Quasielastic Neutrino Nucleus Scattering

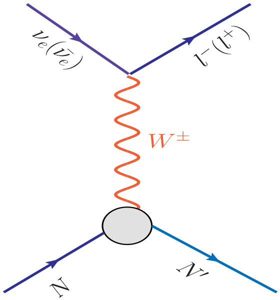
S. K. Singh

Aligarh Muslim University
Aligarh, India

Quasielastic and Elastic $\nu-$scattering processes on Nucleons and Nuclei

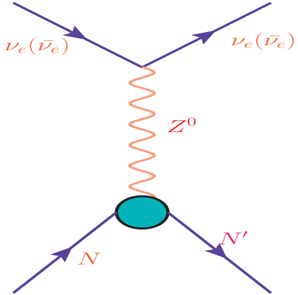
Basic Processes on Nucleon Target

Quasi elastic(CC)



$N = n, p$ and $N' = p, n$

Elastic(NC)

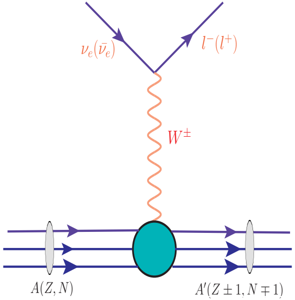


$N, N' = n, p$

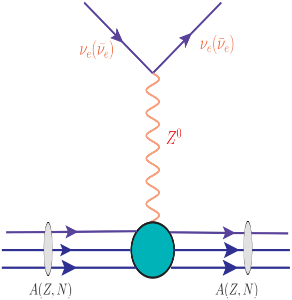
Quasielastic and Elastic $\nu-$scattering processes on Nucleons and Nuclei

Nuclear Target

Quasi elastic(CC)



Elastic(NC)



Why are they important ??

Structure of hadrons and nuclei

- Nucleon Form factors

Size, charge-distribution, axial charge, Magnetic moment distributions, etc.

- Strangeness content of the Nucleon
- Test the models for Nucleon Structure
- Precision tests of QCD

Non nucleonic degrees of freedom in nuclei

- Short and long range correlations, Meson exchange currents in vector and axial vector sector, sub nucleonic degrees of freedom in nuclei

Why are they important ??

Structure of Weak interaction and its applications

- Tests of CVC and PCAC
- Tests of second class currents: G invariance & T invariance
- Precision tests of standard model and presence of NSI
- Dynamics of core collapse processes
- Tests of models for nucleosynthesis
- Tests for solar models and models for Earth's interior

Why are they important ??

Neutrino properties and interactions

- Neutrino mass, magnetic moment
- Neutrino oscillation parameters
- CP violation in lepton sector
- Mass Hierarchy of neutrino mass Δm_i^2

Specifically in the Context of Neutrino Oscillations

- Most important input in analysis of present oscillation experiments in $\sim$ GeV region

Specifically in the Context of Neutrino Oscillations

- Most important input in analysis of present oscillation experiments in $\sim$ GeV region
- Sources of largest signal in

Specifically in the Context of Neutrino Oscillations

- Most important input in analysis of present oscillation experiments in $\sim$ GeV region
- Sources of largest signal in
  - appearance channel
    in which you produce neutrinos of one type but observe neutrinos of a different type – the signal is the observation of a non-zero number of this different type (after allowing for any backgrounds).

$$\nu_l(\bar{\nu}_l) \rightarrow \nu_{l'}(\bar{\nu}_{l'}) \qquad\qquad l \neq l'$$

Specifically in the Context of Neutrino Oscillations

- Most important input in analysis of present oscillation experiments in $\sim$ GeV region
- Sources of largest signal in
  - appearance channel
    in which you produce neutrinos of one type but observe neutrinos of a different type – the signal is the observation of a non-zero number of this different type (after allowing for any backgrounds).

    $$\nu_l(\bar{\nu}_l) \to \nu_{l'}(\bar{\nu_{l'}}) \qquad l \neq l'$$

  - disappearance channel
    in which you know how many neutrinos of a specified type you produce, and you count the number of that same type of neutrinos that you detect at distance L – the oscillation signal is a deficit in the number observed;

    $$\nu_l(\bar{\nu}_l) \to \nu_l(\bar{\nu}_l)$$

Specifically in the Context of Neutrino Oscillations

- Validation of $\nu_\mu(\bar{\nu_\mu})$ flux from cross section measurements in ND for various nuclei like $^{12}C,^{16}O,^{37}Ar,^{56}Fe$.
- Determination of E_ν energy in terms of measured E_μ and θ.

$$E_\nu = \frac{2M_n E_l - m_l^2 - (M_p^2 - M_n^2)}{2\left(M_n - E_l + |\vec{k}'|\cos\theta\right)}$$

  Nuclear effects arise due to
  - Binding energy
  - Fermi motion
  - QE like events(meson exchange currents, multi nucleon correlation...)
- E_ν is important in $\nu-$oscillation analysis because $P(\nu_e \to \nu_\mu) = \sin^2(2\theta)\sin^2\left(1.27 \bigtriangleup m^2 \frac{L(km)}{E_\nu(GeV)}\right)$

Specifically in the Context of Neutrino Oscillations

- A comparative study of Neutrino- Nucleon cross section for $\nu_\mu(\bar{\nu_\mu})$ in appearance channel and $\nu_e(\bar{\nu_e})$ in disappearance channel induced reactions are important for studies of CP violation, Mass Hierarchy.
- A comparative study of $\nu_\mu(\nu_e)$ and $\bar{\nu_\mu}(\bar{\nu_e})$ are important for the determination of neutrino oscillation parameters.
- A knowledge of QE like events gives information about various nuclear effects beyond single particle Model like
  - Short range and long range correlations
  - Meson Exchange currents
  - Multi-nucleon contributions
- A precise knowledge of QE and QE like events helps to reduce systematic errors in the analysis of all neutrinos oscillation experiments.

QE $\nu-$Nucleus scattering in the historical perspective

- First Calculation of QE $\nu-$Nucleus Scattering
  In 1934 Bethe and Peierls after Pauli(1930), proposed ν. Fermi(1933) proposed theory of β decay. Similar matrix elements are involved in nuclear transitions :

QE $\nu-$Nucleus scattering in the historical perspective

- First Calculation of QE $\nu-$Nucleus Scattering
  In 1934 Bethe and Peierls after Pauli(1930), proposed ν. Fermi(1933) proposed theory of β decay. Similar matrix elements are involved in nuclear transitions :

$$\sigma = \frac{A}{T_{\frac{1}{2}}}; \quad A = Area \times time \approx R^2 \times \frac{R}{C}$$

QE $\nu-$Nucleus scattering in the historical perspective

- First Calculation of QE $\nu-$Nucleus Scattering
  In 1934 Bethe and Peierls after Pauli(1930), proposed ν. Fermi(1933) proposed theory of β decay. Similar matrix elements are involved in nuclear transitions :

$$\sigma = \frac{A}{T_{\frac{1}{2}}}; \quad A = Area \times time \approx R^2 \times \frac{R}{C}$$

$R = \frac{\hbar}{m_e C} \longrightarrow \sigma \approx 10^{-44} cm^2$

QE $\nu-$Nucleus scattering in the historical perspective

- First Calculation of QE $\nu-$Nucleus Scattering
  In 1934 Bethe and Peierls after Pauli(1930), proposed ν. Fermi(1933) proposed theory of β decay. Similar matrix elements are involved in nuclear transitions :

$$\sigma = \frac{A}{T_{\frac{1}{2}}}; \quad A = Area \times time \approx R^2 \times \frac{R}{C}$$

$R = \frac{\hbar}{m_e C} \longrightarrow \sigma \approx 10^{-44} cm^2 \longrightarrow$ unobservable

QE $\nu-$Nucleus scattering in the historical perspective

- First Calculation of QE $\nu-$Nucleus Scattering
  In 1934 Bethe and Peierls after Pauli(1930), proposed ν. Fermi(1933) proposed theory of β decay. Similar matrix elements are involved in nuclear transitions :

$$\sigma = \frac{A}{T_{\frac{1}{2}}}; \quad A = Area \times time \approx R^2 \times \frac{R}{C}$$

  $R = \frac{\hbar}{m_e C} \longrightarrow \sigma \approx 10^{-44} cm^2 \longrightarrow$ unobservable
- 1946 Pontecorvo proposed experiments at reactors with high $\bar{\nu}_e$ fluxes

QE $\nu-$Nucleus scattering in the historical perspective

- First Calculation of QE $\nu-$Nucleus Scattering
  In 1934 Bethe and Peierls after Pauli(1930), proposed ν. Fermi(1933) proposed theory of β decay. Similar matrix elements are involved in nuclear transitions :

$$\sigma = \frac{A}{T_{\frac{1}{2}}}; \quad A = Area \times time \approx R^2 \times \frac{R}{C}$$

  $R = \frac{\hbar}{m_e C} \longrightarrow \sigma \approx 10^{-44} cm^2 \longrightarrow$ unobservable
- 1946 Pontecorvo proposed experiments at reactors with high $\bar{\nu}_e$ fluxes
- 1953-56 Reines and Cowan $\bar{\nu_e} + p \rightarrow n + e^+$

QE $\nu-$Nucleus scattering in the historical perspective

- First Calculation of QE $\nu-$Nucleus Scattering
  In 1934 Bethe and Peierls after Pauli(1930), proposed ν. Fermi(1933) proposed theory of β decay. Similar matrix elements are involved in nuclear transitions :

$$\sigma = \frac{A}{T_{\frac{1}{2}}}; \quad A = Area \times time \approx R^2 \times \frac{R}{C}$$

  $R = \frac{\hbar}{m_e C} \longrightarrow \sigma \approx 10^{-44} cm^2 \longrightarrow$ unobservable
- 1946 Pontecorvo proposed experiments at reactors with high $\bar{\nu}_e$ fluxes
- 1953-56 Reines and Cowan $\bar{\nu_e} + p \rightarrow n + e^+$
- 1956 Davis $\nu_e \neq \bar{\nu_e}$; $\quad \bar{\nu_e} + Cl \nrightarrow Ar + e^-$

Both experiments are performed at Savanah river reactor site

$$\sigma = \frac{1}{Tc}\left(\frac{\hbar}{m_e c}\right)^3 \approx 10^{-44} cm^2$$

$$\sigma = \frac{1}{Tc}\left(\frac{\hbar}{m_e c}\right)^3 \approx 10^{-44} cm^2$$

Fission and S.N. reaction

$$\bar{\nu_e} + p \longrightarrow n + e^+;$$

$$\bar{\nu_e} + d \longrightarrow n + n + e^+$$

$$\sigma = \frac{1}{Tc}\left(\frac{\hbar}{m_e c}\right)^3 \approx 10^{-44} cm^2$$

Fission and S.N. reaction

$$\bar{\nu_e} + p \longrightarrow n + e^+;$$

$$\bar{\nu_e} + d \longrightarrow n + n + e^+$$

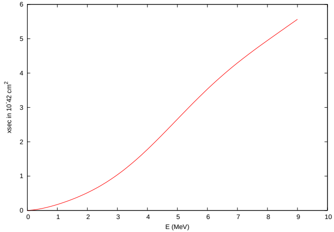
Later at accelerator neutrinos, the following reaction was studied in detail

$$\nu_\mu + d \longrightarrow \mu^- + p + p$$

■ Problem :

Calculate reaction cross section for the reactions.

$$\bar{\nu}_e + p \longrightarrow n + e^+;$$



- After this many people proposed experiments with accelerator neutrinos in late 1950's Cowan(1956), Markov(1960), Pontecorvo(1958),Schwartz(1960), Asarav(1960)
- Theoretical calculations first done in Fermi theory with V-A interactions for free nucleons, Lee and Yang(1960), Cabibbo and Gatto(1960), Yamaguchi(1960)
- With nuclear effects calculated by, Berman(1961), Loveseth(1963), Bell and Llewellyn-Smith(1964)
- Many experiments have been done on QE $\nu-$ Nucleus scattering in the entire energy region of neutrinos with reactor, solar, atmospheric and accelerator neutrinos.
  A list is given below:
  - Low energy $\nu-$N scattering with reactor, solar, atmospheric and accelerator neutrinos.
  - Intermediate and high energy experiments with accelerator neutrino

Reactor $\bar{\nu}$	Target	Year
Savannah River	$^{37}Cl,\ d$	1953-56, 1979
ILL Greneble	$H,\ d$	1981, 1995
Gosgen	H	1986
Krasnoyavsk	$H,\ d$	1987, 2000
Rovno	$H,\ d$	1991
Bugey	H	1994-95
Palo Verde	$H,\ d$	2001
CHOOZ	$H,\ d$	2003
KamLAND	H_2O	2011
Daya Bay	H_2O	2011
RENO	H_2O	2011
JUNO	H_2O	2014

Solar ν	**Target**	**Year**
Homestake	^{37}Cl	1967
SAGE	^{71}Ga	1990
GALLEX	^{71}Ga	1990
SNO, SNO+	D_2O	1999

Atmospheric ν	**Target**	**Year**
KGF	$Fe,\ Ne$	1965
KAMIOKA	H_2O	1983
SuperK	H_2O	1999
ICECUBE	Ice	2006

Other sources of ν	**Target**	**Year**
LAMPF	H	1980
GALLEX	^{71}Ga	1991
KARMEN	$^{12}C,\ ^{56}Fe$	1991-2005
LSND	$^{12}C,\ ^{127}I$	1997-2003
SAGE	^{71}Ga	1999-2006

Accelerator ν	Target	Year
GGM(CERN)	$C_2H_6,\ CF_3Dr$	1964-79
ANL	$Fe,\ D_2$	1969-1982
BNL	$D_2,\ H_2$	1980-83
FNAL	$D_2,\ Ne-H_2$	1982-84
Serpukhov	Al	1985
SKAT	CF_3Br	1988-92
BEBC	D_2	1990
NOMAD	CH_2	2000
MiniBooNE	CH_2	2002
K2K	$CH_2,\ H_2O$	2003-2004
SciBooNE	CH	2007
ArgoNeuT	Ar	2009
MicroBooNE	Ar	2009
MINERνA	C, Fe, Pb	2009
NOνA	CH	2010
T2K	H_2O	2010
LBNO	Ar, CH_2	future
DUNE	Ar, CH_2	future

- After this many people proposed experiments with accelerator neutrinos in late 1950's Cowan(1956), Markov(1960), Pontecorvo(1958),Schwartz(1960), Asarav(1960)
- Theoretical calculations first done in Fermi theory with V-A interactions for free nucleons, Lee and Yang(1960), Cabibbo and Gatto(1960), Yamaguchi(1960)
- With nuclear effects calculated by, Berman(1961), Loveseth(1963), Bell and Llewellyn-Smith(1964)
- Many experiments have been done on QE $\nu-$ Nucleus scattering in the entire energy region of neutrinos with reactor, solar, atmospheric and accelerator neutrinos.

