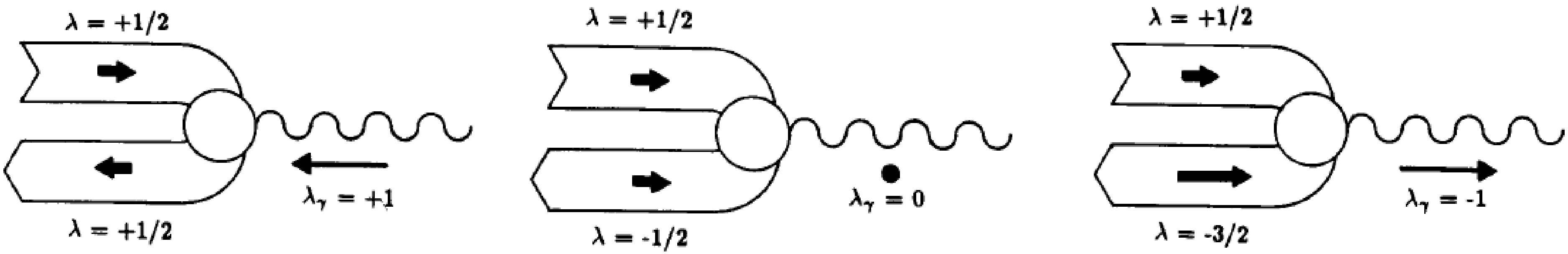
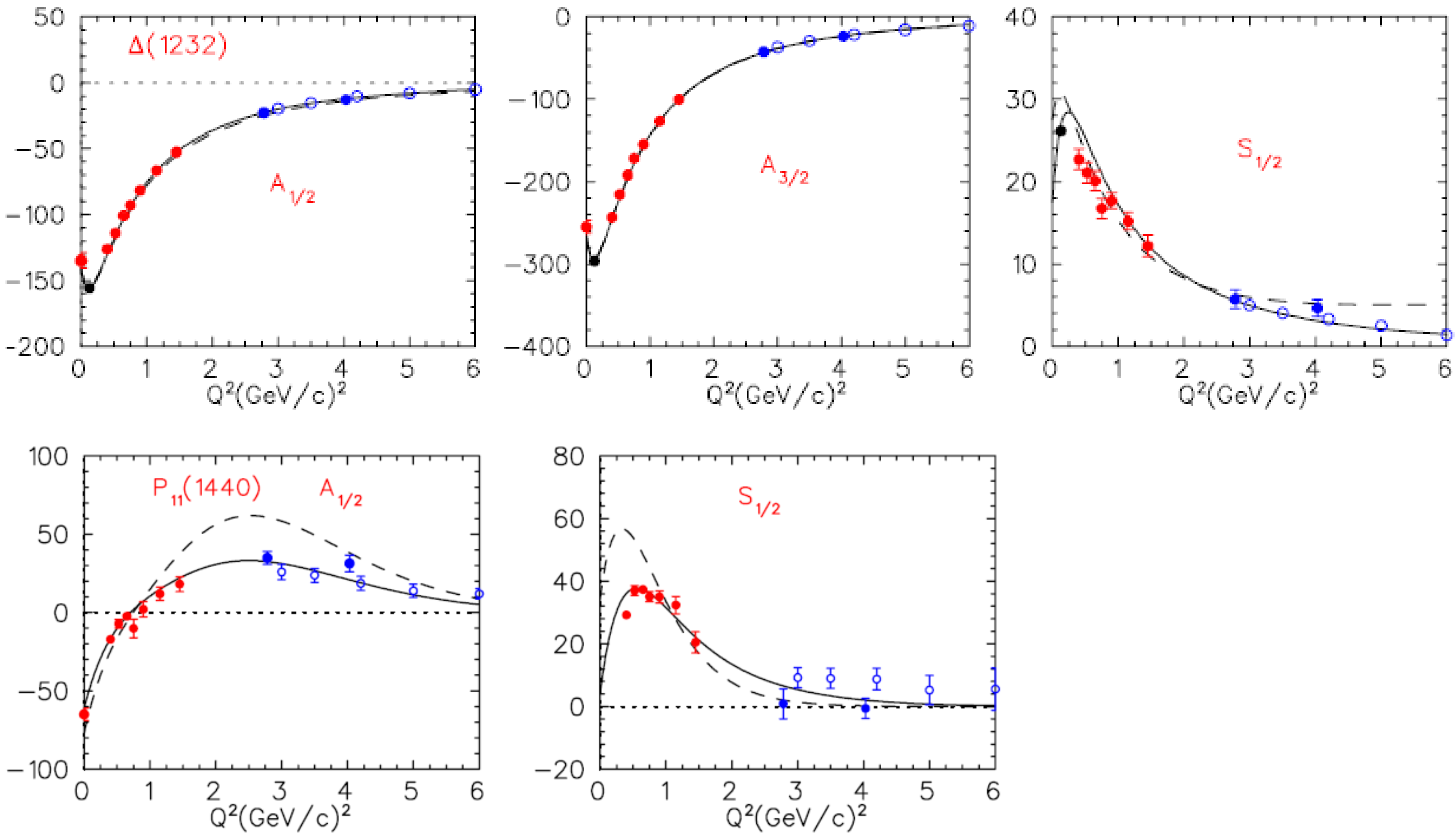


