Some thoughts on inflation and dark energy

RESEARCH CENTER FOR THE EARLY UNIVERSE
THE UNIVERSITY OF TOKYO
RESCEU

Department of Physics
SCHOOL OF SCIENCE
THE UNIVERSITY OF TOKYO

KAVLI IPMU

Jun’ichi Yokoyama

NB This is NOT a project-related talk, but just a wild talk (与太話) or a stimulus.

1 First I'm afraid I must say…

2 Having said that, however,…

3 An alternative idea about the beginning of the universe

4 Assignments

a) To Hazumi-san

b) To Miyazaki-san & Takada-san

c) To myself

1 First I’m afraid I must say…

新学術領域「なぜ宇宙は加速するのか？

- 徹底的究明と将来への挑戦-」

本来、科学の研究は、根源的な理由を問うことはしない。例えば、万有引力の法則を考える際には、重力がどのように働くか、つまり距離の何乗に比例して減るとか、質量に対してどのような依存性を持つか、とかいうことだけが研究の対象となり、「そもそもなぜこの世の中の万物には、重力が働くのか？」などという問いは、自然科学の範疇を超えたものである。

<<「輪廻する宇宙」（講談社）より

By nature, scientific research does not pursue fundamental reasons.
Questions asking WHY… may not be scientific but philosophical.
We SHOULD NOT ask philosophical questions because Science must be universal.

2 Having said that, however, once I have also thought about “Why there exists dark energy or cosmological constant Λ ?” and “What is the use of Λ?” when I was asked to give a talk entitled “From Cosmos to Particles” at a joint symposium of the Physical Society of Japan in 2007.

22pZE-2

宇宙から素粒子へ

東大理 横山順一
From Cosmos To Particles
Univ. of Tokyo Jun'ichi Yokoyama

現在、私たちの宇宙をめぐる研究は爆発的な勢いで急速に進展している。それを支える基盤を提供しているのが素粒子物理学である。すなわち今日、壮大な時空を探求する宇宙物理学と極微の世界を探求する素粒子物理学は融合し、大きな学問体系として展開しつつある。

I did so even if I knew they are not proper scientific questions, because the dark energy problem is such a difficult problem, and unconventional approach may be useful.

The Future of the Universe Dominated by Λ

- N-body simulation of the future of the nearby Universe

(Nagamine&Loeb 02)

☆ Evolution of LSS continues for 28Gyrs.

☆ Our local group will not be bound to the Virgo cluster.

☆ Our galaxy is likely to merge with Andromeda galaxy within Hubble time.

☆ This will be the only galaxy within the horizon 100Gyr later.

Extragalactic astronomy/Cosmology must be solved within 100Gyr!

The Future of the Universe Dominated by Λ

- Asymptotically De Sitter expansion

☆ Event horizon at 5.1Gpc.

☆ We cannot go to a galaxy with $z>1.8$. (Starobinsky 00)

☆ We may observe a galaxy at $z=5$ only for 6.4Gyr. (Loeb 02)

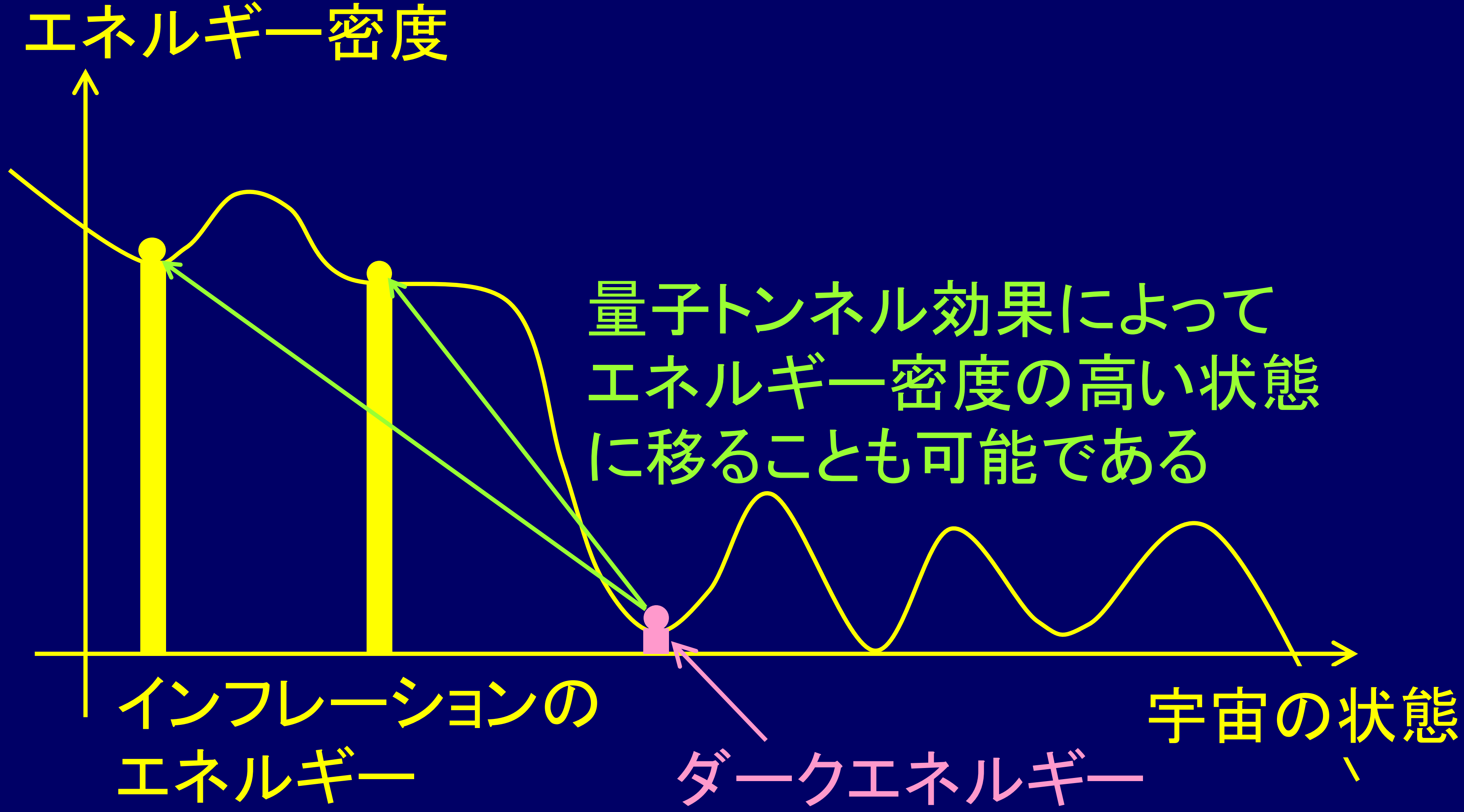
☆ Protons may decay eventually. $\tau_p > 10^{34}$ yr
Dark matter will be diluted or may decay.

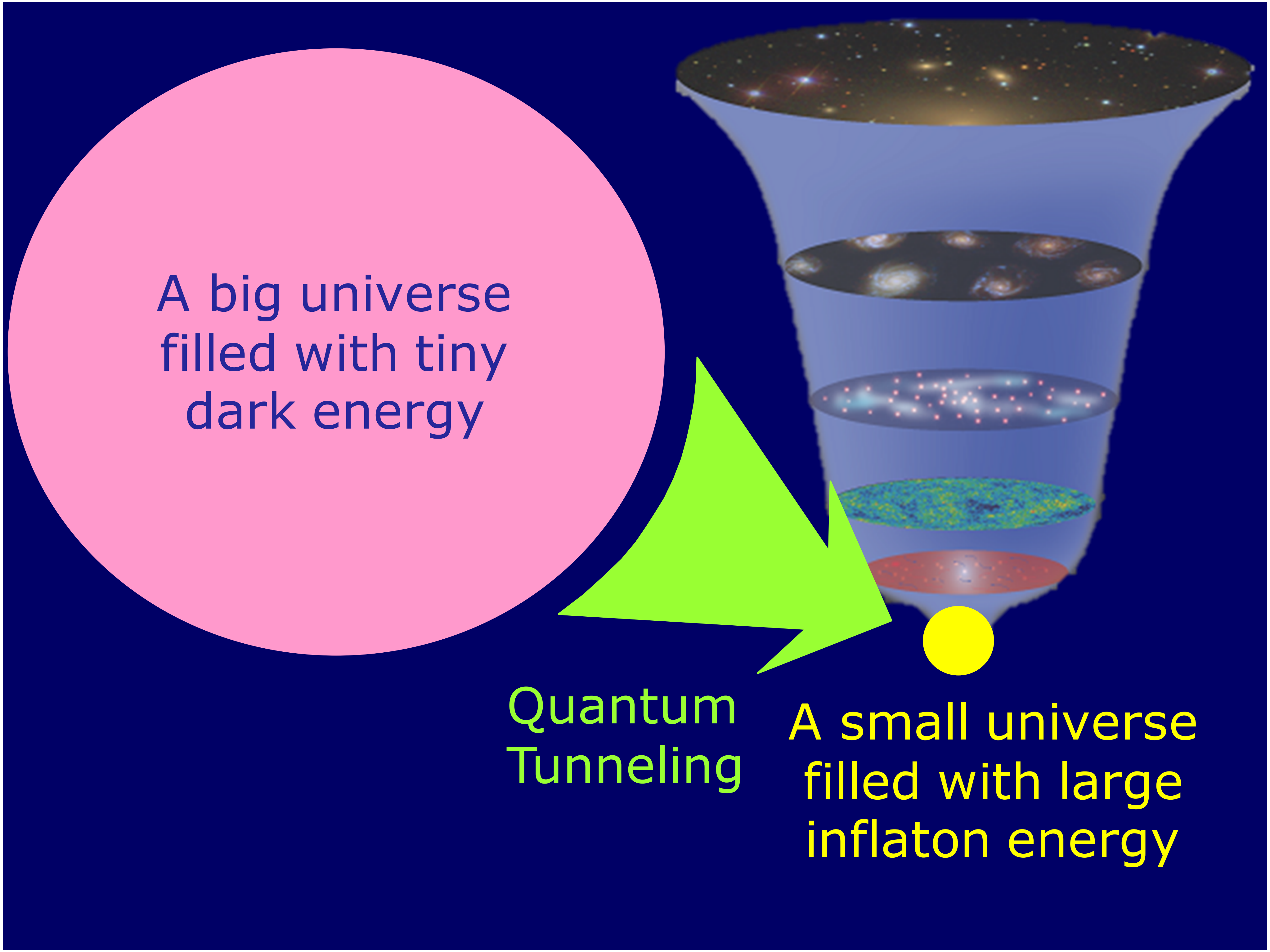
Black holes will evaporate. $\tau_{BH}(M) = 10^{64}\left(\frac{M}{M_{\odot}}\right)^3$ yr

⇨ **de Sitter Universe with $\Lambda = 10^{-120} M_{Pl}^4$**

A de Sitter universe can tunnel to another de Sitter space with a larger Λ and smaller radius.

初期宇宙のインフレーション時代もダークエネルギーと同じく「薄まらないエネルギー」（インフレーションのエネルギー）が宇宙を満たしていた。




A big universe filled with tiny dark energy
Quantum Tunneling
A small universe filled with large inflaton energy

Reincarnation of the universe

The universe can recycle itself to another universe with possibly different properties.

