Exploitation of a pixel-based parametric maximum-likelihood component separation: a LiteBIRD example

Stompor, Leach, Stivoli and Baccigalupi (**2009**) MNRAS, Volume 392, Issue 1, pp. 216-232
Stivoli, Grain, Leach, Tristram, Baccigalupi and Stompor (**2010**) MNRAS , Volume 408, Issue 4, pp. 2319-2335
JE, Stivoli and Stompor (**2011**) Physical Review D, vol. 84, Issue 6, id. 063005
JE. and Stompor (**2012**) Physical Review D, vol. 85, Issue 8, id. 083006
JE., Feeney, Peiris and Jaffe (**2015**) arXiv:1509.06770

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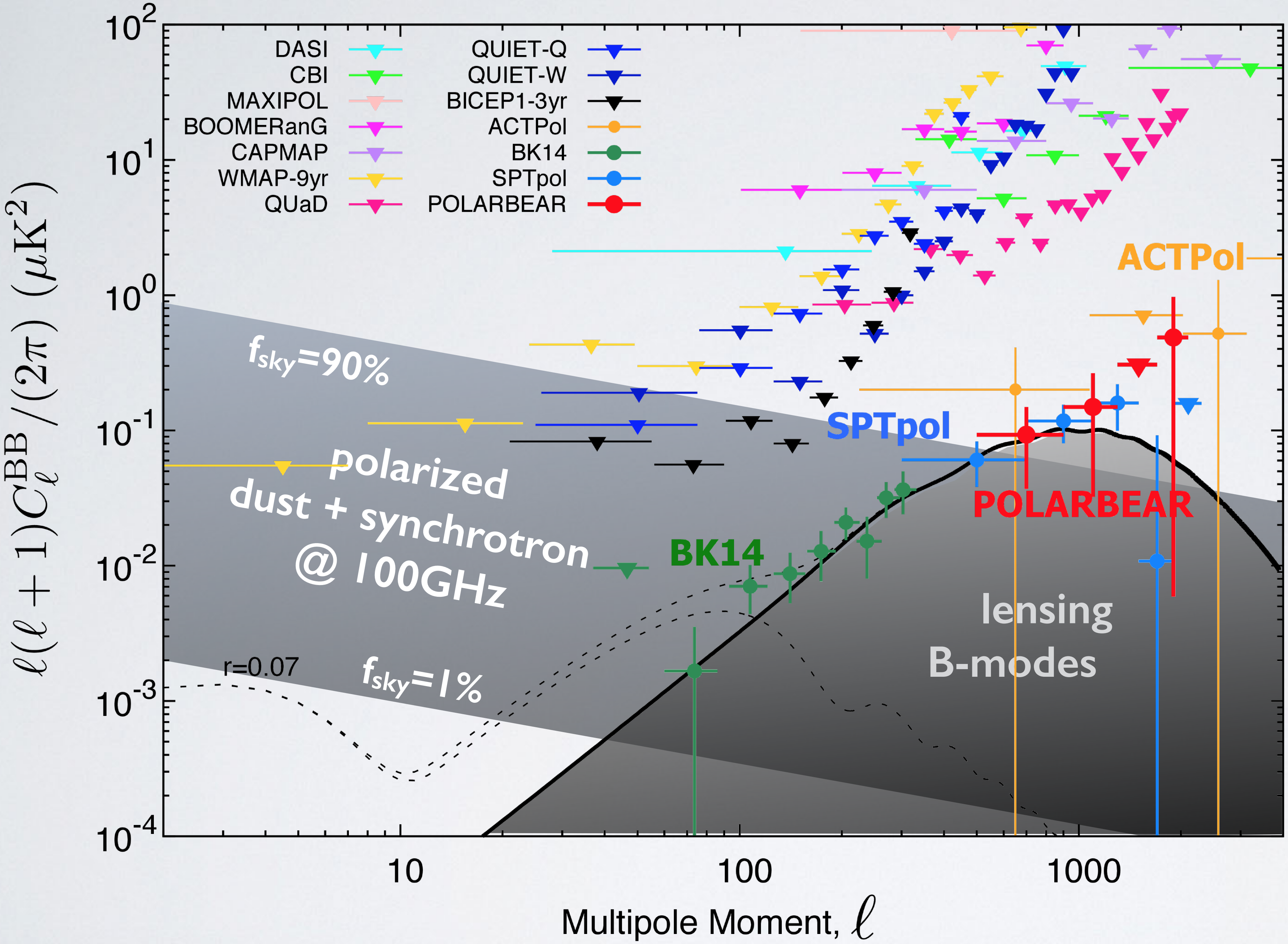
B-modes from space — IPMU — 12/11/2015



A Measurement of the Cosmic Microwave Background B-Mode Polarization Power Spectrum at Sub-degree Scales with POLARBEAR
The POLARBEAR Collaboration
The Astrophysical Journal, Volume 794, 171 (**2014**)

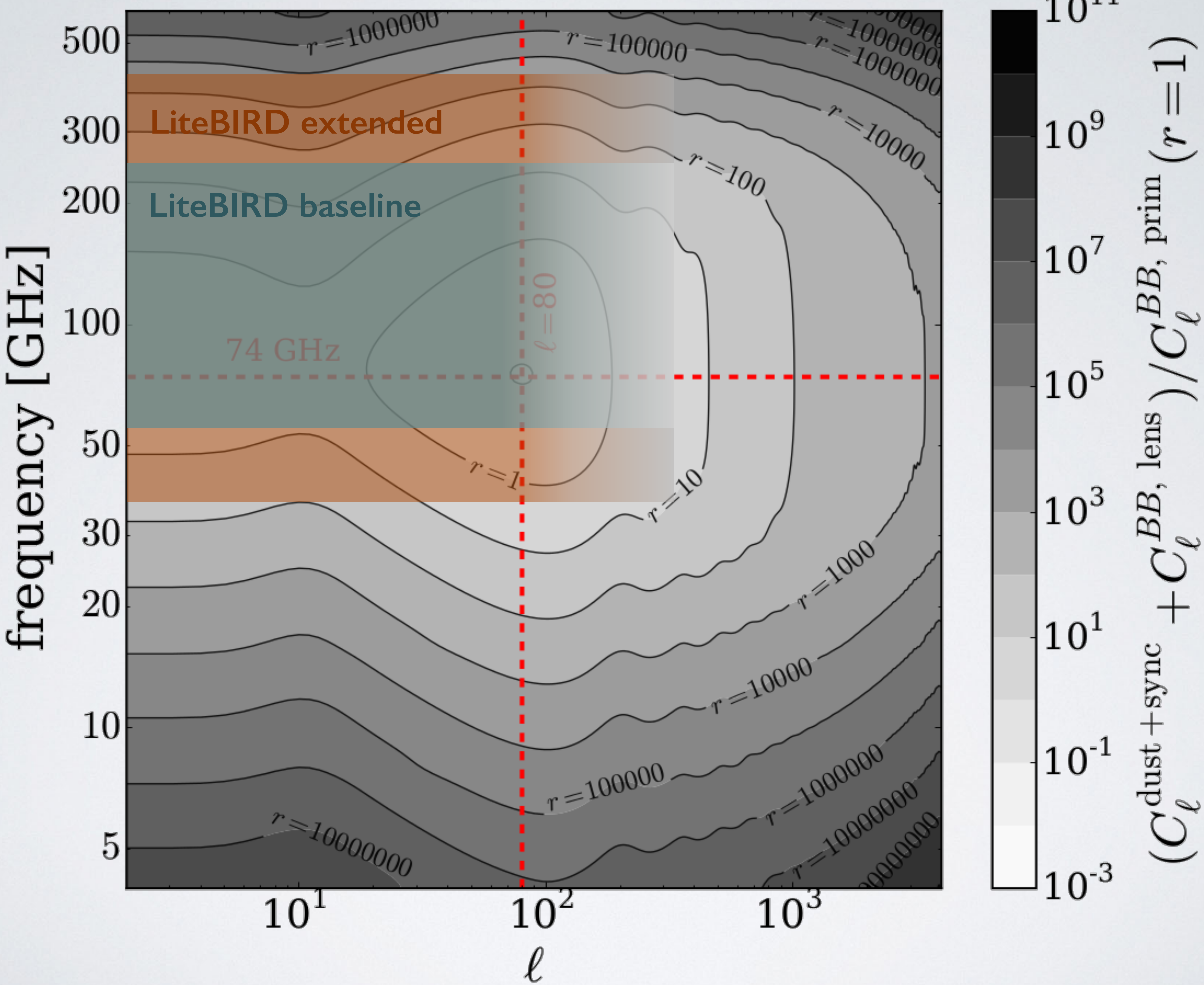
Measurements of Sub-degree B-mode Polarization in the Cosmic Microwave Background from 100 Square Degrees of SPTpol Data
R. Keisler et al.
The Astrophysical Journal, Volume 807, Issue 2, article id. 151, 18 pp. (**2015**)

BK-VI: Improved Constraints On Cosmology and Foregrounds When Adding 95 GHz Data From Keck Array
The Keck Array and BICEP2 Collaborations (**2015**)

The Atacama Cosmology Telescope: CMB polarization at 200 < l < 9000
S. Naess et al.
Journal of Cosmology and Astroparticle Physics, Issue 10, article id. 007, pp. (**2014**)

dust+sync+lensing / primordial B-modes

f_{sky}=50%



outline

1 - Brief rendition of the parametric max-likelihood formalism [e.g., Brandt et al (1994)] **and Gaussian approximation of the likelihood** [JE. et al 2011+2012]

2 - Forecasts for LiteBIRD baseline, LiteBIRD extended and LiteBIRD extended x Stage-IV in various cases:

- Single spectral index over the entire sky
- Simple sky variation of spectral indices (independent patches)
- Modeling of the pixel-to-pixel spatial variation (first order expansion of the mixing matrix)
- CMB + synchrotron + **[single or cold+hot dust]**
- Calibration errors

3 - Conclusions and discussion

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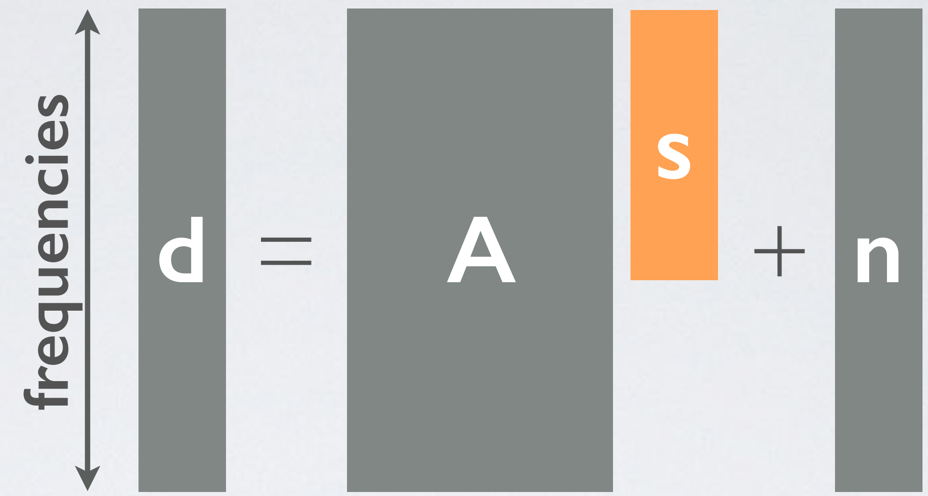
Rendition of parametric max-L component separation

$$d_i(p) = A_{ij}\, s_j(p) + n_i(p)$$

data modeling
for each sky pixel:



1. estimation of the mixing matrix A

$$A^{\rm raw}_{\rm sync}(\nu, \nu_{\rm ref}) \equiv \left(\frac{\nu}{\nu_{\rm ref}}\right)^{\beta_s}$$

$$A^{\rm raw}_{\rm dust}(\nu, \nu_{\rm ref}) \equiv \left(\frac{\nu}{\nu_{\rm ref}}\right)^{\beta_d+1} \frac{e^{\frac{h\nu_{\rm ref}}{k\,T_d}} - 1}{e^{\frac{h\nu}{kT_d}} - 1}$$

$$\mathbf{A} \equiv \mathbf{A}(\beta = \beta_d, \beta_s, \ldots) \longrightarrow \max\left(\mathcal{L}(\beta)\right)$$

not perfect recovery of input spectral parameters ➤ foregrounds residuals

2. solve for s [rather general to any comp sep method]

$$\mathbf{s} = \left(\mathbf{A}^T\mathbf{N}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{N}^{-1}\mathbf{d}$$

