Probing Small-Scale Non-Gaussianity from Anisotropies in Acoustic Reheating

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$c = \hbar = 1, \quad M_G = 1/\sqrt{8\pi G} \sim 2.4 \times 10^{18}\mathrm{GeV}.$

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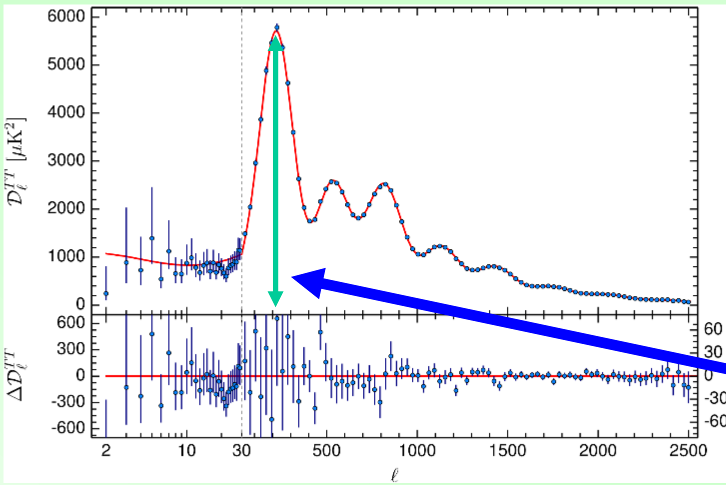
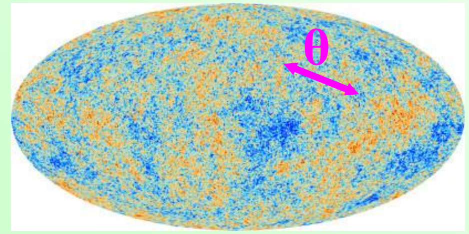
Introduction

Observations of CMB anisotropies

- **Planck TT correlation :** **0.07°**





red line : prediction by inflation

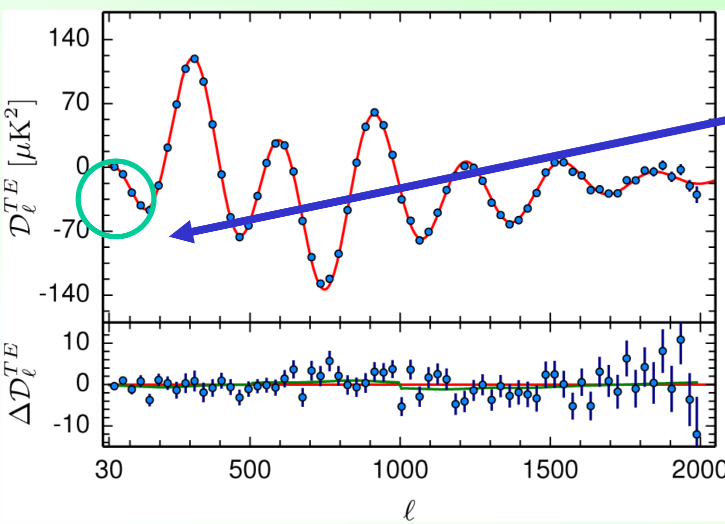
blue points : observation by PLANCK

Angle θ ~ 180° / l

Total energy density ↔ Geometry of our Universe

Our Universe is spatially flat !!

- **Planck TE correlation :**



Superhorizon models

Almost scale invariant, Gaussian, adiabatic curvature perturbations

Though primordial tensor perturbations have not yet been observed, inflation is strongly supported by the observations of CMB anisotropies.

Constraints on scalar and tensor perturbations from the PLANCK satellite

Observational constraints :

$$\left\{\begin{aligned} \Delta_\zeta(k_0) &= 2.137^{+0.063}_{-0.061}\times 10^{-9},\\ n_s &= 0.968\pm 0.006,\\ r &< 0.11,\\ &k_0 = 0.002\mathrm{Mpc}^{-1}. \end{aligned}\right.$$

(TT+lowP+lensing)

Theoretical predictions :

$$\left\{\begin{aligned} \Delta_\zeta(k) &\simeq \frac{1}{8\pi^2\epsilon}\left(\frac{H}{M_G}\right)^2,\\ n_s-1 &= \frac{d\ln\Delta_\zeta(k)}{d\ln k}\simeq -2\epsilon-2\eta,\\ \Delta_h(k) &\simeq \frac{2}{\pi^2}\left(\frac{H}{M_G}\right)^2, \quad n_T=\frac{d\ln\Delta_h(k)}{d\ln k}\simeq -2\epsilon,\\ r &\equiv \frac{\Delta_h(k)}{\Delta_\zeta(k)}\simeq 16\epsilon\,(=-8n_T)\,. \end{aligned}\right.$$

