

Inflationary Universe

(project A01)

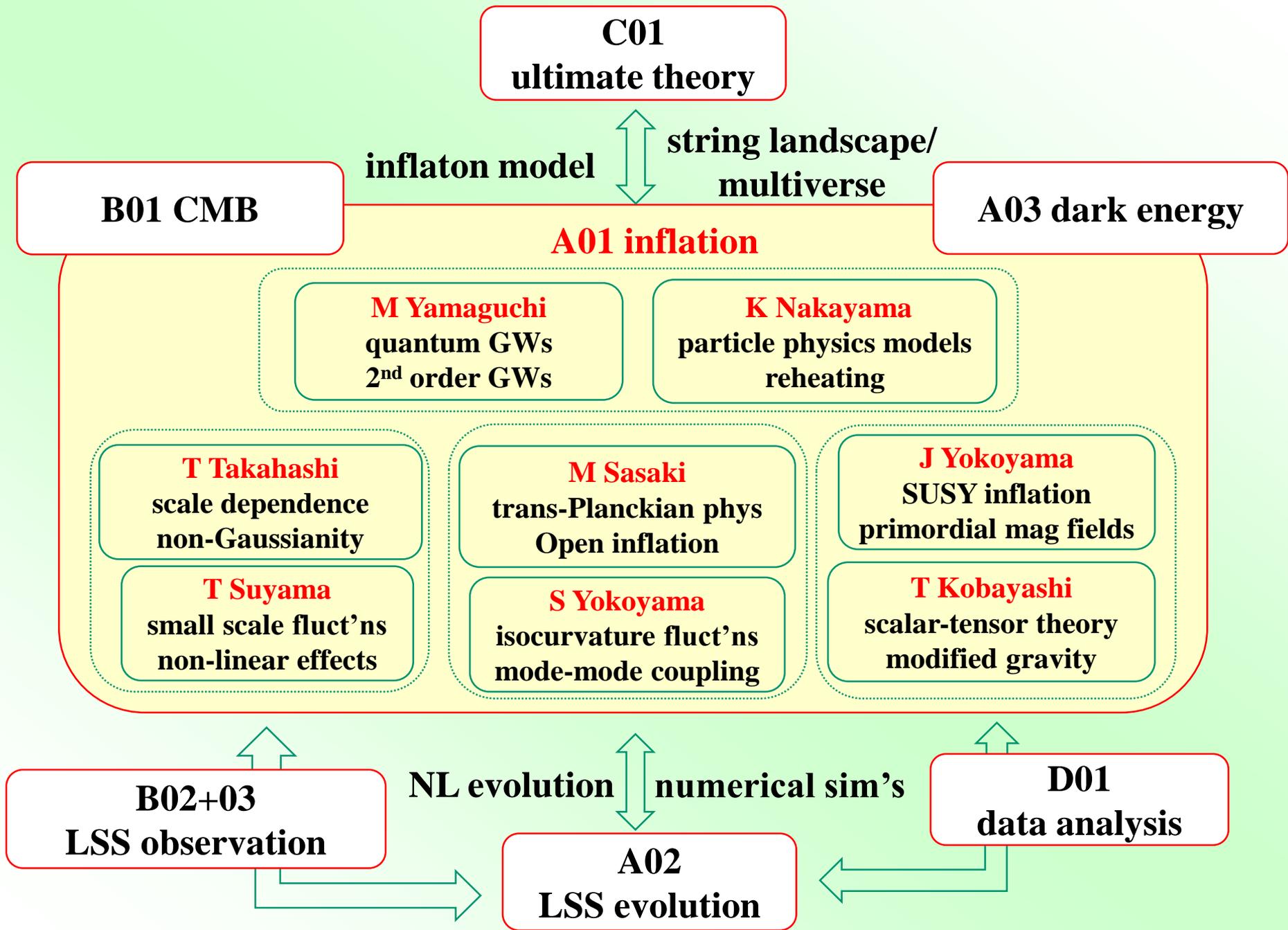
Toward Understanding Physics/Mechanism of Inflation

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(Original version by Masahide Yamaguchi)

$$c = \hbar = 1, \quad M_G = 1/\sqrt{8\pi G} \sim 2.4 \times 10^{18} \text{GeV}.$$



Introduction

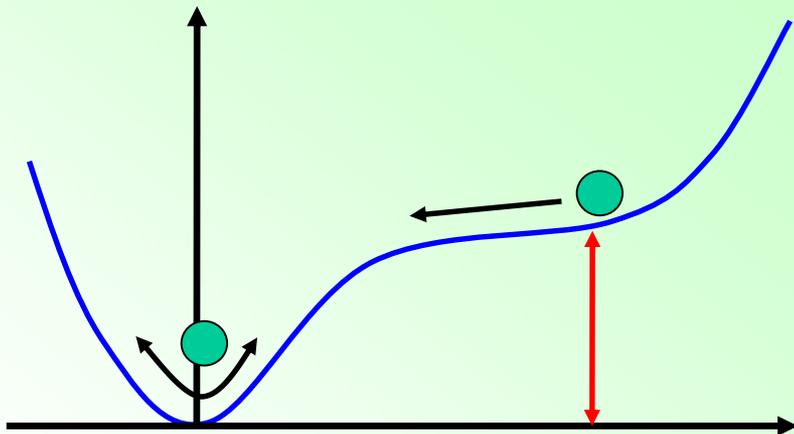
Inflation

The Universe rapidly expanded thanks to the vacuum energy density in the early stage.

