Foreground removal: pixel domain

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In collaboration with
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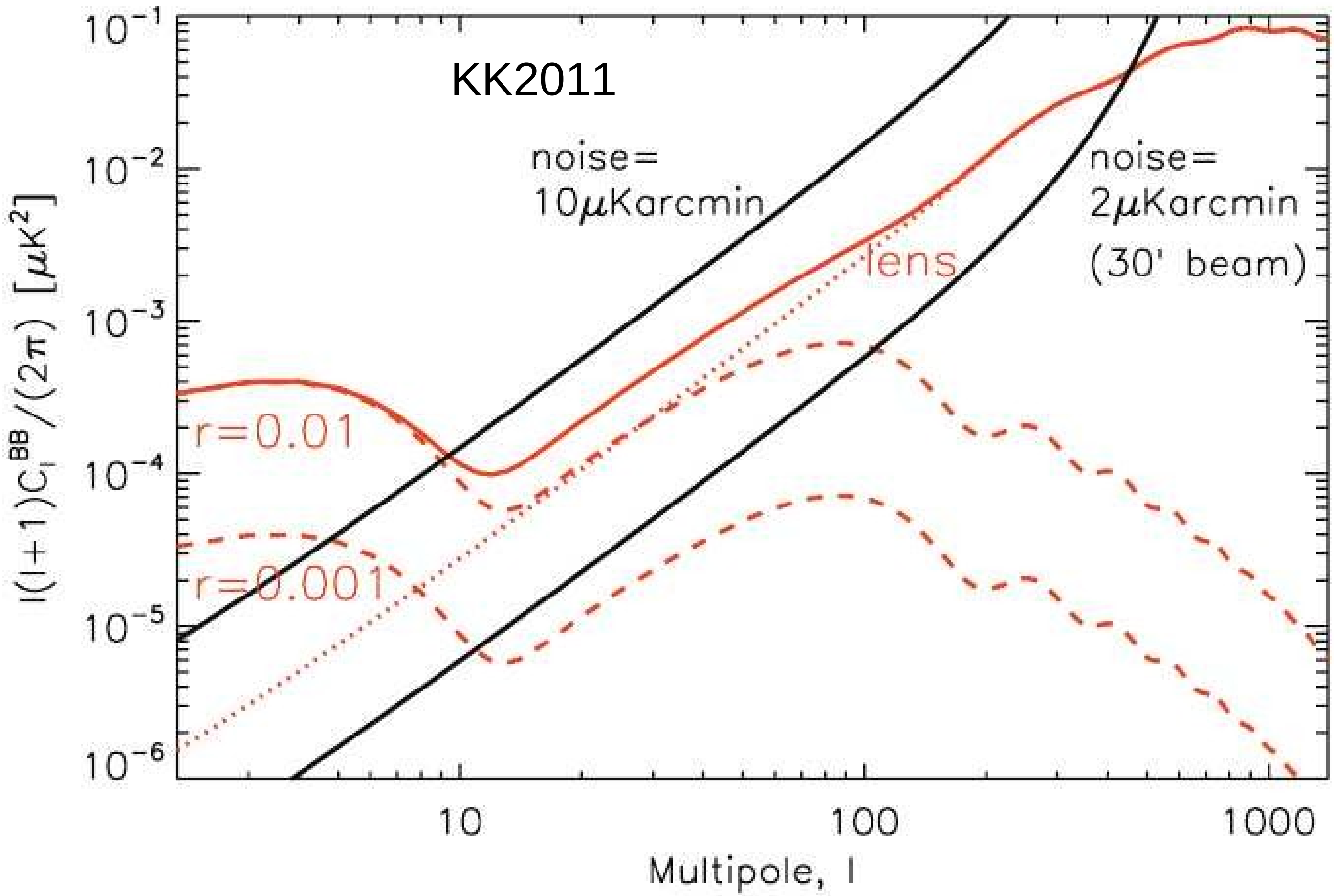
S. Ishino, T. Matsumura, and the LiteBIRD WG

overview

- **Introduction**
  - Foregrounds & B-mode characteristics
  - message from *Katayama & Komatsu '11*
- **A simple foreground cleaning method in pixel space**
  - Algorithm for the
    'delta map' and 'double delta map'
- **Some results**
- **Conclusion**

Target: B-mode



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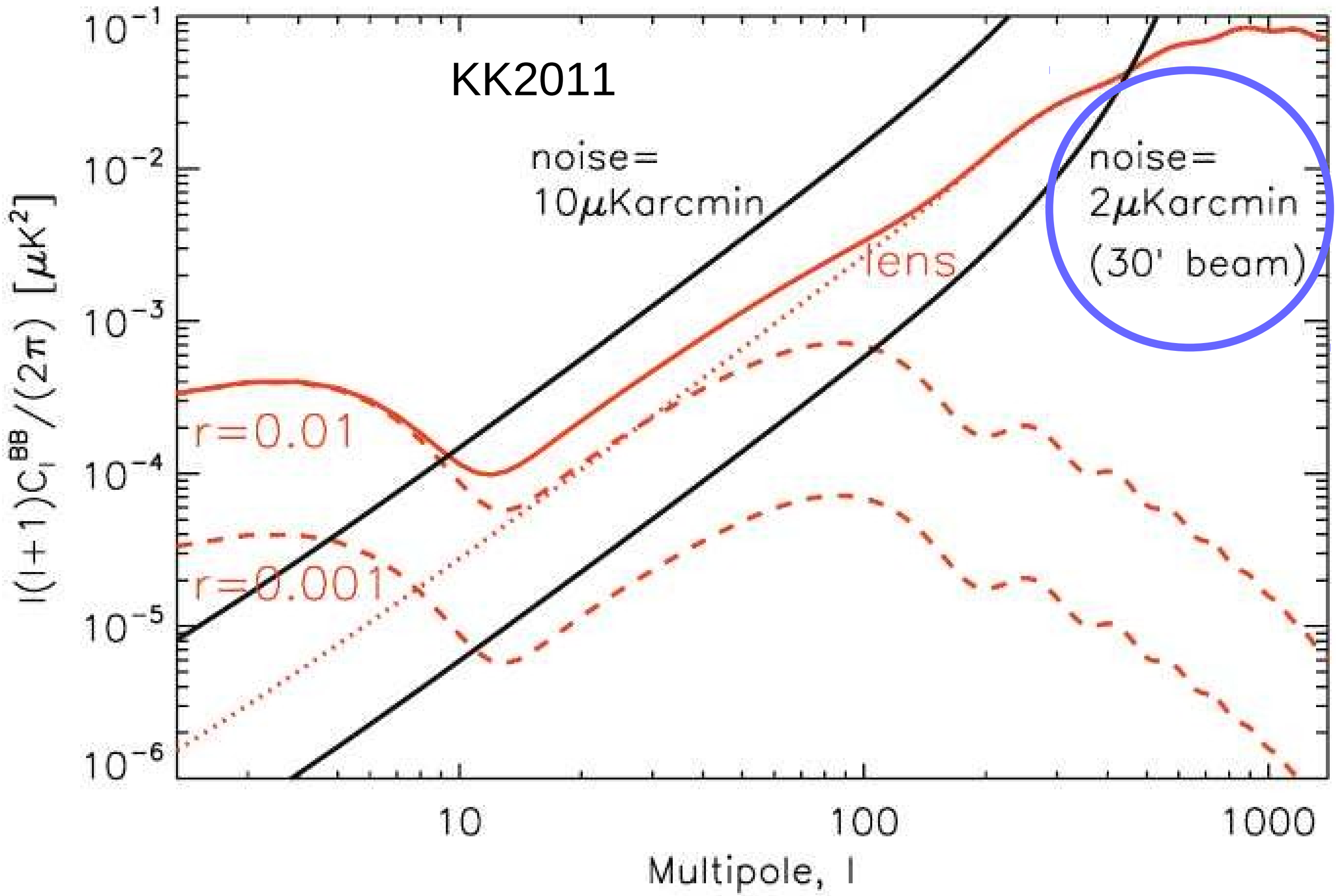

