Simulation for LiteBIRD

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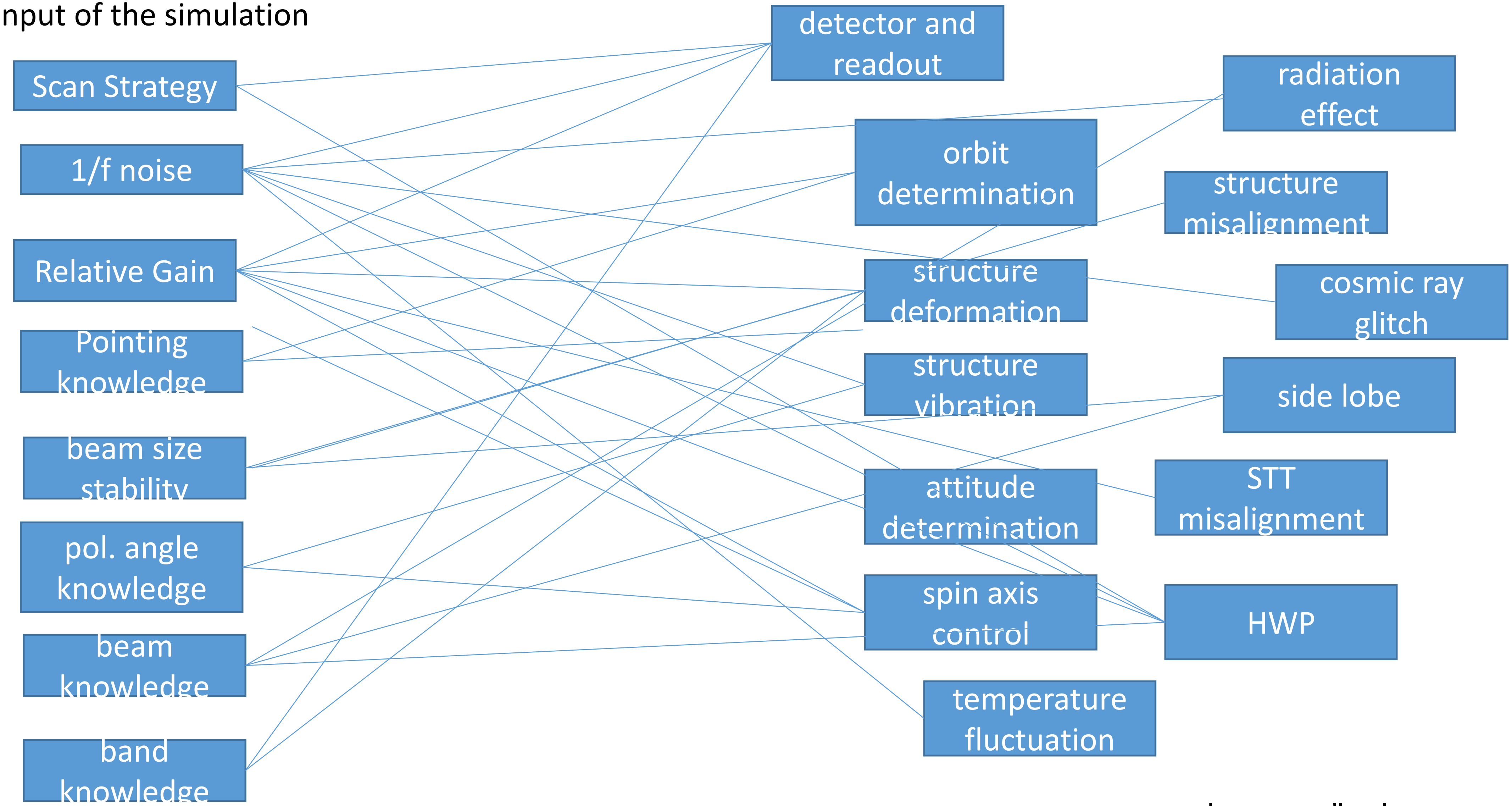
Introduction

- LiteBIRD aims to measure delta r < 0.001
- Ingredients of the delta r:
  - Statistics
  - Lensing
  - Foreground
  - Systematics
- Important to know the systematics
  - related with the requirement on the specifications of the satellite.
  - analytical estimation -> Ryo Nagata
  - estimation with the simulation.

Systematics and hardware

Input of the simulation



may be more "unknown unknowns"

Estimation of the systematics with simulation

- We are developing a LieBIRD simulation to estimate the systematics
  - Tomo, Hirokazu
- The specification of the current version:
  - All sky survey with the baseline scan strategy:
    - precession angle 65 deg. with 90 min.
    - spin angle 30 deg. with 0.1rpm with HWP
  - Focal plane detector
    - only LFT with JAXA MDR design (337 det. pixels, two orthogonal pol. in each det. pix.)
  - Sampling rate: (based on EPIC calculation method)
    - three pol. modulation per beam size sweep with HWP -> 9.3Hz
    - twice of Nyquist per beam size sweep without HWP -> 9Hz
  - Noise
    - inject noise to time-ordered data (TOD) with the sampling rate
    - 50uK rts x sqrt(sampling rate) for white with f_knee for 1/f noise.
    - We use "absrand" for 1/f noise generation

Simulation: TOD to a sky map, power spectrum

- In the current version, noise correlation matrix and filtering are not included in the process
  - being improved
  - map-making with a maximum likelihood to each sky pixel assuming white noise
  - with and without HWP
- HWP
  - a simple model making use of a 3x3 matrix to transfer (I, Q, U) vector
- Power spectrum
  - using Healpix functions

Method of TOD to map (1): w/o HWP

$$\begin{aligned} p &= \frac{1}{2}I + \frac{1}{2}Q\cos 2\psi + \frac{1}{2}U\sin 2\psi + n \\ &= \boldsymbol{w}\cdot\boldsymbol{s} + n, \end{aligned}$$

p: power received by a single detector sensitive to one polarization direction.

$$\boldsymbol{w}^t = 1/2(1,\ \cos 2\psi,\ \sin 2\psi)$$

$$\boldsymbol{s}^t = (I,\ Q,\ U)$$

ψ: angle between the detector pol. and an axis fixed on the sky sphere.

$$\chi^2 = \sum_{i=1}^{N} \frac{1}{\sigma_i^2}\left(p_i - \boldsymbol{w}_i\cdot\boldsymbol{s}\right)^2$$

i: i-th sampling of one sky pixel.
For each sky pixel, we minimize the chi-squire with respect to the vector ***s***