We may not have to consider the real beginning of the universe.

This scenario prefers a pure cosmological constant/vacuum energy with $w = -1$.

Later I found such a scenario had been studied by Garriga and Vilenkin in 1998, almost ten years before I realized.

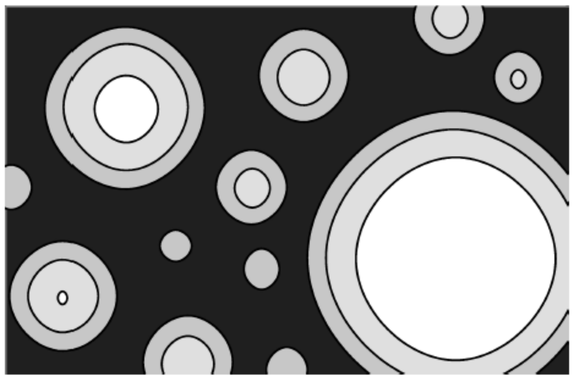


FIG. 1. True vacuum bubbles (white) nucleating in false vacuum (black). The shaded rings represent slow roll regions (external ring) and matter or radiation dominated regions (internal ring).

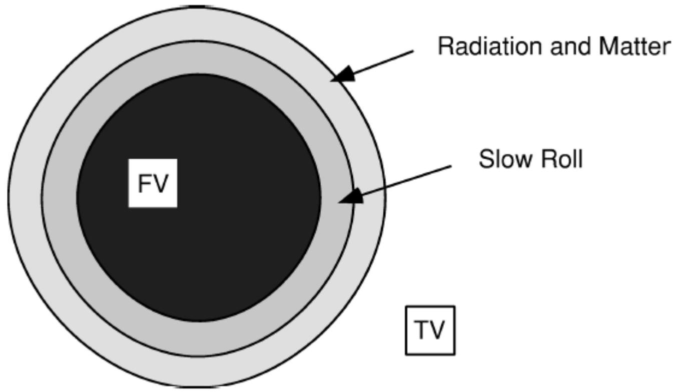


FIG. 4. A false vacuum bubble (black) nucleating in true vacuum (white). Regions of slow roll and of matter and radiation domination surrounding the bubble are indicated.

PHYSICAL REVIEW D VOLUME 57, NUMBER 4 15 FEBRUARY 1998

Recycling universe

Jaume Garriga
IFAE, Departament de Fisica, Universitat Autonoma de Barcelona, 08193 Bellaterra (Barcelona), Spain

Alexander Vilenkin
Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155
(Received 29 July 1997; published 12 January 1998)

If the effective cosmological constant is nonzero, our observable universe may enter a stage of exponential expansion. In such a case, regions of it may tunnel back to the false vacuum of an inflaton scalar field, and inflation with a high expansion rate may resume in those regions. An ''ideal'' eternal observer would then witness an infinite succession of cycles from false vacuum to true, and back. Within each cycle, the entire history of a hot universe would be replayed. If there were several minima of the inflaton potential, our ideal observer would visit each one of these minima with a frequency which depends on the shape of the potential. We generalize the formalism of stochastic inflation to analyze the global structure of the universe when this ''recycling'' process is taken into account. [S0556-2821(98)02904-X]

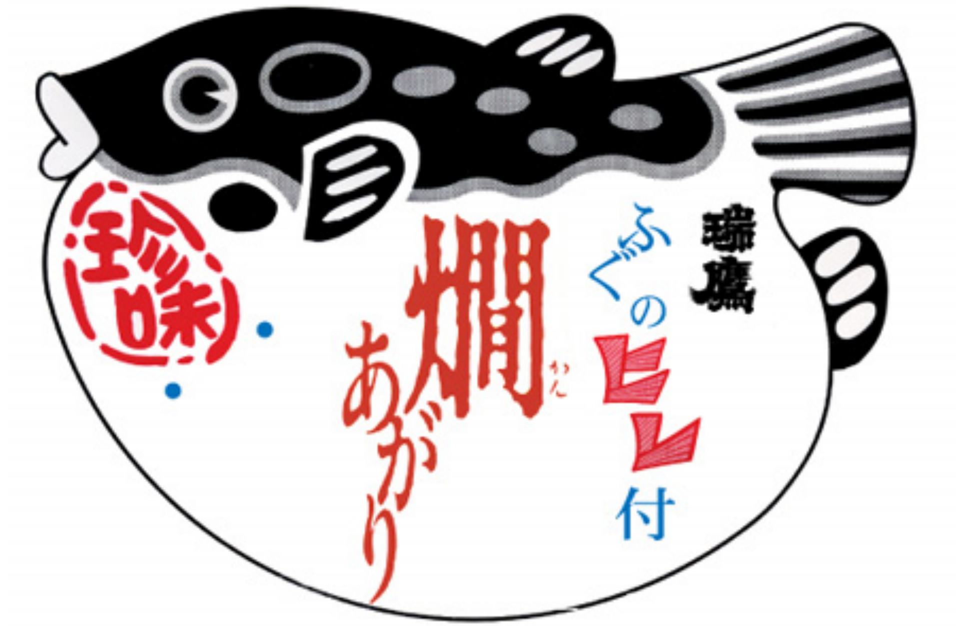
PACS number(s): 98.80.Hw, 98.80.Bp, 98.80.Cq

An "ideal" eternal observer would then witness an infinite succession of cycles from false vacuum to true, and back.

GOD??

Asking Why type question is thus very dangerous.
We may have to stop being a scientist.
But which is better or worse, adopting anthropic principle, or assuming an intelligent designer??

Such lines of thoughts may be poisonous on one hand but very interesting on the other.

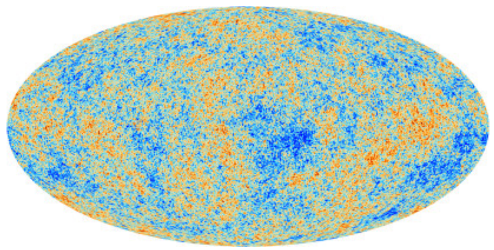


重要な註） ここまで聞いてきてヨコヤマもついに焼きが回ってきたかなどと思わないように。ここから先はまじめなはなしに戻ります。(もっとヘンかもしれないけど)

3 An alternative idea about the beginning of the universe

Despite the great advancements in precision observations such as those conducted by WMAP, Planck etc., there is no single observational result that is in contradiction with inflationary cosmology even 35 years after its proposal.

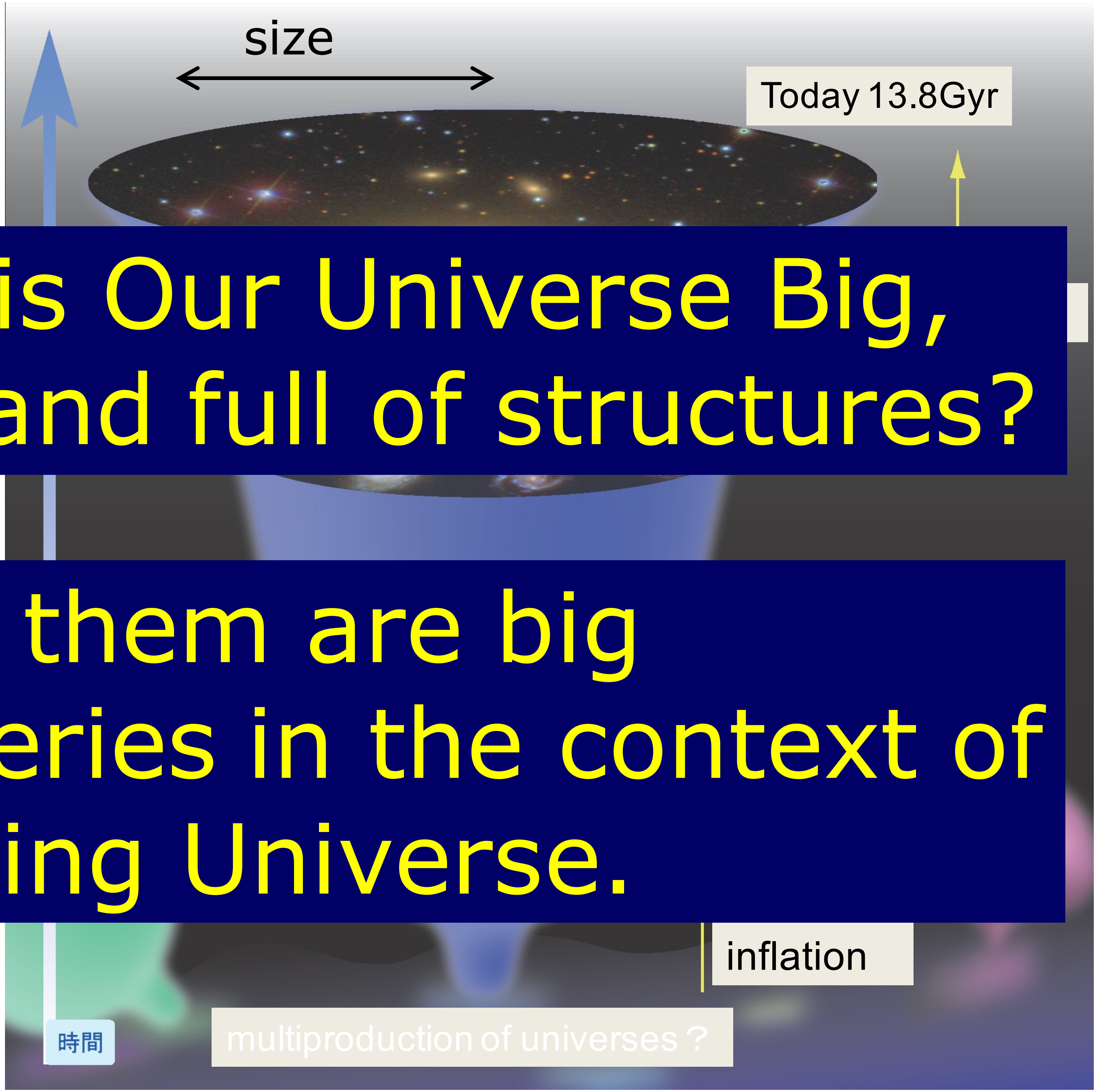
(Starobinsky 1980, Sato 1981, Guth 1981, Linde 1982,....)