CMB lensing is the limiting noise if instrumental noise is less than $5\mu\mathrm{K\ arcmin}$

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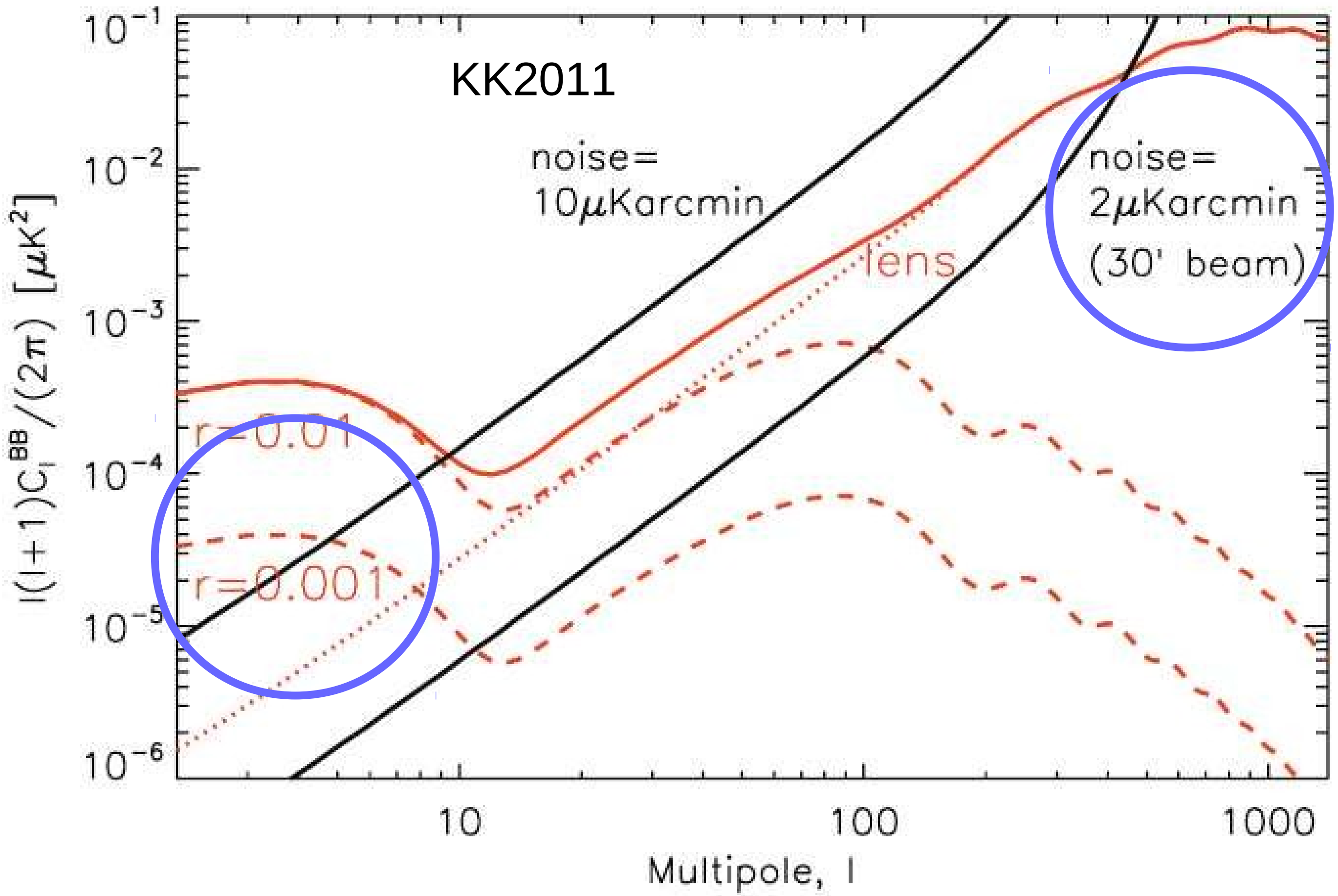
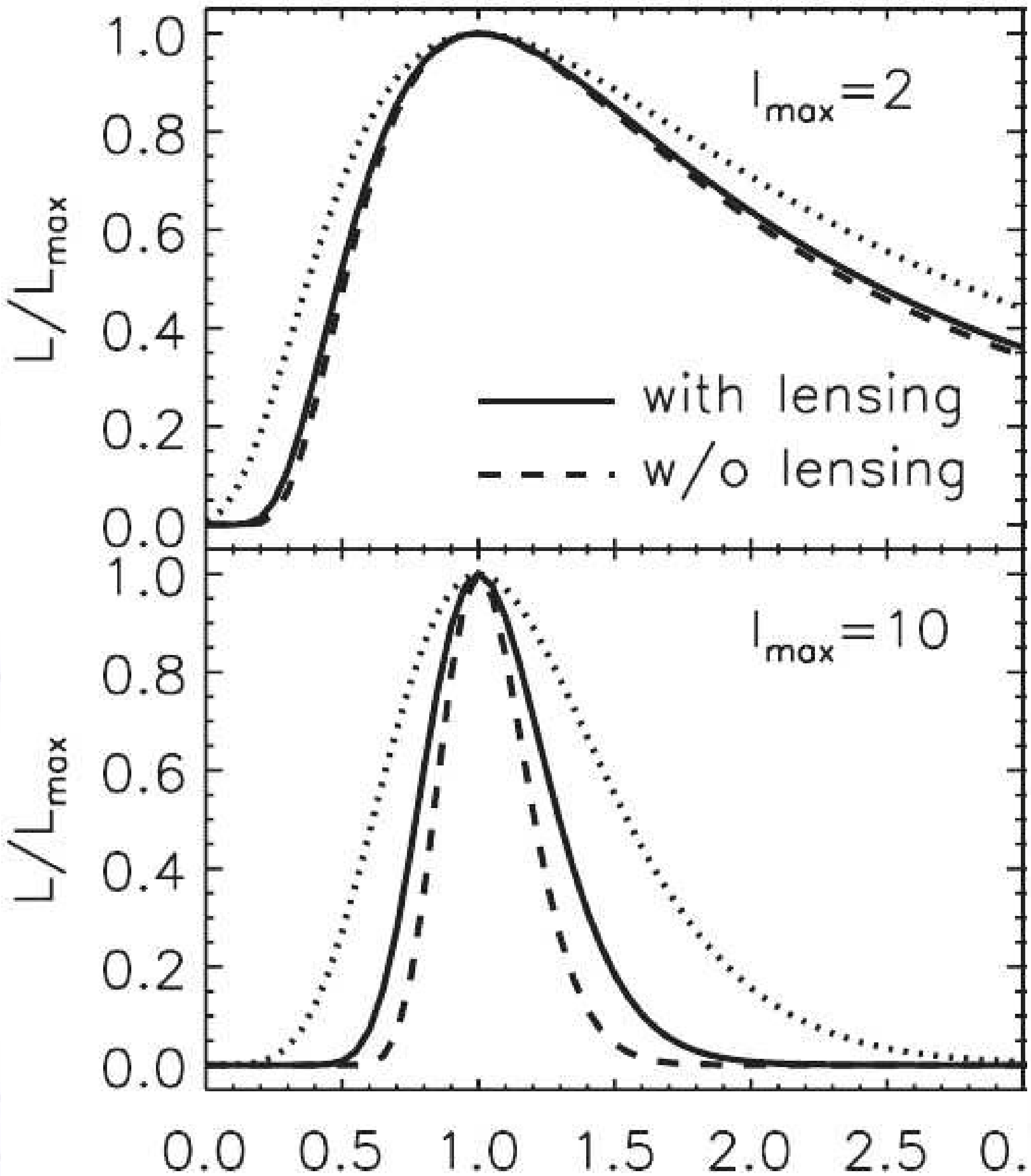
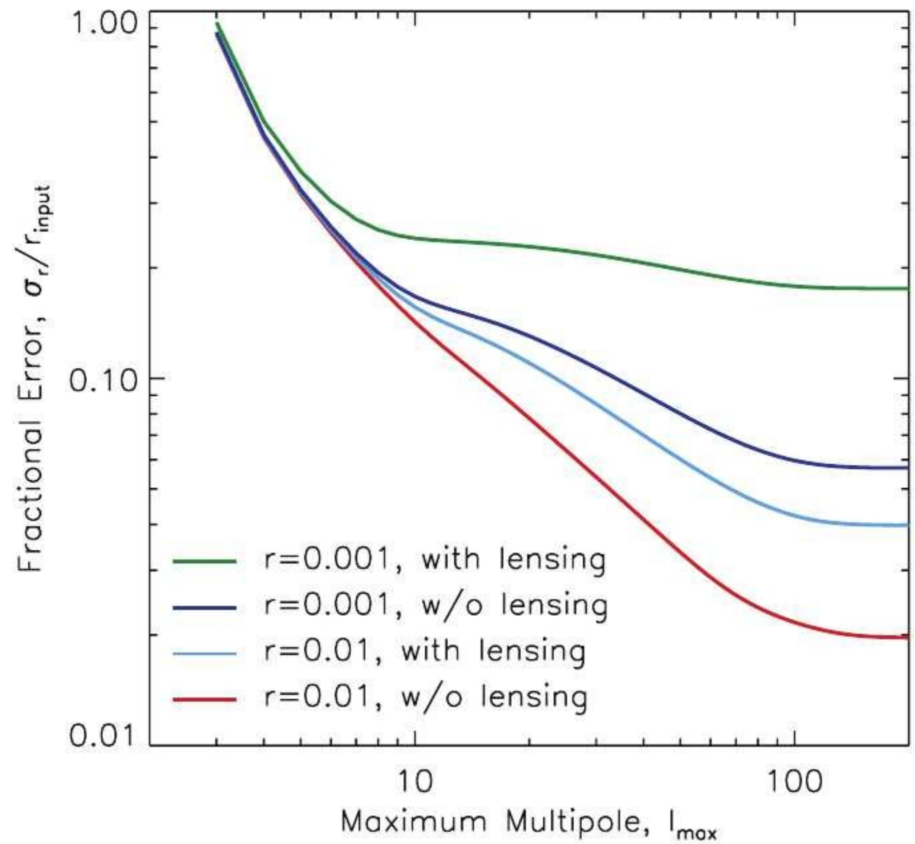


