Why do we need dr < 0.001
-- Lyth bound etc.--

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Messages to the LiteBIRD collaboration

- A detection of $r \sim O(10^{-3})$ tests the (trans-)Planck-scale physics of the field values in large-field inflation models

- By LiteBIRD, we will be able to test the Starobinsky model, and check a variety of Starobinsky-type models with combining 21cm line observation by SKA

- By LiteBIRD, we can measure n_t within a sensitivity of 0.2

Lyth bound (rough estimation)

Boubekeur and Lyth (2005)

- E-folding number

$$N(\phi_{\rm CMB}) = \int_0^{N(\phi_{\rm CMB})} dN \simeq \int_{\phi_{\rm end}}^{\phi_{\rm CMB}} \frac{3H^2}{V'} d\phi, = \int_{\phi_{\rm end}}^{\phi_{\rm CMB}} \frac{V}{V'} \frac{d\phi}{M_{\rm P}^2}, = \int_{\phi_{\rm end}}^{\phi_{\rm CMB}} \frac{1}{\sqrt{\epsilon(\phi)}} \frac{d\phi}{M_{\rm P}}$$

- Tensor to scalar ratio at the CMB scale

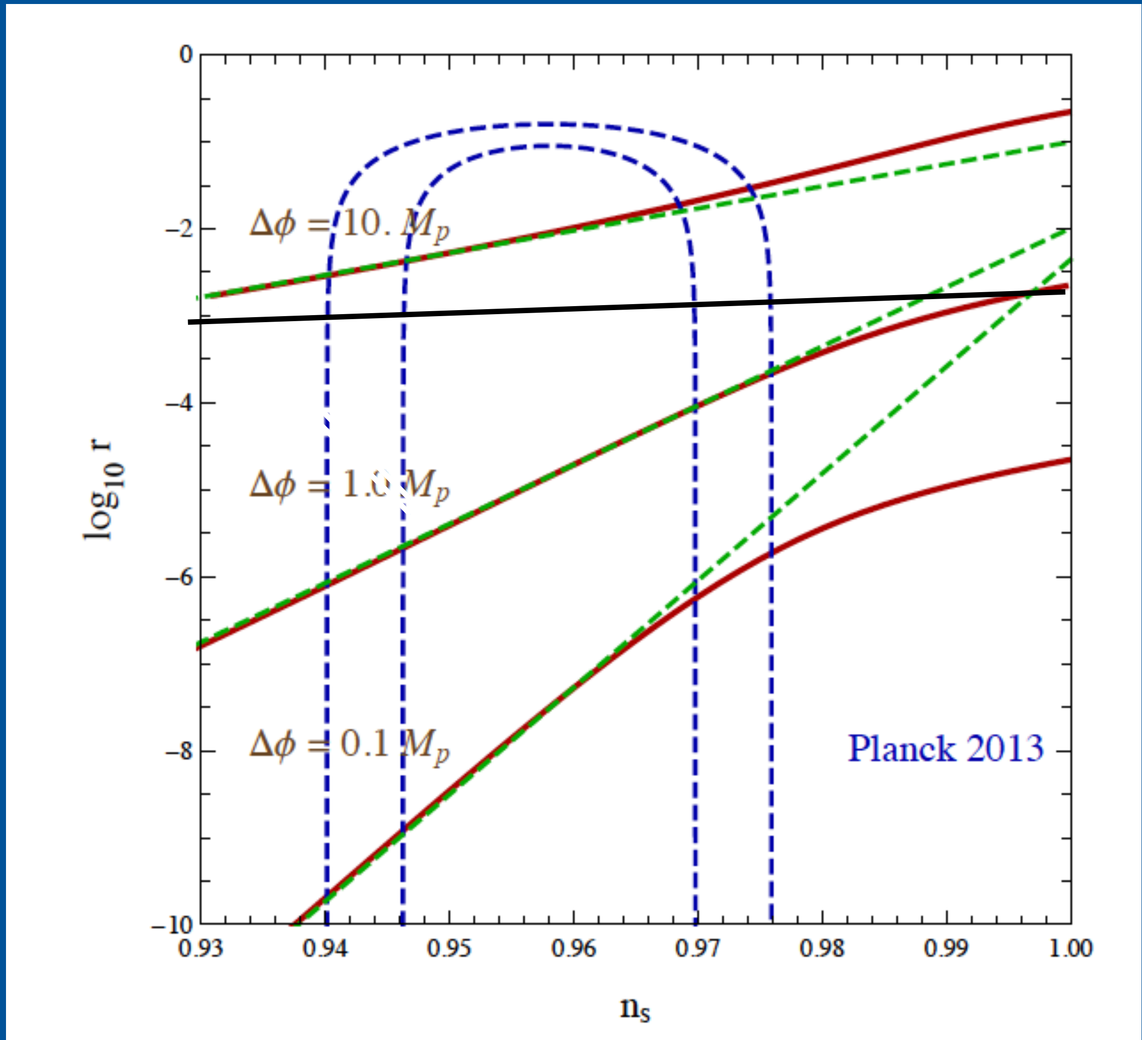
$$\boxed{r = 16\varepsilon_{CMB} \lesssim 16\varepsilon(\phi)}$$