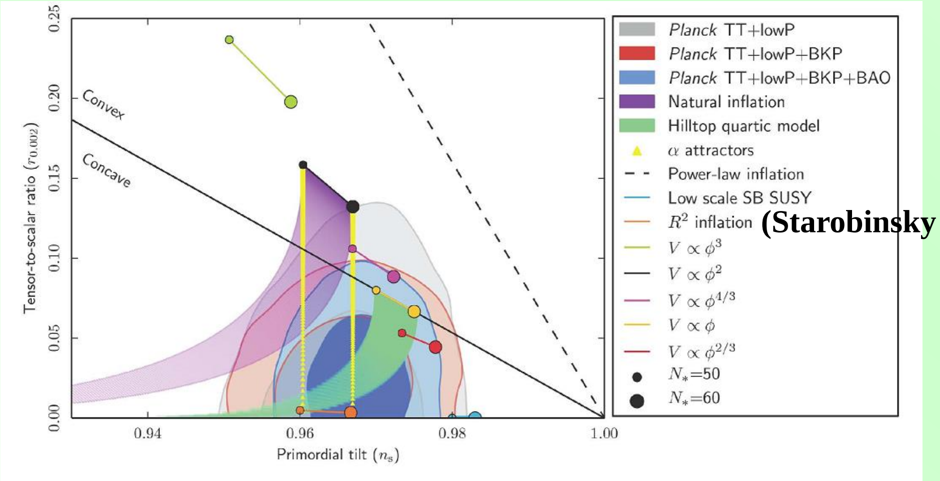


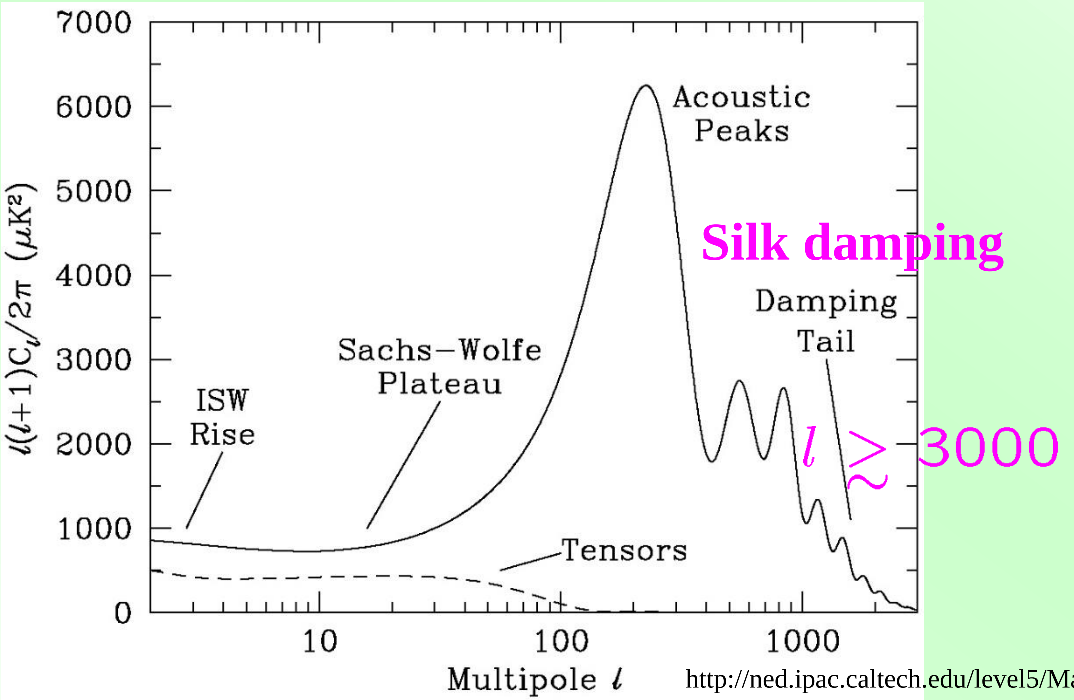
Fig. 54. Marginalized joint 68 % and 95 % CL regions for n_s and $r_{0.002}$ from *Planck* alone and in combination with its cross-correlation with BICEP2/Keck Array and/or BAO data compared with the theoretical predictions of selected inflationary models.

Planck 2015 results. XX

One may wonder how many scales these features apply for.

How small scales of primordial perturbations can be (directly) probed by CMB anisotropies ?

CMB anisotropies cannot directly probe relatively small scales



http://ned.ipac.caltech.edu/level5/March05/Scott/Scott4.html

Due to Silk damping, the information on the scales smaller than l ~ 3000 is lost apparently.

- **mean free path of the photons in the baryons :** $\lambda_C = \dot{\tau}^{-1}, \quad \dot{\tau}^{-1} = \sigma_T x_e n_e a.$

  Thomson scattering cross section

  fraction

- **diffusion scale (Silk damping scale) in the random walk approximation :**

$$\left\{\begin{aligned} \lambda_D &= \sqrt{N}\lambda_C = \sqrt{\eta/\lambda_C}\,\lambda_C = \sqrt{\eta\lambda_C} \\ &\simeq 9\,(\Omega_m/0.3)^{-1/4}(\Omega_b/0.05)^{-3/2}(h/0.75)^{-3/2}\ \mathrm{Mpc} \\ l_D &\simeq 3\times 10^3(\Omega_m/0.3)^{-1/4}(\Omega_b/0.05)^{1/2}(h/0.75)^{1/2} \end{aligned}\right.$$

Perturbations on small scales are frontier

CMB anisotropies can probe only 5-6 e-folds directly.

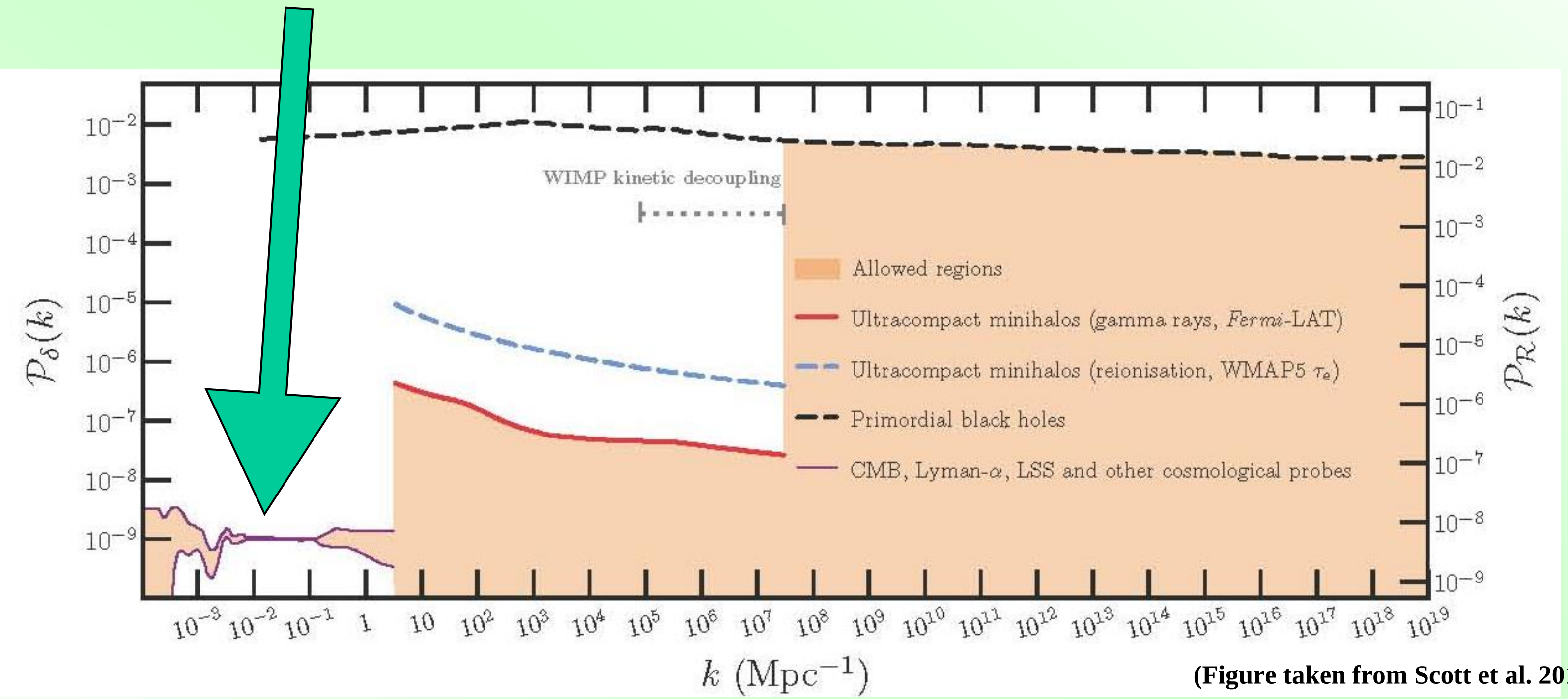
However, in order to solve the horizon and flatness problems, the inflation must last at least during 50-60 e-folds.