(accelerated expansion: $\ddot{a} > 0$)

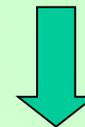
Brout, Englert & Gunzig '78, Starobinsky '79, Sato '80, Guth '80, ...

Vacuum energy density



State of vacuum
(expectation value of scalar field)

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho \simeq \text{const.}$$

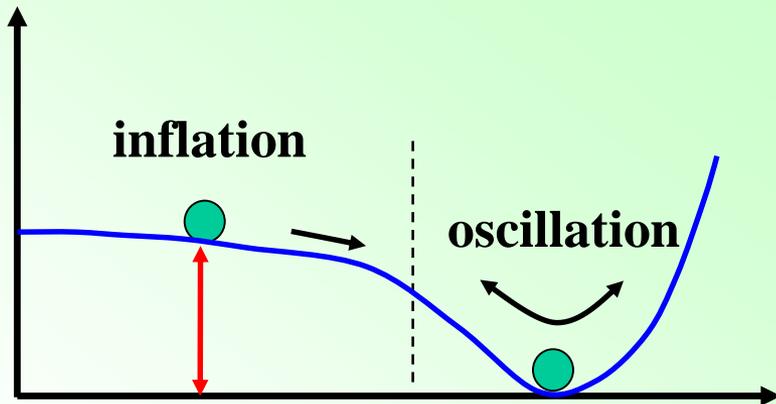


$$a(t) \propto \exp[Ht]$$

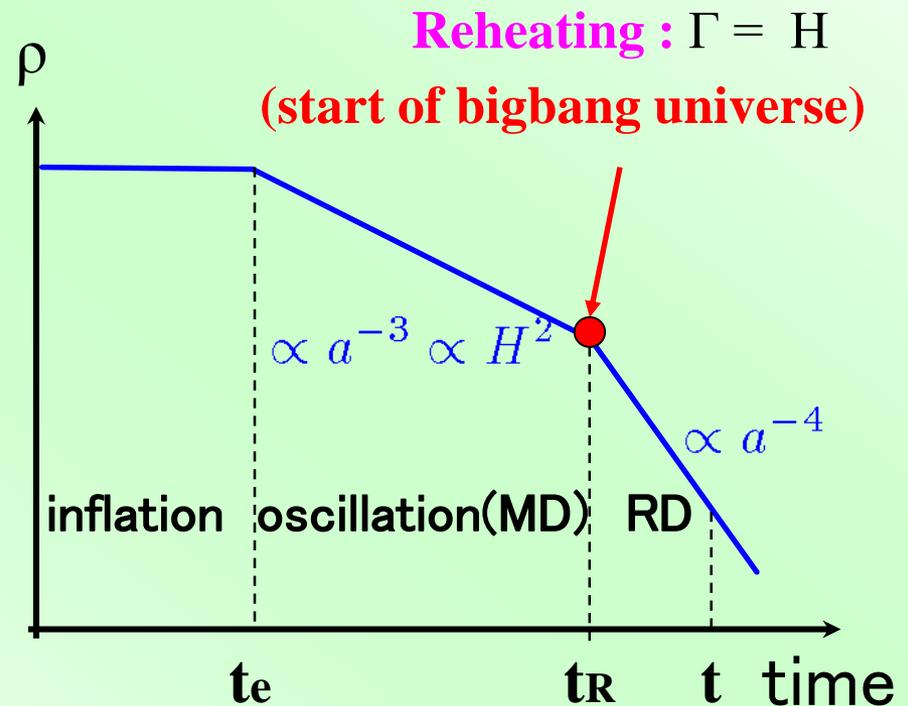
From inflation to bigbang

At the end of inflation, the vacuum energy is released as **latent heat** (called “**re**”heating) and **hot Bigbang Universe** is realized.