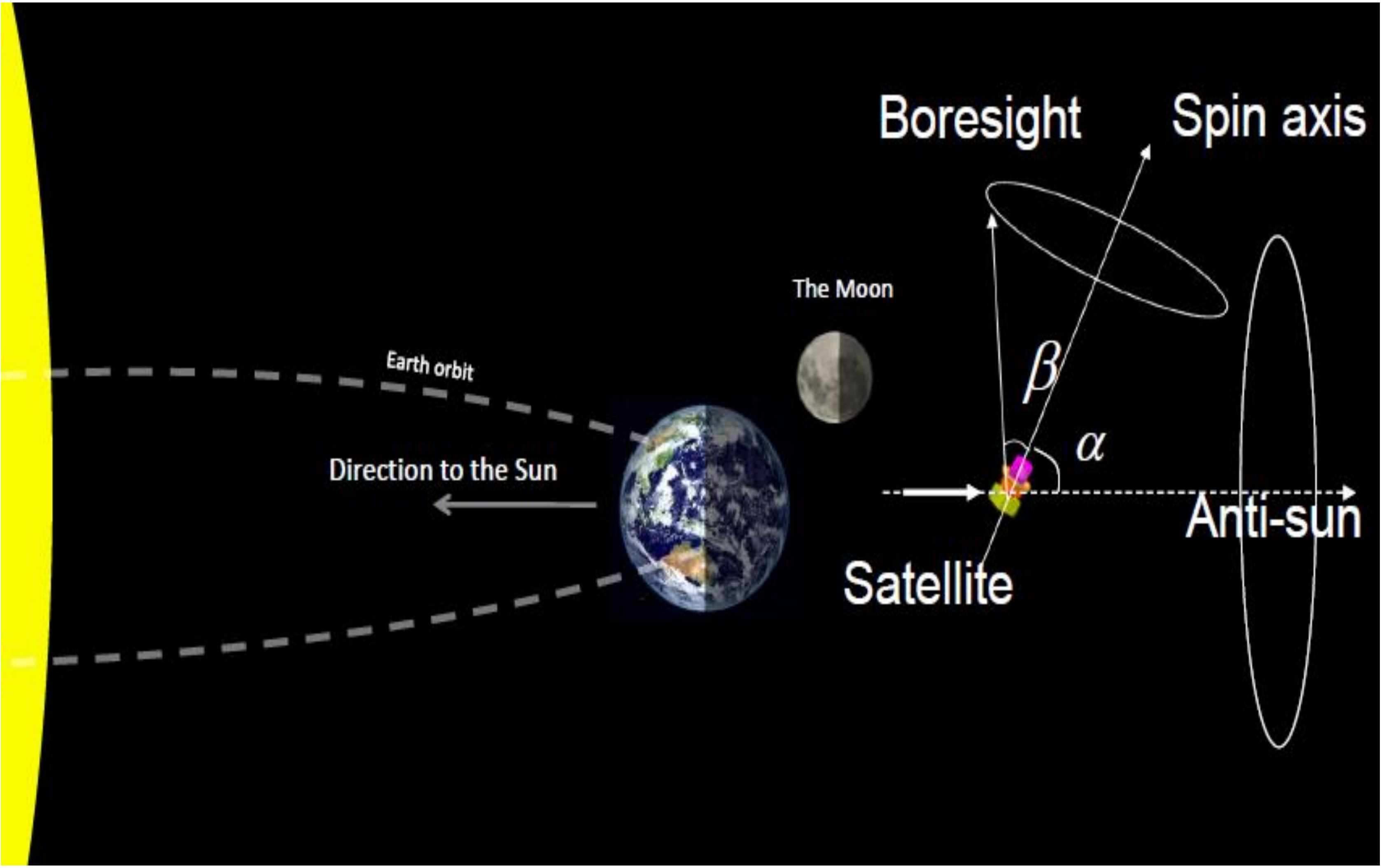
Method of TOD to map (1): w/o HWP

$$\hat{\boldsymbol{s}} = \left(\sum_i \boldsymbol{W}_i\right)^{-1} \sum_i p_i \boldsymbol{w}_i.$$

$$\sum_i p_i \boldsymbol{w}_i = \frac{1}{2}\begin{pmatrix} \sum_i p_i \\ \sum_i p_i \cos 2\psi_i \\ \sum_i p_i \sin 2\psi_i \end{pmatrix}$$

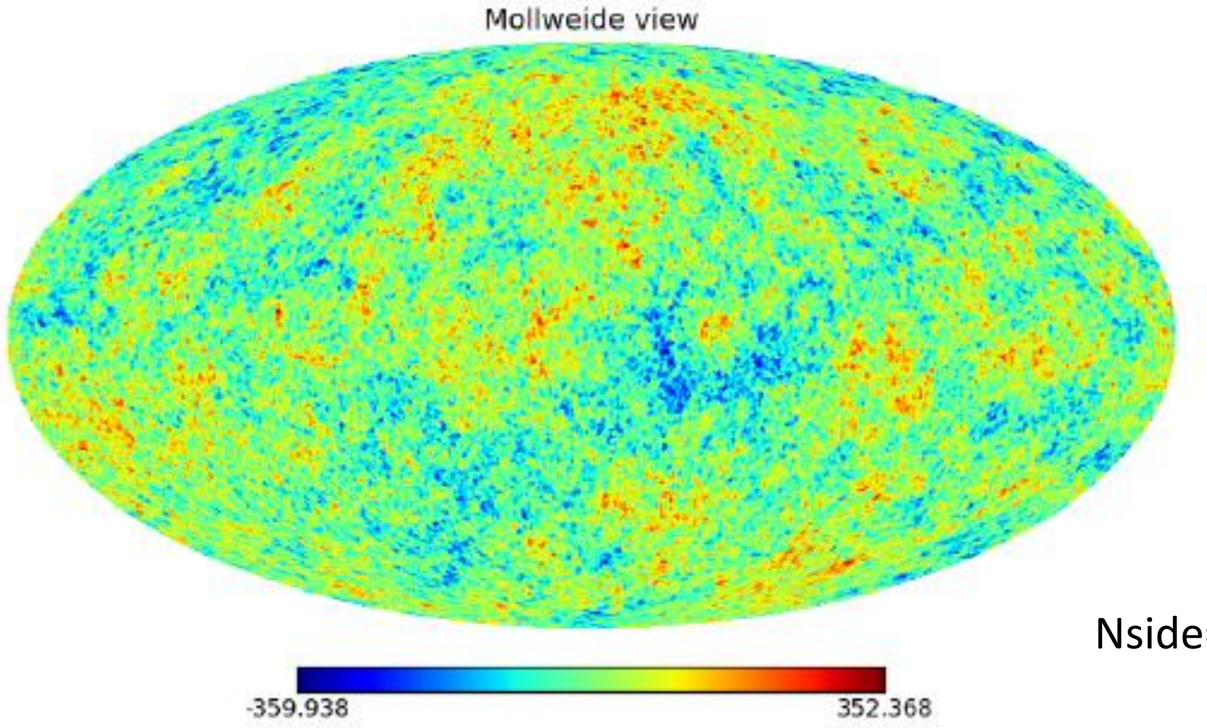
$$\sum_i \boldsymbol{W}_i = \frac{1}{4}\begin{pmatrix} \sum_i & \sum_i \cos 2\psi_i & \sum_i \sin 2\psi_i \\ \sum_i \cos 2\psi_i & \sum_i \cos^2 2\psi_i & \sum_i \cos 2\psi_i \sin 2\psi_i \\ \sum_i \sin 2\psi_i & \sum_i \cos 2\psi_i \sin 2\psi_i & \sum_i \sin^2 2\psi_i \end{pmatrix}$$

