Weak Pion Production

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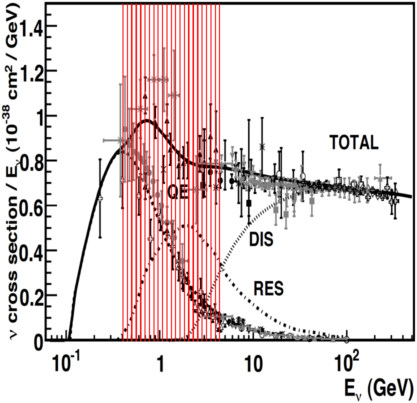
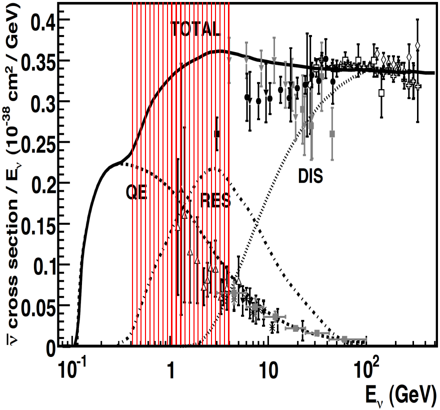
Introduction

Neutrino



Anti-neutrino



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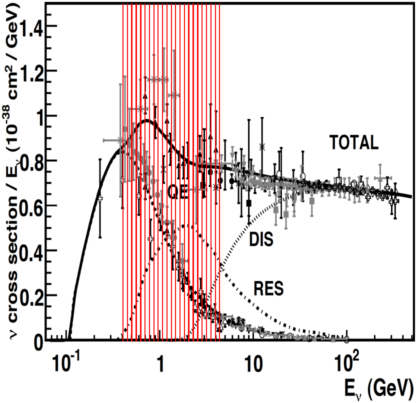

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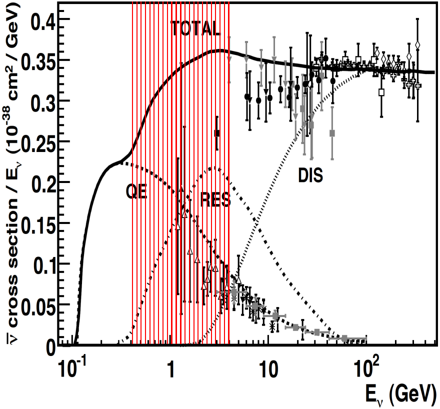

$$\sigma^{Total} = \sigma^{QE}_{\nu_l n \to l^- p} + \sigma^{Inelastic} + \sigma^{DIS}$$

$$\sigma^{Inelastic} = \sigma^{1\pi} + \sigma^{2\pi} + \cdots + \sigma^{YK} + \sigma^{1K} + \sigma^{1Y} + \sigma^{\eta} + \cdots$$

$\nu/\bar{\nu}$ induced single pion production(SPP)

Charged current(CC)

$$\nu_l p \rightarrow l^- p\pi^+ \qquad \bar{\nu}_l n \rightarrow l^+ n\pi^-$$
$$\nu_l n \rightarrow l^- n\pi^+ \qquad \bar{\nu}_l p \rightarrow l^+ p\pi^-$$
$$\nu_l n \rightarrow l^- p\pi^0 \qquad \bar{\nu}_l p \rightarrow l^+ n\pi^0 \qquad l = e, \mu$$

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  $\rightarrow$ $\Delta(1232)$
  $\rightarrow$ Isospin $\mathrm{I} = \frac{3}{2}$
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Resonances R_{IJ}	M_R [GeV]	Γ_0^{tot} (GeV)	πN branching ratio (%)
$P_{33}(1232)$	1.232	0.117	100
$P_{11}(1440)$	1.430	0.350	$55-75$
$D_{13}(1520)$	1.515	0.115	$55-65$
$S_{11}(1535)$	1.535	0.150	$35-55$
$S_{31}(1620)$	1.630	0.140	$20-30$
$S_{11}(1650)$	1.655	0.140	$50-90$
$D_{15}(1675)$	1.675	0.150	$35-45$
$F_{15}(1680)$	1.685	0.130	$65-70$
$D_{33}(1700)$	1.700	0.150	12
$P_{13}(1720)$	1.720	0.250	11
$F_{35}(1905)$	1.880	0.330	$9-15$
$P_{31}(1910)$	1.890	0.280	$15-30$
$F_{37}(1950)$	1.930	0.285	$35-45$

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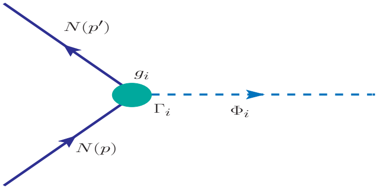
Lagrangian of nucleon interacting with a meson field

$$\mathcal{L} = g_i \bar{\Psi}(p') \Gamma_i \Psi(p) \Phi_i$$



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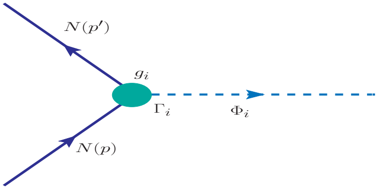

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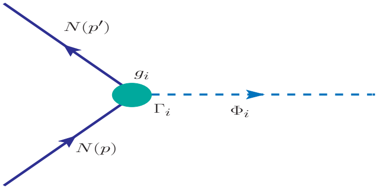

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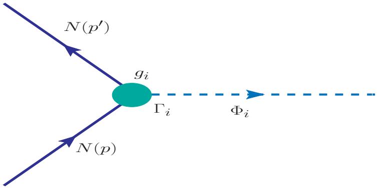

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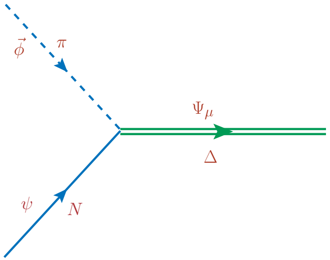
iii interaction with a vector field, $\rho - meson$

$$\mathcal{L}_v = g_v \bar{\Psi}(p')\gamma^\mu \Psi(p)\Phi_v$$

$N\pi\Delta$ Lagrangian

Interaction among N, Δ, π is given by the following Lagrangian



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It is a collection of three 4x2 matrices (one matrix for each pion field) that connects the nucleon field with the corresponding Rarita-Schwinger field of the Δ's; each 4x2 matrix is basically a Clebsch-Gordan array.

$\times 1$ nucleon doublet
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Interaction of pseudoscalar meson with external field

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Pion fields(ϕ_x, ϕ_y, ϕ_z) in the spherical coordinates:

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The nomalization of the transition operator $T_\lambda^\dagger$ is:

$$\langle\frac{3}{2},M|\,T_\lambda^\dagger|\frac{1}{2},m\rangle = \left(1,\frac{1}{2},\frac{3}{2}|\lambda,m,M\right)$$

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On expanding the scalar product of $(\vec{T}^\dagger \cdot \partial^\mu \vec{\phi})$ in spherical basis, Lagrangian can be rewritten as

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$$T_0^\dagger = \begin{pmatrix} (1, \frac{1}{2}, \frac{3}{2} | 0, +\frac{1}{2}, +\frac{3}{2}) & (1, \frac{1}{2}, \frac{3}{2} | 0, \frac{-1}{2}, +\frac{3}{2}) \\ (1, \frac{1}{2}, \frac{3}{2} | 0, +\frac{1}{2}, +\frac{1}{2}) & (1, \frac{1}{2}, \frac{3}{2} | 0, \frac{-1}{2}, +\frac{1}{2}) \\ (1, \frac{1}{2}, \frac{3}{2} | 0, +\frac{1}{2}, \frac{-1}{2}) & (1, \frac{1}{2}, \frac{3}{2} | 0, \frac{-1}{2}, \frac{-1}{2}) \\ (1, \frac{1}{2}, \frac{3}{2} | 0, +\frac{1}{2}, \frac{-3}{2}) & (1, \frac{1}{2}, \frac{3}{2} | 0, \frac{-1}{2}, \frac{-3}{2}) \end{pmatrix}$$

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$$T_{+1}^\dagger = \begin{pmatrix} (1, \frac{1}{2}, \frac{3}{2} | +1, +\frac{1}{2}, +\frac{3}{2}) & (1, \frac{1}{2}, \frac{3}{2} | +1, -\frac{1}{2}, +\frac{3}{2}) \\ (1, \frac{1}{2}, \frac{3}{2} | +1, +\frac{1}{2}, +\frac{1}{2}) & (1, \frac{1}{2}, \frac{3}{2} | +1, -\frac{1}{2}, +\frac{1}{2}) \\ (1, \frac{1}{2}, \frac{3}{2} | +1, +\frac{1}{2}, -\frac{1}{2}) & (1, \frac{1}{2}, \frac{3}{2} | +1, -\frac{1}{2}, -\frac{1}{2}) \\ (1, \frac{1}{2}, \frac{3}{2} | +1, +\frac{1}{2}, -\frac{3}{2}) & (1, \frac{1}{2}, \frac{3}{2} | +1, -\frac{1}{2}, -\frac{3}{2}) \end{pmatrix}$$

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Focusing only on the part of the Lagrangian which is with $\partial^\mu \phi_{-1}$:

Lagrangian,

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Focusing only on the part of the Lagrangian which is with $\partial^\mu \phi_{-1}$:

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after substituting the values of Clebsch-Gordan coefficients:

$$\mathcal{L} = -\frac{f^*}{m_\pi} \bar{\Psi}_\mu \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{3}} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \psi \, \partial^\mu \phi_{-1}$$

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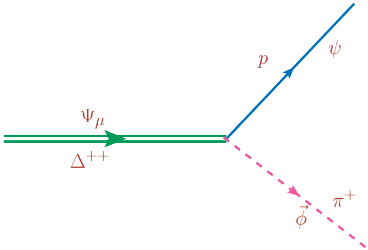
solving further

$$\mathcal{L} = -\frac{f^*}{m_\pi} \partial^\mu \phi_{-1} \left(\bar{\Delta}^{++}_\mu p + \frac{1}{\sqrt{3}} \bar{\Delta}^{+}_\mu n \right) - \frac{f^*}{m_\pi} \partial^\mu \phi^\dagger_{-1} \left(\bar{p} \, \Delta^{++}_\mu + \frac{1}{\sqrt{3}} \bar{n} \, \Delta^{+}_\mu \right)$$

Decay rate of $\Delta^{++} \rightarrow p\pi^+$

Tutorial problem



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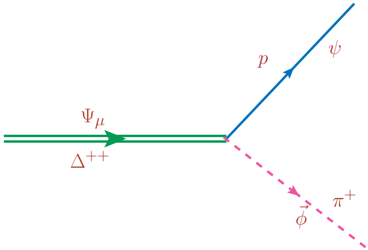

The Lagrangian is:

$$\mathcal{L} = -\frac{f^*}{m_\pi}\partial^\mu \phi_{+1}\left(\bar{p}\,\Delta_\mu^{++} + \frac{1}{\sqrt{3}}\bar{n}\,\Delta_\mu^{+}\right)$$

$\nu/\bar{\nu}$ *induced charged current single pion production*

$$\nu(k) + N(p) \rightarrow l^-(k') + N'(p') + \pi^i(k_\pi)$$

- quantities in the parenthesis represent four momenta of the corresponding particle
- conservation of four momenta: $k + p = k' + p' + k_\pi$

Differential scattering cross section

$$d\sigma = \frac{1}{4ME_\nu(2\pi)^5}\frac{d\vec{k}'}{(2E_l)}\frac{d\vec{p}\,'}{(2E_p')}\frac{d\vec{k}_\pi}{(2E_\pi)}\delta^4(k+p-k'-p'-k_\pi)\bar{\Sigma}\Sigma|\mathcal{M}|^2,$$

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Fermi coupling constant

Leptonic Current

Differential scattering cross section

$$d\sigma = \frac{1}{4ME_\nu(2\pi)^5}\frac{d\vec{k}'}{(2E_l)}\frac{d\vec{p}\,'}{(2E'_p)}\frac{d\vec{k}_\pi}{(2E_\pi)}\delta^4(k+p-k'-p'-k_\pi)\bar{\Sigma}\Sigma|\mathcal{M}|^2,$$

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Leptonic Current

Hadronic Current

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$$j_\mu^{(L)} = \bar{u}(k')\gamma_\mu(1 \pm \gamma_5)u(k)$$

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$$W^i + N \to N' + \pi$$

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  - $CC : (W^i \equiv W^\pm \, ; i = \pm)$
  - $NC : (W^i \equiv Z^0 \, ; i = 0)$

Resonant Contribution

- Pion production through resonance excitation is

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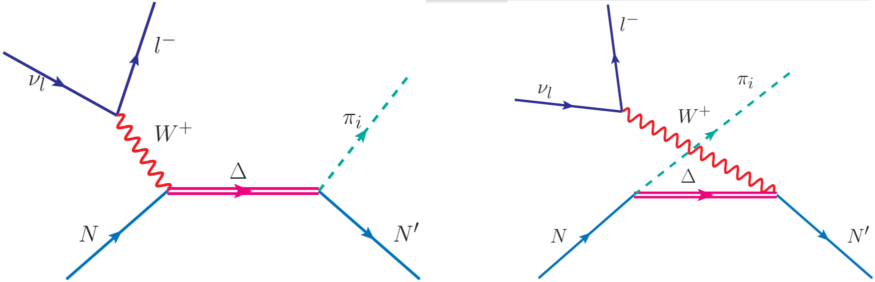
Δ Resonance or Higher Resonance

Neutrino induced charged current single pion production process



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$$J_\mu^{\frac{3}{2}} \quad = \quad < \Delta(p')|J_\mu^{\frac{3}{2}}(0)|N(p) >$$

$$J_\mu^{\frac{3}{2}} = \bar{\psi}^\nu(p')\Gamma^{\frac{3}{2}}_{\nu\mu}u(p)$$

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N-Δ transition vertex

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Rarita-Schwinger Spinor   N-Δ transition vertex   Dirac Spinor

- For $J = \frac{3}{2}$ resonance excitation hadronic current is

$$J^{\frac{3}{2}}_{\mu} \quad = \quad < \Delta(p')|J^{\frac{3}{2}}_{\mu}(0)|N(p) >$$

$$J^{\frac{3}{2}}_{\mu} = \bar{\psi}^{\nu}(p')\Gamma^{\frac{3}{2}}_{\nu\mu} u(p)$$

Rarita-Schwinger Spinor N-Δ transition vertex Dirac Spinor

Transition vertex

- For Positive Parity states:

$$\Gamma^{\frac{3}{2}^+}_{\nu\mu} = \left[V^{\frac{3}{2}}_{\nu\mu} - A^{\frac{3}{2}}_{\nu\mu} \right] \gamma_5$$

