Neutrino-Nucleus Interactions From Elastic to Quasi-Elastic Region (1-100MeV)

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Outline

- 1. Neutrino Interactions for Neutrinos from Supernova
- 2. Form Factors (FF)
 - ✓ Nuclear FF (Electric and magnetic)
 - ✓ Nucleon FF
 - ✓ Hosstadter's results
 - ✓ Exercises
- 3. Neutrino Interactions in 10-100MeV
- 4. E398 ¹⁶O,¹²C(p,p' γ) Experiment at RCNP
- 5. Summary

Importance of Neutral-Current γ-production

---Neutrinos from SN explosion@10kpc---



1. vCH cross sections in the Supernova energy region



2. Form Factors

- Many of you will participate in NuInt15 Workshop and you will hear a lot about words like ,"Form Factor (FF)", "Dipole", "Vector and Axial Vector Form Factor", etc, there. In NuSTEC News-Letters, there have been many hot e-mail discussions over "Form Factors" since last week.
- It is very instructive to review the definition of "Form Factor", though it is a clasical topics.

Charge and Magnitization Density Distribution of the nucleons and nucleus –Form Factors

- Discovery of the structure of the nucleons---R.Hofstadter (Nobel Prize in 1961)
 - ✓ Rutherford started the study of nuclear size by studying deviations from Coulomb scattering of α -particles in 1919. Nuclear Radius~10⁻⁵ x (Atom).
 - ✓ Hofstadter et al used electron-nucleus elastic scattering, since (they thought) electron, or electromagnetic interaction, is "simpler" and better-understood than α -particle. 1953-. E_e=190MeV. Later E_e=420MeV.

De Broglie wavelength $\lambda = hc/E = 200 MeV \cdot fm/(E[MeV])$ must be <1fm.

- ✓ They started with meadium and heavy nuclei and and found, Radius $R=r_0xA^{1/3}$. ($r_0=1.1\pm0.1$ fm) and skin thickness t=2.4±0.3fm.
- ✓ They also found that this method can be applied to nucleons (proton and neutrons) and discovered the charge and magnetic structure of the nucleons.

→Internal Structure $\rho(\mathbf{r}) \approx$ Form Factor F(q) [Fourier Transform]

- The historical results are quoted by many textbooks. Let's look at the formalism and typical plots from the experiments, where Form Factors play a central role.
 - R.Hofstadter, Electron Scattering and Nuclear Structure, RMP.28,214-254(1956).
 - H.de Vries, C.W.de Jager and C.de Vries, Atom.Data Nucl.Data Tabl. 36,495 (1987).
 - T.W.Donnelly and I.Sick, RMP 56, 461(1984). For Updates.

Form Factor F(q) and Charge distribution $\rho(r)$

- How was it introduced?? \rightarrow Matrix Element in a Born approximation: $U_{fi} = \langle f | U | i \rangle = \frac{1}{V} \int_{V} e^{-i\vec{p}_{f}\cdot\vec{r}} U(\vec{r}) e^{i\vec{p}_{i}\cdot\vec{r}} d^{3}\vec{r} = \frac{1}{V} \int_{V} U(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^{3}\vec{r}, \quad with \ \vec{q} = \vec{p}_{i} - \vec{p}_{f}.$ (1)
- If the nucleus has a finite (charge) density distribution ρ(r'), an interaction (potential) U(r) is the sum of point interaction (potential) V(r) over each charge at r'. V(R) is a Coulomb potential.

$$U(\vec{r}) = \int_{V'} V(\vec{r} - \vec{r}') \rho(\vec{r}') d^{3}\vec{r}'$$

• Put U(r) into (1) and set $\vec{R} = \vec{r} - \vec{r}$ ', we obtain

$$U_{fi} = \frac{1}{V} \int_{V} U(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^{3}\vec{r} = \frac{1}{V} \int_{V} V(\vec{R}) e^{i\vec{q}\cdot\vec{R}} d^{3}\vec{R} \int_{V'} \rho(\vec{r}') e^{i\vec{q}\cdot\vec{r}'} d^{3}\vec{r}' = V_{fi}F(q)$$

• Where $F(q) = \int_{V} \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^{3}\vec{r}$ (3) is called a nuclear form factor. Then, charge distribution is obtained by Fourier transformation.