Vacuum energy density

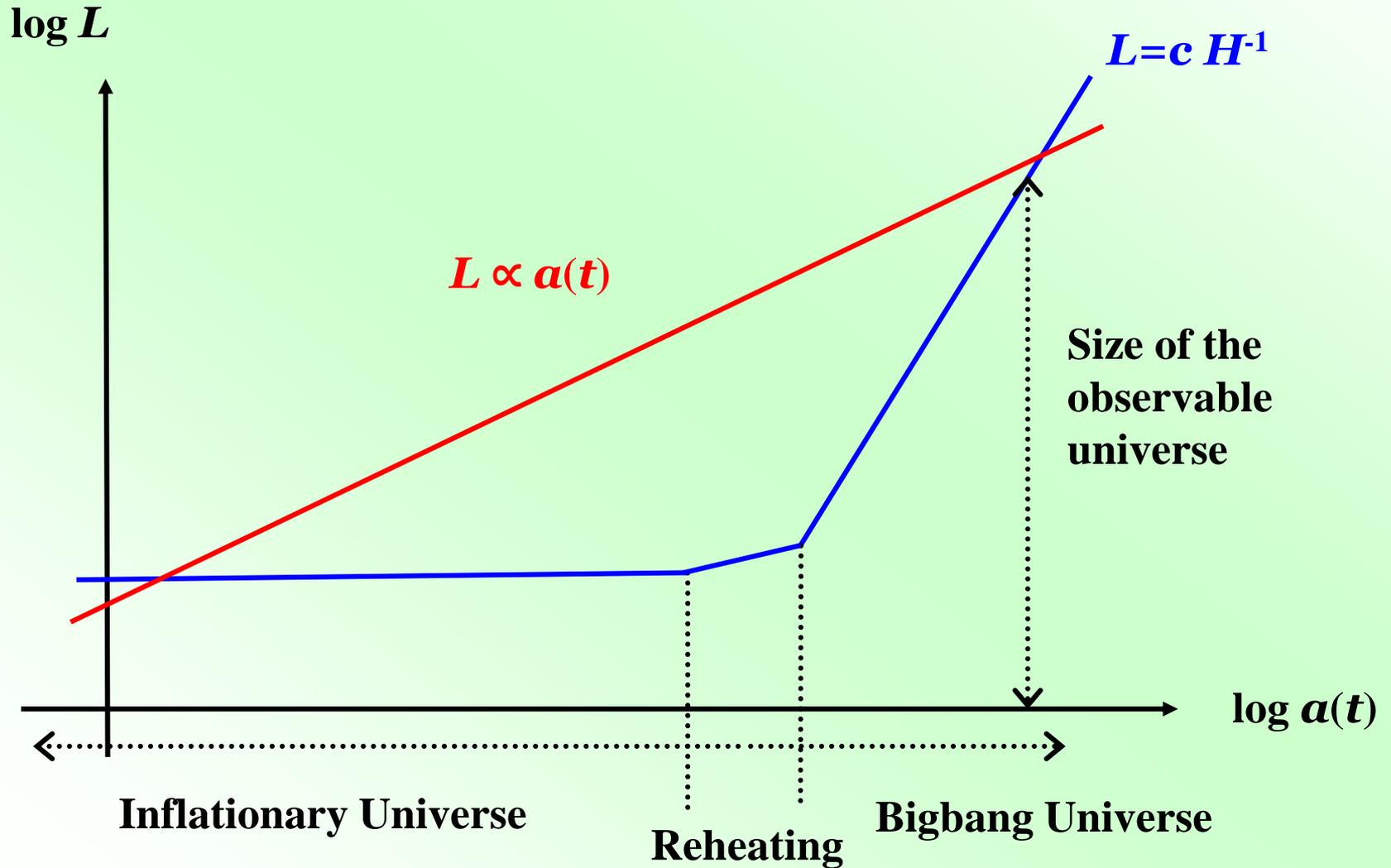


State of vacuum
(expectation value of scalar field)



Kinematics

Length scales of inflationary universe



Flatness

small universe



expands by a
factor $>10^{30}$

Size of our observable universe



looks perfectly
flat

Birth of a gigantic
universe

Flatness can be explained **only by Inflation**

Dynamics

seeds of cosmological perturbations

Mukhanov '81,

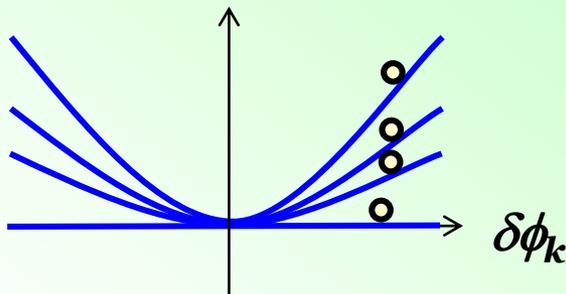
Zero-point (vacuum) fluctuations of ϕ :

$$\delta\phi = \sum_k \delta\phi_k(t) e^{ik \cdot x}$$

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \omega^2(t)\delta\phi_k = 0 ; \quad \omega^2(t) = \frac{k^2}{a^2(t)} \equiv \left(\frac{2\pi c}{\lambda(t)} \right)^2$$

physical wavelength $\nearrow \lambda(t) \propto a(t)$

harmonic oscillator with friction term and time-dependent ω



$$\delta\phi_k \rightarrow \text{const.}$$

... frozen when $\lambda > c H^{-1}$
(on superhorizon scales)

tensor (gravitational wave) modes also satisfy the same eq.