CMB lensing is the limiting noise if instrumental noise is less than $5\mu\mathrm{K}$ arcmin

r=0.001 B-mode signal is visible at $\ell \lesssim 10$





Message from KK2011 for r=0.001 GWs

A single multipole l=2 is sufficient to detect GWs (if foreground free)

If the lensing is the limiting factor, information l<10 ↔ Nside=4

Foregrounds

K.Ichiki, PTEP, '14

Table 1: Summary of Galactic emissions

component	spectrum	polarization
Synchrotron	power law with $\beta \approx -3$; possible curvature by ageing of CRs	$\approx 20\%$, become larger at higher latitudes up to $\approx 40\%$
Spinning dust	$\beta \approx -2.5$ around 20-30 GHz, falls rapidly above 60 GHz [14, 58]	less than 0.5% for $\gtrsim 30$ GHz
Free-free	power-law with $\beta \approx -2.1$	unpolarized; with an upper limit $\lesssim 3.4\%$ [58]
Thermal dust	approximately two temperature components with $\beta_{(1,2)} = (1.67, 2.70)$ [79]	≈ 3.6 % [47]

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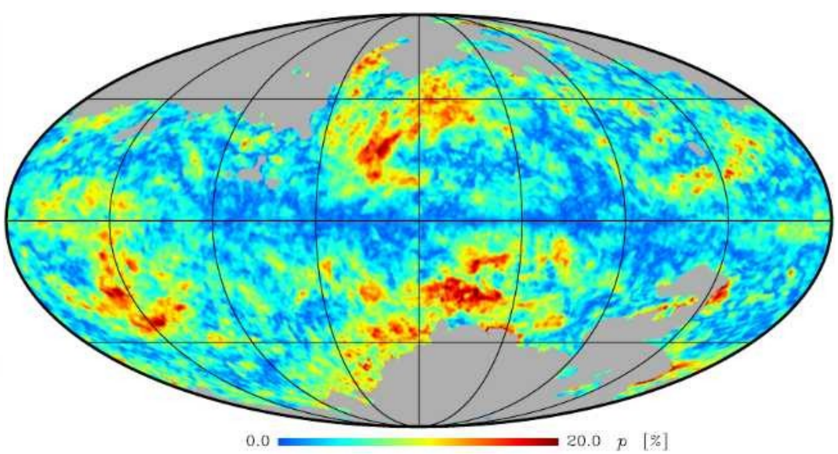