  A list is given below:
  - Low energy $\nu-$N scattering with reactor, solar, atmospheric and accelerator neutrinos.
  - Intermediate and high energy experiments with accelerator neutrino

Theory of QE $\nu-$ Nucleus scattering

Two ingredients are needed

1. Theory of $\nu-$Nucleon scattering

- Standard model
- Non Standard Interaction and Beyond Standard Model physics

Theory of QE $\nu-$ Nucleus scattering

Two ingredients are needed

1. Theory of $\nu-$Nucleon scattering

- Standard model
- Non Standard Interaction and Beyond Standard Model physics

2. Nuclear model to describe Nucleons bound in Nucleus

- Fermi Gas Model and its various versions
- Shell Model with two body correlations treated in various ways.
- Relativistic Mean Field theoretical Models
- Relativistic Green Function approach with complex optical potential
- Superscaling Approximation(SuSA)

Theory of QE $\nu-$ Nucleon scattering

$\nu-$N scattering in the Standard Model

Quasi elastic

$$|\Delta S| = 0$$

$$\nu_l + n \longrightarrow l^- + p$$

$$\bar{\nu}_l + p \longrightarrow l^+ + n$$

$$|\Delta S| = 1$$

$$\bar{\nu}_l + p \longrightarrow l^+ + \Lambda^0$$

$$\bar{\nu}_l + p \longrightarrow l^+ + \Sigma^0$$

$$\bar{\nu}_l + n \longrightarrow l^+ + \Sigma^-$$

Elastic scattering

$$\nu_l(\bar{\nu}_l) + p \longrightarrow \nu_l(\bar{\nu}_l) + p$$

$$\nu_l(\bar{\nu}_l) + n \longrightarrow \nu_l(\bar{\nu}_l) + n$$

Theory of QE $\nu-$ Nucleon scattering

ν–N scattering in the Standard Model

Quasi elastic

$$\nu_l + n \quad \longrightarrow \quad l^- + \Sigma^+ ; \Delta Q = -\Delta S$$

$$\left.\begin{array}{l} \nu_l(\bar{\nu}_l) + p \longrightarrow \nu_l(\bar{\nu}_l) + \Sigma^0 \\ \nu_l(\bar{\nu}_l) + p \longrightarrow \nu_l(\bar{\nu}_l) + \Lambda^0 \end{array}\right\} FCNC$$

Elastic

Not allowed in standard model:
Beyond Standard Model ??

$$\nu_l(\bar{\nu}_l) + n \longrightarrow \nu_l(\bar{\nu}_l) + n$$

Theory of QE $\nu-$ Nucleon scattering

Kinematics: Free nucleons and nuclear effects

$$\nu_l(k) + n(p) \longrightarrow l^-(k') + p(p')$$

$$q_\mu = (k - k')_\mu = (p' - p)_\mu;$$

$$q^2 = m_\mu^2 - 2E_\nu E_l + 2E_\nu |\vec{k}'| \cos\theta$$

Theory of QE $\nu-$ Nucleon scattering

Kinematics: Free nucleons and nuclear effects

$$\nu_l(k) + n(p) \longrightarrow l^-(k') + p(p')$$

$$q_\mu = (k-k')_\mu = (p'-p)_\mu;$$

$$q^2 = m_\mu^2 - 2E_\nu E_l + 2E_\nu|\vec{k}'|\cos\theta$$

Q.E. kinematics: $p'^2 = p^2 \Longrightarrow (p+q)^2$

Theory of QE $\nu-$ Nucleon scattering

Kinematics: Free nucleons and nuclear effects

$$\nu_l(k) + n(p) \longrightarrow l^-(k') + p(p')$$

$$q_\mu = (k-k')_\mu = (p'-p)_\mu;$$

$$q^2 = m_\mu^2 - 2E_\nu E_l + 2E_\nu |\vec{k}'| \cos\theta$$

Q.E. kinematics: $p'^2 = p^2 \Longrightarrow (p+q)^2$

In lab frame

$$q_0 = \frac{-q^2 + M_p^2 - M_n^2}{2M_n}$$

$$E_\nu = \frac{2M_n E_l - m_l^2 - (M_p^2 - M_n^2)}{2\left(M_n - E_l + \sqrt{E_l^2 - m_l^2}\cos\theta\right)}$$

$$E_\nu = \frac{E_l}{1 - \frac{E_l(1-cos\theta)}{M}}$$

$$E_l = \frac{E_\nu}{1 + \frac{E_\nu(1-cos\theta)}{M}}$$

In the limit $m_l = 0$

Theory of QE $\nu-$ Nucleon scattering

Kinematics: Free nucleons and nuclear effects

$$\nu_l(k) + n(p) \longrightarrow l^-(k') + p(p')$$

$$q_\mu = (k - k')_\mu = (p' - p)_\mu;$$

$$q^2 = m_\mu^2 - 2E_\nu E_l + 2E_\nu |\vec{k}'| \cos\theta$$

Q.E. kinematics: $p'^2 = p^2 \Longrightarrow (p+q)^2$

In lab frame

$$q_0 = \frac{-q^2 + M_p^2 - M_n^2}{2M_n}$$

$$E_\nu = \frac{2M_n E_l - m_l^2 - (M_p^2 - M_n^2)}{2\left(M_n - E_l + \sqrt{E_l^2 - m_l^2}\cos\theta\right)}$$

$$E_\nu = \frac{E_l}{1 - \frac{E_l(1-cos\theta)}{M}}$$

$$E_l = \frac{E_\nu}{1 + \frac{E_\nu(1-cos\theta)}{M}}$$

In the limit $m_l = 0$

Problem : Derive expressions for E_ν & $E_l(m_l \neq 0)$

- The quasi-elastic peak will be at

$$q_0 = \Delta E = \frac{-q^2}{2M}$$

- But in nuclei
    -

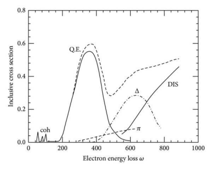
- The quasi-elastic peak will be at

$$q_0 = \Delta E = \frac{-q^2}{2M}$$

- But in nuclei
  - The peak will be shifted due to binding energy

$$E_\nu = \frac{2(M_n - E_B)E_l' - m_l^2 - E_B(E_B - 2M_n) - (M_p^2 - M_n^2)}{2\left(M_n - E_B - E_l + |\vec{k}'|\cos\theta\right)}$$

  - Instead of δ function, peak at $q_0 = \frac{-q^2}{2M}$, the peak will be broadened due to Fermi motion and width will be a measure of Fermi momentum distribution of nuclei $\simeq q_0 \simeq \frac{(\vec{p}+\vec{q})^2}{2M_n}$
  - It is indeed so in the case of electron scattering.



  - In case of neutrino scattering where neutrinos have a energy spectrum, the situation is more complicated.

E_ν reconstruction is affected by the nuclear effects in the following way:

- Smearing due to Fermi motion may introduce an error of about 60 MeV.
- More important are the QE like events, whose energy dependence is quite different than genuine QE events. May lead to an energy shift of about 150MeV around $E_\nu \sim 1GeV$.
- The lepton energy(and angles) of these QE like events correspond to the scattering from strongly correlated nucleons(not quasi-free) or mesons in flight or nucleons in excited state for which the above kinematics does not hold.
- One needs theoretical inputs for description of QE like events to make correction for those events in determination of E_ν.

Neutrino nucleon scattering

Interaction Lagrangian and matrix element in SM

$$\mathcal{L}_{int} = -\frac{g}{2\sqrt{2}} J_\mu^{CC} W^{+\mu} + h.c - \frac{g}{2\cos\theta_w} J_\mu^{NC} Z^\mu$$

$$\frac{G_f}{\sqrt{2}} = \frac{g^2}{8M_w^2}, \frac{M_W}{M_Z} = \cos\theta_w$$

Neutrino nucleon scattering

Interaction Lagrangian and matrix element in SM

$$\mathcal{L}_{int} = -\frac{g}{2\sqrt{2}} J_\mu^{CC} W^{+\mu} + h.c - \frac{g}{2\cos\theta_w} J_\mu^{NC} Z^\mu$$

$$\frac{G_f}{\sqrt{2}} = \frac{g^2}{8M_w^2}, \frac{M_W}{M_Z} = \cos\theta_w$$

with

$$J_\mu^{CC} = \bar{P}\gamma_\mu(1-\gamma_5) V_{CKM} N + \bar{\nu}_l\gamma_\mu(1-\gamma_5)e + h.c.$$

with $P = (u, c, t),\ N = (d, s, c),\ V_{CKM} = CKM\ \ matrix$

Neutrino nucleon scattering

Interaction Lagrangian and matrix element in SM

$$\mathcal{L}_{int} = -\frac{g}{2\sqrt{2}} J_\mu^{CC} W^{+\mu} + h.c - \frac{g}{2\cos\theta_w} J_\mu^{NC} Z^\mu$$

$$\frac{G_f}{\sqrt{2}} = \frac{g^2}{8M_w^2}, \frac{M_W}{M_Z} = \cos\theta_w$$

with

$$J_\mu^{CC} = \bar{P}\gamma_\mu(1-\gamma_5)V_{CKM}N + \bar{\nu_l}\gamma_\mu(1-\gamma_5)e + h.c.$$

with $P = (u,c,t),\ N = (d,s,c),\ V_{CKM} = CKM\ \ matrix$

$$J_\mu^{NC} = \Sigma_i \bar{\Psi}_i \gamma_\mu (I_{3\,i} - Q_i \sin^2\theta_w)\Psi_i$$

where i runs over all fermions of SM.

Neutrino nucleon scattering

Interaction Lagrangian and matrix element in SM

$$\mathcal{L}_{int} = -\frac{g}{2\sqrt{2}} J_\mu^{CC} W^{+\mu} + h.c - \frac{g}{2\cos\theta_w} J_\mu^{NC} Z^\mu$$

$$\frac{G_f}{\sqrt{2}} = \frac{g^2}{8M_w^2}, \frac{M_W}{M_Z} = \cos\theta_w$$

with

$$J_\mu^{CC} = \bar{P}\gamma_\mu(1-\gamma_5) V_{CKM} N + \bar{\nu_l}\gamma_\mu(1-\gamma_5) e + h.c.$$

with $P = (u, c, t),\ N = (d, s, c),\ V_{CKM} = CKM\ \ matrix$

$$J_\mu^{NC} = \Sigma_i \bar{\Psi}_i \gamma_\mu (I_{3\,i} - Q_i \sin^2\theta_w) \Psi_i$$

where i runs over all fermions of SM.i.e. $\begin{pmatrix} \nu_l \\ e^- \end{pmatrix}_L,\ (\nu_l,\ l)_R,\ l = e, \mu, \tau$

I_3 is weak isospin and Q is the charge of i^{th} fermion.
Quark doublets of $L-$handed quraks and singlet for $R-$handed quarks for all generators.

Interaction Lagrangian and matrix element in SM

- Translated at Nucleon level and assuming M_W^2, $M_Z^2 >> q^2$ in the W,Z propagator we obtain the SM Lagrangian at Nucleon level:

$$\mathcal{L}_{int} = -\frac{G_F}{\sqrt{2}} a J^h_\mu l^\mu + h.c.$$

for CC

Interaction Lagrangian and matrix element in SM

- Translated at Nucleon level and assuming $M_W^2, M_Z^2 >> q^2$ in the W,Z propagator we obtain the SM Lagrangian at Nucleon level:

$$\mathcal{L}_{int} = -\frac{G_F}{\sqrt{2}} a J_\mu^h l^\mu + h.c.$$

  for CC

-

$$J_\mu^h = V_\mu^{1+i2} - A_\mu^{1+i2}$$

  for $\Delta S = 0$ CC, $a = \cos\theta_c$

$$J_\mu^h = V_\mu^{4+i5} - A_\mu^{4+i5}$$

  for $\Delta S = 1$ CC, $a = \sin\theta_c$

Interaction Lagrangian and matrix element in SM

- Translated at Nucleon level and assuming M_W^2, $M_Z^2 >> q^2$ in the W,Z propagator we obtain the SM Lagrangian at Nucleon level:

$$\mathcal{L}_{int} = -\frac{G_F}{\sqrt{2}} a J_\mu^h l^\mu + h.c.$$

  for CC

- $$J_\mu^h = V_\mu^{1+i2} - A_\mu^{1+i2}$$

  for $\Delta S = 0$ CC, $a = \cos\theta_c$

$$J_\mu^h = V_\mu^{4+i5} - A_\mu^{4+i5}$$

  for $\Delta S = 1$ CC, $a = \sin\theta_c$

- $V_\mu^{NC} - A_\mu^{NC}$

$$V_\mu^{NC} = V_\mu^3 - 2\sin^2\theta_w J_\mu^{e\mu} - \frac{1}{2}(V_\mu^S - A_\mu^S),$$

  for $\Delta S = 0$, NC, a=1

$$A_\mu^{NC} = A_\mu^3 - \frac{1}{2} A_\mu^S$$

Interaction Lagrangian and matrix element in SM

- Translated at Nucleon level and assuming M_W^2, $M_Z^2 >> q^2$ in the W,Z propagator we obtain the SM Lagrangian at Nucleon level:

$$\mathcal{L}_{int} = -\frac{G_F}{\sqrt{2}} a J_\mu^h l^\mu + h.c.$$

for CC

- $$J_\mu^h = V_\mu^{1+i2} - A_\mu^{1+i2}$$

for $\Delta S = 0$ CC, $a = \cos\theta_c$

$$J_\mu^h = V_\mu^{4+i5} - A_\mu^{4+i5}$$

for $\Delta S = 1$ CC, $a = \sin\theta_c$

- $V_\mu^{NC} - A_\mu^{NC}$

$$V_\mu^{NC} = V_\mu^3 - 2\sin^2\theta_w J_\mu^{e\mu} - \frac{1}{2}(V_\mu^S - A_\mu^S),$$

for $\Delta S = 0$, NC, a=1

$$A_\mu^{NC} = A_\mu^3 - \frac{1}{2} A_\mu^S$$

- Superscript i in V_μ^i and A_μ^i refer to SU(3) index and S refers to the strangeness current which are isoscalar.

