Inflation in viscous fluid models

新学術領域研究「なぜ宇宙は加速するのか? –
徹底的究明と将来への挑戦 –」発足シンポジウム

Kavli IPMU, University of Tokyo

20th September, 2015

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References

[1] arXiv:1508.05451 [gr-qc]

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[2] Phys. Rev. D 90, 124061 (2014) [arXiv:1410.3993 [hep-th]]

Collabolators:

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I. Introduction

INFLATION   DARK ENRGY

(1) Scalar field theories

Chaotic inflation

X-matter, Quintessence

[Linde, Phys. Lett. B 91, 99 (1980)]

[Chiba, Sugiyama and Nakamura, Mon. Not. Roy. Astron. Soc. 289, L5 (1997)]

[Caldwell, Dave and Steinhardt, Phys. Rev. Lett. 80, 1582 (1998)]

(2) Modifications of gravity

R^2 **(Starobinsky) inflation**

$F(R)$ **gravity**

[Starobinsky, Phys. Lett. B 91, 99 (1980)]

[Capozziello, Cardone, Carloni and Troisi, Int. J. Mod. Phys. D 12, 1969 (2003)]

[Carroll, Duvvuri, Trodden and Turner, Phys. Rev. D 70, 043528 (2004)]

[Nojiri and Odintsov, Phys. Rev. D 68, 123512 (2003)]

Motivations and Purposes

INFLATION　　DARK ENRGY

(3) Fluid models

THIS WROK

Cf. [Barrow and Mimoso, Phys. Rev. D 50, 3746 (1994)]

Chaplygin gas

[Kamenshchik, Moschella and Pasquier, Phys. Lett. B 511, 265 (2001)]

Viscous fluid

[Brevik, Obukhov and Timoshkin, Astrophys. Space Sci. 355, 399 (2015)]

We explore viscous fluid models to explain inflation.

Planck 2015 results

[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]

(1) Spectral index of power spectrum of the curvature perturbations

$$n_{\mathrm{s}} = 0.968 \pm 0.006 \quad (68\%\,\mathrm{CL})$$

(2) Tensor-to-scalar ratio

$$r < 0.11\,(95\%\,\mathrm{CL})$$

(3) Running of the spectral index k : Wave number

$$\alpha_{\mathrm{s}} \equiv \frac{dn_{\mathrm{s}}}{d\ln k} = -0.003 \pm 0.007 \ (68\%\,\mathrm{CL})$$

R^2 (Starobinsky) inflation

Action: $S = \int d^4x \sqrt{-g}\ \frac{1}{2\kappa^2}(R + \beta R^2\ )$

g : Determinant of the metric $g_{\mu\nu}$ $\kappa^2 \equiv 8\pi G_{\mathrm{N}}$

R : Scalar curvature, β : Constant , G_{N} : Gravitatioanl constant

- $N = 60 \longrightarrow n_{\mathrm{s}} = 0.967,\quad r = 3.33 \times 10^{-3}$

$$\alpha_{\mathrm{s}} = -\,5.56 \times 10^{-4}$$

N : Number of *e*-folds during inflation

Cf. [Hinshaw *et al.*, Astrophys. J. Suppl. 208, 19 (2013)]

Inflationary models with $n_{\mathrm{s}} - 1 = -\frac{2}{N}$

- R^2 **(Starobinsky) inflation**

  [Starobinsky, Phys. Lett. B 91, 99 (1980)]

- **Chaotic inflation** **[Linde, Phys. Lett. B 91, 99 (1980)]**

- **Higgs inflation with its non-minimal coupling**

  [Salopek, Bond and Bardeen, Phys. Rev. D 40, 1753 (1989)]

  [Bezrukov and Shaposhnikov, Phys. Lett. B 659, 703 (2008)]

- α **-attractor** **[Kallosh and Linde, JCAP 1307, 002 (2013); Phys. Rev. D 91, 083528 (2015); arXiv:1503.06785 [hep-th]]**

Subjects

- **From the spectral index of $n_{\mathrm{s}} - 1 = -\frac{2}{N}$, the inflaton potential V of a scalar field theory has been reconstructed.**

[Chiba, arXiv:1504.07692 [astro-ph.CO]]

By applying this procedure to fluids, we reconstruct viscous fluid models.

Assumption

- **We suppose that the representation of inflation in viscous fluid models can be equivalent to that of the so-called slow-roll inflation driven by inflaton potential in scalar field theories.**

⇨ **Under this assumption, even in the viscous fluid description, it is considered that the three observables of the inflationary universe** $(n_{\mathrm{s}}, r, \alpha_{\mathrm{s}})$ **can be defined.**

Flat Friedmann-Lemaitre-Robertson-Walker (FLRW) space-time

$$ds^2 = -dt^2 + a^2(t) \sum_{i=1,2,3} \left(dx^i\right)^2$$

$a(t)$: Scale factor

$H = \dot{a}/a$: Hubble parameter

* The dot shows the time derivative.

II. Fluid descriptions

Gravitational field equations

$$\frac{3}{\kappa^2}\left(H(N)\right)^2 = \rho \,, \qquad -\frac{2}{\kappa^2}H(N)H'(N) = \rho + P$$

* The prime denotes the derivative with respect to *N*.

Equation of state (EoS)

$$P(N) = -\rho(N) + f(\rho)$$

ρ, P : Energy density and pressure of a fluid

$f(\rho)$: Arbitrary function of ρ

Conservation law

$$0 = \rho'(N) + 3\left(\rho(N) + P(N)\right)$$

III. Reconstruction of EoS of the viscous fluid from the spectral index

For scalar field theories with the potential $V(N)$,

$$n_s - 1 = \frac{d}{dN}\left[\ln\left(\frac{1}{V^2(N)}\frac{dV(N)}{dN}\right)\right]$$

$$r = \frac{8}{V(N)}\frac{dV(N)}{dN} \quad , \quad \alpha_s = -\frac{d^2}{dN^2}\left[\ln\left(\frac{1}{V^2(N)}\frac{dV(N)}{dN}\right)\right]$$

- $H = H(N)$
- **Gravitational field equations**

[Chiba, arXiv:1504.07692 [astro-ph.CO]]

We reconstruct the EoS of viscous fluids.

