Dark Energy and Modified Gravities

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Preliminary

Universe can be regarded as isotropic and homogeneous in the scale larger than the clusters of galaxies
$\Rightarrow$ **Friedmann-Robertson-Walker (FRW) metric:**

$$ds^2 = \sum_{\mu,\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 \sum_{i,j=1}^{3} \tilde{g}_{ij} dx^i dx^j \, .$$

$a(t)$: scale factor, $\tilde{g}_{ij}$: spacial metric
$\tilde{R}_{ij} = 2K\tilde{g}_{ij}$ ($\tilde{R}_{ij}$: Ricci curvature given by $\tilde{g}_{ij}$)
$K > 0$: sphere, $K < 0$: hyperboloid, $K = 0$: flat space

$$\begin{cases} da(t)/dt > 0 & : \text{expanding universe} \\ d^2a(t)/dt^2 > 0 & : \text{accelerating expansion} \end{cases}$$

Assume the Universe is filled with perfect fluids.
1st FRW equation: (t,t) component of the Einstein eq.

$$0 = -\frac{3}{\kappa^2}H^2 - \frac{3K}{\kappa^2 a^2} + \rho\,, \quad \kappa^2 \equiv 8\pi G$$

2nd FRW equation: (i,j) component

$$0 = \frac{1}{\kappa^2}\left(2\frac{dH}{dt} + 3H^2\right) + \frac{K}{\kappa^2 a^2} + p\,,$$

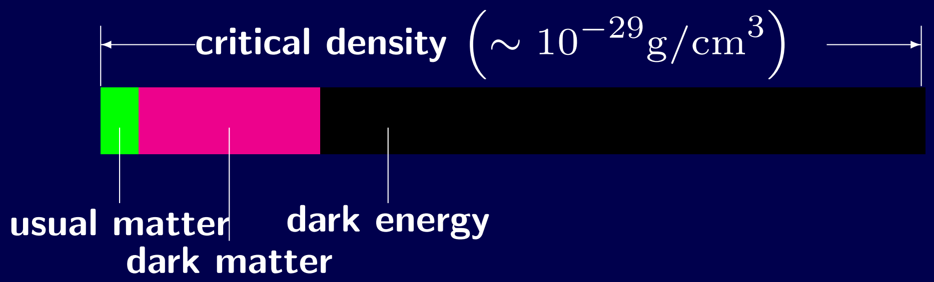
ρ**: energy density,** p**: pressure,** $H \equiv (1/a)\,da(t)/dt$**: Hubble rate**
The Hubble constant H_0**: the present value of** H**.**
$H_0 \sim 70\,\mathrm{km\,s^{-1}Mpc^{-1}} \sim 10^{-33}\,\mathrm{eV}$ **in the unit** $\hbar = c = 1$**.**

Dark Energy

Cosmic Microwave Background Radiation (CMB) $\Rightarrow K \sim 0$
(When we choose $a(t) = 1$ **for the present universe)**

$$\rho \sim \rho_c \equiv \frac{3}{\kappa^2} H_0^2 \sim \left(10^{-3}\,\mathrm{eV}\right)^4 \sim 10^{-29} \mathrm{g/cm}^3 \,.$$

ρ_c: **critical density. Flat universe** $\Rightarrow \rho \sim \rho_c$
Density of usual matter $\sim 4.9\%$ **, dark matter** $\sim 26.8\%$ **of** ρ_c
$\Longrightarrow$ **something unknown** $\sim 68.3\% \cdots$ **dark energy**



Planck satellite

Type Ia Supernovae

$\Rightarrow$ **accelerating expansion started about 5 billion years ago.**

1st and 2nd FRW eqs. $\Rightarrow \quad \frac{1}{a}\frac{d^2a(t)}{dt^2} = \frac{dH}{dt} + H^2 = -\frac{\kappa^2}{6}\left(\rho + 3p\right)$.

accelerating expansion $\Rightarrow p < -\rho/3$

$\Rightarrow$ **Dark energy: large negative pressure**

Equation of state (EoS) parameter: $w \equiv \frac{p}{\rho}$

Dark energy: $w \sim -1$

Radiation: $w = 1/3$,

Usual matter, cold dark matter (CDM): $w \sim 0$ **(dust),**

(pressure $\ll$ **rest energy)**

Cosmological constant: $w = -1$

Dark energy $=$ Cosmological constant??

Fine-tuning problem and/or Coincidence problem

Fine-tuning problem, Coincidence problem:
The definitions slightly depend on persons.

A. 1st and 2nd FRW equations ($K = 0$)

$$0 = -\frac{3}{\kappa^2}H^2 + \frac{\Lambda}{2\kappa^2} + \rho_{\mathrm{matter}}\,, \quad 0 = \frac{1}{\kappa^2}\left(2\frac{dH}{dt} + 3H^2\right) - \frac{\Lambda}{2\kappa^2} + p_{\mathrm{matter}}\,,$$

Λ: **cosmological constant**
If the dark energy comes from the cosmological term, the cosmological constant is unnaturally small.

$$\sqrt{\Lambda} \sim 10^{-33}\,\mathrm{eV} \ll M_{\mathrm{Planck}} \sim 1/\kappa \sim 10^{19}\,\mathrm{GeV} = 10^{28}\,\mathrm{eV}$$

Strange, especially from the viewpoint of the particle physics.