Matrix elements and Form factors

- With the Lagrangian, we define the matrix elements corresponding to CC and NC processes.
- CC processes:

$$\nu_l(k) + n(p) \longrightarrow l^-(k') + p(p')$$

$$\mathcal{M} = \frac{G_f \cos\theta_c (\sin\theta_c)}{\sqrt{2}} \left\langle p' \mid J_\mu^{cc} \mid p \right\rangle \bar{\nu}_l \gamma^\mu (1-\gamma_5) l$$

$$\left\langle p' \mid J_\mu^{CC} \mid p \right\rangle = \bar{u}(p') \left[\gamma_\mu F_1^V(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2M} F_2^V(q^2) + \frac{q_\mu}{M} F_3^V(q^2) \right.$$
$$\left. + \gamma_\mu \gamma_5 F_A(q^2) + \frac{p_\mu + p'_\mu}{M} \gamma_5 F_3^A(q^2) + \frac{q_\mu}{M} \gamma_5 F_P(q^2) \right] u(p)$$

$$Problem: \; Show \; that \; this \; is \; the \; most \; general \; structure \; of \; the \; M.E.$$

- where $F_1^V(q^2), F_2^V(q^2), F_3^V(q^2)$ are vector form factors.
- $F_A(Q^2), F_P(Q^2), F_3^A(Q^2)$ are axial vector form factors.

Matrix elements and Form factors

- With the Lagrangian, we define the matrix elements corresponding to CC and NC processes.
- CC processes:

$$\nu_l(k) + n(p) \longrightarrow l^-(k') + p(p')$$

$$\mathcal{M} = \frac{G_f \cos\theta_c(\sin\theta_c)}{\sqrt{2}} \left\langle p' \mid J_\mu^{cc} \mid p \right\rangle \bar{\nu}_l \gamma^\mu (1-\gamma_5) l$$

$$\left\langle p' \mid J_\mu^{CC} \mid p \right\rangle = \bar{u}(p') \left[\gamma_\mu F_1^V(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2M} F_2^V(q^2) + \frac{q_\mu}{M} F_3^V(q^2) \right.$$
$$\left. + \gamma_\mu \gamma_5 F_A(q^2) + \frac{p_\mu + p'_\mu}{M} \gamma_5 F_3^A(q^2) + \frac{q_\mu}{M} \gamma_5 F_P(q^2) \right] u(p)$$

$$\mathit{Problem:\ Show\ that\ this\ is\ the\ most\ general\ structure\ of\ the\ M.E.}$$

- where $F_1^V(q^2), F_2^V(q^2), F_3^V(q^2)$ are vector form factors.
- $F_A(Q^2), F_P(Q^2), F_3^A(Q^2)$ are axial vector form factors.

Matrix elements and Form factors

- With the Lagrangian, we define the matrix elements corresponding to CC and NC processes.
- CC processes:

$$\nu_l(k) + n(p) \longrightarrow l^-(k') + p(p')$$

$$\mathcal{M} = \frac{G_f \cos\theta_c (\sin\theta_c)}{\sqrt{2}} \left\langle p' \mid J_\mu^{cc} \mid p \right\rangle \bar{\nu}_l \gamma^\mu (1-\gamma_5) l$$

$$\left\langle p' \mid J_\mu^{CC} \mid p \right\rangle = \bar{u}(p') \left[\gamma_\mu F_1^V(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2M} F_2^V(q^2) + \frac{q_\mu}{M} F_3^V(q^2) \right.$$

$$\left. + \gamma_\mu \gamma_5 F_A(q^2) + \frac{p_\mu + p'_\mu}{M} \gamma_5 F_3^A(q^2) + \frac{q_\mu}{M} \gamma_5 F_P(q^2) \right] u(p)$$

$$\textit{Problem: Show that this is the most general structure of the M.E.}$$

- where $F_1^V(q^2), F_2^V(q^2), F_3^V(q^2)$ are vector form factors.
- $F_A(Q^2), F_P(Q^2), F_3^A(Q^2)$ are axial vector form factors.

Matrix elements and Form factors

- With the Lagrangian, we define the matrix elements corresponding to CC and NC processes.
- CC processes:

$$\nu_l(k) + n(p) \longrightarrow l^-(k') + p(p')$$

$$\mathcal{M} = \frac{G_f \cos\theta_c (\sin\theta_c)}{\sqrt{2}} \left\langle p' \mid J_\mu^{cc} \mid p \right\rangle \bar{\nu}_l \gamma^\mu (1-\gamma_5) l$$

$$\left\langle p' \mid J_\mu^{CC} \mid p \right\rangle = \bar{u}(p') \left[\gamma_\mu F_1^V(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2M} F_2^V(q^2) + \frac{q_\mu}{M} F_3^V(q^2) \right.$$
$$\left. + \gamma_\mu \gamma_5 F_A(q^2) + \frac{p_\mu + p'_\mu}{M} \gamma_5 F_3^A(q^2) + \frac{q_\mu}{M} \gamma_5 F_P(q^2) \right] u(p)$$

Problem : Show that this is the most general structure of the M.E.

- where $F_1^V(q^2), F_2^V(q^2), F_3^V(q^2)$ are vector form factors.
- $F_A(Q^2), F_P(Q^2), F_3^A(Q^2)$ are axial vector form factors.

Matrix elements and Form factors

- With the Lagrangian, we define the matrix elements corresponding to CC and NC processes.
- CC processes:

$$\nu_l(k) + n(p) \longrightarrow l^-(k') + p(p')$$

$$\mathcal{M} = \frac{G_f \cos\theta_c(\sin\theta_c)}{\sqrt{2}} \left\langle p' \mid J_\mu^{cc} \mid p \right\rangle \bar{\nu}_l \gamma^\mu (1-\gamma_5) l$$

$$\left\langle p' \mid J_\mu^{CC} \mid p \right\rangle = \bar{u}(p') \Bigg[\gamma_\mu F_1^V(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2M} F_2^V(q^2) + \frac{q_\mu}{M} F_3^V(q^2)$$

$$+ \gamma_\mu \gamma_5 F_A(q^2) + \frac{p_\mu + p'_\mu}{M} \gamma_5 F_3^A(q^2) + \frac{q_\mu}{M} \gamma_5 F_P(q^2) \Bigg] u(p)$$

Problem : Show that this is the most general structure of the M.E.

- where $F_1^V(q^2), F_2^V(q^2), F_3^V(q^2)$ are vector form factors.
- $F_A(Q^2), F_P(Q^2), F_3^A(Q^2)$ are axial vector form factors.

Matrix elements and Form factors

- We define the matrix element for NC processes on protons and neutrons i.e.

$$\nu_l(k) + p(p) \longrightarrow \nu_l(k') + p(p')$$

$$\nu_l(k) + n(p) \longrightarrow \nu_l(k') + n(p')$$

Matrix elements and Form factors

- We define the matrix element for NC processes on protons and neutrons i.e.

$$\nu_l(k) + p(p) \longrightarrow \nu_l(k') + p(p')$$

$$\nu_l(k) + n(p) \longrightarrow \nu_l(k') + n(p')$$

- In terms of NC form factors $F_i^Z(q^2)(i = 1, 2, 3, A, P, 3A)$ defined for protons and neutrons.

Matrix elements and Form factors

- We define the matrix element for NC processes on protons and neutrons i.e.

$$\nu_l(k) + p(p) \longrightarrow \nu_l(k') + p(p')$$

$$\nu_l(k) + n(p) \longrightarrow \nu_l(k') + n(p')$$

- In terms of NC form factors $F_i^Z(q^2)(i = 1, 2, 3, A, P, 3A)$ defined for protons and neutrons.

$$\begin{aligned} \left\langle p' \mid J_\mu^{NC} \mid p \right\rangle_p \quad = \quad & \bar{u}(p') \Bigg[\gamma_\mu \tilde{F}_1^p + \frac{i\sigma_{\mu\nu} q^\nu \tilde{F}_2^p}{2M_p} + \frac{q_\mu}{M} \gamma_5 \tilde{F}_3^{V,p}(q^2) \\ & + \gamma_\mu \gamma_5 \tilde{F}_A^p + \frac{q_\mu \gamma_5 \tilde{F}_P^p}{M_p} + \frac{(p_\mu + p'_\mu)}{M} \gamma_5 \tilde{F}_3^{A,p}(q^2) \Bigg] u(p) \end{aligned}$$

Matrix elements and Form factors

- We define the matrix element for NC processes on protons and neutrons i.e.

$$\nu_l(k) + p(p) \longrightarrow \nu_l(k') + p(p')$$

$$\nu_l(k) + n(p) \longrightarrow \nu_l(k') + n(p')$$

- In terms of NC form factors $F_i^Z(q^2)(i = 1, 2, 3, A, P, 3A)$ defined for protons and neutrons.

$$\begin{aligned}\left\langle p' \mid J_\mu^{NC} \mid p \right\rangle_p &= \bar{u}(p')\left[\gamma_\mu \tilde{F}_1^p + \frac{i\sigma_{\mu\nu} q^\nu \tilde{F}_2^p}{2M_p} + \frac{q_\mu}{M}\gamma_5 \tilde{F}_3^{V,p}(q^2)\right. \\ &\left. +\gamma_\mu\gamma_5 \tilde{F}_A^p + \frac{q_\mu \gamma_5 \tilde{F}_P^p}{M_p} + \frac{(p_\mu + p'_\mu)}{M}\gamma_5 \tilde{F}_3^{A,p}(q^2)\right] u(p)\end{aligned}$$

$$\begin{aligned}\left\langle p' \mid J_\mu^{NC} \mid p \right\rangle_n &= \bar{u}(p')\left[\gamma_\mu \tilde{F}_1^n + \frac{i\sigma_{\mu\nu} q^\nu \tilde{F}_2^n}{2M_n} + \frac{q_\mu}{M}\gamma_5 \tilde{F}_3^{V,n}(q^2)\right. \\ &\left. +\gamma_\mu\gamma_5 \tilde{F}_A^n + \frac{q_\mu \gamma_5 \tilde{F}_P^n}{M_n} + \frac{(p_\mu + p'_\mu)}{M}\gamma_5 \tilde{F}_3^{A,n}(q^2)\right] u(p)\end{aligned}$$

- Vector form factors are

$$\tilde{F}^{p}_{1,2} = (\frac{1}{2} - 2sin^2\theta_W)F^{p}_{1,2} - \frac{1}{2}F^{n}_{1,2} - \frac{1}{2}F^{s}_{1,2}$$

$$\tilde{F}^{n}_{1,2} = (\frac{1}{2} - 2sin^2\theta_W)F^{n}_{1,2} - \frac{1}{2}F^{p}_{1,2} - \frac{1}{2}F^{s}_{1,2}$$

- Vector form factors are

$$\tilde{F}^{p}_{1,2} = (\frac{1}{2} - 2sin^{2}\theta_{W})F^{p}_{1,2} - \frac{1}{2}F^{n}_{1,2} - \frac{1}{2}F^{s}_{1,2}$$

$$\tilde{F}^{n}_{1,2} = (\frac{1}{2} - 2sin^{2}\theta_{W})F^{n}_{1,2} - \frac{1}{2}F^{p}_{1,2} - \frac{1}{2}F^{s}_{1,2}$$

- Axial form factors for nucleons

$$\tilde{F}^{A}_{p,n} = \pm\frac{1}{2}F_{A} - \frac{1}{2}F^{s}_{A}$$

- F^s_1, F^s_2 and F^s_A are strangeness vector and axial vector form factors.

Determination of Form factors using properties of hadronic currents

- Conserved vector current follows from SU(2) symmetry of strong interaction (keeping u and d in same doublets) and can be generalized to SU(3) symmetry to describe $\Delta S = 1$ processes.
  V_μ^+, V_μ^- along with V_μ^3 of EM current form an isotriplet leading to CVC.

Determination of Form factors using properties of hadronic currents

- Conserved vector current follows from SU(2) symmetry of strong interaction (keeping u and d in same doublets) and can be generalized to SU(3) symmetry to describe $\Delta S = 1$ processes.
  V_μ^+, V_μ^- along with V_μ^3 of EM current form an isotriplet leading to CVC.
- $\Longrightarrow F_1^V$ and F_2^V are related to isovector E.M. FF and $F_3^V(q^2) = 0$

$$F_1^V(q^2) = F_{1,2}^p(q^2) - F_{1,2}^n(q^2)$$

Determination of Form factors using properties of hadronic currents

- Conserved vector current follows from SU(2) symmetry of strong interaction (keeping u and d in same doublets) and can be generalized to SU(3) symmetry to describe $\Delta S = 1$ processes.
  V_μ^+, V_μ^- along with V_μ^3 of EM current form an isotriplet leading to CVC.