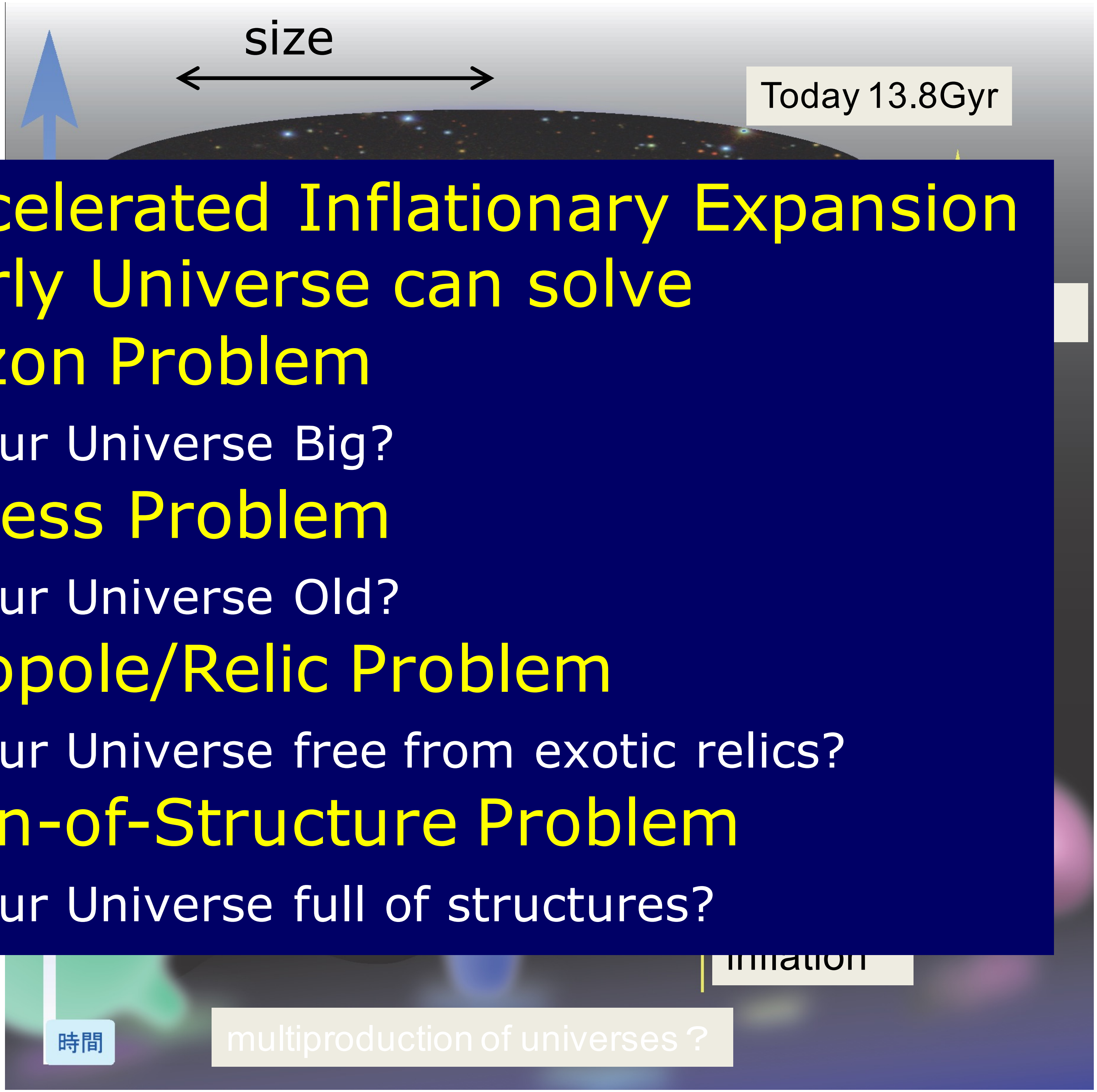


Indeed inflation in the early universe answers to such questions as

Why is Our Universe Big, Old, and full of structures?

All of them are big mysteries in the context of evolving Universe.





Rapid Accelerated Inflationary Expansion in the early Universe can solve

The Horizon Problem

Why is our Universe Big?

The Flatness Problem

Why is our Universe Old?

The Monopole/Relic Problem

Why is our Universe free from exotic relics?

The Origin-of-Structure Problem

Why is our Universe full of structures?

Does inflation solve all the problems in the early Universe?

Certainly NO. The initial singularity problem…

In order to achieve full understanding of the evolution of the Universe, we need to clarify what was there before inflation.

How did our Universe begin ?

Recycling? but if inflation lasts long enough, the beginning of inflation cannot be observed.

A more observationally relevant question:

Is inflation falsifiable?

Here, I would first like to consider the second question

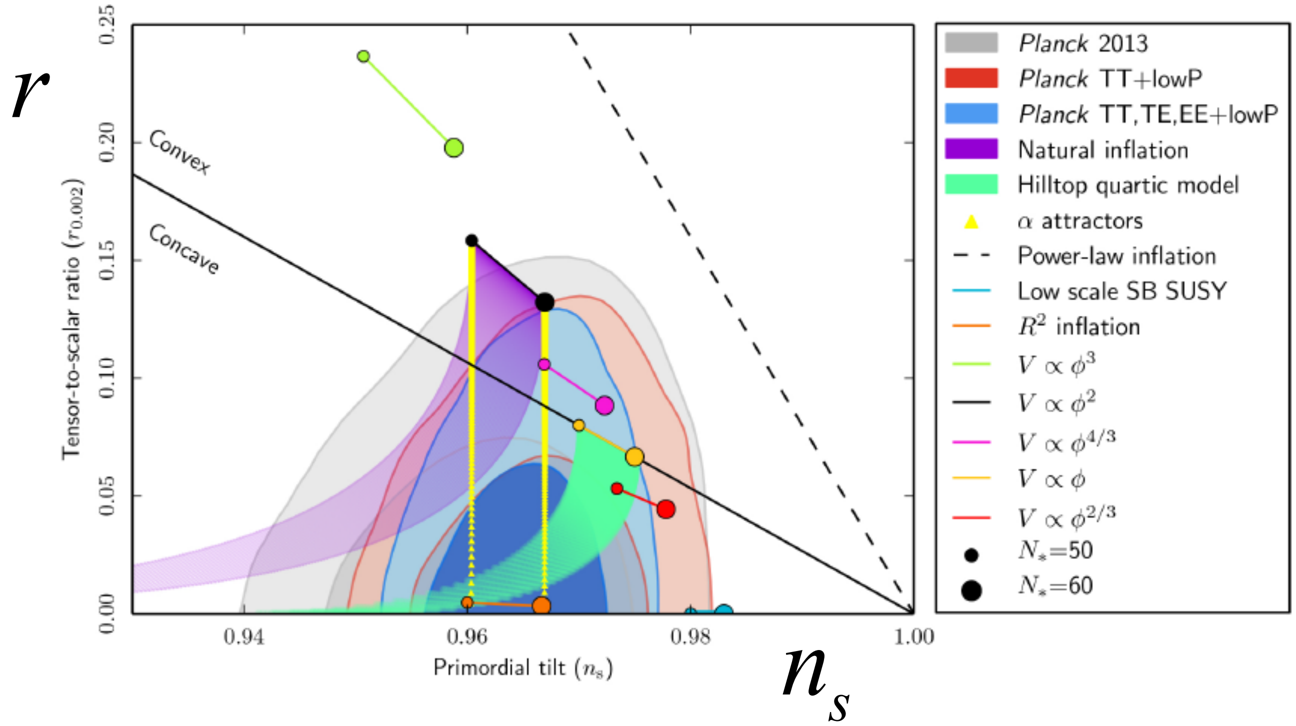
Is inflation falsifiable?

and then approach the first one

How did our Universe begin ?

apart from the case of recycling universe.

It is often claimed if...



Tensor perturbation and its spectral index in standard inflation

$$\left\langle h_{ij}h^{ij}(k)\right\rangle = \mathcal{P}_T(k) = \left.\frac{2H^2}{\pi^2 M_{Pl}^2}\right|_{t_k} \qquad n_t = \frac{d\ln\mathcal{P}_T(k)}{d\ln k} = -2\varepsilon_H = 2\frac{\dot{H}}{H^2}$$

It is often claimed that positive n_t would falsify inflation.

But this is based on a prejudice that both energy density and the Hubble parameter decrease in time in expanding universe.

$$\dot{\rho} = -3H(\rho + p). \qquad \dot{H} = -4\pi G(\rho + p)$$

$\dot{\rho} \leq 0$ in the expanding universe provided $\rho + p \geq 0$.

Null energy condition (NEC)

$T_{\mu\nu}\xi^{\mu}\xi^{\nu} \geq 0$ for any null vector ξ^{μ}

$$(g_{\mu\nu}\xi^{\mu}\xi^{\nu} = 0)$$

For perfect fluid $T_{\mu\nu} = (\rho + p)\, u_{\mu}u_{\nu} - g_{\mu\nu}p$

the null energy condition is equivalent with $\rho + p \geq 0$

As long as the NEC is satisfied,
the Universe cannot start from a low energy state.

How robust is the NEC ?

- Canonical scalar field with a potential:

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi). \quad S = \int \mathcal{L}\sqrt{-g}\,d^4x \quad T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}$$

$$\Rightarrow \begin{cases} \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases} \Rightarrow \rho + p = \dot{\phi}^2 \geq 0.$$

(NEC is satisfied)

- Noncanonical scalar field such as those used in K-inflation

(Armendariz-Picon, Damour, Mukhanov 1999)

$$\mathcal{L} = K(\phi, X), \qquad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.$$

$$T_{\mu\nu} = K_X\nabla_\mu\phi\nabla_\nu\phi - Kg_{\mu\nu} \Leftrightarrow T_{\mu\nu} = (\rho + p)\,u_\mu u_\nu - g_{\mu\nu}p$$

$$K_X = \frac{\partial K}{\partial X} \qquad u_\mu = \frac{\nabla_\mu\phi}{\sqrt{2X}}$$

$$\Rightarrow \begin{cases} \rho = 2XK_X - K \\ p = K \end{cases} \Rightarrow \rho + p = 2XK_X.$$

If $K_X < 0$, it violates the NEC. But…

Curvature perturbations in K-inflation

(Garriga & Mukhanov 1999)

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R+K(X,\phi)\right]$$

$$\mathrm{d}s^2=-N^2\mathrm{d}t^2+h_{ij}(\mathrm{d}x^i+N^i\mathrm{d}t)(\mathrm{d}x^j+N^j\mathrm{d}t),\quad h_{ij}\equiv a^2(t)e^{2\varsigma}\delta_{ij},$$

Curvature perturbation ς satisfies the following action

$$S_2=M_{Pl}^2\int \mathrm{d}t\mathrm{d}^3x a^3\left[\frac{\Sigma}{H^2}\dot{\varsigma}^2-\varepsilon_H\frac{(\partial\varsigma)^2}{a^2}\right],\quad \varepsilon_H\equiv-\frac{\dot{H}}{H^2}$$

$$M_{Pl}^2\Sigma\equiv XK_X+2X^2K_{XX}$$

If $\dot{H}=-4\pi G(\rho+p)>0$ and ε_H is negative, the square sound speed is negative and small scale perturbations grows rapidly, causing gradient instability.

K-inflation violating the NEC suffers from gradient instability

Indeed in all potential-driven inflation and viable K-inflation, the NEC is satisfied and the Hubble parameter decreases with time.

$n_t < 0$ in these models

$n_t > 0$ would falsify these models.

The story is different in theories with higher derivative action but with 2nd order field equations.