EoS of viscous fluid

$$P = -\rho + A\rho^{\beta} + \zeta(H)$$

$$\zeta(H) = \bar{\zeta} H^{\gamma}$$

$A, \beta, \gamma, \bar{\zeta}$

: Constants

$f(\rho)$

Viscosity term

$$\Rightarrow f(\rho) = A\rho^{\beta} + \zeta(H(\rho)) = A\rho^{\beta} + \bar{\zeta}\left(\frac{\kappa}{\sqrt{3}}\right)^{\gamma}\rho^{\gamma/2}$$

Friedmann equation

Inflationary models with $n_{\mathrm{s}} - 1 = -\frac{2}{N}$ (2)

$$\Rightarrow \quad V(N) = \frac{1}{(C_1/N) + C_2}$$

$C_1 (> 0), C_2$: Constants

$$r = \frac{8}{N\left[1 + (C_2/C_1)\, N\right]}\,, \qquad \alpha_{\mathrm{s}} = -\frac{2}{N^2}$$

[Chiba, arXiv:1504.07692 [astro-ph.CO]]

Suppose $\left(3/\kappa^2\right)\left(H(N)\right)^2 = \rho(N) \approx V(N)$ **,**

$$\longrightarrow \quad N \approx \frac{C_1 \rho}{1 - C_2 \rho}\,, \qquad H(N) \approx \kappa \sqrt{\frac{1}{3\left[(C_1/N) + C_2\right]}}$$

$$(C_1/N) + C_2 > 0$$

Inflationary models with $n_{\mathrm{s}} - 1 = -\frac{2}{N}$ (3)

Equation of state

$$P = -\rho - \frac{2}{\kappa^2} H(N) H'(N) \approx -\rho - \frac{3C_1}{N^2 \kappa^4} H^4$$

$f(\rho)$

$$\Rightarrow \quad f(\rho) \approx -\frac{1}{3C_1} \left(1 - 2C_2 \rho + C_2^2 \rho^2\right)$$

By comparing with this expression, we can obtain the EoS of visous fluid.

Reconstructed viscous fluid models

Model (a) $\left(|C_2\rho| \gg 1\right)$

$$P = -\rho + \left(\frac{2C_2}{3C_1}\right)\rho - \left(\frac{3C_2^2}{C_1\kappa^4}\right)H^4$$

Model (b) $\left(|C_2\rho| \gg 1\right)$

$$P = -\rho - \left(\frac{C_2^2}{3C_1}\right)\rho^2 + \left(\frac{2C_2}{C_1\kappa^2}\right)H^2$$

Model (c) $\left(|C_2\rho| \ll 1\right)$

$$P = -\rho - \left(\frac{1}{3C_1}\right) + \left(\frac{2C_2}{C_1\kappa^2}\right)H^2$$

Model (d) $\left(|C_2\rho| \ll 1\right)$

$$P = -\rho + \left(\frac{2C_2}{3C_1}\right)\rho - \left(\frac{1}{3C_1}\right)$$

Observables $(n_\mathrm{s}, r, \alpha_\mathrm{s})$ of the inflationary universe

(1) $n_\mathrm{s} - 1 = -\dfrac{2}{N}$ ⇨ $N = 60 \longrightarrow n_\mathrm{s} = 0.967$

(2) $r = \dfrac{8}{N\left[1 + (C_2/C_1)\,N\right]}$

Models (a), (b)
Models (c), (d) with $C_2 < 0$ ⇨ $N \gtrsim 73\,,\ \ r < 0.11$

Models (c), (d) with $C_2 > 0$ ⇨ $N \gtrsim 60\,,\ \ r < 0.11$

(3) $\alpha_\mathrm{s} = -\dfrac{2}{N^2}$ ⇨ $N = 60 \longrightarrow \alpha_\mathrm{s} = -5.56 \times 10^{-4}$

IV. Graceful exit from inflation

Perturbation of the de Sitter solution

$$H = H_{\inf} + \underline{H_{\inf}\delta(t)}$$

$$|\delta(t)| \ll 1$$

$$H_{\inf}(> 0)$$

: Hubble parameter at the inflationary stage

Gravitational field equation

$$\longrightarrow \ddot{H} - \frac{\kappa^4}{2}\left[\beta A^2 \left(\frac{3}{\kappa^2}\right)^{2\beta} H^{4\beta-1} + \left(\beta + \frac{\gamma}{2}\right) A\bar{\zeta}\left(\frac{3}{\kappa^2}\right)^{\beta} H^{2\beta+\gamma-1} + \frac{\gamma}{2}\bar{\zeta}^2 H^{2\gamma-1}\right] = 0$$

Instability of the de Sitter solutions

Perturbation: $\delta(t) = \exp(\lambda t)$

$$\Rightarrow \quad \lambda^2 - \frac{1}{2}\frac{\kappa^4}{H_{\rm inf}^2}\mathcal{Q} = 0 \qquad \lambda \text{ : Constant}$$

$$\mathcal{Q} \equiv \beta\left(4\beta - 1\right) A^2 \left(\frac{3}{\kappa^2}\right)^{2\beta} H_{\rm inf}^{4\beta}$$

$$+ \left(\beta + \frac{\gamma}{2}\right)\left(2\beta + \gamma - 1\right) A\bar{\zeta}\left(\frac{3}{\kappa^2}\right)^{\beta} H_{\rm inf}^{2\beta+\gamma} + \frac{\gamma}{2}\left(2\gamma - 1\right)\bar{\zeta}^2 H_{\rm inf}^{2\gamma}$$

$\longrightarrow$ **Solution:** $\lambda = \lambda_{\pm} \equiv \pm\frac{1}{\sqrt{2}}\frac{\kappa^2}{H_{\rm inf}}\sqrt{\mathcal{Q}}$

Instability of the de Sitter solutions (2)

⇨ **It is possible to exist a positive solution.**

$$\lambda = \lambda_+ > 0$$

→ **The de Sitter solution can be unstable.**

⇩

The universe can gracefully exit from inflation.

Conditions for the existence of $\lambda = \lambda_+ > 0$

$$\longrightarrow \quad \mathcal{Q} > 0$$

Model (a): **No condition**

Model (b): **No condition**

Model (c): $C_2 < 0$ **or** $C_2 > \frac{1}{36}\left(\frac{\kappa}{H_{\rm inf}}\right)^2$

Model (d): $C_2 < 0$ **or** $C_2 > \frac{1}{18}\left(\frac{\kappa}{H_{\rm inf}}\right)^2$

V. Singular inflation

[Barrow and Graham, Phys. Rev. D 91, 083513 (2015)]

[Nojiri, Odintsov and Oikonomou, Phys. Lett. B 747, 310 (2015)]

Hubble parameter and scale factor

$$H = H_{\mathrm{inf}} + \bar{H}\left(t_{\mathrm{s}} - t\right)^{q}, \qquad q > 1$$

$$a = \bar{a}\exp\left[H_{\mathrm{inf}}t - \frac{\bar{H}}{q+1}\left(t_{\mathrm{s}} - t\right)^{q+1}\right]$$

$\bar{H},\ \bar{a},\ q$: Constants

Gravitational field equations

$$\rho = \frac{3H^2}{\kappa^2}, \qquad P = -\frac{2\dot{H} + 3H^2}{\kappa^2}$$

Singular inflation (2)

⇨ **When $t \to t_{\rm s}$, all of $a, \rho,$ and P become finite values, but higher derivatives of H diverge.**

$\longrightarrow$ **Type IV singularity appears at $t = t_{\rm s}$.**

[Nojiri, Odintsov and Tsujikawa, Phys. Rev. D 71, 063004 (2005)]

Equation of state

$$P = -\rho + f(\rho)\,, \quad f(\rho) = \frac{2q\bar{H}^{1/q}}{\kappa^2}\left(\kappa\sqrt{\frac{\rho}{3}} - H_{\rm inf}\right)^{(q-1)/q}$$

EoS of viscous fluid models

For $H_{\mathrm{inf}}/\sqrt{\kappa^2\rho/3} = H_{\mathrm{inf}}/H \ll 1$ **,**

$$\Rightarrow \quad f(\rho) \approx \frac{2}{3^{(q-1)/(2q)}} \frac{\bar{H}^{1/q}}{\kappa^{(q+1)/q}} \times \left[\rho^{(q-1)/(2q)} - \frac{\sqrt{3}\,(q-1)}{q} \frac{H_{\mathrm{inf}}}{\kappa} \rho^{-1/(2q)} \right]$$

Linear combination of two kinds of power of ρ

A kind of the viscous fluid models reconstructed

We have taken the first order quantity of $\left(H_{\mathrm{inf}}/\sqrt{(\kappa^2\rho)/3}\right)$.