Trans-Planckian?

$$r \lesssim 2.2 \times 10^{-3} \left(\frac{\Delta N_{\rm slow}}{60}\right)^{-2} \left(\frac{\Delta\phi_{\rm slow}}{M_{\rm P}}\right)^2$$

For a bit more specific models

Juan Garcia-Bellido, Diederik Roest, Marco Scalisi, Ivonne Zavala (2014)



$$\epsilon = \frac{\beta}{N^p}$$

$$r = \frac{16\beta}{N^p}$$

$$n_s = \begin{cases} 1 - \frac{2\beta+1}{N}\,, & p = 1 \\ 1 - \frac{p}{N}\,, & p > 1 \end{cases}$$

Observations

- Planck2015 satellite reported;

$$\epsilon \equiv \frac{1}{2}\left(M_G \frac{V'}{V}\right)^2 = 2\left(\frac{M_G}{\phi}\right)^2$$

$$\eta \equiv M_G^2 \frac{V''}{V} = 2\left(\frac{M_G}{\phi}\right)^2$$

$$\xi^{(2)} \equiv V''' V' m_G^4 / V^2$$

  - Power spectrum of curvature perturbation $M_G = M_p / \sqrt{8\pi}$

$$P_\zeta = \frac{V}{24\pi^2 m_G \varepsilon} = (3.091 \pm 0.025) \times 10^{-10} \sim (\Delta T / T)^2$$

  - Spectral index

$$n = \frac{d \ln P_\zeta}{d \ln k} + 1 == 1 + 2\eta - 6\varepsilon = 0.9639 \pm 0.0047$$