linear combination of various frequency maps ➤ boosted noise

Rendition of parametric max-L component separation

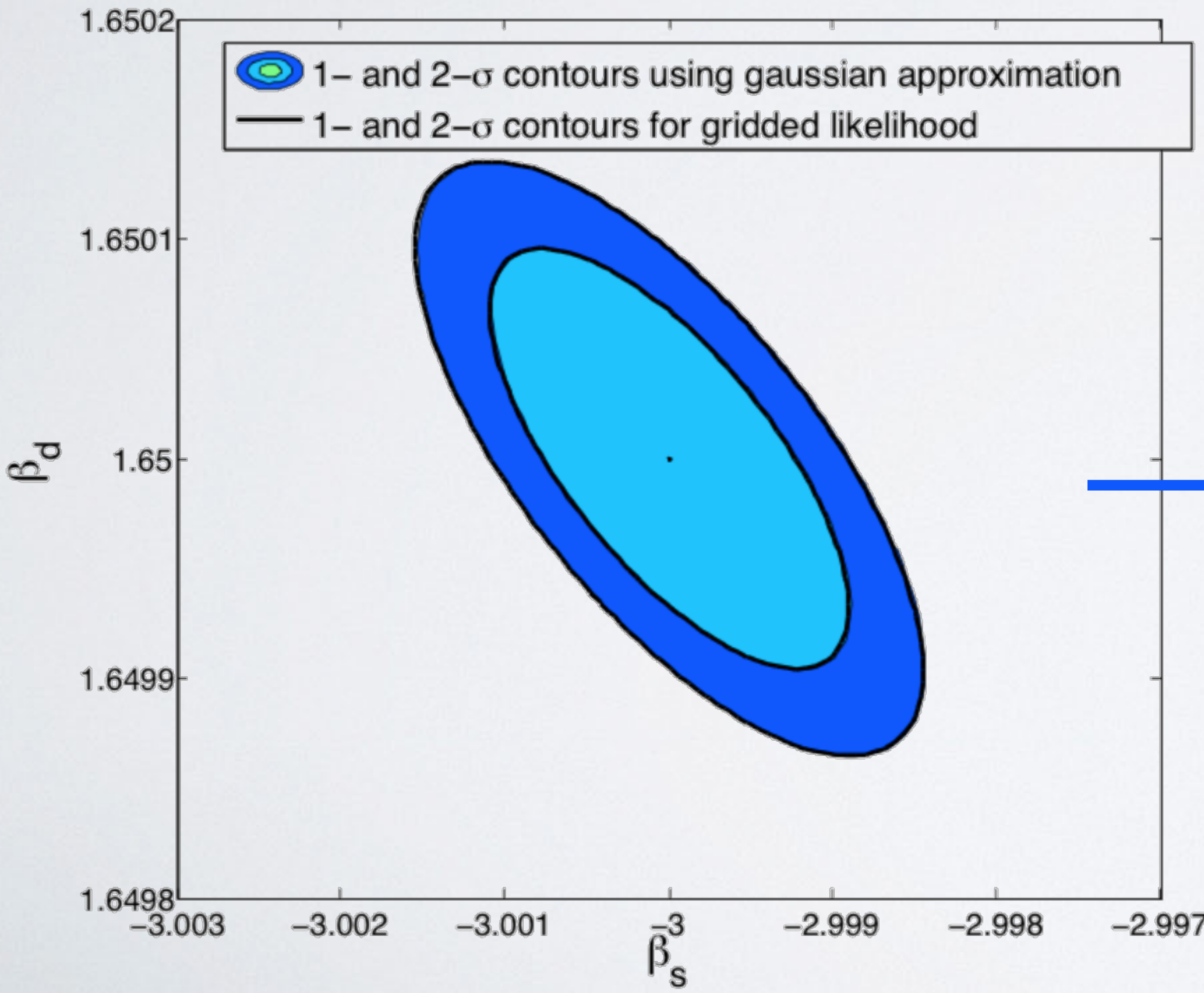
[Brandt et al. (1994), Ericksen et al. (2006), Stompor et al. (2009)]

$$-2\log\mathcal{L}(\mathbf{s},\beta) = \text{constant} + \sum_p (d_p - \mathbf{A}_p s_p)^T \mathbf{N}_p^{-1} (d_p - \mathbf{A}_p \mathbf{s}_p)$$

$$-2\log\mathcal{L}_{\text{marg}}(\beta) = -2\log\int d\mathbf{s}\exp\left[-\frac{1}{2}(\mathbf{d}-\mathbf{A}\mathbf{s})^T\mathbf{N}^{-1}(\mathbf{d}-\mathbf{A}\mathbf{s})\right]$$

$$= \text{constant} - (\mathbf{A}^T\mathbf{N}^{-1}\mathbf{d})^T(\mathbf{A}^T\mathbf{N}^{-1}\mathbf{A})^{-1}(\mathbf{A}^T\mathbf{N}^{-1}\mathbf{d}) + \log\left|(\mathbf{A}^T\mathbf{N}^{-1}\mathbf{A})^{-1}\right|$$

$$-2\log\mathcal{L}_{\text{spec}}(\beta)$$



[JE., F. Stivoli and R. Stompor (2011)]

$-2\log\mathcal{L}_{\text{spec}}(\beta)$ turns out to be **often** well-approximated by a Gaussian at its peak

Rendition of parametric max-L component separation

$$\boldsymbol{\Sigma}^{-1} \simeq -\left\langle \frac{\partial^2 \mathcal{L}}{\partial\beta\partial\beta'} \right\rangle_{\rm noise} \Bigg|_{\rm true\ \beta}$$

[JE. et al (2011)]

$$= -\,{\rm tr}\left\{ \left[\frac{\partial \mathbf{A}}{\partial\beta}^T \mathbf{N}^{-1}\mathbf{A} \left(\mathbf{A}^T\mathbf{N}^{-1}\mathbf{A}\right)^{-1} \mathbf{A}^T\mathbf{N}^{-1}\frac{\partial \mathbf{A}}{\partial\beta'} - \frac{\partial \mathbf{A}}{\partial\beta}^T \mathbf{N}^{-1}\frac{\partial \mathbf{A}}{\partial\beta'} \right] \sum_p \mathbf{s}(p)\,\mathbf{s}^T(p) \right\}$$

$$C_\ell^{\rm fg\ \ res} \equiv \sum_{k,k'} \sum_{j,j'} \Sigma_{kk'}\,\kappa_{kk'}^{jj'} C_\ell^{jj'}$$

noise in the reconstructed maps

[Stivoli et al (2010)]

information about sky components

↳ prediction of error bars for parametric methods like COMMANDER

not perfect recovery of input spectral parameters ➤ foregrounds residuals

$$\sum_{\ell=20}^{200} C_\ell^{BB,\,{\rm prim.}}(r_{eff}) = \sum_{\ell=20}^{200} C_\ell^{\rm fg,\ \ res},$$

linear combination of various frequency maps ➤ boosted noise

$$\Delta \equiv \left(\frac{\sigma_{\rm CMB}}{\sigma_{\rm quad}}\right)^2 \geq 1$$

post component separation noise

simple quadratic combination of sensitivities in all channels

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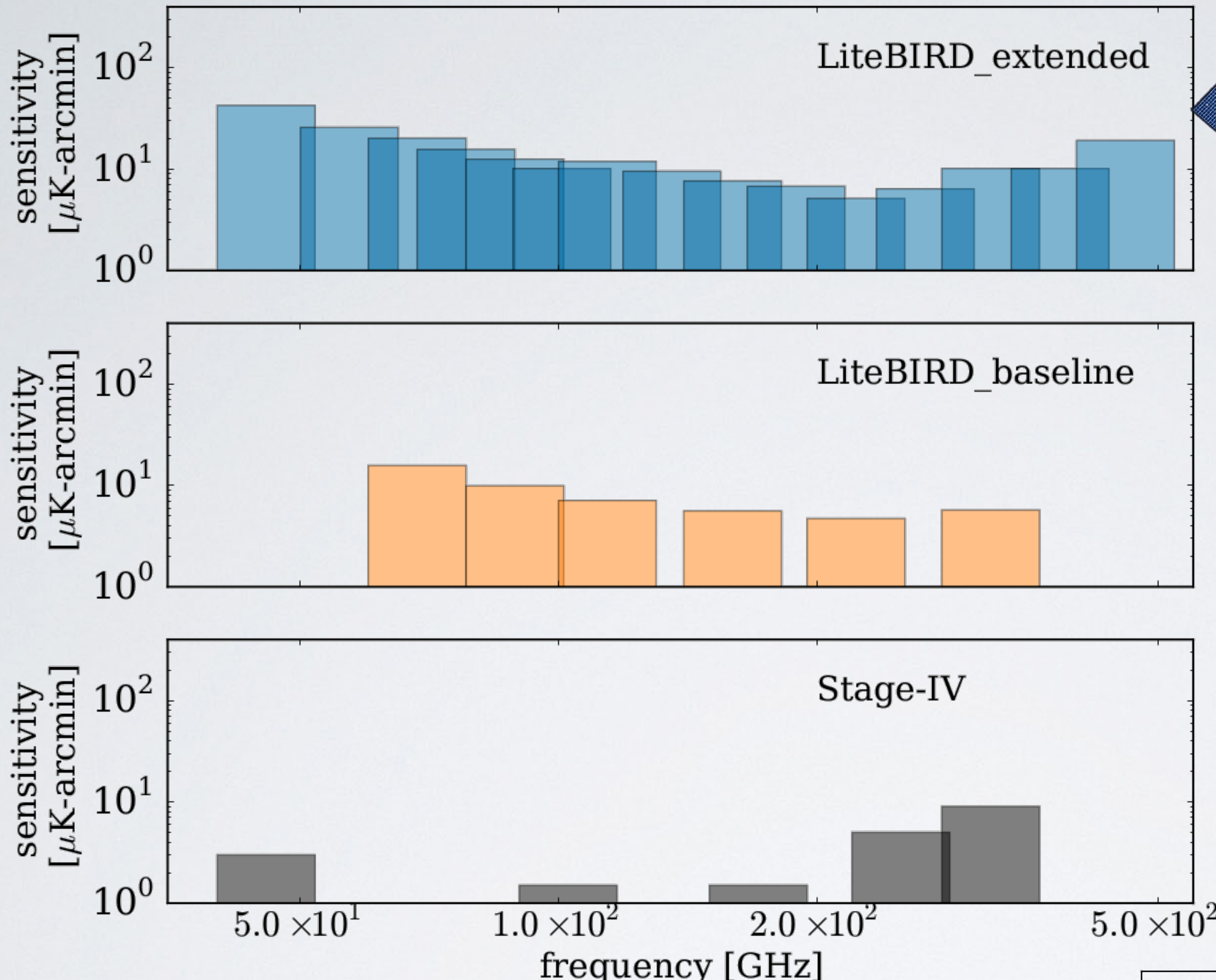
3 - Conclusions and discussion

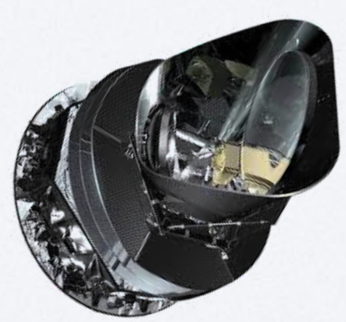
Assumed instrumental specifications

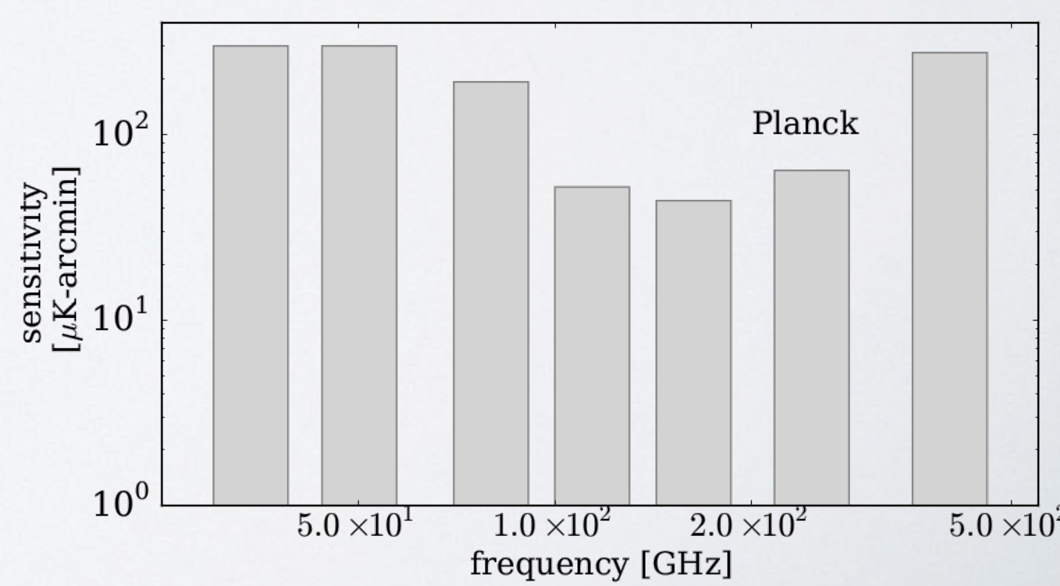


f_{sky}=70%

[JE., S. Feeney et al (2015)]
[K. Wu, JE. et al (2014)]