G Inflation

(Kobayashi, Yamaguchi, JY 2010)

$$\mathcal{L}_{\phi} = K(\phi, X) - G(\phi, X)\Box\phi \qquad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi.$$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + \mathcal{L}_\phi\right]$$

Energy density and Pressure

$$\rho = 2K_X X - K + 3G_X H\dot{\phi}^3 - 2G_\phi X$$

$$p = K - 2(G_\phi + G_X\ddot{\phi})X.$$

Inflationary solution can easily be found in a similar way as K-inflation

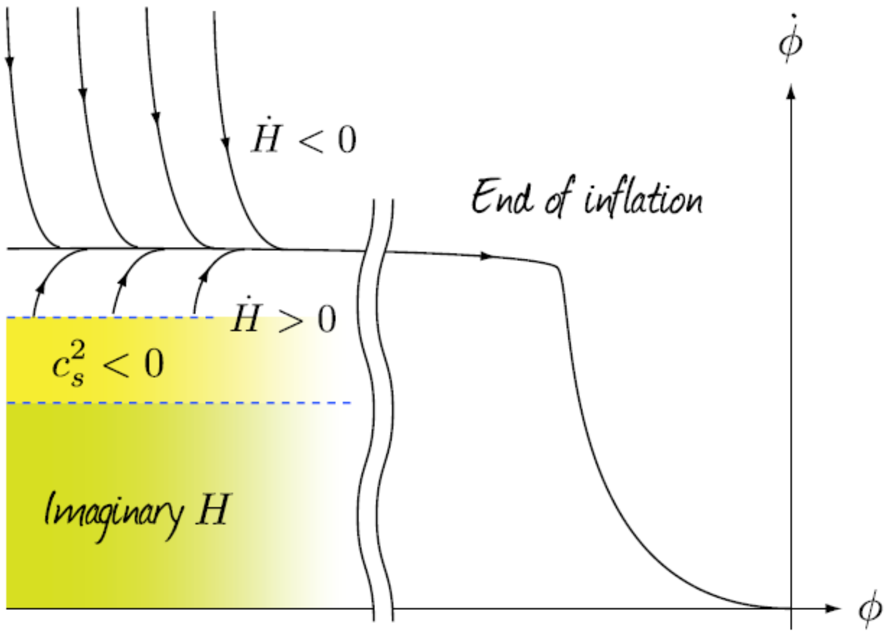
Crucial difference in the action of curvature perturbation

$$S^{(2)} = \frac{1}{2}\int \mathrm{d}\tau \mathrm{d}^3x\, z^2 \left[\mathcal{G}\varsigma'^2 - \mathcal{F}\left(\nabla\varsigma\right)^2\right] \qquad z := \frac{a\dot{\phi}}{H - G_X\dot{\phi}^3/2M_{\rm pl}^2},$$

$$\mathcal{F} := K_X + 2G_X(\ddot{\phi} + 2H\dot{\phi}) - 2\frac{G_X^2}{M_{\rm pl}^2}X^2 + 2G_{XX}X\ddot{\phi} - 2(G_\phi - XG_{\phi X}),$$

$$\mathcal{G} := K_X + 2XK_{XX} + 6G_XH\dot{\phi} + 6\frac{G_X^2}{M_{\rm pl}^2}X^2 - 2(G_\phi + XG_{\phi X}) + 6G_{XX}HX\dot{\phi},$$

The sound speed $c_s^2 = \mathcal{F}/\mathcal{G}$ can be positive even if the NEC is violated.



Inflation with $\dot{H} > 0$ is possible without instabilities.

n_t may be positive in G inflation.

If inflation with $\dot{H} > 0$ is possible,
the universe may start from a low energy,
asymptotically Minkowski space,
so that the initial singularity problem
of inflationary cosmology may be solved…

4a) Assignment to Hazumi-san

Try to observe a positive tensor spectral index,

$$n_t > 0 \ !$$

**If inflation with $\dot{H} > 0$ is possible,
the universe may start from a low energy,
asymptotically Minkowski space,
so that the initial singularity problem
of inflationary cosmology may be solved…**

Construction of such a model had been our objective ever since we proposed G inflation,
but we had not been successful for a long time;
meanwhile a number of relevant work was done by a number of authors…

Galilean Genesis

(Creminelli et al. 2010)

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_G^2 R + f^2 e^{2\phi} (\partial\phi)^2 + \frac{f^3}{\Lambda^3} (\partial\phi)^2 \Box\phi + \frac{f^3}{2\Lambda^3} (\partial\phi)^4 \right]$$

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} \qquad T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu - g_{\mu\nu} p$$

$$\rho = -f^2 \left(e^{2\phi} \dot\phi^2 - \frac{3}{2} \frac{f}{\Lambda^3} \dot\phi^4 - 6H \frac{f}{\Lambda^3} \dot\phi^3 \right),$$

$$p = -f^2 \left(e^{2\phi} \dot\phi^2 - \frac{1}{2} \frac{f}{\Lambda^3} \dot\phi^4 + 2 \frac{f}{\Lambda^3} \dot\phi^2 \ddot\phi \right).$$

Based on Galileon: higher derivative theories with Galilean invariance $\partial_\mu \phi \longrightarrow \partial_\mu \phi + b_\mu$ whose field equations are 2nd order.

(Nicolis, Rattazzi, & Trincherini 2009)

Its extension to curved spacetime yields Generalized Galileon or Horndeski theory, again with 2nd order field equations.

(Deffayet, Gao, Steer, Zahariade, 2011)
(Horndeski, 1974) (Kobayashi, Yamaguchi & JY 2011)

Background solution from $t \to -\infty$

$$e^{\phi} \simeq \frac{1}{\sqrt{2Y_0}}\frac{1}{(-t)}, \quad H \simeq \frac{h_0}{(-t)^3}, \quad a(t) \simeq 1 + \frac{h_0}{2(-t)^2} \quad \left(Y_0 \equiv \frac{\Lambda^3}{3f}, \quad h_0 \equiv \frac{1}{2M_G^2}\frac{f^3}{\Lambda^3}\right)$$

The Hubble parameter increases in time with NEC violation

$$\rho + p \simeq -\frac{f^3}{\Lambda^3}\frac{4}{(-t)^4} < 0.$$

There is no instability, since the action for the curvature perturbation ζ reads

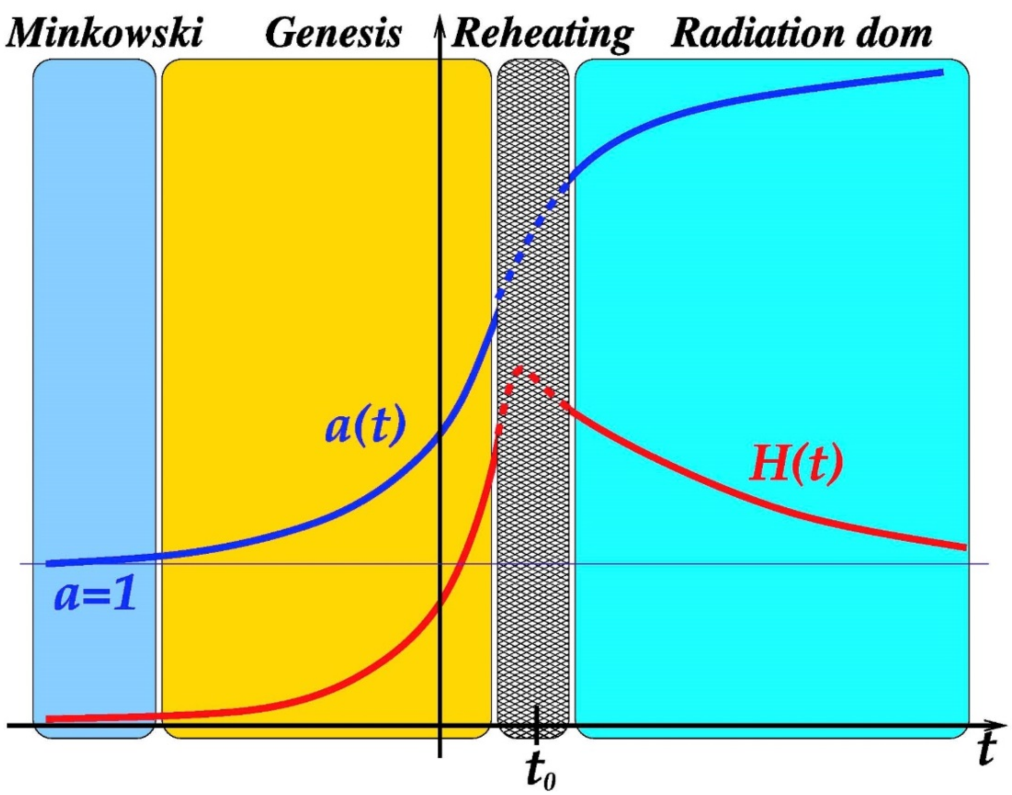
$$S_S^{(2)} = \frac{1}{2}\int dt d^3x a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\nabla\zeta)^2\right],$$

with

$$\mathcal{G}_s = \mathcal{F}_s \simeq 6M_G^4 \frac{\lambda^3}{f^3}(-t)^2 > 0.$$

(But the spectrum of curvature perturbation is blue, $n_s \approx 3$.)

But this model breaks down before turning into radiation dominated regime.



(figure taken from Creminelli et al. 1007.0027)

The scale factor would diverge in a finite time just like the big rip singularity in dark energy models with w < -1.

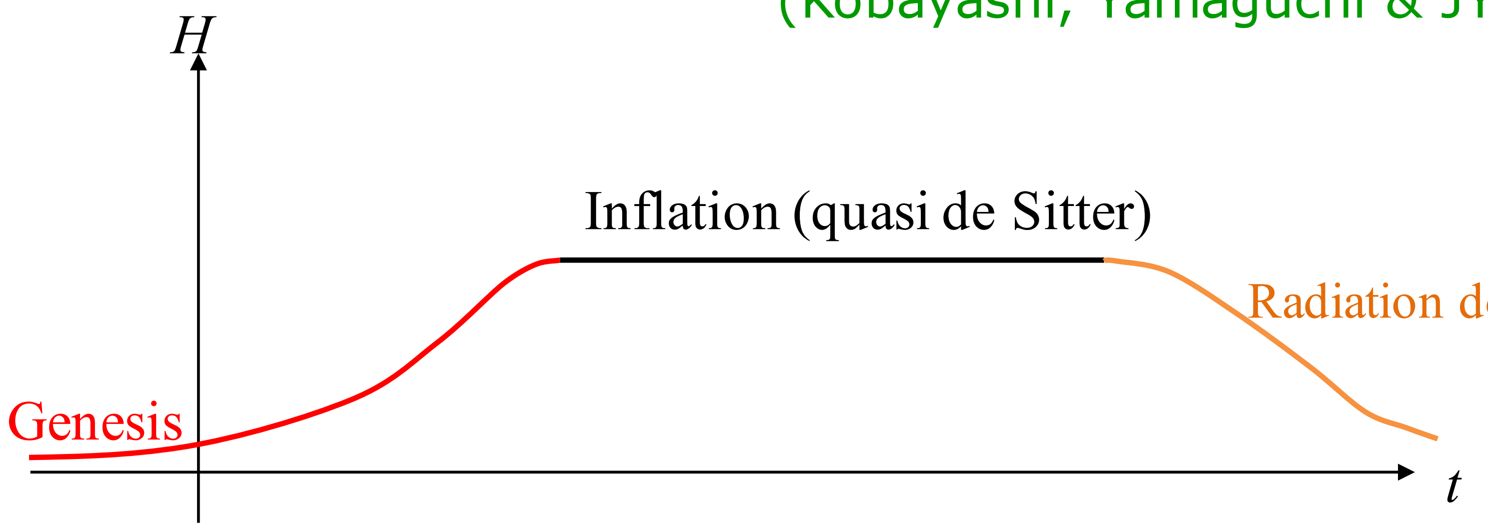
Difficult to realize genesis-big-bang transition in a sensible manner without any instabilities.