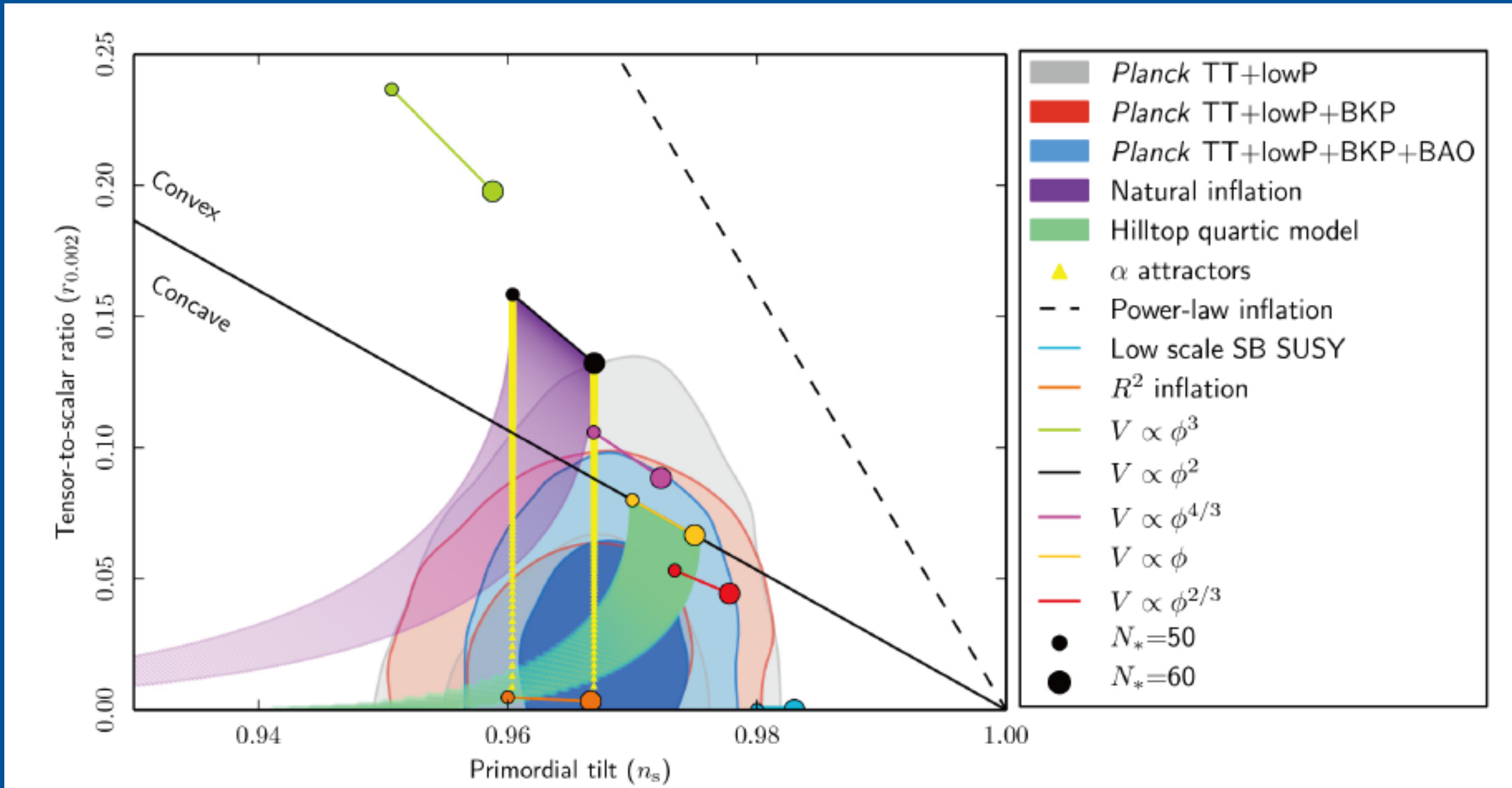
n-1 ~ - 0.04

  - Running of n(k)

$$\alpha_s = \frac{dn}{d \ln k} = 24\varepsilon^2 - 16\varepsilon\eta - 2\xi^{(2)} = 0.009 \pm 0.010 \ (1\sigma)$$

Tensor to scalar ratio
Planck and BICEP/Keck + BAO

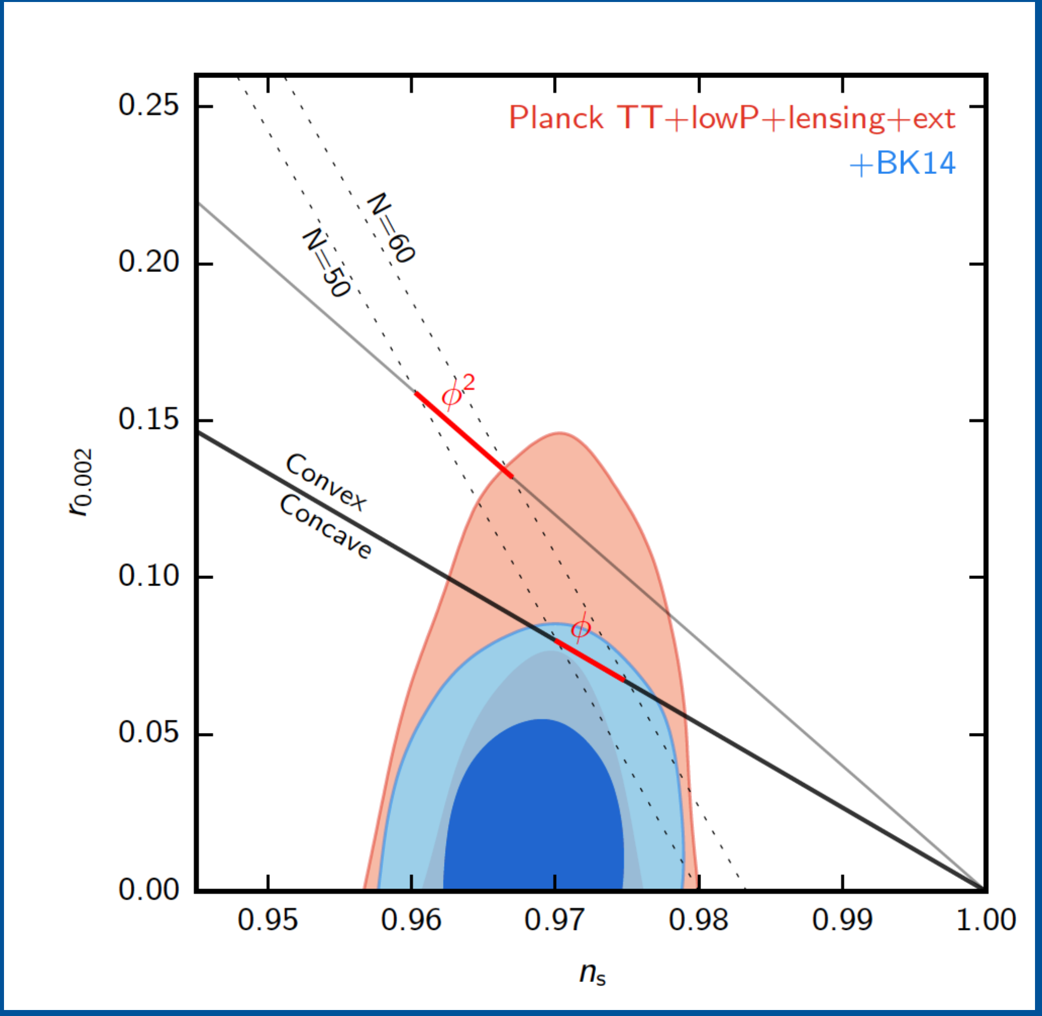
Planck 2015 results. XX. Constraints on inflation