$A_{1/2}$ $S_{1/2}$ $A_{3/2}$

MAID

- Transition N-R e.m. helicity amplitudes extracted for all 4-star resonaces with W < 1.8 GeV
- For example:

Tiator et al., EPJ Special Topics 198 (2011)



MAID

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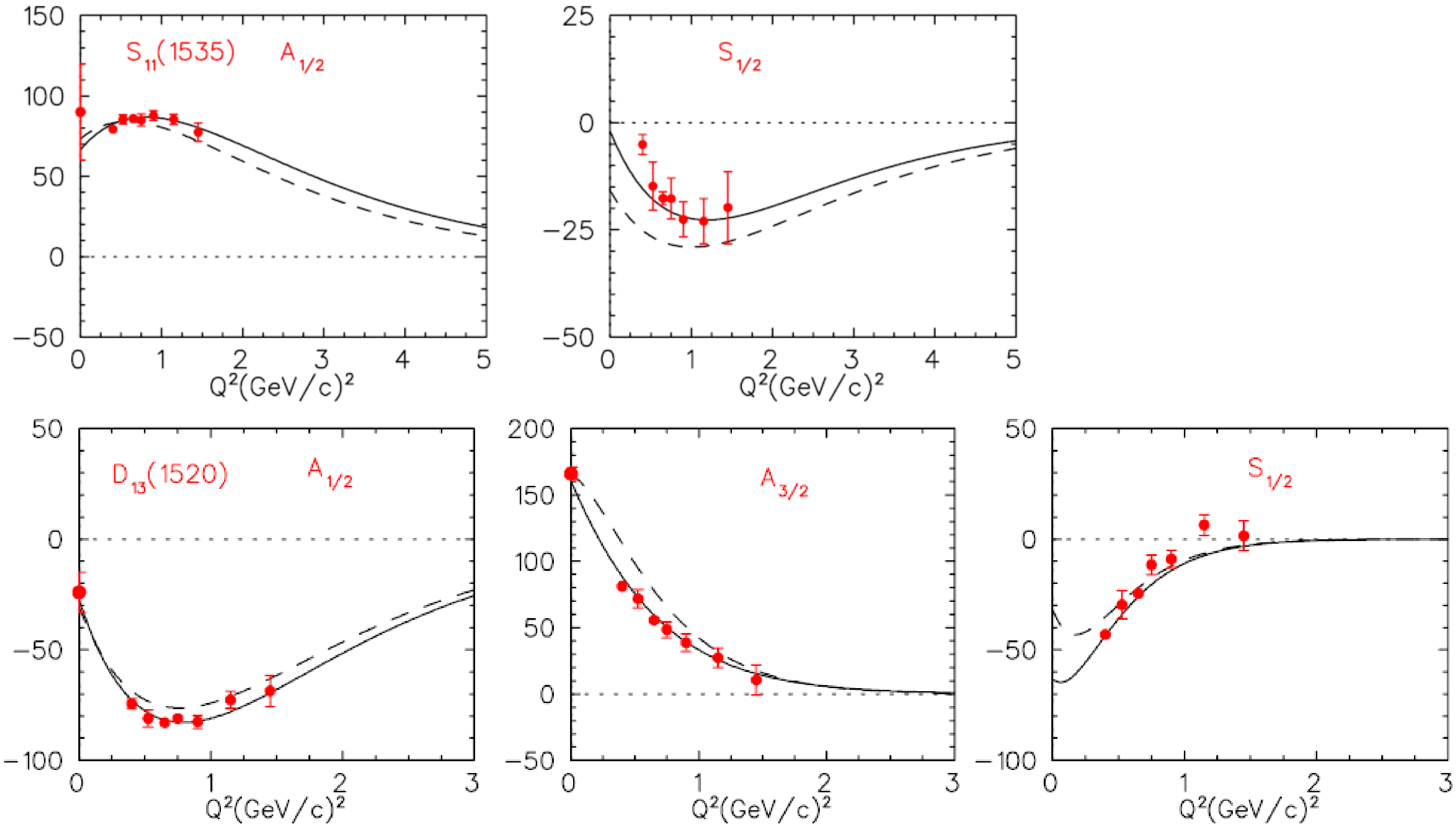