The smaller scale perturbations are frontier and have a lot of fruitful information.

The information of perturbations itself is very important. In addition, we can extract the dynamics of the inflation at the corresponding periods.

Constraints on smaller scale perturbations

CMB anisotropies can probe only 5-6 e-folds directly.



(Figure taken from Scott et al. 2012)

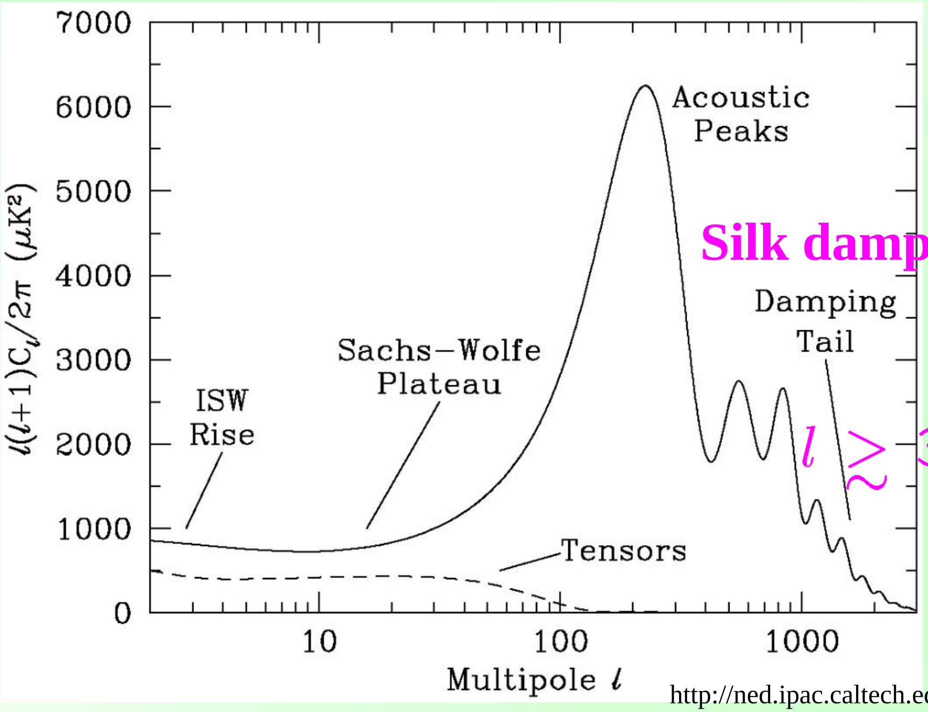
Small scale perturbations are still unknown.

Spectral distortions, PBH, UCMH are powerful tools to probe them.

In this talk, we discuss another method.

Acoustic reheating

Silk damping & acoustic reheating



Silk damping

$l \gtrsim 3000$

http://ned.ipac.caltech.edu/level5/March05/Scott/Scott4.html

Once a mode, k, of the perturbations enters the horizon,

→ **It becomes dynamical and oscillates (acoustic wave)**

→ **It dissipates its energy by diffusion effects (Silk damping)**

Silk damping → **Energy transfer from perturbations to background photons (Acoustic reheating of the Universe)**

e.g. baryon-to-photon ratio can be different between BBN & LSS.

→ **New constraints on small scale powerspectrum**

(Jeong, Pradler, Chluba, Kamionkowski 2014, Nakama, Suyama, Yokoyama 2014)

$\Delta^2_{\mathcal{R}} < \mathcal{O}(0.01)$ **for** $10^4\ \mathrm{Mpc}^{-1} < k < 10^5\ \mathrm{Mpc}^{-1}$

Inhomogeneities (Anisotropies) of acoustic reheating

Since diffusion and thermalization scales are limited, acoustic reheating is not homogeneous over the present horizon scale.

⇒ **Inhomogeneities (anisotropies) of acoustic reheating**

⇒ **Additional contribution to temperature anisotropies**

⇒ **Such (additional) temperature anisotropies cannot exceed those observed by the CMB experiments.**

⇒ **Constraints on small scale perturbations**

(Similar analysis for anisotropies in CMB distortions, Pajer & Zaldarriaga 2012)

How much temperature rise and fluctuations are induced by acoustic reheating

Consider a perturbed system of photon, $T(\boldsymbol{x}) = \overline{T}(1 + \Theta_i(\boldsymbol{x}, \hat{n}))$
and assume that acoustic reheating happens instantaneously,

(fractional) temperature rise :

$$\Delta(\boldsymbol{x}) = \frac{\langle\rho\rangle_{\boldsymbol{x}}^{\frac{1}{4}} - \langle\rho^{\frac{1}{4}}\rangle_{\boldsymbol{x}}}{a_B^{\frac{1}{4}}\overline{T}} \qquad \left(T = (\rho/a_B)^{1/4},\ a_B = \pi^2/15\right)$$

$$\langle X\rangle_{\boldsymbol{x}} = \int d^3\boldsymbol{x}'\ W_{r_T}(\boldsymbol{x}')X(\boldsymbol{x}+\boldsymbol{x}')$$

: spatial average through $W_{r_T}(x)$
(r_T : (largest) thermalization radius)

$$\Delta(\boldsymbol{x}) = \frac{3}{2}\langle\Theta_i^2\rangle_{\boldsymbol{x}}$$

(Thermalization makes the system isotropic in the box)