$1/2m^2\varphi^2$ Chaotic inflation model was excluded at 95%

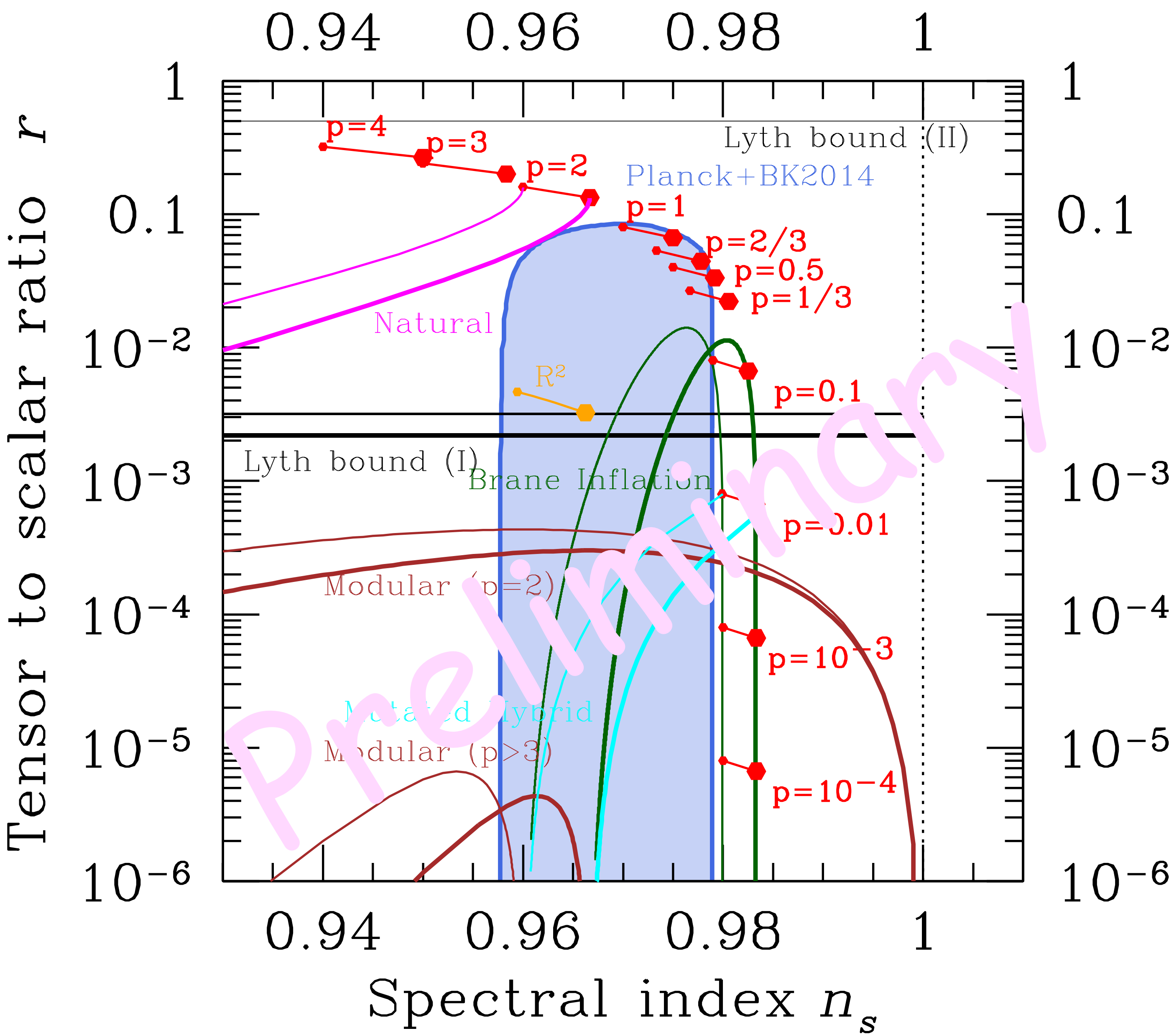
(February, 2015)

PlanckTT+lowP+lensing+ext. +Bicep2/Keck Array 150Hz+95Hz



r < 0.07 at 95% at 0.05Mpc-1

Keck Array and BICEP2 Collaborations: P. A. R. Ade, et al, arXiv:1510.09217 [astro-ph.CO]



0.94
0.96
0.98
1
Lyth bound (II)
Planck+BK2014
p=4
p=3
p=2
p=1
p=2/3
p=0.5
p=1/3
Natural
R²
p=0.1
Lyth bound (I)
Brane Inflation
p=0.01
Modular (p=2)
p=10^-3
Mutated Hybrid
Modular (p>3)
p=10^-4
Preliminary
Tensor to scalar ratio r
Spectral index n_s
10^-2
10^-3
10^-4
10^-5
10^-6
0.1