Scan strategy (current baseline method)



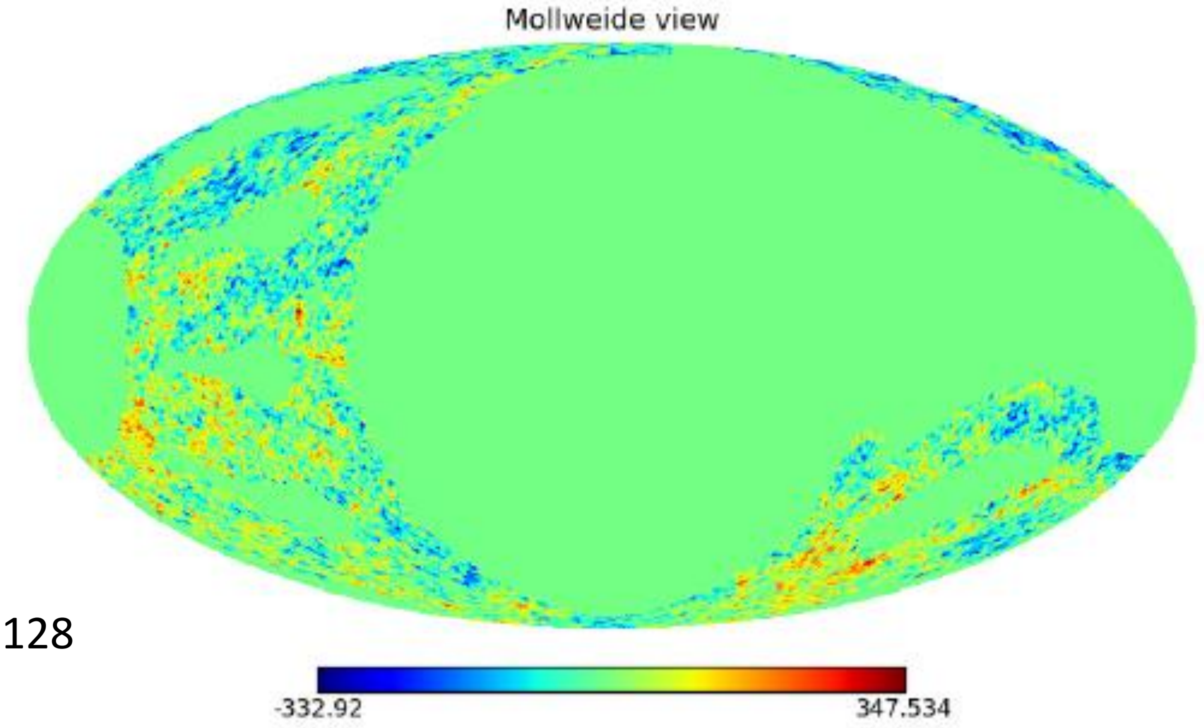
- Sun-Earth L2 hallo orbit
- Scan strategy
  - $\alpha = 65$ deg.
  - 1.5 hours precession
  - $\beta = 30$ deg.
  - 10 min. spin (w/ HWP)
    - 0.1 rpm
  - 3.3 min. spin (w/o HWP)
    - 0.3 rpm
  - HWP : 35 rpm

Demonstration of the sky scanning (w/ HWP)