(4)

$$\rho(\vec{r}) = \frac{1}{(2\pi)^3} \iiint F(q) e^{-i\vec{q}\cdot\vec{r}} d\vec{q}$$

■ (Matric element)²= $|U_{fi}|^2 = |V_{fi}|^2 |F(q)|^2$ →

$$\frac{d\sigma}{d\Omega} = \left[\frac{d\sigma}{d\Omega}\right]_{Point} |F(q)|^2$$

(2)

 \vec{r} $\vec{R} = \vec{r} - \vec{r}'$

Form Factor

• If the nucleus (nucleon) has a structure $\rho(r)$, then the cross section is reduced by $F(q)^2$, as compared to that for no structure ($F(q)^2=1$.).

$$\frac{d\sigma}{d\Omega} = \left[\frac{d\sigma}{d\Omega}\right]_{Point} |F(q)|^2$$

- 1) Point, 2) Exponential->Dipole, 3) Gauss,
- 4) Uniform
- 5) Fermi Type: two parameters (c , t), c=radius and t=skin.

$$\rho(\vec{r}) = \frac{\rho_0}{1 + \exp[(r - c)/t]}$$

6) A Born approximation fails. A phase shift analysis must $b_{\rm -}$

Formulas

$$F(q) = \int_{V} \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} dV = \iiint_{V} \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} dx dy dz$$

$$F(q) = \int_{0}^{\infty} \frac{\sin(qr)}{qr} \rho(r) 4\pi r^{2} dr \qquad \iiint_{V} \rho(\vec{r}) dx dy dz = 1$$

$$\rho(\vec{r}) = \frac{1}{(2\pi)^{3}} \iiint_{V} F(q) e^{-i\vec{q}\cdot\vec{r}} d\vec{q}$$





Exercises

• Calculate F(q) for typical 1-4) distributions.

Hofstadter's results

- For medium and heavy nuclei,
 (c,t) were obtained.
 - C=(1.07+-0.02) fm
 - t=2.4+-0.3 fm







Size of the nucleus and the cross section

- If a is the size, FF is a function of qa.
- qa=2Easin(θ/2)
- To look for a finite structure {|F(q)|<1}, the region, qa<1, contributes to the cross section.
 - De Broglie wavelength $\lambda = hc/E = 200 MeV \cdot fm/(E[MeV])$ must be <1fm.
 - If E is low, q=0, then, F(0)=1. It looks like a point.
 - qa=2Easin(θ /2)<2 contributes.
 - If E becomes very large, Ea>>1, then, only forward angle θ~0, contributes to the cross section. The cross section saturates. F(q)²



Review of the formula

1) e-p (Point Charge)

-Rutherford Scattering

- 2) e-p (e:Dirac, p:point charge) -Mott Scattering
- 3) e-p (e:Dirac, p:Dirac)

-No Name. Still, interacion is point.

4) e-p (e:Dirac, p:Dirac)

$$\begin{split} \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} &= \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \\ \left(\frac{d\sigma}{d\Omega}\right)_{Mott} &= \frac{4\alpha^2}{q^4} E_e^{\prime 2} \cos^2 \frac{\theta}{2} \\ &= \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cos^2 \frac{\theta}{2} \\ \frac{d\sigma}{d\Omega} &= \frac{4\alpha^2 E_e^{\prime 2}}{q^4} \frac{E_e^{\prime}}{E_e} \cos^2 \frac{\theta}{2} \left[1 + 2\tau \tan^2(\theta/2)\right] \\ &= \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E_e^{\prime}}{E_e} \left[1 + 2\tau \tan^2(\theta/2)\right] \end{split}$$

-Proton has charge and magnetic distributions (Form Factors, F_1 , $F_2/G_E, G_M$). ->Rothenbluth Formula $da_{10} = 4a^2 E'_2 E'_2 = a^2 F_1 = a^2 F_2$

$$\frac{d\sigma_{eN}}{d\Omega} = \frac{4\alpha^2 E_e'^2}{q^4} \frac{E_e'}{E_e} \cos^2 \frac{\theta}{2} \left[F_1^2 + \tau \chi^2 F_2^2 + 2\tau (F_1 + \chi F_2)^2 \tan^2(\theta/2) \right]$$

Note:

*Gordon Decomposition for Dirac Current :

*Consider "structure" (Form Factors):

$$\begin{split} \bar{u}_{f}\gamma^{\mu}u_{I} &= \frac{1}{2m}\bar{u}_{f}((p_{f}+p_{I})^{\mu}+i\sigma^{\mu\nu}(p_{f}-p_{I})_{\nu})u_{I}\\ J_{\alpha} &= \bar{u}_{f}\left[F_{1}(q^{2})\frac{(p_{f}+p_{I})_{\alpha}}{2M} + \frac{F_{1}(q^{2})+\chi F_{2}(q^{2})}{2M}i\sigma_{\alpha\beta}q^{\beta}\right]u_{I}e^{i(p'-p)x}\\ &= \bar{u}_{f}\left[F_{1}(q^{2})\gamma_{\alpha} + \frac{\chi F_{2}(q^{2})}{2M}i\sigma_{\alpha\beta}q^{\beta}\right]u_{I}e^{i(p'-p)x} \end{split}$$