Specification of LiteBIRD

(1) JAXA baseline

Band (GHz)	Beam (ar-cmin)	NET ($\mu K\sqrt{s}$)	Pixels per wafer	N_{wf}	N_{bolo}	NET_{arr} ($\mu K\sqrt{s}$)	Sens. (μK·arcmin)	Sens. with margin (μK·arcmin)	Band
60	54.1	94	19	8	304	5.4	9.6	15.7	X
78	55.5	59	19	8	304	3.4	6.0	9.9	X
100	56.8	42	19	8	304	2.4	4.3	7.1	Y
140	40.5	37	37	5	370	1.9	3.4	5.6	Y
195	38.4	31	37	5	370	1.6	2.9	4.7	Z
280	37.7	38	37	5	370	2.0	3.5	5.7	Z
total					2022		1.6	2.6	

LiteBIRD + Simons Array

- LiteBIRD single: without lensing for ell<=1500 for TT, TE, EE, BB

- LiteBIRD + Simons Array: with lensing for ell<=3000, + Temp.-fluctuation

Forecast of r v.s. n_s by LiteBIRD

Preliminary

Oyama, Kohri, Hazumi et al, in preparation

Forecast of *r* v.s. n_t by LiteBIRD

Preliminary

$$n_t = -2\varepsilon$$

Oyama, Kohri, Hazumi et al, in preparation

Starobinsky Inflation Model

- Action

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(-\frac{1}{2}M_{\mathrm{P}}^2 R + \frac{M_{\mathrm{P}}^2}{12m^2}R^2\right)$$

See also K. Maeda, Phys. Rev. D 37, 858 (1988).

$\frac{M_{\mathrm{P}}^2}{12m^2} \simeq 5 \times 10^8$ is a big number

Transformation from Jordan to Einstein frame

- Action

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R)$$ $$f(R) = R + F(R)$$

- After Legendre transformation

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\varphi R - U(\varphi)\right]$$ $$\varphi \equiv 1 + F_{,\chi}(\chi)$$

- Effective Potential

$$U(\varphi) = \frac{(\varphi - 1)\chi(\varphi) - F(\chi(\varphi))}{2\kappa^2}$$

Transformation from Jordan to Einstein frame

- Conformal transformation

$$g^E_{\mu\nu} = \varphi g_{\mu\nu}$$

$$S_E = \int d^4x \sqrt{-g_E} \left[\frac{1}{2\kappa^2} R_E - \frac{1}{2} g_E^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

- Potential

$$V(\phi) = \frac{1}{2\kappa^2 \varphi^2} \left[(\varphi - 1)\chi(\varphi) - F(\chi(\varphi)) \right]$$