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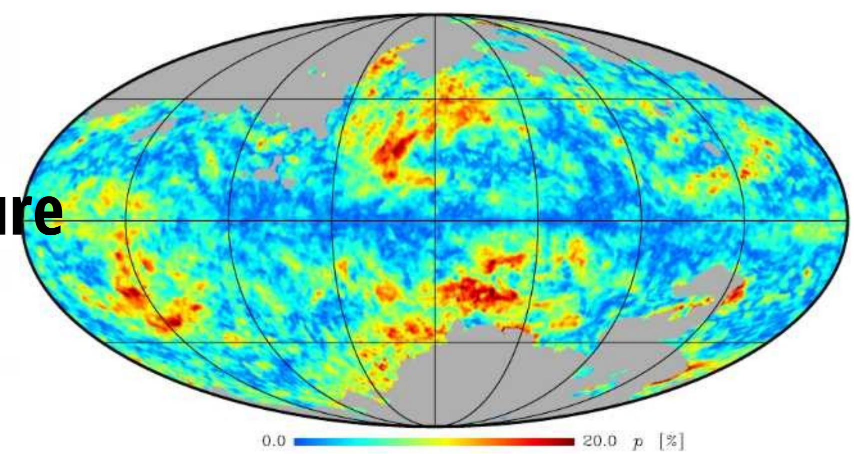
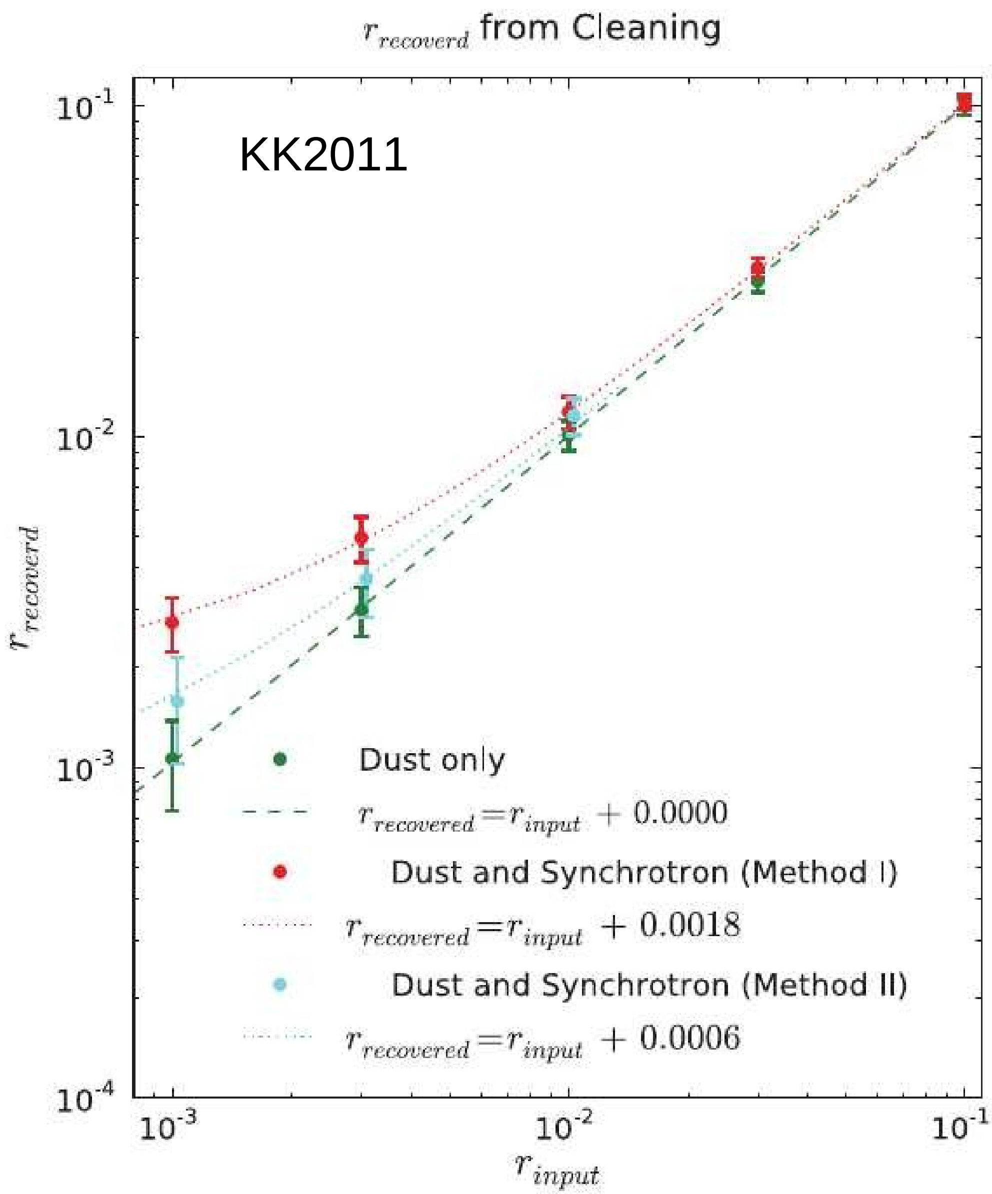
Key words

Synch ... lower ν, spatial variation, curvature

Dust ... higher ν, large pol. degree





Message from KK2011 for the simple cleaning method:

Directional variation of synchrotron $\beta_{\rm sync}(\hat{n})$ will cause a significant bias in the estimate of "r".

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A simple foreground cleaning method (e.g., KK2011)

- **high angular resolution not necessary**
  - use the full likelihood in pixel space

$$\mathcal{L}(r,s,\bar{\beta}) \propto \frac{\exp\left[-\frac{1}{2}[Q,U]^T \boldsymbol{C}^{-1}[Q,U]\right]}{\sqrt{|\boldsymbol{C}(r,s,\bar{\beta})|}}$$

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- **work directly with Q,U stokes parameters in pixel space**
  - simple implementation of foreground masks
    (see, page et al., WMAP3 polarization paper)
  - free from non-local E-B mode mixing

Algorithm (1)

- **Sky model (ignoring dust for the moment):**

$$Q(\nu,\hat{n}) = \mathrm{CMB}(\hat{n}) + g_\nu \left(\frac{\nu}{\nu_*}\right)^{\beta(\hat{n})} Q^{\mathrm{sync}}(\nu_*,\hat{n}) + \mathrm{noise}$$

Taylor expansion $\beta(\hat{n}) \equiv \bar{\beta} + \delta\beta(\hat{n})$

$$Q(\nu,\hat{n}) \approx \mathrm{CMB}(\hat{n}) + g_\nu \left(\frac{\nu}{\nu_*}\right)^{\bar{\beta}} \left[1 + \ln\left(\frac{\nu}{\nu_*}\right)\delta\beta(\hat{n})\right] Q^{\mathrm{sync}}(\nu_*,\hat{n}) + \mathrm{noise}$$

Let us estimate $\delta\beta(\hat{n})Q^{\mathrm{sync}}(\nu_*,\hat{n})$ **and subtract it !**

Algorithm (2)

- **Use two bands (60GHz, 78GHz) to calculate a '*delta map*', and add it to the target sky (100GHz)**

$$Q(\nu_0) + \boxed{\alpha_1 Q(\nu_1) + \alpha_2 Q(\nu_2)} = (1 + \alpha_1 + \alpha_2)\mathrm{CMB}$$

$$+ f(\alpha_1, \alpha_2, \bar{\beta}) Q^{\mathrm{synch}}(\nu_*)$$

$$+ g(\alpha_1, \alpha_2, \bar{\beta}) \delta\beta Q^{\mathrm{synch}}(\nu_*)$$

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- Demand that $f = 0$, $g = 0$ and solve α_1, α_2 as a function of $\bar{\beta}$

Algorithm (3)

- **Demand that $f = 0, \;\; g = 0$, and solve $\alpha_1, \;\; \alpha_2$ as a function of $\bar{\beta}$**

$$\alpha_1 = -\frac{g_\nu}{g_{\nu_1}} \left(\frac{\nu_0}{\nu_1} \right)^{\bar{\beta}} \frac{\ln\left(\frac{\nu_0}{\nu_2}\right)}{\ln\left(\frac{\nu_1}{\nu_2}\right)}$$

$$\alpha_2 = -\frac{g_\nu}{g_{\nu_2}} \left(\frac{\nu_0}{\nu_2} \right)^{\bar{\beta}} \frac{\ln\left(\frac{\nu_0}{\nu_1}\right)}{\ln\left(\frac{\nu_2}{\nu_1}\right)}$$

$(\hat{n}, \nu_*)$ **independent**

If β is running ...

- $\beta(\hat{n}) \to \beta(\hat{n}, \nu) = \beta(\hat{n}) + \gamma(\hat{n}) \ln(\nu/\nu_*)$

If β is running ...

- $\beta(\hat{n}) \to \beta(\hat{n},\nu) = \beta(\hat{n}) + \gamma(\hat{n}) \ln(\nu/\nu_*)$
- **The sky model is modified to:**

$$Q(\nu,\hat{n}) \approx \mathrm{CMB}(\hat{n})$$
$$+g_\nu \left(\frac{\nu}{\nu_*}\right)^{\bar{\beta}} \left[1 + \ln(\frac{\nu}{\nu_*})\delta\beta(\hat{n}) + (\ln(\frac{\nu}{\nu_*}))^2\gamma(\hat{n})\right] Q^{\mathrm{sync}}(\nu_*,\hat{n})$$
$$+\mathrm{noise}$$

Again estimate $\delta\beta(\hat{n})Q^{\mathrm{sync}}(\nu_*,\hat{n})$ **and** $\gamma(\hat{n})Q^{\mathrm{sync}}(\nu_*,\hat{n})$ **and subtract them ! (We now need four bands)**