- $\Longrightarrow F_1^V$ and F_2^V are related to isovector E.M. FF and $F_3^V(q^2) = 0$

$$F_1^V(q^2) = F_{1,2}^p(q^2) - F_{1,2}^n(q^2)$$

$$\left\langle p' \mid J_\mu^{CC} \mid p \right\rangle = \bar{u}(p') \left[\gamma_\mu F_1^V(q^2) + \frac{i\sigma_{\mu\nu} q^\nu}{2M} F_2^V(q^2) + \frac{q_\mu}{M} F_3^V(q^2) \right.$$
$$\left. + \gamma_\mu \gamma_5 F_A(q^2) + \frac{p_\mu + p'_\mu}{M} \gamma_5 F_3^A(q^2) + \frac{q_\mu}{M} \gamma_5 F_P(q^2) \right] u(p)$$

- T invariance: All Form factors are real

- T invariance: All Form factors are real
- G invariance: Isospin and charge conjugation

- T invariance: All Form factors are real
- G invariance: Isospin and charge conjugation

$$G = C\, e^{i\pi I_2}, \qquad\qquad p \longrightarrow n$$

under G

  - First class currents (F_1, F_2, F_A, F_P)

$$V^\mu \longrightarrow V^\mu$$

$$A^\mu \longrightarrow -A_\mu$$

  - Second class currents (F_3^V, F_3^A)

$$V^\mu \longrightarrow -V^\mu$$

$$A^\mu \longrightarrow A_\mu$$

- Therefore assuming G-invariance $F_3^V = F_3^A = 0$

- T invariance: All Form factors are real
- G invariance: Isospin and charge conjugation

$$G = C\, e^{i\pi I_2}, \qquad\qquad p \longrightarrow n$$

under G

- First class currents (F_1, F_2, F_A, F_P)

$$V^\mu \longrightarrow V^\mu$$

$$A^\mu \longrightarrow -A_\mu$$

- Second class currents (F_3^V, F_3^A)

$$V^\mu \longrightarrow -V^\mu$$

$$A^\mu \longrightarrow A_\mu$$

- Therefore assuming G-invariance $F_3^V = F_3^A = 0$
- PCAC, Pion Pole dominance and G–T relation:

$$F_p(q^2) = \frac{2MF_A(q^2)}{q^2 - m_\pi^2}$$

Parametrisation of Form factors

- $F_1^V(q^2)$ and $F_2^V(q^2)$ are given in terms of Sachs Form factors $G_E(q^2)$ and $G_M(q^2)$ which are parameterised in dipole form.

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2}F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$\begin{aligned}
G_E^p(q^2) &= \frac{1}{(1-q^2/M_v^2)^2} = G_D(q^2) \\
G_M^p(q^2) &= (1+\mu_p)G_E^p(q^2) \\
G_M^n(q^2) &= \mu_n G_E^p(q^2) \\
G_E^n(q^2) &= (\frac{q^2}{4M^2})\mu_n G_E^p(q^2)\xi_n \\
\xi_n &= \frac{1}{(1-\lambda_n \frac{q^2}{4M^2})}
\end{aligned}$$

The strangeness vector form factors $F_{1,2}^s(q^2)$ are parameterised in a way similar to $F_{1,2}^V(q^2)$ in terms of corresponding Sach's form factors in the strange sector

$$G_E^{(s)}(q^2) = \rho_s \tau G_D(q^2)$$
$$G_M^{(s)}(q^2) = \mu_s G_D(q^2)$$

$$\rho_s = \left.\frac{dG_E^{(s)}}{d\tau}\right|_{\tau=0}$$

- $\mu_p = 1.7927\mu_N, \ \mu_n = -1.913\mu_N, \ M_v = 0.84GeV, \ \lambda_n = 5.6$
- Other parameterisations in recent years: Gari-Krüempelmann, Kelly, Alberico et al, BBA03, BBBA05, BBBA07 etc.

Parametrisation of Form factors

- $F_1^V(q^2)$ and $F_2^V(q^2)$ are given in terms of Sachs Form factors $G_E(q^2)$ and $G_M(q^2)$ which are parameterised in dipole form.

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2} F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$\begin{aligned}
G_E^p(q^2) &= \frac{1}{(1 - q^2/M_v^2)^2} = G_D(q^2) \\
G_M^p(q^2) &= (1 + \mu_p) G_E^p(q^2) \\
G_M^n(q^2) &= \mu_n G_E^p(q^2) \\
G_E^n(q^2) &= (\frac{q^2}{4M^2}) \mu_n G_E^p(q^2) \xi_n \\
\xi_n &= \frac{1}{(1 - \lambda_n \frac{q^2}{4M^2})}
\end{aligned}$$

The strangeness vector form factors $F_{1,2}^s(q^2)$ are parameterised in a way similar to $F_{1,2}^V(q^2)$ in terms of corresponding Sach's form factors in the strange sector

$$G_E^{(s)}(q^2) = \rho_s \tau G_D(q^2)$$
$$G_M^{(s)}(q^2) = \mu_s G_D(q^2)$$
$$\rho_s = \left. \frac{dG_E^{(s)}}{d\tau} \right|_{\tau=0}$$

- $\mu_p = 1.7927 \mu_N, \; \mu_n = -1.913 \mu_N, \; M_v = 0.84 GeV, \; \lambda_n = 5.6$
- Other parameterisations in recent years: Gari-Krüempelmann, Kelly, Alberico et al, BBA03, BBBA05, BBBA07 etc.

Parametrisation of Form factors

- $F_1^V(q^2)$ and $F_2^V(q^2)$ are given in terms of Sachs Form factors $G_E(q^2)$ and $G_M(q^2)$ which are parameterised in dipole form.

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2} F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$\begin{aligned}
G_E^p(q^2) &= \frac{1}{(1 - q^2/M_v^2)^2} = G_D(q^2) \\
G_M^p(q^2) &= (1 + \mu_p) G_E^p(q^2) \\
G_M^n(q^2) &= \mu_n G_E^p(q^2) \\
G_E^n(q^2) &= (\frac{q^2}{4M^2}) \mu_n G_E^p(q^2) \xi_n \\
\xi_n &= \frac{1}{(1 - \lambda_n \frac{q^2}{4M^2})}
\end{aligned}$$

The strangeness vector form factors $F_{1,2}^s(q^2)$ are parameterised in a way similar to $F_{1,2}^V(q^2)$ in terms of corresponding Sach's form factors in the strange sector

$$G_E^{(s)}(q^2) = \rho_s \tau G_D(q^2)$$
$$G_M^{(s)}(q^2) = \mu_s G_D(q^2)$$

$$\rho_s = \left. \frac{dG_E^{(s)}}{d\tau} \right|_{\tau=0}$$

- $\mu_p = 1.7927 \mu_N, \ \mu_n = -1.913 \mu_N, \ M_v = 0.84 GeV, \ \lambda_n = 5.6$
- Other parameterisations in recent years: Gari-Krüempelmann, Kelly, Alberico et al, BBA03, BBBA05, BBBA07 etc.

Axial vector FF

- Axial vector form factors are parameterised in dipole form as:

$$F_A(Q^2) = \frac{F_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2} \; ; \qquad F_A^s(q^2) = \frac{\Delta s}{(1 - \frac{q^2}{M_A^2})^2}$$

Axial vector FF

- Axial vector form factors are parameterised in dipole form as:

$$F_A(Q^2) = \frac{F_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2} \; ; \qquad F_A^s(q^2) = \frac{\Delta s}{(1 - \frac{q^2}{M_A^2})^2}$$

- where $F_A(0) = 1.2$ for β decay and $M_A = 1.016$

Axial vector FF

- Axial vector form factors are parameterised in dipole form as:

$$F_A(Q^2) = \frac{F_A(0)}{\left(1 - \frac{q^2}{M_A^2}\right)^2} \; ; \qquad F_A^s(q^2) = \frac{\Delta s}{(1 - \frac{q^2}{M_A^2})^2}$$

- where $F_A(0) = 1.2$ for β decay and $M_A = 1.016$
- To be determined from experimental data in total cross section and angular distributions of leptons in QE scattering from nucleus and nucleons.

Axial form factors for nucleons

$$\tilde{F}^A_{p,n} = \pm\frac{1}{2} F_A - \frac{1}{2} F_A^s$$

$\Delta s \equiv$ strange sea quark contribution to nucleon spin
Representative values of Δs are from 0 $\rightarrow$ -0.15

Parameterisation from threshold Electroproduction

$$\left.\frac{dE}{dq^2}\right|_{q^2=0} = \left\langle r_A^2 \right\rangle + \frac{3}{M^2}\left(k^2 + \frac{1}{2}\right) + \frac{3}{64 f_\pi^2}\left(1 - \frac{12}{\pi^2}\right)$$

For free nucleon

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE_l'} = \frac{M^2}{E_n E_p}\frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}L_{\mu\nu}J^{\mu\nu}\delta(q_0+E_n-E_p)$$

where $J^{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[(\not{p}'+M)\Gamma^\mu(\not{p}+M)\tilde{\Gamma}^\nu\right]$

$$\frac{d\sigma}{dq^2} = \frac{G^2M^2\cos^2\theta_c}{8\pi E_\nu^2}\left[A(q^2)\pm\frac{s-u}{M^2}B(q^2)+\frac{(s-u)^2}{M^4}C(q^2)\right]$$

where

$$s-u=(k+p)^2-(k'-p)^2=4ME_\nu-q^2-m_e^2$$

$$
\begin{aligned}
A &= 2x'|F_1^V + F_2^V|^2 - (1+x')|F_1^V|^2 - x'(1+x')|F_2^V|^2 + (1+x')|F_A|^2 \\
&\quad -4x'(1+x')|F_3^A|^2 - \kappa^2\left[|F_1^V + F_2^V|^2 + |F_A + 2F_P|^2 - 4(1+x')(|F_A|^2 + |F_P|^2)\right], \\
B &= \mp 4x'\mathrm{Re}\left[F_A^*(F_1^V + F_2^V)\right] + 4\kappa^2\mathrm{Re}\left[F_3^{A*}\left(F_A - x'F_P\right) - F_3^{V*}\left(F_1^V - x'F_2^V\right)\right], \\
C &= \frac{1}{4}\left(|F_1^V|^2 + x'|F_2^V|^2 + |F_A|^2 + 4x'|F_3^A|^2\right), \\
s &= (k+p)^2 = 2ME_\nu + M^2, \\
u &= \left(k'-p\right)^2 = m_\ell^2 - 2ME_\ell = m_\ell^2 - 2ME_\nu - q^2, \\
x' &= \frac{-q^2}{4M^2}.
\end{aligned}
$$

$$
\begin{aligned}
A &= 2x'|F_1^V+F_2^V|^2-(1+x')|F_1^V|^2-x'(1+x')|F_2^V|^2+(1+x')|F_A|^2 \\
&\quad -4x'(1+x')|F_3^A|^2-\kappa^2\left[|F_1^V+F_2^V|^2+|F_A+2F_P|^2-4(1+x')(|F_A|^2+|F_P|^2)\right], \\
B &= \mp 4x'\mathrm{Re}\left[F_A^*(F_1^V+F_2^V)\right]+4\kappa^2\mathrm{Re}\left[F_3^{A*}\left(F_A-x'F_P\right)-F_3^{V*}\left(F_1^V-x'F_2^V\right)\right], \\
C &= \frac{1}{4}\left(|F_1^V|^2+x'|F_2^V|^2+|F_A|^2+4x'|F_3^A|^2\right), \\
s &= (k+p)^2=2ME_\nu+M^2, \\
u &= \left(k'-p\right)^2=m_\ell^2-2ME_\ell=m_\ell^2-2ME_\nu-q^2, \\
x' &= \frac{-q^2}{4M^2}.
\end{aligned}
$$

Problem : Derive the above equations

- A similar expression is derived for elastic NC scattering cross sections from proton and neutron targets.
- Replace $F_{1,2}^{V} \longrightarrow F_{1,2}^{Z}$ and $F_A \longrightarrow F_A^Z$ for protons and neutrons.

- A similar expression is derived for elastic NC scattering cross sections from proton and neutron targets.
- Replace $F_{1,2}^{V} \longrightarrow F_{1,2}^{Z}$ and $F_A \longrightarrow F_A^Z$ for protons and neutrons.
- Charge radius for proton is

$$< r^2 >_p = \frac{6}{G_E^p(0)} \frac{dG_E^p}{dq^2}\Big|_{q^2=0},$$

and similar expressions for radius for magnetic moment distribution and axial charge distribution. For neutron radius for charge distribution is defined as

$$< r^2 >_n = 6\, \frac{dG_E^n}{dq^2}\Big|_{q^2=0}$$

and similar expression for radius of strangeness distribution.

Information about strangeness content of nucleon is obtained from NC, elastic neutrino nucleon scattering.

- It is done by writing the cross section $\frac{d\sigma}{dq^2}$ as an expansion in small variables at low q^2 in terms of small variables like $\frac{q^2}{M^2}$, $\frac{q^2}{ME}$:

$$\frac{d\sigma^{NC}}{dq^2} = \frac{1}{8\pi} G^2 \left[R + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

Information about strangeness content of nucleon is obtained from NC, elastic neutrino nucleon scattering.