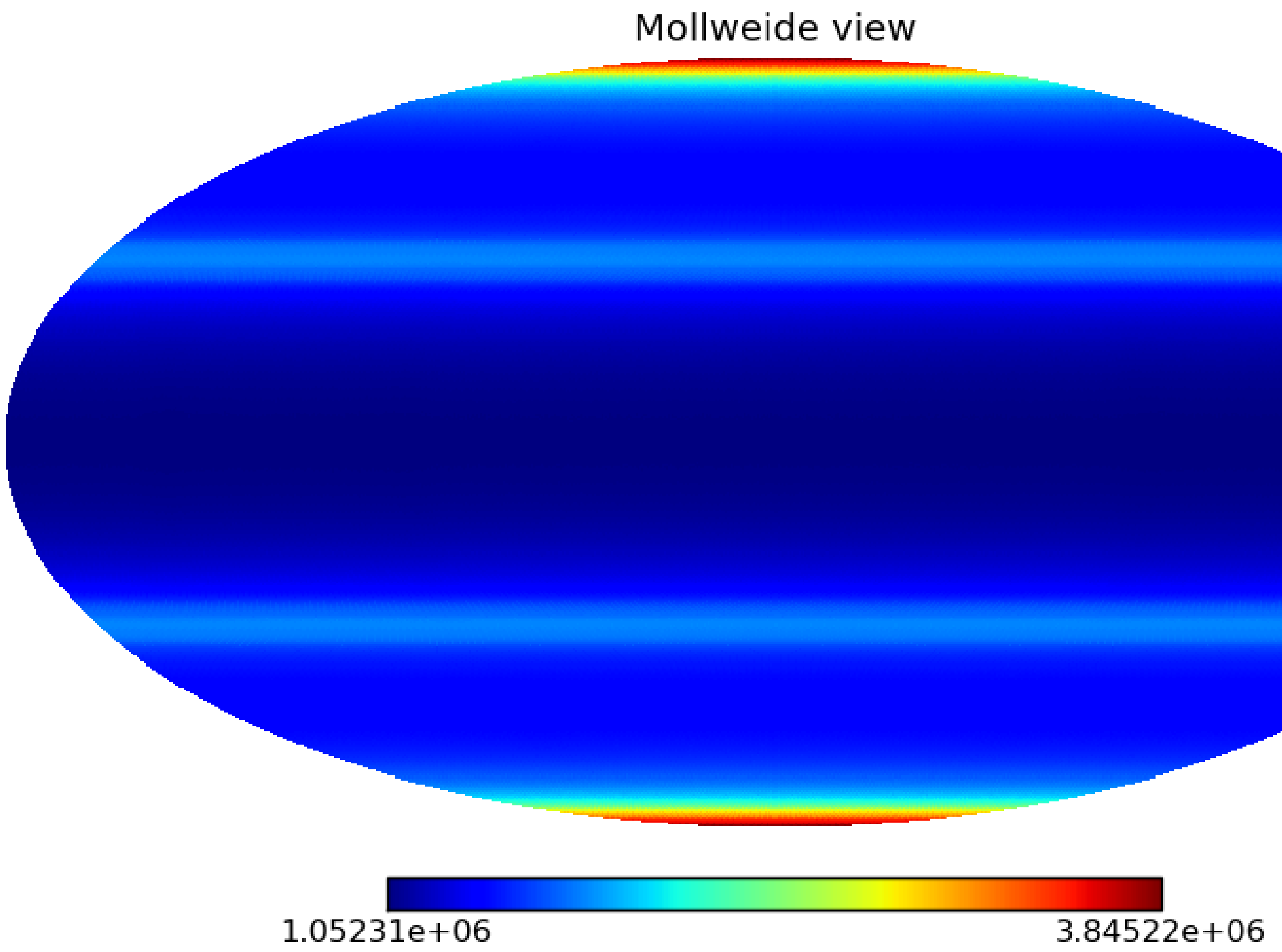
Original CMB temperature map

Nside=128



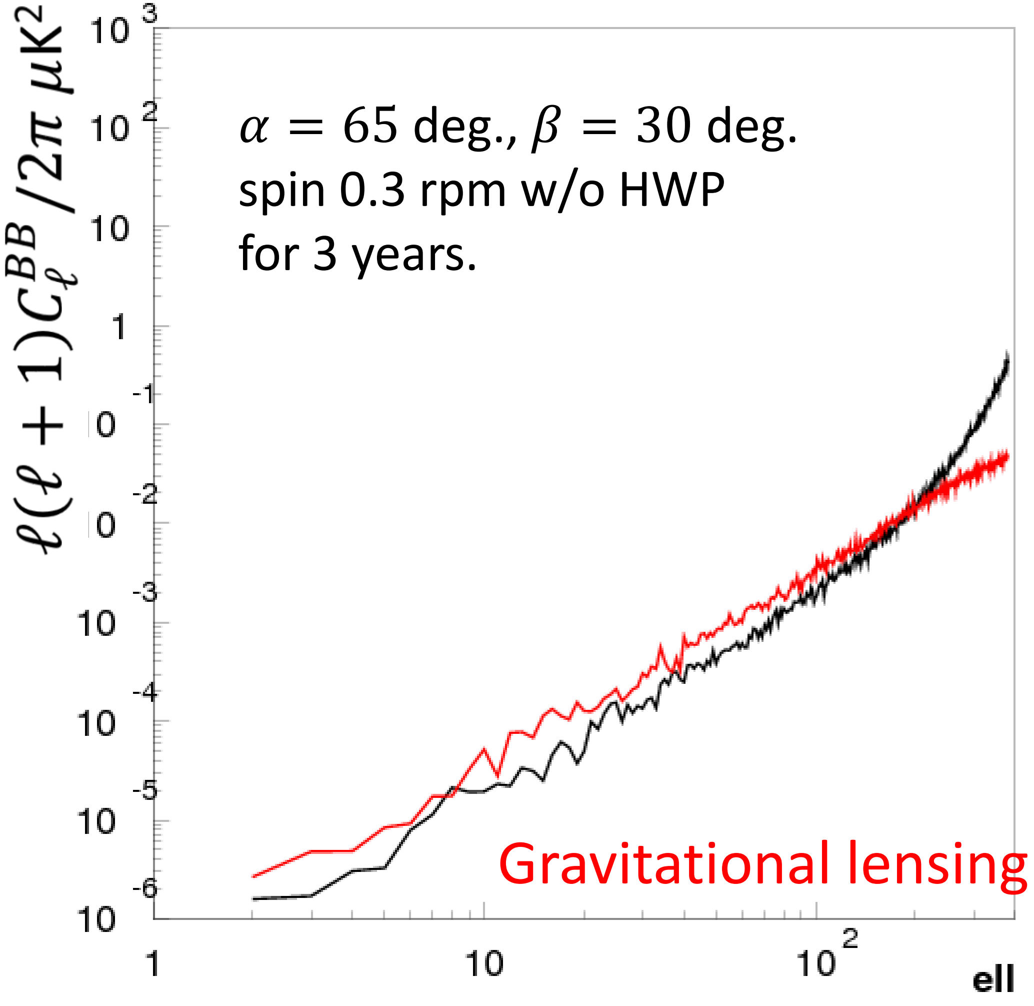
One hour scan of the map

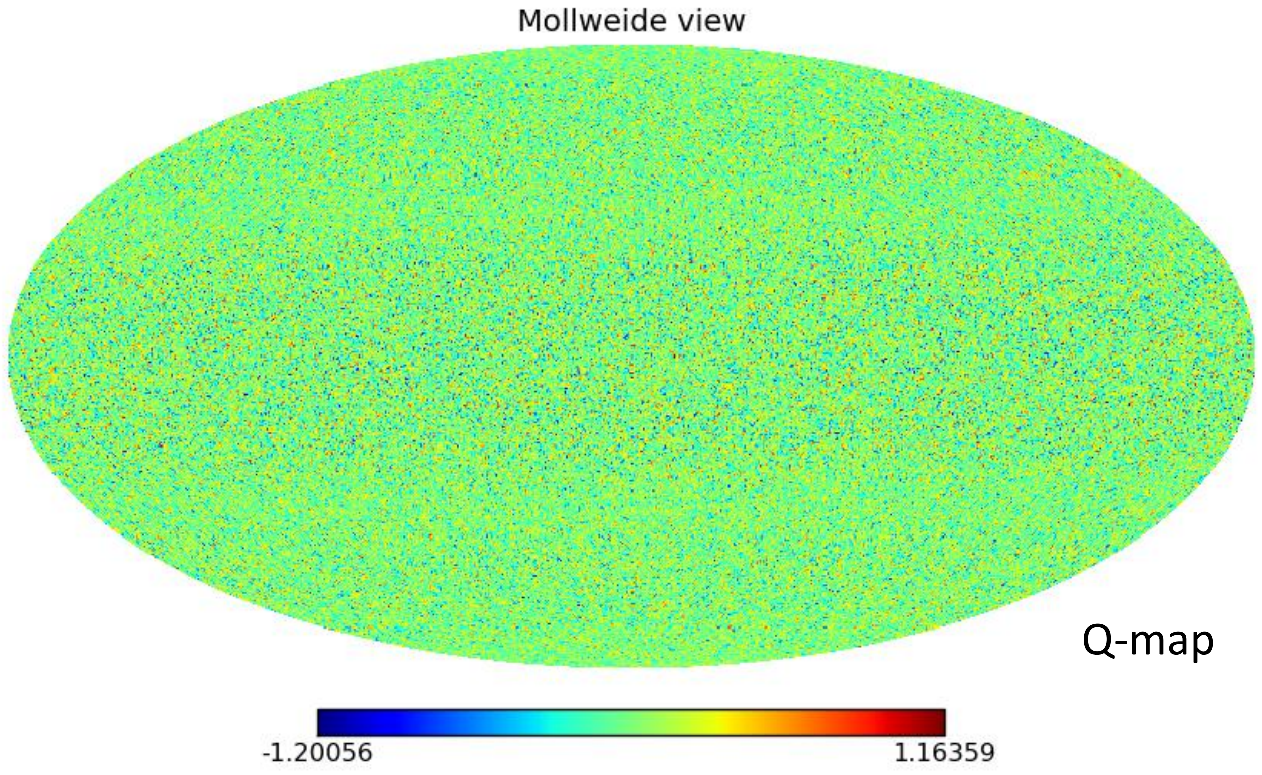
Hit map for three years (337 x 2 detectors)