- For Negative Parity states

$$\Gamma^{\frac{3}{2}^-}_{\nu\mu} = V^{\frac{3}{2}}_{\nu\mu} - A^{\frac{3}{2}}_{\nu\mu}$$

$$V^{\frac{3}{2}}_{\nu\mu} = \left[\frac{\tilde{C}^V_3}{M}(g_{\mu\nu}\not{q} - q_\nu\gamma_\mu) + \frac{\tilde{C}^V_4}{M^2}(g_{\mu\nu}\,q\cdot p' - q_\nu p'_\mu) + \frac{\tilde{C}^V_5}{M^2}(g_{\mu\nu}\,q\cdot p - q_\nu p_\mu) + g_{\mu\nu}\,\tilde{C}^V_6 \right]$$

Vector Form Factors $\Longrightarrow$ CVC $\longrightarrow C^V_6(Q^2) = 0$

$$V^{\frac{3}{2}}_{\nu\mu} = \left[\frac{\tilde{C}_3^V}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}_4^V}{M^2} (g_{\mu\nu}\, q \cdot p' - q_\nu p'_\mu) + \frac{\tilde{C}_5^V}{M^2} (g_{\mu\nu}\, q \cdot p - q_\nu p_\mu) + g_{\mu\nu} \tilde{C}_6^V \right]$$

Vector Form Factors $\Longrightarrow$ CVC $\longrightarrow C_6^V(Q^2) = 0$

$$\begin{aligned}
C_3^V(Q^2) &= \frac{2.13}{(1 + Q^2/M_V^2)^2} \times \frac{1}{1 + \frac{Q^2}{4M_V^2}}, \\
C_4^V(Q^2) &= \frac{-1.51}{(1 + Q^2/M_V^2)^2} \times \frac{1}{1 + \frac{Q^2}{4M_V^2}}, \\
C_5^V(Q^2) &= \frac{0.48}{(1 + Q^2/M_V^2)^2} \times \frac{1}{1 + \frac{Q^2}{0.776 M_V^2}}
\end{aligned}$$

$$A^{\frac{3}{2}}_{\nu\mu} = -\left[\frac{\tilde{C}^A_3}{M}(g_{\mu\nu}\not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}^A_4}{M^2}(g_{\mu\nu}\, q\cdot p' - q_\nu\, p'_\mu) + \tilde{C}^A_5\, g_{\mu\nu} + \frac{\tilde{C}^A_6}{M^2} q_\nu\, q_\mu \right] \gamma_5$$

$$A^{\frac{3}{2}}_{\nu\mu} = -\left[\frac{\tilde{C}^A_3}{M}(g_{\mu\nu}\not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}^A_4}{M^2}(g_{\mu\nu}\, q\cdot p' - q_\nu p'_\mu) + \tilde{C}^A_5 g_{\mu\nu} + \frac{\tilde{C}^A_6}{M^2} q_\nu q_\mu \right]\gamma_5$$

Axial Vector Form Factors $\Longrightarrow$ Dominant Contribution from $C^A_5(Q^2)$

$$A^{\frac{3}{2}}_{\nu\mu} = -\left[\frac{\tilde{C}^A_3}{M} (g_{\mu\nu} \not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}^A_4}{M^2} (g_{\mu\nu}\, q \cdot p' - q_\nu p'_\mu) + \tilde{C}^A_5 g_{\mu\nu} + \frac{\tilde{C}^A_6}{M^2} q_\nu q_\mu \right] \gamma_5$$

Axial Vector Form Factors $\Longrightarrow$ Dominant Contribution from $C_5^A(Q^2)$

The Q^2 dependence of $C_5^A(Q^2)$ is parameterized by Schreiner and von Hippel in the Adler's model and is given by $a = -1.21$ and $b = 2$

$$C_5^A(Q^2) = \frac{C_5^A(0)\left(1 + \frac{a\,Q^2}{b\ +\ Q^2}\right)}{\left(1 + Q^2/M^2_{A\Delta}\right)^2}$$

$$A^{\frac{3}{2}}_{\nu\mu} = -\left[\frac{\tilde{C}^A_3}{M}(g_{\mu\nu}\not{q} - q_\nu \gamma_\mu) + \frac{\tilde{C}^A_4}{M^2}(g_{\mu\nu}\, q\cdot p' - q_\nu p'_\mu) + \tilde{C}^A_5 g_{\mu\nu} + \frac{\tilde{C}^A_6}{M^2} q_\nu q_\mu \right]\gamma_5$$

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Modified dipole form

$$C^A_5(Q^2) = \frac{C^A_5(0)}{\left(1 + Q^2/M^2_{A\Delta}\right)^2}\, \frac{1}{1 + Q^2/(3M^2_{A\Delta})}.$$

Δ *Resonance*

PCAC hypothesis $\Rightarrow$ divergence should vanish in the limit when $m_\pi \to 0$.

Axial part of the current

$$
\begin{aligned}
< \Delta^{++} | A_\mu^{\frac{3}{2}} | p > \quad &= \sqrt{3}\bar{\psi}_\nu(p') \left[\frac{C_3^A}{M} (g_{\nu\mu} \not{q} - q_\nu \gamma_\mu) + \right. \\
&\left. \frac{C_4^A}{M^2} (g_{\nu\mu}\, q \cdot p' - q_\nu\, p'_\mu) + C_5^A\, g_{\mu\nu} + \frac{C_6^A}{M^2}\, q_\nu\, q_\mu \right] u(p)
\end{aligned}
$$

A

$$
\begin{aligned}
< \Delta^{++}|i\partial_\mu A_\mu|p > &= \sqrt{3}\bar{\psi}_\nu(p')q_\mu\left[C_5^A g_{\mu\nu} + \frac{C_6^A}{M^2}q_\nu q_\mu\right]u(p) \\
&= \bar{\psi}_\nu(p')q^\nu\left[C_5^A + \frac{C_6^A}{M^2}q^2\right]u(p)
\end{aligned}
$$

Pion pole dominance demands that C_6^A must have a pion pole $\tilde{C}_6^A(q^2) = \frac{f_{\pi N\Delta}f_\pi}{2\sqrt{3}M}\frac{M^2}{m_\pi^2 - q^2}$ In the limit $q^2 \to 0$ and $m_\pi \to 0$ one has

$$
< \Delta^{++}|i\partial_\mu A_\mu|p > = \sqrt{3}\bar{\psi}_\nu(p')q^\nu\left[C_5^A(0) - \frac{f_{\pi N\Delta}f_\pi}{2\sqrt{3}M}\right]u(p)
$$

Δ *Resonance*

Axial Vector Form Factors $\Longrightarrow$ PCAC & G-T relation
Adler's Model

$$C_6^A(Q^2) = C_5^A(Q^2)\frac{M^2}{Q^2+m_\pi^2}$$

$$C_5^A(0) = f_\pi \frac{f_{\Delta N\pi}}{2\sqrt{3}M},$$

$C_5^A(0) = 1.0$ GeV

$M_{A\Delta} = 1.026$ GeV

$$C_4^A(Q^2) = -\frac{1}{4}C_5^A(Q^2); \qquad C_3^A(Q^2) = 0.$$

Hadronic current for spin $\frac{3}{2}$ resonance

$$j^\mu\Big|^{\frac{3}{2}}_{R} = i\,a\,\mathcal{C}^{\mathcal{R}}\frac{k_\pi^\alpha}{p_R^2 - M_R^2 + iM_R\Gamma_R}\bar{u}(\vec{p}\,')P^{3/2}_{\alpha\beta}(p_R)\Gamma^{\beta\mu}_{\frac{3}{2}}(p,q)u(\vec{p}), \quad p_R = p+q$$

$$j^\mu\Big|^{\frac{3}{2}}_{CR} = i\,a\,\mathcal{C}^{\mathcal{R}}\frac{k_\pi^\beta}{p_R^2 - M_R^2 + iM_R\Gamma_R}\bar{u}(\vec{p}\,')\hat{\Gamma}^{\mu\alpha}_{\frac{3}{2}}(p',-q)P^{3/2}_{\alpha\beta}(p_R)u(\vec{p}), \quad p_R = p'-q$$

Hadronic current for spin $\frac{3}{2}$ resonance

$$j^\mu\Big|_R^{\frac{3}{2}} = i\, a\, \mathcal{C}^{\mathcal{R}} \frac{k_\pi^\alpha}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}\,')P_{\alpha\beta}^{3/2}(p_R)\Gamma_{\frac{3}{2}}^{\beta\mu}(p,q)u(\vec{p}), \quad p_R = p + q$$

$$j^\mu\Big|_{CR}^{\frac{3}{2}} = i\, a\, \mathcal{C}^{\mathcal{R}} \frac{k_\pi^\beta}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}\,')\hat{\Gamma}_{\frac{3}{2}}^{\mu\alpha}(p',-q)P_{\alpha\beta}^{3/2}(p_R)u(\vec{p}), \quad p_R = p' - q$$

- $a = \cos\theta_c$: the charged current process

Hadronic current for spin $\frac{3}{2}$ resonance

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\begin{aligned}
j^\mu\Big|_R^{\frac{3}{2}} &= i\, a\, \mathcal{C}^{\mathcal{R}} \frac{k_\pi^\alpha}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}\,') P_{\alpha\beta}^{3/2}(p_R) \Gamma_{\frac{3}{2}}^{\beta\mu}(p,q) u(\vec{p}), \quad p_R = p + q \\
j^\mu\Big|_{CR}^{\frac{3}{2}} &= i\, a\, \mathcal{C}^{\mathcal{R}} \frac{k_\pi^\beta}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}\,') \hat{\Gamma}_{\frac{3}{2}}^{\mu\alpha}(p',-q) P_{\alpha\beta}^{3/2}(p_R) u(\vec{p}), \quad p_R = p' - q
\end{aligned}
$$

- $a = \cos\theta_c$: the charged current process
- $a = 1$: the neutral current process

Hadronic current for spin $\frac{3}{2}$ resonance

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- $a = \cos\theta_c$: the charged current process
- $a = 1$: the neutral current process
- $\mathcal{C}^{\mathcal{R}}$: coupling strength for $\mathcal{R} \to N\pi$

Hadronic current for spin $\frac{3}{2}$ resonance

$$
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j^\mu\Big|^{\frac{3}{2}}_{R} &= i\,a\,\mathcal{C}^{\mathcal{R}}\frac{k_\pi^\alpha}{p_R^2-M_R^2+iM_R\Gamma_R}\bar{u}(\vec{p}\,')P^{3/2}_{\alpha\beta}(p_R)\Gamma^{\beta\mu}_{\frac{3}{2}}(p,q)u(\vec{p}), \quad p_R=p+q \\
j^\mu\Big|^{\frac{3}{2}}_{CR} &= i\,a\,\mathcal{C}^{\mathcal{R}}\frac{k_\pi^\beta}{p_R^2-M_R^2+iM_R\Gamma_R}\bar{u}(\vec{p}\,')\hat{\Gamma}^{\mu\alpha}_{\frac{3}{2}}(p',-q)P^{3/2}_{\alpha\beta}(p_R)u(\vec{p}), \quad p_R=p'-q
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- $P^{3/2}_{\alpha\beta}$: spin three-half projection operator

Hadronic current for spin $\frac{3}{2}$ resonance

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- $P_{\alpha\beta}^{3/2}$: spin three-half projection operator

$$P_{\alpha\beta}^{3/2} = -(\not{p}' + M_R)\left(g_{\alpha\beta} - \frac{2}{3}\frac{p'_\alpha p'_\beta}{M_R^2} + \frac{1}{3}\frac{p'_\alpha\gamma_\beta - p'_\beta\gamma_\alpha}{M_R} - \frac{1}{3}\gamma_\alpha\gamma_\beta\right)$$

The hadronic tensor

$$
\begin{aligned}
H^{\mu\nu}_{\frac{3}{2}} &= \frac{1}{2}\sum_{s_i}\sum_{s_f} J^{\mu\,\dagger}_{\frac{3}{2}} J^{\nu}_{\frac{3}{2}} \\
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- It was introduced by Gell-Mann and Levy in 1960 to study chiral symmetry in the pion-nucleon system. Later the spontaneous symmetry breaking and PCAC were incorporated.
- The Lagrangian is written in such a way that it is a Lorentz-scalar and invariant under vector(Λ_V) and axial-vector(Λ_A)transformations.

Linear sigma model Lagrangian

$$\mathcal{L} = i\bar{\psi}\ \partial\!\!\!/\ \psi - g_\pi(\bar{\psi}\gamma_5\vec{\tau}\psi\vec{\pi} + \bar{\psi}\psi\sigma)$$

$$+\frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{\lambda}{4}\left((\pi^2+\sigma^2) - f_\pi^2\right)^2$$

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Lagrangian of

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Kinetic energy term
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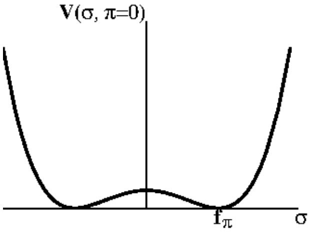
Pion-sigma
potential

The shape of the potential



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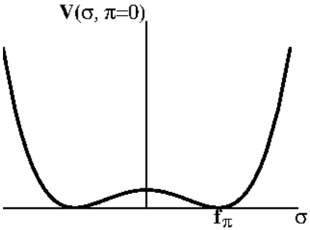
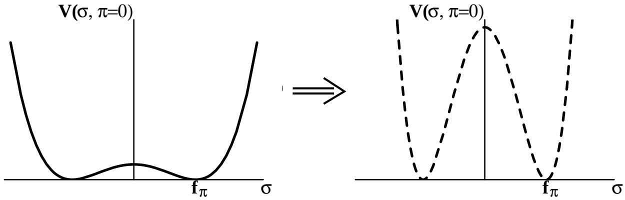


Linear sigma-model has massive $\sigma-$field, which is not identified with any existing particle.