B. Anthropic principle?

$\frac{\Lambda}{2\kappa^2} \sim \rho_{\rm matter}$ **(including dark matter)** **Very accidental! if Λ is a constant**

Age of the Universe: 13.7 billion years

$$\sim \left(10^{-33}\,{\rm eV}\right)^{-1} \sim \Lambda^{-\frac{1}{4}}$$

Present temperature of the Universe: ($3K$)

$$\sim 10^{-3}\,{\rm eV} \sim \left(\rho_{\rm matter}\right)^{1/4} \sim \left(\frac{\Lambda}{2\kappa^2}\right)^{1/4}$$

$\Rightarrow$ **Dark energy might be dynamical?**

C. Initial condition?
If the dark energy is a perfect fluid whose EoS parameter $w \sim -1$,

$$\rho_{\rm DE} = \rho_{\rm DE\,0} a^{-3(1+w)} \sim \rho_{\rm DE\,0}$$

Usual matter or CDM (dust with $w = 0$)

$$\rho_{\rm matter} = \rho_{\rm matter\,0} a^{-3}$$

Ratio of densities of the dark energy to usual matter and dark matter

$$\rho_{\rm DE}/\rho_{\rm matter} \sim \left(\rho_{\rm DE\,0}/\rho_{\rm matter\,0}\right) a^{-3}$$

In order that $\rho_{\mathrm{DE}\,0} \sim \rho_{\mathrm{matter}\,0}$ in the present Universe,

because the ratio is given by $\rho_{\mathrm{DE}}/\rho_{\mathrm{matter}} \sim a^{-3}$,
when transparent to radiation ($a \sim 10^{-3}$), for example:

$$\rho_{\mathrm{DE}}/\rho_{\mathrm{matter}} \sim 10^{-9}$$

We need to fine-tune the initial condition of the ratio.

There might be a model where the dark matter interacts with dark energy and there is a transition between them?
The EoS parameter of the dark energy changes dynamically depending on the expansion (tracker model)?

D. If the dark energy is the vacuum energy,

the quantum corrections from the matter diverge $\sim \Lambda_{\mathrm{cutoff}}^4$.

$$\rho_{\mathrm{vacuum}} = \frac{1}{(2\pi)^3}\int d^3k \frac{1}{2}\sqrt{k^2+m^2} \sim \Lambda_{\mathrm{cutoff}}^4 \, .$$

$\Lambda_{\mathrm{cutoff}}$**: cutoff scale**

If the supersymmetry is restored in the high energy,
the vacuum energy by the quantum corrections $\sim \Lambda_{\mathrm{cutoff}}^2 \Lambda_{\cancel{\mathrm{SUSY}}}^2$

$$\rho_{\mathrm{vacuum}} = \frac{1}{(2\pi)^3}\int d^3k \frac{1}{2}\left(\sqrt{k^2+m_{\mathrm{boson}}^2} - \sqrt{k^2+m_{\mathrm{fermion}}^2}\right) \sim \Lambda_{\mathrm{cutoff}}^2 \Lambda_{\cancel{\mathrm{SUSY}}}^2 \, .$$

$\Lambda_{\cancel{\mathrm{SUSY}}}$**: the scale of the supersymmetry breaking.**
$\Lambda_{\cancel{\mathrm{SUSY}}}^2 = m_{\mathrm{boson}}^2 - m_{\mathrm{fermion}}^2$.

If we use the counter term in order to obtain the very small vacuum energy $\left(10^{-3}\,\mathrm{eV}\right)^4$, we need very very fine-tuning and extremely unnatural.

Maybe we do not understand quantum gravity?

The above problems could be clues to understand the gravity.

Modified gravities

The Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 T_{\mu\nu}$$

If the dark energy is a perfect fluid filling the Universe
$\cdots$ **modification of the energy momentum tensor $T_{\mu\nu}$ of matters (r.h.s. in the Einstein equation).**

Many models to consider the modification of the Einstein tensor (l.h.s.)
$\cdots$ **Modified gravity models**

$F(R)$ **gravity, scalar-tensor theory (Brans-Dicke type model), Gauss-Bonnet gravity,** $F(G)$ **gravity, massive gravity, bigravity...**

Scalar-tensor theory

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right\} , \quad V(\phi) \sim \phi^{-n}\text{: } \textbf{tracker} .$$

Brans-Dicke type ($\Leftarrow$ higher dimensional theory?)

$$S = \int d^4x\sqrt{-g}\left\{f(\phi)R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right\} .$$

k**-essence (**X**-matter)**

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R - F\left(\partial_\mu\phi\partial^\mu\phi\right)\right\} ,$$

$$F\left(\partial_\mu\phi\partial^\mu\phi\right) \ \rightarrow\ F\left(\partial_\mu\phi\partial^\mu\phi, \phi\right)\text{: } \textbf{generalized } k\textbf{-essence} .$$

Scalar-Gauss-Bonnet gravity ($\Leftarrow \alpha'$ correction in string theory?)

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - f(\phi)G\right\} ,$$
$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} .$$

$F(R)$ **gravity**
(low energy effective theory after integrating heavy fields?)

$$S = \int d^4x\sqrt{-g}F(R) .$$

$F(G)$ **gravity**

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R + F(G)\right\} .$$

Galileon model, Born-Infeld gravity, massive gravity, bigravity...

Summary

Dark energy

Too many problems, too many models.

???????

Anyway problem

Any clue for the gravity beyond general relativity

Some peoples often said that some models died, but always incorrect. Loopholes in logic or revive by some modifications.

We need non-linear cosmological perturbation to know which model could be realistic.

From the viewpoint of particle physics

(Unified theory including gravity like string)

Gravity beyond Einstein's general relativity

Quantum gravity