Our study on inflation starting from Minkowski space was preceded by Pirtskhalva et al 1410.0882
but their model yields a negative square sound speed temporarily, which must be avoided even for a tiny period because short wave perturbations grows exponentially.

$$\propto e^{\mathrm{Im}(c_s)kt}$$

We have finally succeeded in constructing a model to realize Genesis-Inflation-Big Bang transition without any instabilities, making use of the Beyond-Horndeski Theories .

(Kobayashi, Yamaguchi & JY 2015)



Our study on inflation starting from Minkowski space was preceded by Pirtskhalva et al 1410.0882
but their model yields a negative square sound speed temporarily, which must be avoided even for a tiny period because short wave perturbations grows exponentially.

$$\propto e^{\mathrm{Im}(c_s)kt}$$

We have finally succeeded in constructing a model to realize Genesis-Inflation-Big Bang transition without any instabilities, making use of the Beyond-Horndeski Theories .

Beyond-Horndeski Theory

Horndeski theory or Generalized Galileon gives 2nd order field equations both in time and spatial coordinates, but in order to avoid instabilities, we only need field equations 2nd order in time, not necessarily in space.

From Horndeski theory

$$\mathcal{L}_2 = K(\phi, X),$$
$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi,$$
$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$
$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].$$

Apply the ADM decomposition taking $\phi =$const surface as t =const surface

$$ds^2 = -N^2dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

Then $\phi = \phi(t),\ X = \frac{\dot{\phi}^2(t)}{2N^2}$ so that functions of (ϕ, X) can be regarded as those of (t, N) provided $\dot{\phi}N^{-1}$ does not vanish.

In this representation, the Horndeski theory reads

$$\mathcal{L} = \sqrt{\gamma}N\sum_a L_a$$

$$(t, N) \overset{\dot{\phi}N^{-1} \neq 0}{\Longleftrightarrow} (\phi, X)$$

$$L_2 = A_2(t, N),$$
$$L_3 = A_3(t, N)K,$$
$$L_4 = A_4(t, N)\left(K^2 - K_{ij}^2\right) + B_4(t, N)R^{(3)},$$

with $A_4 = -B_4 - N\frac{\partial B_4}{\partial N}, \quad A_5 = \frac{N}{6}\frac{\partial B_5}{\partial N}.$

$$L_5 = A_5(t, N)\left(K^3 - 3KK_{ij}^2 + 2K_{ij}^3\right) + B_5(t, N)K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2}g_{ij}R^{(3)}\right).$$

Geometrical expression of the Beyond Horndeski theory

$$\mathcal{L} = \sqrt{\gamma} N \sum_a L_a$$

$R_{ij}^{(3)}$: spatial curvature

$$K_{ij} = \frac{1}{2N}\left(\frac{\partial \gamma_{ij}}{\partial t} - D_i N_j - D_j N_i\right)$$: extrinsic curvature

$$L_2 = A_2(t, N),$$
$$L_3 = A_3(t, N)K,$$
$$L_4 = A_4(t, N)\left(K^2 - K_{ij}^2\right) + B_4(t, N)R^{(3)},$$
$$L_5 = A_5(t, N)\left(K^3 - 3KK_{ij}^2 + 2K_{ij}^3\right) + B_5(t, N)K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2}g_{ij}R^{(3)}\right).$$

In terms of a vector n_μ normal to the $\phi = const$ surface, $n_\mu = -\frac{\nabla_\mu \phi}{\sqrt{2X}}$, the extrinsic curvature can be expressed as

$$K_{\mu\nu} = -\frac{\nabla_\nu \nabla_\mu \phi}{\sqrt{2X}} + n_\mu n^\rho \nabla_\rho n_\nu + n_\nu n^\rho \nabla_\rho n_\mu + \frac{n_\mu n_\nu n^\rho \nabla_\rho X}{2X}$$

so $K = -\frac{1}{\sqrt{2X}}\left(\Box\phi - \frac{\nabla^\rho \phi \nabla_\rho X}{2X}\right)$ etc. reproduces derivative interactions

of ϕ in (Beyond) Horndeski theory.

$$\mathcal{L} = \sqrt{\gamma} N \sum_a L_a$$

$$L_2 = A_2(t, N),$$
$$L_3 = A_3(t, N) K,$$
$$L_4 = A_4(t, N) \left(K^2 - K_{ij}^2 \right) + B_4(t, N) R^{(3)},$$
$$L_5 = A_5(t, N) \left(K^3 - 3 K K_{ij}^2 + 2 K_{ij}^3 \right) + B_5(t, N) K^{ij} \left(R_{ij}^{(3)} - \frac{1}{2} g_{ij} R^{(3)} \right).$$

$R_{ij}^{(3)}$: spatial curvature

K_{ij} : extrinsic curvature

In fact, this theory yields field equations 2nd order in time even without the relations

$$A_4 = -B_4 - N \frac{\partial B_4}{\partial N}, \qquad A_5 = \frac{N}{6} \frac{\partial B_5}{\partial N}.$$

(Gleyzed, Langlois, Piazza, Vernizzi 1404.6495 hereafter referred to as GLPV)

Further generalization is also possible. (Gao 1406.0822)

$$\mathcal{L} = \sqrt{\gamma} N \Big[d_0 + d_1 R^{(3)} + d_2 \left(R^{(3)} \right)^2 + \cdots + (a_0 + a_1 R^{(3)} + \cdots) K$$
$$+ \left(a_2 R^{(3)ij} + \cdots \right) K_{ij} + b_1 K^2 + b_2 K_{ij} K^{ij} + \cdots \Big]$$

The Hamiltonian analysis of these theories shows that they have three propagating modes as they should.

Our setup

$$\mathcal{L} = \sqrt{\gamma} N \sum_a L_a$$

$$\begin{cases} L_2 = A_2(t, N), \\ L_3 = A_3(t, N) K, \\ L_4 = A_4(t, N)\left(\lambda_1 K^2 - K_{ij}^2\right) + B_4(t, N) R^{(3)}, \\ L_5 = A_5(t, N)\left(\lambda_2 K^3 - 3\lambda_3 K K_{ij}^2 + 2K_{ij}^3\right) \\ \qquad + B_5(t, N) K^{ij}\left(R_{ij}^{(3)} - \frac{1}{2} g_{ij} R^{(3)}\right). \end{cases}$$

(The GLPV theory corresponds to the case with $\lambda_1 = \lambda_2 = \lambda_3 = 1$.)

Homogeneous and isotropic part

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\boldsymbol{x}^2 \quad H = \frac{1}{N}\frac{\dot{a}}{a}$$

$$\eta_4 := (3\lambda_1 - 1)/2$$

$$\eta_5 := (9\lambda_2 - 9\lambda_3 + 2)/2$$

$$\mathcal{L}^{(0)} = N a^3 \left(A_2 + 3A_3 H + 6\eta_4 A_4 H^2 + 6\eta_5 A_5 H^3\right)$$

Field equations

$$-\mathcal{E} := (N A_2)' + 3N A_3' H + 6\eta_4 N^2 (N^{-1} A_4)' H^2 + 6\eta_5 N^3 (N^{-2} A_5)' H^3 = 0,$$

$$\mathcal{P} := A_2 - 6\eta_4 A_4 H^2 - 12\eta_5 A_5 H^3 - \frac{1}{N}\frac{\mathrm{d}}{\mathrm{d}t}\left(A_3 + 4\eta_4 A_4 H + 6\eta_5 A_5 H^2\right) = 0,$$

Some remarks

In the standard cosmology with $ds^2 = -N^2(t)dt^2 + a^2(t)d\mathbf{x}^2, \;\; N(t) = 1$
00 component of the Einstein equation yields the Friedman equation

$$H^2 = \frac{8\pi G}{3}\rho \qquad (1)$$

and $j\,j$ component yields 2nd order evolution equation

$$\frac{\ddot{a}}{a} + \frac{H^2}{2} = -4\pi G p \qquad (2)$$

Matter field equation is given from the conservation law of the energy momentum tensor.

$$\dot{\rho} + 3H(\rho + p) = 0 \qquad \text{or} \qquad \ddot{\phi} + 3H\dot{\phi} + V'[\phi] = 0 \qquad (3)$$

As is well known, only two of the three are independent.
We usually solve (1) and (3),
but here we are using equations corresponding to (1) and (2) instead.

Perturbations

$$ds^2 = -N^2dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

$$N = \overline{N}(t)\left(1 + \delta n\right),$$
$$N_i = \overline{N}(t)\partial_i\chi,$$
$$\gamma_{ij} = a^2(t)e^{2\zeta}\left(e^h\right)_{ij}.$$

curvature perturbations

tensor perturbations

$\left(h_{ii} = h_{ij,j} = 0\right)$

- Tensor perturbations :

$$\mathcal{L}_T^{(2)} = \frac{\overline{N}a^3}{8}\left[\frac{\mathcal{G}_T}{\overline{N}^2}\dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\partial h_{ij})^2\right]$$

$$\mathcal{G}_T := -2A_4 - 6\left(3\lambda_3 - 2\right)A_5 H,$$
$$\mathcal{F}_T := 2B_4 + \frac{1}{\overline{N}}\frac{dB_5}{dt}.$$

- Curvature perturbations :

higher order spatial derivative

$$\mathcal{L}_S^{(2)} = \overline{N}a^3\left[\mathcal{G}_S\frac{\dot{\zeta}^2}{\overline{N}^2} + \zeta\left(\mathcal{F}_S\frac{\partial^2}{a^2} - \mathcal{H}_S\frac{\partial^4}{a^4}\right)\zeta\right]$$

∂^2 : Laplacian in the Cartesian coordinate

⇩ Dispersion relation

$$\omega^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S}k^2 + \frac{\mathcal{H}_S}{\mathcal{G}_S}\frac{k^4}{a^2}.$$

$$\mathcal{G}_S := \frac{\Sigma\mathcal{G}_T^2}{\Theta^2+\Sigma\mathcal{C}} + 3\mathcal{G}_T,$$

$$\mathcal{F}_S := \frac{1}{\overline{N}a}\frac{d}{dt}\left(\frac{a\Theta\mathcal{G}_B\mathcal{G}_T}{\Theta^2+\Sigma\mathcal{C}}\right) - \mathcal{F}_T,$$

$$\mathcal{H}_S := \frac{\mathcal{G}_B^2\mathcal{C}}{\Theta^2+\Sigma\mathcal{C}}.$$

$$\begin{aligned}
\Sigma &:= \overline{N}A_2' + \frac{1}{2}\overline{N}^2A_2'' + \frac{3}{2}\overline{N}^2A_3''H \\
&\quad +3\eta_4\left(2A_4 - 2\overline{N}A_4' + \overline{N}^2A_4''\right)H^2 \\
&\quad +3\eta_5\left(6A_5 - 4\overline{N}A_5' + \overline{N}^2A_5''\right)H^3, \\
\Theta &:= \frac{\overline{N}A_3'}{2} - 2\eta_4\left(A_4 - \overline{N}A_4'\right)H \\
&\quad -3\eta_5\left(2A_5 - \overline{N}A_5'\right)H^2, \\
\mathcal{G}_A &:= -2\eta_4A_4 - 6\eta_5A_5H, \\
\mathcal{G}_B &:= 2\left(B_4 + \overline{N}B_4'\right) - H\overline{N}B_5', \\
\mathcal{C} &:= (1-\lambda_1)A_4 - (6+9\lambda_2-15\lambda_3)A_5H. \\
\eta_4 &:= (3\lambda_1-1)/2,\ \eta_5 := (9\lambda_2-9\lambda_3+2)/2
\end{aligned}$$