- It is done by writing the cross section $\frac{d\sigma}{dq^2}$ as an expansion in small variables at low q^2 in terms of small variables like $\frac{q^2}{M^2}$, $\frac{q^2}{ME}$:

$$\frac{d\sigma^{NC}}{dq^2} = \frac{1}{8\pi} G^2 \left[R + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

$$\begin{aligned} R_{\rm NC}^{(p)} &= \alpha_{\rm V}^2 + (g_A - \Delta s)^2 \,, \\ T_{\rm NC}^{(p)} &= \alpha_{\rm V}^2 - (g_A - \Delta s)^2 + 2\frac{E_\nu}{M}\left[\alpha_{\rm V} \mp (g_A - \Delta s)\right]^2 \mp 4\frac{E_\nu}{M}(g_A - \Delta s)\kappa_{\rm NC}^{(p)} - \left(\frac{E_\nu}{M}\kappa_{\rm NC}^{(p)}\right)^2 \\ &+ 4E_\nu^2 \left\{ \alpha_{\rm V} \left[\frac{1}{3}\left(\alpha_{\rm V}\langle r_p^2\rangle - \langle r_n^2\rangle - \langle r_s^2\rangle\right) - \frac{1}{2M^2}\kappa_{\rm NC}^{(p)} \right] + \frac{1}{3}(g_A - \Delta s)\left(g_A\langle r_A^2\rangle - \Delta s\langle r_{As}^2\rangle\right) \right\} \end{aligned}$$

with

$\kappa_{\rm NC}^{(p)} = \alpha_{\rm V}(\mu_p - 1) - \mu_n - \mu_s$

$\alpha_{\rm V} = 1 - 4\sin^2\theta_W$

Information about strangeness content of nucleon is obtained from NC, elastic neutrino nucleon scattering.

- It is done by writing the cross section $\frac{d\sigma}{dq^2}$ as an expansion in small variables at low q^2 in terms of small variables like $\frac{q^2}{M^2}$, $\frac{q^2}{ME}$:

$$\frac{d\sigma^{NC}}{dq^2} = \frac{1}{8\pi} G^2 \left[R + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

$$\begin{aligned}
R_{\mathrm{NC}}^{(n)} &= 1 + (g_A + \Delta s)^2 , \\
T_{\mathrm{NC}}^{(n)} &= 1 - (g_A + \Delta s)^2 + 2\frac{E_\nu}{M} \left[1 \mp (g_A + \Delta s)\right]^2 \pm 4\frac{E_\nu}{M} (g_A + \Delta s)\kappa_{\mathrm{NC}}^{(n)} - \left(\frac{E_\nu}{M} \kappa_{\mathrm{NC}}^{(n)} \right)^2 \\
&+ 4E_\nu^2 \left\{ -\frac{1}{3} \left(\alpha_{\mathrm{V}} \langle r_n^2 \rangle - \langle r_p^2 \rangle - \langle r_s^2 \rangle \right) + \frac{1}{2M^2} \kappa_{\mathrm{NC}}^{(n)} + \frac{1}{3} (g_A + \Delta s) \left(g_A \langle r_A^2 \rangle + \Delta s \langle r_{As}^2 \rangle \right) \right\}
\end{aligned}$$

with
$\kappa_{\mathrm{NC}}^{(n)} = 1 - \mu_p + \alpha_{\mathrm{V}} \mu_n - \mu_s$.
$\alpha_{\mathrm{V}} = 1 - 4\sin^2\theta_W$

Quasi elastic scattering from Nuclei

- The interaction (of W and Z bosons) takes place with bound nucleons which are off shell and interacting with other nucleons through exchange of mesons

Quasi elastic scattering from Nuclei

- The interaction (of W and Z bosons) takes place with bound nucleons which are off shell and interacting with other nucleons through exchange of mesons
- These may lead to an interaction of W/Z with additional degrees of freedom in nuclei, which may be present due to nucleon interactions

Quasi elastic scattering from Nuclei

- The interaction (of W and Z bosons) takes place with bound nucleons which are off shell and interacting with other nucleons through exchange of mesons
- These may lead to an interaction of W/Z with additional degrees of freedom in nuclei, which may be present due to nucleon interactions
- Moreover after the Interaction, new particles may be produced which are subsequently absorbed in the nucleus, leaving only leptons leading to QE like events

Theory of QE $\nu-$ Nucleus scattering

Nuclear calculations are generally done in Nucleon only Impulse Approximation(NOIA). The following nuclear effects are taken into account:

- Pauli Blocking of Nucleons
- Fermi motion of Nucleons.

Theory of QE $\nu-$ Nucleus scattering

Nuclear calculations are generally done in Nucleon only Impulse Approximation(NOIA). The following nuclear effects are taken into account:

- Pauli Blocking of Nucleons
- Fermi motion of Nucleons.

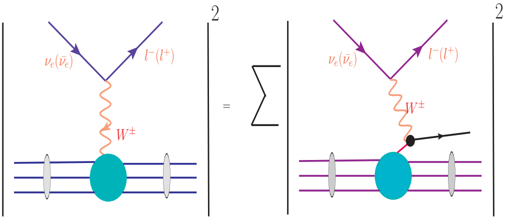
Beyond Impulse Approximation:

- short range and long range correlations
- Meson Exchange currents
- Initial state interactions, spectral functions
- Final state interactions(FSI) of nucleons and pions in nuclear medium

QE Neutrino nucleus scattering from nuclear targets are studied in the entire energy region of ν-energy

- In the low energy scattering few nuclear states are excited. Calculations are done using the specified final states and a sum over all the final states are performed.
- Simplest calculations are done using shell model (with its various extensions like RPA, CRPA, QRPA) for describing the initial and final state of nucleus.

  In Impulse Approximation, it is assumed that the cross section is given as incoherent sum of scattering from individual nucleons



In low energy region the cross sections are calculated in terms of multipole expansion.

Multipole expansion of nuclear matrix elements and cross section

The matrix element for the weak transition from $|i\rangle$ to $|f\rangle$ in the process $\nu(\bar{\nu}) + A(Z,N) \rightarrow l^-(l^+) + A(Z\pm 1, N\mp 1)$ is written as,

$$\langle f|\,H\,|i\rangle = \frac{G_F}{\sqrt{2}} l^\mu \int d\vec{x}\, e^{-i\vec{q}.\vec{x}} \langle f|\,\mathcal{J}_\mu\,|i\rangle$$

$$\langle f|\,H\,|i\rangle = \frac{G_F}{\sqrt{2}} \int d\vec{x}\, e^{-i\vec{q}.\vec{x}} \langle f|\,l_0\mathcal{J}_0 - \vec{l}.\vec{\mathcal{J}}\,|i\rangle$$

$$l^\mu = \bar{u}\gamma^\mu(1-\gamma^5)u$$

and $\mathcal{J}_\mu = \mathcal{J}_\mu^{CC}$ *for* CC process

$= \mathcal{J}_\mu^{NC}$ *for* NC process

with

$$\mathcal{J}_\mu^{CC} = V_\mu^1 + i\,V_\mu^2 - (A_\mu^1 + i\,A_\mu^2)$$

$$\mathcal{J}_\mu^{NC} = (1-2\sin^2\theta_W)\,V_\mu^3 - \sin^2\theta_W\,V_\mu^Y - \frac{1}{2}V_\mu^S - \left(A_\mu^3 + \frac{1}{2}A_\mu^S\right)$$

in general; $\mathcal{J}_\mu^{CC,NC} = \sum_i \mathcal{J}_\mu^i + \sum_{i<j} \mathcal{J}_\mu^{ij}$

We make a Multipole analysis of hadronic matrix element.

- The matrix element will involve $l_0 \mathcal{J}_0$ and $\vec{l}.\vec{\mathcal{J}}$.
- Writing any vector $\vec{l} = \sum l_\lambda . \vec{e}^\dagger_\lambda$, $l_\lambda = \vec{e}_\lambda . \vec{l}$ we make a multipole expansion for

$$e^{(i\vec{q}.\vec{x})} = \sum_{J=0}^{\infty} [4\pi(2J+1)]^{\frac{1}{2}} i^J j_J(kx) Y_{J0}(\Omega_x) \qquad k = |\vec{q}|$$

$$e_{q\lambda} e^{(i\vec{q}.\vec{x})} = -\frac{i}{k} \sum_{J=0}^{\infty} [4\pi(2J+1)]^{\frac{1}{2}} i^J \nabla(j_J(kx) Y_{J0}(\Omega_x)), for \lambda = 0$$

$$= -\sum_{J\geq 1}^{\infty} [2\pi(2J+1)]^{\frac{1}{2}} i^J \left[\lambda_{jJ}(kx) \mathcal{Y}_{JJ1\lambda} + \frac{1}{k} \nabla \times (j_J(kx) \mathcal{Y}_{JJ1\lambda})\right], for \lambda = \pm 1$$

- Using these we write

$$\left\langle f \mid \hat{H}_W \mid i \right\rangle = -\frac{G}{\sqrt{2}} \left\langle f \mid \{-\sum_{\lambda \pm 1} l_\lambda \sum_{J\geq 1}^{\infty} [2\pi(2J+1)^{\frac{1}{2}}](-i)^J \times [\lambda \hat{\mathcal{J}}_{J-\lambda^{mag}}(k) + \hat{\mathcal{J}}_{J-\lambda^{e1}}(k)]\right.$$

$$\left. + \sum_{J=0}^{\infty} [4\pi(2J+1)]^{\frac{1}{2}} (-i)^J [l_3 \hat{\mathcal{L}}_{J0}(k) - l_0 \hat{\mathcal{M}}_{J0}(k)]\} \mid i \right\rangle$$

The multipole operators are defined by

$$\hat{\mathcal{M}}_{JM}(k) \equiv M^V_{JM} + M^A_{JM} \equiv \int dx[j_J(kx)\,Y_{JM}(\Omega_x)]\hat{\mathcal{J}}_0(x)$$

$$\hat{\mathcal{L}}_{JM}(k) \equiv L^V_{JM} + L^A_{JM} \equiv \frac{i}{k}\int dx[\vec{\nabla}(j_J(kx)\,Y_{JM}(\Omega_x)].\hat{\mathcal{J}}(x)$$

$$\hat{\mathcal{J}}_{JM^{e1}}(k) \equiv T^V_{JM^{e1}} + T^A_{JM^{e1}} \equiv \frac{1}{k}\int dx[\vec{\nabla} \times (j_J(kx)\vec{\mathcal{Y}}_{JJ1^M}].\hat{\mathcal{J}}(x)$$

$$\hat{\mathcal{J}}_{JM^{mag}}(k) \equiv T^V_{JM^{mag}} + T^A_{JM^{mag}} \equiv \int dx[(j_J(kx)\vec{\mathcal{Y}}_{JJ1^M}].\hat{\mathcal{J}}(x)$$

Assuming that initial and final states are good states of total angular momenta $|i\rangle \equiv |J_i, M_i\rangle$, $|f\rangle \equiv |J_f, M_f\rangle$ and applying Wigner-Eckart theorem,

$$< J_f M_f \mid \hat{\mathcal{J}}_{JM} \mid J_i M_i >= (-1)^{J_f - M_f} \begin{pmatrix} J_f & J & J_i \\ -M_f & M & M_i \end{pmatrix} < J_f \mid\mid \hat{\mathcal{J}}_J \mid\mid J_i >$$

$$\frac{1}{2J+1}\sum_{M_i}\sum_{M_f}|<f\mid \hat{H_W}\mid i>|^2$$

$$= \frac{G^2}{2}\frac{4\pi}{2J_i+1}\left\{\sum_{J\geq 1}^{\infty}\left[\frac{1}{2}(l.l^* - l_3 l_3^*)(|< J_f||\hat{\mathcal{J}}_{J^{mag}}||J_i>|^2\right.\right.$$

$$\left. + |< J_f||\hat{\mathcal{J}}_{J^{e1}}||J_i>|^2) - \frac{i}{2}(l\times l^*)_3 2Re < J_f||\hat{\mathcal{J}}^{mag}||J_i>< J_f||\hat{\mathcal{J}}_{J^{e1}}||J_i>^*\right]$$

$$+ \sum_{J=0}^{\infty}\left[l_3 l_3^*| < J_f||\hat{\mathcal{L}}||J_i> |^2 + l_0 l_0^*| < J_f||\hat{\mathcal{M}}_J||J_i> |^2\right.$$

$$\left.\left. - 2Re l_3 l_0^* < J_f||\hat{\mathcal{L}}_J||J_i>< J_f||\hat{\mathcal{M}}_J||J_i>^*\right]\right\}$$

This leads to

$$
\left(\frac{d\sigma}{d\Omega}\right)_{\nu} = \left(\frac{k}{\in}\right)\frac{G^2\in^2}{4\pi^2}\frac{4\pi}{2J_i+1}\{\{\sum_{J=0}^{\infty}\{(1+\hat{\nu}.\vec{\beta})|<J_f||\hat{\mathcal{M}}_J||J_i>|^2
$$

$$
+[1-\hat{\nu}.\vec{\beta}+2(\hat{\nu}.q)(\hat{q}.\vec{\beta})]|<J_f||\hat{\mathcal{L}}_J||J_i>|^2
$$

$$
-[\hat{q}.(\hat{\nu}+\vec{\beta})]2Re<J_f||\hat{\mathcal{L}}_J||J_i><J_f||\hat{\mathcal{M}}_{\mathcal{J}}||J_i>^*\}
$$

$$
+\sum_{J\geq 1}^{\infty}\{[1-(\hat{\nu}.\hat{q})(\hat{q}.\vec{\beta})][|<J_f||\hat{\mathcal{J}}_{Jmag}||J_i>|^2+|<J_f||\hat{\mathcal{J}}_{Je1}||J_i>|^2]
$$

$$
\pm[\hat{q}.(\nu-\vec{\beta})]2Re<J_f||\hat{\mathcal{J}}_{Jmag}||J_i><J_f||\hat{\mathcal{J}}_{Je1}||J_i>^*\}\}\}
$$