Nside=128

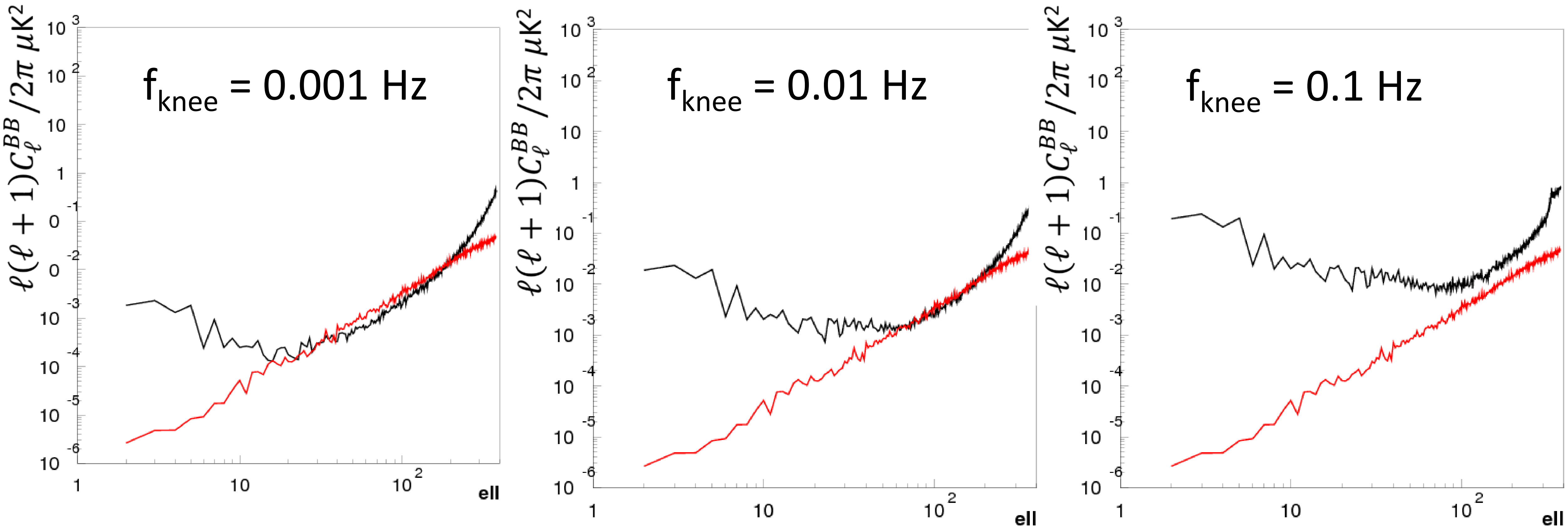
White noise only





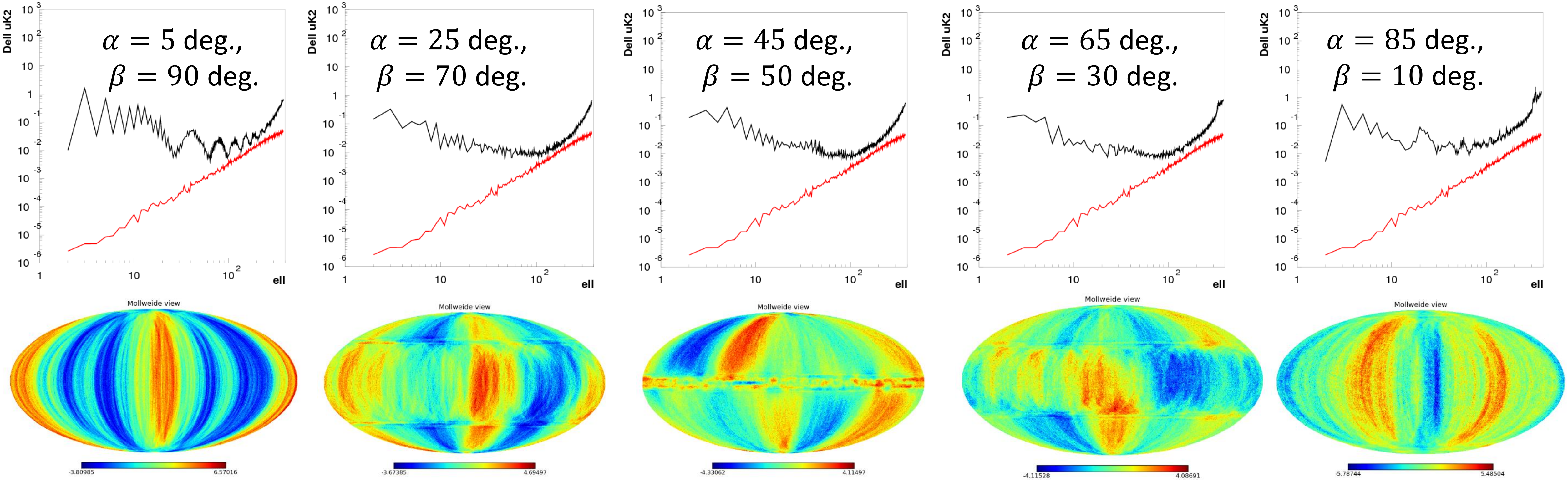
1/f noise only

$\alpha = 65$ deg., $\beta = 30$ deg. spin 0.3 rpm w/o HWP, for 3 years.



Scan dependence of the 1/f noise map

f_{knee} = 0.1 Hz, spin 0.3 rpm w/o HWP, for 3 years.



Q-map, Nside=128

No dependence on the scan strategy for 1/f noise power spectrum with the method (1).