+





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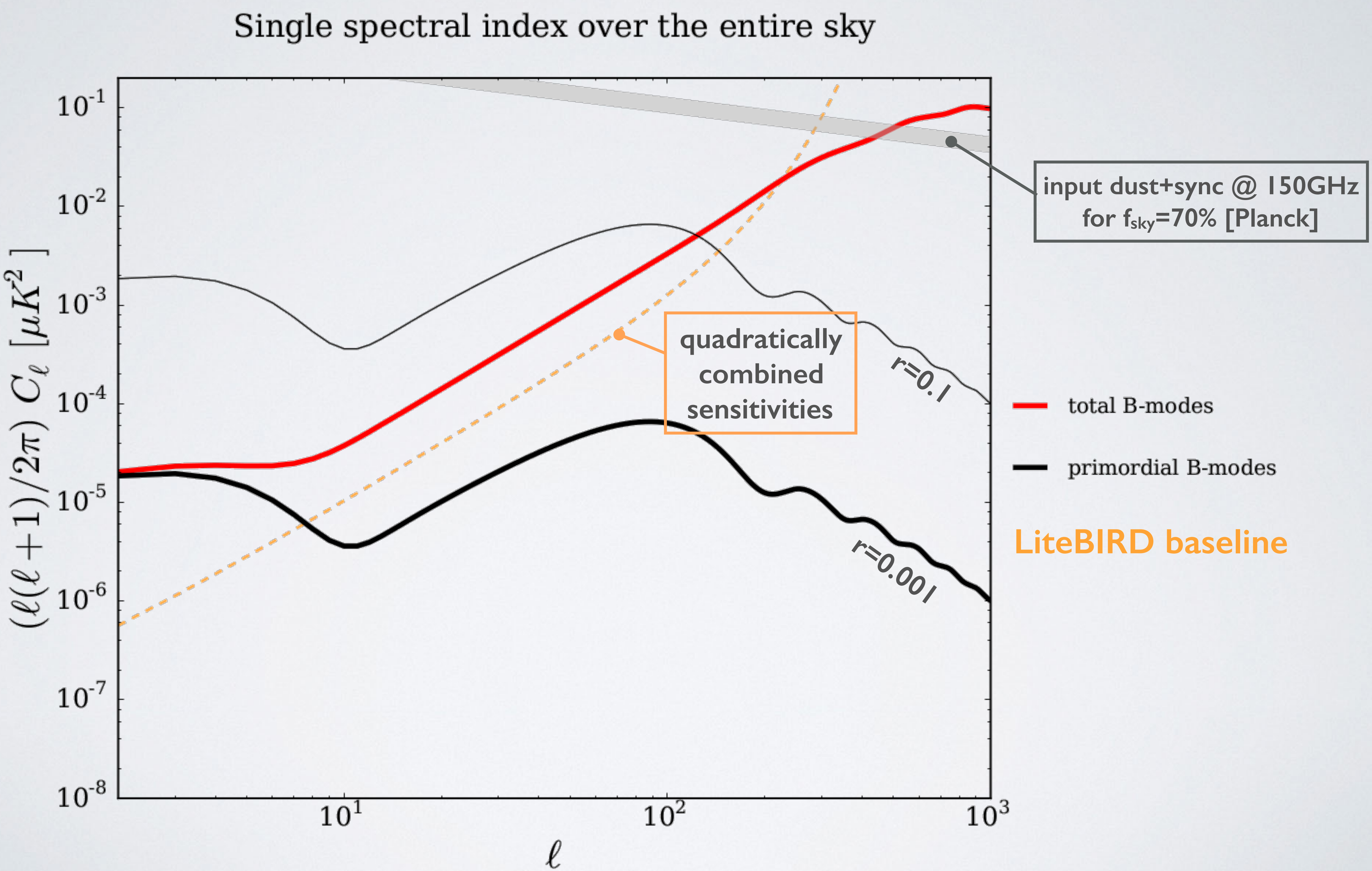
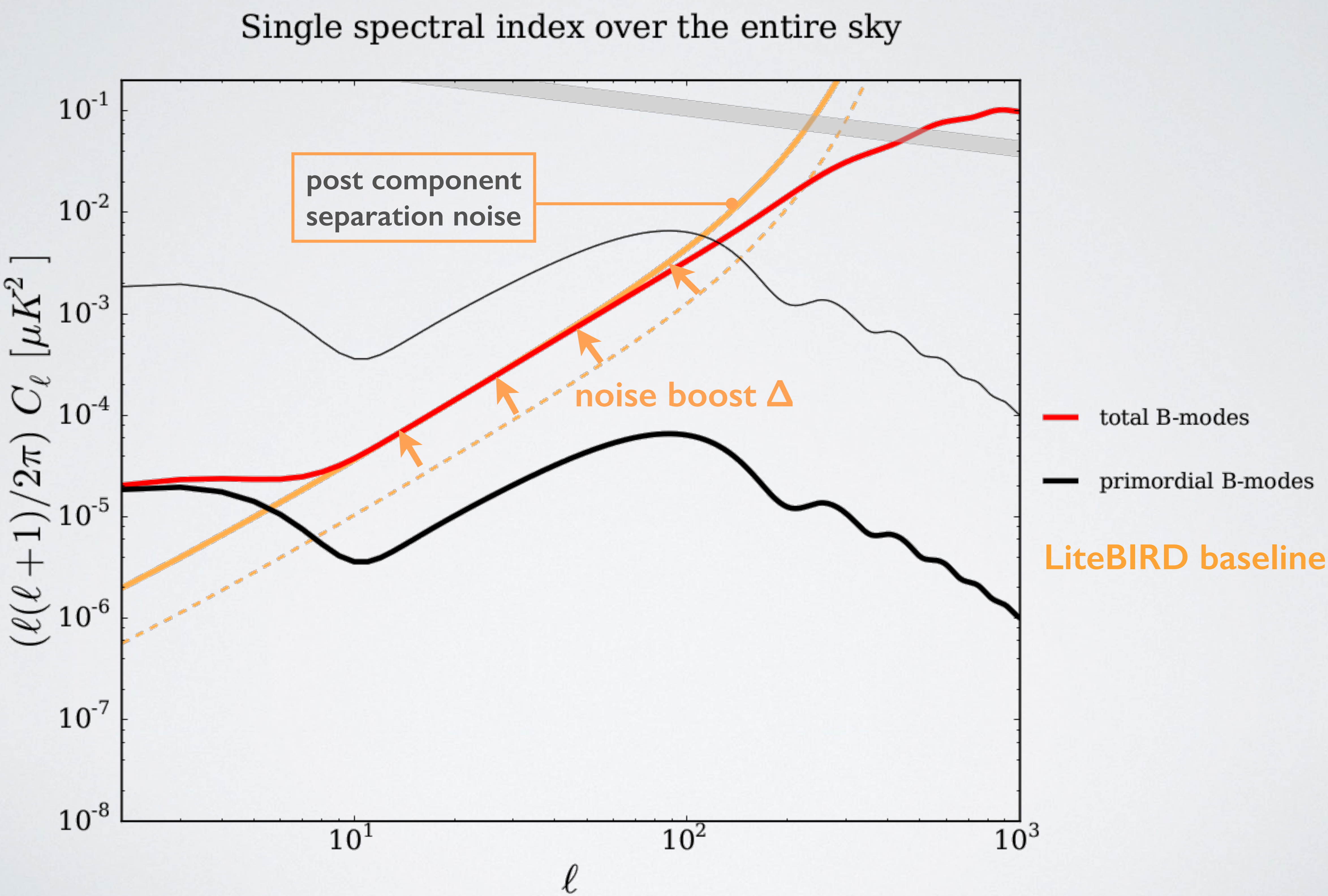
complexity

3 - Conclusions and discussion

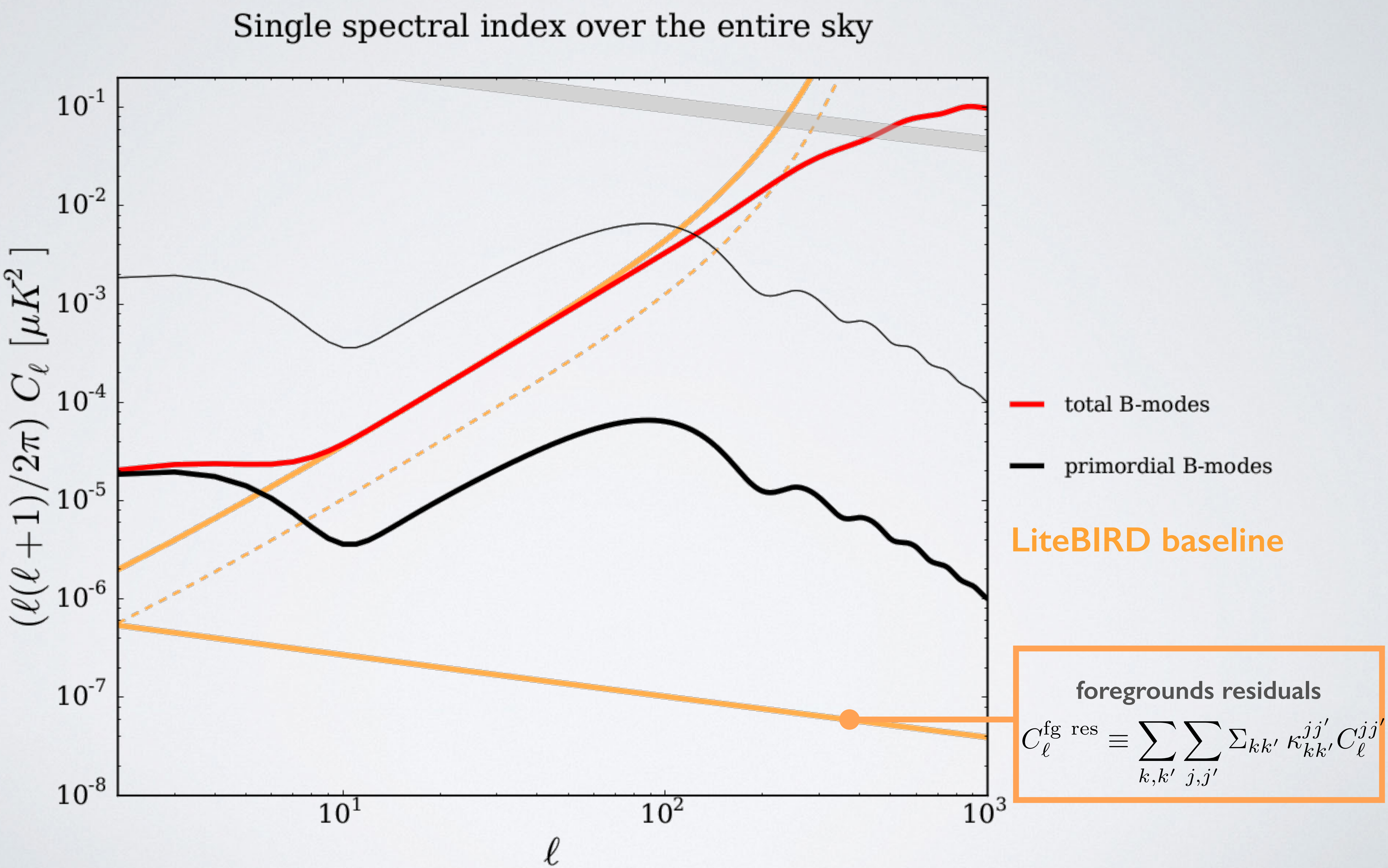
single spectral indices over the entire sky



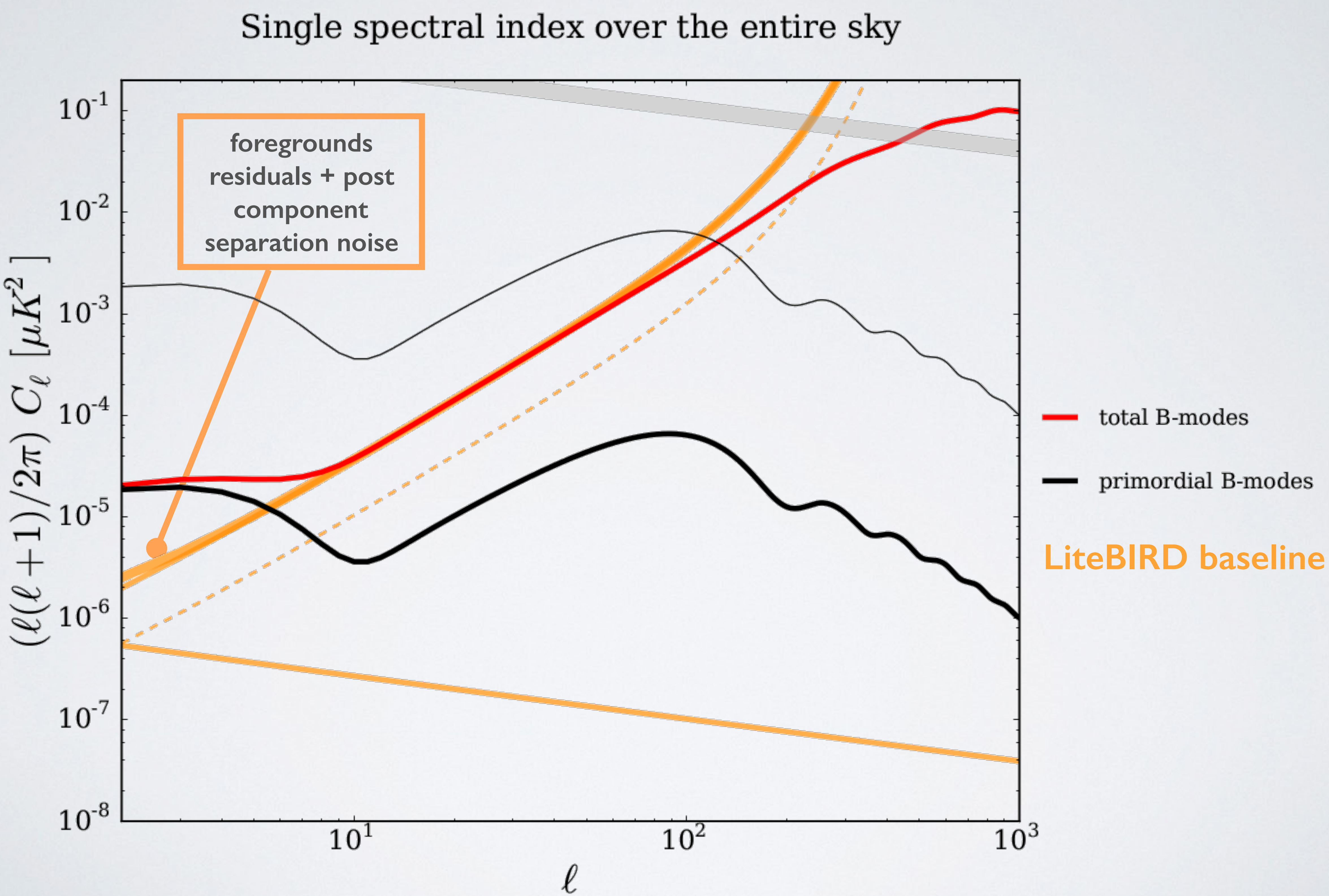
single spectral indices over the entire sky