- Use 3 bands to calculate a '**double delta map**'

$$Q(\nu_0) + \boxed{\alpha_1 Q(\nu_1) + \alpha_2 Q(\nu_2) + \alpha_3 Q(\nu_3)} = (1 + \alpha_1 + \alpha_2 + \alpha_3)\mathrm{CMB}$$

$$+ f(\alpha_1, \alpha_2, \alpha_3, \bar{\beta}) Q^{\mathrm{sync}}(\nu_*) \quad + g(\alpha_1, \alpha_2, \alpha_3, \bar{\beta}) \delta\beta Q^{\mathrm{sync}}(\nu_*)$$

$$+ h(\alpha_1, \alpha_2, \alpha_3, \bar{\beta}) \gamma Q^{\mathrm{sync}}(\nu_*)$$

demand $f = g = h = 0$

$$\alpha_1 = -\frac{g_\nu}{g_{\nu_1}} \left(\frac{\nu}{\nu_2}\right)^{\bar{\beta}} \frac{\ln(\nu_3/\nu\ )\ln(\nu_2/\nu\ )}{\ln(\nu_3/\nu_1)\ln(\nu_2/\nu_1)}$$

$$\alpha_2 = -\frac{g_\nu}{g_{\nu_2}} \left(\frac{\nu}{\nu_2}\right)^{\bar{\beta}} \frac{\ln(\nu_3/\nu\ )\ln(\nu_1/\nu\ )}{\ln(\nu_3/\nu_2)\ln(\nu_1/\nu_2)}$$

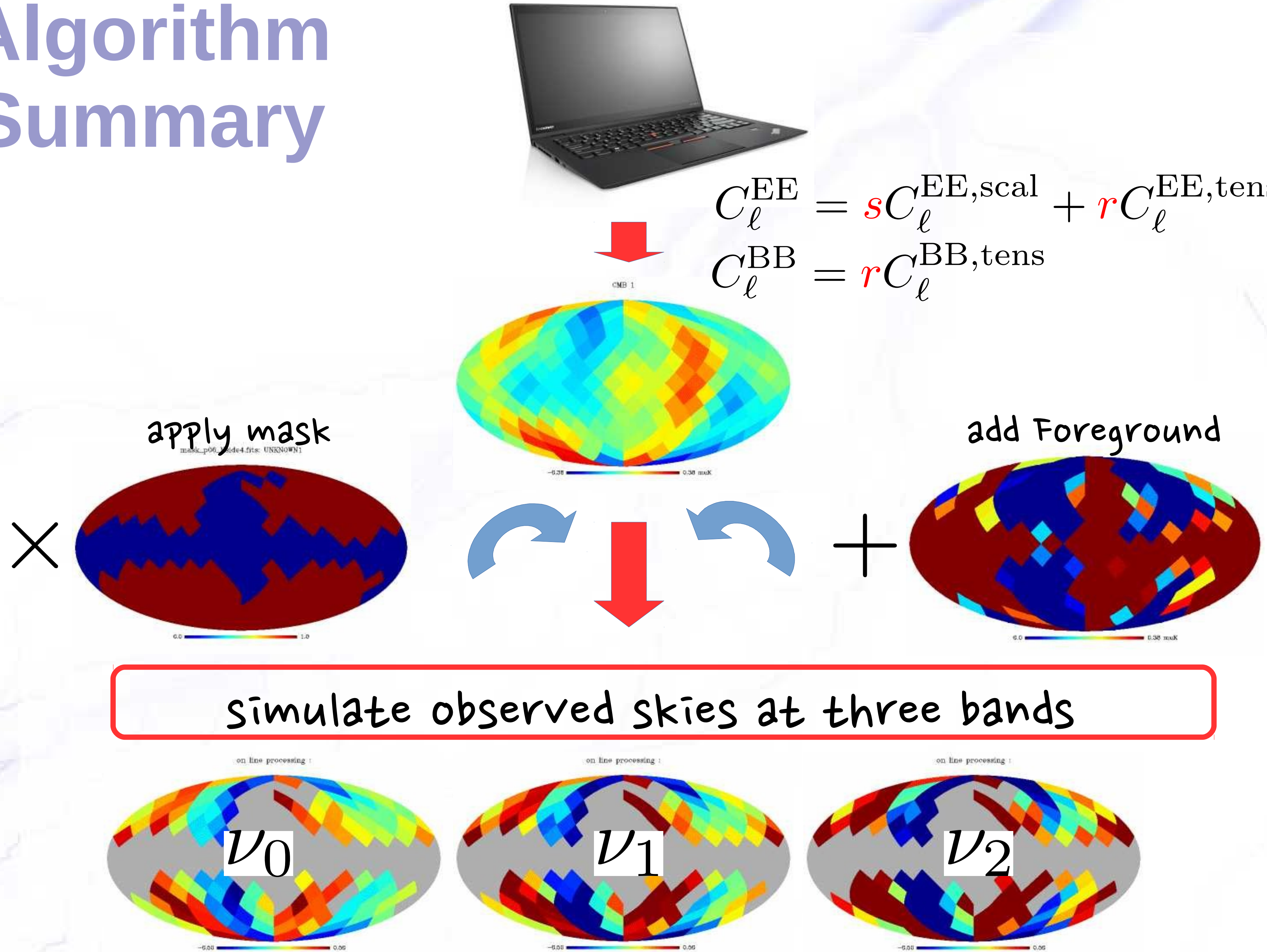
$$\alpha_3 = -\frac{g_\nu}{g_{\nu_3}} \left(\frac{\nu}{\nu_3}\right)^{\bar{\beta}} \frac{\ln(\nu_2/\nu\ )\ln(\nu_1/\nu\ )}{\ln(\nu_2/\nu_3)\ln(\nu_1/\nu_3)}$$