In the actual system with $T(\eta, \boldsymbol{x}, \hat{\boldsymbol{n}}) = \overline{T}(\eta)\big(1 + \Theta(\eta, \boldsymbol{x}, \hat{\boldsymbol{n}})\big),$

$$\Delta(\eta, \boldsymbol{x}) = \frac{3}{2}\langle\Theta_i^2 - \Theta^2(\eta)\rangle_{\boldsymbol{x}}$$

$$\frac{d\Delta}{d\eta} = -\frac{3}{2}\frac{d\langle\Theta^2\rangle_{\boldsymbol{x}}}{d\eta}.$$

For $5 \times 10^4 < z < 2 \times 10^6$ (μ-era)

$$\left(\frac{d\Delta}{d\eta} \simeq -2.277\frac{d\langle\Theta^2\rangle_{\boldsymbol{x}}}{d\eta}\right)$$

Evolution of silk damping effects

$$T(\eta, \boldsymbol{x}, \hat{\boldsymbol{n}}) = \overline{T}(\eta)\left[1 + \Theta(\eta, \boldsymbol{x}, \hat{\boldsymbol{n}}) + \Delta(\eta, \boldsymbol{x})\right]$$

(Δ consists of only a monopole component because of the thermalization effects)

Time evolution of the temperature perturbations in the conformal Newtonian gauge :

$$\frac{d}{d\eta}\Theta = \dot{\phi} - (\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}})\psi + \dot{\tau}\left[\Theta - \Theta_0 + \frac{1}{2}P_2(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}})(\Theta_2 + \Theta_2^P + \Theta_0^P) - (\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}})v\right]$$

$\left(\Theta = \sum_l (-i)^l (2l+1) P_l(\hat{k}\cdot\hat{n})\Theta_l\right)$ (Ma & Bertschinger 1995)

$$\left(\frac{d\Delta}{d\eta} = -\alpha_n \frac{d\langle\Theta^2\rangle_{\boldsymbol{x}}}{d\eta},\ \ \Theta_2 \simeq -\frac{8k}{15\sqrt{3}\dot{\tau}}\sin(kr_s)\exp\left(-k^2/k_D^2\right)\mathcal{R}_{\boldsymbol{k}}\right)$$

Solve diff eq. in Fourier space

$$\Delta(\eta_*, \boldsymbol{k}) = 2\sum_{n=1}^{5}\alpha_n \int\frac{d^3\boldsymbol{k}_1}{(2\pi)^3}\int\frac{d^3\boldsymbol{k}_2}{(2\pi)^3} 5P_2(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{n}})P_2(\hat{\boldsymbol{k}}_2\cdot\hat{\boldsymbol{n}})(2\pi)^3\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2)$$

$$\times\ \mathcal{W}_{r_T}(k)\langle\sin(k_1 r_s)\sin(k_2 r_s)\rangle_p\left[\exp\left(-\frac{k_1^2+k_2^2}{k_D^2(z)}\right)\right]_{z_n}^{z_{n-1}}\mathcal{R}_{\boldsymbol{k}_1}\mathcal{R}_{\boldsymbol{k}_2}$$

last scattering

(2nd order effects)

(periodic average over the duration longer than the oscillation period. Only for k1= k2, it is non-zero and 1/2)

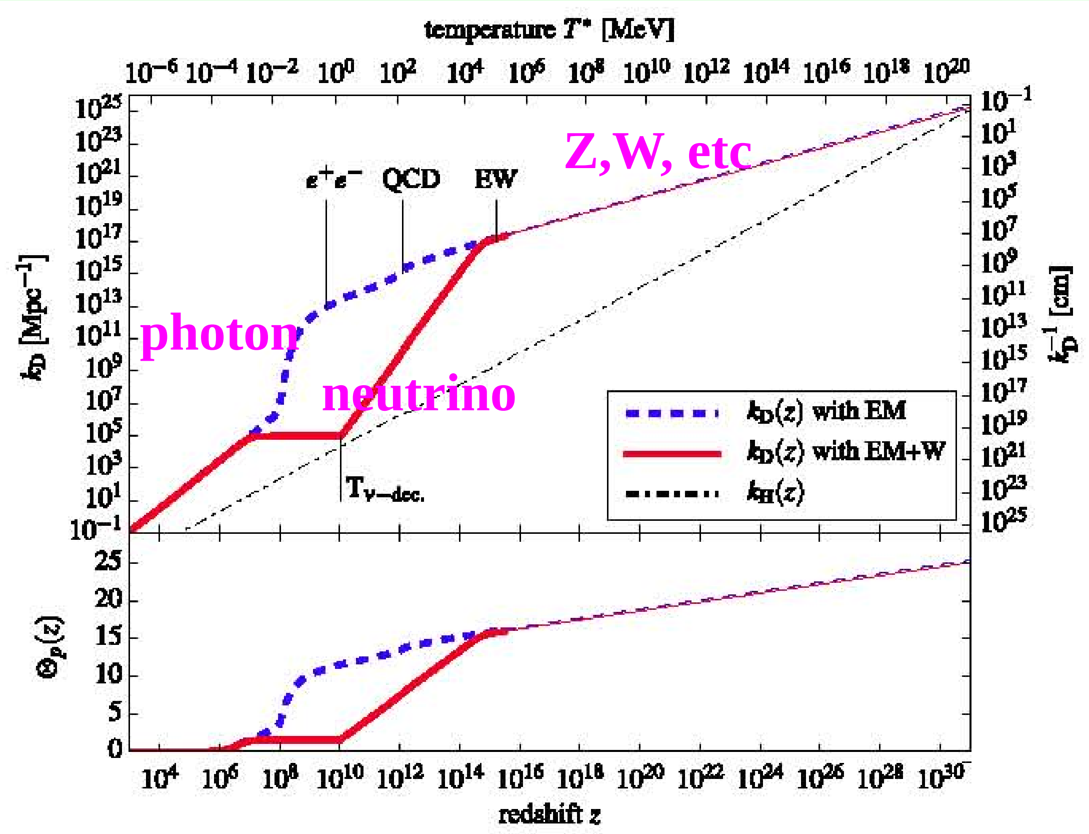
(inverse) diffusion scale

The temperature rise due to acoustic reheating is smoothed over the diffusion radius, $r_T \sim 1/k_D$, which implies that only the sum of vectors **k**1 and **k**2 must be small, while magnitudes k1 and k2 can be large.

Information on small scale perturbations

Diffusion scales

Diffusion scales change according to the cosmic times :



(figure taken from Jeong et al. 2014)

z0 = 10^{30},
z1 = 1.5 x 10^{14},
z2 = 5.5 x 10^{9},
z3 = 8.4 x 10^{6},
z4 = 2 x 10^{6}, ⟵ **μ era**
z5= 5 x 10^{4}.