Nonlinear sigma-model



- To remove $\sigma - \mathit{field}$, we take an infinitely large coupling λ in the linear $\sigma-$model.
- Infinitely large coupling results in infinite mass of a σ-meson.
- Potential gets infinitely steep in the sigma-direction.
- Minimum of the potential defines a circle(Chiral circle), which includes the dynamics

$$\sigma^2 + \pi^2 = f_\pi^2$$

Chiral circle restricts the dynamics to rotate on the circle. The fields can be expressed in terms of angles $\vec{\Phi}$,

$$\begin{aligned} \sigma(x) &= f_\pi \; cos\left(\frac{\Phi(x)}{f_\pi}\right) = f_\pi + \mathcal{O}(\Phi^2) \\ \vec{\pi}(x) &= f_\pi \; \hat{\Phi} \; sin\left(\frac{\Phi(x)}{f_\pi}\right) = \vec{\Phi} + \mathcal{O}(\Phi^3) \end{aligned}$$

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Complex notation for the fields,

$$U(x) = e^{i\frac{\vec{\tau}\vec{\Phi}(x)}{f_\pi}} = cos\left(\frac{\Phi(x)}{f_\pi}\right) + i\vec{\tau}\hat{\Phi} \; sin\left(\frac{\Phi(x)}{f_\pi}\right) = \frac{1}{f_\pi}(\sigma + i\vec{\tau}\vec{\pi})$$

$U(x)$ represents a 2x2 unitary matrix.

Tutorial Problem

If a SU(3) valued field is given by $U(x) = e^{i\frac{\vec{\tau}\vec{\Phi}(x)}{f_\pi}}$ then show that the Lagrangian

$$\mathcal{L}_{eff} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\partial_\mu \mathrm{U}^\dagger \partial^\mu \mathrm{U}\right].$$

gives rise to kinetic energy term: $\frac{1}{2}\partial_\mu \phi_k \partial^\mu \phi_k$

If a SU(N) valued field is given by $U(x) = e^{i\frac{\Phi_a(x)\Lambda_a}{f_\pi}}$, where Λ_a are traceless matrices and Φ_a are real fields then show that

$$(i) \quad Tr\left[\partial_\mu U U^\dagger\right] = 0$$

$$(ii) \quad U^\dagger \partial_\mu U = -\partial_\mu U^\dagger U$$

In terms of redefined fields $U(x)$, the Lagrangian of Linear sigma model gets modified as:

$$\mathcal{L} = i\bar{\psi} \not{\partial} \psi - g_\pi(\bar{\psi}\gamma_5\vec{\tau}\psi\vec{\pi} + \bar{\psi}\psi\sigma)$$

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$$\bar{\psi}_W(i\,\partial\!\!\!/ + \gamma^\mu V_\mu + \gamma^\mu\gamma_5 A_\mu)\psi_W$$

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where $\Lambda \equiv e^{i\gamma_5\frac{\vec{\tau}\vec{\Phi}(x)}{2f_\pi}}$, $\bar{\psi}_W = \bar{\psi}\Lambda$ and $\psi_W = \Lambda\psi$

Lagrangian of the nonlinear sigma-model

Also referred to as the Weinberg-Lagrangian,

$$\mathcal{L}_W = \bar{\psi}_W(i\not{\partial} + \gamma_\mu V_\mu + \gamma^\mu\gamma_5 A_\mu - M_N)\psi_W + \frac{f_\pi^2}{4} Tr(\partial_\mu U^\dagger \partial^\mu U)$$

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- $\vec{\Phi}$ is identified with the pion field
- σ-field has disappeared
- coupling between nucleons and pions has been changed to a pseudovector

Meson-Meson Interaction

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$SU(3)$ representation of pseudoscalar fields:

$$\Phi(x) = \sum_{k=1}^{8} \phi_k(x) \lambda_k = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

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Relation between Cartesian components and Pseudoscalar meson fields.

$\phi_1 + i\phi_2 \rightarrow$	$\sqrt{2}\pi^+$	$\phi_1 - i\phi_2 \rightarrow$	$\sqrt{2}\pi^-$	$\phi_3 \rightarrow$	π^0
$\phi_4 + i\phi_5 \rightarrow$	$\sqrt{2}K^+$	$\phi_4 - i\phi_5 \rightarrow$	$\sqrt{2}K^-$	$\phi_8 \rightarrow$	η
$\phi_6 + i\phi_7 \rightarrow$	$\sqrt{2}K^0$	$\phi_6 - i\phi_7 \rightarrow$	$\sqrt{2}\bar{K}^0$		

Interaction of pseudoscalar meson with external field

We replace the partial derivatives ∂^μ by covariant derivatives D^μ to make the effective chiral Lagrangian invariant under the local transformations:

$$\mathcal{L}_M^{(2)} = \frac{f_\pi^2}{4}\mathrm{Tr}[D_\mu U (D^\mu U)^\dagger] + ...$$

$$\begin{aligned} D^\mu U &\equiv \partial^\mu U - i r^\mu U + i U l^\mu \\ D^\mu U^\dagger &\equiv \partial^\mu U^\dagger + i U^\dagger r^\mu - i l^\mu U^\dagger, \end{aligned}$$

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- l^μ and r^μ are left handed and right handed currents:
  - $l^\mu = \frac{1}{2}(v^\mu - a^\mu)$
  - $r^\mu = \frac{1}{2}(v^\mu + a^\mu)$,
- a^μ : axial-vector field
- v^μ : vector field

- For the Charged Current(CC) case l^μ and r^μ are given by

$$l_\mu = -\frac{g}{\sqrt{2}}(W_\mu^+ T_+ + W_\mu^- T_-), \quad r_\mu = 0$$

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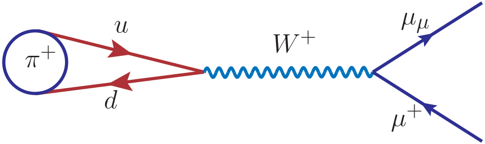
with $W^{\pm}$, the W boson fields and

$$T_{+} = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad T_{-} = \begin{pmatrix} 0 & 0 & 0 \\ V_{ud} & 0 & 0 \\ V_{us} & 0 & 0 \end{pmatrix}.$$

Here, V_{ij} are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

For example: We take an example of pion decay, $\pi^+ \to \mu^+ \nu_\mu$:



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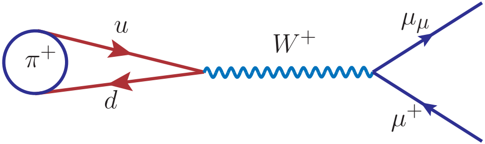

- Coupling of the W bosons to the leptons is given by:

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} \left[W_\rho^+ \bar{\nu}_\mu \gamma^\rho (1-\gamma_5)\mu + W_\rho^- \bar{\mu} \gamma^\rho (1-\gamma_5)\nu_\mu \right]$$

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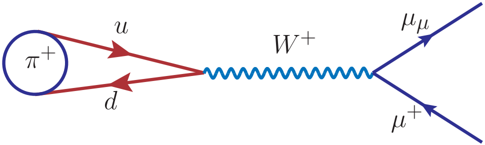

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- Interaction term of a single Goldstone boson with a W field:

$$\begin{aligned}\mathcal{L}_{W\phi} &= \frac{F_0}{2} Tr(l_\mu \partial^\mu \phi) \\ &= -\frac{g}{\sqrt{2}}\frac{F_0}{2} Tr[(W_\mu^+ T_+ + W_\mu^- T_-)\partial^\mu \phi]\end{aligned}$$

Considering $Tr(T_+\partial^\mu\phi)$ first,

$$
\begin{aligned}
Tr & \quad (T_+\partial^\mu\phi) \\
= & \quad Tr\left[\begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\partial^\mu\begin{pmatrix} \pi^0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}\right] \\
= & \quad V_{ud}\sqrt{2}\,\partial^\mu\pi^- + V_{us}\sqrt{2}\,\partial^\mu K^-
\end{aligned}
$$

Interaction of pseudoscalar meson with external field

The complex scalar field $\Phi(x)$, represents particles with charge q and antiparticles with charge $-q$.

The field operator, $\hat{\Phi}(x)$:

$$\hat{\Phi}(x) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega_k}} \left[\hat{a}(\vec{k})e^{-ip^\mu x_\mu} + \hat{b}^\dagger(\vec{k})e^{ip^\mu x_\mu}\right]$$

annihilates a particle — creates a particle

Adjoint field operator, $\hat{\Phi}^\dagger(x)$:

$$\hat{\Phi}^\dagger(x) = \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2\omega_k}} \left[\hat{a}^\dagger(\vec{k})e^{ip^\mu x_\mu} + \hat{b}(\vec{k})e^{-ip^\mu x_\mu}\right]$$

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Similarly considering next term, $Tr(T_{-}\partial^{\mu}\phi)$,

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Tr \quad & (T_{-}\partial^{\mu}\phi) \\
= \quad & Tr\left[\begin{pmatrix} 0 & 0 & 0 \\ V_{ud} & 0 & 0 \\ V_{us} & 0 & 0 \end{pmatrix} \partial^{\mu} \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}\right] \\
= \quad & V_{ud}\sqrt{2}\,\partial^{\mu}\pi^+ + V_{us}\sqrt{2}\,\partial^{\mu}K^+
\end{aligned}
$$

Similarly considering next term, $Tr(T_{-}\partial^{\mu}\phi)$,

$$
\begin{aligned}
& Tr \quad (T_{-}\partial^{\mu}\phi) \\
= & \quad Tr\left[\begin{pmatrix} 0 & 0 & 0 \\ V_{ud} & 0 & 0 \\ V_{us} & 0 & 0 \end{pmatrix} \partial^{\mu} \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}\right] \\
= & \quad V_{ud}\sqrt{2}\,\partial^{\mu}\pi^+ + V_{us}\sqrt{2}\,\partial^{\mu}K^+
\end{aligned}
$$

Then Lagrangian becomes:

$$
\begin{aligned}
\mathcal{L}_{W\phi} \quad = & \quad -g\frac{F_0}{2}\left[W_{\mu}^{+}\left(V_{ud}\,\partial^{\mu}\pi^{-} + V_{us}\,\partial^{\mu}K^{-}\right)\right. \\
+ & \quad \left. W_{\mu}^{-}\left(V_{ud}\,\partial^{\mu}\pi^{+} + V_{us}\,\partial^{\mu}K^{+}\right)\right]
\end{aligned}
$$

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$$\begin{aligned}\mathcal{M} &= i\left[-\frac{g}{2\sqrt{2}}\bar{u}_{\nu_\mu}\gamma^\rho(1-\gamma_5)\nu_{\mu^+}\right]\times\frac{ig_{\rho\sigma}}{M_W^2}\times i\left[-g\frac{F_0}{2}V_{ud}(-ip^\sigma)\right]\\ &= -G_F V_{ud} F_0 \bar{u}_{\nu_\mu}\not{p}(1-\gamma_5)\nu_{\mu^+}\end{aligned}$$

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- p denotes the 4-momentum of the pion
- G_F Fermi coupling constant

Interaction of pseudoscalar fields with baryons

The methods of χPT can be extended to describe the low energy interaction of baryons with the Goldstone bosons as well as with the external fields

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We consider the octet of $\frac{1}{2}^+$ baryons. With each member of the octet we associate a complex, four-component Dirac field

$$B(x) = \sum_{k=1}^{8} \frac{1}{\sqrt{2}} b_k(x) \lambda_k = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix},$$

$$B(x) = \sum_{k=1}^{8} \frac{1}{\sqrt{2}} b_k(x)\lambda_k = \frac{1}{\sqrt{2}} \begin{pmatrix} b_3 + \frac{b_8}{\sqrt{3}} & b_1 - ib_2 & b_4 - ib_5 \\ b_1 + ib_2 & \frac{b_8}{\sqrt{3}} - b_3 & b_6 - ib_7 \\ b_4 + ib_5 & b_6 + ib_7 & -2\frac{b_8}{\sqrt{3}} \end{pmatrix}$$

Relation between cartesian components and baryon octet fields.

$b_1 + ib_2 \rightarrow$	$\sqrt{2}\Sigma^+$	$b_1 - ib_2 \rightarrow$	$\sqrt{2}\Sigma^-$	$b_3 \rightarrow$	Σ^0
$b_4 + ib_5 \rightarrow$	$\sqrt{2}\Xi^0$	$b_4 - ib_5 \rightarrow$	$\sqrt{2}p$	$b_8 \rightarrow$	Λ
$b_6 + ib_7 \rightarrow$	$\sqrt{2}\Xi^-$	$b_6 - ib_7 \rightarrow$	$\sqrt{2}n$		

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- If we take nucleons as massive matter fields which couple to external currents and Goldstone fields(pions etc.), we have to then expand the Lagrangian according to their increasing number of momenta.
- For this, one defines a new unitary matrix

$$u = \sqrt{U} \equiv exp\left(i\frac{\Phi(x)}{2f_\pi}\right)$$

The lowest-order chiral Lagrangian for the baryon octet in the presence of an external current may be written in terms of the SU(3) matrix B as,

$$\begin{aligned}\mathcal{L}_{MB}^{(1)} &= \mathrm{Tr}\left[\bar{B}\left(i\not{D}-M\right)B\right]-\frac{D}{2}\mathrm{Tr}\left(\bar{B}\gamma^{\mu}\gamma_5\{u_\mu,B\}\right)\\ &- \frac{F}{2}\mathrm{Tr}\left(\bar{B}\gamma^{\mu}\gamma_5[u_\mu,B]\right),\end{aligned}$$

covariant derivative of B:

$$D_\mu B=\partial_\mu B+[\Gamma_\mu,B],$$

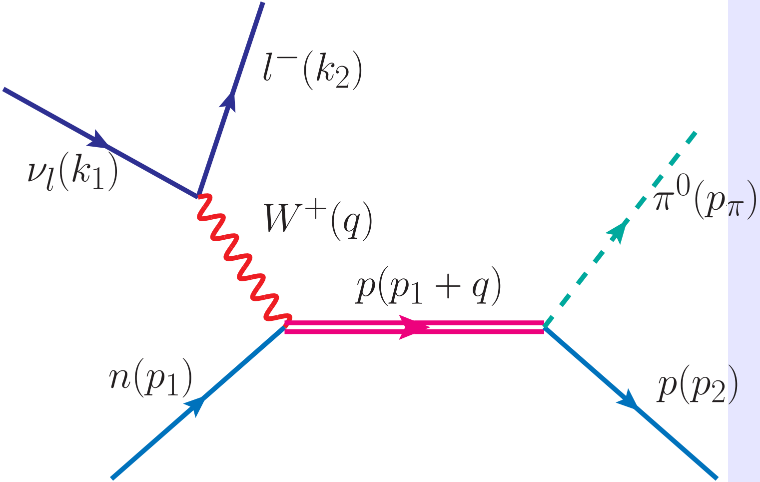
$$\Gamma^\mu=\frac{1}{2}\left[u^\dagger(\partial^\mu-ir^\mu)u+u(\partial^\mu-il^\mu)u^\dagger\right]$$