VI. Conclusions

- **We have explicitly reconstructed the EoS of the viscous fluid models from the spectral index** $n_{\mathrm{s}} - 1 = -2/N$ **.**

- **We have shown that the spectral index** n_{s} **, the tensor-to-scalar ratio** r**, and the running** α_{s} **of the spectral index can be consistent with the recent Planck results.**

- **We have demonstrated that the universe can gracefully exit from inflation.**

Backup slides

Discussions

- The existence of an unstable de Sitter solution is only the necessary conditions for inflationary cosmology based on fundamental physics, in which the reheating stage occurs after the end of inflation.

- The viscous fluid models reconstructed in this work are phenomenological one.

→ It is important to obtain some clues to connect such phenomenological models to basic physics in the future works.

Models (a) and (b)

- $|C_2\rho| \gg 1 \longrightarrow f(\rho) \approx \frac{2C_2}{3C_1}\rho - \frac{C_2^2}{3C_1}\rho^2$
- $C_2 < 0 \longrightarrow (-C_2)/C_1 \approx 1/N \ll 1$

$$\Rightarrow \quad w = \frac{P}{\rho} \approx -1 + \frac{1}{3N}(-2 - C_2\rho)$$

If $|C_2\rho| = \mathcal{O}(10)$ **and** $N \gtrsim 60$ **,**

$w \approx -1$ $\Rightarrow$ **(Quasi-)de Sitter inflation can occur.**

Models (c) and (d)

- $|C_2\rho| \ll 1$ $\longrightarrow$ $f(\rho) \approx -\frac{1}{3C_1} + \frac{2C_2}{3C_1}\rho$
- $C_1\rho \approx N \gg 1$ $\longrightarrow$ $|C_2|/C_1 \ll 1$

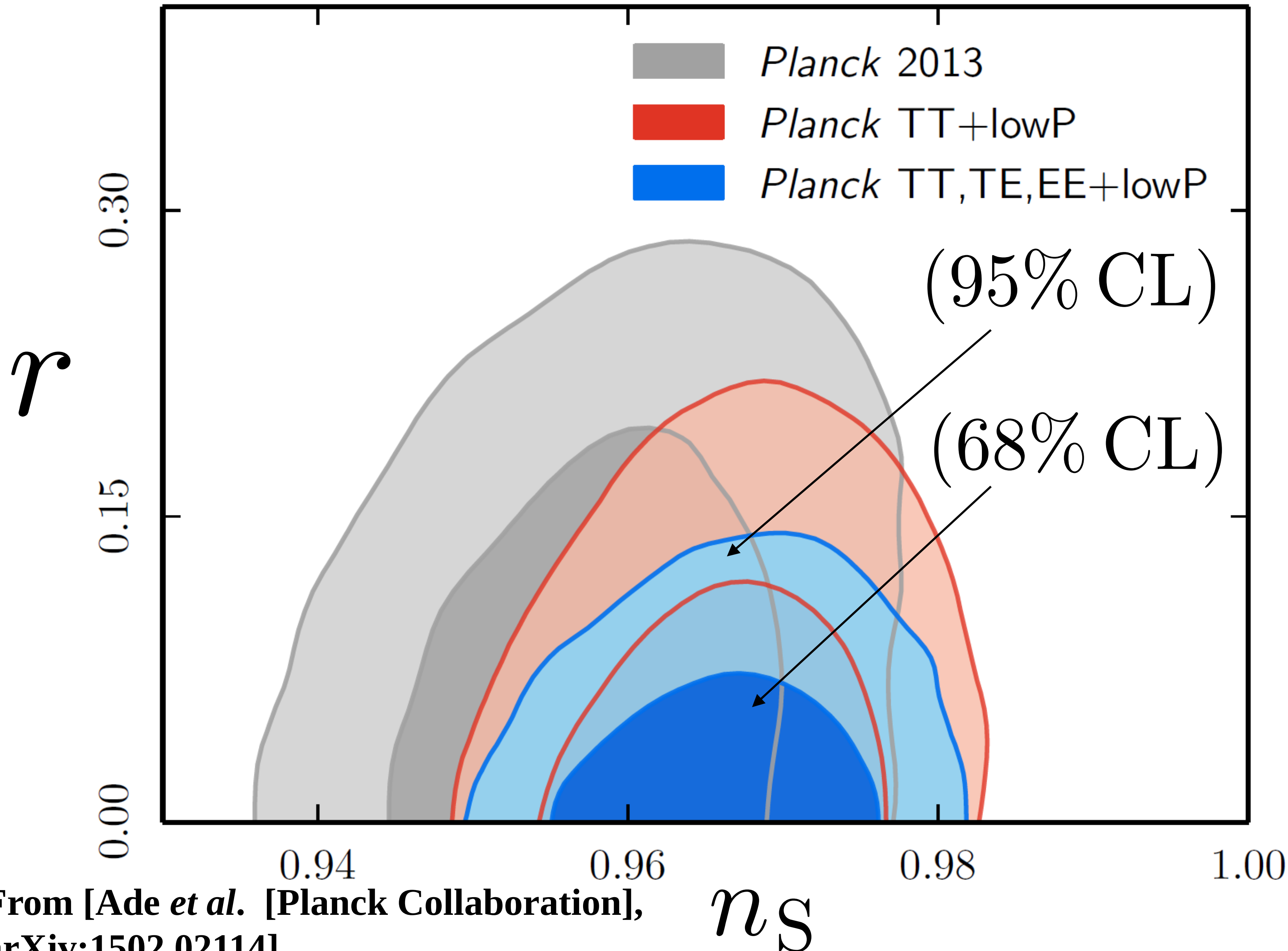
$$\Rightarrow \quad w = \frac{P}{\rho} \approx -1 + \frac{1}{3}\left(-\frac{1}{N} + 2\frac{C_2}{C_1}\right)$$

$$\longrightarrow \quad w \approx -1$$

$\Downarrow$

(Quasi-)de Sitter inflation can be realized.