- Curvature perturbations :

higher order spatial derivative

$$\mathcal{L}_S^{(2)} = \overline{N} a^3 \left[\mathcal{G}_S \frac{\dot{\zeta}^2}{\overline{N}^2} + \zeta \left(\mathcal{F}_S \frac{\partial^2}{a^2} - \mathcal{H}_S \frac{\partial^4}{a^4} \right) \zeta \right]$$

∂^2 : Laplacian in the Cartesian coordinate

Dispersion relation

$$\omega^2 = \frac{\mathcal{F}_S}{\mathcal{G}_S} k^2 + \frac{\mathcal{H}_S}{\mathcal{G}_S} \frac{k^4}{a^2}.$$

c_s^2 may take a negative value temporarily.

$$\mathcal{G}_S := \frac{\Sigma \mathcal{G}_T^2}{\Theta^2 + \Sigma \mathcal{C}} + 3\mathcal{G}_T,$$

$$\mathcal{F}_S := \frac{1}{\overline{N} a} \frac{d}{dt} \left(\frac{a \Theta \mathcal{G}_B \mathcal{G}_T}{\Theta^2 + \Sigma \mathcal{C}} \right) - \mathcal{F}_T,$$

$$\mathcal{H}_S := \frac{\mathcal{G}_B^2 \mathcal{C}}{\Theta^2 + \Sigma \mathcal{C}}.$$

$$\mathcal{C} := (1 - \lambda_1) A_4 - (6 + 9\lambda_2 - 15\lambda_3) A_5 H.$$

If this higher-order term is always positive, namely, $\mathcal{H}_S / \mathcal{G}_S > 0$, higher wavenumber modes do not grow anomalously even when the square sound velocity $c_s^2 = \mathcal{F}_S / \mathcal{G}_S$ takes a negative value.

If $\lambda_1 = \lambda_2 = \lambda_3 = 1$, we find $\mathcal{C} = \mathcal{H}_S = 0$, so GLPV theory does not work.

Specific Example

$$A_2 = M_2^4 f^{-2(\alpha+1)} a_2(N),$$
$$A_3 = M_3^3 f^{-(2\alpha+1)} a_3(N),$$
$$A_4 = -\frac{M_{\rm Pl}^2}{2} + M_4^2 f^{-2\alpha} a_4(N),$$
$$A_5 = M_5 f a_5(N),$$

$a_a(N)$ dimensionless function of N

M_a mass scales (later identified with the reduced Planck mass)

$M_{\rm Pl}$ reduced Planck scale $(\alpha > 0)$

$B_4 = \frac{\beta M_{\rm Pl}^2}{2}, \quad B_5 = 0$ affect only on perturbations.

$f = f(t)$ is a function of time. It is NOT a dynamical variable.

Determination of its functional form is part of the definition of the model.
$(t, N) \Longleftrightarrow (\phi, X)$

① Genesis phase (start expansion from Minkowski space)

$$f \approx \dot{f}_0 t \quad (\dot{f}_0 = \text{const} < 0) \quad -\infty \to t < t_0 \quad f \gg 1$$

② Inflation

$$f = f_1 = \text{const} \qquad t \gtrsim t_0 \quad \text{until} \quad t_{\rm end}$$

③ Graceful exit and reheating

$$f \sim t^{1/(\alpha+1)} \quad \text{after sufficient inflation} \quad t \gtrsim t_{\rm end}$$

① Genesis phase (start expansion from Minkowski space)

$$-\mathcal{E} = M_2^4 f^{-2(\alpha+1)}(Na_2)' + \mathcal{O}(f^{-4\alpha-2}) = 0,$$

➡ N is a constant N_0 satisfying $a_2(N_0) + N_0 a_2'(N_0) = 0$

$$\mathcal{P} = -\frac{1}{N}\frac{\mathrm{d}}{\mathrm{d}t}\left(M_3^3 f^{-(2\alpha+1)} a_3 - 2\eta_4 M_{\rm Pl}^2 H\right) + M_2^4 f^{-2(\alpha+1)} a_2 + \mathcal{O}(f^{-4\alpha-2}) = 0.$$

➡ $\frac{2\eta_4 M_{\rm Pl}^2}{N_0}\frac{\mathrm{d}H}{\mathrm{d}t} + f^{-2(\alpha+1)}\hat{p} = 0,$ with $\hat{p} = M_2^4 a_2(N_0) + (2\alpha+1) M_3^3 a_3(N_0)\frac{\dot{f}_0}{N_0}$

Solution

$$H = -\frac{\hat{p}}{2(2\alpha+1)\eta_4 M_{\rm Pl}^2}\frac{N_0}{|\dot{f}_0|} f^{-(2\alpha+1)} \sim \frac{1}{(-t)^{2\alpha+1}}$$

$$a = 1 - \frac{\hat{p}}{4\alpha(2\alpha+1)\eta_4 M_{\rm Pl}^2}\frac{N_0^2}{\dot{f}_0^2} f^{-2\alpha}.$$

② Inflation

$$f = f_1 = \text{const} \qquad N = N_{\rm inf} = \text{const} \qquad H = H_{\rm inf} = \text{const}$$

They satisfy

$$\begin{aligned} -\mathcal{E} &= (N_{\rm inf}A_2)' + 3N_{\rm inf}A_3'H_{\rm inf} + 6\eta_4 N_{\rm inf}^2 (N_{\rm inf}^{-1}A_4)'H_{\rm inf}^2 \\ &\quad + 6\eta_5 N_{\rm inf}^3 (N_{\rm inf}^{-2}A_5)'H_{\rm inf}^3 = 0, \\ \mathcal{P} &= A_2 - 6\eta_4 A_4 H_{\rm inf}^2 - 12\eta_5 A_5 H_{\rm inf}^3 = 0. \end{aligned}$$

③ Graceful exit

Assuming $f \sim t^{1/(\alpha+1)}$? 1

we find $N = N_{\rm e} = \text{const}$ and $H^2 \sim 1/t^2 \sim f^{-2(\alpha+1)} \sim A_2$

satisfying

$$-\mathcal{E} = (N_{\rm e}A_2)' + 3\eta_4 M_{\rm Pl}^2 H^2 + \mathcal{O}(f^{-(3\alpha+2)}) = 0, \quad \Rightarrow \quad (N_{\rm e}a_2)' < 0$$

$$\mathcal{P} = A_2 + 3\eta_4 M_{\rm Pl}^2 H^2 + \frac{2\eta_4 M_{\rm Pl}^2}{N_{\rm e}}\frac{\mathrm{d}H}{\mathrm{d}t} + \mathcal{O}(f^{-(3\alpha+2)}) = 0,$$

We find

$$H^2 \propto \frac{1}{a^m}, \quad m := \frac{3N_{\rm e}a_2'}{(N_{\rm e}a_2)'} \qquad m > 0 \Leftrightarrow a_2' < 0$$

③-2 Reheating

In our setup the scalar field may not oscillate because ϕ may not vanish.

The Universe can be reheated through gravitational particle production due to the change of the cosmic expansion law from exponential to power-law.

(Ford 1987, Kunimitsu & JY 2013)

$$H^2 \propto \frac{1}{a^m}, \quad m := \frac{3N_e a_2'}{(N_e a_2)'}$$

For this mechanism to operate m must be significantly larger than 4 so that the scalar field dissipates its energy sufficiently rapidly compared with radiation.

$$\rho_r = \sigma H_{\rm inf}^4$$

$$\sigma_1 = \frac{9}{32\pi^2} \ln\left(\frac{1}{H\Delta t}\right), \qquad (m=6)$$

$$\sigma_1 = \frac{1}{8\pi^2} \ln\left(\frac{1}{H\Delta t}\right), \qquad (m=4)$$

$$T_R = \left(\frac{30}{\pi^2 g_*}\right)^{\frac{1}{4}} \left(\frac{\sigma^{\frac{m}{4}}}{3}\right)^{\frac{1}{m-4}} \left(\frac{H_{\rm inf}}{M_{\rm Pl}}\right)^{\frac{2}{m-4}} H_{\rm inf}$$

Perturbations

$$\mathcal{L}_S^{(2)} = \overline{N}a^3\left[\mathcal{G}_S\frac{\dot{\zeta}^2}{\overline{N}^2} + \zeta\left(\mathcal{F}_S\frac{\partial^2}{a^2} - \mathcal{H}_S\frac{\partial^4}{a^4}\right)\zeta\right]$$

$$\mathcal{G}_S := \frac{\Sigma\mathcal{G}_T^2}{\Theta^2+\Sigma\mathcal{C}} + 3\mathcal{G}_T,$$

$$\mathcal{F}_S := \frac{1}{\overline{N}a}\frac{d}{dt}\left(\frac{a\Theta\mathcal{G}_B\mathcal{G}_T}{\Theta^2+\Sigma\mathcal{C}}\right) - \mathcal{F}_T,$$

$$\mathcal{H}_S := \frac{\mathcal{G}_B^2\mathcal{C}}{\Theta^2+\Sigma\mathcal{C}}.$$

① Genesis phase

$$\mathcal{G}_T \simeq M_{\rm Pl}^2, \quad \Sigma \simeq \frac{M_2^4}{2}f^{-2(\alpha+1)}\left(N_0^2a_2'\right)',$$

$$\Theta \simeq \frac{M_3^3}{2}f^{-(2\alpha+1)}N_0a_3' + \eta_4M_{\rm Pl}^2H,$$

$$\mathcal{F}_T = \beta M_{\rm Pl}^2, \quad \mathcal{C} \simeq \frac{M_{\rm Pl}^2}{2}(\lambda_1-1).$$

$$\mathcal{G}_S \simeq \frac{\mathcal{G}_T^2}{\mathcal{C}} + 3\mathcal{G}_T, \quad \mathcal{H}_S \simeq \frac{\mathcal{G}_B^2}{\Sigma}.$$

$$\mathcal{F}_S \simeq 2\beta M_{\rm Pl}^2\left[\frac{M_2^4a_2+(2\alpha+1)M_3^3(\dot{f}_0/N_0)(N_0a_3)'}{(2\alpha+1)(\lambda_1-1)M_2^4(N_0^2a_2')'} - \frac{1}{2}\right] = \text{const.} \tag{58}$$

Positivity of coefficients require

$$\lambda_1 > 1 \qquad \left(N_0^2a_2'\right)' > 0$$

Most importantly, $a_3(N)$ may not vanish just as in original G-inflation.

$$k^3|\zeta_k|^2 \sim \frac{k_*^2}{\mathcal{G}_S}\left(\frac{k}{k_*}\right)^{(3\alpha+4)/(\alpha+2)}$$

Blue spectrum

② Inflation

$\mathcal{G}_T$, $\mathcal{F}_T$, $\mathcal{G}_S$, $\mathcal{F}_S$, and $\mathcal{H}_S$ are time-independent.