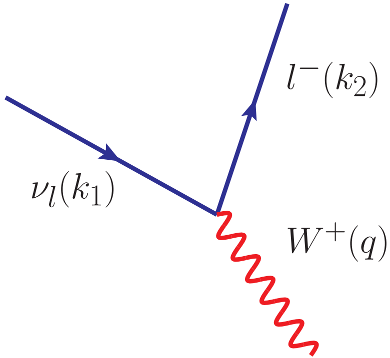
Tutorial problem:Consider the lowest order $\pi - N$ Lagrangian

$$\bar{\Psi}(i \not{D} - m + \frac{g_A}{2}\gamma_\mu \gamma_5 u_\mu)\Psi$$

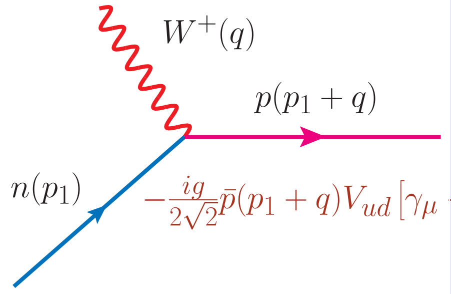
in the absence of external currents show that by expanding $u = exp\left(\frac{i\vec{\phi}\cdot\vec{\tau}}{2f_\pi}\right)$, pion nucleon interaction Lagrangian may be written as

$$\mathcal{L}_{\pi NN} = -\frac{g_A}{2f_\pi}\bar{\Psi}\gamma_\mu\gamma_5\partial^\mu\phi_a\tau_a\Psi.$$

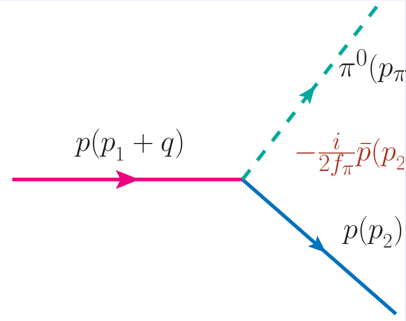

$l^-(k_2)$
$\nu_l(k_1)$
$W^+(q)$
$\pi^0(p_\pi)$
$p(p_1 + q)$
$n(p_1)$
$p(p_2)$



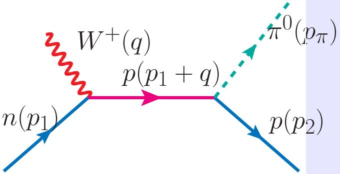
$$-\frac{\mathbf{ig}}{\mathbf{2\sqrt{2}}}\gamma^{\mu}(\mathbf{1}-\gamma_{\mathbf{5}})$$



$$-\frac{ig}{2\sqrt{2}}\bar{p}(p_1 + q)V_{ud}\left[\gamma_\mu - (D + F)\gamma_\mu\gamma_5\right] n(p_1)W_+$$

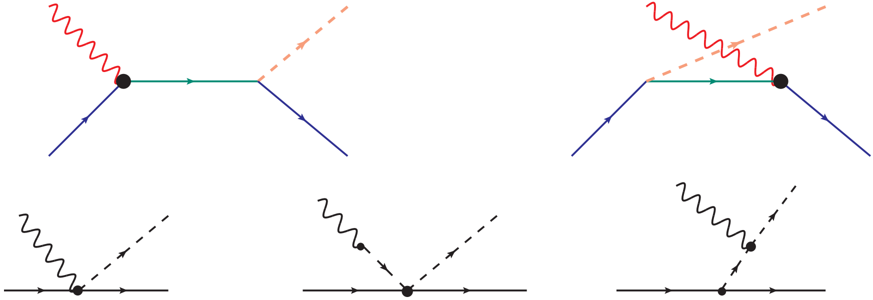


$$-\frac{i}{2f_\pi}\bar{p}(p_2)\left[(D+F)\gamma_\theta\gamma_5\right]p(p_1+q)(ip_\pi^\theta)$$

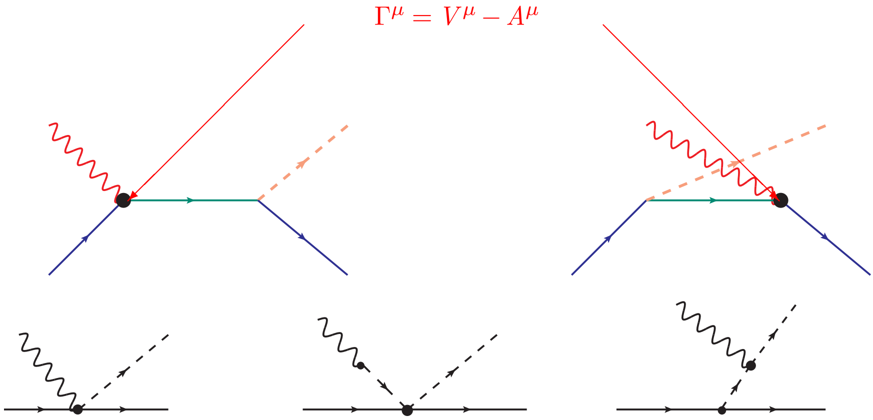


$$-\frac{1}{2f_\pi}\left(-\frac{g}{2\sqrt{2}}\right)\bar{p}(p_2)\left\{\not{p}_\pi(D+F)\gamma_5\frac{\not{p}_1+\not{q}+M}{(p_1+q)^2-M^2}V_{ud}\left[\gamma_\mu-(D+F)\gamma_\mu\gamma_5\right]\right\}n(p_1)$$

Non-resonant background



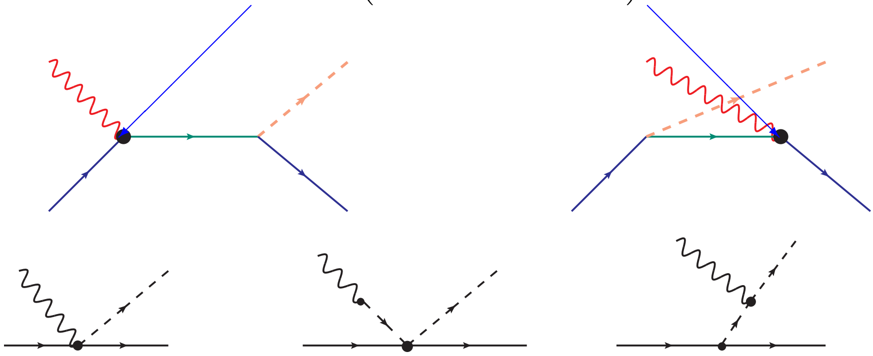
Non-resonant background



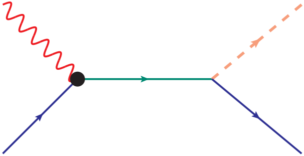
Non-resonant background

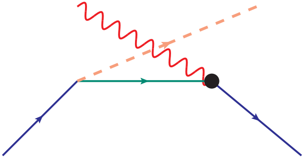
$$V^\mu \longrightarrow \tilde{f}_1(Q^2)\gamma^\mu + \tilde{f}_2(Q^2)i\sigma^{\mu\nu}\frac{q_\nu}{2M}$$

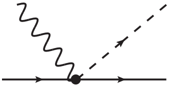
$$A^\mu \longrightarrow \left(\tilde{f}_A(Q^2)\gamma^\mu + \tilde{f}_P(Q^2)\frac{q^\mu}{M}\right)\gamma^5$$



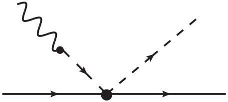
Non-resonant background



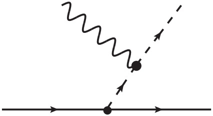




$\Gamma^\mu \sim \gamma^\mu - g_A \gamma^\mu \gamma^5$



$\Gamma^\mu \sim \gamma^\mu$



$\Gamma^\mu \sim \gamma^\mu \gamma^5$

Charged Current Process

$$\tilde{f}_{1,2}(Q^2) \longrightarrow f_{1,2}^V(Q^2) = f_{1,2}^p(Q^2) - f_{1,2}^n(Q^2),$$

Axial form factor($\tilde{f}_A(Q^2)$) is generally taken to be of dipole form,

$$\tilde{f}_A(Q^2) = f_A(Q^2) = f_A(0)\ \left[1 + \frac{Q^2}{M_A^2}\right]^{-2},$$

$$f_P(Q^2) = \frac{2M^2\, f_A(Q^2)}{m_\pi^2 + Q^2}.$$

$$
\begin{aligned}
j^\mu\big|_{NP} &= V_{ud}\ \mathcal{A}^{NP}\,\bar{u}(\vec{p}\,')\not{k}_\pi\gamma_5\,\frac{\not{p}+\not{q}+M}{(p+q)^2-M^2+i\epsilon}\left[V_N^\mu(q)-A_N^\mu(q)\right]u(\vec{p}\,),\\
j^\mu\big|_{CP} &= V_{ud}\ \mathcal{A}^{CP}\,\bar{u}(\vec{p}\,')\left[V_N^\mu(q)-A_N^\mu(q)\right]\frac{\not{p}'-\not{q}+M}{(p'-q)^2-M^2+i\epsilon}\not{k}_\pi\gamma_5\,u(\vec{p}\,),\\
j^\mu\big|_{CT} &= V_{ud}\ \mathcal{A}^{CT}\,\bar{u}(\vec{p}\,')\gamma^\mu\left(g_A f_{CT}^V(Q^2)\gamma_5 - f_\rho\left((q-k_\pi)^2\right)\right)u(\vec{p}\,),\\
j^\mu\big|_{PP} &= V_{ud}\ \mathcal{A}^{PP} f_\rho\left((q-k_\pi)^2\right)\frac{q^\mu}{m_\pi^2+Q^2}\,\bar{u}(\vec{p}\,')\ \not{q}\ u(\vec{p}\,),\\
j^\mu\big|_{PF} &= V_{ud}\ \mathcal{A}^{PF} f_{PF}(Q^2)\,\frac{(2k_\pi-q)^\mu}{(k_\pi-q)^2-m_\pi^2}\,2M\,\bar{u}(\vec{p}\,')\gamma_5\,u(\vec{p}\,),
\end{aligned}
$$

with $V_{ud}=\cos\theta_C$ for C. C. process and $V_{ud}=1$ for N. C. process.

Constant term →	$\mathcal{A}$(CC ν)			$\mathcal{A}$(CC $\bar{\nu}$)			$\mathcal{A}$(NC $\nu(\bar{\nu})$)			
Final states →	$p\pi^+$	$n\pi^+$	$p\pi^0$	$n\pi^-$	$n\pi^0$	$p\pi^-$	$n\pi^+$	$p\pi^0$	$p\pi^-$	$n\pi^0$
NP	0	$\frac{-ig_A}{\sqrt{2}f_\pi}$	$\frac{-ig_A}{f_\pi}$	0	$\frac{ig_A}{\sqrt{2}f_\pi}$	$\frac{-ig_A}{f_\pi}$	$\frac{-ig_A}{\sqrt{2}f_\pi}$	$\frac{-ig_A}{f_\pi}$	$\frac{-ig_A}{\sqrt{2}f_\pi}$	$\frac{ig_A}{f_\pi}$
CP	$\frac{-ig_A}{f_\pi}$	0	$\frac{ig_A}{\sqrt{2}f_\pi}$	$\frac{-ig_A}{f_\pi}$	$\frac{-ig_A}{\sqrt{2}f_\pi}$	0	$\frac{-ig_A}{\sqrt{2}f_\pi}$	$\frac{-ig_A}{f_\pi}$	$\frac{-ig_A}{\sqrt{2}f_\pi}$	$\frac{ig_A}{f_\pi}$
CT	$\frac{-i}{\sqrt{2}f_\pi}$	$\frac{i}{\sqrt{2}f_\pi}$	$\frac{i}{2f_\pi}$	$\frac{-i}{\sqrt{2}f_\pi}$	$\frac{i}{f_\pi}$	$\frac{i}{\sqrt{2}f_\pi}$	-	-	-	-
PP	$\frac{i}{\sqrt{2}f_\pi}$	$\frac{-i}{\sqrt{2}f_\pi}$	$\frac{-i}{2f_\pi}$	$\frac{i}{\sqrt{2}f_\pi}$	$\frac{i}{f_\pi}$	$\frac{-i}{\sqrt{2}f_\pi}$	-	-	-	-
PF	$\frac{-i}{\sqrt{2}f_\pi}$	$\frac{i}{\sqrt{2}f_\pi}$	$\frac{i}{2f_\pi}$	$\frac{-i}{\sqrt{2}f_\pi}$	$\frac{-i}{f_\pi}$	$\frac{-i}{\sqrt{2}f_\pi}$	-	-	-	-

Spin $\frac{1}{2}$ resonance

1. $S_{11}(1535)$
2. $S_{11}(1620)$

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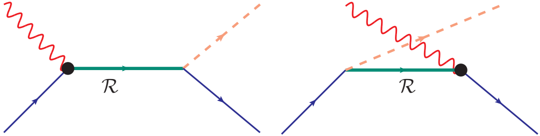

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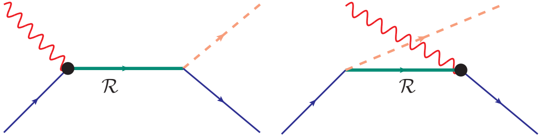

$$j^{\mu}_{\frac{1}{2}} = \bar{u}(p')\Gamma^{\mu}_{\frac{1}{2}} u(p),$$

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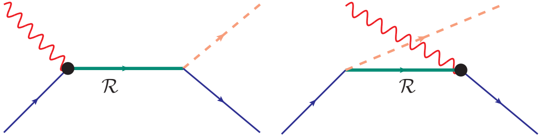

$$j^{\mu}_{\frac{1}{2}} = \bar{u}(p')\Gamma^{\mu}_{\frac{1}{2}} u(p),$$

Positive parity state $\Gamma^{\mu}_{\frac{1}{2}^+} = V^{\mu}_{\frac{1}{2}} - A^{\mu}_{\frac{1}{2}}$

Negative parity state $\Gamma^{\mu}_{\frac{1}{2}^-} = \left[V^{\mu}_{\frac{1}{2}} - A^{\mu}_{\frac{1}{2}}\right]\gamma_5$

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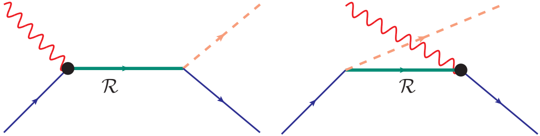