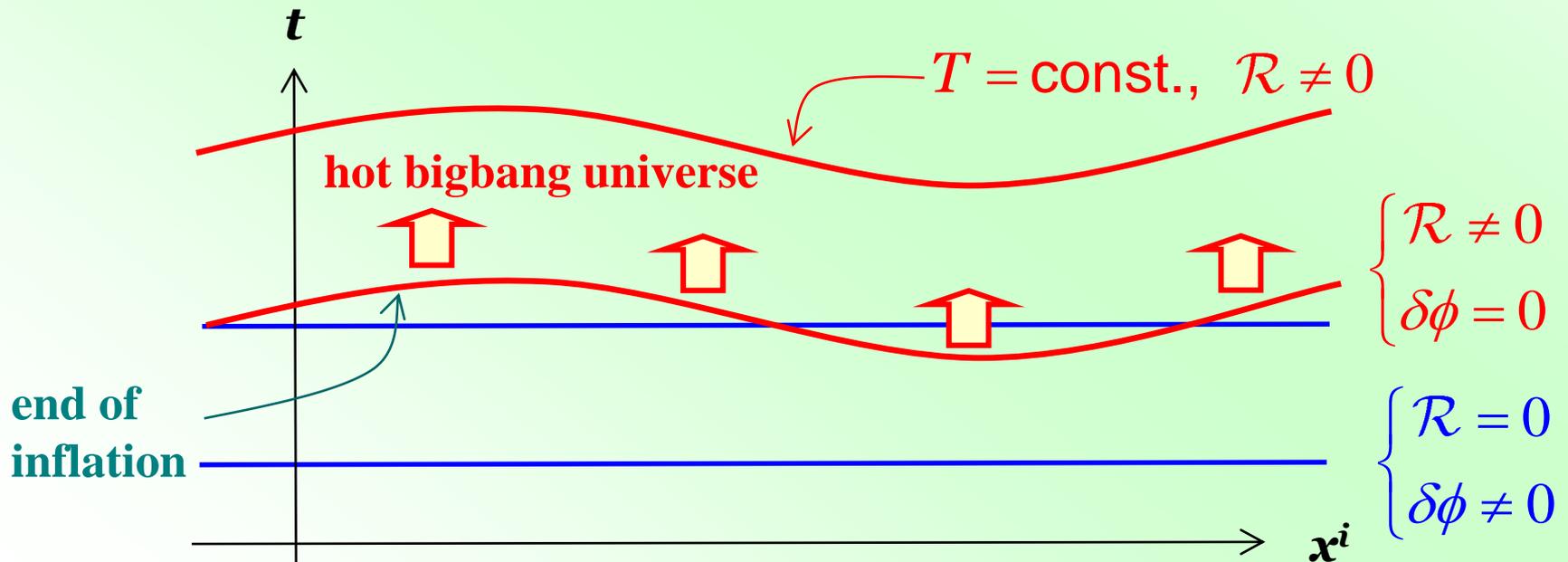
Starobinsky '79

generation of curvature perturbation

Mukhanov '81, '85; MS '86, ...

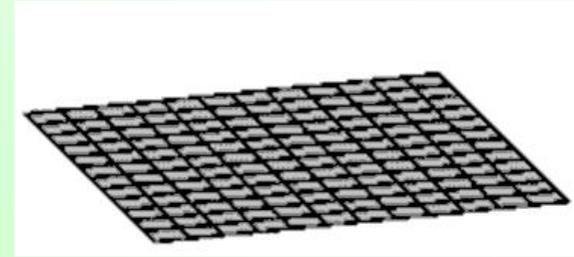
curvature perturbation $\mathcal{R} \approx$ gravitational potential Ψ

- $\delta\phi$ is frozen on “flat” ($\mathcal{R}=0$) 3-surface ($t=\text{const.}$ hypersurface)
- Inflation ends/damped osc starts on $\phi=\text{const.}$ 3-surface.

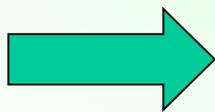


Generic predictions of inflation

- **Spatially flat universe**



- **Almost scale invariant, adiabatic, and Gaussian primordial density fluctuations**
- **Almost scale invariant and Gaussian primordial tensor fluctuations**



**Generates anisotropy of CMBR.
Origin of galaxies, stars, ...**

- **Amplitude of curvature perturbation:**

$$\mathcal{R} = \frac{H^2}{2\pi\dot{\phi}} \Big|_{k/a=H} \quad \text{Mukhanov (1985), MS (1986)}$$

- **Power spectrum index:**

$$M_{pl} \equiv \frac{1}{\sqrt{8\pi G}} \sim 2.4 \times 10^{18} \text{ GeV: Planck mass}$$

$$\frac{4\pi k^3}{(2\pi)^3} P_{\mathcal{R}}(k) = Ak^{n_s-1} ; \quad n_s - 1 = M_{pl}^2 \left(2 \frac{V''}{V} - 3 \frac{V'^2}{V^2} \right)$$

- **Tensor (gravitational wave) spectrum:**

$$\frac{4\pi k^3}{(2\pi)^3} P_T(k) = Ak^{n_T} ; \quad n_T = -3 \frac{\dot{\phi}^2}{V} = -\frac{1}{8} \frac{P_{\mathcal{R}}(k)}{P_T(k)} \quad \text{Liddle-Lyth (1992)}$$