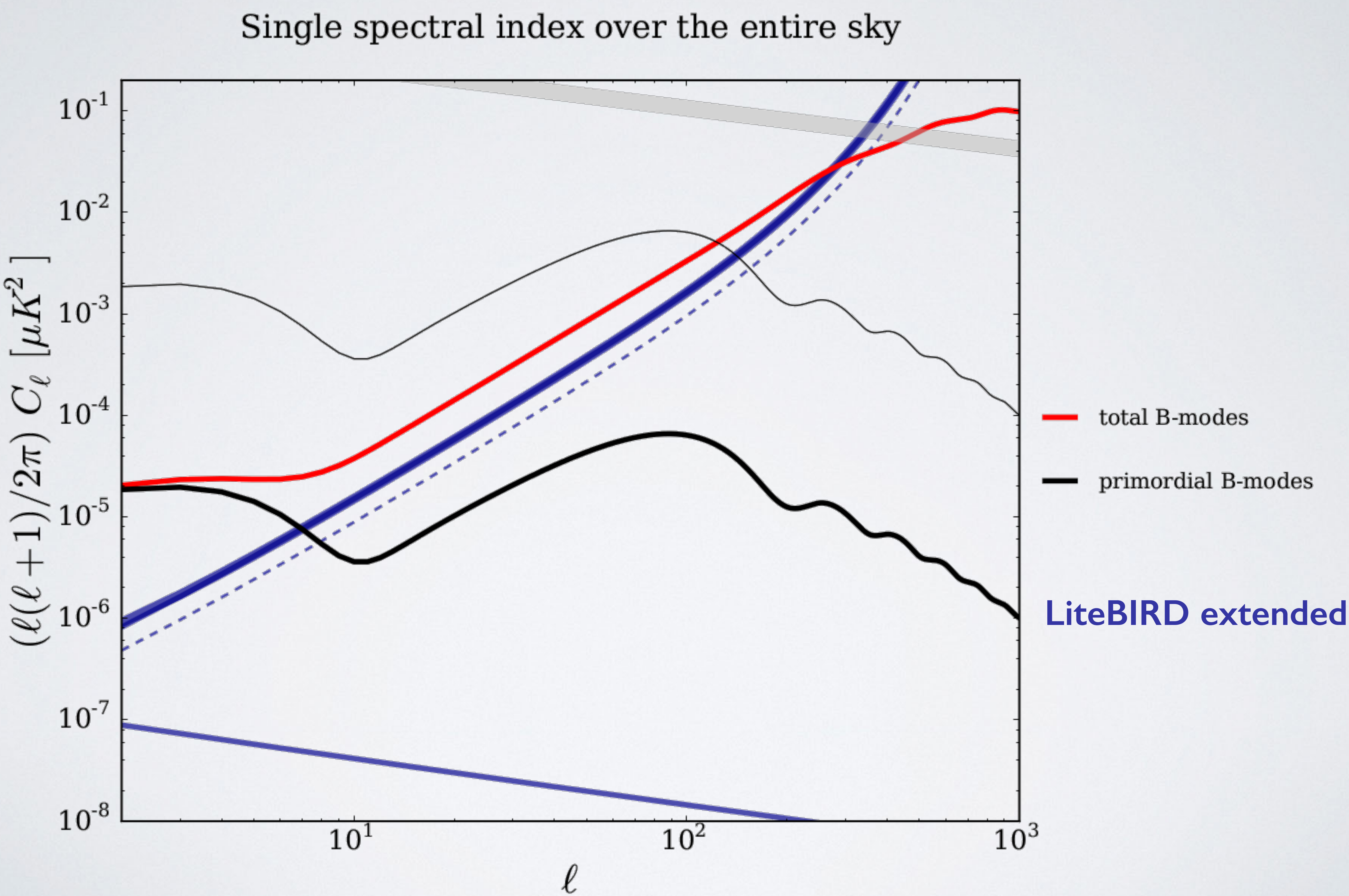
single spectral indices over the entire sky



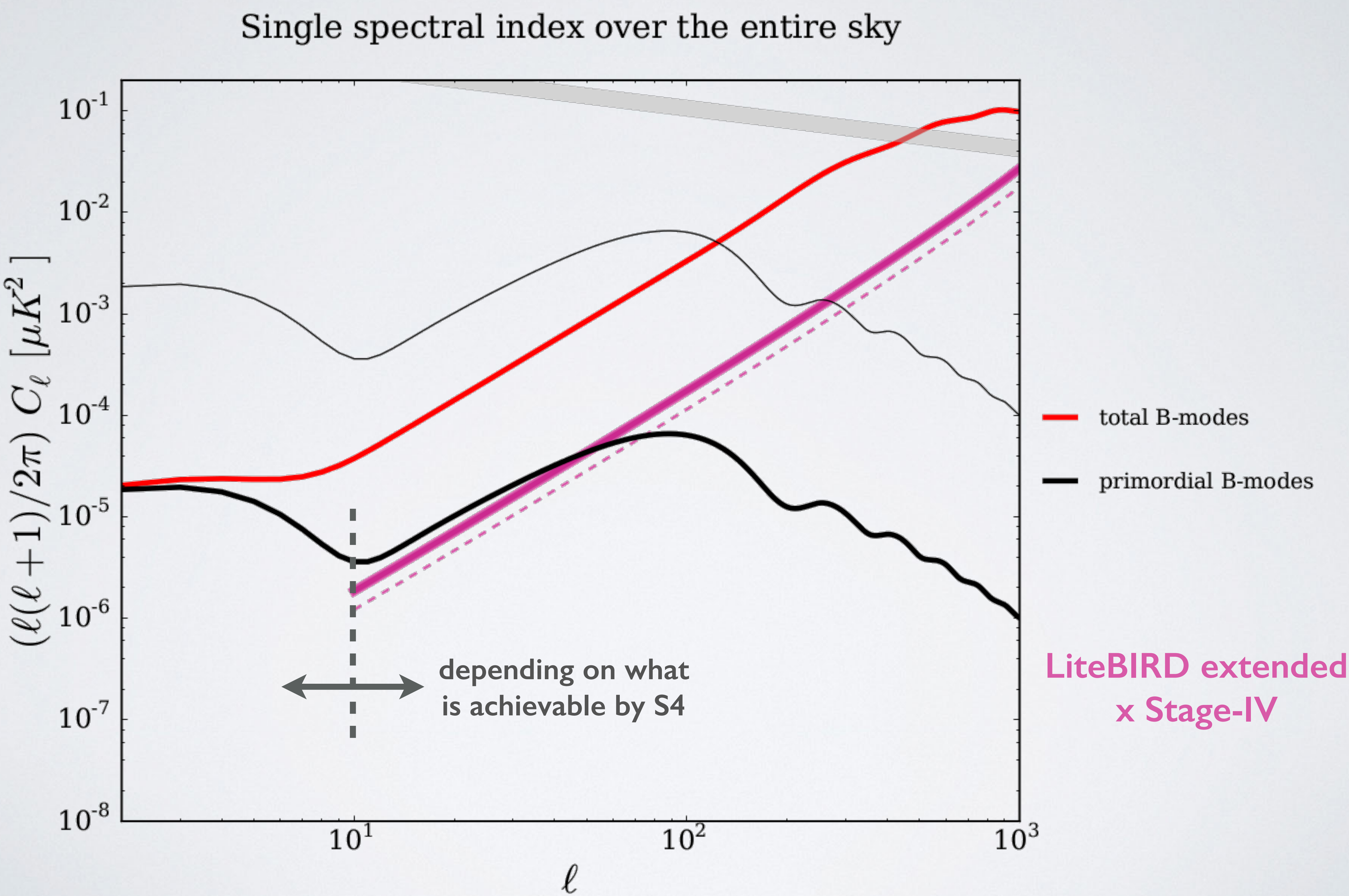
single spectral indices over the entire sky



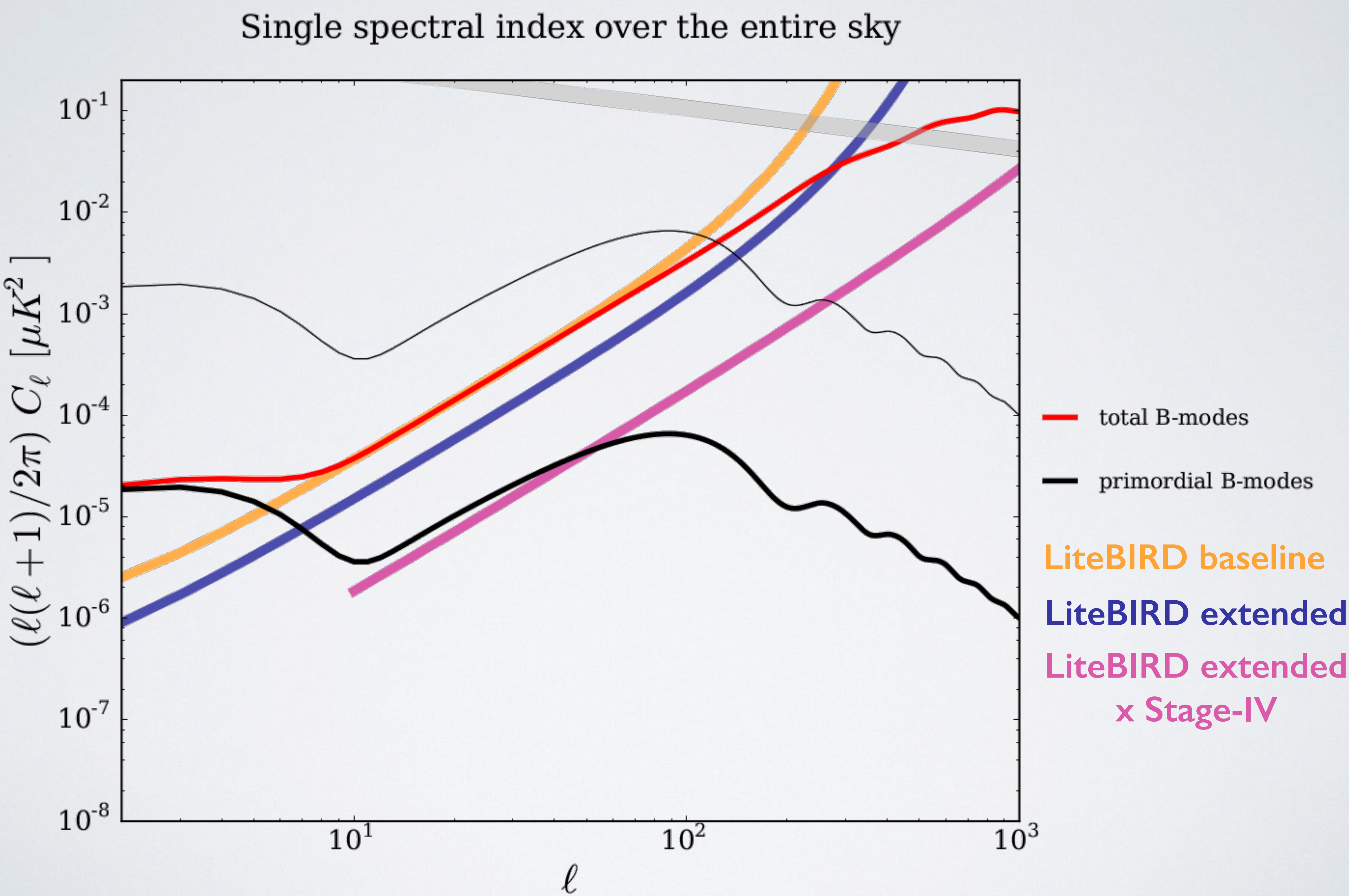
single spectral indices over the entire sky



single spectral indices over the entire sky



single spectral indices over the entire sky



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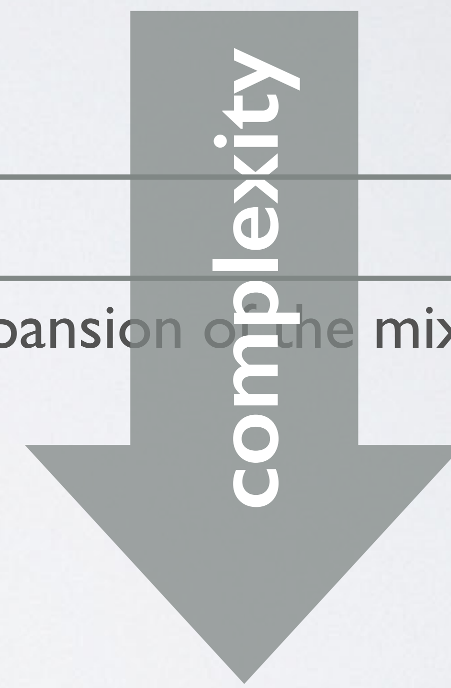
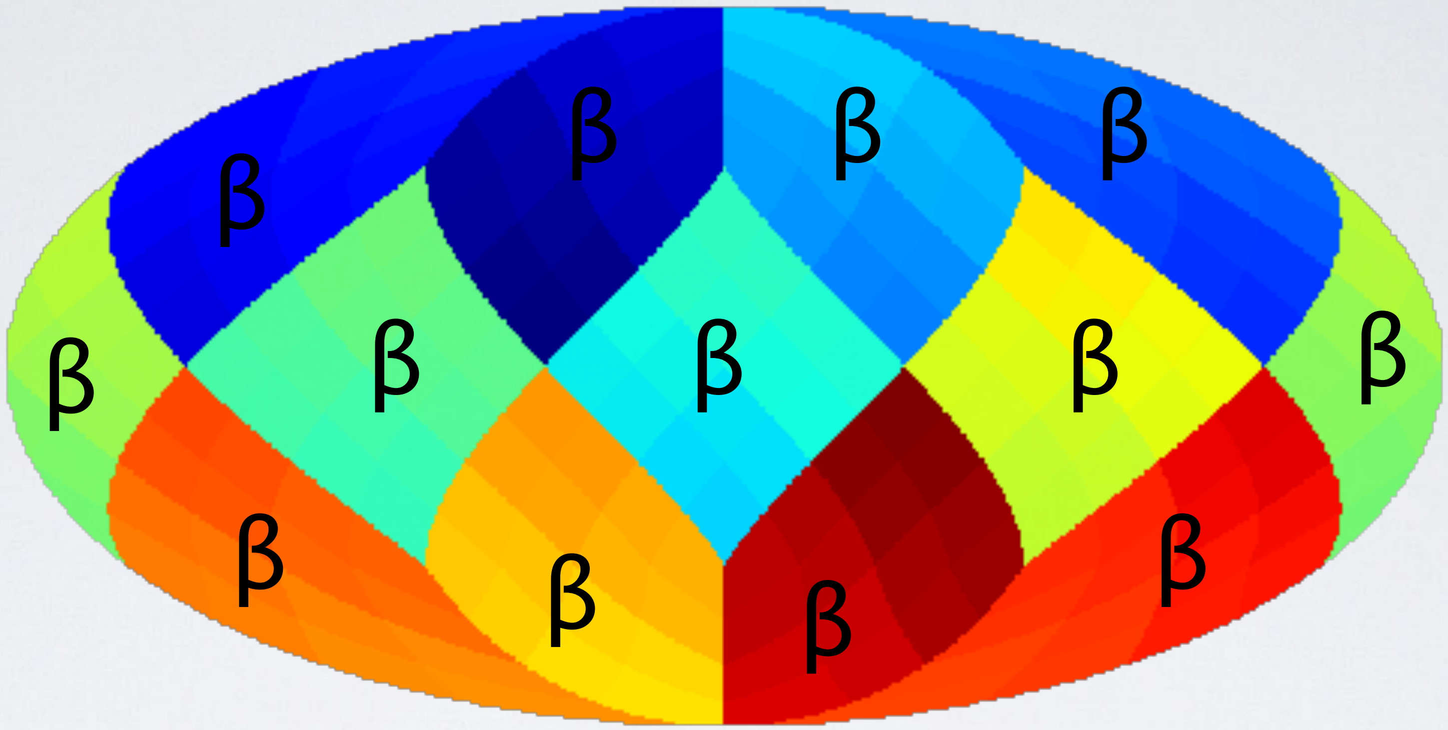