$(\hat{n}, \nu_*)$ **independent**

Algorithm Summary

$$C_\ell^{\rm EE} = sC_\ell^{\rm EE,scal} + rC_\ell^{\rm EE,tens}$$

$$C_\ell^{\rm BB} = rC_\ell^{\rm BB,tens}$$



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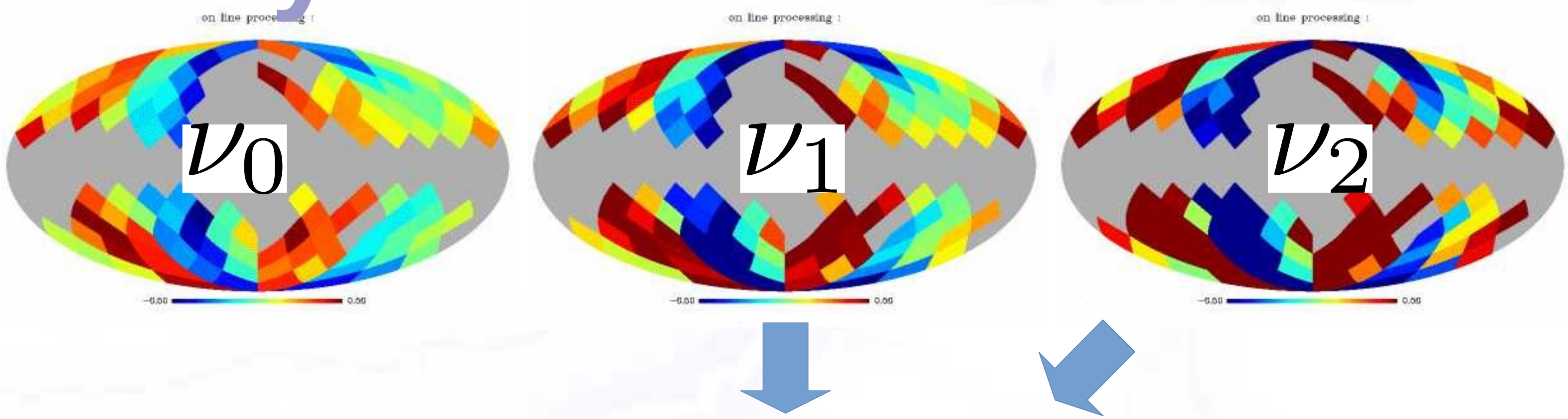
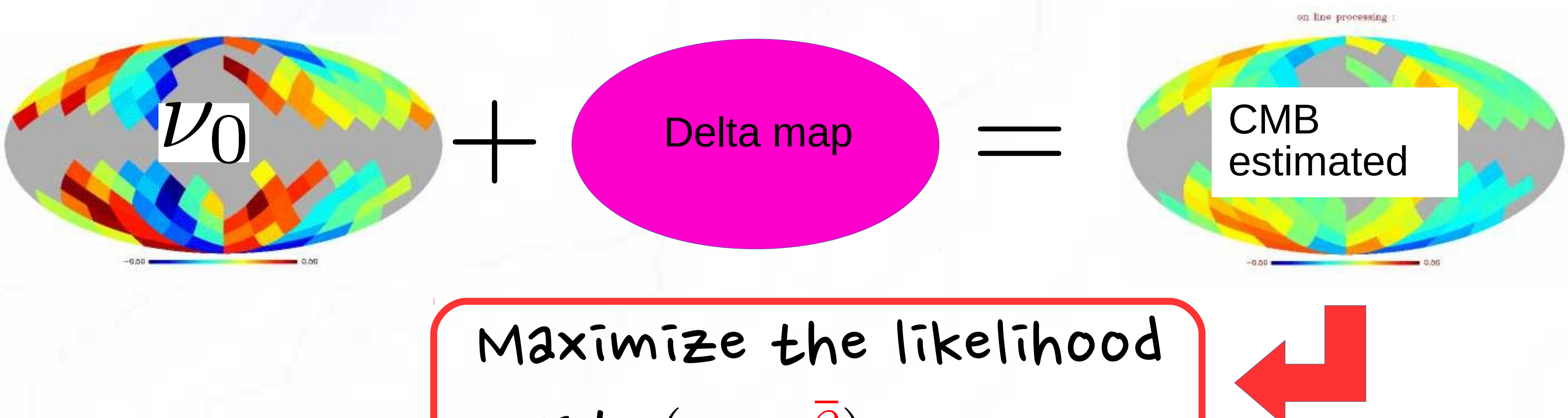


calculate a delta map with a free parameter $\bar{\beta}$,
And add it to the target sky



Maximize the likelihood
w.r.t $(r, s, \bar{\beta})$

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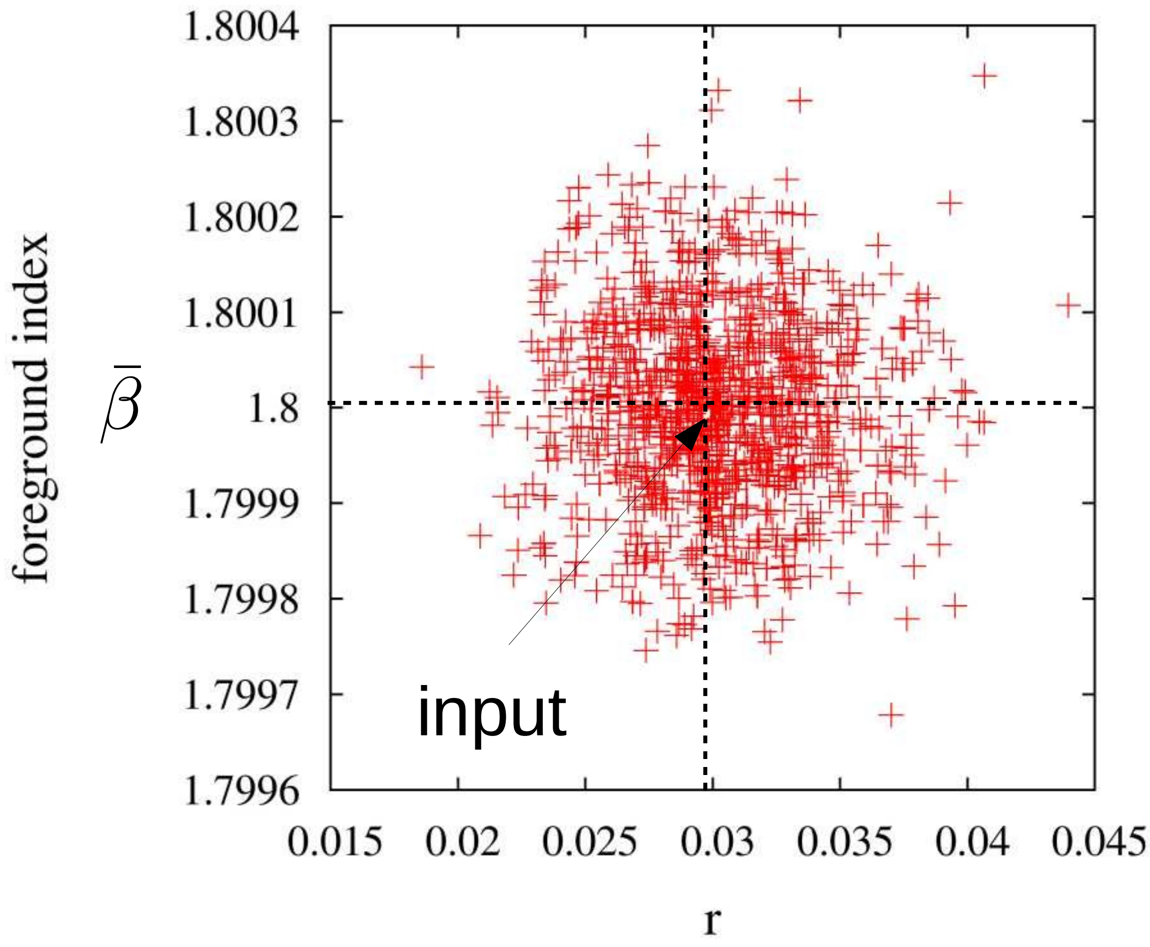
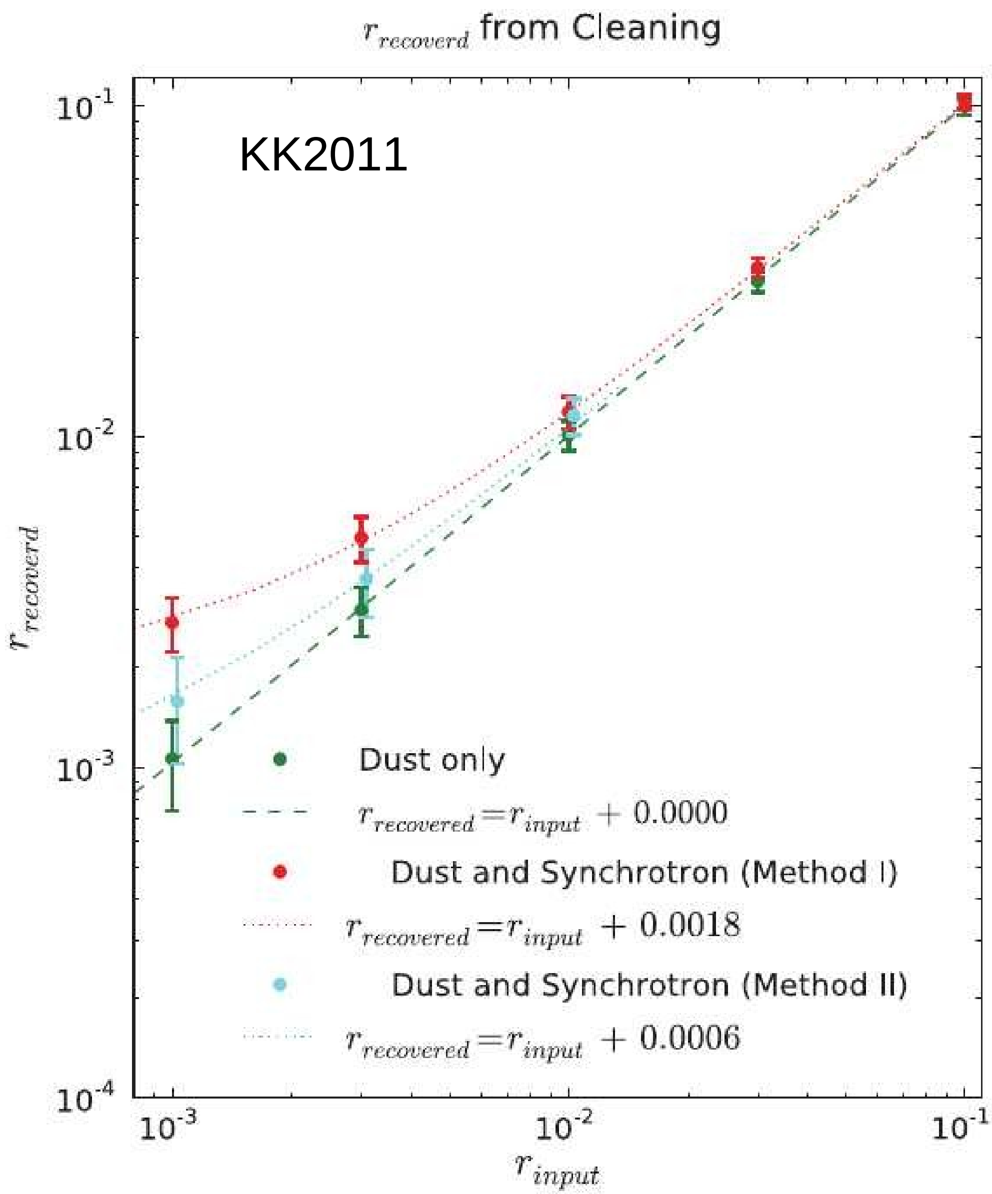
performance test