$(n_{\rm s}, r)$ Contours [Planck 2015]

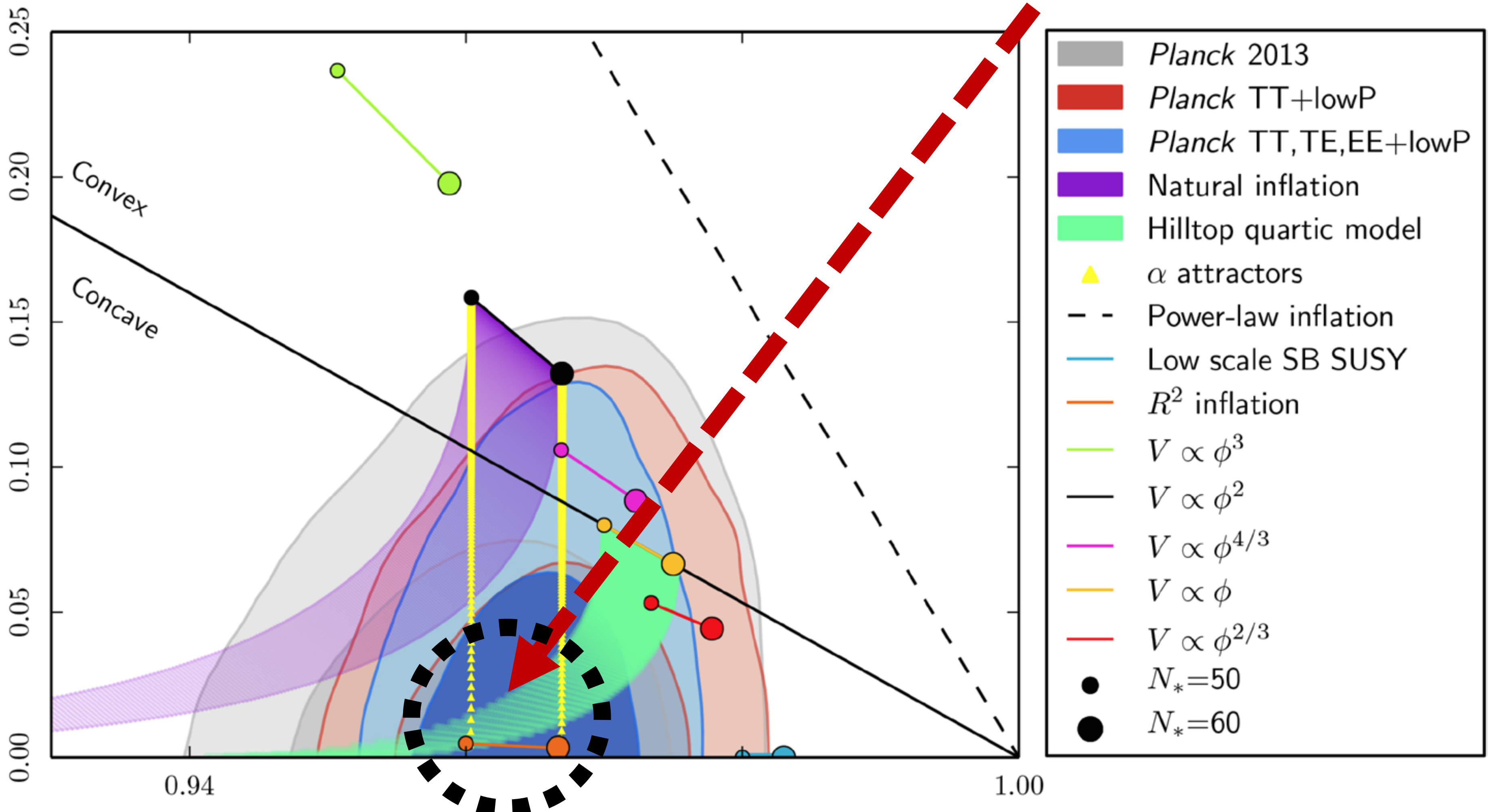


From [Ade *et al*. [Planck Collaboration], arXiv:1502.02114].

Constraints on inflationary models

$r_{0.002}$ $(k = 0.002\,\mathrm{Mpc}^{-1})$

R^2 **Inflation**



n_S

From [Ade *et al*. [Planck Collaboration], arXiv:1502.02114].

Subjects

- **From the spectral index of** $n_{\rm s} - 1 = -\frac{2}{N}$**,** **the inflaton potential *V* of a scalar field theory has been reconstructed.**

[Chiba, arXiv:1504.07692 [astro-ph.CO]]

Action of a scalar field theory

ϕ : Scalar field

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \right)$$

⇩

By applying this procedure to fluids, we reconstruct viscous fluid models.

Viscous fluid models and inflation

$$P = -\rho + f(\rho) = -\rho + A\rho^{\beta} + \bar{\zeta}\left(\frac{\kappa}{\sqrt{3}}\right)^{\gamma}\rho^{\gamma/2}$$

Case 1: $|C_2\rho| \gg 1 \qquad (-C_2)/C_1 \approx 1/N \ll 1$

$$C_2 < 0$$

$$f(\rho) \approx \frac{2C_2}{3C_1}\rho - \frac{C_2^2}{3C_1}\rho^2$$

$$a(t) = a_{\mathrm{i}} \exp\left[H_{\mathrm{inf}}(t - t_{\mathrm{i}})\right]$$

$$w = \frac{P}{\rho} \approx -1 - \frac{2}{3}\left(-\frac{C_2}{C_1}\right) + \frac{1}{3}\left(-\frac{C_2}{C_1}\right)(-C_2\rho) \approx -1 + \frac{1}{3N}(-2 - C_2\rho)$$

流体モデル

$$C_1 \rho \approx N \gg 1$$

Case 2: $|C_2 \rho| \ll 1$

$$|C_2| / C_1 \ll 1$$

$$f(\rho) \approx -\frac{1}{3C_1} + \frac{2C_2}{3C_1}\rho$$

$$w = \frac{P}{\rho} \approx -1 - \frac{1}{3}\frac{1}{C_1\rho} + \frac{2}{3}\left(\frac{C_2}{C_1}\right) \approx -1 + \frac{1}{3}\left(-\frac{1}{N} + 2\frac{C_2}{C_1}\right)$$

$$C_2 > 0 \qquad\qquad C_2 = (2/3)\, C_1$$

$$\bar{\zeta} = 4/\left(3\kappa^2\right) \quad A = 4/9$$

[Starobinsky, Phys. Lett. B <u>91</u>, 99 (1980)]

流体モデル

Case	Model	A	$\bar{\zeta}$	β	γ
(i)	(a)	$2C_2/\left(3C_1\right)$	$-3C_2^2/\left(C_1\kappa^4\right)$	1	4
(i)	(b)	$-C_2^2/\left(3C_1\right)$	$2C_2/\left(C_1\kappa^2\right)$	2	2
(ii)	(c)	$-1/\left(3C_1\right)$	$2C_2/\left(C_1\kappa^2\right)$	0	2
(ii)	(d)	$2C_2/\left(3C_1\right)$	$-1/\left(3C_1\right)$	1	0