3 - Conclusions and discussion

np-approach

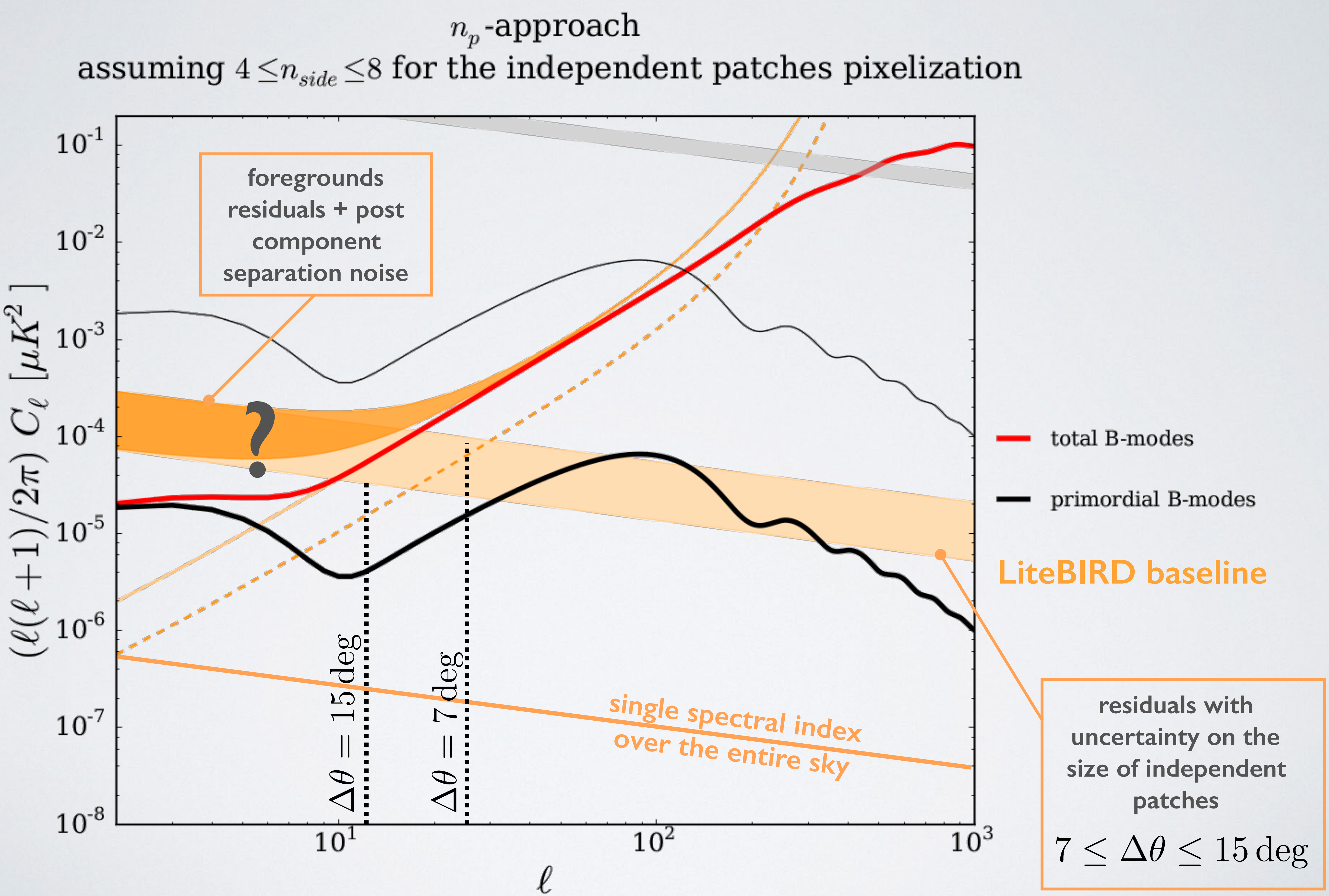
“n_p-approach”



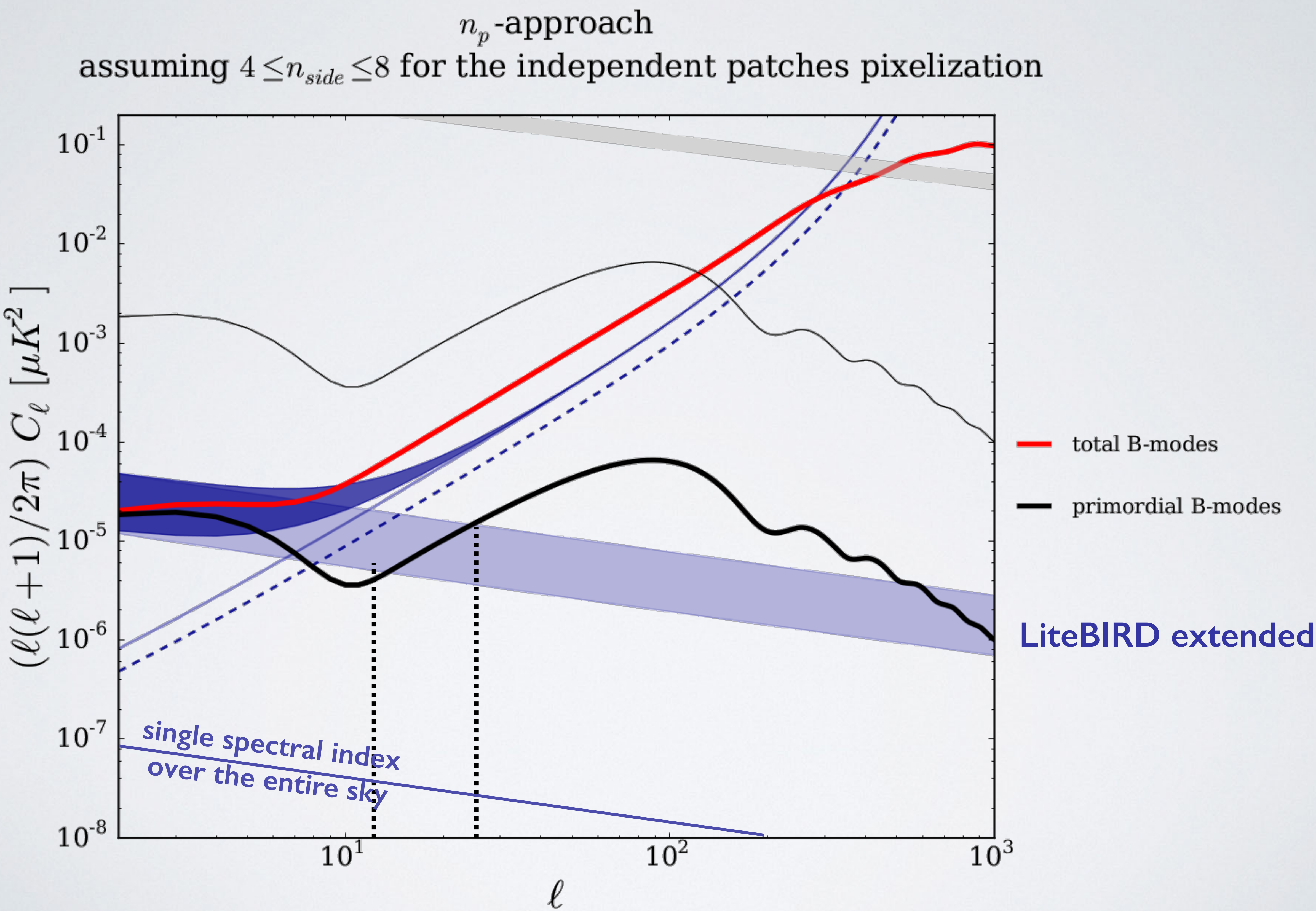
$$\left\{\begin{array}{l} n_{\text{patch}} \equiv \lfloor 12 \times f_{\text{sky}} \times 4^2 \rfloor \\ n_{\text{patch}} \equiv \lfloor 12 \times f_{\text{sky}} \times 8^2 \rfloor \end{array}\right. \longrightarrow \Delta\theta \left\{\begin{array}{r} \sim 15\,\text{deg} \\ \sim 7\,\text{deg} \end{array}\right.$$

$$\sigma(\beta) \propto \sqrt{n_p} \Leftrightarrow C_\ell^{\text{fg, res}} \propto n_p$$

np-approach

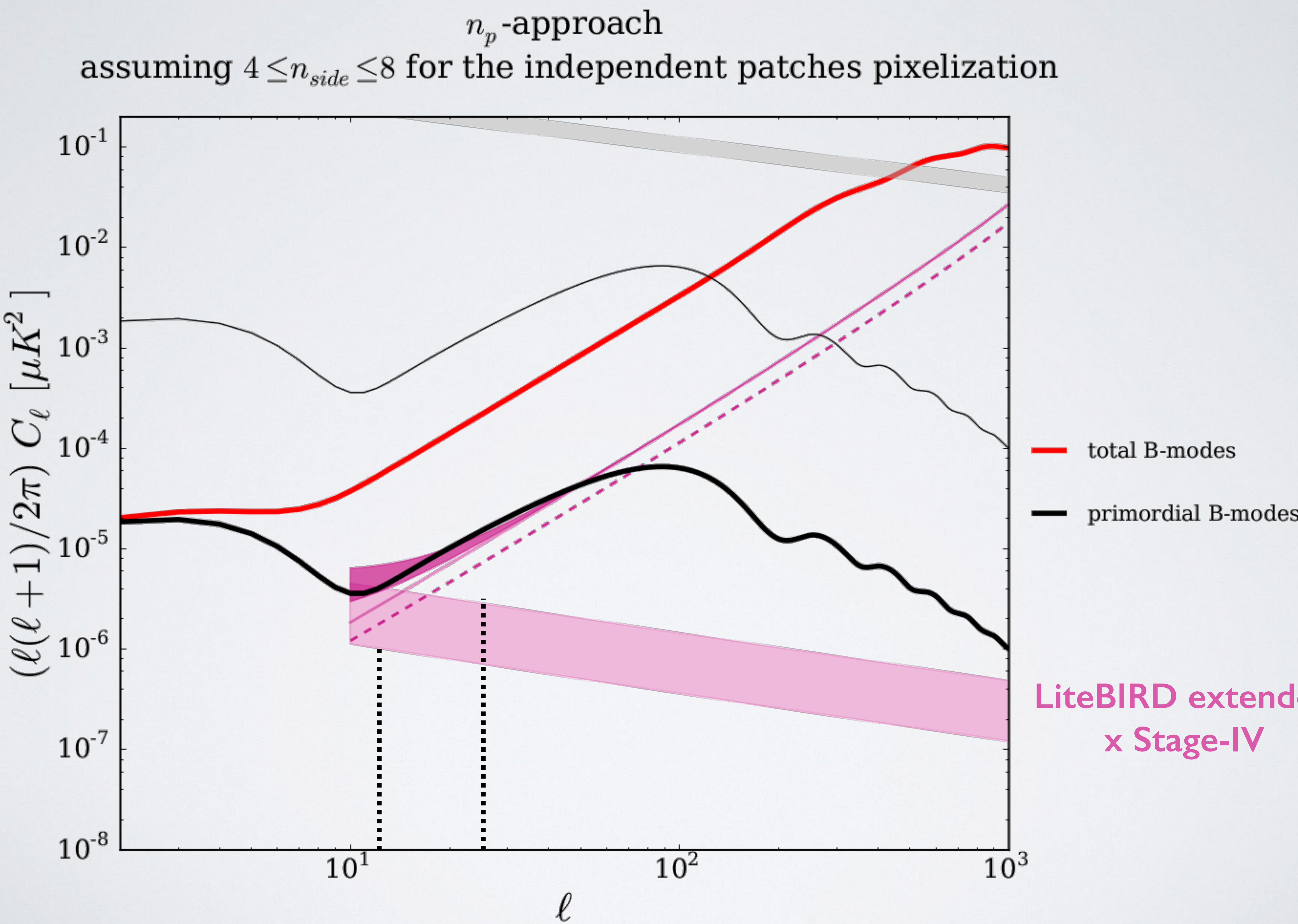

n_p-approach
assuming $4 \leq n_{side} \leq 8$ for the independent patches pixelization
foregrounds residuals + post component separation noise
?
$\Delta\theta = 15\,\mathrm{deg}$
$\Delta\theta = 7\,\mathrm{deg}$
single spectral index over the entire sky
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{1}
10^{2}
10^{3}
ℓ
$(\ell(\ell+1)/2\pi)\ C_\ell\ [\mu K^2]$
total B-modes
primordial B-modes
LiteBIRD baseline
residuals with uncertainty on the size of independent patches
$7 \leq \Delta\theta \leq 15\,\mathrm{deg}$