Foreground and CMB parameters are estimated simultaneously

Result



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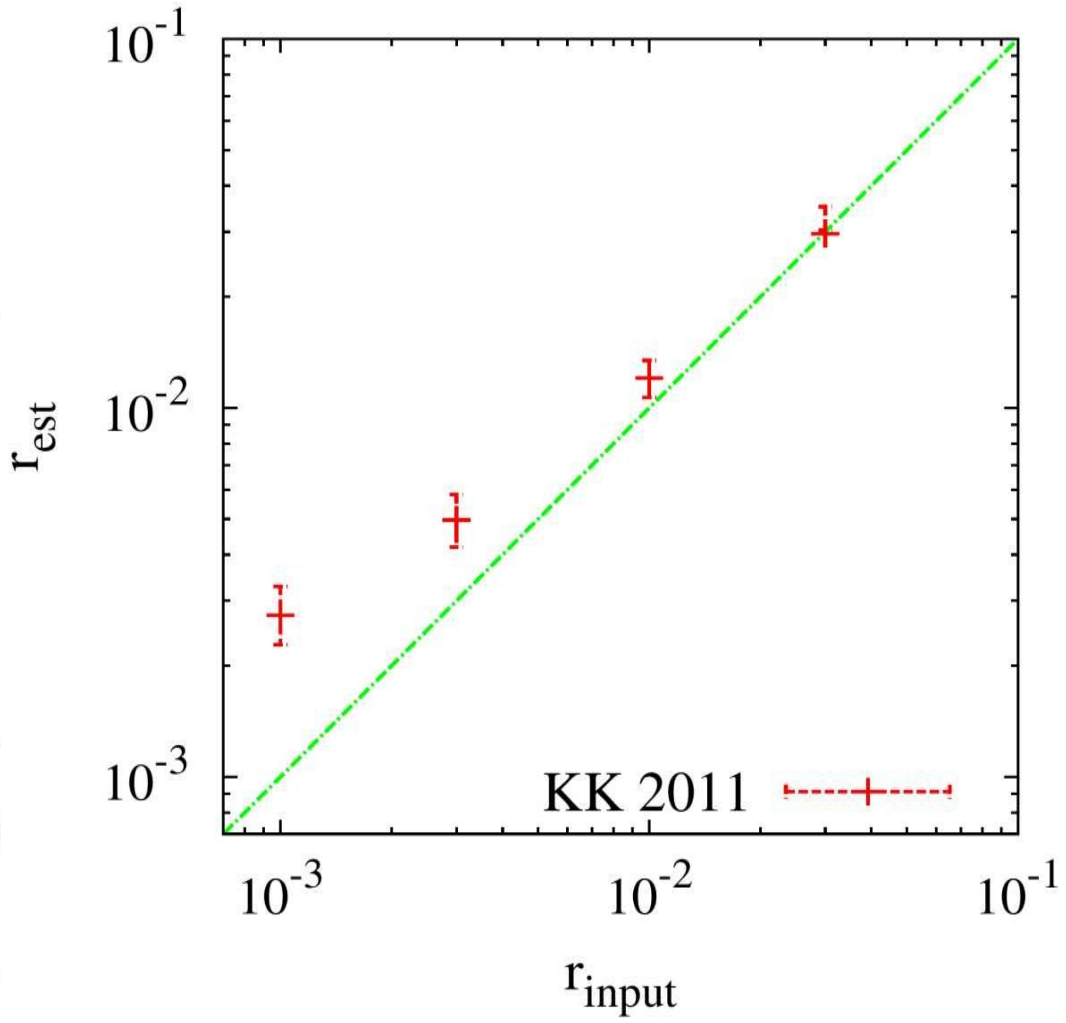

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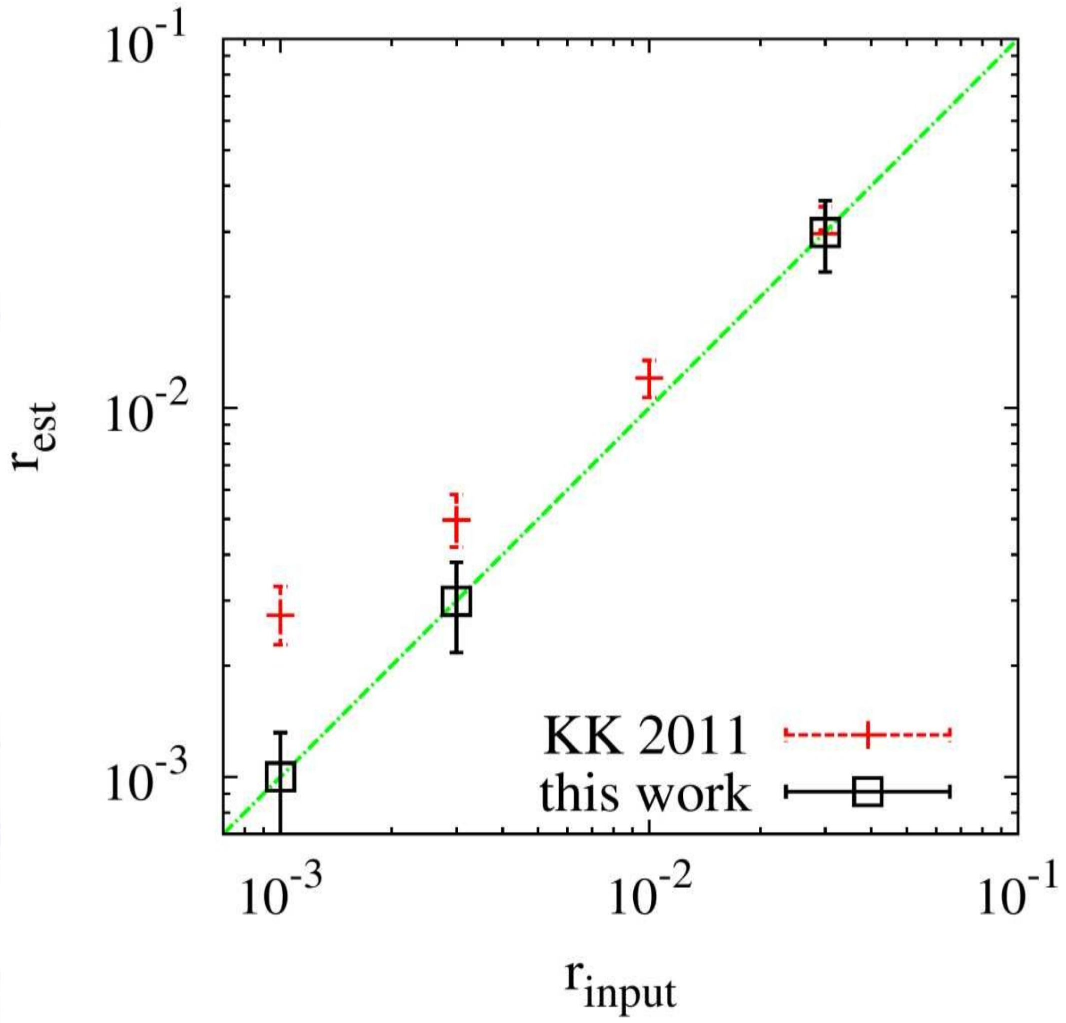