Method of TOD to map (2): w/o HWP

$$p_i = \boldsymbol{w}_i \cdot \boldsymbol{H}_i \boldsymbol{s} + n_i = \boldsymbol{d}_i \cdot \boldsymbol{s} + n_i,$$

H is the 3x3 HWP transfer matrix

$$\boldsymbol{d} = \frac{1}{2}\begin{pmatrix} 1 \\ \cos(4\phi - 2\psi) \\ \sin(4\phi - 2\psi) \end{pmatrix}$$

$\boldsymbol{d} = \boldsymbol{Hs}$,
in ideal case the vector d becomes a simple form as a function of ψ and ϕ,where ϕ is the HWP matrix

$$\begin{pmatrix} I_{\rm out} \\ Q_{\rm out} \\ U_{\rm out} \end{pmatrix} = f \begin{pmatrix} T_{II} & T_{IQ} & T_{IU} \\ T_{QI} & T_{QQ} & T_{QU} \\ T_{UI} & T_{UQ} & T_{UU} \end{pmatrix} \begin{pmatrix} I_{\rm in} \\ Q_{\rm in} \\ U_{\rm in} \end{pmatrix}$$

more general form of the HWP transfer
(the notation may be different from those used in published literatures.)

$$\begin{aligned} T_{II} &= 1 + t^2 \\ T_{IQ} &= 2t\cos 2\phi \\ T_{IU} &= 2t\sin 2\phi \\ T_{QI} &= 2t\cos 2\phi \end{aligned}$$

$$\begin{aligned} T_{QQ} &= (1+t^2)\cos^2 2\phi + (1-t^2)\sin^2 2\phi\cos(\Delta\varphi) \\ T_{QU} &= \cos 2\phi \sin 2\phi[1 + t^2 - (1-t^2)\cos(\Delta\varphi) \\ T_{UI} &= 2t\sin 2\phi \\ T_{UQ} &= \cos 2\phi \sin 2\phi[1 + t^2 - (1-t^2)\cos(\Delta\varphi) \\ T_{UU} &= (1+t^2)\sin^2 2\phi + (1-t^2)\cos^2 2\phi\cos(\Delta\varphi), \end{aligned}$$

This is taken into account for future studies of the systematics.
More systematics related beam, non-uniformity etc. may also be included.

Method of TOD to map (2): w/ HWP

$$\chi^2 = \sum_i \frac{1}{2\sigma_i^2}(\Delta p_i - \Delta \boldsymbol{d}_i \cdot \boldsymbol{s})^2 + \sum_i \frac{1}{\sigma_i^2}(p_i - \boldsymbol{d}_i \cdot \boldsymbol{s})^2$$

We pick up a pair of the successive samplings in time, and calculate the difference when the samples are in the same sky pixel. Other case we use it as a single measurement included in the second term.

$$\hat{\boldsymbol{s}} = \left(\frac{1}{2}\sum_i \boldsymbol{\mathcal{D}}_i + \sum_i \boldsymbol{D}_i\right)^{-1}\left(\sum_i \frac{1}{2}\Delta p_i \Delta \boldsymbol{d}_i + \sum_i p_i \boldsymbol{d}_i\right)$$

$$\boldsymbol{\mathcal{D}}_i = \sin^2 2\Delta\phi \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sin^2 2\Psi & \sin 2\Psi \cos 2\Psi \\ 0 & \sin 2\Psi \cos 2\Psi & \cos^2 2\Psi \end{pmatrix}$$

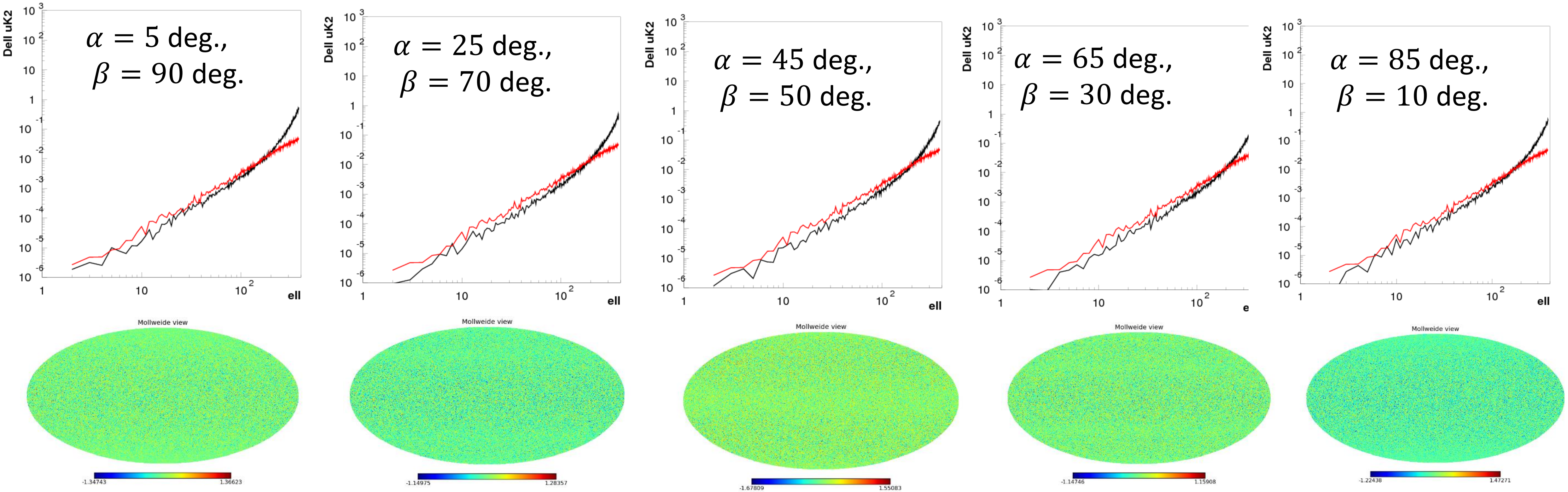
$$\Delta\phi = \phi_i - \phi_j$$

$$\Psi = \psi - \phi_i - \phi_j.$$

$$\boldsymbol{D}_i = \boldsymbol{d}_i \boldsymbol{d}_i^t = \boldsymbol{H}_i \boldsymbol{W}_i \boldsymbol{H}_i^t$$

Scan dependence of the 1/f noise map

f_{knee} = 0.1 Hz, spin 0.1 rpm w/ HWP, for 3 years.



Q-map, Nside=128

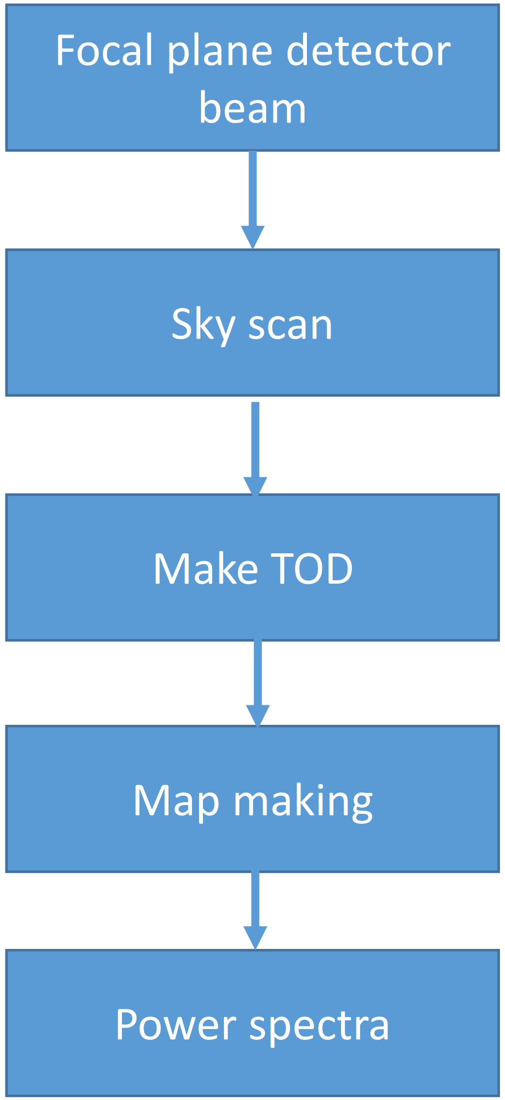
With the simple HWP model and the method (2), the 1/f noise is removed

A consideration why the method (2) removes the 1/f noise.