np-approach



n_p-approach
assuming $4 \leq n_{side} \leq 8$ for the independent patches pixelization
$(\ell(\ell+1)/2\pi)\ C_\ell\ [\mu K^2]$
ℓ
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^1
10^2
10^3
total B-modes
primordial B-modes
LiteBIRD extended
single spectral index over the entire sky

np-approach



LiteBIRD extended
x Stage-IV

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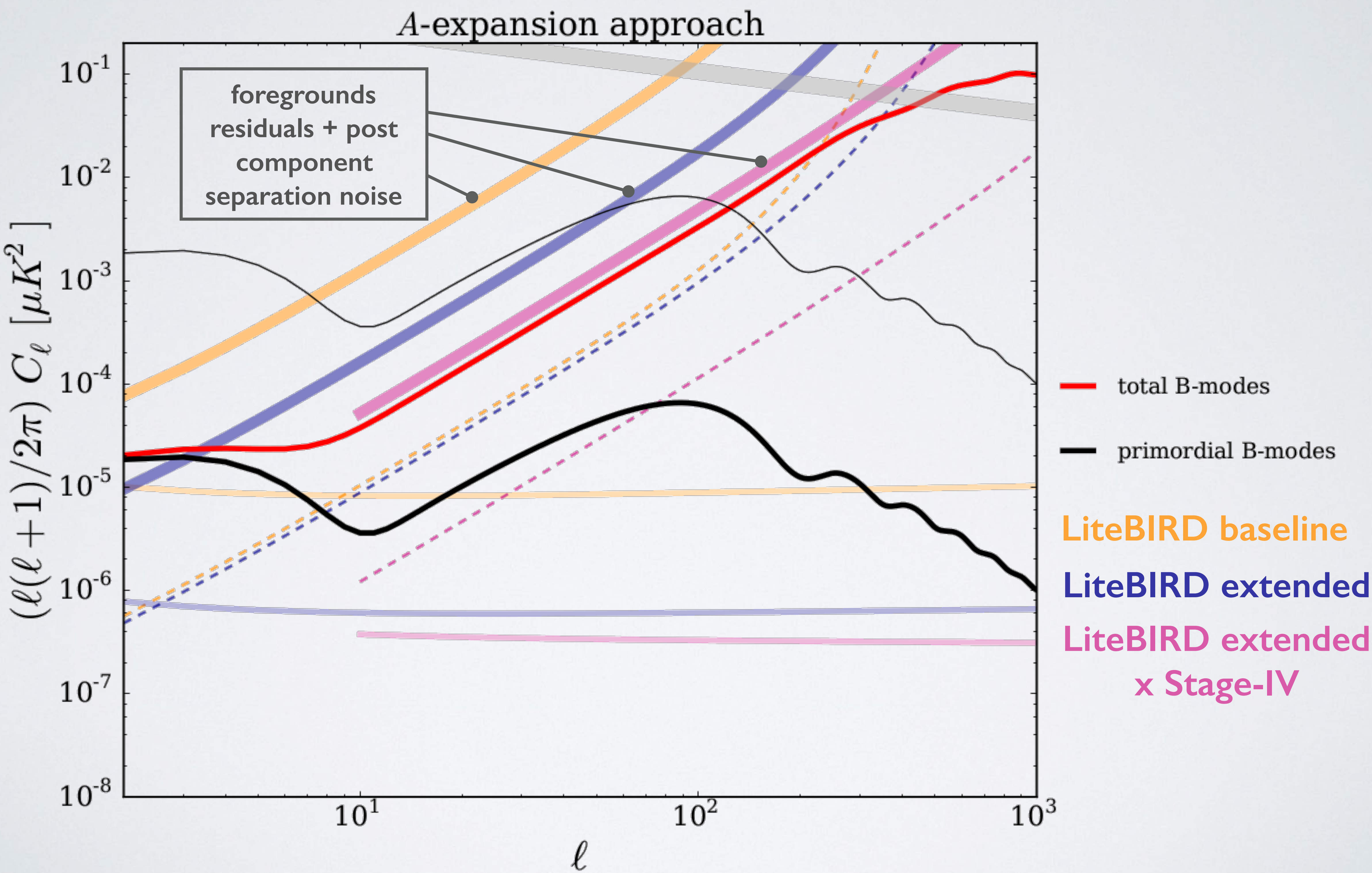
complexity

3 - Conclusions and discussion

"A-expansion approach" [e.g. Stolyarov et al (2005)]

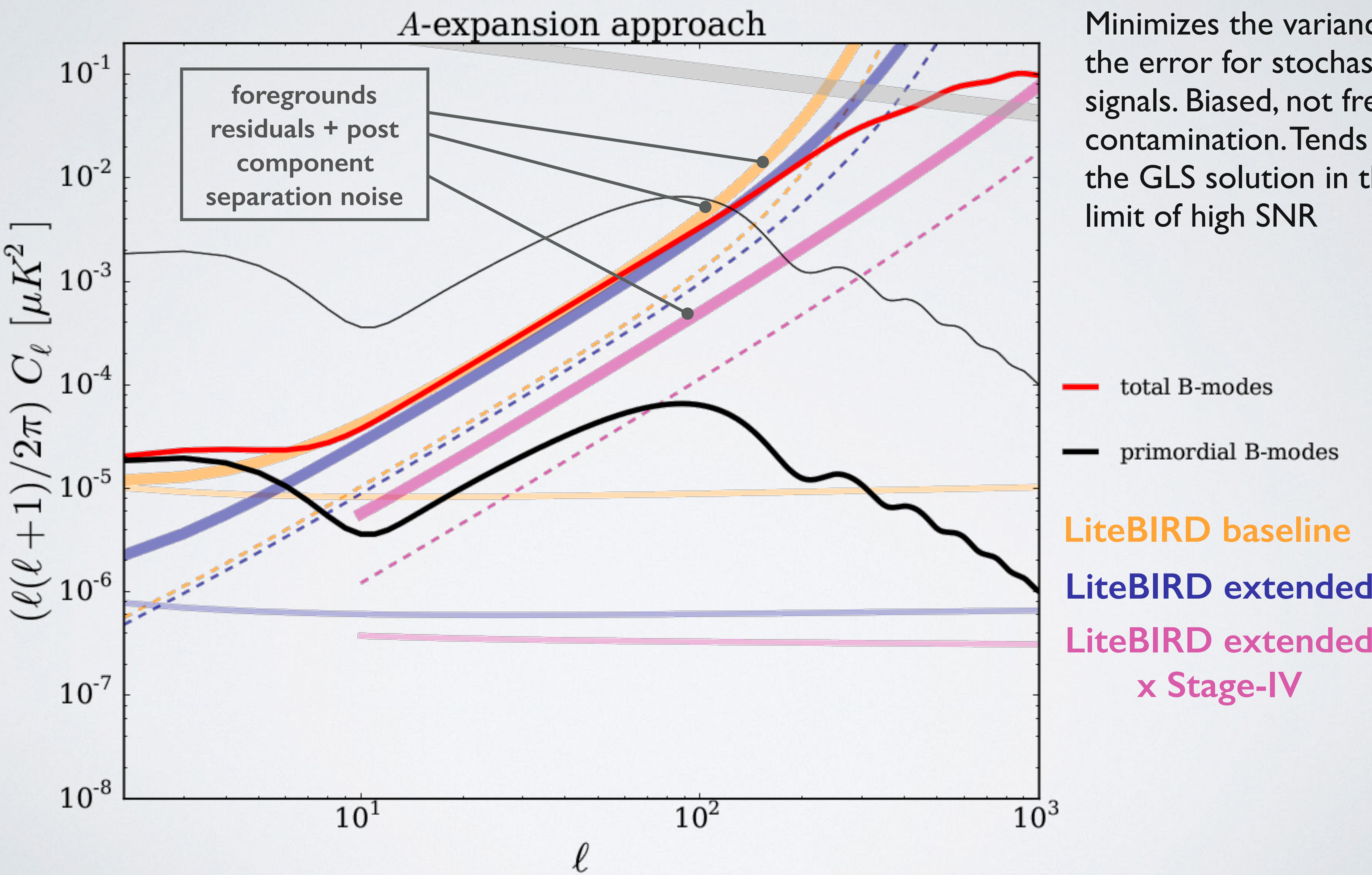
$$\mathbf{A}(\beta) \approx \mathbf{A}(\hat{\beta}) + \delta\beta(p)\,\frac{\partial \mathbf{A}}{\partial \beta}\bigg|_{\hat{\beta}} + \mathcal{O}\left(\delta\beta(p)^2\right) \longrightarrow \mathbf{A} = \left[\mathbf{A}_{\rm cmb}, \mathbf{A}_{\rm dust}, \frac{\partial \mathbf{A}_{\rm dust}}{\partial \beta_d}, \mathbf{A}_{\rm sync}, \frac{\partial \mathbf{A}_{\rm sync}}{\partial \beta_s}\right]$$



"A-expansion approach"

[e.g., Tegmark et al (1996), Bouchet et al (1999)]

$$\left(\mathbf{A}^T\mathbf{N}^{-1}\mathbf{A}\right) \xrightarrow[\text{filtering}]{\text{Wiener}} \left(\mathbf{A}^T\mathbf{N}^{-1}\mathbf{A}+\mathbf{S}^{-1}\right)$$



Minimizes the variance of the error for stochastic signals. Biased, not free of contamination. Tends to the GLS solution in the limit of high SNR

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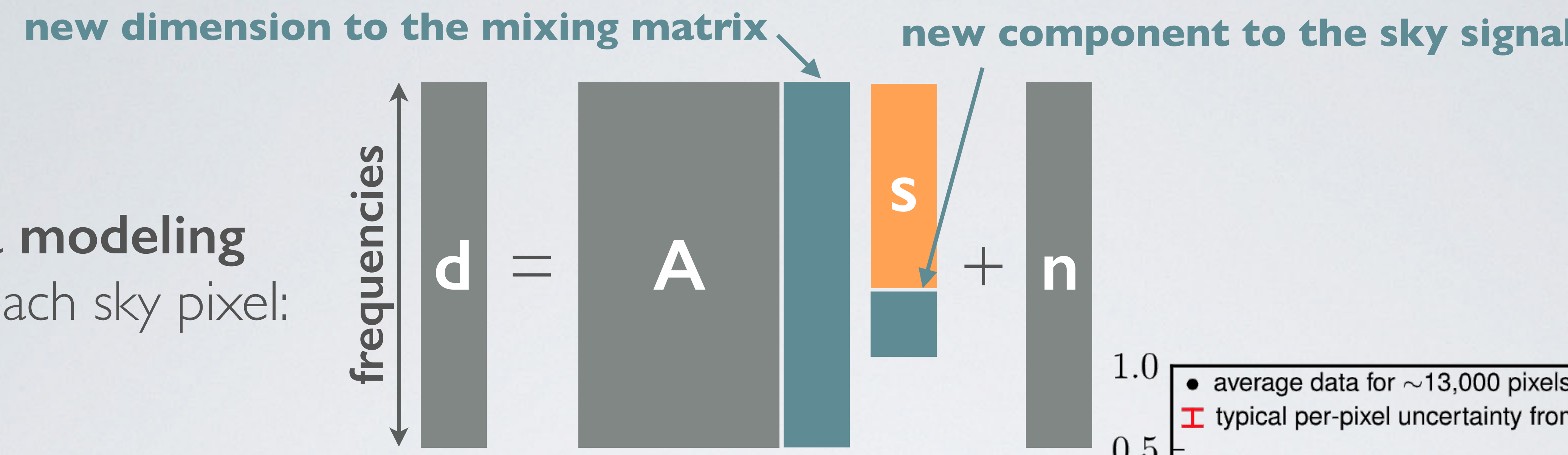
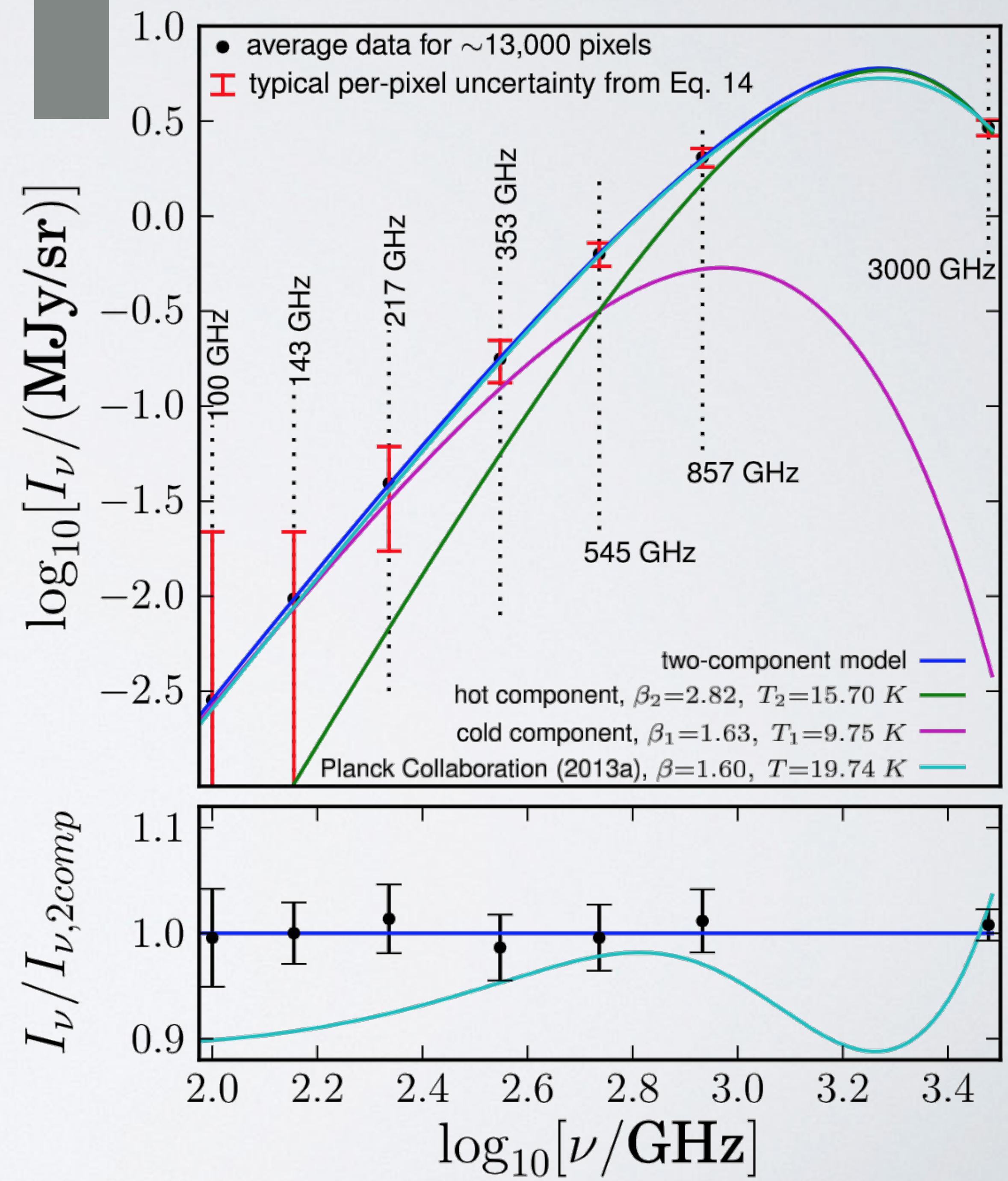
3 - Conclusions and discussion