流体モデル

Model	EoS
(a)	$P = -\rho + \left[2C_2/\left(3C_1\right)\right]\rho - \left[3C_2^2/\left(C_1\kappa^4\right)\right]H^4$
(b)	$P = -\rho - \left[C_2^2/\left(3C_1\right)\right]\rho^2 + \left[2C_2/\left(C_1\kappa^2\right)\right]H^2$
(c)	$P = -\rho - \left[1/\left(3C_1\right)\right] + \left[2C_2/\left(C_1\kappa^2\right)\right]H^2$
(d)	$P = -\rho + \left[2C_2/\left(3C_1\right)\right]\rho - \left[1/\left(3C_1\right)\right]$

⇒ **不安定なde Sitter解が存在し、インフレーションは終了可能である。**

IV. インフレーションの終了

de Sitter解の摂動

$$|\delta(t)| \ll 1$$

$$H = H_{\rm inf} + \underline{H_{\rm inf}\delta(t)} \qquad H_{\rm inf}(>0)$$

重力場の方程式

$$\longrightarrow \ddot{H} - \frac{\kappa^4}{2}\left[\beta A^2 \left(\frac{3}{\kappa^2}\right)^{2\beta} H^{4\beta-1} \right.$$

$$\left. + \left(\beta + \frac{\gamma}{2}\right) A\bar{\zeta}\left(\frac{3}{\kappa^2}\right)^{\beta} H^{2\beta+\gamma-1} + \frac{\gamma}{2}\bar{\zeta}^2 H^{2\gamma-1}\right] = 0$$

de Sitter解の不安定性

摂動: $\delta(t) = \exp(\lambda t)$

$$\longrightarrow \quad \lambda^2 - \frac{1}{2}\frac{\kappa^4}{H_{\rm inf}^2}\mathcal{Q} = 0$$

$$\mathcal{Q} \equiv \beta\,(4\beta - 1)\,A^2\left(\frac{3}{\kappa^2}\right)^{2\beta} H_{\rm inf}^{4\beta}$$

$$+\left(\beta + \frac{\gamma}{2}\right)(2\beta + \gamma - 1)\,A\bar{\zeta}\left(\frac{3}{\kappa^2}\right)^{\beta} H_{\rm inf}^{2\beta+\gamma} + \frac{\gamma}{2}(2\gamma - 1)\,\bar{\zeta}^2 H_{\rm inf}^{2\gamma}$$

$\longrightarrow$ 解: $\lambda = \lambda_\pm \equiv \pm\frac{1}{\sqrt{2}}\frac{\kappa^2}{H_{\rm inf}}\sqrt{\mathcal{Q}}$

⇒ $\lambda = \lambda_+ > 0$ の解が存在する。

流体モデル

Model (a):

$$\mathcal{Q} = 2\left(\frac{C_2}{C_1}\right)^2 \left(\frac{H_{\rm inf}}{\kappa}\right)^4 \left[6 - 45C_2\left(\frac{H_{\rm inf}}{\kappa}\right)^2 + 63C_2^2\left(\frac{H_{\rm inf}}{\kappa}\right)^4\right] > 0$$

Model (b):

$$\mathcal{Q} = 6\left(\frac{C_2}{C_1}\right)^2 \left(\frac{H_{\rm inf}}{\kappa}\right)^4 \left[2 - 15C_2\left(\frac{H_{\rm inf}}{\kappa}\right)^2 + 21C_2^2\left(\frac{H_{\rm inf}}{\kappa}\right)^4\right] > 0$$

流体モデル

Model (c):

$$\mathcal{Q} = \left(\frac{C_2}{C_1}\right)^2 \left(\frac{H_{\inf}}{\kappa}\right)^2 \left[-\frac{1}{3C_2} + 12\left(\frac{H_{\inf}}{\kappa}\right)^2\right]$$

$$C_2 < 0 \quad \textbf{or} \quad C_2 > \frac{1}{36}\left(\frac{\kappa}{H_{\inf}}\right)^2$$

Model (d):

$$\mathcal{Q} = 2\left(\frac{C_2}{C_1}\right)^2 \left(\frac{H_{\inf}}{\kappa}\right)^2 \left[6\left(\frac{H_{\inf}}{\kappa}\right)^2 - \frac{1}{3C_2}\right]$$

$$C_2 < 0 \quad \textbf{or} \quad C_2 > \frac{1}{18}\left(\frac{\kappa}{H_{\inf}}\right)^2$$

II. スカラー場の理論

作用 $$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

ϕ :インフラトン

$V(\phi)$:ポテンシャル

g :計量 $g_{\mu\nu}$ の行列式

R :スカラー曲率

$\kappa^2 \equiv 8\pi/{M_{\rm Pl}}^2$

$M_{\rm Pl}$:プランク質量

平坦な Friedmann-Lemaître-Robertson-Walker (FLRW) 時空

$$ds^2 = -dt^2 + a^2(t)\sum_{i=1,2,3}\left(dx^i\right)^2$$

$a(t)$:スケールファクター

スローロールインフレーション

スローロールパラメーター: $(\epsilon,\ \eta,\ \xi^2)$

$$\epsilon \equiv \frac{1}{2\kappa^2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \quad \eta \equiv \frac{1}{\kappa^2}\frac{V''(\phi)}{V(\phi)}, \quad \xi^2 \equiv \frac{1}{\kappa^4}\frac{V'(\phi)V'''(\phi)}{(V(\phi))^2}$$

＊ プライム: 各関数の引数に関する微分を表す。

($V'(\phi) \equiv \partial V(\phi)/\partial\phi$)

インフレーションに関する観測量: $(n_{\mathrm{S}}, r, \alpha_{\mathrm{S}})$

$$n_{\mathrm{s}} - 1 = -6\epsilon + 2\eta, \qquad r = 16\epsilon$$

$$\alpha_{\mathrm{s}} \equiv \frac{dn_{\mathrm{s}}}{d\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2$$

k : 波数

重力場の方程式

$$\frac{3}{\kappa^2}H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$-\frac{1}{\kappa^2}\left(3H^2 + 2\dot{H}\right) = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$H = \dot{a}/a$: ハッブルパラメーター　　$\dot{} = \partial/\partial t$

⇨ **場の再定義:** $\phi = \phi(\varphi)$

N : e-folds数

$$\varphi = N$$

$$H = H(N)$$

Description of a perfect fluid

- **Second gravitational equation**

- **Conservation law**

$$\Rightarrow \quad \frac{2}{\kappa^2}\left(H(N)\right)^2\left[\left(\frac{H'(N)}{H(N)}\right)^2+\frac{H''(N)}{H(N)}\right]=3f'(\rho)f(\rho)$$

$$f'(\rho)\equiv df(\rho)/d\rho$$

It is possible to express $H(N)$ and its derivatives with respect to *N* only with $\rho(N)$ and $f(\rho(N))$.

Description of a perfect fluid (2)

The slow-roll parameters can be described in terms of $H(N)$ and its derivatives with respect to N .