$$\mathcal{P}_T = 8\frac{\mathcal{G}_T^{1/2}}{\mathcal{F}_T^{3/2}}\frac{H_{\rm inf}^2}{4\pi^2}$$

$$\mathcal{P}_\zeta = \frac{H_{\rm inf}^2}{2\mathcal{G}_S c_s^3}\frac{1}{F(c_s^2/\epsilon)} \qquad F(x) := \frac{4}{\pi}x^{-3/2}e^{\pi x/4}\left|\Gamma(5/4 - ix/4)\right|^2$$

$$u_k := \sqrt{2\mathcal{G}_S}a\zeta_k \qquad \frac{{\rm d}^2 u_k}{{\rm d}\tau^2} + \left(\omega^2 - \frac{2}{\tau^2}\right)u_k = 0,$$

$$\omega^2 = c_s^2 k^2 + \epsilon^2 k^4 \tau^2$$

$$c_s^2 = \mathcal{F}_S/\mathcal{G}_S \text{ and } \epsilon := H_{\rm inf}\mathcal{H}_S^{1/2}/\mathcal{G}_S^{1/2}$$

$$u_k = \frac{e^{-\pi c_s^2/8\epsilon}W_{ic_s^2/4\epsilon,3/4}(-i\epsilon k^2\tau^2)}{(-2\epsilon k^2\tau)^{1/2}}$$

(Ashoorioon, Chialva, & Danielsson 2011)

③ Graceful exit

$$\mathcal{G}_T \simeq M_{\rm Pl}^2,\ \mathcal{F}_T = \beta M_{\rm Pl}^2,$$

$$\mathcal{F}_S \simeq \beta M_{\rm Pl}^2 \frac{-\lambda_1 + 1 + \ell m/2}{\lambda_1 - 1 + \ell},$$

$$\mathcal{G}_S \simeq M_{\rm Pl}^2 \frac{3\lambda_1 - 1}{\lambda_1 - 1 + \ell},$$

$$\mathcal{H}_S \simeq \frac{\beta^2 M_{\rm Pl}^2}{H^2} \frac{\lambda_1 - 1}{3\lambda_1 - 1} \frac{\ell}{\lambda_1 - 1 + \ell},$$

$$\ell := -\frac{4}{3}\frac{(N_{\rm e} a_2)'}{(N_{\rm e}^2 a_2')'}$$

If $\ell m > 2(\lambda_1 - 1)$ all the coefficients are positive.

Choice of the functions

$$a_2 = -\frac{1}{N^2} + \frac{1+5\Delta^2}{3}\frac{N_0^2}{N^4} - \Delta^2\frac{N_0^4}{N^6}, \qquad a_3 = \frac{\gamma}{N^3},$$