If we assume the noise correlation as $\langle n(t+\tau)n(t)\rangle = R(\tau)$

$$
\begin{aligned}
&\langle (n(t+\Delta t) - n(t))(n(t+\Delta t+\tau) - n(t+\tau))\rangle \\
&= \langle n(t+\Delta t)n(t+\Delta t+\tau)\rangle - \langle n(t+\Delta t)n(t+\tau)\rangle \\
&\qquad - \langle n(t)n(t+\Delta t+\tau)\rangle + \langle n(t)n(t+\tau)\rangle \\
&= R(\tau) - R(\tau-\Delta t) - R(\tau+\Delta t) + R(\tau) \\
&\simeq R(\tau) - R(\tau) + \frac{dR}{d\tau}\Delta t - R(\tau) - \frac{dR}{d\tau}\Delta t + R(\tau) \\
&= 0,
\end{aligned}
$$

Needed items for systematic studies



The current version: JAXA MDR design -> LFT/HFT
Need to approximate or realistic beam pattern to include beam eliptisity, side lobe etc.

Tuning of the scan parameters by taking into account 1/f noise, scan synchronous, cross-link, etc.
Inclusion of the pointing and angle knowledge systematics.

Inclusion of the systematic effects of the HWP, detector gain fluctuation, cosmic ray glitches etc.
Not only CMB signal and noise, but also pointing sources, foregrounds, galactic plane, etc.

Inclusion of the noise correlation matrix and filtering in the pipeline.
Taking into account the uncertainties of the calibration.

Foreground removal taking into account the systematics (band mismatch etc.).

Semi-analytic results for isotropic scan patterns

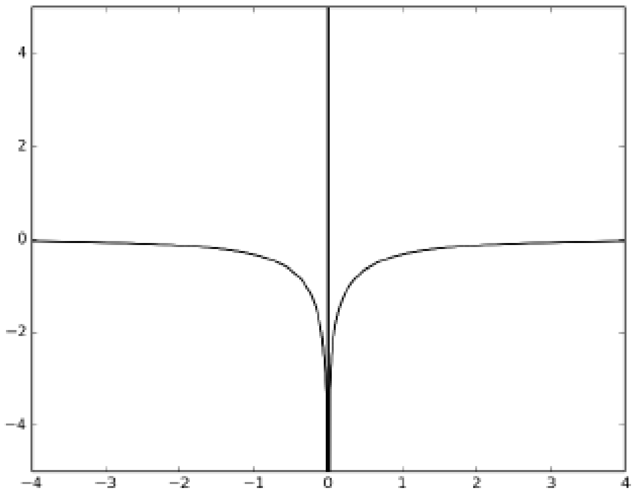
Martin Bucher

Map Making equation :

$$\mathbf{m}_{ML} = (\mathbf{A}^T\mathbf{N}^{-1}\mathbf{A})^{-1}(\mathbf{A}^T\mathbf{N}^{-1})\mathbf{d}$$

$\mathbf{m}_{ML}$ = optimal (maximum likelihood) map. $\mathbf{d}$ = time-ordered data vector.
$\mathbf{A}$=pointing matrix, $\mathbf{N}^{-1}$ detector inverse noise matrix.

High-pass map making filter



$N^{-1}(t)$
Time ($T = \nu_{knee} t$)

(Only differences in temperature are recorded)

White noise + 1/f noise

$$N(\nu) = N_{white}\left(1 + \frac{\nu_{knee}}{\nu}\right)$$

For an **isotropic** scan pattern, the **map inverse noise matrix** is **diagonal** :

$$(\mathbf{A}^T\mathbf{N}^{-1}\mathbf{A})^{-1} = {N_{w,map}}^{-1}\sum_{\ell,m} w_\ell Y^*_{\ell m}(\hat{\Omega})Y^*_{\ell m}(\hat{\Omega}')$$

Semi-analytic expressions for w_ℓ (inverse noise boost factor)

Martin Bucher

$$(\text{Boost factor}) \equiv \frac{\text{Noise(with 1/f included)}}{\text{Noise(only white noise)}}$$

Temperature case :

$$w_\ell^T \quad = \quad 1 - \sum_{i=1}^{n} f_i \int_{-\infty}^{+\infty} dt\ (2\nu_{knee})\ g(2\pi\nu_{knee}|t|)\ P_\ell\Big(\cos[\theta_i(t)]\Big)$$