The leptonic factors are

Summed	General $result^a$	ERL $\mid \vec{\beta} \mid \longrightarrow 1$
$\frac{1}{2}(l.l^* - l_3 l_3^*)$	$1 - (\hat{v}.\hat{q})(\vec{\beta}.\hat{q})$	$\frac{q^2}{2q^2}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}$
$l_0 l_0^*$	$1 + \hat{v}.\vec{\beta}$	$2cos^2\frac{\theta}{2}$
$l_3 l_3^*$	$1 - \hat{v}.\vec{\beta} + 2(\hat{v}.\hat{q})(\vec{\beta}.\hat{q})$	$\frac{q_0^2}{q^2} 2cos^2\frac{\theta}{2}$
$-l_3 l_0^*$	$-\hat{q}.(\hat{\nu} + \vec{\beta})$	$-\frac{q_0}{\lvert q \rvert} 2\cos^2\frac{\theta}{2}$
$-\frac{i}{2}(l \times l^*)$	$-S_1 \hat{q}.(\hat{\nu} - \vec{\beta})$	$\frac{2\sin\frac{\theta}{2}}{\lvert q \rvert}\left(q^2\cos^2\frac{\theta}{2} + q^2\sin^2\frac{\theta}{2}\right)^{\frac{1}{2}} S_1 S_2$

S_1 and $S_1 S_2$ is ± 1 and ∓ 1 for neutrino and antineutrino respectively.
In relativistic case for $(m_\mu \longrightarrow 0)$. This gives

$$\left(\frac{d\sigma}{d\Omega}\right)_\nu = \frac{G^2 \in^2}{2\pi^2}\frac{4\pi}{2J_i+1}\Big\{ \cos^2\frac{\theta}{2}\Big[\sum_{J=0}^{\infty} \mid < J_f || \hat{\mathcal{M}}_J - \frac{q_0}{|q|}\hat{\mathcal{L}}_J || J_i > \mid^2\Big] \quad + \Big[\frac{q^2}{2q^2}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\Big]$$

$$\times \Big[\sum_{J\geq 1}^{\infty} (\mid < J_f || \hat{\mathcal{J}}_{Jmag} || J_i > \mid^2 + \mid < J_f || \hat{\mathcal{J}}_J^{e1} || J_i > \mid^2\Big]$$

$$\mp \sin\frac{\theta}{2}\frac{1}{|q|}\left(q^2\cos^2\frac{\theta}{2} + q^2\sin^2\frac{\theta}{2}\right)^{\frac{1}{2}}$$

$$\times \Big[\sum_{J\geq 1}^{\infty} 2Re < J_f || \hat{\mathcal{J}}_{Jmag} || J_i >< J_f || \hat{\mathcal{J}}_J^{e1} || J_i >^* \Big]\Big\}$$

The leptonic factors are

Summed	General $result^a$	ERL $\mid \vec{\beta} \mid \longrightarrow 1$
$\frac{1}{2}(l.l^* - l_3 l_3^*)$	$1-(\hat{v}.\hat{q})(\vec{\beta}.\hat{q})$	$\frac{q^2}{2q^2}\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}$
$l_0 l_0^*$	$1+\hat{v}.\vec{\beta}$	$2\cos^2\frac{\theta}{2}$
$l_3 l_3^*$	$1-\hat{v}.\vec{\beta}+2(\hat{v}.\hat{q})(\vec{\beta}.\hat{q})$	$\frac{q_0^2}{q^2}2\cos^2\frac{\theta}{2}$
$-l_3 l_0^*$	$-\hat{q}.(\hat{\nu}+\vec{\beta})$	$-\frac{q_0}{\lvert q\rvert}2\cos^2\frac{\theta}{2}$

This form is used for numerical evaluations with $\mathcal{J}_0, \vec{\mathcal{J}}$: obtained as nonrelativistic expansion of $\mathcal{J}_\mu^{CC}$ and $\mathcal{J}_\mu^{NC}$ and $|i>$ and $|f>$ calculated with some NN potential.

$$\left(\frac{d\sigma}{d\Omega}\right)_\nu = \frac{G \quad \epsilon}{2\pi^2}\frac{4\pi}{2J_i+1}\left\{\cos^2\frac{\theta}{2}\left[\sum_{J=0}|<J_f||\hat{\mathcal{M}}_J-\frac{q_0}{|q|}\hat{\mathcal{L}}_J||J_i>|^2\right] + \left[\frac{q}{2q^2}\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\right]\right.$$

$$\times\left[\sum_{J\geq 1}^{\infty}(|<J_f||\hat{\mathcal{J}}_{Jmag}||J_i>|^2+|<J_f||\hat{\mathcal{J}}_J^{el}||J_i>|^2\right]$$

$$\mp\sin\frac{\theta}{2}\frac{1}{|q|}\left(q^2\cos^2\frac{\theta}{2}+q^2\sin^2\frac{\theta}{2}\right)^{\frac{1}{2}}$$

The leptonic factors are

Intermediate and Higher Energies

As the energy of neutrino increases a large number of states are excited. It is not easy to calculate all the excited state wave function from a nuclear Hamiltonian needed to calculate these matrix elements. Therefore, other models for nuclei calculation of cross section are used.

In relativistic case for ($m_\mu \longrightarrow 0$). This gives

$$\left(\frac{d\sigma}{d\Omega}\right)_\nu = \frac{G^2 \in^2}{2\pi^2}\frac{4\pi}{2J_i+1}\left\{\cos^2\frac{\theta}{2}\left[\sum_{J=0}^{\infty}|<J_f||\hat{\mathcal{M}}_J - \frac{q_0}{|q|}\hat{\mathcal{L}}_J||J_i>|^2\right] \quad + \left[\frac{q^2}{2q^2}\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\right]\right.$$

$$\times\left[\sum_{J\geq 1}^{\infty}(|<J_f||\hat{\mathcal{J}}_{Jmag}||J_i>|^2+|<J_f||\hat{\mathcal{J}}_J^{e1}||J_i>|^2\right]$$

$$\mp \sin\frac{\theta}{2}\frac{1}{|q|}\left(q^2\cos^2\frac{\theta}{2}+q^2\sin^2\frac{\theta}{2}\right)^{\frac{1}{2}}$$

Some of the models are

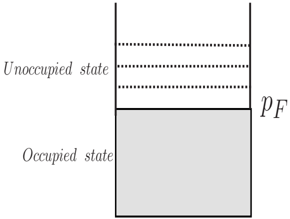
- Fermi Gas Model(with various versions)
- Relativistic Mean field
- Relativistic Green Function approach
- SuSA

Fermi Gas Model

In this model it is assured that the nucleons in a nucleus (or nuclear matter) occupy one nucleon per unit cell in phase space so that the total number of Nucleons N is given by

$$N = 2V \int \frac{d^3p}{(2\pi)^3}$$

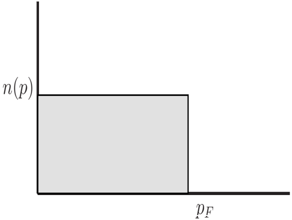
where a factor of two to account spin degree of freedom. All states upto a maximum momentum p_F $(p < p_F)$ are filled. The momentum states higher than $p > p_F$ are unoccupied.



Fermi Gas Model(with various versions)

Such that the occupation number n(p) is defined as:

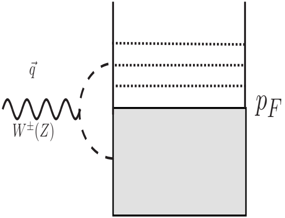
$$n(p) = 1, p < p_F$$
$$= 0, p > p_F$$
$$\Longrightarrow \rho = \frac{N}{V} = \frac{p_f^3}{3\pi^2}$$
$$\therefore p_F = (3\pi^2 \rho)^{\frac{1}{3}}$$



Protons and neutrons are supposed to have different Fermi sphere so

$$\rho_p^F = (3\pi\rho_p)^{\frac{1}{3}} \qquad \rho_n^F = (3\pi\rho_n)^{\frac{1}{3}}$$

Under a weak interaction induced by $\nu/\bar{\nu}$ a nucleon is excited from an occupied state to an unoccupied state i.e.



Creating a hole in the Fermi sea and a particle above the sea. This is known as 1p1h excitation, with the condition that:

- initial momentum: $p < p_F^i$
- final momentum: $|\vec{p} + \vec{q}| > p_F^f$

This condition could be incorporated in the expression for the free nucleon cross section.

For free nucleon

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE_l'} = \frac{M^2}{E_n E_p}\frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}L_{\mu\nu}J^{\mu\nu}\delta(q_0 + E_n - E_p)$$

where $J^{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[(\not{p}' + M)\Gamma^\mu(\not{p} + M)\tilde{\Gamma}^\nu\right]$

Inside the nucleus

$$\left.\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE_l'}\right|_{Nucleus} = \frac{G^2}{4\pi^2}\int\frac{M^2}{E_n E_p}2d\vec{p}\frac{1}{(2\pi)^3}n_n(\vec{p})(1 - n(|\,\vec{p} + \vec{q}\,|))\frac{|\vec{k}'|}{|\vec{k}\,|}$$
$$\times\,\delta(q_0 + E_n - E_p)L_{\mu\nu}J^{\mu\nu}$$

- The final nucleon has to be created with a momentum $p' = \mid \vec{p} + \vec{q} \mid > p_{F_f}$
- Initial nucleon is at rest.
- The hadronic tensor $J_{\mu\nu}$ in equation has to be integrated over the Fermi momentum of initial nucleon subject to the above conditions i.e. $J_{\mu\nu}$ is replaced by

$$\frac{M^2}{E_n E_p} J_{\mu\nu} \delta(q_0 + E_n - E_p) \longrightarrow \int f(q,p) J_{\mu\nu}(p) \frac{d^3 p}{(2\pi)^3}$$

$$f(q,p) = n(\mid \vec{p} \mid)(1 - n(\mid \vec{p} + \vec{q} \mid) \frac{M^2}{E_n E_p} \delta(q_0 + E_n - E_p)$$

$$n(p) = \theta(p_F^i - p)$$
$$1 - n(p+q) = \theta(\mid p+q \mid - p_F^f)$$

- $J_{\mu\nu}$ involves terms like $g_{\mu\nu}, q_\mu q_\nu, p_\mu p_\nu$ and $p_\mu p_\nu$.
  Now $\int f(q,p) J_{\mu\nu}(p) \frac{d^3 p}{(2\pi)^3}$ can be evaluated explicitly.

  These are the main features of **Smith and Moniz** RgFG model.

A simpler model is given by Gaisser O'Connel, in which

$$\frac{d^2\sigma}{d\Omega_l dE'_l} = R(q,\omega)\frac{d^2\sigma_0}{d\Omega_l dE'_l}$$

- $\frac{d^2\sigma_0}{d\Omega_l dE'_l}$ is free nucleon cross section and
- $R(q,\omega)$ is nuclear correction given by

  $$R(q,\omega) = \frac{1}{\frac{4}{3}\pi p_F^3}\int \frac{d^3p}{E_n E_p} M^2 \delta(E_N + \omega' - E_{N'})\theta(p_F - p)\theta(|p+q| - p_F)$$

  with $\omega' = \omega - E_B$
- Note that in this model $\vec{p}$ is not included in $\frac{d\sigma}{d\Omega_l}$ which is still evaluated for initial nucleus at rest.

In the local Fermi gas model we replace:

$$
\begin{aligned}
p_F \longrightarrow p_F(r) =& (3\pi^2 \rho(r))^{\frac{1}{3}} \\
\rho(r) =& \rho_n(r) \quad \text{for } \nu \text{ reactions and} \\
=& \rho_p(r) \quad \text{for } \bar{\nu} \text{ reactions}
\end{aligned}
$$

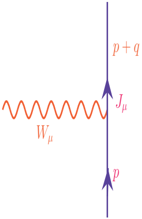
However, the nuclear densities $\rho_{n,p}(r)$ taken from electron scattering experiments with appropriate corrections where available and the integration is performed over whole volume as the original normalization was one nucleon per unit volume i.e.

Inside the nucleus with local density approximation

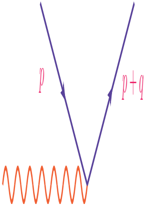
$$
\left. \frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE_l'} \right|_{Nucleus} = \frac{G^2}{4\pi^2} \frac{|\vec{k}'|}{|\vec{k}\,|} \int \frac{M^2}{E_n E_p} 2 d\vec{r} d\vec{p} \frac{1}{(2\pi)^3} n_n(\vec{p},\vec{r})(1 - n(|\,\vec{p}+\vec{q}\,|,\vec{r})
$$
$$
\times \delta(q_0 + E_n - E_p) L_{\mu\nu} J^{\mu\nu}(\vec{p},\vec{r})
$$

p-h excitation:

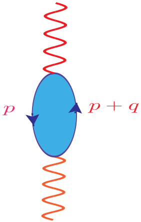
- Initial Nucleon has a momentum(distribution) such that $p < p_F^f$ Fermi momentum of initial nucleon.
- Final Nucleon should be outside the Fermi level so $p = \mid \vec{p} + \vec{q} \mid > p_F^f$ Fermi momentum of final nucleon
- In the interaction (of W or Z) with the nucleon, a hole is created in Fermi sea and excited to a particular state $W + n \longrightarrow p$
- Creation of 1p1h state:Diagrammatically



Feynman Diagram



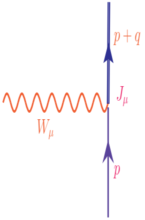
1p1h excitation matrix element



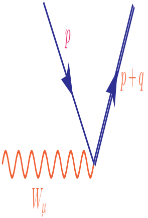
1p1h response

Many body theory: language of p-h excitation's:

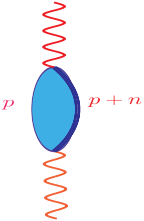
Creation of 1Δ1h state



Feynman Diagram matrix element



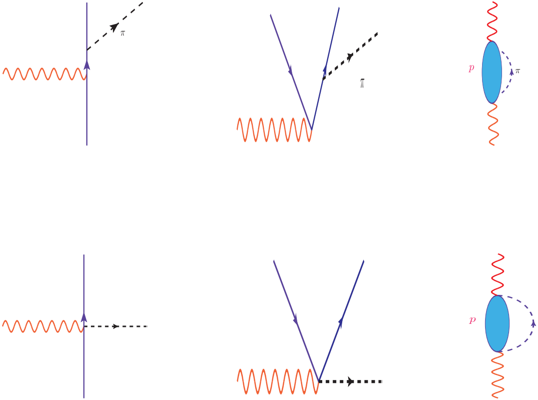
1Δ1h excitation



1Δ1h response

Many body theory: language of p-h excitation:

Creation of pion state



Beyond 1p1h excitation

The nucleons move under a nuclear potential inside the nucleus and dynamics is described by Hamiltonian(H) :

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{ij}(r) + \sum_{i,j,k} V_{ijk}(r)$$

Beyond 1p1h excitation

The nucleons move under a nuclear potential inside the nucleus and dynamics is described by Hamiltonian(H) :

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{ij}(r) + \sum_{i,j,k} V_{ijk}(r)$$

where V_{ij} and V_{ijk} are two body and three body interaction potentials.