$N_0 = 1$

$\Delta = 0.05$

$\gamma = 10$

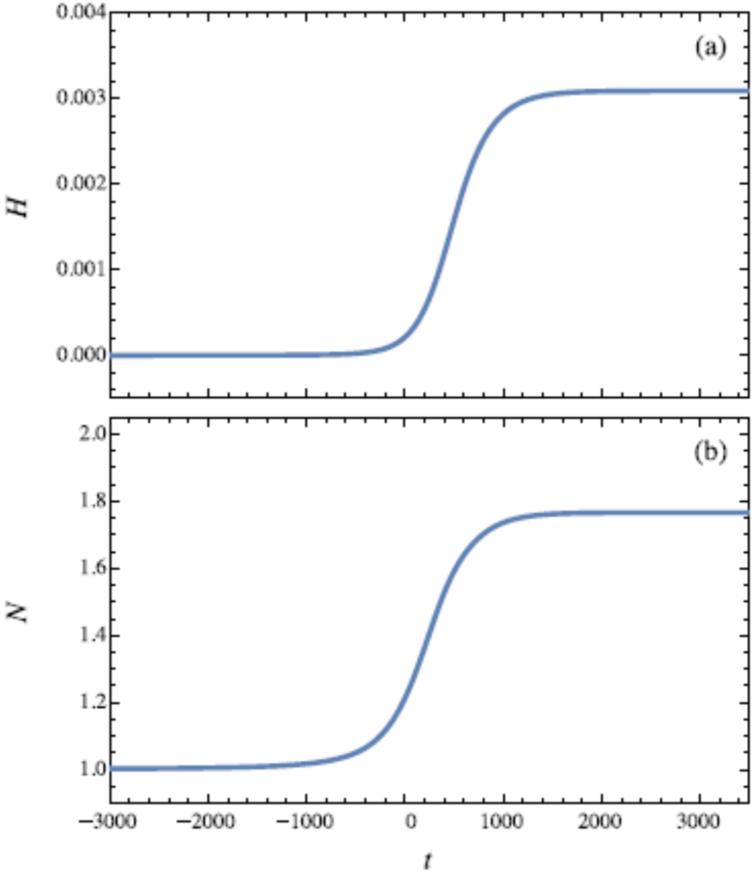
$\beta = 1$

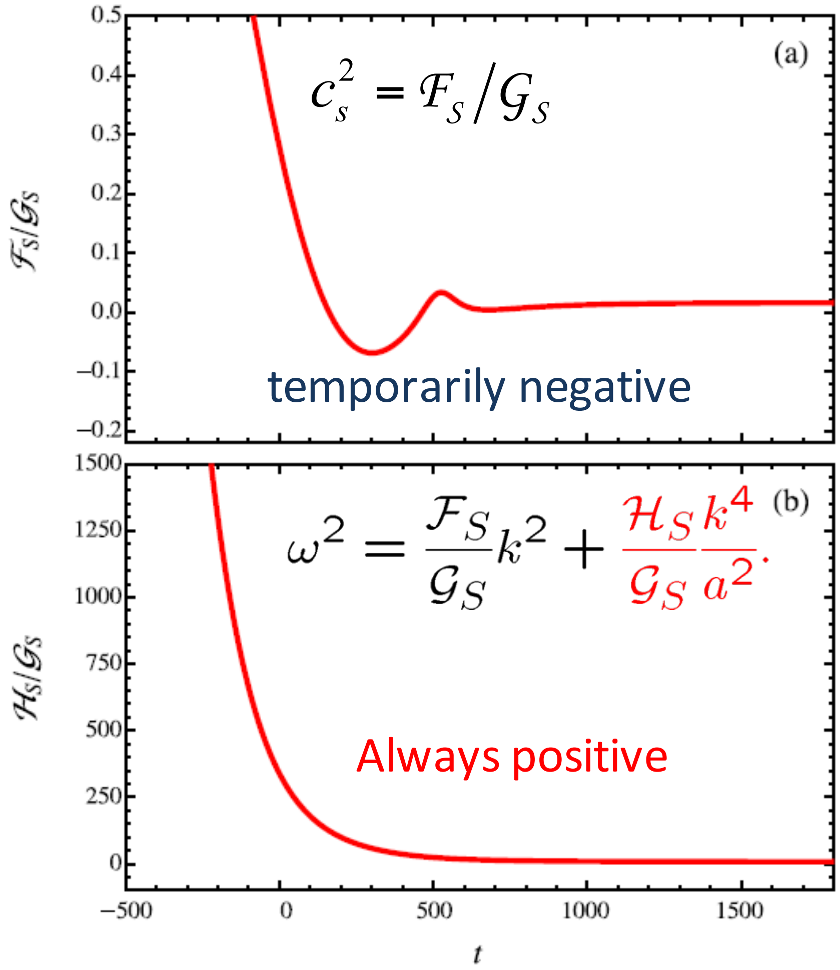
Genesis de Sitter transition

$$\dot{f}_0 = -10^{-1}, \; f_1 = 10, \; \text{and} \; s = 2\times 10^{-3}$$

$$f = \frac{\dot{f}_0}{2}\left[t - \frac{\ln(2\cosh(st))}{s}\right] + f_1,$$

$M_{Pl} = 1$



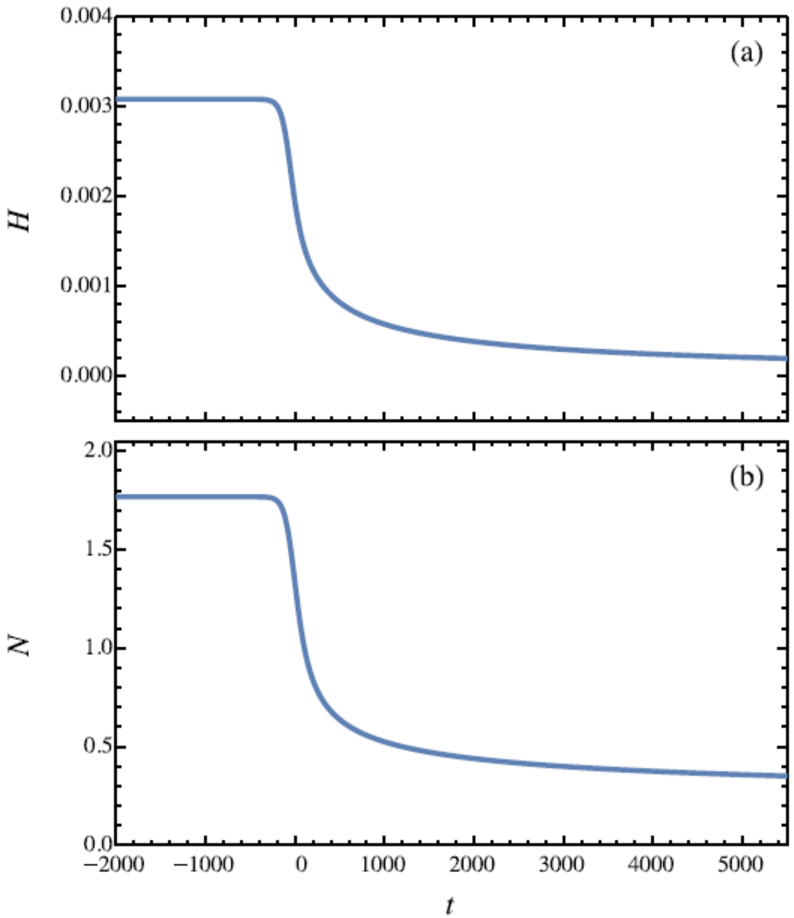


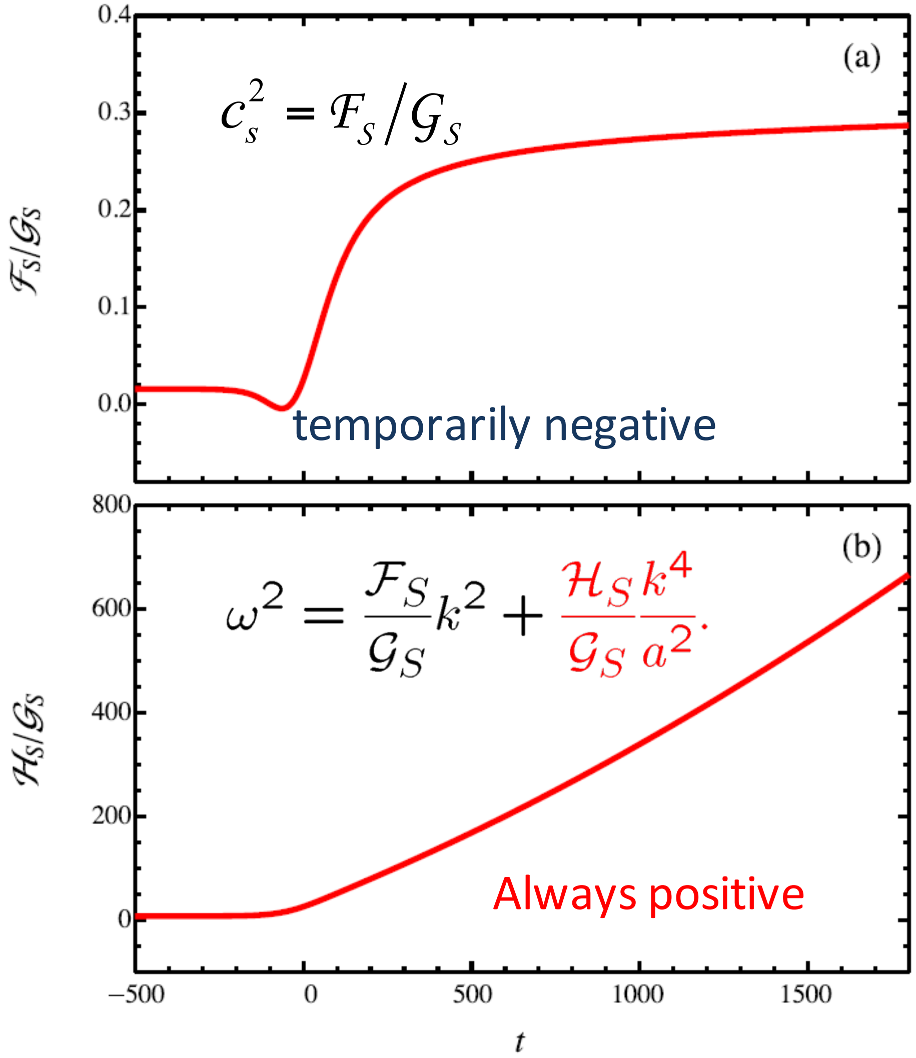
From inflation to graceful exit

$$f=\left\{f_1^{\alpha+1}+\frac{v}{2}\left[t+\frac{\ln\left(2\cosh(s't)\right)}{s'}\right]\right\}^{1/(\alpha+1)}$$

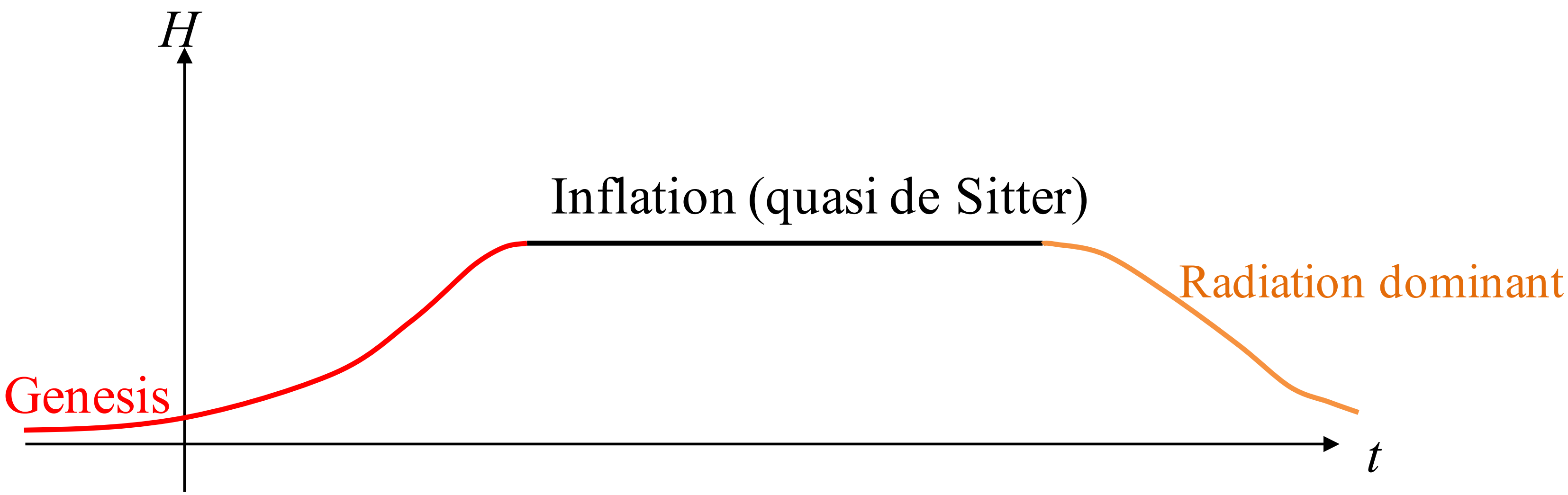
(The origin of time has been shifted by the duration of inflation.)

$$\alpha=1,\ \lambda_1=1+10^{-3},\ s'=10^{-2},\ v=6$$





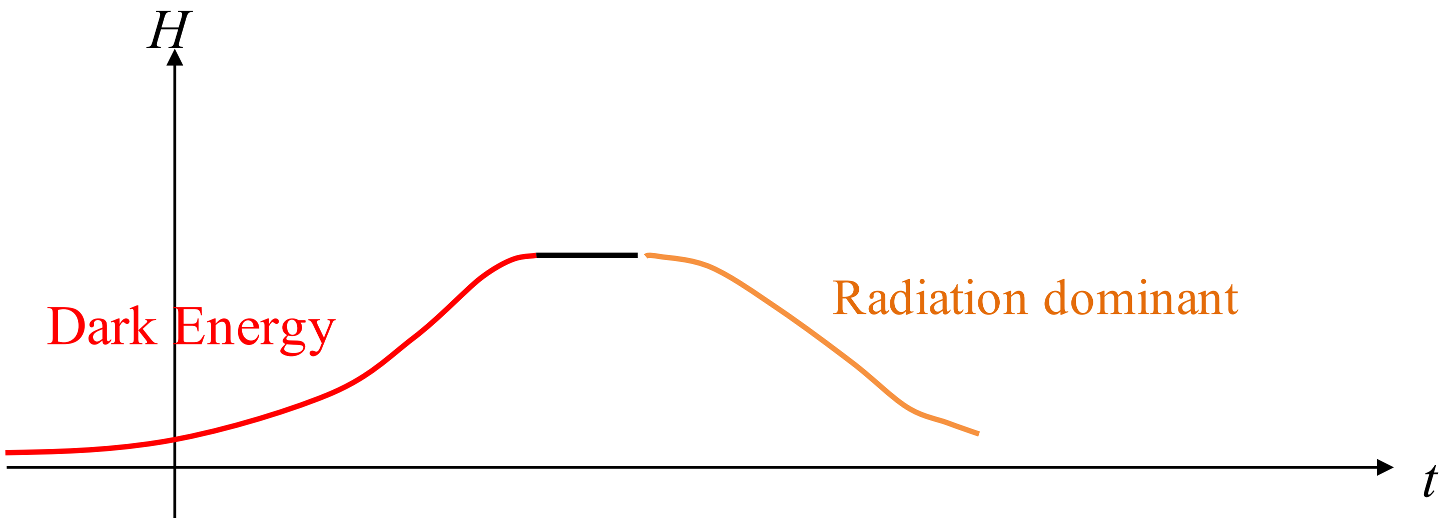
Our model works and realize transitions from Genesis to Inflation and then to Radiation domination without any instabilities.



A similar mechanism may apply for dark energy to realize w < -1 without phantom menace or big-rip singularity.

NB Galileonlike dark energy was first studied by Deffayet, Pujolas, Sawicki, and Vikman 2010 with the title kinetic gravity braiding. Their paper appeared in arXiv only 1 day before our G inflation paper.

If we apply our Beyond-Horndeski model of inflation to dark energy we may find w < -1 without any instabilities,



and the second generation of radiation dominated universe may follow after dark energy domination.

Unlike Garriga-Vilenkin type recycling universe, this universe keeps some memories of the previous generation (=our universe).

4b) Assignment to Takada-san and Miyazaki-san

Try to find $w < -1$!

or more proper statement would be

DO NOT get embarrassed even if you found

$$w < -1$$

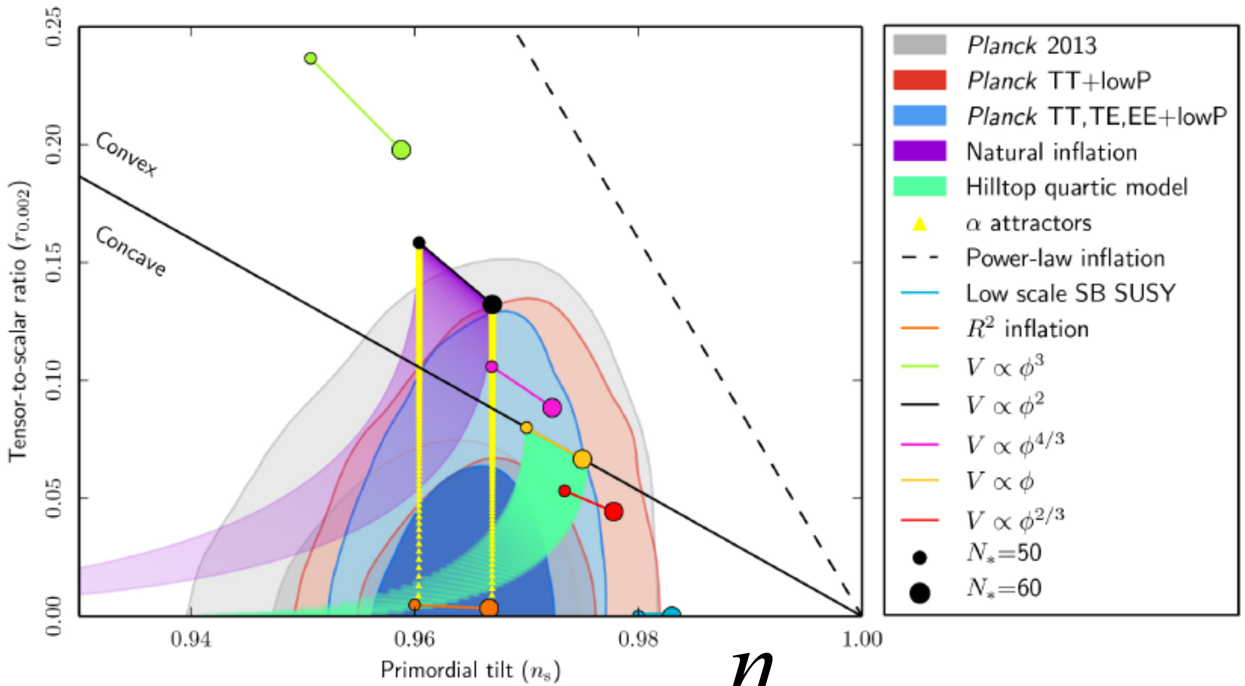
4c) Assignment to myself


上州無智亦無才
剛毅木訥易被欺
唯以正直接萬人
至誠依神期勝利
鑑三

4c) Assignment to myself

Nevertheless, I would bet $n_t < 0$ and $w = -1$.

r



n_s