The observables of the inflationary models $(n_{\mathrm{S}}, r, \alpha_{\mathrm{S}})$ can be represented with $\rho(N)$ and $f(\rho(N))$.

Observables of the inflationary models

- **For** $|f(\rho)/\rho(N)| \ll 1$**,**

$$\longrightarrow \quad n_{\rm s} \approx 1 - 6\frac{f(\rho)}{\rho(N)}\,, \qquad r \approx 24\frac{f(\rho)}{\rho(N)}$$

$$\alpha_{\rm s} \approx -9\left(\frac{f(\rho)}{\rho(N)}\right)^2$$

- **If** $f(\rho)/\rho(N) = 4.35 \times 10^{-3}$**,**

$$(n_{\rm s}, r, \alpha_{\rm s}) = (0.974, 0.104, -1.70 \times 10^{-4})$$

Viscous fluid models

$$\frac{f(\rho)}{\rho} = A\rho_{\mathrm{c}}^{\beta-1}\left(\frac{\rho}{\rho_{\mathrm{c}}}\right)^{\beta-1} + \bar{\zeta}\left(\frac{\kappa}{\sqrt{3}}\right)^{\gamma}\rho_{\mathrm{c}}^{\gamma/2-1}\left(\frac{\rho}{\rho_{\mathrm{c}}}\right)^{\gamma/2-1}$$

$$= A\rho_{\mathrm{c}}^{\beta-1}\left(\frac{H_{\mathrm{inf}}}{H_0}\right)^{2(\beta-1)} + \bar{\zeta}\left(\frac{\kappa}{\sqrt{3}}\right)^{\gamma}\rho_{\mathrm{c}}^{\gamma/2-1}\left(\frac{H_{\mathrm{inf}}}{H_0}\right)^{\gamma-2}$$

H_{inf} : Hubble parameter at the inflationary stage

$$\rho_{\mathrm{c}} \equiv 3H_0^2/\kappa^2 = 8.10 \times 10^{-47}\,\mathrm{GeV}^4$$

: Critical density

$$H_0 = 100h\,\mathrm{km}\,\mathrm{sec}^{-1}\,\mathrm{Mpc}^{-1} = 2.13h \times 10^{-42}\mathrm{GeV}$$

: Current Hubble parameter $h = 0.678$

Viscous fluid models (2)

For simplicity, if $\gamma = 2\beta$ **,**

$$\frac{f(\rho)}{\rho} = J\left(\frac{H_{\rm inf}}{H_0}\right)^{2(\beta-1)}, \quad J \equiv \left[A + \bar{\zeta}\left(\frac{\kappa}{\sqrt{3}}\right)^{2\beta}\right]\rho_{\rm c}^{\beta-1}$$

- $\beta = 1\,,\;\; J = 4.35 \times 10^{-3}$
- $\beta = 2\,,\;\; (H_{\rm inf}, J) = (1.0 \times 10^{10}\,{\rm GeV}, 9.10 \times 10^{-107})$

  $(1.0 \times 10^{5}\,{\rm GeV}, 9.10 \times 10^{-97})$

$\Longrightarrow$ $f(\rho)/\rho(N) = 4.35 \times 10^{-3}$

$\longrightarrow$ **The Planck results can be realized.**

EoS of viscous fluid models (2)

$$\frac{f(\rho)}{\rho} \approx \frac{2q}{3} \bar{H}^{1/q} \left(\frac{\kappa^2 \rho}{3} \right)^{-(q+1)/(2q)} \left[1 - \frac{(q-1)}{q} \frac{H_{\mathrm{inf}}}{\sqrt{\kappa^2 \rho / 3}} \right]$$

$$= \frac{2q}{3} \left(\frac{\bar{H}}{H^{q+1}} \right)^{1/q} \left[1 - \frac{(q-1)}{q} \frac{H_{\mathrm{inf}}}{H} \right]$$

$$\bar{H} / H^{q+1} \ll 1 \quad \Longrightarrow \quad f(\rho)/\rho \ll 1$$

Cf. [Nojiri, Odintsov, Oikonomou and Saridakis, arXiv:1503.08443 [gr-qc]]

[Odintsov and Oikonomou, arXiv:1507.05273 [gr-qc]]

Planck 2015 results

[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]

(1) Spectral index of power spectrum of the curvature perturbations

$$n_s = 0.968 \pm 0.006 \quad (68\% \, \mathrm{CL})$$

(2) Tensor-to-scalar ratio

$$r < 0.11 \, (95\% \, \mathrm{CL})$$

⇨ (n_s, r) **can be used to constrain inflationary models.**

I. 序

Planck 2015 の結果

[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]