Weak Resonance excitation

- Axial transition form factors
  - Poorly known (if at all…)
  - PCAC: $q^\alpha A_\alpha \approx 0$
  - π-pole dominance of the pseudoscalar form factor: F_P

- N*(1440) $J^P=1/2^+$

PCAC $\Rightarrow$ $F_P = -\frac{(M^* + M_N)M_N}{q^2} F_A$

Using $\mathcal{L}_{N^*N\pi} = -\frac{g_{N^*N\pi}}{f_\pi} \bar{N}^* \gamma_\mu \gamma_5 (\partial^\mu \vec{\pi}) \vec{\tau} N$ $\quad g_{N^*N\pi} \Leftrightarrow \Gamma(N^* \to N\pi)$

π-pole dominance $\Rightarrow F_P = -2 g_{N^*N\pi} F(q^2) \frac{(M^* + M_N)M_N}{q^2 - m_\pi^2}$ $\quad F(0) = 1$

Therefore $F_A(0) = 2 g_{N^*N\pi}$ $\leftarrow$ Goldberger-Treiman relation

Educated guess: $F_A(q^2) = F_A(0) \left(1 - \frac{q^2}{{M_A}^2}\right)^{-2}$ $\quad M_A = 1 \text{ GeV}$

Weak Resonance excitation

- Axial transition form factors
  - Poorly known (if at all...)
  - PCAC: $q^\alpha A_\alpha \approx 0$
  - π-pole dominance of the pseudoscalar form factor: F_P

- N*(1535) $J^P=1/2^-$

PCAC $\Rightarrow$ $$F_P = -\frac{(M^* - M_N)M_N}{q^2} F_A$$

Using $$\mathcal{L}_{N^*N\pi} = -\frac{g_{N^*N\pi}}{f_\pi}\bar{N}^*\gamma_\mu(\partial^\mu \vec{\pi})\vec{\tau}N \qquad g_{N^*N\pi} \Leftrightarrow \Gamma(N^* \to N\pi)$$

π-pole dominance $\Rightarrow$ $$F_P = -2g_{N^*N\pi}F(q^2)\frac{(M^* - M_N)M_N}{q^2 - m_\pi^2} \qquad F(0) = 1$$

Therefore $F_A(0) = 2g_{N^*N\pi}$ $\leftarrow$ Goldberger-Treiman relation

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Weak Resonance excitation

- Axial transition form factors
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  - π-pole dominance of the pseudoscalar form factor: C_6^A