Observational results

Map of CMBR by PLANCK

Temperature anisotropy

Feb 2015

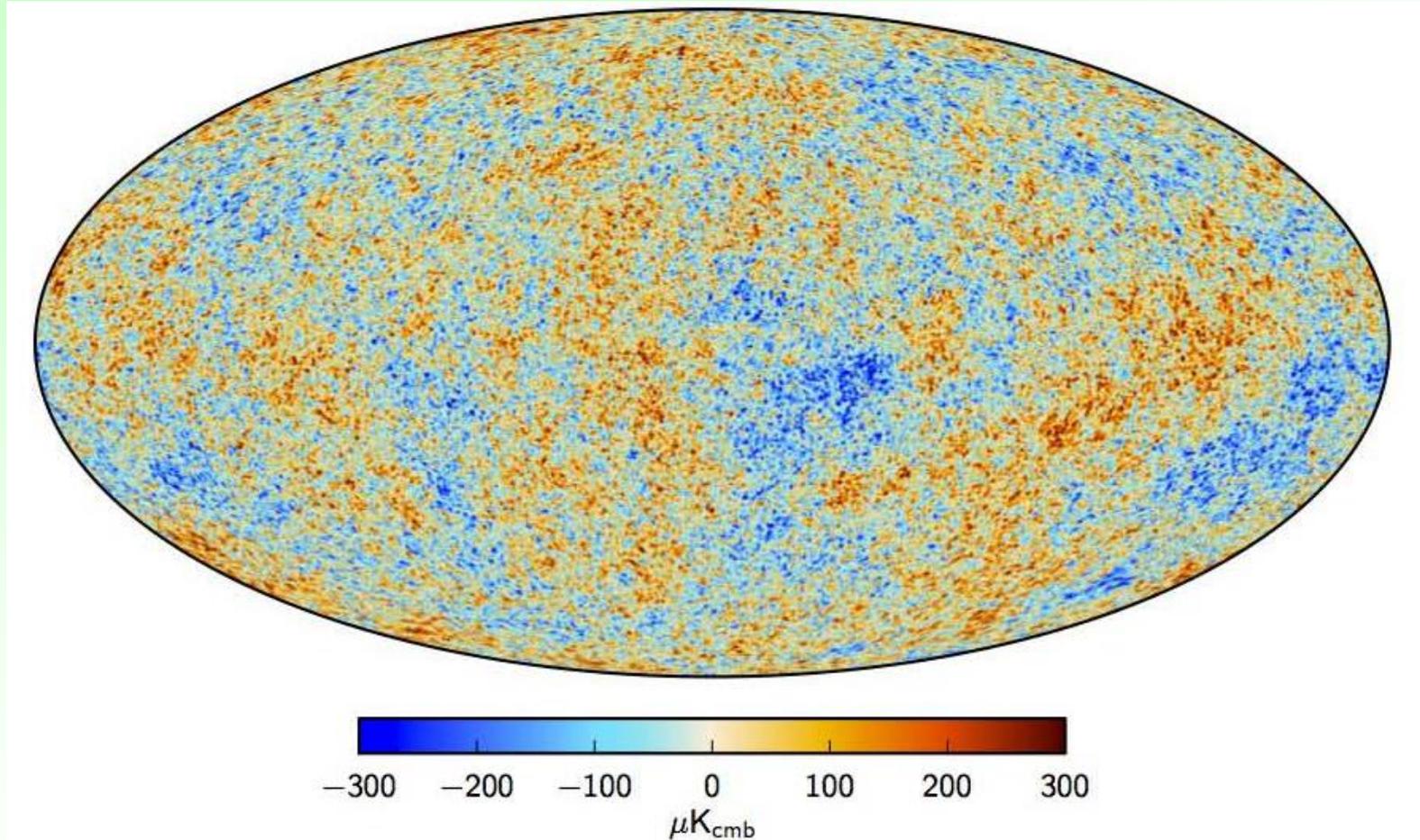
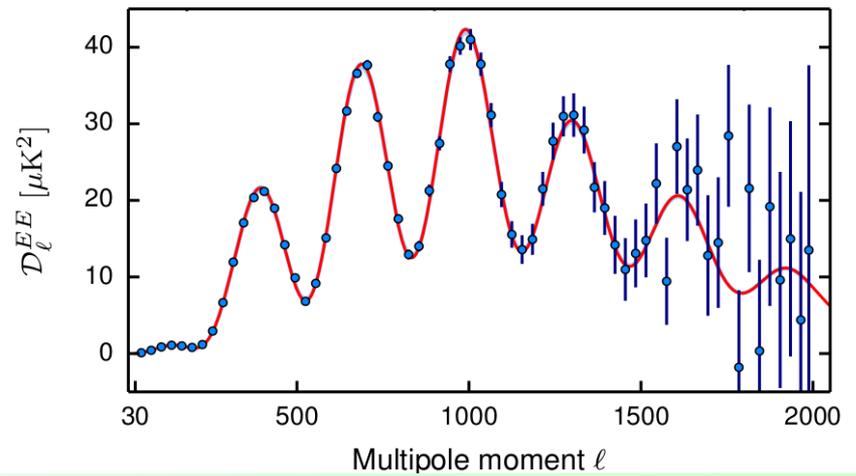
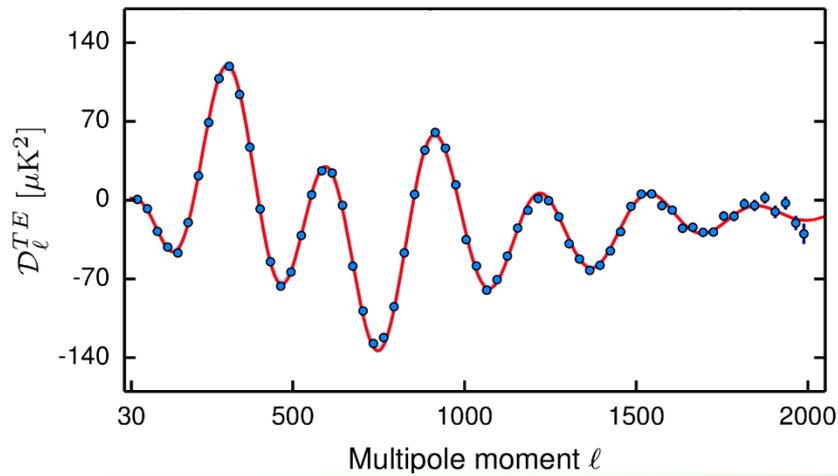
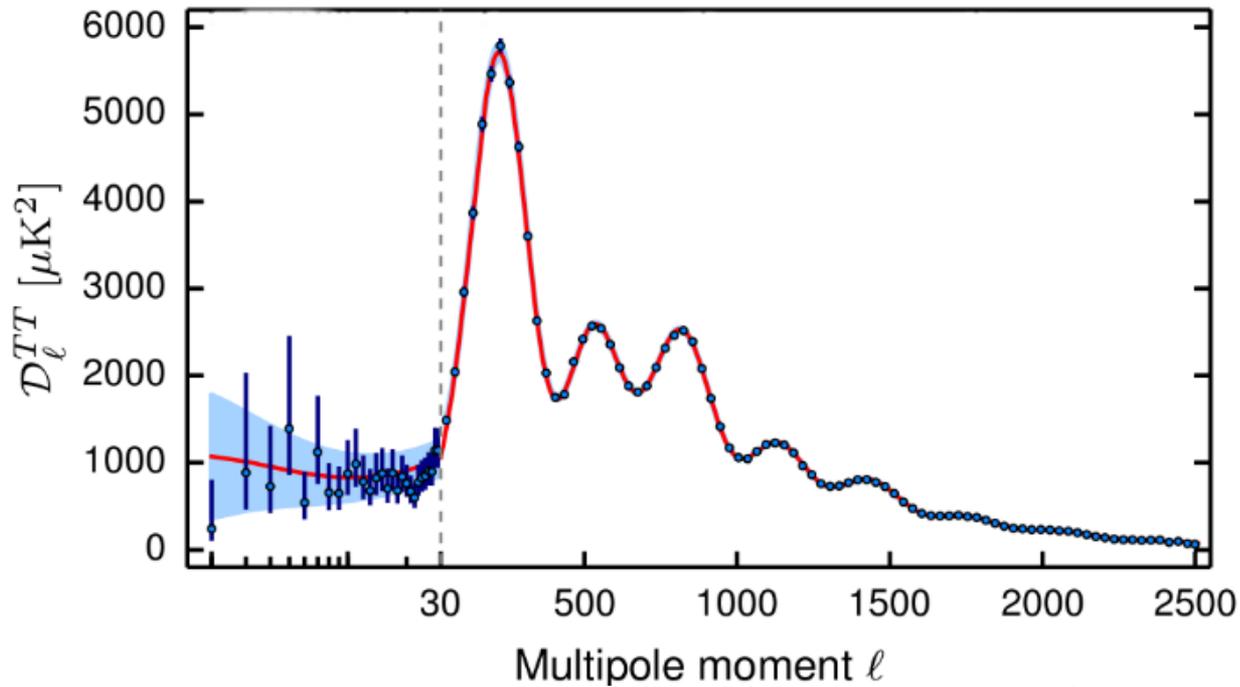