$V_{ij}(r)$ is generally described in terms of π,ρ,ω exchange potential which generate long range and short range potentials. Also, supplemented by hard core/soft core central potential.

Beyond 1p1h excitation

The nucleons move under a nuclear potential inside the nucleus and dynamics is described by Hamiltonian(H) :

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{ij}(r) + \sum_{i,j,k} V_{ijk}(r)$$

where V_{ij} and V_{ijk} are two body and three body interaction potentials.

$V_{ij}(r)$ is generally described in terms of π,ρ,ω exchange potential which generate long range and short range potentials. Also, supplemented by hard core/soft core central potential.

In general nucleon-nucleon potential is

- Spin dependent
- Isospin Independent
- Tensor Forces

Beyond 1p1h excitation

The nucleons move under a nuclear potential inside the nucleus and dynamics is described by Hamiltonian(H) :

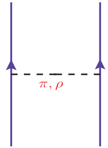
$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} V_{ij}(r) + \sum_{i,j,k} V_{ijk}(r)$$

where V_{ij} and V_{ijk} are two body and three body interaction potentials.

$V_{ij}(r)$ is generally described in terms of π,ρ,ω exchange potential which generate long range and short range potentials. Also, supplemented by hard core/soft core central potential.

These potentials give rise to:

- Nucleon-Nucleon Correlations
- Meson Exchange(Two body) currents.



Beyond 1p1h excitation

Continuity Equation

$$\nabla . J + i \left[H, J^0 \right] = 0$$

Beyond 1p1h excitation

Continuity Equation

H has isospin dependent two body potential

$$\left[H, J^0\right] \neq 0$$

$$\nabla . J + i\left[H, J^0\right] = 0$$

Beyond 1p1h excitation

Continuity Equation

H has isospin dependent two body potential

$$\left[H, J^0\right] \neq 0$$

Two Body Terms

$$\nabla . J + i\left[H, J^0\right] = 0$$

Beyond 1p1h excitation

Continuity Equation

H has isospin dependent two body potential

Two Body Terms

$$\left[H, J^0\right] \neq 0$$

$$\nabla . J + i\left[H, J^0\right] = 0$$

$$J_\mu = \sum_i J_\mu^i + \sum_{i>j} J_\mu^{ij}$$

Beyond 1p1h excitation

Continuity Equation

H has isospin dependent two body potential

Two Body Terms

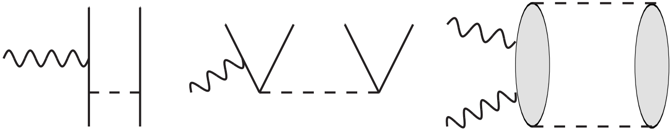
$$\left[H, J^0\right] \neq 0$$

$$\nabla.J + i\left[H, J^0\right] = 0$$

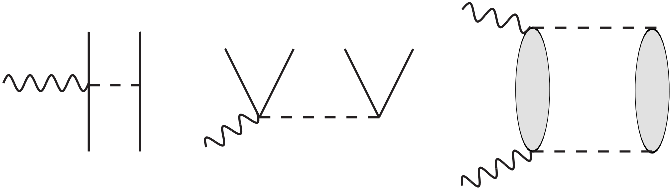
These give rise to 2p-2h and higher order excitation in nuclear medium

$$J_\mu = \sum_i J_\mu^i + \sum_{i>j} J_\mu^{ij}$$

Examples 2p–2h



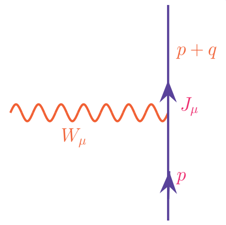
Nucleon nucleon correlation

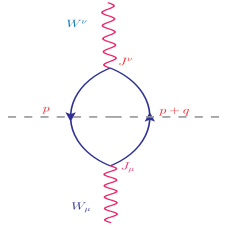


Meson exchange current

Cross section from p–h excitation response

Consider 1p–1h excitation





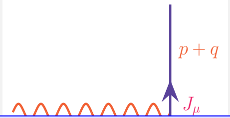
$$d\sigma \propto \sum_{\text{spins}} W^\mu W^\nu J_{\mu\nu} n(p)(1-n(p+q)) \times \frac{d^3p}{(2\pi)^3} \delta(q_0 + E_n - E_p)$$

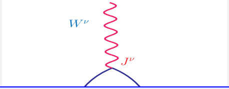
$$\text{Response} \propto \sum_{\text{spins}} W^\mu W^\nu \Pi_{\mu\nu}$$

$$\Pi_{\mu\nu} \propto \int \frac{d^3p}{(2\pi)^3} J_{\mu\nu} D(p) D(p+q) n(p)(1-n(p+q))$$

Cross section from p–h excitation response

Consider 1p–1h excitation





- Consider nonrelativistic case in which
- $\Pi_{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{J_{\mu\nu} n(p)(1-n(p+q))}{q_0 + E_n - E_p + i\epsilon}$
- $\text{Im } \Pi_{\mu\nu} = -\pi \int \frac{d^3p}{(2\pi)^3} J_{\mu\nu} n(p)(1-n(p+q)) \delta(q_0 + E_n - E_p) = -\pi W_{\mu\nu}$
- $W_{\mu\nu} = -\frac{1}{\pi} Im\Pi_{\mu\nu}$

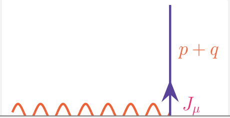
$$\times \frac{d^3p}{(2\pi)^3} \delta(q_0 + E_n - E_p)$$

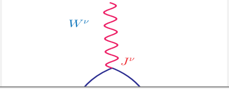
spins

$$\Pi_{\mu\nu} \propto \int \frac{d^3p}{(2\pi)^3} J_{\mu\nu} D(p) D(p+q) n(p)(1-n(p+q))$$

Cross section from p–h excitation response

Consider 1p–1h excitation





$$W_{\mu\nu} = -\frac{1}{\pi} Im\Pi_{\mu\nu}$$

where $\Pi_{\mu\nu}$ is the polarization tensor for p-h excitation.
This is rigorous result in the analytic theory of Feynman propagators.
In general $\Pi_{\mu\nu}$ is obtained using rules of covariant perturbative theory for any given diagram.

- $W_{\mu\nu} = -\frac{1}{\pi} Im\Pi_{\mu\nu}$

$\times \frac{d^3p}{(2\pi)^3}\delta(q_0+E_n-E_p)$

spins

$$\Pi_{\mu\nu} \propto \int \frac{d^3p}{(2\pi)^3} J_{\mu\nu} D(p) D(p+q) n(p)(1-n(p+q))$$

$\mathrm{Im}\Pi_{\mu\nu}$ is obtained by putting propagator particle onshell by cutting the loop diagram using Cutkosky rules.

Main features of Cutkosky rules:

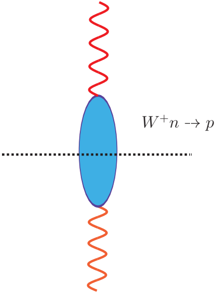
- Cut through the diagram in any way that can put all of the cut propagators on-shell without violating momentum conservation.
- For each cut, replace $\frac{1}{p^2-m^2+i\epsilon} \rightarrow -2i\pi\delta(p^2-m^2)\theta(p^0)$.
- Sum over all cuts.
- The result is the discontinuity of the diagram, where $Disc(i\mathcal{M})$= -2 Im $\mathcal{M}$.

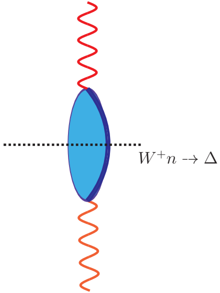
Examples

Main features of Cutkosky rules:

- Cut through the diagram in any way that can put all of the cut propagators on-shell without violating momentum conservation.
- For each cut, replace $\frac{1}{p^2-m^2+i\epsilon} \rightarrow -2i\pi\delta(p^2-m^2)\theta(p^0)$.
- Sum over all cuts.
- The result is the discontinuity of the diagram, where $Disc(i\mathcal{M})$= -2 Im $\mathcal{M}$.

Examples

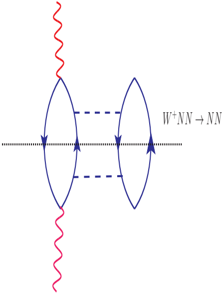


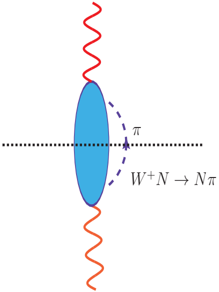


Main features of Cutkosky rules:

- Cut through the diagram in any way that can put all of the cut propagators on-shell without violating momentum conservation.
- For each cut, replace $\frac{1}{p^2-m^2+i\epsilon} \rightarrow -2i\pi\delta(p^2-m^2)\theta(p^0)$.
- Sum over all cuts.
- The result is the discontinuity of the diagram, where $Disc(i\mathcal{M})$= -2 Im $\mathcal{M}$.

Examples

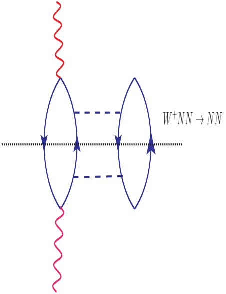


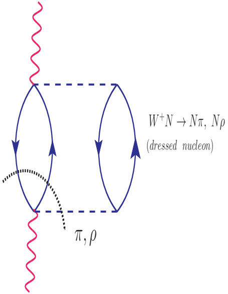


Main features of Cutkosky rules:

- Cut through the diagram in any way that can put all of the cut propagators on-shell without violating momentum conservation.
- For each cut, replace $\frac{1}{p^2-m^2+i\epsilon} \rightarrow -2i\pi\delta(p^2 - m^2)\theta(p^0)$.
- Sum over all cuts.
- The result is the discontinuity of the diagram, where $Disc(i\mathcal{M})$= -2 Im $\mathcal{M}$.

Examples

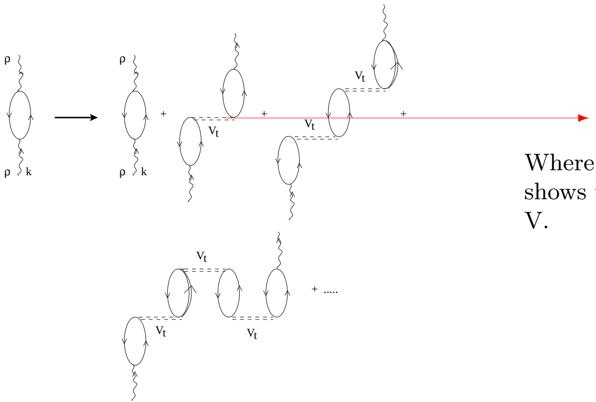




RPA Correlations(Treated Nonrelativistically)

The nucleons in a nucleus interact through two body NN potential(simply modeled with π and ρ exchange.)

Once 1p1h are excited by an external probe, they can interact through the NN–potential(π and ρ exchange) any number of times. In fact in this interaction they can also produce Δ leading to ph–Δh interaction which can be depicted as:



Where the dotted line shows the N-N potential V.

In general we have a N-N potential whose structure is

$$V(\vec{r_1},\vec{r_2}) = C_0\left\{f_0 + f'{}_0\vec{\tau}_1.\vec{\tau}_2 + g_0\vec{\sigma}_1.\vec{\sigma}_2 + g'{}_0\vec{\sigma}_1.\vec{\sigma}_2\vec{\tau}_1.\vec{\tau}_2\right\}$$

f_0 & g_0 is strength of the NN-potential in isoscalar spin-dependent and spin-independent channel.

f_0' & g_0' is strength of the NN-potential in isovector spin-independent and spin-dependent channel.