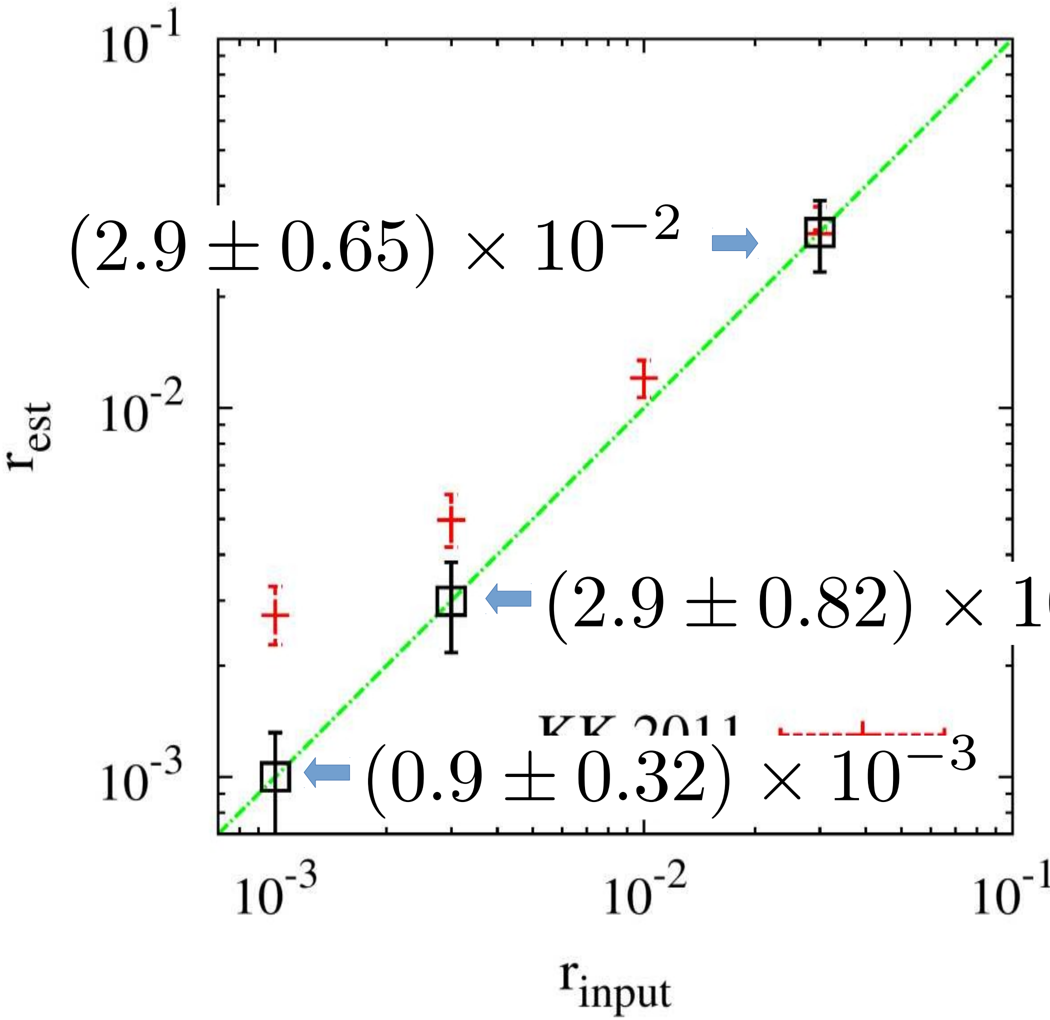
Delta-map Method parameters

$$N_{\rm side} = 4$$

$$\begin{pmatrix} \nu_0 \\ \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} 100 \\ 78 \\ 60 \end{pmatrix} \text{GHz}$$

Mask: p06 mask

Result



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For a more detailed analysis, see Yamashita-kun's poster

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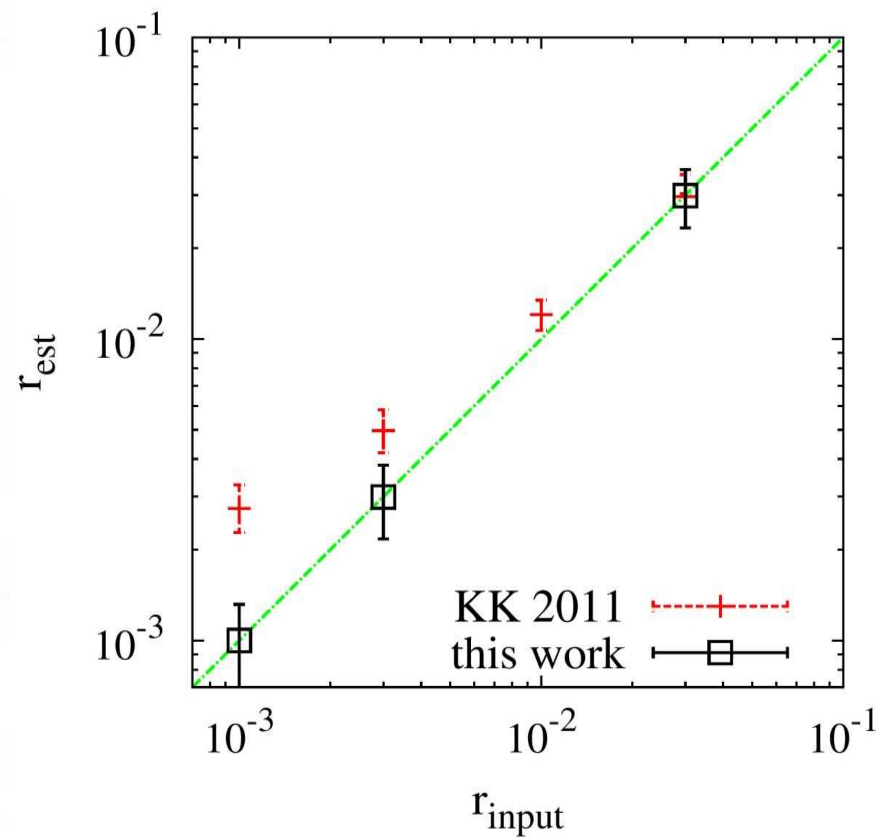
- **Use the delta-map method to eliminate the bias caused by the spatial variation of $\beta_{\mathrm{sync}}(\hat{n})$**



CONCLUSION