α1 = 3/2,
α2 = 3/2,
α3 = 3/2,
α4 = 3/2,
α5= 2.277.

Angular powerspectrum

$$\begin{cases} a_{lm}^{\Theta} = 4\pi(-i)^l \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} Y_{lm}^*(\hat{\boldsymbol{k}}) \mathcal{T}_l(k,\eta_0)\Theta(\eta_*,\boldsymbol{k}), \\ a_{lm}^{\Delta} = 4\pi(-i)^l \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} Y_{lm}^*(\hat{\boldsymbol{k}}) \mathcal{T}_l(k,\eta_0)\Delta(\eta_*,\boldsymbol{k}), \end{cases}$$

$$\left(a_{lm}^{\Delta} = \int d\hat{\boldsymbol{n}} Y_{lm}^*(\hat{\boldsymbol{n}})\Delta(\boldsymbol{x}=0,\hat{\boldsymbol{n}}) \right)$$

(Θ and Δ above are evolved from the last scattering surface to the observer (x =0) and hence Δ has n dependence.)

Transfer function (for small l) :

$$\mathcal{T}_l \simeq [\Theta_0(\eta_*) + \Psi(\eta_*)] j_l[k(\eta_0 - \eta_*)] \simeq -3 j_l(k\eta_0)/5$$

$$C_l^{XY} \equiv \frac{1}{2l+1} \sum_{m=-l}^{l} \langle a_{lm}^{X*} a_{lm}^{Y} \rangle = \frac{36\pi}{25} \int d\ln k \; j_l^2(k\eta_0) \mathcal{P}_{XY}(k)$$

$$\left(\langle X_{\boldsymbol{k}} Y_{\boldsymbol{k}'}^* \rangle = (2\pi)^3 \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') P_{XY}(k), \quad \mathcal{P}_{XY}(k) = \frac{k^3}{2\pi^2} P_{XY}(k), \quad X,Y = \Theta_{\boldsymbol{k}} \text{ or } \Delta_{\boldsymbol{k}} \right)$$

$$C_l^{TT} = C_l^{\Theta\Theta} + 2C_l^{\Theta\Delta} + C_l^{\Delta\Delta} \lesssim \mathcal{O}(10^{-10})$$

Constraints on
small scale non-Gaussianity

Θ-Δ cross powerspectrum

$$\left(C_l^{\Theta\Delta} = \frac{36\pi}{25}\int d\ln k\; j_l^2(k\eta_0)\mathcal{P}_{\Theta\Delta}(k)\right)$$

$$\langle \Theta_{\boldsymbol{k}} \Delta^*_{\boldsymbol{k}'} \rangle = -\frac{2}{3}\sum_{n=1}^{5}\alpha_n \int \frac{d^3\boldsymbol{k}_1}{(2\pi)^3}\int \frac{d^3\boldsymbol{k}_2}{(2\pi)^3}(2\pi)^3\delta^{(3)}(\boldsymbol{k}'+\boldsymbol{k}_1+\boldsymbol{k}_2)5P_2(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{n}})P_2(\hat{\boldsymbol{k}}_2\cdot\hat{\boldsymbol{n}})$$
$$\times \mathcal{W}_{r_T}(k)\ \langle \sin(k_1 r_s)\sin(k_2 r_s)\rangle_p \left[\exp\left(-\frac{k_1^2+k_2^2}{k_D^2(z)}\right)\right]_{z_n}^{z_{n-1}} \langle \mathcal{R}_{\boldsymbol{k}}\mathcal{R}_{\boldsymbol{k}_1}\mathcal{R}_{\boldsymbol{k}_2}\rangle.$$

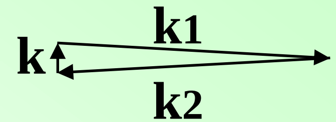
Θ-Δ cross correlation depends on bispectrum of primordial curvature perturbations

$$\langle \mathcal{R}_{\boldsymbol{k}}\mathcal{R}_{\boldsymbol{k}_1}\mathcal{R}_{\boldsymbol{k}_2}\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}+\boldsymbol{k}_1+\boldsymbol{k}_2)B_{\mathcal{R}}(k,k_1,k_2)$$

$$\Rightarrow\quad \langle \Theta_{\boldsymbol{k}} \Delta^*_{\boldsymbol{k}'} \rangle = -\frac{2}{3}\sum_{n=1}^{5}\alpha_n \int \frac{d^3\boldsymbol{k}_1}{(2\pi)^3}(2\pi)^3\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')B_{\mathcal{R}}(k,k_1,k_2)5P_2(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{n}})P_2(\hat{\boldsymbol{k}}_2\cdot\hat{\boldsymbol{n}})$$
$$\times \mathcal{W}_{r_T}(k)\ \langle \sin(k_1 r_s)\sin(k_2 r_s)\rangle_p \left[\exp\left(-\frac{k_1^2+k_2^2}{k_D^2(z)}\right)\right]_{z_n}^{z_{n-1}}. \qquad (\boldsymbol{k}_2 = -\boldsymbol{k}-\boldsymbol{k}_1)$$

Since the acoustic reheating effects are substantially suppressed on scales k_1, $k_2 \gg r_T^{-1} = O(10\ \mathrm{Mpc}^{-1})$, we have taken $k \ll k_1 \sim k_2$.

CMB scales →



$\Rightarrow$ **Can probe squeezed configuration**

Scale-dependent non-Gaussianity (bispectrum)

$$\langle \mathcal{R}_{\boldsymbol{k}_1}\mathcal{R}_{\boldsymbol{k}_2}\mathcal{R}_{\boldsymbol{k}_3}\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_{\mathcal{R}}(k_1,k_2,k_3)$$

Since we are interested in squeezed configurations, we consider local type.

$$B_{\mathcal{R}}(k_1,k_2,k_3) = -\frac{6}{5}f_{\rm NL}(k_1,k_2,k_3)P_{\mathcal{R}}^2(k_0)(x_1^{n_s-4}x_2^{n_s-4}+2\text{perms.})$$

$x_i = k_i/k_0,$ (k0 : pivot (CMB) scale)

Scale dependence:

(squeezed isosceles triangles, k << k1)