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$$V^{\mu}_{\frac{1}{2}} = \left[\frac{F_1(Q^2)}{(2M)^2} \left(Q^2 \gamma^{\mu} + \not{q} q^{\mu} \right) + \frac{F_2(Q^2)}{2M} i\sigma^{\mu\alpha} q_{\alpha} \right] \gamma_5$$

$$A^{\mu}_{\frac{1}{2}} = -F_A(Q^2)\gamma^{\mu} - \frac{F_P(Q^2)}{M} q^{\mu}$$

Spin $\frac{1}{2}$ resonance

$$j^\mu\Big|_R^{\frac{1}{2}} = i\,V_{ud}\,\mathcal{C}^{\mathcal{R}}\,\bar{u}(\vec{p}\,')\not{k}_\pi\gamma_5\,\frac{\not{p}+\not{q}+M}{(p+q)^2-M^2+i\epsilon}\Gamma^\mu_{\frac{1}{2}}\,u(\vec{p}),$$

$$j^\mu\Big|_{CR}^{\frac{1}{2}} = i\,V_{ud}\,\mathcal{C}^{\mathcal{R}}\,\bar{u}(\vec{p}\,')\Gamma^\mu_{\frac{1}{2}}\,\frac{\not{p}'-\not{q}+M}{(p'-q)^2-M^2+i\epsilon}\not{k}_\pi\gamma_5\,u(\vec{p}),$$

Spin $\frac{1}{2}$ resonance

$$
\begin{aligned}
j^\mu\Big|_R^{\frac{1}{2}} &= i\, V_{ud}\, \mathcal{C}^{\mathcal{R}}\, \bar{u}(\vec{p}\,')\not{k}_\pi \gamma_5 \frac{\not{p}+\not{q}+M}{(p+q)^2 - M^2 + i\epsilon} \Gamma^\mu_{\frac{1}{2}}\, u(\vec{p}), \\
j^\mu\Big|_{CR}^{\frac{1}{2}} &= i\, V_{ud}\, \mathcal{C}^{\mathcal{R}}\, \bar{u}(\vec{p}\,')\Gamma^\mu_{\frac{1}{2}} \frac{\not{p}'-\not{q}+M}{(p'-q)^2 - M^2 + i\epsilon} \not{k}_\pi \gamma_5\, u(\vec{p}),
\end{aligned}
$$

Vector form factor

$$
\begin{aligned}
A^{p,n}_{\frac{1}{2}} &= \sqrt{\frac{2\pi\alpha}{M} \frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}} \left[\frac{Q^2}{4M^2} F_1^{p,n} + \frac{M_R \pm M}{2M} F_2^{p,n} \right] \\
S^{p,n}_{\frac{1}{2}} &= \mp \sqrt{\frac{\pi\alpha}{M} \frac{(M \pm M_R)^2 + Q^2}{M_R^2 - M^2}} \frac{(M_R \mp M)^2 + Q^2}{4 M_R M} \left[\frac{M_R \pm M}{2M} F_1^{p,n} - F_2^{p,n} \right]
\end{aligned}
$$

Spin $\frac{1}{2}$ resonance

$$
\begin{aligned}
j^\mu\Big|^{\frac{1}{2}}_{R} &= i\, V_{ud}\, \mathcal{C}^{\mathcal{R}}\, \bar{u}(\vec{p}\,')\not{k}_\pi \gamma_5 \frac{\not{p}+\not{q}+M}{(p+q)^2-M^2+i\epsilon}\Gamma^\mu_{\frac{1}{2}}\, u(\vec{p}),\\
j^\mu\Big|^{\frac{1}{2}}_{CR} &= i\, V_{ud}\, \mathcal{C}^{\mathcal{R}}\, \bar{u}(\vec{p}\,')\Gamma^\mu_{\frac{1}{2}} \frac{\not{p}'-\not{q}+M}{(p'-q)^2-M^2+i\epsilon}\not{k}_\pi \gamma_5\, u(\vec{p}),
\end{aligned}
$$

Spin $\frac{1}{2}$ resonance

$$j^\mu\Big|_{R}^{\frac{1}{2}} = i\, V_{ud}\, \mathcal{C}^{\mathcal{R}}\, \bar{u}(\vec{p}\,')\not{k}_\pi \gamma_5 \frac{\not{p}+\not{q}+M}{(p+q)^2-M^2+i\epsilon}\Gamma^\mu_{\frac{1}{2}}\, u(\vec{p}),$$

$$j^\mu\Big|_{CR}^{\frac{1}{2}} = i\, V_{ud}\, \mathcal{C}^{\mathcal{R}}\, \bar{u}(\vec{p}\,')\Gamma^\mu_{\frac{1}{2}} \frac{\not{p}'-\not{q}+M}{(p'-q)^2-M^2+i\epsilon}\not{k}_\pi \gamma_5\, u(\vec{p}),$$

Vector form factor

$$A^{p,n}_{\frac{1}{2}} = \sqrt{\frac{2\pi\alpha}{M}\frac{(M_R \mp M)^2+Q^2}{M_R^2-M^2}}\left[\frac{Q^2}{4M^2}F_1^{p,n}+\frac{M_R \pm M}{2M}F_2^{p,n}\right]$$

Upper sign for Positive & lower sign for Negative parity state.

$$S^{p,n}_{\frac{1}{2}} = \mp\sqrt{\frac{\pi\alpha}{M}\frac{(M \pm M_R)^2+Q^2}{M_R^2-M^2}}\,\frac{(M_R \mp M)^2+Q^2}{4M_R M}\left[\frac{M_R \pm M}{2M}F_1^{p,n}-F_2^{p,n}\right]$$

Spin $\frac{1}{2}$ resonance

$$j^\mu\Big|_R^{\frac{1}{2}} = i\, V_{ud}\, \mathcal{C}^{\mathcal{R}}\, \bar{u}(\vec{p}\,')\not{k}_\pi \gamma_5 \frac{\not{p}+\not{q}+M}{(p+q)^2 - M^2 + i\epsilon} \Gamma^\mu_{\frac{1}{2}}\, u(\vec{p}),$$

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$$A^{p,n}_{\frac{1}{2}} = \sqrt{\frac{2\pi\alpha}{M} \frac{(\boxed{M_R} \mp M)^2 + Q^2}{\boxed{M_R^2} - M^2}} \left[\frac{Q^2}{4M^2} F_1^{p,n} + \frac{\boxed{M_R} \pm M}{2M} F_2^{p,n} \right]$$

Mass of corresponding resonance

$$S^{p,n}_{\frac{1}{2}} = \mp \sqrt{\frac{\pi\alpha}{M} \frac{(M \pm \boxed{M_R})^2 + Q^2}{\boxed{M_R^2} - M^2} \frac{(\boxed{M_R} \mp M)^2 + Q^2}{4 \boxed{M_R} M}} \left[\frac{\boxed{M_R} \pm M}{2M} F_1^{p,n} - F_2^{p,n} \right]$$

Spin $\frac{1}{2}$ resonance

$$j^\mu\Big|_R^{\frac{1}{2}} = i\, V_{ud}\, \mathcal{C}^{\mathcal{R}}\, \bar{u}(\vec{p}\,')\not{k}_\pi \gamma_5 \frac{\not{p}+\not{q}+M}{(p+q)^2 - M^2 + i\epsilon}\Gamma^\mu_{\frac{1}{2}}\, u(\vec{p}),$$

$$j^\mu\Big|_{CR}^{\frac{1}{2}} = i\, V_{ud}\, \mathcal{C}^{\mathcal{R}}\, \bar{u}(\vec{p}\,')\Gamma^\mu_{\frac{1}{2}} \frac{\not{p}'-\not{q}+M}{(p'-q)^2 - M^2 + i\epsilon}\not{k}_\pi \gamma_5\, u(\vec{p}),$$

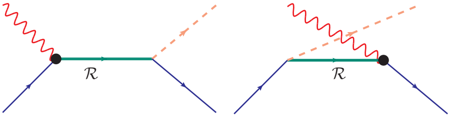
$$A^{p,n}_{\frac{1}{2}} = \sqrt{\frac{2\pi\alpha}{M}\frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}} \left[\frac{Q^2}{4M^2}\boxed{F_1^{p,n}} + \frac{M_R \pm M}{2M}\boxed{F_2^{p,n}}\right]$$

Electromagnetic transition form factors

$$S^{p,n}_{\frac{1}{2}} = \mp \sqrt{\frac{\pi\alpha}{M}\frac{(M \pm M_R)^2 + Q^2}{M_R^2 - M^2}}\, \frac{(M_R \mp M)^2 + Q^2}{4 M_R M} \left[\frac{M_R \pm M}{2M}\boxed{F_2^{p,n}} - \boxed{F_2^{p,n}}\right]$$

Spin $\frac{3}{2}$ & Isospin $\frac{1}{2}$ resonance

1. $D_{13}(1520)$
2. $P_{13}(1720)$



Spin $\frac{3}{2}$ & Isospin $\frac{1}{2}$ resonance

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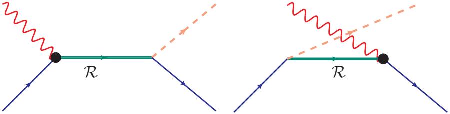

$$j^{\mu}_{\frac{3}{2}} = \bar{u}_{\nu}(p')\Gamma^{\nu\mu}_{\frac{3}{2}}u(p),$$

Positive parity state $\Gamma^{\frac{3}{2}^{+}}_{\nu\mu} = \left[V^{\frac{3}{2}}_{\nu\mu} - A^{\frac{3}{2}}_{\nu\mu}\right]\gamma_5$

Negative parity state $\Gamma^{\frac{3}{2}^{-}}_{\nu\mu} = V^{\frac{3}{2}}_{\nu\mu} - A^{\frac{3}{2}}_{\nu\mu}$

Spin $\frac{3}{2}$ & Isospin $\frac{1}{2}$ resonance

1. $D_{13}(1520)$
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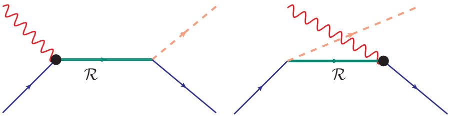

$$j^{\mu}_{\frac{3}{2}} = \bar{u}_{\nu}(p')\Gamma^{\nu\mu}_{\frac{3}{2}}u(p),$$

$$V^{\frac{3}{2}}_{\nu\mu} = \left[\frac{\tilde{C}^V_3}{M}(g_{\mu\nu}\not{q} - q_{\nu}\gamma_{\mu}) + \frac{\tilde{C}^V_4}{M^2}(g_{\mu\nu}\,q\cdot p' - q_{\nu}p'_{\mu}) + \frac{\tilde{C}^V_5}{M^2}(g_{\mu\nu}\,q\cdot p - q_{\nu}p_{\mu}) + g_{\mu\nu}\tilde{C}^V_6\right]$$

$$A^{\frac{3}{2}}_{\nu\mu} = -\left[\frac{\tilde{C}^A_3}{M}(g_{\mu\nu}\not{q} - q_{\nu}\gamma_{\mu}) + \frac{\tilde{C}^A_4}{M^2}(g_{\mu\nu}\,q\cdot p' - q_{\nu}p'_{\mu}) + \tilde{C}^A_5 g_{\mu\nu} + \frac{\tilde{C}^A_6}{M^2}q_{\nu}q_{\mu}\right]\gamma_5$$

$$\tilde{C}^V_i = C^P_i - C^N_i$$

$$
\begin{aligned}
j^\mu\Big|_R^{\frac{3}{2}} &= i\, V_{ud}\, \mathcal{C}^{\mathcal{R}} \frac{k_\pi^\alpha}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}\,')P^{3/2}_{\alpha\beta}(p_R)\Gamma^{\beta\mu}_{\frac{3}{2}}(p,q)u(\vec{p}), \quad p_R = p + q, \\
j^\mu\Big|_{CR}^{\frac{3}{2}} &= i\, a\, \mathcal{C}^{\mathcal{R}} \frac{k_\pi^\beta}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}\,')\hat{\Gamma}^{\mu\alpha}_{\frac{3}{2}}(p',-q)P^{3/2}_{\alpha\beta}(p_R)u(\vec{p}), \quad p_R = p' - q,
\end{aligned}
$$

$$j^\mu\Big|_R^{\frac{3}{2}} = i\, V_{ud}\, \mathcal{C}^{\mathcal{R}} \frac{k_\pi^\alpha}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}\,') P_{\alpha\beta}^{3/2}(p_R) \Gamma_{\frac{3}{2}}^{\beta\mu}(p,q) u(\vec{p}), \quad p_R = p + q,$$

$$j^\mu\Big|_{CR}^{\frac{3}{2}} = i\, a\, \mathcal{C}^{\mathcal{R}} \frac{k_\pi^\beta}{p_R^2 - M_R^2 + iM_R\Gamma_R} \bar{u}(\vec{p}\,') \hat{\Gamma}_{\frac{3}{2}}^{\mu\alpha}(p',-q) P_{\alpha\beta}^{3/2}(p_R) u(\vec{p}), \quad p_R = p' - q,$$

Vector form factor $\Longrightarrow$ Helicity Ansatz

$$A_{\frac{3}{2}}^{p,n} = \sqrt{\frac{\pi\alpha}{M} \frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}} \left[\frac{C_3^{p,n}}{M}(M \pm M_R) \pm \frac{C_4^{p,n}}{M^2} \frac{M_R^2 - M^2 - Q^2}{2} \pm \frac{C_5^{p,n}}{M^2} \frac{M_R^2 - M^2 + Q^2}{2} \right]$$

$$A_{\frac{1}{2}}^{p,n} = \sqrt{\frac{\pi\alpha}{3M} \frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}} \left[\frac{C_3^{p,n}}{M} \frac{M^2 + MM_R + Q^2}{M_R} - \frac{C_4^{p,n}}{M^2} \frac{M_R^2 - M^2 - Q^2}{2} - \frac{C_5^{p,n}}{M^2} \frac{M_R^2 - M^2 + Q^2}{2} \right]$$

$$S_{\frac{1}{2}}^{p,n} = \pm\sqrt{\frac{\pi\alpha}{6M} \frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}} \frac{\sqrt{Q^4 + 2Q^2(M_R^2 + M^2) + (M_R^2 - M^2)^2}}{M_R^2}$$

$$\times \left[\frac{C_3^{p,n}}{M} M_R + \frac{C_4^{p,n}}{M^2} M_R^2 + \frac{C_5^{p,n}}{M^2} \frac{M_R^2 + M^2 + Q^2}{2} \right],$$

+ sign for Positive & − sign for Negative parity state.