Two dust components, with varying correlation

data modeling
for each sky pixel:



[M. Meisner and D. P. Finkbeiner (2014)] →



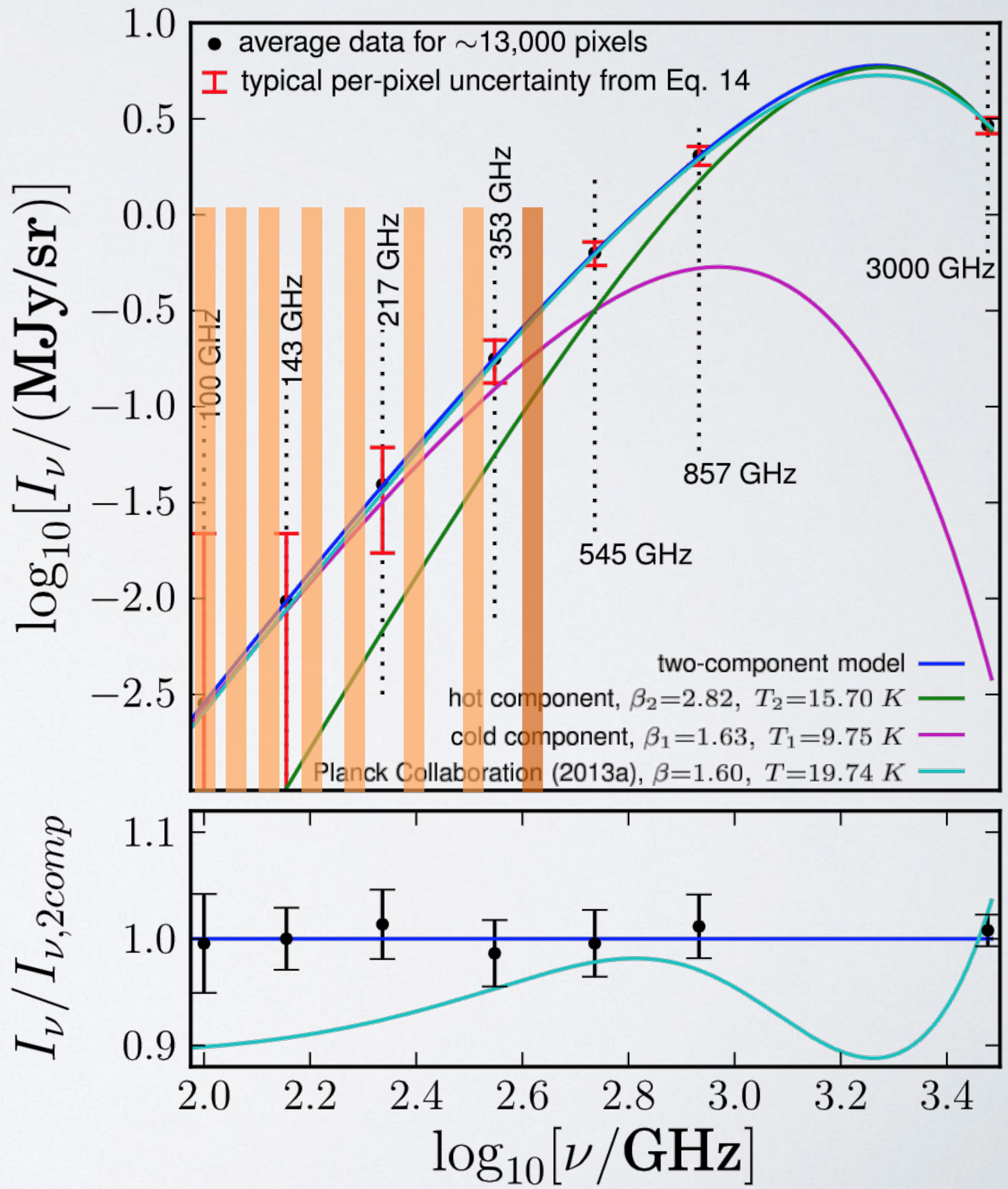
hot dust T_d=15.70K, β_d=2.82
cold dust T_d=9.75K, β_d=1.63
(single dust T_d=19.74K, β_d=1.60)

correlation between 2 dusts ∈ [0.0,1.0]

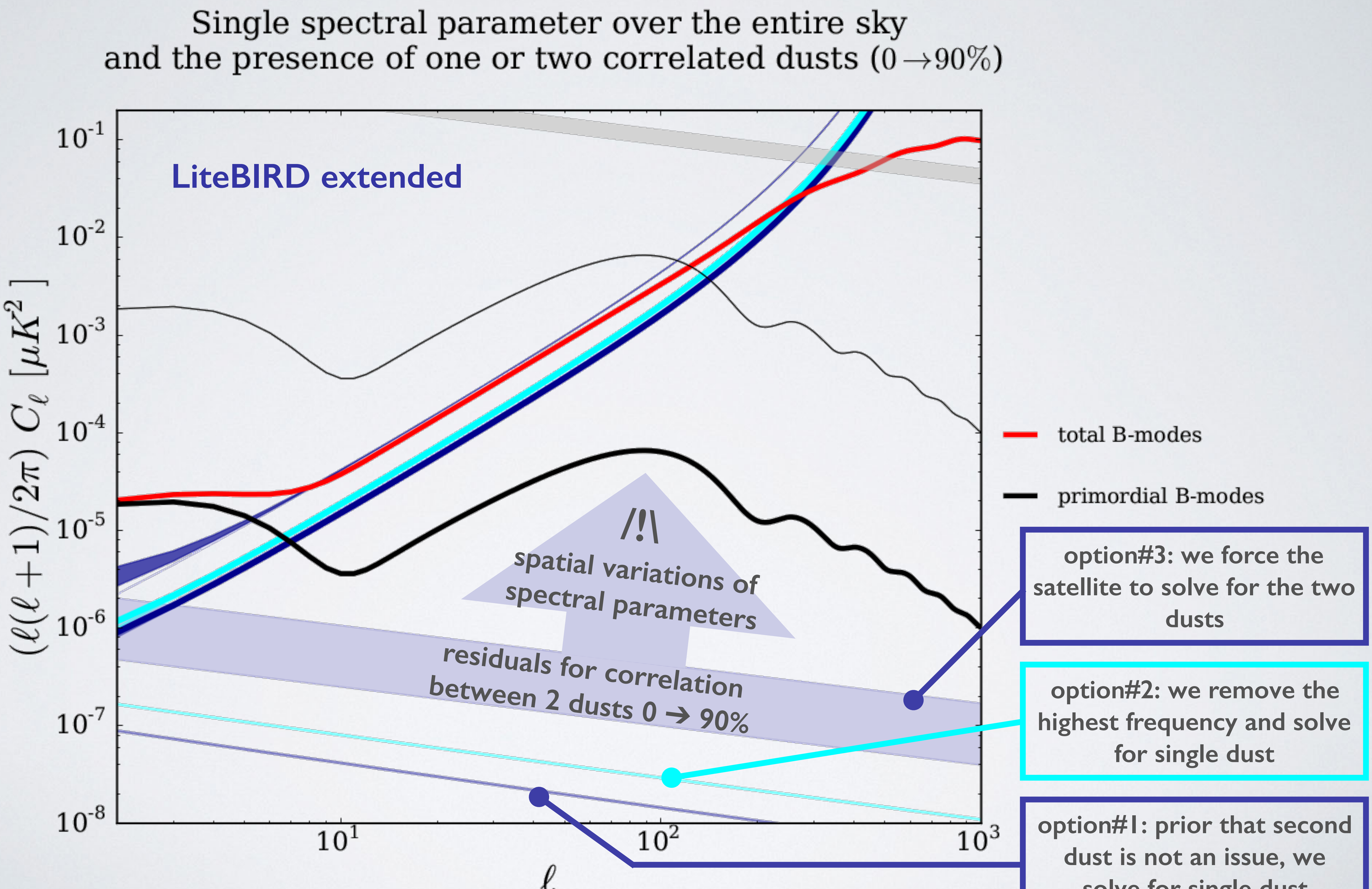
Two dust components, with varying correlation

We have several options:

- prior that second dust is not important: we consider a single dust
- we remove the highest frequency (400GHz) and treat a single dust with the remaining channels
- we realize that we have no other choice than solving for two dusts



Two dust components, with varying correlation



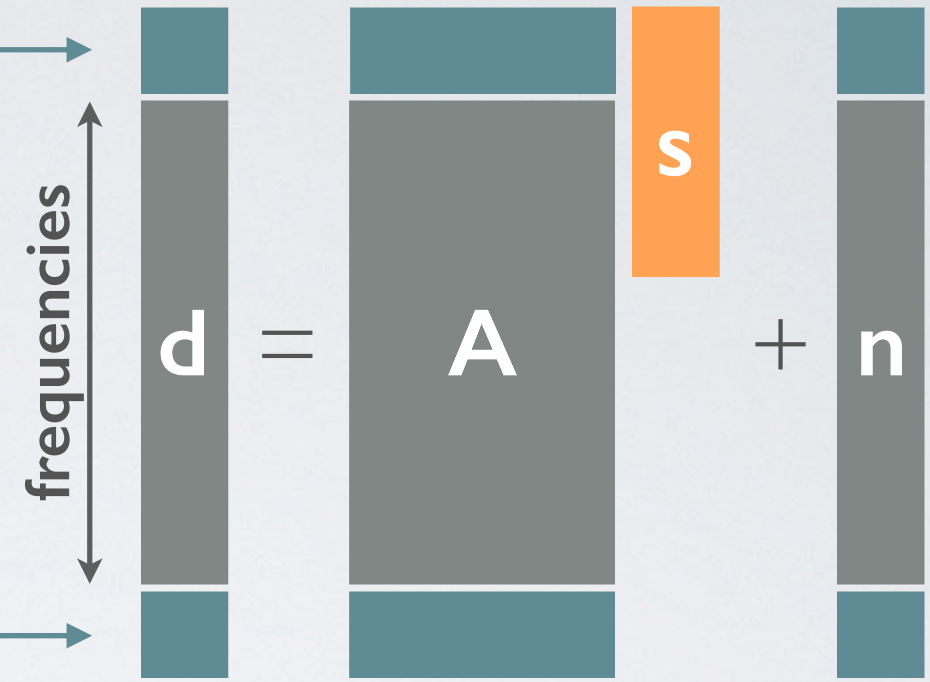
external tracers for extra components?

$$d_i(p) = A_{ij}\, s_j(p) + n_i(p)$$

e.g. C-BASS, Quijote

data modeling
for each sky pixel:

e.g. EBEX10K, Plan B



C-BASS
http://www.astro.caltech.edu/cbass/posters/Dickinson_CBASS_Okinawa_June2013.pdf