Fig. 7. Maximum posterior CMB intensity map at 5' resolution derived from the joint baseline analysis of *Planck*, WMAP, and 408 MHz observations. A small strip of the Galactic plane, 1.6% of the sky, is filled in by a constrained realization that has the same statistical properties as the rest of the sky.

Planck TT, TE & EE spectrum



- **Amplitude of curvature perturbation:**

$$\mathcal{R} = \frac{H^2}{2\pi\dot{\phi}} \Big|_{k/a=H} \quad \text{Mukhanov (1985), MS (1986)}$$

$$\mathcal{R}_{obs} \sim 10^{-5} \Rightarrow V^{1/4}(\phi) \sim 10^{16} \text{ GeV}$$

- **Power spectrum index:**

$$M_{pl} \equiv \frac{1}{\sqrt{8\pi G}} \sim 2.4 \times 10^{18} \text{ GeV: Planck mass}$$

$$\frac{4\pi k^3}{(2\pi)^3} P_{\mathcal{R}}(k) = Ak^{n_s-1} ; \quad n_s - 1 = M_{pl}^2 \left(2 \frac{V''}{V} - 3 \frac{V'^2}{V^2} \right)$$

$$n_{S, \text{Planck}} - 1 = -0.032 \pm 0.006 \Leftrightarrow n_s - 1 \sim -0.04 \text{ for a typical model}$$