Axial–Vector form factors

Data is not available for the axial form-factors

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Dipole–form $\Longrightarrow$

$$F_A(Q^2) = \frac{F_A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} \; ; \qquad C_5^A(Q^2) = \frac{C_5^A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

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PCAC & Goldberger-Trieman relation

$$F_A(0) = -2f_\pi \frac{f_{R\frac{1}{2}}}{m_\pi}, \qquad C_5^A(0) = -2f_\pi \frac{f_{R\frac{3}{2}}}{m_\pi},$$

$$F_P(Q^2) = \frac{(MM_R \pm M^2)}{m_\pi^2 + Q^2} F_A(Q^2) \qquad C_6^A(Q^2) = \frac{(MM_R \pm M^2)}{m_\pi^2 + Q^2} C_5^A(Q^2)$$

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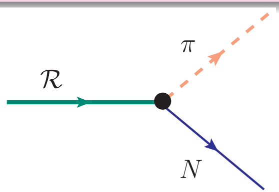
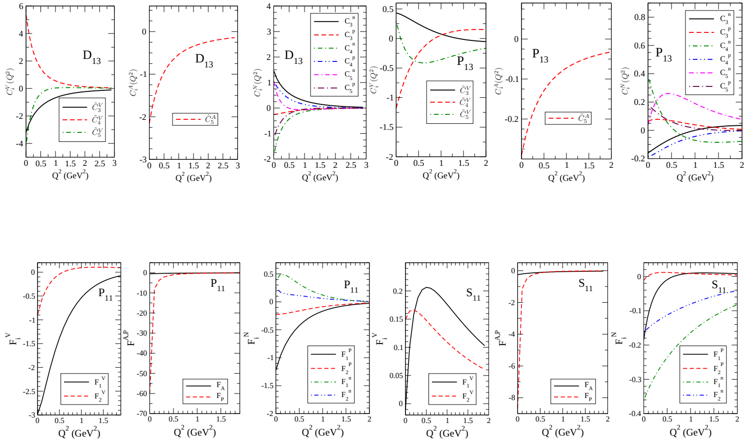
$$F_A(Q^2) = \frac{F_A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2} \;; \qquad C_5^A(Q^2) = \frac{C_5^A(0)}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

The couplings $f_{R\frac{3}{2}}$ & $f_{R\frac{1}{2}}$ are determined from the $\mathcal{R} \to N\pi$ decay rate



form–factors



Couplings of Resonances

⋆ Uncertainty in $\mathcal{R}N\pi$ coupling at $\mathcal{R} \to N\pi$ vertex

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  Lack of experimental data

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- ⋆ Uncertainty in $\mathcal{R}N\pi$ coupling at $\mathcal{R} \rightarrow N\pi$ vertex

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  Data of branching ratio
  Decay width of resonances

Couplings of Resonances

$$\mathcal{L}_{R_{\frac{1}{2}} N\pi} = \frac{f_{R\frac{1}{2}}}{m_\pi} \bar{\Psi}_{R_{\frac{1}{2}}} \, \Gamma^{\mu}_{\frac{1}{2}} \, \partial_\mu \phi^i T_i \Psi$$

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$\mathcal{R}N\pi$ coupling strength

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$\mathcal{R}N\pi$ coupling strength

Field for spin $\frac{1}{2}$ resonances

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Field for spin $\frac{1}{2}$ resonances

Nucleon field

Couplings of Resonances

$$\mathcal{L}_{R_{\frac{1}{2}}N\pi} = \frac{f_{R\frac{1}{2}}}{m_\pi} \bar{\Psi}_{R_{\frac{1}{2}}} \Gamma^\mu_{\frac{1}{2}} \partial_\mu \phi^i T_i \Psi$$

$\mathcal{R}N\pi$ coupling strength

Field for spin $\frac{1}{2}$ resonances

Nucleon field

$$\Gamma_{R_{\frac{1}{2}} \to \pi N} = \frac{\mathcal{C}}{4\pi} \left(\frac{f_{R\frac{1}{2}}}{m_\pi} \right)^2 \left(M_R \pm M \right)^2 \frac{E_N \mp M}{M_R} |\vec{q}_{\text{cm}}|$$

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- upper sign represents positive parity resonant state
- lower sign represents negative parity resonant state

$$\mathcal{L}_{R_{\frac{3}{2}}N\pi} = \frac{f_{R\frac{3}{2}}}{m_\pi} \bar{\Psi}_{R_{\frac{3}{2}}} \Gamma^{\mu}_{\frac{3}{2}} \partial^{\mu} \phi^{i} T_i \Psi$$

$$\mathcal{L}_{R_{\frac{3}{2}}N\pi} = \frac{f_{R_{\frac{3}{2}}}}{m_\pi}\bar{\Psi}_{R_{\frac{3}{2}}}\,\Gamma^{\mu}_{\frac{3}{2}}\,\partial^{\mu}\phi^{i}\,T_i\,\Psi$$

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Field for spin $\frac{3}{2}$ resonances

$$\mathcal{L}_{R_{\frac{3}{2}} N\pi} = \frac{f_{R_{\frac{3}{2}}}}{m_\pi} \bar{\Psi}_{R_{\frac{3}{2}}} \Gamma^{\mu}_{\frac{3}{2}} \partial^{\mu} \phi^{i} T_i \Psi$$

$\mathcal{R}N\pi$ coupling strength

Field for spin $\frac{3}{2}$ resonances

Pionic field

$$\mathcal{L}_{R_{\frac{3}{2}} N\pi} = \frac{f_{R_{\frac{3}{2}}}}{m_\pi} \bar{\Psi}_{R_{\frac{3}{2}}} \, \Gamma^{\mu}_{\frac{3}{2}} \, \partial^{\mu} \phi^{i} \, T_i \, \Psi$$

$\mathcal{R}N\pi$ coupling strength

Field for spin $\frac{3}{2}$ resonances

Isospin operator

Pionic field

$$\mathcal{L}_{R_{\frac{3}{2}} N\pi} = \frac{f_{R_{\frac{3}{2}}}}{m_\pi} \bar{\Psi}_{R_{\frac{3}{2}}} \, \Gamma^{\mu}_{\frac{3}{2}} \, \partial^{\mu} \phi^{i} \, T_i \, \Psi$$

$\mathcal{R}N\pi$ coupling strength

Field for spin $\frac{3}{2}$ resonances

Pionic field

Isospin operator

Nucleon field

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$\mathcal{R}N\pi$ coupling strength

Field for spin $\frac{3}{2}$ resonances

Pionic field

Isospin operator

Nucleon field

- In the rest frame resonance decay width

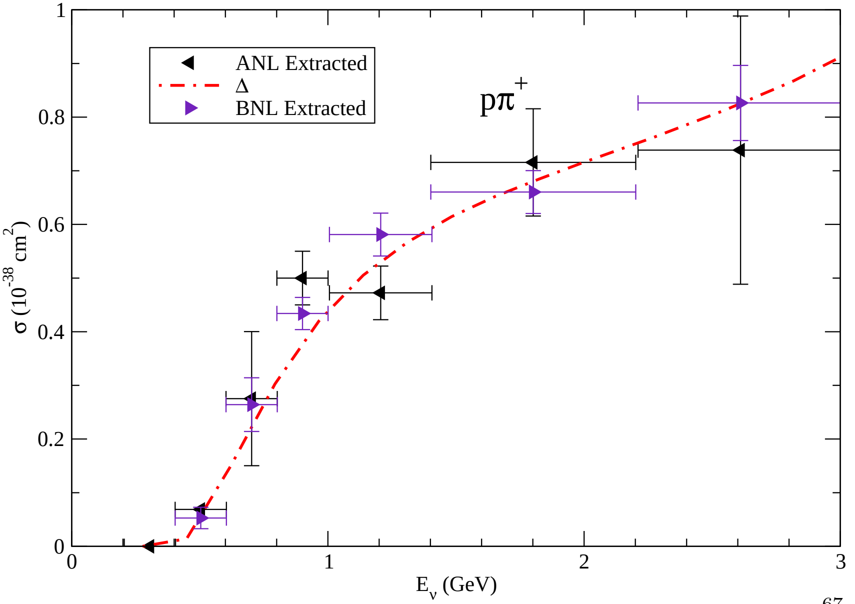
$$\Gamma_{R_{\frac{3}{2}}\to\pi N} = \frac{\mathcal{C}}{12\pi}\left(\frac{f_{R\frac{3}{2}}}{m_\pi}\right)^2 \frac{E_N \pm M}{M_R}|\vec{q}_{cm}|^3$$

Process	$\mathcal{C}^{\mathcal{R}}(\mathrm{CC}\ \nu)$			$\mathcal{C}^{\mathcal{R}}(\mathrm{CC}\ \bar{\nu})$		
	$p\pi^+$	$n\pi^+$	$p\pi^0$	$n\pi^-$	$n\pi^0$	$p\pi^-$
$P_{33}(1232)$	$\frac{\sqrt{3}f^\star}{m_\pi}$	$\sqrt{\frac{1}{3}}\frac{f^\star}{m_\pi}$	$-\sqrt{\frac{2}{3}}\frac{f^\star}{m_\pi}$	$\frac{\sqrt{3}f^\star}{m_\pi}$	$\sqrt{\frac{2}{3}}\frac{f^\star}{m_\pi}$	$\sqrt{\frac{1}{3}}\frac{f^\star}{m_\pi}$
$P_{11}(1440)$	0	$-\sqrt{\frac{1}{2}}\frac{f^\star}{f_\pi}$	$-\frac{1}{2}\frac{f^\star}{f_\pi}$	0	$-\frac{1}{2}\frac{f^\star}{f_\pi}$	$\sqrt{\frac{1}{2}}\frac{f^\star}{f_\pi}$
$D_{13}(1520)$	0	$-\sqrt{\frac{1}{2}}\frac{f^\star}{m_\pi}$	$-\frac{1}{2}\frac{f^\star}{m_\pi}$	0	$-\frac{1}{2}\frac{f^\star}{m_\pi}$	$\sqrt{\frac{1}{2}}\frac{f^\star}{m_\pi}$
$S_{11}(1535)$	0	$-\sqrt{\frac{1}{2}}\frac{f^\star}{f_\pi}$	$-\frac{1}{2}\frac{f^\star}{f_\pi}$	0	$-\frac{1}{2}\frac{f^\star}{f_\pi}$	$\sqrt{\frac{1}{2}}\frac{f^\star}{f_\pi}$
$S_{11}(1650)$	0	$-\sqrt{\frac{1}{2}}\frac{f^\star}{f_\pi}$	$-\frac{1}{2}\frac{f^\star}{f_\pi}$	0	$-\frac{1}{2}\frac{f^\star}{f_\pi}$	$\sqrt{\frac{1}{2}}\frac{f^\star}{f_\pi}$
$P_{13}(1720)$	0	$-\sqrt{\frac{1}{2}}\frac{f^\star}{m_\pi}$	$-\frac{1}{2}\frac{f^\star}{m_\pi}$	0	$-\frac{1}{2}\frac{f^\star}{m_\pi}$	$\sqrt{\frac{1}{2}}\frac{f^\star}{m_\pi}$

Resonances R_{IJ}	M_R [GeV]	J	I	P	Γ_0^{tot} (GeV)	πN branching ratio (%)	$F_A(0)$ or $\tilde{C}_5^A(0)$	$f^\star$
$P_{33}(1232)$	1.232	$\frac{3}{2}$	$\frac{3}{2}$	$+$	0.117	100	1.0	2.14
$P_{11}(1440)$	1.430	$\frac{1}{2}$	$\frac{1}{2}$	$+$	0.350	$55-75$	-0.43	0.215
$D_{13}(1520)$	1.515	$\frac{3}{2}$	$\frac{1}{2}$	$-$	0.115	$55-65$	-2.08	1.575
$S_{11}(1535)$	1.535	$\frac{1}{2}$	$\frac{1}{2}$	$-$	0.150	$35-55$	0.184	0.092
$S_{11}(1650)$	1.655	$\frac{1}{2}$	$\frac{1}{2}$	$-$	0.140	$50-90$	-0.21	-0.105
$P_{13}(1720)$	1.720	$\frac{3}{2}$	$\frac{1}{2}$	$+$	0.250	11	-0.195	0.147

Process	$\mathcal{C}^{\mathcal{R}}(\mathrm{CC}\ \nu)$			$\mathcal{C}^{\mathcal{R}}(\mathrm{CC}\ \bar{\nu})$		
$S_{11}(1650)$	0	$-\sqrt{\frac{1}{2}}\,\frac{f}{f_\pi}$	$-\frac{1}{2}\,\frac{f}{f_\pi}$	0	$-\frac{1}{2}\,\frac{f}{f_\pi}$	$\sqrt{\frac{1}{2}}\,\frac{f}{f_\pi}$
$P_{13}(1720)$	0	$-\sqrt{\frac{1}{2}}\,\frac{f^\star}{m_\pi}$	$-\frac{1}{2}\,\frac{f^\star}{m_\pi}$	0	$-\frac{1}{2}\,\frac{f^\star}{m_\pi}$	$\sqrt{\frac{1}{2}}\,\frac{f^\star}{m_\pi}$

$\nu_\mu p \to \mu^- p \pi^+$



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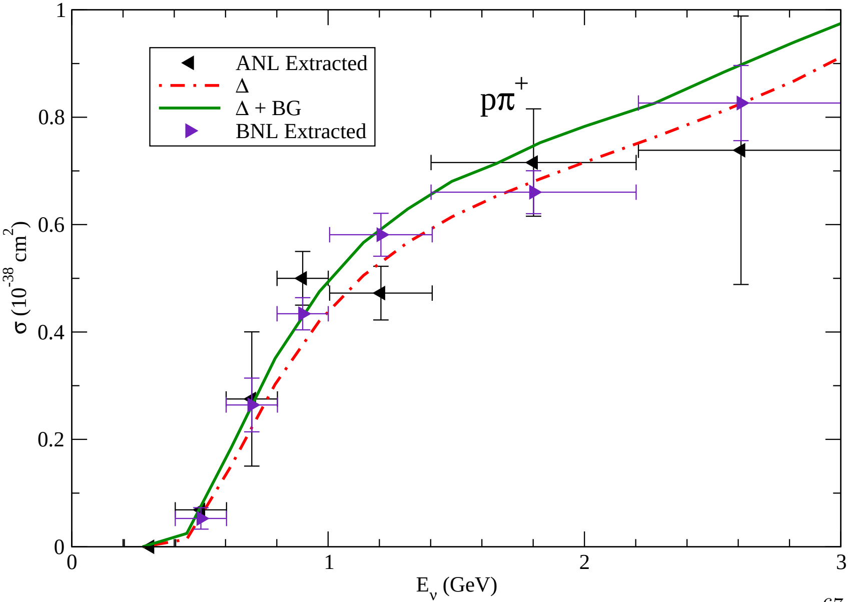