$$\varphi \equiv 1 + F_{,\chi}(\chi)$$

$$\varphi = e^{\sqrt{2/3}\kappa\phi}$$

Starobinsky Inflation in Einstein frame

- Action

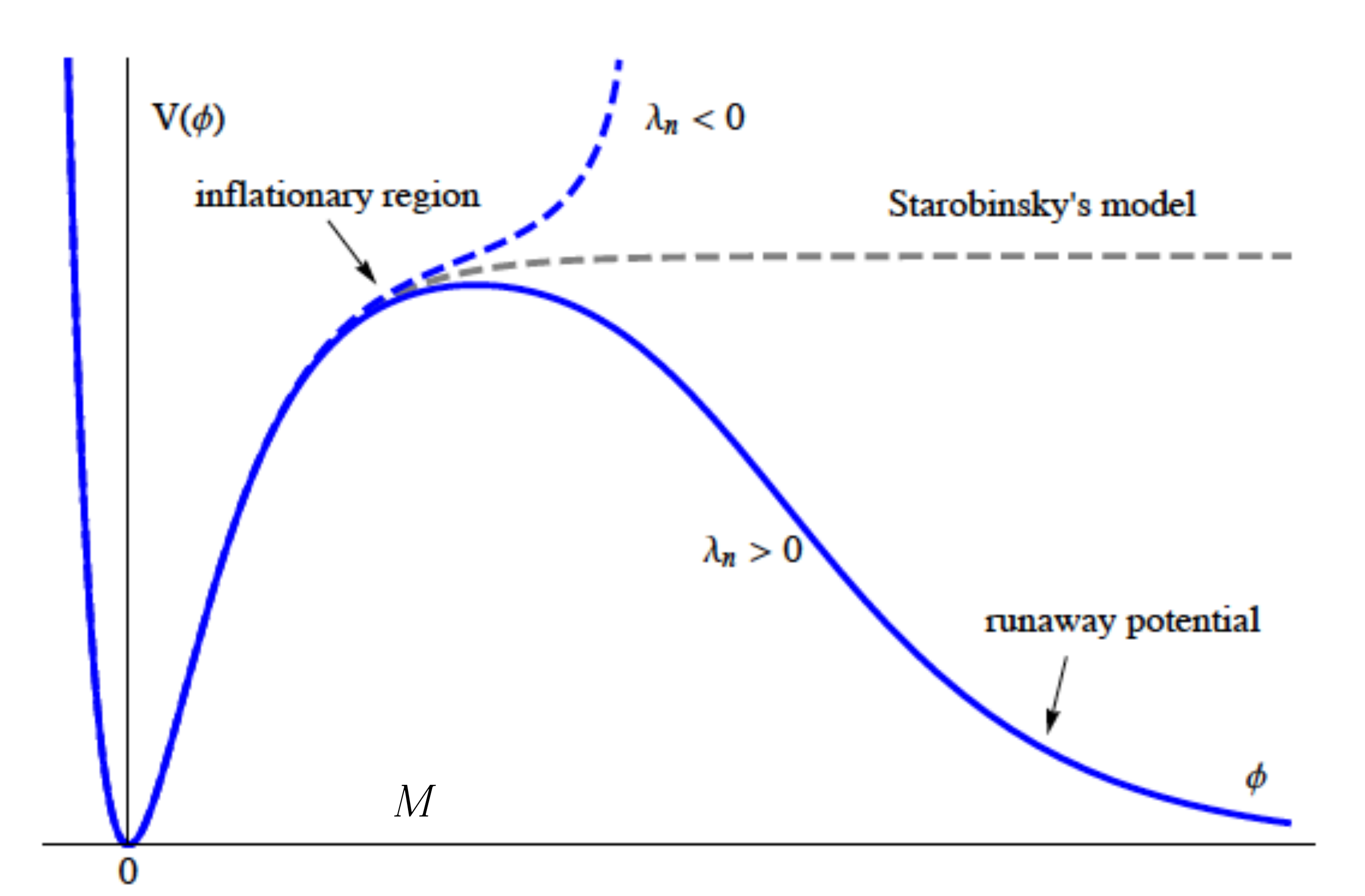
$$S_E = \int d^4x\sqrt{-g_E}\left[\frac{1}{2\kappa^2}R_E - \frac{1}{2}g_E^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

- Potential

$$V_{\text{Starobinsky}} = \frac{3}{4}m^2M_{\text{P}}^2\left(1 - e^{-\sqrt{2/3}\phi/M_{\text{P}}}\right)^2$$

General potential shapes

$$f(R) = R + \frac{R^2}{6M^2} + \frac{\lambda_n}{2n}\frac{R^n}{(3M^2)^{n-1}}$$



Q.-G. Huang, 2013

Starobinsky-like model from extra dimensions

Asaka, Iso, Kawai, Kohri, Noumi, Terada, 2015

- Action in D-dimension

$$S = \Lambda^D \int \mathrm{d}^D x \sqrt{-g} \sum_{n=0} b_n \left(\frac{R}{\Lambda^2} \right)^n$$

$D = 10$

Λ is a cutoff scale

- Action in 4D

$$S = c \int \mathrm{d}^4 x \sqrt{-g} \sum_{n=0} b_n \Lambda^4 \left(\frac{R}{\Lambda^2} \right)^n$$

$$c \equiv V_{D-4} \Lambda^{D-4} \qquad L \equiv V_6^{1/6}$$

- If we take

$L=30/\Lambda$

$$c = \frac{M_{\mathrm{P}}^2}{12 m^2} \simeq 5 \times 10^8$$

Starobinsky-like Inflation

Asaka, Iso, Kawai, Kohri, Noumi, Terada, 2015

- Einstein term

$$c b_1 \Lambda^2 = -\frac{M_{\rm P}^2}{2}$$

$$b_1 \sim O(10^{-4})$$

- Including higher-order terms

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(-\frac{1}{2} M_{\rm P}^2 R + \frac{M_{\rm P}^2}{12 m^2} \left(R^2 + \sum_{n=3}^{\infty} b_n \left(-\frac{6m^2}{b_1} \right)^{2-n} R^n \right) \right)$$

$$b = b_1 b_3$$

$$V = \frac{m^2}{9b^2} e^{-2\sqrt{2/3}\phi} \left(\sqrt{1 + 3b \left(e^{\sqrt{2/3}\phi} - 1 \right)} - 1 \right) \left(1 + 6b \left(e^{\sqrt{2/3}\phi} - 1 \right) - \sqrt{1 + 3b \left(e^{\sqrt{2/3}\phi} - 1 \right)} \right)$$

Starobinsky-like Inflation

- Potential

Asaka, Iso, Kawai, Kohri, Noumi, Terada, 2015

$$V = V_{\text{Starobinsky}} \times \left(1 - \frac{b}{2} e^{\sqrt{2/3}\phi} \left(1 - e^{-\sqrt{2/3}\phi}\right)\right) + \mathcal{O}(b^2),$$