★ $$f_{\rm NL}^{G}(k_1,k_2,k_3) = f_{\rm NL}^{\rm CMB}X_G^{n_f} \Longrightarrow f_{\rm NL}^{\rm CMB}x_1^{\frac{2n_f}{3}}x^{\frac{n_f}{3}}$$

(geometric average) $\left(X_G = (x_1x_2x_3)^{\frac{1}{3}}\right)$

★ $$\begin{aligned} f_{\rm NL}^{\rm tophat}(k_1,k_2,k_3) \;=\; & f_{\rm NL}^{\rm CMB} + \tilde{f}_{\rm NL}^{(1)}(W(x_1)+2\text{perms.}) \\ & +\tilde{f}_{\rm NL}^{(2)}(W(x_1)W(x_2)+2\text{perms.}) \\ & +\tilde{f}_{\rm NL}^{(3)}W(x_1)W(x_2)W(x_3) \end{aligned} \Longrightarrow f_{\rm NL}^{\rm CMB} + \tilde{f}_{\rm NL}W(x_1)$$

(top hat) $\left(\tilde{f}_{\rm NL} = 2\tilde{f}_{\rm NL}^{(1)} + \tilde{f}_{\rm NL}^{(2)}\right)$

$\left(W(x) = \theta(x-x_i) - \theta(x-x_f)\right)$

(Trispectrum is defined in a similar manner)

$$\Longrightarrow \mathcal{P}_{\Theta\Delta}(k) \simeq \frac{4}{5}\mathcal{W}_{r_T}(k)x^{n_s-1}\mathcal{P}_{\mathcal{R}}^2(k_0)\times \sum_{n=1}^{5}\alpha_n\left[\int d\ln k_1\; f_{\rm NL}(k_1,k_1,k)\; x_1^{n_s-1}\exp\left(-\frac{2k_1^2}{k_D^2(z)}\right)\right]_{z_n}^{z_{n-1}}$$

Δ-Δ auto powerspectrum

$$\left(C_l^{\Delta\Delta} = \frac{36\pi}{25}\int d\ln k\; j_l^2(k\eta_0)\mathcal{P}_{\Delta\Delta}(k)\right)$$

$$\begin{aligned}\langle \Delta_{\boldsymbol{k}}\Delta^*_{\boldsymbol{k}'}\rangle \;=\;& 4\sum_{n,m}\alpha_n\alpha_m\int\frac{d^3\boldsymbol{k}_1}{(2\pi)^3}\int\frac{d^3\boldsymbol{k}_2}{(2\pi)^3}\int\frac{d^3\boldsymbol{k}_3}{(2\pi)^3}\int\frac{d^3\boldsymbol{k}_4}{(2\pi)^3}\\ &\times 5P_2(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{n}})P_2(\hat{\boldsymbol{k}}_2\cdot\hat{\boldsymbol{n}})5P_2(\hat{\boldsymbol{k}}_3\cdot\hat{\boldsymbol{n}})P_2(\hat{\boldsymbol{k}}_4\cdot\hat{\boldsymbol{n}})\;(2\pi)^3\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}_1-\boldsymbol{k}_2)(2\pi)^3\delta^{(3)}(\boldsymbol{k}'+\boldsymbol{k}_3+\boldsymbol{k}_4)\\ &\times \mathcal{W}_{r_T}(k)\langle\sin(k_1r_s)\sin(k_2r_s)\rangle_p\left[\exp\left(-\frac{k_1^2+k_2^2}{k_D^2(z)}\right)\right]_{z_n}^{z_{n-1}}\;\mathcal{W}_{r_T}(k')\langle\sin(k_3r_s)\sin(k_4r_s)\rangle_p\left[\exp\left(-\frac{k_3^2+k_4^2}{k_D^2(z)}\right)\right]_{z_m}^{z_{m-1}}\\ &\times\langle\mathcal{R}_{\boldsymbol{k}_1}\mathcal{R}_{\boldsymbol{k}_2}\mathcal{R}_{\boldsymbol{k}_3}\mathcal{R}_{\boldsymbol{k}_4}\rangle\end{aligned}$$

Δ-Δ auto correlation depends on four point correlation functions of primordial curvature perturbations.

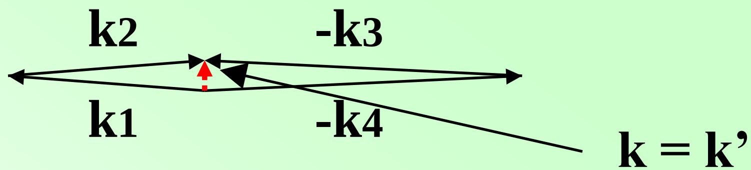
$$\langle\mathcal{R}_{\boldsymbol{k}_1}\mathcal{R}_{\boldsymbol{k}_2}\mathcal{R}_{\boldsymbol{k}_3}\mathcal{R}_{\boldsymbol{k}_4}\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4)T_{\mathcal{R}}(k_1,k_2,k_3,k_4)$$

$$\begin{aligned}\Rightarrow\quad\langle \Delta_{\boldsymbol{k}}\Delta^*_{\boldsymbol{k}'}\rangle \;=\;& 4\sum_{n,m}\alpha_n\alpha_m\int\frac{d^3\boldsymbol{k}_1}{(2\pi)^3}\int\frac{d^3\boldsymbol{k}_3}{(2\pi)^3}\,5P_2(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{n}})P_2(\hat{\boldsymbol{k}}_2\cdot\hat{\boldsymbol{n}})5P_2(\hat{\boldsymbol{k}}_3\cdot\hat{\boldsymbol{n}})P_2(\hat{\boldsymbol{k}}_4\cdot\hat{\boldsymbol{n}})\\ &\times(2\pi)^3\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')T_{\mathcal{R}}(k_1,k_2,k_3,k_4)\\ &\times \mathcal{W}_{r_T}(k)\langle\sin(k_1r_s)\sin(k_2r_s)\rangle_p\left[\exp\left(-\frac{k_1^2+k_2^2}{k_D^2(z)}\right)\right]_{z_n}^{z_{n-1}}\\ &\times \mathcal{W}_{r_T}(k')\langle\sin(k_3r_s)\sin(k_4r_s)\rangle_p\left[\exp\left(-\frac{k_3^2+k_4^2}{k_D^2(z)}\right)\right]_{z_m}^{z_{m-1}}.\end{aligned}$$

$$(\boldsymbol{k}_2=\boldsymbol{k}-\boldsymbol{k}_1)$$
$$(\boldsymbol{k}_4=-\boldsymbol{k}'-\boldsymbol{k}_3)$$

k = k' << k1 ~ k2, k3 ~ k4 ⇒

(CMB scales) (acoustic reheating scales)