- **Tensor (gravitational wave) spectrum:**

$$\frac{4\pi k^3}{(2\pi)^3} P_T(k) = Ak^{n_T} ; \quad n_T = -3 \frac{\dot{\phi}^2}{V} = -\frac{1}{8} \frac{P_{\mathcal{R}}(k)}{P_T(k)} \quad \text{Liddle-Lyth (1992)}$$

to be observed by **LiteBIRD/...**

PLANCK constraints

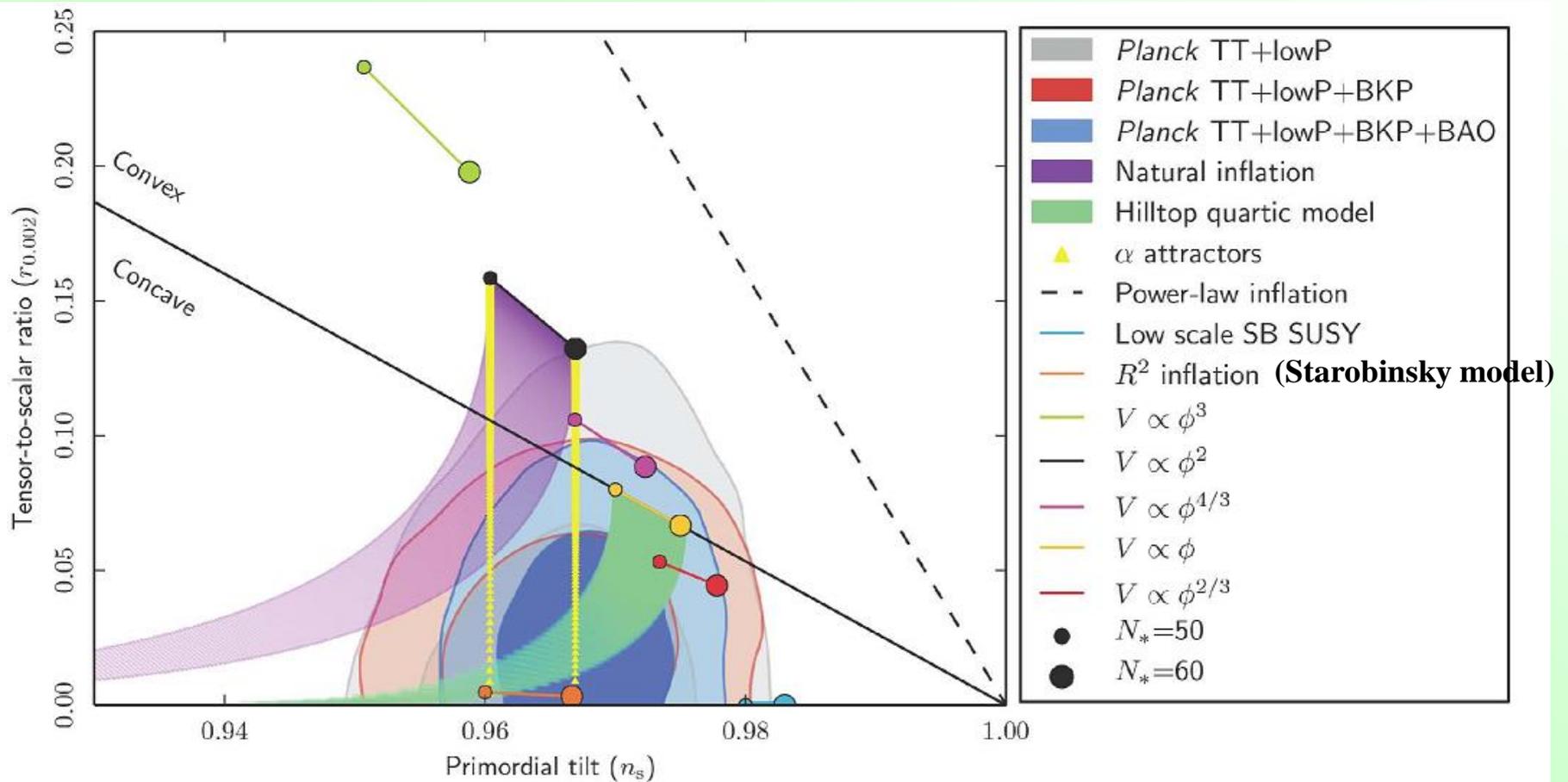
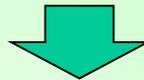


Fig. 54. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* alone and in combination with its cross-correlation with BICEP2/Keck Array and/or BAO data compared with the theoretical predictions of selected inflationary models.

Implications

Planck implications

- scalar spectral index: $n_s < 1$ at $\sim 5 \sigma$
- tensor/scalar ratio: $r < 0.1$ implies $E_{\text{inflation}} < 10^{16} \text{ GeV}$
- simple, **canonical models** are **on verge of extinction**
($m^2\phi^2$ model excluded at $> 2 \sigma$)
- R^2 (Starobinsky) model seems to fit best. **But why?**
(large R^2 correction but negligible higher order terms)
- $f_{\text{NL}}^{\text{local}} < O(1)$ suggests (effectively) **single-field slow-roll**
(but non-slow-roll models with $f_{\text{NL}}^{\text{local}} = O(1)$ not excluded)



perhaps elements of **non-canonicity** is needed

non-canonical single-field models

- **Non-canonical kinetic term? ($c_s < 1$?)**

$$P_{\mathcal{R}} \propto \frac{1}{c_s} \quad (c_s: \text{sound speed}), \quad f_{\text{NL}}^{\text{equil}} \propto \frac{1}{c_s^2}$$

Planck: $c_s > 0.024$ at 95% CL

- **non-minimal coupling to gravity?**

$$V(\phi) + \xi \phi^2 R \quad \longrightarrow \quad r = \frac{P_T(k)}{P_{\mathcal{R}}(k)} \propto \frac{1}{\xi}$$

Planck: $\xi > \mathcal{O}(10)$?