Quijote
arxiv: 1401.4690

dust monitor
e.g. 1.5mK.arcmin @ 600GHz

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Calibration errors

$$d_i(p) \;=\; \Omega_{ij}\; A_{jk}\; s_k \;+\; n_i$$

data modeling
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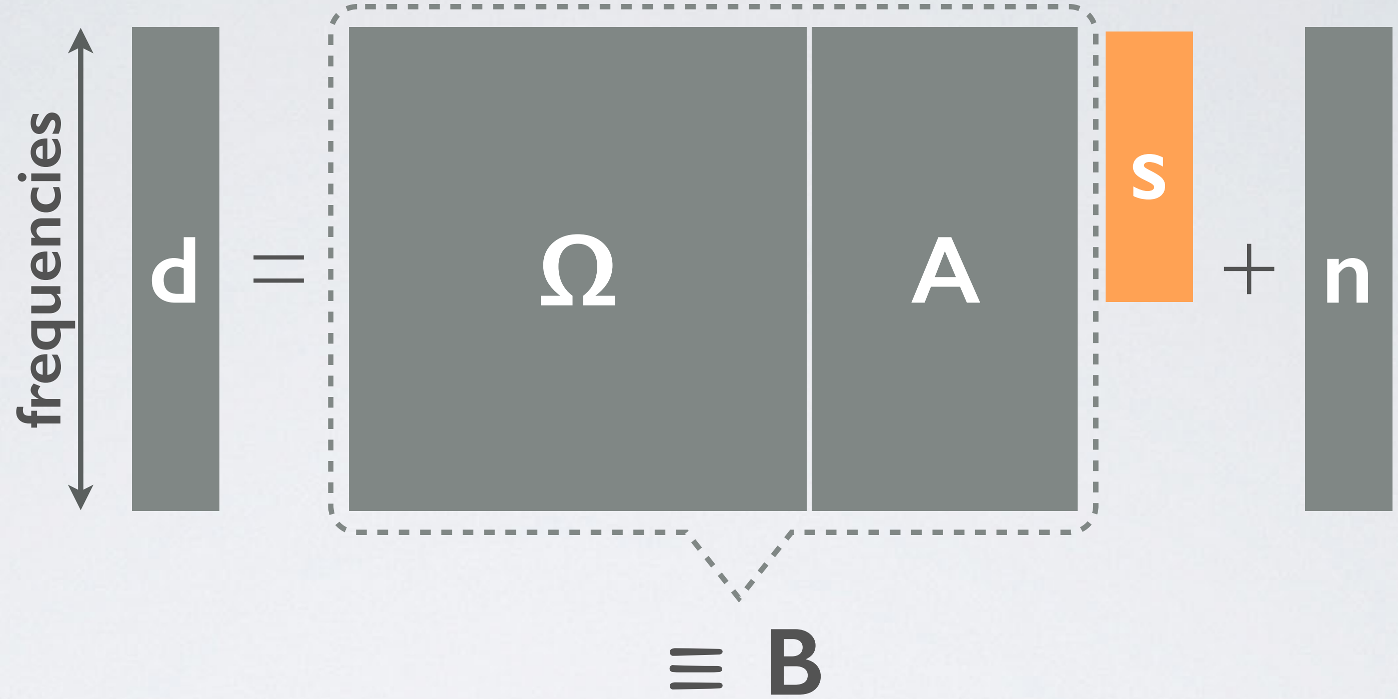
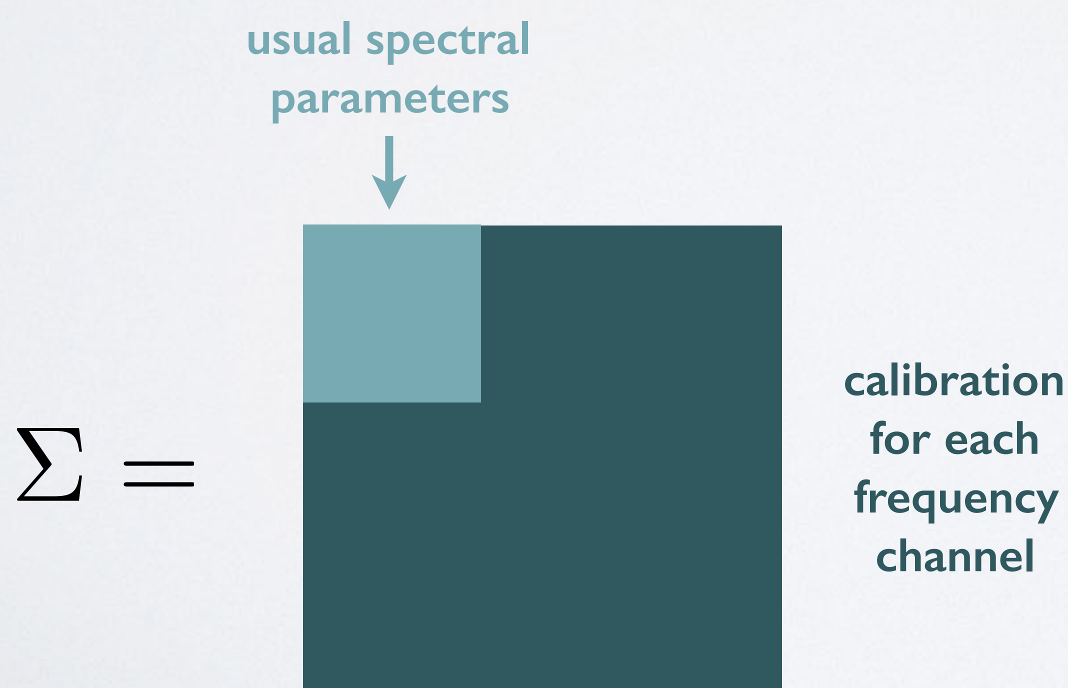


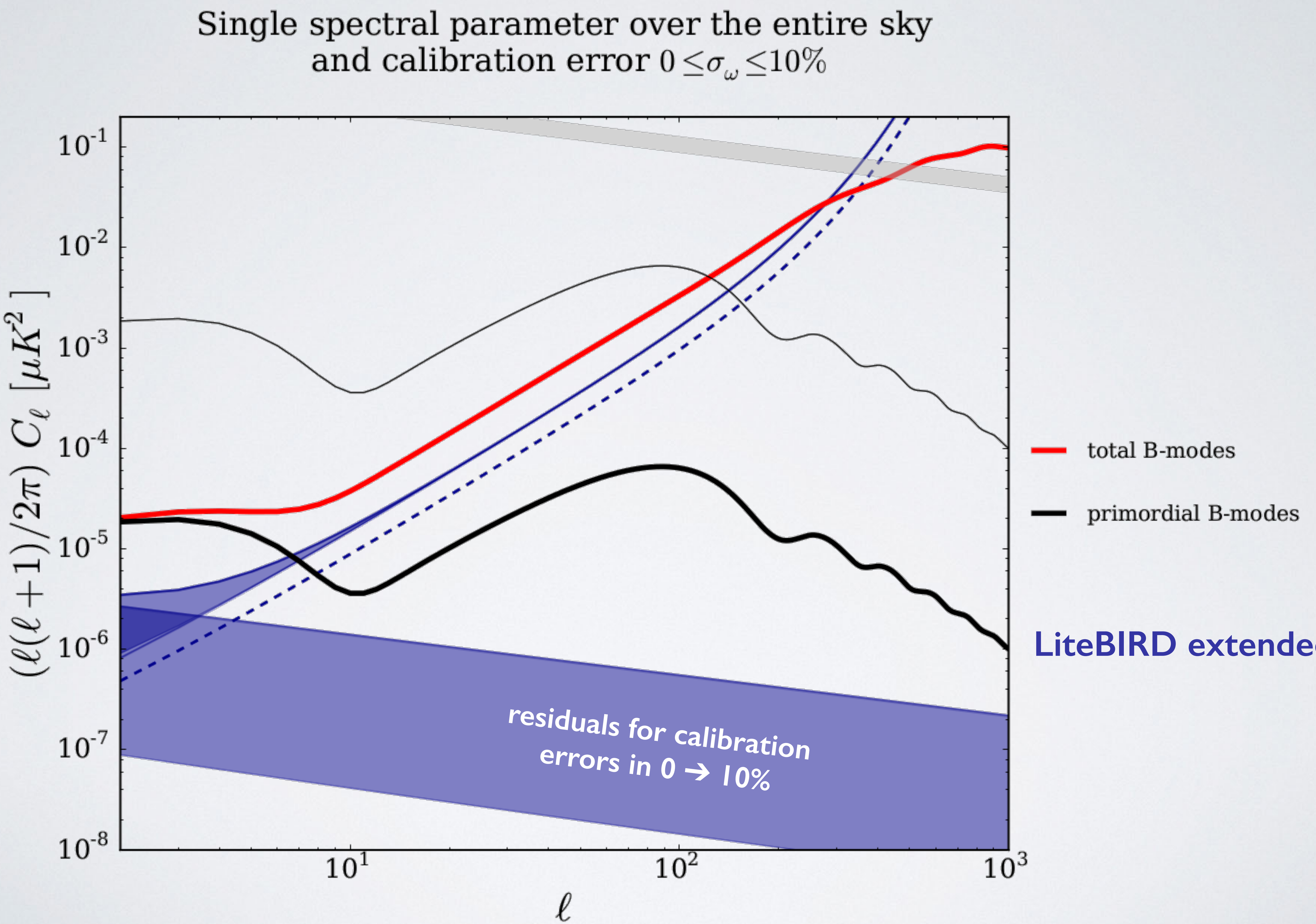
[Stompor et al (2009) + JE and Stompor (2012)]

$$\boldsymbol{\Sigma}^{-1}_{ij} = n_{pix}\;\mathrm{tr}\left\{\left[\boldsymbol{B}^t_{,i}\boldsymbol{N}^{-1}\boldsymbol{B}\left(\boldsymbol{B}^t\boldsymbol{N}^{-1}\boldsymbol{B}\right)^{-1}\boldsymbol{B}^t\boldsymbol{N}^{-1}\boldsymbol{B}_{,j} - \boldsymbol{B}^t_{,i}\boldsymbol{N}^{-1}\boldsymbol{B}_{,j}\right]\hat{\boldsymbol{F}}\right\} + \left[(\boldsymbol{\omega}-\bar{\boldsymbol{\omega}})^t\,\boldsymbol{\Xi}^{-1}\,(\boldsymbol{\omega}-\bar{\boldsymbol{\omega}})\right]_{,ij}\Big|_{\hat{\gamma}}$$

$$diag(\frac{1}{\sigma_\omega^2})$$



Calibration errors



LiteBIRD extended

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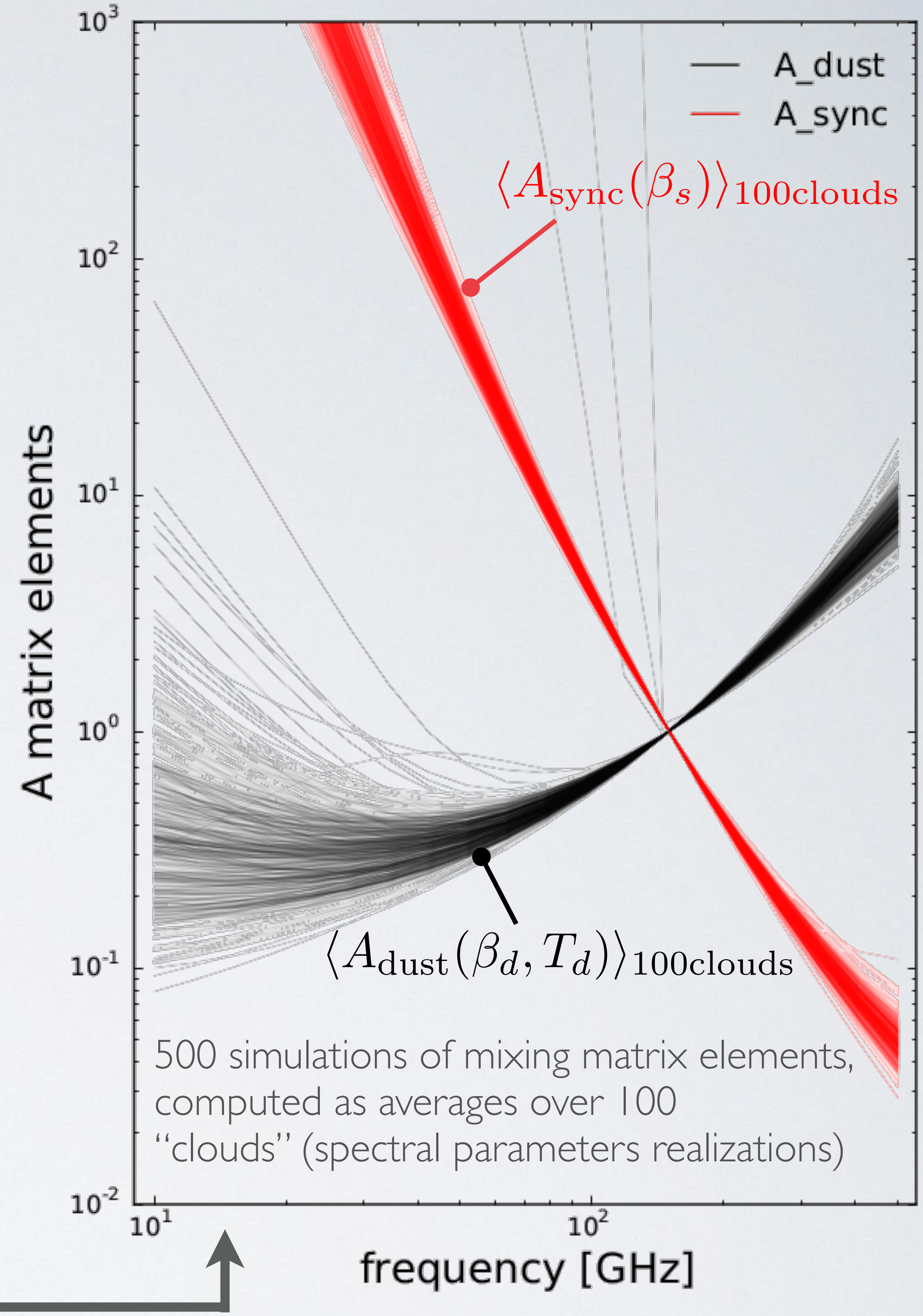
3 - Conclusions and discussion

Conclusions & discussion

- Broad frequency-coverage + balanced sensitivities leads to low noise boost and control of foregrounds residuals
- Spatial variation of spectral indices requires **sensitive foregrounds monitors**
  /!\ inter-calibration, band-mismatch, etc.
- cf. *Fantaye et al (2011)* for debiasing the estimation of r in the presence of foregrounds residuals
- More results about the consequence of component separation on delensing and science extraction in [*JE., S. Feeney et al (2015)*, software accessible at **turkey.lbl.gov**]
- Need for realistic sky simulations



- Under the parametric assumptions, I think the main challenge would be to derive an **effective modeling** vs **realistic modeling of foregrounds emissions** (cf. talks by E. Wilcots, F. Boulanger)
  ↳ effect of angular resolution on galactic structures?
- Need for a generalized formalism to incorporate systematics due to modeling assumptions [*Remazeilles et al (2015), Armitage-Caplan et al (2012), Stivoli et al (2010)*] and instrumental systematics [working with D. Poletti @ APC]