- Predictions

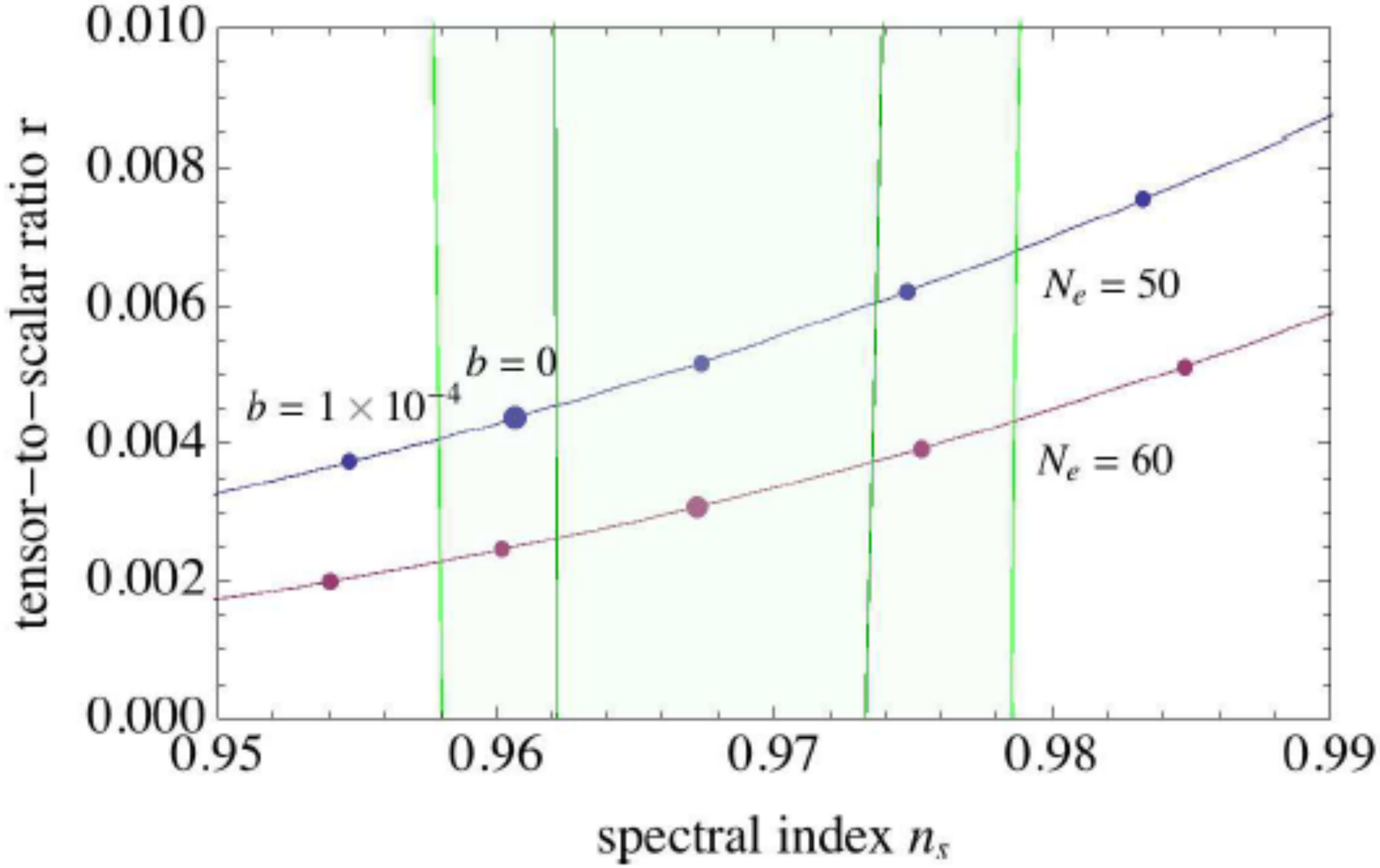
$$b = b_1 b_3$$

$$\begin{aligned}
1 - n_s &\simeq \frac{2}{N}\left(1 + \frac{16}{27} b N^2\right) = \frac{2}{N} + \frac{32}{27} b N, \\
r &\simeq \frac{12}{N^2}\left(1 - \frac{16}{27} b N^2\right) = \frac{12}{N^2} - \frac{64}{9} b, \\
\alpha_s &\simeq -\frac{2}{N^2}\left(1 - \frac{16}{27} b N^2\right) = -\frac{2}{N^2} + \frac{32}{27} b \\
\beta_s &\simeq -\frac{4}{N^3}\left(1 + \frac{4}{9} b N\right) = -\frac{4}{N^3} - \frac{16}{9N^2} b
\end{aligned}$$