(1) 曲率揺らぎのスペクトル指数

$$n_{\rm s} = 0.968 \pm 0.006 \quad (68\%\,\mathrm{CL})$$

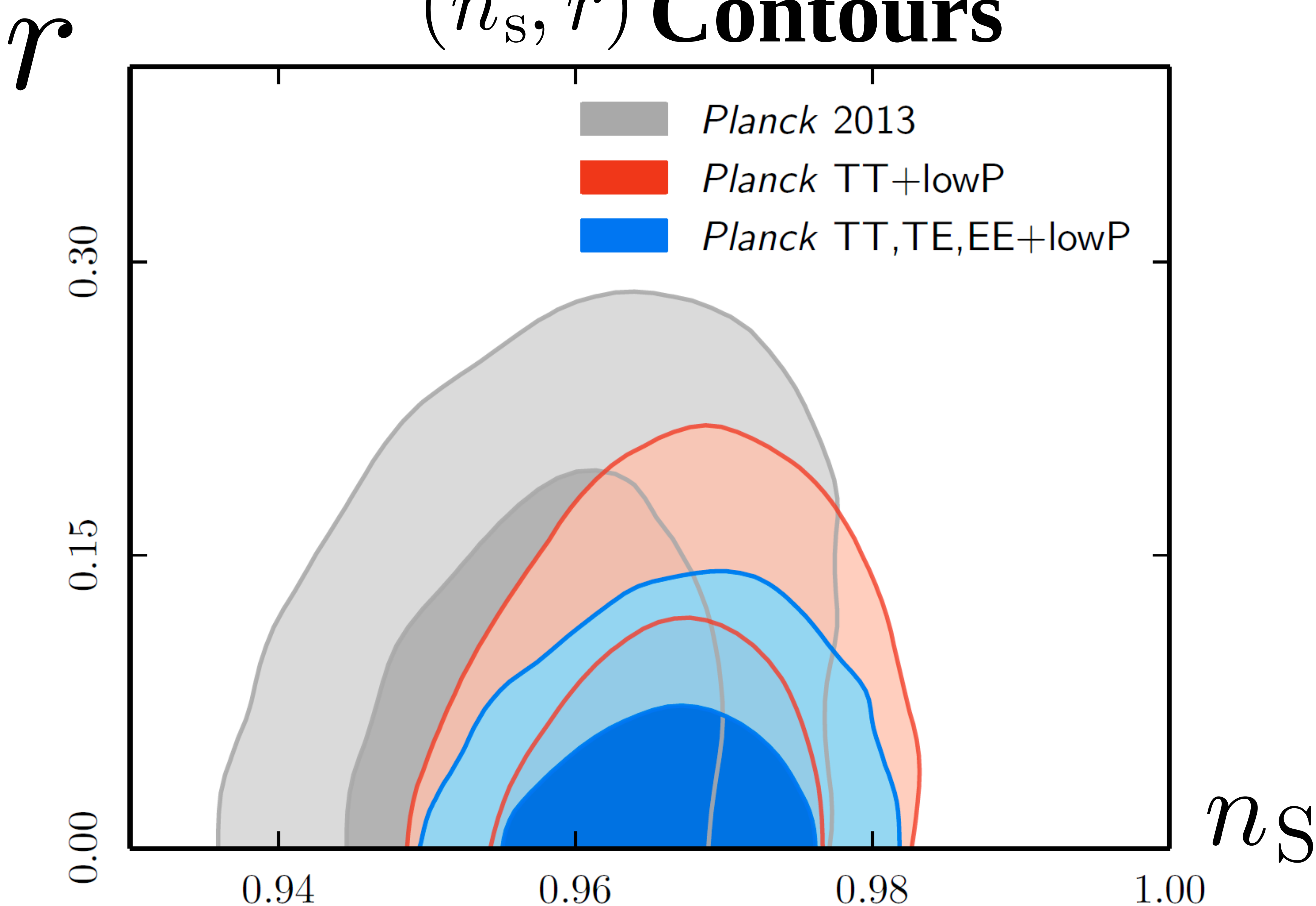
(2) テンソル／スカラー比

$$r < 0.11\,(95\%\,\mathrm{CL})$$

(3) スペクトル指数のランニング

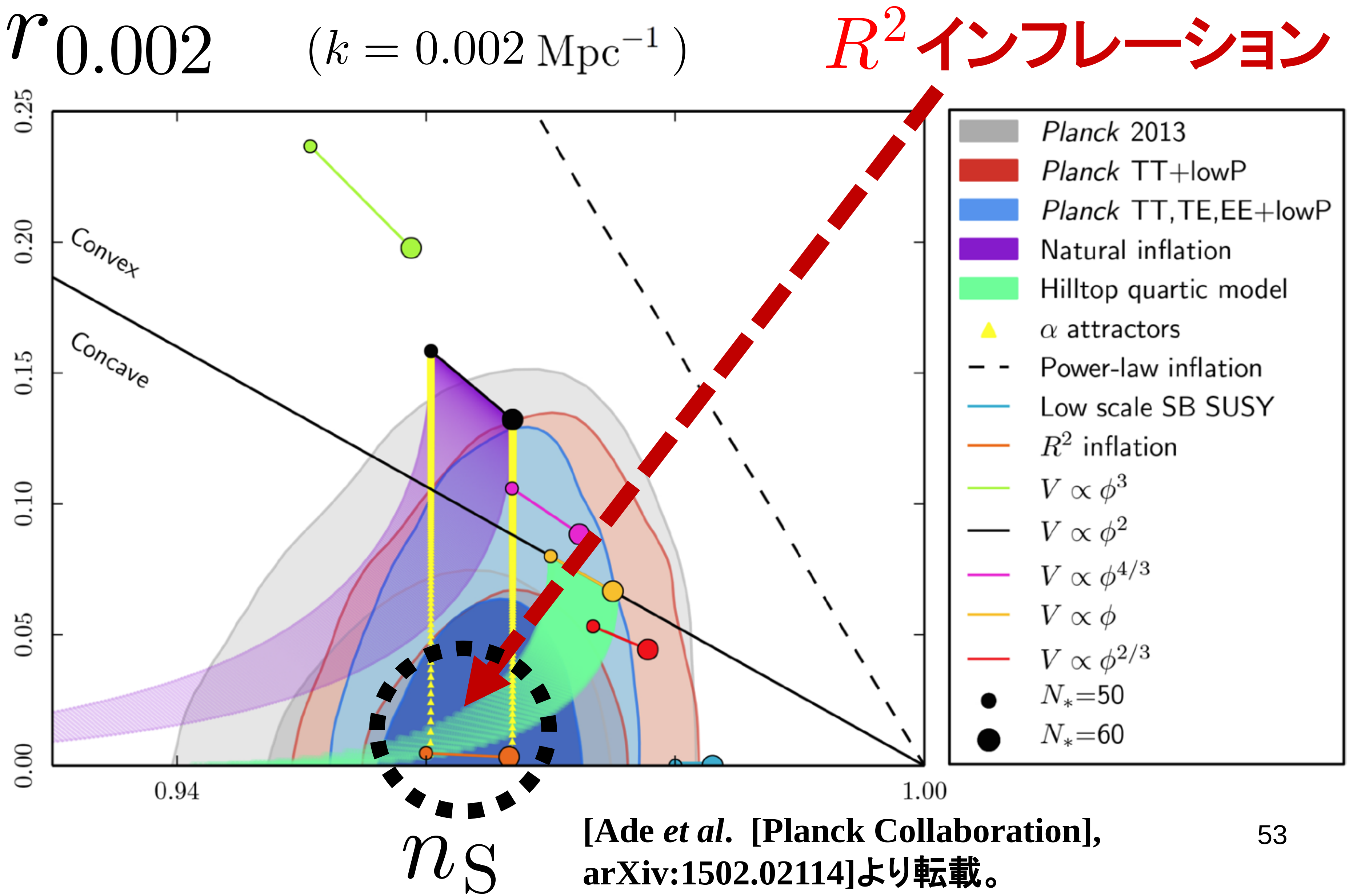
$$\alpha_{\rm S} = -0.003 \pm 0.007\;(68\%\,\mathrm{CL})$$

(n_{s}, r) Contours



[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]より転載。

インフレーションモデルへの制限



[Ade *et al*. [Planck Collaboration], arXiv:1502.02114]より転載。

Starobinsky(R^2)インフレーション

作用積分 $S = \int d^4x \sqrt{-g} \frac{1}{2\kappa^2}(R + \alpha_S R^2)$

[Starobinsky, Phys. Lett. B 91, 99 (1980)]

$\kappa^2 \equiv 8\pi / {M_{Pl}}^2$　　M_{Pl} :プランク質量　　α_S :定数

- $N = 60 \longrightarrow$ $n_S = 0.967, \quad r = 3.33 \times 10^{-3}$

$\alpha_s = -5.56 \times 10^{-4}$

N :インフレーション期のe-folds数

Cf. [Hinshaw *et al.*, Astrophys. J. Suppl. 208, 19 (2013)]

重力場の方程式(2)

$$\longrightarrow \quad \frac{3}{\kappa^2}(H(N))^2 = \frac{1}{2}\underline{\omega(\varphi)}(H(N))^2 \underline{+ V(\phi(\varphi))}$$

$$-\frac{1}{\kappa^2}\left(3(H(N))^2 + 2H'(N)H(N)\right) = \frac{1}{2}\underline{\omega(\varphi)}(H(N))^2 \underline{- V(\phi(\varphi))}$$