Hofstadter's results

- He started e-p scattering in 1954.
 - Consider magnetic scattering.
 Rothenbluth Formula.
 - Found the structure of nucleon.
 - Latest (2005)



Observable	Value \pm error						
$\langle (r_E^p)^2 \rangle^{1/2}$	$0.895\pm0.018{\rm fm}$						
$\langle (r_M^p)^2 \rangle^{1/2}$	$0.855\pm0.035\text{fm}$						
$\langle (r_E^n)^2 \rangle$	$-0.119\pm0.003~\text{fm}^2$						
$\langle (r_M^n)^2\rangle^{1/2}$	$0.87\pm0.01fm$						

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Hofstadter's results

TABLE VI. This table gives the radial parameters for the nuclei of column 1 and the appropriate charge (and magnetic) distributions. All quantities used in the table are defined in the text, except the parameters of the Hill model (used only for $_{32}Pb^{$038}$). All distances are given in units of 10^{-13} cm (one fermi unit). The accuracy in surface thickness parameter is about $\pm 10\%$ and may be somewhat poorer for the lighter elements where it is less well defined. The accuracy of the radial parameters is about $\pm 2\%$ except, possibly, in the case of Ta. The accuracy for gold is better than $\pm 2\%$. ρ_U in column 9 is the charge density in proton charge per cubic fermi for the equivalent uniform model and may be compared with Fig. 1 (b). The results for lithium and beryllium are to be considered preliminary.

Nucleus (1)	Type of charge distribution (see Table I) (2)	rms radius (3)	Radius of equivalent uniform model (R) (4)	$r_0 = \frac{R}{A^{\frac{1}{2}}}$ (5)	Skin thick- ness (6)	Half- density Radius c (7)	$r_1 = \frac{c}{A^{\frac{1}{2}}}$ (8)	р и (9)	A [‡] (10)	Comments (11)	Reference number (12)
1H1	III, IV, VI, VII mag- netic distribution similar	0.77±0.10	1.00	1.00				0.239	1.00	The charge distribu- tions in column 2 are equivalent to each other. The rms radius is a mean value for all. The magnetic dis- tribution is the same as that of the charge. The fact that $R=1.00$ in column 4 is acci- dental.	42, 55, 68
1D2	Charge distribution calculated from deu- teron wave function for Hulthèn, etc.,	1,96	2.53	2,01	•••	•••		0.0147	1.26		71
2He4 3Li ⁶ 3Li ⁷ 4Be ⁹ 6C ¹² 12Mg ²⁴ 14S ²³ 20Ca ⁴⁰ 22V ⁵¹ 27Co ³⁰ 46In ¹¹⁵ 41Sb ¹⁰² 72Ta ¹⁶¹	III XII XII XII XII <i>gU</i> <i>gU</i> <i>gU</i> <i>gU</i> Fermi Fermi Fermi Fermi Fermi Fermi Fermi Fermi pole	$\begin{array}{c} 1.61 \\ 2.78 \\ 2.71 \\ 3.04 \\ 2.37 \\ 2.98 \\ 3.04 \\ 3.19 \\ 3.52 \\ 3.59 \\ 3.83 \\ 4.50 \\ 4.63 \\ 5.50 \end{array}$	2.08 3.59 3.92 3.04 3.84 3.92 4.12 4.54 4.63 4.94 5.80 5.97 ~ 7.10	1.31 1.98 1.83 1.33 1.33 1.29 1.30 1.32 1.25 1.27 1.19 1.20 ~1.25	~2.0 2.6 2.8 2.6 2.5 2.2 2.5 2.3 2.5 ~2.8 ~2.8	~2.3 2.85 2.95 3.28 3.64 3.98 4.09 5.24 5.32 ~6.45	1.00 0.99 0.97 1.03 1.06 1.07 1.08 1.07 -1.14	0.053 0.0153 0.0167 0.0157 0.051 0.056 0.055 0.055 0.0662 0.0605 0.0572 0.0691	$1.59 \\ 1.82 \\ 1.19 \\ 2.08 \\ 2.29 \\ 2.88 \\ 3.04 \\ 3.18 \\ 3.42 \\ 3.71 \\ 3.89 \\ 4.87 \\ 4.96 \\ 5.65 \\ 1.55 \\ $	$\alpha = 4/3$ The radial distances should be considered "effective" radii in view of the quadru-	42, 49 75 75 75 77 46 46 46 33 33 33 33 33 33 61, 88
78Au ¹⁹⁷ 82Pb ²⁰⁸	Fermi Hill ϵt al. (reference 9) $n = 10$, $s = 0$	$^{5.32}_{\sim 5.42}$	$\sim^{6.87}_{-7.0}$	$\substack{1.180\\1.18}$	$\sim^{2.32}_{\sim 2.3}$	6.38 ~6.5	$^{1.096}_{\sim 1.09}$	0.0581 0.057	5.82 5.93	pole effects. The model of Hill et al. is similar to the	33 9
saBi209	Fermi	5.52	7.13	1,20	2.7	6.47	1.09	0.054	5,935	Permi model	33