Four-point correlation functions (local type)

$$\langle \mathcal{R}_{\boldsymbol{k}_1}\mathcal{R}_{\boldsymbol{k}_2}\mathcal{R}_{\boldsymbol{k}_3}\mathcal{R}_{\boldsymbol{k}_4}\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4)T_{\mathcal{R}}(k_1,k_2,k_3,k_4).$$

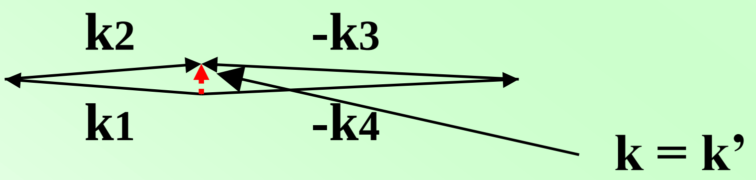
Connected:

$$\langle \mathcal{R}_{\boldsymbol{k}_1}\mathcal{R}_{\boldsymbol{k}_2}\mathcal{R}_{\boldsymbol{k}_3}\mathcal{R}_{\boldsymbol{k}_4}\rangle_c = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4) \times \Big[\tau_{\rm NL}(k_1,k_2,k_3,k_4)(P_{\mathcal{R}}(k_{12})P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_3) + 11\text{perms.}) + \frac{54}{25}g_{\rm NL}(k_1,k_2,k_3,k_4)(P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(k_3) + 3\text{perms.})\Big]$$

$$\left(P_{\mathcal{R}}(k) = \frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(k), \quad k_{12} \equiv |\boldsymbol{k}_1+\boldsymbol{k}_2|\right)$$

Disconnected:

$$\langle \mathcal{R}_{\boldsymbol{k}_1}\mathcal{R}_{\boldsymbol{k}_2}\mathcal{R}_{\boldsymbol{k}_3}\mathcal{R}_{\boldsymbol{k}_4}\rangle_{dc} = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_3-\boldsymbol{k}_4)\Big[(2\pi)^3\delta^{(3)}(\boldsymbol{k}_3+\boldsymbol{k}_4)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_3) + (2\pi)^3\delta^{(3)}(\boldsymbol{k}_2-\boldsymbol{k}_4)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + (2\pi)^3\delta^{(3)}(\boldsymbol{k}_2-\boldsymbol{k}_3)P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)\Big].$$



Only the terms with τ_{NL} is sensitive to the above configuration, while the terms with g_{NL} and the disconnected part are insensitive and no useful constraints are obtained from them.

$$\mathcal{P}^{\tau}_{\Delta\Delta}(k) \simeq 4\mathcal{P}_{\mathcal{R}}(k)\mathcal{W}^2_{r_T}(k)\int d\ln k_1 \int d\ln k_2\ \tau_{\rm NL}(k_1,k_1,k_2,k_2)\ \mathcal{P}_{\mathcal{R}}(k_1)\mathcal{P}_{\mathcal{R}}(k_2) \times \left(\sum \alpha_n \left[\exp\left(-\frac{2k_1^2}{k_D^2(z)}\right)\right]^{z_{n-1}}_{z_n}\right)\left(\sum \alpha_m \left[\exp\left(-\frac{2k_2^2}{k_D^2(z)}\right)\right]^{z_{m-1}}_{z_m}\right),$$

New constraints on scale dependent non-Gaussianities (geometric average)

$$C_l^{\Theta\Delta},\ C_l^{\Delta\Delta}\ <\ C_l^{\Theta\Theta} = \mathcal{O}(10^{-10})$$

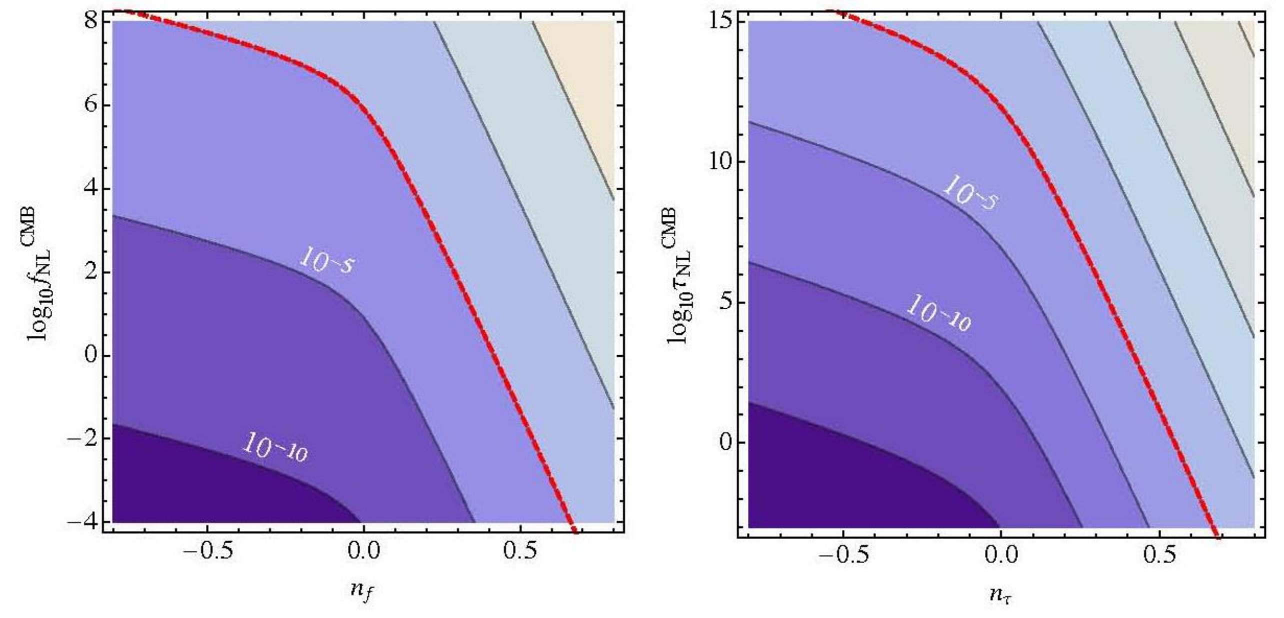


Figure 1. The constraints on n_f-$f_{\rm NL}^{\rm CMB}$ plane (left) and n_τ-$\tau_{\rm NL}^{\rm CMB}$ plane (right). The interval of the contours of the $2C_{50}^{\Theta\Delta}/C_{50}^{\Theta\Theta}$ (left) and $C_{50}^{\Delta\Delta}/C_{50}^{\Theta\Theta}$ (right) are 10^5, and red dashed lines correspond to the lines of $2C_{50}^{\Theta\Delta} = C_{50}^{\Delta\Delta} = C_{50}^{\Theta\Theta}$. Only the region below the red line is allowed.