- N*(1520) $J^P=3/2^-$

PCAC $\Rightarrow$ $C_6^A = -\dfrac{M_N^2}{q^2 - m_\pi^2} C_5^A$

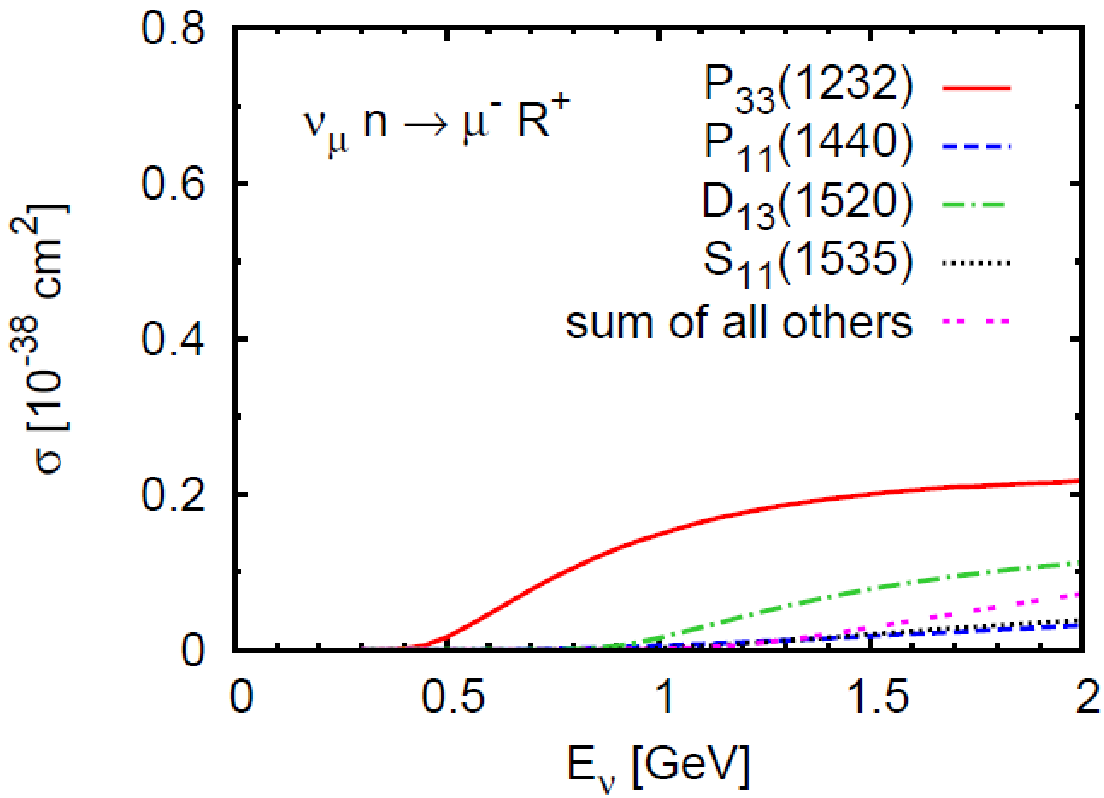
Using $\mathcal{L}_{N^* N\pi} = -\dfrac{g_{N^* N\pi}}{f_\pi} \bar{N}_\mu^* \gamma_5 (\partial^\mu \vec{\pi}) \vec{\tau} N$ $\quad g_{N^* N\pi} \Leftrightarrow \Gamma(N^* \to N\pi)$

π-pole dominance $\Rightarrow$ $C_6^A = 2 g_{N^* N\pi} F(q^2) \dfrac{M_N^2}{q^2 - m_\pi^2}$ $\quad F(0) = 1$

Therefore $C_5^A(0) = -2 g_{N^* N\pi}$ $\leftarrow$ Goldberger-Treiman relation

Educated guess: $C_5^A(q^2) = C_5^A(0) \left(1 - \dfrac{q^2}{{M_A}^2}\right)^{-2}$ $M_A = 1 \text{ GeV}$ $\quad C_3^4 = C_4^A = 0$

Inclusive resonance production



T. Leitner, O. Buss, LAR, U. Mosel, PRC 79 (2009)
T. Leitner, PhD Thesis, 2009

- At E_ν = 2 GeV, CCN*(1520)/CCΔ $\sim$ 0.5, CCN*(1440,1535)/CCΔ $\sim$ 0.22
- N*(1520) is important for $\nu_l \, N \to l \, N' \, \pi$

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