Proton, Neutron form factors (BBBA5@NuInt05)

• FFs (GE, GM) are not a simple dipole.

$$G(Q^{2}) = \frac{\sum_{k=0}^{2} a_{k} \tau^{k}}{1 + \sum_{k=1}^{4} b_{k} \tau^{k}}$$

R.Bradford et al., Nucl. Phys. Proc. Suppl.159:127-132 (2006).

BBBA05/ Dipole (G/D)



2. Overall Picture of the v-A cross section



3. vC cross sections in the SN energy region



Axial Vector is dominant in low energy nuclear excitations in Both CC and NC v-¹⁶O,¹²C reactions



•CC reactions too.



Emergence of Giant Resonances



NC v-¹⁶O,¹²C reacting in , Co', NPA761('05)



- E398: I. Ou, Y. Yamada, D.Fukuda, T.Shirahige, T. Yano, T. Mori, Y. Koshio, M. Sakuda, (Okayama), A. Tamii, N.Aoi, M.Yosoi, E. Ideguchi, T. Suzuki, T. Hashimoto, C. Iwamoto, K. Miki, T. Ito, T. Yamamoto (RCNP), H. Akimune (Konan), T.Kawabata (Kyoto)
- [Goal]: We measure the probability of γ -ray emission (E $_{\gamma}$ >2 MeV) from giant resonances of ¹⁶O and ¹²C, at ±1% stat. accuracy, as the functions of excitation energy (E $_x$).
 - Definition: the γ -ray emission probability (E_v>2MeV)=
 - − (Number of γ-rays observed for E_{γ} >2 MeV)/(Number of events excited in the range Ex=15-30 MeV, each Ex bin) →Fig.
- [Importance]: Data for $vO \rightarrow vO^* \rightarrow \gamma$ and $vC \rightarrow vC^* \rightarrow \gamma$ do not exist and they are very important to neutrino physics. RCNP Grand-Raiden is the best place for this experiment.
- Proposal was approved in March, 2013 and Experiment was finished in May, 2014.

2. ¹⁶O, ¹²C(p,p' γ) experimental setup



E398 (May 16-27, 2014)



(1) Analysis of Grand Raiden: Excitation Energy Spectrum -I.Ou (Okayama)@JPS 2015.09.27



Values of peak energy agree with the known levels within calibration error (~40 keV).

Ref. Table of Isotopes, 8th ed.

(2) γ -rays from giant resonance are seen:¹⁶O



emit 5~9MeV-γ rays agree with statistical calc by Langanke.

(5) Status of E398 Quick look at γ-ray spectrum from giant resonances (1/10 of the total data used)

- Our data agree qualitativly with theoretical picture by Langanke et. al. "γ-rays are emitted from daughter nuclei after p/n decay"
- Next Step: Quantitative determination of emission probability(Pr)
- Future: After we publish the result, we would like to do an experiment at **GRFBL** (4-15 Degrees).

Summary

- Form Factors Charge and Magtetization density distributions
 - H.de Vries, C.W.de Jager and C.de Vries, Atom.Data Nucl.Data Tabl. 36,495 (1987).
 - T.W.Donnelly and I.Sick, RMP 56, 461(1984). For Updates.
 - Nucleon FF Recent NuInt Proceedings
 - R.Hofstadter, Electron Scattering and Nuclear Structure, RMP.28,214-254(1956).
- Neutrino Interactions in 110-100 MeV
- We would like to finilise our analysis and give new experimental data on O*/C*->gamma emission probability.