CC interaction in non-relativistic case is dominated by $\vec{\sigma}.\vec{\tau}$ term which is most affected by NN-potential in the spin-isospin channel. We calculate NN-potential in π and ρ exchange model i.e.

$$V = V_\pi + V_\rho$$

$$V_\pi = \frac{f_\pi^2}{m_\pi^2}\frac{\vec{\sigma}_1.\vec{q}\vec{\sigma}_2.\vec{q}}{q_0^2 - \vec{q}^2 - m_\pi^2 + i\in} \qquad f_\pi = \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2}$$

$$V_\rho = \frac{f_\rho^2}{m_\rho^2}\frac{\vec{\sigma}_1\times\vec{q}\ \ \vec{\sigma}_2\times\vec{q}}{q_0^2 - \vec{q}^2 - m_\rho^2 + i\in}\vec{\tau}.\vec{\tau} \qquad f_\rho = \frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2}$$

$$\Lambda_\pi = 1.1\,GeV \qquad \Lambda_\rho = 2.5\,GeV$$

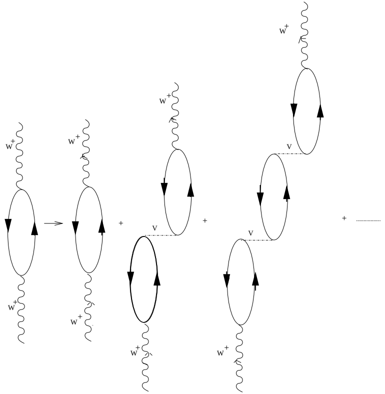
We express V in terms of longitudinal and transverse components as

$$V_{ij} = V_l + V_t$$

$$V_l = \frac{f_\pi^2}{m_\pi^2} \frac{\vec{\sigma}_{1i}\vec{\sigma}_{2j}\hat{q}_{1i}\hat{q}_{2j}}{q_0^2 - \vec{q}^2 - m_\pi^2} q^2$$

$$V_t = \frac{f_\rho^2}{m_\rho^2} \frac{\vec{\sigma}_{1i}\vec{\sigma}_{2j}(\delta_{ij} - \hat{q_1}_i \hat{q_2}_j)}{q_0^2 - \vec{q}^2 - m_\pi^2} q^2$$

We sum the series in ladder approximation



- The ladder diagram may be represented as

$$U_l(q) = U(q)[1 + V_l U(q) + V_l U(q) V_l U(q) + \cdots]\hat{q}_i\hat{q}_j\sigma_i\sigma_j\tau_1 \cdot \tau_2$$

- This is an geometric series which can be summed over separately in longitudinal and transverse channel giving rise to

$$U(q) \longrightarrow \frac{U\hat{q}_i\hat{q}_j}{1 - V_l U} + \frac{U(\delta_{ij} - \hat{q}_i\hat{q}_j)}{1 - V_t U}$$

- and the imaginary part may be expressed as

$$ImU \longrightarrow \frac{ImU\hat{q}_i\hat{q}_j}{\mid 1 - V_l U \mid^2} + \frac{ImU(\delta ij - \hat{q}_i\hat{q}_j)}{\mid 1 - V_t U \mid^2}$$

Now consider a term like F_A^2

In nonrelativistic limit F_A^2 term comes after squaring $F_A\vec{\sigma}$ term $\propto F_A^2\sigma_i\sigma_j$
write $F_A^2 \longrightarrow \frac{1}{6}F_A^2\delta_{ij}\;\; Tr(\sigma_i\sigma_j)$

Replace

$$F_A^2\delta ij \longrightarrow F_A^2\left[(\hat{q}_i\hat{q}_j)+(\delta_{ij}-\vec{q}_i\vec{q}_j)\right]$$

$$F_A^2 \longrightarrow \frac{1}{6}F_A^2\left[(\hat{q}_i\hat{q}_j)+(\delta_{ij}-\hat{q}_i\hat{q}_j)\right]$$

$$F_A^2 \longrightarrow \frac{1}{6}F_A^2 ImU\left[\frac{2\hat{q}_i\hat{q}_j\delta_{ij}}{|1-V_lU|^2}+\frac{2\delta_{ij}(\delta_{ij}-\hat{q}_i\hat{q}_j)}{|1-UV_t|^2}\right]$$

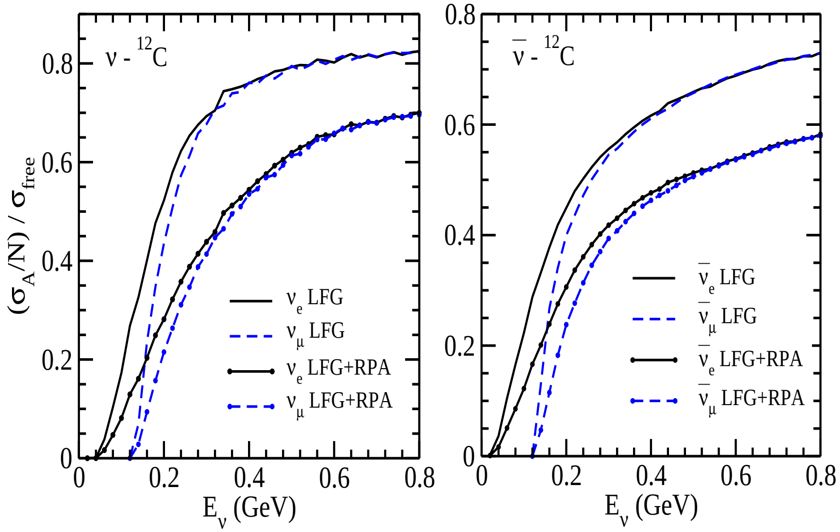
$$F_A^2 \longrightarrow \frac{1}{6}F_A^2 ImU\left[\frac{1}{3}\frac{1}{|1-V_lU|^2}+\frac{2}{3}\frac{1}{|1-V_tU|^2}\right]$$

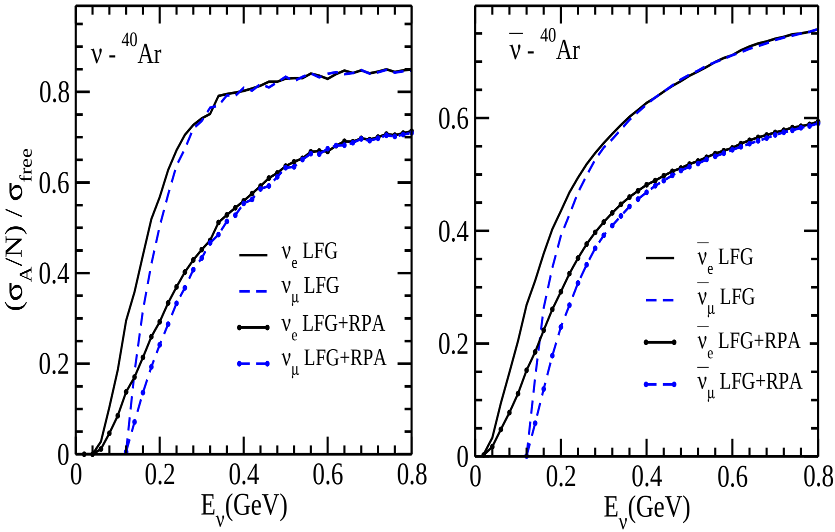
With these modifications in the leading terms of $W_{\mu\nu}$, the cross section is calculated.

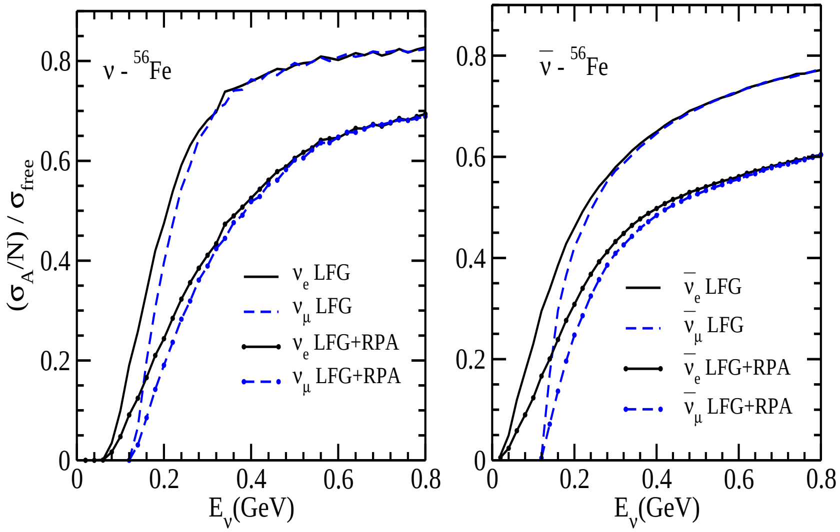
	Pauli+Q	RPA	SM	SM	CRPA	Exp	
						LSND'95	LSND'97
$\overline{\sigma}\ (\nu_\mu, \mu^-)$	20.7	11.9	13.2	15.2	19.2	$8.3 \pm 0.7 \pm 1.6$	$11.2 \pm 0.3 \pm 1.8$
						KARMEN	LSND
$\overline{\sigma}\ (\nu_e, e^-)$	0.19	0.14	0.12	0.16	0.15	$0.15 \pm 0.01 \pm 0.01$	$0.15 \pm 0.01 \pm 0.01$

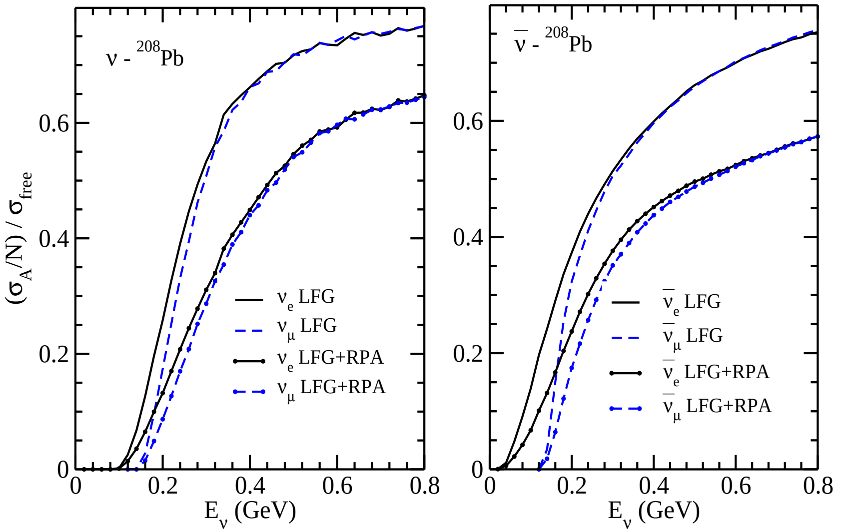
Table : Experimental and theoretical flux averaged $^{12}\mathrm{C}(\nu_\mu, \mu^-)X$ and $^{12}\mathrm{C}(\nu_e, e^-)X$ cross sections in 10^{-40} cm^2 units.

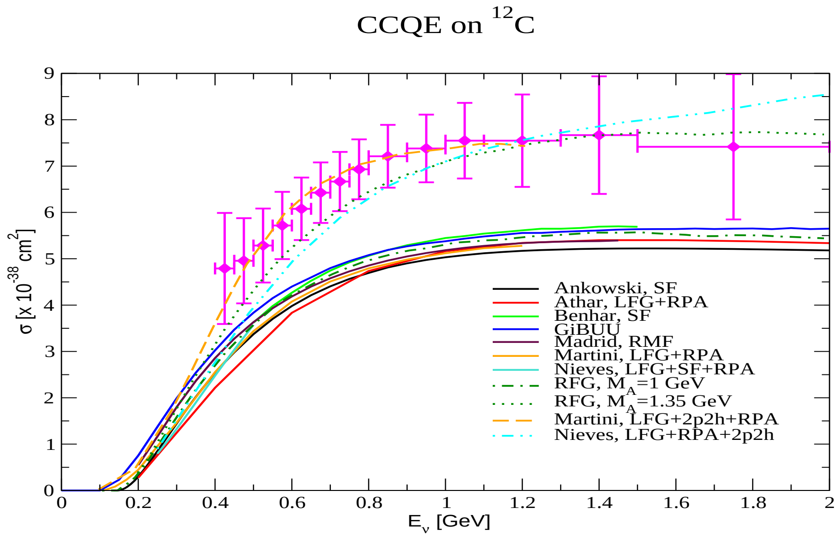
Phys. Rev. C **70**, 055503 (2004) [Phys. Rev. C **72**, 019902 (2005)]


ν - 12C
ν̄ - 12C
(σA/N) / σfree
Eν (GeV)
0
0.2
0.4
0.6
0.8
νe LFG
νμ LFG
νe LFG+RPA
νμ LFG+RPA
ν̄e LFG
ν̄μ LFG
ν̄e LFG+RPA
ν̄μ LFG+RPA


ν - 40Ar
$(\sigma_A/N) / \sigma_{free}$
E_ν(GeV)
ν_e LFG
ν_μ LFG
ν_e LFG+RPA
ν_μ LFG+RPA
$\bar{\nu}$ - 40Ar
E_ν(GeV)
$\bar{\nu}_e$ LFG
$\bar{\nu}_\mu$ LFG
$\bar{\nu}_e$ LFG+RPA
$\bar{\nu}_\mu$ LFG+RPA


ν - ^{56}Fe
$\bar{\nu}$ - ^{56}Fe
$(\sigma_A/N) / \sigma_{free}$
E_ν(GeV)
ν_e LFG
ν_μ LFG
ν_e LFG+RPA
ν_μ LFG+RPA
$\bar{\nu}_e$ LFG
$\bar{\nu}_\mu$ LFG
$\bar{\nu}_e$ LFG+RPA
$\bar{\nu}_\mu$ LFG+RPA


ν - ^{208}Pb
$(\sigma_A/N) / \sigma_{free}$
0
0.2
0.4
0.6
0.8
E_ν (GeV)
ν_e LFG
ν_μ LFG
ν_e LFG+RPA
ν_μ LFG+RPA
$\bar{\nu}$ - ^{208}Pb
$\bar{\nu}_e$ LFG
$\bar{\nu}_\mu$ LFG
$\bar{\nu}_e$ LFG+RPA
$\bar{\nu}_\mu$ LFG+RPA


CCQE on ^{12}C
0
1
2
3
4
5
6
7
8
9
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
σ [x 10^{-38} cm^2]
E_ν [GeV]
Ankowski, SF
Athar, LFG+RPA
Benhar, SF
GiBUU
Madrid, RMF
Martini, LFG+RPA
Nieves, LFG+SF+RPA
RFG, M_A=1 GeV
RFG, M_A=1.35 GeV
Martini, LFG+2p2h+RPA
Nieves, LFG+RPA+2p2h