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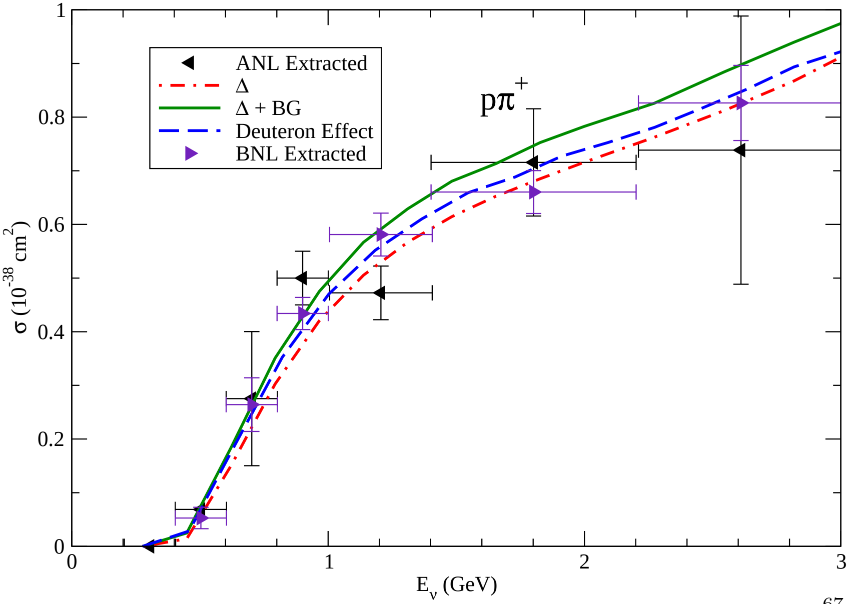
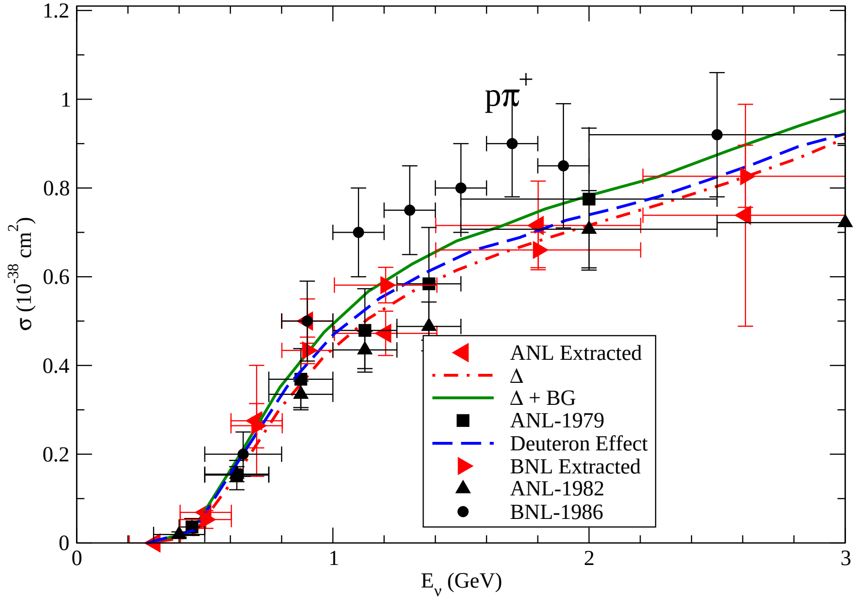
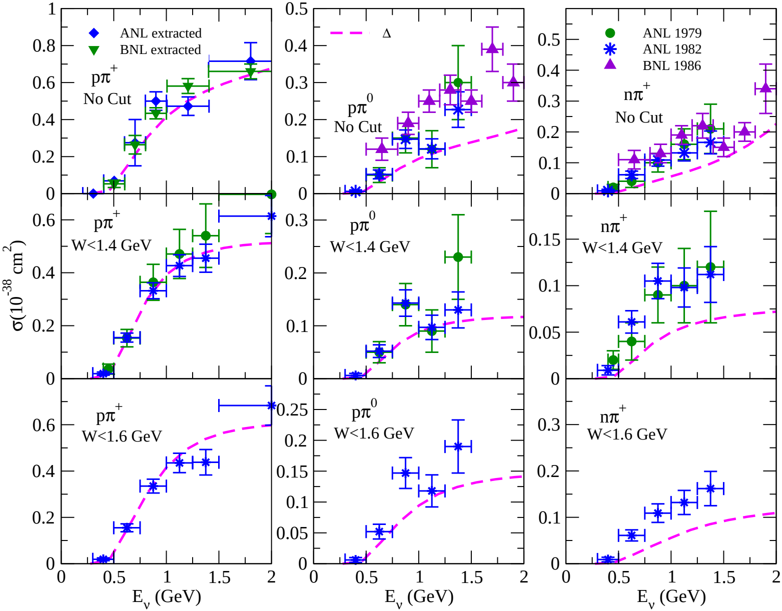


$\nu_\mu p \to \mu^- p \pi^+$



Neutrino induced Charged-current pion producion



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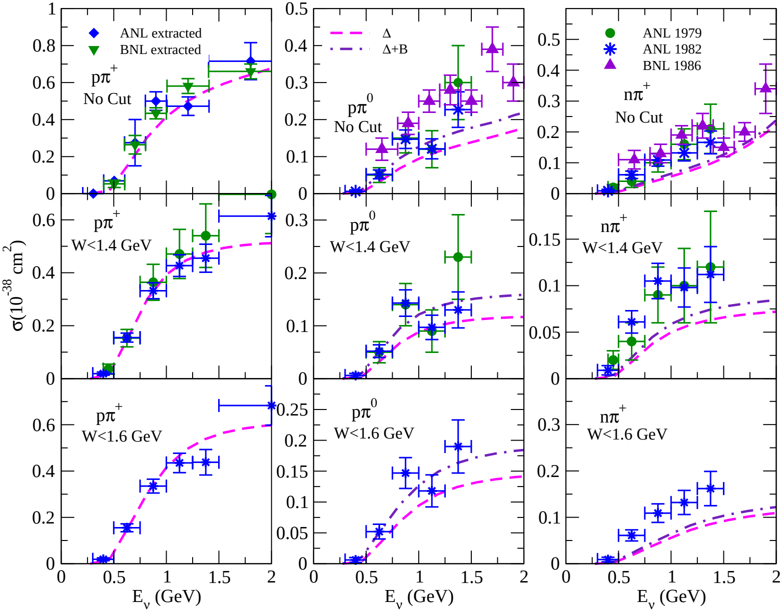

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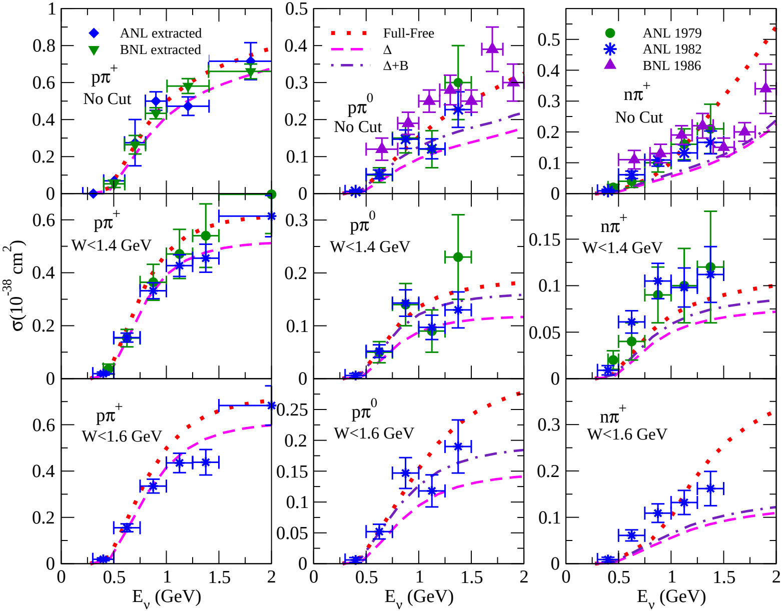

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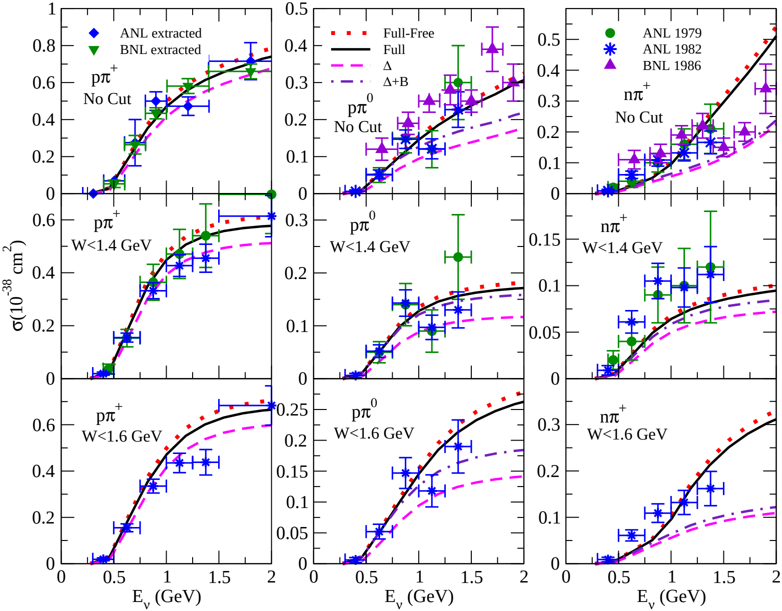
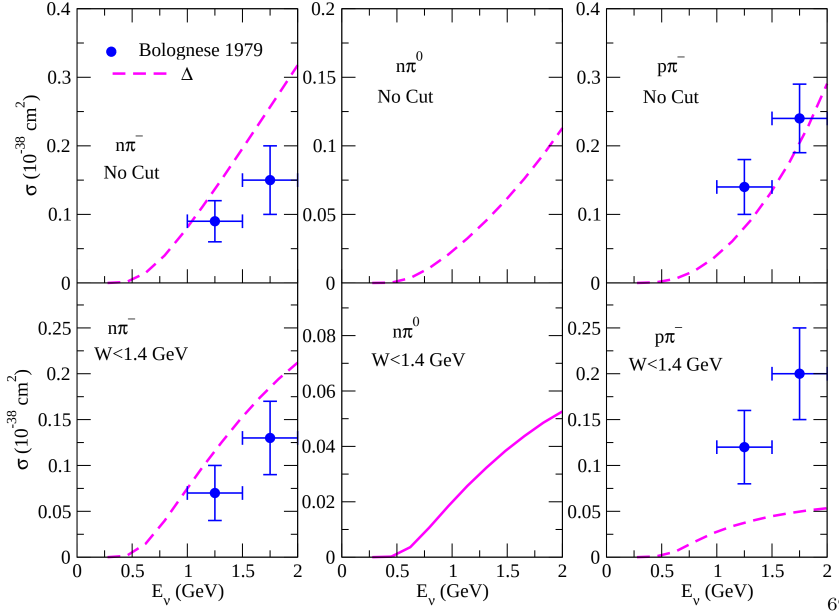
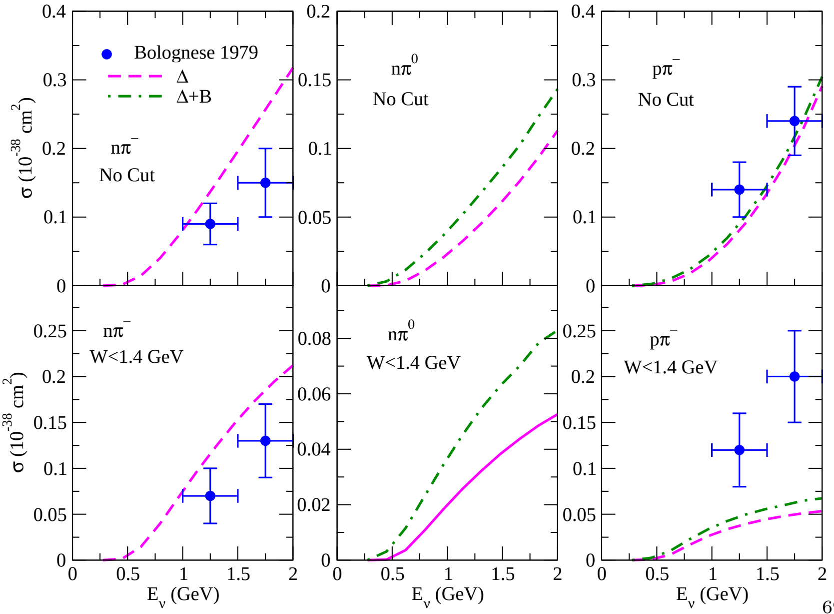


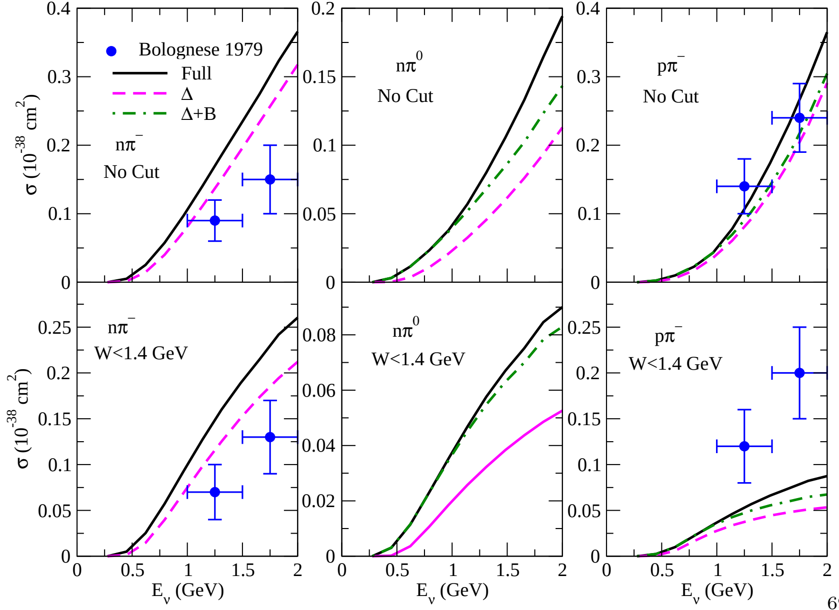
Antineutrino induced Charged-current pion producion



Antineutrino induced Charged-current pion producion



Antineutrino induced Charged-current pion producion



Inside the nucleus

Inside the nucleus, neutrino interacts with a bound nucleon.

Local Density Approximation

Cross section is evaluated as a function of local Fermi momentum($p_F(r)$) and integrated over the size of whole nucleus.