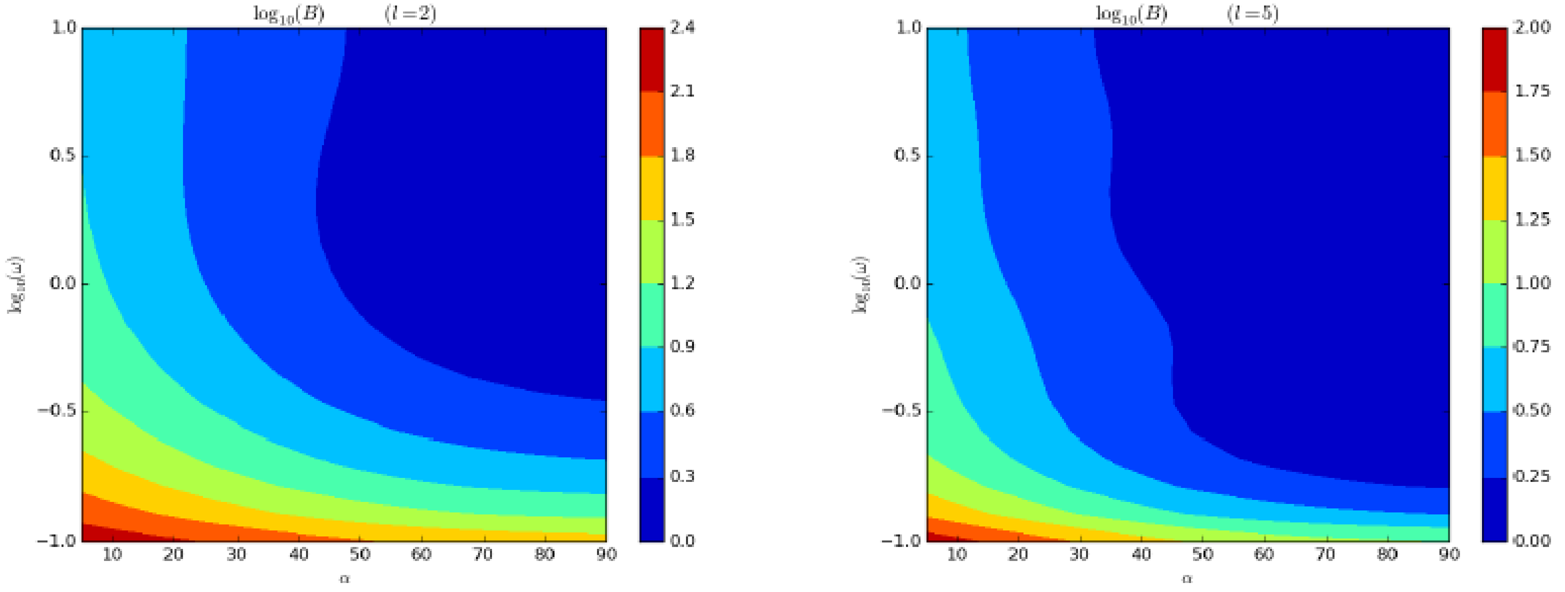
Polarization case :

$$\begin{aligned} w_\ell^P = 1 - \sum_i\ f_i \int_{-\infty}^{+\infty} dt\ (2\nu_{knee})\ g(2\pi\nu_{knee}|t|) \\ \times \frac{1}{2} \frac{1}{\sqrt{Q_\ell{}^2(1) + U_\ell{}^2(1)}} \Bigg[\Big(Q_{\ell 2}(\cos\theta_i(t)) + i\ U_{\ell 2}(\cos\theta_i(t)) \Big) \\ \times \exp\Big[2i\{\phi_i(t) - \chi_i(t) + \chi_i(t=0)\} \Big] \\ + \text{c.c.} \Bigg] \end{aligned}$$

Results for log_{10}(Boost factor) : temperature case

Martin Bucher

$$(\text{Boost factor}) \equiv \frac{\text{Noise(with 1/f included)}}{\text{Noise(only white noise)}}$$



$$\alpha = (\text{radius of scanning circle})$$

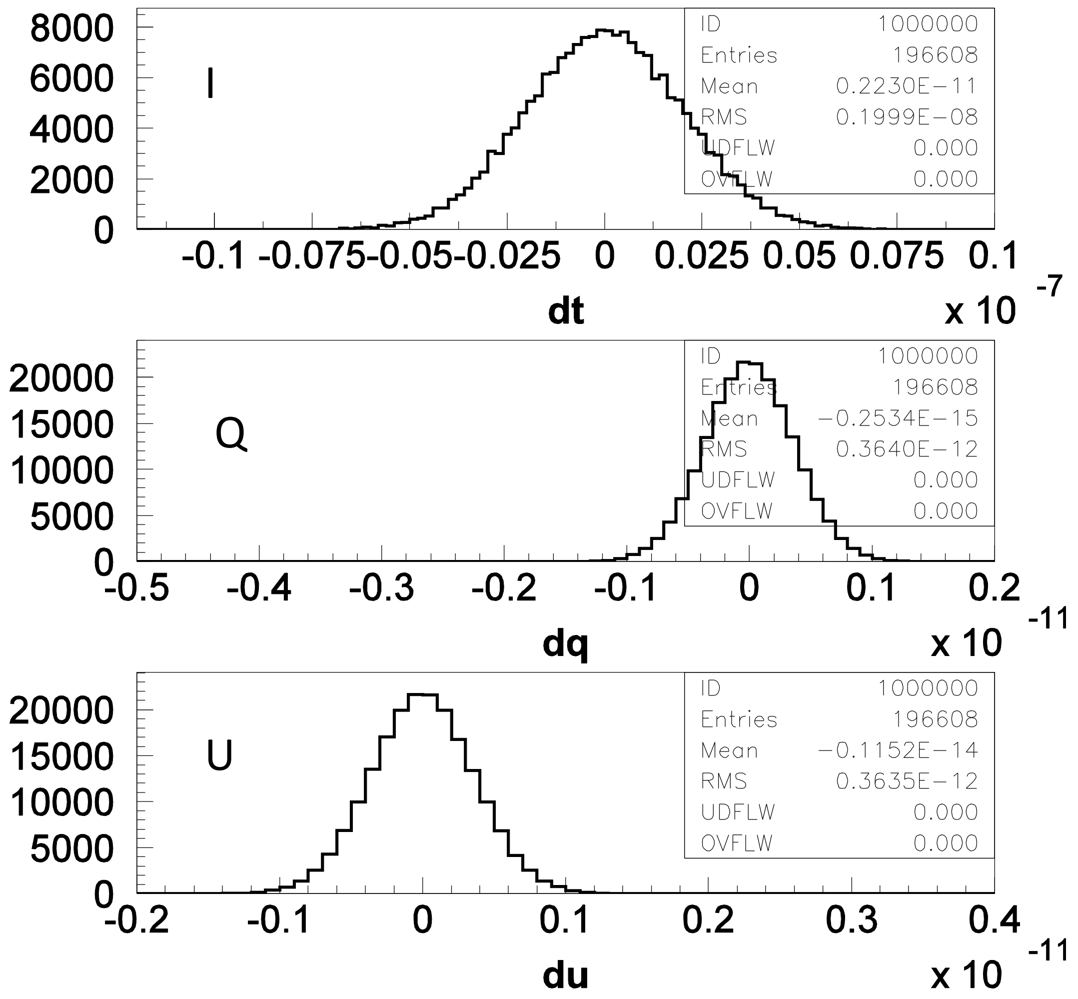
$$\omega = \Omega_{spin}/(2\pi\nu_{knee})$$

Summary

- We are developing a LiteBIRD simulator
  - JAXA MDR baseline design (372 pairs).
  - Sky scanning and map making using simple methods w/ and w/o HWP.
  - Power spectra with 1/f noise for various scan patterns are presented.
    - W/O HWP and method (1) shows large noise in the lower ell region.
    - W/ HWP and method (2) removes the 1/f noise
  - The systematic effects will be included in the simulation for further studies.
    - Semi-analytic scan strategy study by Martin is on-going.
- Mitigation of systematics is of importance
  - Any contributions from the heritage of WMAP and Planck are appreciated.
  - New ideas are welcome.

backup slides

CMB signal reproducibility with the method (2)



Calculated - generated in uK