- **scalar-tensor with derivative couplings (Hordeski) ?**

$$c_s < 1, \quad c_{s,T} < 1, \quad c_s \neq c_{s,T}$$



tensor propagation speed

non-existence of
Einstein frame?

other possibilities

WMAP/Planck anomalies:

suppression of $\delta T/T$ at $l < 10$?

hemispherical asymmetry of $\delta T/T$ at $l < 30$?



- **featured models:** heavy fields, particle creation, trans-Planckian, ...
- **open inflation**, supercurvature modulation, ...

still viable if $\Omega_K > 10^{-3}$

$$\lambda \gg R_{\text{curvature}}$$



$$\gg H_0^{-1} : \text{ supercurvature mode}$$

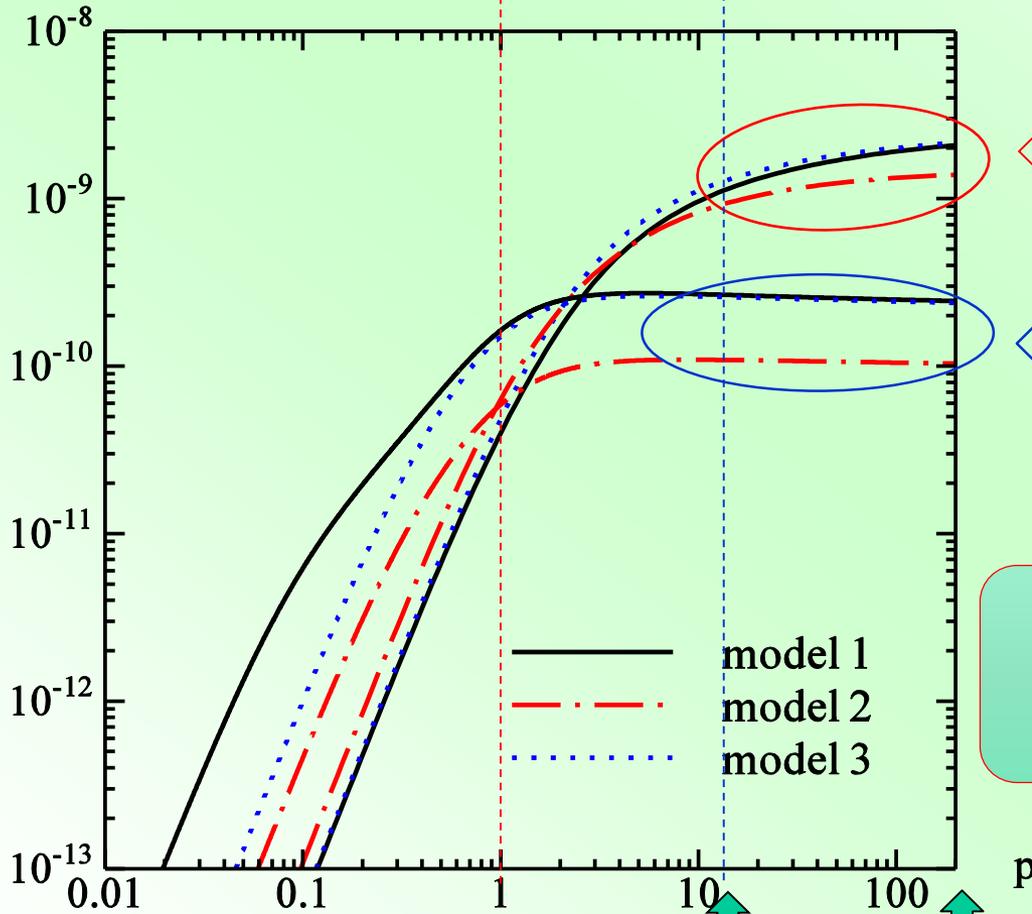
leading order effect on
our universe is **dipolar**

our
universe

scalar & tensor spectrum in open inflation

Linde, MS & Tanaka (1999)
White, Zhang & MS (2014)

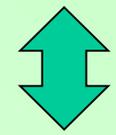
$$(|R_p|^2, |U_p|^2) p^3 / (2\pi^2)$$



← scalar

← tensor
(no suppression)

scalar suppression begins
at smaller scale



curvature
radius

H_0^{-1} if $\Omega_K \approx 0.01$

break scale

Future Issues

- **definition of inflation?** Domenech & MS '15
(conformal trans can give any expansion law)

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

$$d\tilde{s}^2 = \Omega^2(t)ds^2 \Rightarrow d\tilde{t} = \Omega(t)dt, \tilde{a}(\tilde{t}) = \Omega(t)a(t)$$

- **initial condition before inflation, multiverse?**
- **successful reheating?**
- **non-linear effects, non-Gaussianities?**
- **gravitational waves at second order?**
- **massive gravity?**
-

Identification of Inflaton!