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Inside the nucleus, the neutrino interacts with a neutron or proton whose local density in the medium is $\rho_n(r)$ or $\rho_p(r)$, respectively. Corresponding local Fermi momenta for neutron and proton are

$$p_{F_n} = [3\pi^2 \rho_n(r)]^{\frac{1}{3}};\ \ p_{F_p} = [3\pi^2 \rho_p(r)]^{\frac{1}{3}}$$

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Cross section is evaluated as a function of local Fermi momentum($p_F(r)$) and integrated over the size of whole nucleus.

Inside the nucleus, the neutrino interacts with a neutron or proton whose local density in the medium is $\rho_n(r)$ or $\rho_p(r)$, respectively. Corresponding local Fermi momenta for neutron and proton are

$$p_{F_n} = [3\pi^2 \rho_n(r)]^{\frac{1}{3}}; \;\; p_{F_p} = [3\pi^2 \rho_p(r)]^{\frac{1}{3}}$$

Differential scattering cross section

$$\left(\frac{d\sigma}{dE_\pi \, d\Omega_\pi}\right)_{\nu A} = \int d\vec{r} \; \rho_N(r) \; \left(\frac{d\sigma}{dE_\pi \, d\Omega_\pi}\right)_{\nu N}$$

- In a symmetric nuclear matter, each nucleon occupies a volume $(2\pi\hbar)^3$

$$N = 2\,V \int_0^{p_F} \frac{d^3 p_N}{(2\pi\hbar)^3}$$

$$\rho_N = \frac{N}{V} = 2\int \frac{d^3 p_N}{(2\pi)^3}\Theta(p_{F_i} - p_i^N)\Theta(p_F^N - p_{F_f})$$

or

$$\rho_N = 2\int_0^{E_{F_N}} \frac{d^3 p_N}{(2\pi)^3}\Theta(E_F^N(r) - E_N)\Theta(E_N + q_0 - E_\pi - E_F^{N'}(r))$$

with

$$E_F^N(r) = \sqrt{M^2 + (p_F^N(r))^2}; \qquad p_F^N(r) = (3\pi^2 \rho_N(r))^{1/3}$$

$$
\begin{aligned}
\rho_N(r) &\rightarrow \frac{Z}{A}\rho(r); \quad \textit{For proton} \\
\rho_N(r) &\rightarrow \frac{A-Z}{A}\rho(r); \textit{For neutron}
\end{aligned}
$$

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$$
\rho(r) = \frac{\rho(0)}{1 - \exp\left(\frac{r-c}{a}\right)}
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$$\rho(r) = \frac{\rho(0)}{1 - \exp\left(\frac{r-c}{a}\right)}$$

	c		a	
	c^n	c^p	a^n	a^p
^{40}Ar	3.64	3.47	0.569	0.569
^{56}Fe	4.05	3.971	0.5935	0.5935

$$
\begin{aligned}
\left(\frac{d\sigma}{dE_\pi\, dcos\theta_\pi}\right)_A \quad = \quad & 2\int d^3\vec{r} \sum_{N=n,p} \frac{d^3\vec{p}_N}{(2\pi)^3}\; \Theta_1(E_F^N(r) - E_N) \\
& \Theta_2(E_N + q_0 - E_\pi - E_F^{N'}(r)) \times \left(\frac{d\sigma}{dE_\pi\, dcos\theta_\pi}\right)_N
\end{aligned}
$$

$$\left(\frac{d\sigma}{dE_\pi\, dcos\theta_\pi}\right)_A = 2\int d^3\vec{r} \sum_{N=n,p} \frac{d^3\vec{p}_N}{(2\pi)^3}\, \Theta_1(E_F^N(r) - E_N)$$

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$$d^3\vec{p}_N = |\vec{p}_N|^2\; d|\vec{p}_N|\; dcos\theta_N\; d\phi_N$$

$$\left(\frac{d\sigma}{dE_\pi\, dcos\theta_\pi}\right)_A = 2\int d^3\vec{r} \sum_{N=n,p} \frac{d^3\vec{p}_N}{(2\pi)^3}\, \Theta_1(E_F^N(r) - E_N)$$
$$\Theta_2(E_N + q_0 - E_\pi - E_F^{N'}(r)) \times \left(\frac{d\sigma}{dE_\pi\, dcos\theta_\pi}\right)_N$$

$$d^3\vec{p}_N = |\vec{p}_N|^2\ d|\vec{p}_N|\ dcos\theta_N\ d\phi_N$$

- $E_N = \sqrt{|\vec{p}_N|^2 + M^2};\quad \vec{k} + \vec{p}_N = \vec{k}' + \vec{p}\,'_N + \vec{p}_\pi$

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- $E'_N = \sqrt{|\vec{p}'_N|^2 + M^2} = \sqrt{|\vec{q} - \vec{p}_\pi + \vec{p}_N|^2 + M^2}$

$$\left(\frac{d\sigma}{dE_\pi\, dcos\theta_\pi}\right)_A = 2\int d^3\vec{r} \sum_{N=n,p} \frac{d^3\vec{p}_N}{(2\pi)^3}\, \Theta_1(E_F^N(r) - E_N)$$

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- $\Rightarrow E'_N = \sqrt{|\vec{P}|^2 + |\vec{p}_N|^2 + 2|\vec{P}||\vec{p}_N|cos\theta_N + M^2}; \qquad \vec{P} = \vec{q} - \vec{p}_\pi$

$$\left(\frac{d\sigma}{dE_\pi\, dcos\theta_\pi}\right)_N = \frac{G_F^2 cos^2\theta_c}{128 M_N E_\nu}\frac{|\vec{p}_l||\vec{p}_\pi|}{E'_p}\frac{1}{(2\pi)^4}\delta^0(q_0+E_N-E'_p-E_\pi) \times L^{\mu\nu} J_{\mu\nu} \quad dE_l\, d\Omega_l$$

$$
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$$

$$
J^{\mu\nu} = \bar{\Sigma}\Sigma J^{\mu\dagger} J^{\nu}
$$

$$
L^{\mu\nu} = 8 \left[k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}\ k \cdot k' \pm i\epsilon_{\mu\nu\alpha\beta}\ k'^\alpha k^\beta\right]
$$

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$$
\begin{aligned}
\left(\frac{d\sigma}{dE_\pi\, dcos\theta_\pi}\right)_A &= 2\int_0^\infty 4\pi r^2\, dr \int_\epsilon^{E_F^N(r)} \int_0^{2\pi} \frac{|\vec{p}_N| E_N\, dE_N\, dcos\theta_N\, d\phi_N}{(2\pi)^3} \\
&\times \quad \Theta_1(E_F^N(r) - E_N)\ \Theta_2(E_N + q_0 - E_\pi - E_F^{N'}(r)) \\
&\times \quad \frac{G_F^2 cos^2\theta_c}{128 M_N E_\nu} \frac{|\vec{p}_l||\vec{p}_\pi|}{E_p'} \frac{1}{(2\pi)^4} \delta^0(q_0 + E_N - E_p' - E_\pi) \\
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&\times \quad L^{\mu\nu} J_{\mu\nu} \quad dE_l\, d\Omega_l
\end{aligned}
$$

- As the azimuthal angle dependence have been found out to be very small for pion production in electro- and photo- induced processes

$$\int_0^{2\pi} d\phi_N \sim 2\pi$$

Recoil factor

Let

$$f=\delta^0(q_0+\sqrt{|\vec{p}_N|^2+M^2}-\sqrt{|\vec{P}|^2+|\vec{p}_N|^2+2|\vec{P}||\vec{p}_N|cos\theta_N+M^2}-E_\pi)$$

Recoil factor

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$$E_N'^2 = |\vec{P}|^2 + |\vec{p}_N|^2 + 2|\vec{P}||\vec{p}_N|cos\theta_N + M^2$$

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On partially differentiating and using property of delta function

$$\int dx\ \delta[f(x)] = \left|\frac{\partial f}{\partial x}\right|^{-1}_{x=x_0}$$

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we get

$$\left|\frac{\partial E_N'}{\partial cos\theta_N}\right|^{-1} = \frac{E_N'}{|\vec{P}||\vec{p}_N|}$$

From energy conservation

$$q_0 + \sqrt{|\vec{p}_N|^2 + M^2} - \sqrt{|\vec{P}|^2 + |\vec{p}_N|^2 + 2|\vec{P}||\vec{p}_N|cos\theta_N + M^2} - E_\pi = 0$$

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$$abs(\cos\theta_N) \leq 1$$

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as

$$abs(\cos\theta_N) \leq 1$$

$$\frac{P^2 + 2E'_N P_0}{2|\vec{P}||\vec{p}_N|} \leq 1$$

$$\Rightarrow P^2 + 2E'_N P_0 \leq 2|\vec{P}||\vec{p}_N|$$

On squaring both sides and further simplifying

$$E_N'^2 + E_N' P_0 + \frac{P^2}{4} + \frac{M^2|\vec{P}|^2}{P^2} \leq 0$$

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Solving above quadratic equation and as $E_N' \geq M$, *so neglecting negative solution, we get*

$$E_N' = \frac{-P_0 + \sqrt{P_0^2 - 4\left(\frac{P^2}{4} + \frac{M^2|\vec{P}|^2}{P^2}\right)}}{2}$$

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$$\begin{aligned}\left(\frac{d\sigma}{dE_\pi\, dcos\theta_\pi}\right)_A &= \frac{G_F^2 cos^2\theta_c}{64\pi^5}\int_0^\infty r^2\, dr \int dE_l\, d\Omega_l \frac{|\vec{p}_l||\vec{p}_\pi|}{|\vec{P}||\vec{k}|}(E_F^N(r) - \epsilon) \\ &\times \Theta(E_F^N(r) - E_N) \times \Theta(-P^2)\Theta(P_0) L^{\mu\nu} W_{\mu\nu}(\tilde{p}_N, q, k_\pi) \\ &\times \left(\frac{d\sigma}{dE_\pi\, dcos\theta_\pi}\right)_N\end{aligned}$$

In nuclear medium the properties of $\Delta(1232)$ like its mass and decay width get modified

Modifications

Modification in the width $\tilde{\Gamma}$

$$\frac{\tilde{\Gamma}}{2} \rightarrow \frac{\tilde{\Gamma}}{2} - Im\Sigma_\Delta$$

and in mass M_Δ of the Δ resonance

$$M_\Delta \rightarrow M_\Delta + Re\Sigma_\Delta$$

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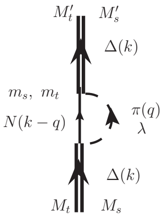
Imaginary part of Δ self energy accounts for

- Quasielastic corrections($WN \rightarrow N\pi$)
- Two body absorption($WNN \rightarrow NN$) and
- Three body absorption($WNNN \rightarrow NNN$)

Δ *self energy in free space*

Δ *Self energy*

Feynman diagram for free Δ self energy containing $\Delta \to N\pi$ decay



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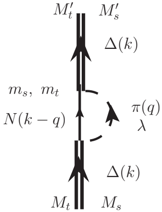

For πNN vertex

$$\delta H_{\pi NN} = i\,\frac{f(q^2)}{\mu}\,\vec{\sigma}\cdot\vec{q}\,\vec{\tau}\cdot\hat{\phi}$$

for $\pi N\Delta$ vertex

$$\delta H_{\pi N\Delta} = i\,\frac{f^*(q^2)}{\mu}\,\vec{S}\cdot\vec{q}\,\vec{T}\cdot\hat{\phi} + h.c.$$

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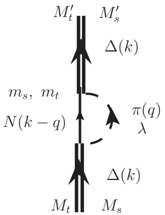

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Coupling Constants

for $\pi N\Delta$ vertex

$$\delta H_{\pi N\Delta} = i\,\frac{f^*(q^2)}{\mu}\,\vec{S}\cdot\vec{q}\,\vec{T}\cdot\hat{\phi} + h.c.$$

Following Feynman rules, Δ self energy for the Feynman diagram

$$\begin{aligned}\langle M_s' M_t' | -i\Sigma(k) | M_s M_t \rangle &= \Sigma_{m_s m_t \Lambda} \int \frac{d^4 q}{(2\pi)^4} \left(\frac{f^*(q^2)}{\mu} \right)^2 \langle M_s' | S^\dagger \cdot q | m_s \rangle \\ &\times \langle m_s' | S \cdot (-q) | M_s \rangle \langle M_t' | T^{\dagger\Lambda} | m_t \rangle \langle m_t | T^\Lambda | M_t \rangle \\ &\times iG_0(k-q) iD_0(q)\end{aligned}$$

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\end{aligned}
$$

G_0 is nucleon propagator

D_0 is meson propagator

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In operator form

$$\Sigma(k) = iS_i^\dagger S_j \int \frac{d^4 q}{(2\pi)^4} \left(\frac{f^*(q^2)}{\mu} \right)^2 G_0(k-q) D_0(q)$$

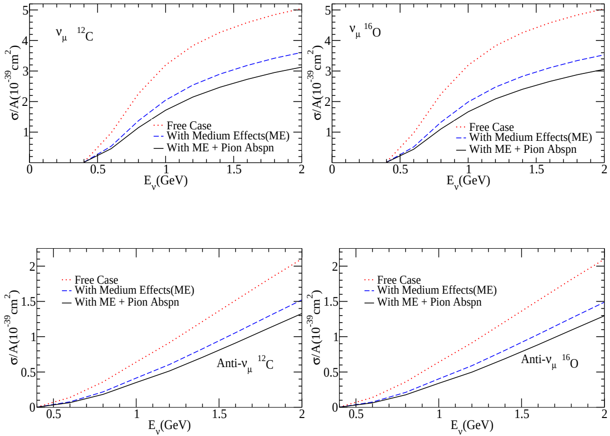
Expressions for the real and the imaginary parts of Σ_Δ

$$\begin{aligned} Re\Sigma_\Delta &= 40\frac{\rho}{\rho_0} MeV \quad and \\ -Im\Sigma_\Delta &= C_Q\left(\frac{\rho}{\rho_0}\right)^\alpha + C_{A2}\left(\frac{\rho}{\rho_0}\right)^\beta + C_{A3}\left(\frac{\rho}{\rho_0}\right)^\gamma \end{aligned}$$

with

$$C(T_\pi) = ax^2 + bx + c;\ x = T_\pi/\mu$$

E. Oset et al., Nucl. Phys. A 468, 631 (1987); C. Garcia Recio et al. Nucl. Phys. A 526, 685 (1991)



νμ 12C
νμ 16O
σ/A(10-39cm2)
Eν(GeV)
Free Case
With Medium Effects(ME)
With ME + Pion Abspn
Anti-νμ 12C
Anti-νμ 16O


DOMO ARIGATO
GOZAIMASU