$$\alpha_s = dn_s / d\ln k$$

$$\beta_s = d\alpha_s / d\ln k$$

Tensor to scalar ratio

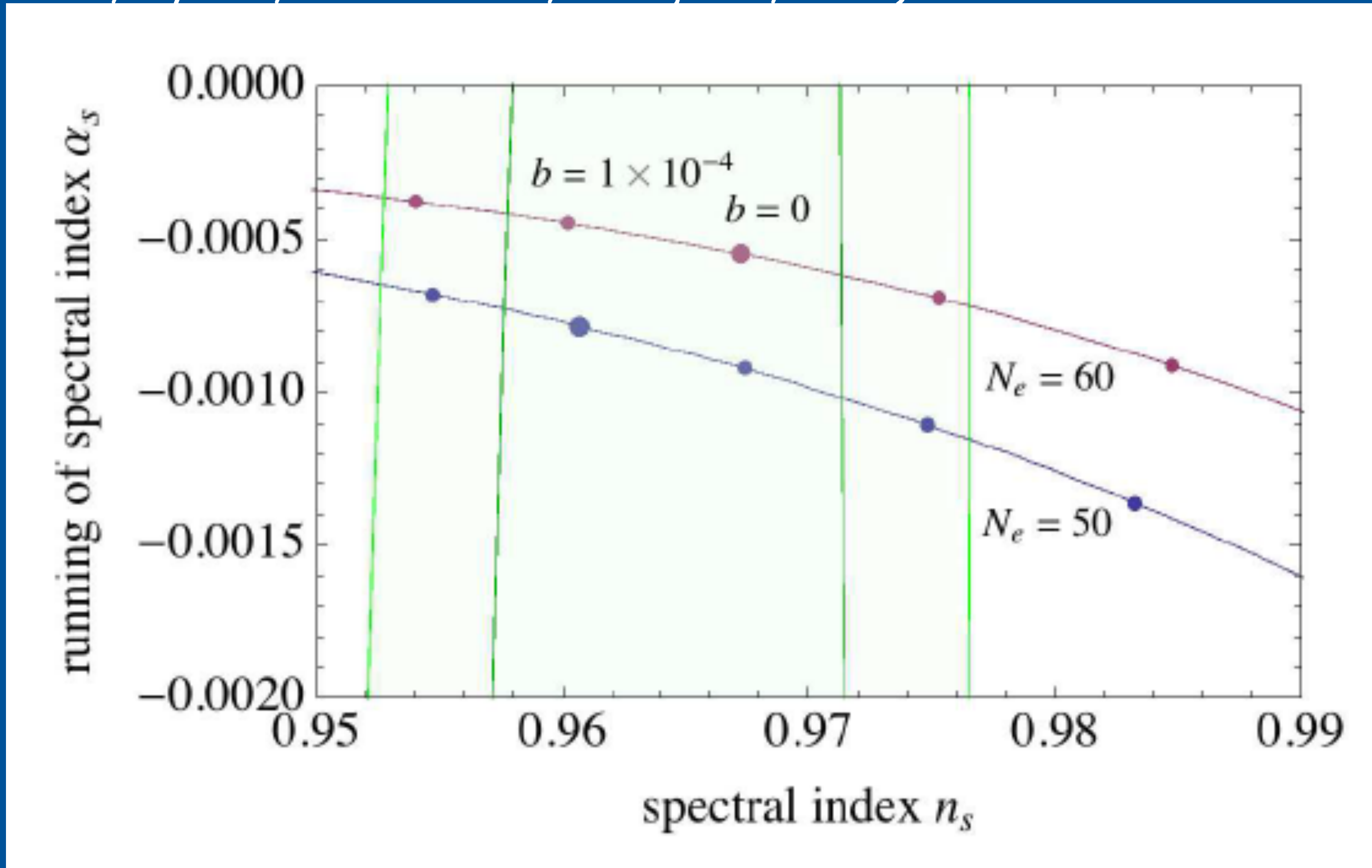


$$b = b_1 b_3$$

Asaka, Iso, Kawai, Kohri, Noumi, Terada, 2015

Starobinsky Inflation

Running will be observed by future 21cm line with the precison of 3.E-4 (Kohri, Oyama, T.Takahashi, Sekiguchi, 2015)



$$b = b_1 b_3$$

Asaka, Iso, Kawai, Kohri, Noumi, Terada, 2015

Cut-off scale of this model

Asaka, Iso, Kawai, Kohri, Noumi, Terada, 2015

- Upper bounds on b_1

$$|b_1| \lesssim 2 \times 10^{-4}$$

- Lower bounds on the cutoff scale

$$\Lambda = m\sqrt{\frac{6}{|b_1|}} \gtrsim 5 \times 10^{15}\mathrm{GeV}$$

Conclusion

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