ns = 1 fixed

★ $$C_l^{\Theta\Delta,G} \simeq \frac{9\pi^2}{125} f_{\rm NL}^{\rm CMB} \mathcal{P}_{\mathcal{R}}^2(k_0)\Gamma(\epsilon^G)\sum_{n=1}^{5}\alpha_n\left[\left(\frac{k_D^2}{2k_0^2}\right)^{\epsilon^G}\right]_{z_n}^{z_{n-1}}\mathcal{G}(l, n_s + n_f/3).$$

★ $$C_l^{\Delta\Delta,G} \simeq \frac{9\pi^2}{50}\tau_{\rm NL}^{\rm CMB}\mathcal{P}_{\mathcal{R}}^3(k_0)\mathcal{G}(l,n_s)\left(\Gamma(\epsilon^G)\sum_{n=1}^{5}\alpha_n\left[\left(\frac{k_D^2}{2k_0^2}\right)^{\epsilon^G}\right]_{z_n}^{z_{n-1}}\right)^2.$$

$$\mathcal{G}(l,n) = \left(\frac{k_0\eta_0}{2}\right)^{1-n}\frac{\Gamma\left(l+\frac{n}{2}-\frac{1}{2}\right)\Gamma(3-n)}{\Gamma\left(l-\frac{n}{2}+\frac{5}{2}\right)\Gamma^2\left(2-\frac{n}{2}\right)}$$

$$\epsilon^G = (n_s + 2n_f/3 - 1)/2$$

New constraints on scale dependent non-Gaussianities (top hat type)

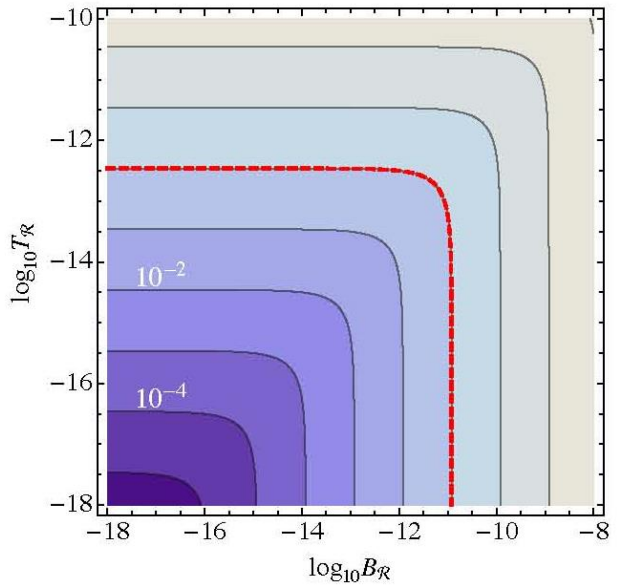


Figure 2. The figure shows the constraints on $B_{\mathcal{R}} = \widetilde{f}_{\rm NL}\mathcal{P}^2_{\mathcal{R}}(k_0)$ and trispectrum $T_{\mathcal{R}} = \widetilde{\tau}_{\rm NL}\mathcal{P}^3_{\mathcal{R}}(k_0)$. The contours of $(2C^{\Theta\Delta}_{50} + C^{\Delta\Delta}_{50})/C^{\Theta\Theta}_{50}$ are inserted at the interval of 10 in the case with $\log_{10} k_i/k_f = 4$, and a red dashed line is the line along which the non-linear correction is equal to $C^{\Theta\Theta}_{50}$. Only the region below the red line is allowed.

ns = 1 fixed

$$C_l^{\Theta\Delta,\rm tophat} \simeq \frac{9\pi^2}{125}\tilde{f}_{\rm NL}\mathcal{P}^2_{\mathcal{R}}(k_0)\mathcal{G}(l,n_s)\sum_{n=1}^{5}\alpha_n\left[x_i^{n_s-1}E_{\frac{1}{2}(3-n_s)}\left(\frac{2k_i^2}{k_D^2(z)}\right) - x_f^{n_s-1}E_{\frac{1}{2}(3-n_s)}\left(\frac{2k_f^2}{k_D^2(z)}\right)\right]_{z_n}^{z_n\ 1} .$$

$$C_l^{\Delta\Delta,\rm tophat} \simeq \frac{9\pi^2}{50}\mathcal{P}^3_{\mathcal{R}}(k_0)\mathcal{G}(l,n_s)\tilde{\tau}_{\rm NL} \times \left(\sum_{n=1}^{5}\alpha_n\left[x_i^{n_s-1}E_{\frac{1}{2}(3-n_s)}\left(\frac{2k_i^2}{k_D^2(z)}\right) - x_f^{n_s-1}E_{\frac{1}{2}(3-n_s)}\left(\frac{2k_f^2}{k_D^2(z)}\right)\right]_{z_n}^{z_{n-1}}\right)^2 ,$$

$$\mathcal{G}(l,n) = \left(\frac{k_0\eta_0}{2}\right)^{1-n}\frac{\Gamma\left(l+\frac{n}{2}-\frac{1}{2}\right)\Gamma(3-n)}{\Gamma\left(l-\frac{n}{2}+\frac{5}{2}\right)\Gamma^2\left(2-\frac{n}{2}\right)}$$

$$\mathrm{E}_n(x) = \int_1^\infty dt\,\frac{e^{-xt}}{t^n}$$

Conclusions and discussions

- Acoustic reheating happened as a consequence of Silk damping and thermalization.

- Such reheating is not homogeneous over the present horizon and leads to additional temperature anisotropies.

- Since such processes are second order effects of perturbations, the cross and the auto powerspectra of additional temperature perturbations give constraints on small scale non Gaussianities.

My message

- **Small scale perturbations are vast frontier and can give us a lot of fruitful information including the dynamics of inflation. But, our methods to probe such scales are very limited.**

- **As long as one sticks to linear theory, each mode evolves independently. But, if one goes into non-linearlity, there are mode-mode couplings so that the information on small scales can be extracted from large scale observables. We need to invent a further new method to probe small scale perturbations.**