スカラー場理論を特徴付ける量

- $\underline{\omega(\varphi) \equiv (d\phi/d\varphi)^2}$ $\Longrightarrow$ $\omega(\varphi) = -\left.\dfrac{2H'(N)}{\kappa^2 H(N)}\right|_{N=\varphi}$

- $\underline{V(\varphi) \equiv V(\phi(\varphi))}$

$$\Longrightarrow \quad V(\varphi) = \frac{1}{\kappa^2}(H(N))^2\left(3 + \frac{H'(N)}{H(N)}\right)\Bigg|_{N=\varphi}$$

動機と目的

- インフレーションに関する観測量を考慮することにより、現在の宇宙の加速膨張を説明する理想流体模型や$F(R)$重力理論に対するさらなる観測的示唆を得る。

⇨ インフレーションに関する三つの観測量 $(n_{\mathrm{S}}, r, \alpha_{\mathrm{S}})$ の表式を用いて、(i) スカラー場理論、(ii) 理想流体模型、(iii) $F(R)$重力理論を記述する。

スローロールパラメーター

$H'''(N) \ll H''(N) \ll H'(N) \ll H(N)$ の場合、

$$\longrightarrow \quad \epsilon(N) \sim -\frac{H'(N)}{H(N)}$$

$$\eta(N) \sim -\frac{3}{2}\frac{H'(N)}{H(N)} \sim \frac{3}{2}\epsilon(N)$$

$$\xi^2 \sim \frac{3}{2}\left(\frac{H'(N)}{H(N)}\right)^2 \sim \frac{3}{2}(\epsilon(N))^2$$

スカラー場理論を特徴付ける量

$$\omega(\varphi) \sim \frac{2}{\kappa^2}\epsilon(N)\Big|_{N=\varphi}$$

$$V(\varphi) \sim \frac{3H_0^2}{\kappa^2}\exp\left(-2\int^N d\hat{N}\epsilon(\hat{N})\right)\Big|_{N=\varphi}$$

H_0 :定数

[KB, Nojiri and Odintsov, Phys. Lett. B <u>737</u>, 374 (2014)]

状態方程式 $P(N) = -\rho(N) + f(\rho)$ (2)

・ **モデル (B)** $f(\rho) = (\rho(N))^{\tau}$ $\tau(\neq 1)$: 定数

[Frampton, Ludwick and Scherrer, Phys. Rev. D 84, 063003 (2011); 85, 083001 (2012)]

[Astashenok, Nojiri, Odintsov and Yurov, Phys. Lett. B 709, 396 (2012)]

$\longrightarrow$ $f(\rho)/\rho(N) \approx \left(3H_{\mathrm{inf}}^2/\kappa^2\right)^{\tau-1} = 6.65 \times 10^{-3}$ **の場合、**

⇨ $(n_{\mathrm{S}}, r, \alpha_{\mathrm{S}})$ **の値は、Planck の結果と整合し得る。**

$(H(N))^2$ の表式

→ ***H(N)*が与えられた場合、理想流体模型での状態方程式が得られる。**

- **線形型**: $(H(N))^2 = G_0 N + G_1$

$G_0 (< 0)$, $G_1 (> 0)$: 定数

⇨ $$f(\rho) = -\frac{G_0}{\kappa^2}$$

$$w(N) \equiv \frac{P(N)}{\rho(N)} = -1 + \frac{f(\rho)}{\rho(N)} = -\frac{(3N+1)\,G_0 + 3G_1}{3\,(G_0 N + G_1)}$$

線形型 $(H(N))^2 = G_0 N + G_1$

指数関数的インフレーション

$$a = \bar{a}\exp\left(H_{\rm inf} t\right)$$

$\bar{a}$:定数

$H_{\rm inf}$:インフレーション期のハッブルパラメーター

→ **N を時間とみなした場合、**

$G_1/G_0 \gg N$ ⇨ **H はほぼ一定に振舞う。**

$(H(N))^2$ の表式(2)

- **指数関数型**: $(H(N))^2 = G_2 \mathrm{e}^{\beta N} + G_3$

$G_2(< 0),\ G_3(> 0),\ \beta(> 0)$:定数

$$\longrightarrow \quad f(\rho) = -\frac{\beta}{3}\rho(N) + \frac{G_3 \beta}{\kappa^2}$$

$$w(N) = -\frac{(3+\beta)\, G_2 \mathrm{e}^{\beta N} + 3 G_3}{3\left(G_2 \mathrm{e}^{\beta N} + G_3\right)}$$

指数関数型 $(H(N))^2 = G_2 e^{\beta N} + G_3$

パワーローインフレーション

$$a = \bar{a} t^{\hat{p}}$$

$\hat{p} \ (> 1)$: 定数

$$\rightarrow \quad H^2 = (\hat{p}/t)^2 = \hat{p}^2 \exp(-2N/\hat{p})$$

$$\Rightarrow \quad G_2 = \hat{p}^2 , \quad \beta = -2/\hat{p} , \quad G_3 = 0$$

の場合に相当する。

状態方程式 $P(N) = -\rho(N) + f(\rho)$

- **モデル (A)** $f(\rho) = \bar{f}\sin\left(\rho(N)/\bar{\rho}\right)$

$\bar{f}$, $\bar{\rho}$: 定数

[Astashenok, Nojiri, Odintsov and Scherrer, Phys. Lett. B 713, 145 (2012)]

$\longrightarrow \quad \rho(N)/\bar{\rho} \ll 1\,, \quad f(\rho)/\rho(N) \approx \bar{f}/\bar{\rho} = 6.65 \times 10^{-3}$

の場合、

⇨ $(n_{\mathrm{S}}, r, \alpha_{\mathrm{S}})$ **の値は、Planck の結果と整合し得る。**

インフレーションに関する観測量

$\longrightarrow$ $w(N) \equiv P(N)/\rho(N) = -1 + f(\rho)/\rho(N) \approx -1,$

即ち $|f(\rho)/\rho(N)| \ll 1$ の場合、

$$n_{\mathrm{s}} \sim 1 - 6\frac{f(\rho)}{\rho(N)} \quad , \qquad r \approx 24\frac{f(\rho)}{\rho(N)}$$

$$\alpha_{\mathrm{s}} \approx -9\left(\frac{f(\rho)}{\rho(N)}\right)^2$$

- $f(\rho)/\rho(N) = 6.65 \times 10^{-3}$ の場合、

$$(n_{\mathrm{s}}, r, \alpha_{\mathrm{s}}) = (0.960, 0.160, -3.98 \times 10^{-4})$$

IV. *F*(*R*)重力理論での描像

作用　$$S_{F(R)} = \int d^4x \sqrt{-g}\, \frac{F(R)}{2\kappa^2}$$

- **重力場の方程式の (0, 0) 成分**

$$0 = -9G\left(N\left(R\right)\right)\left(4G'\left(N\left(R\right)\right) + G''\left(N\left(R\right)\right)\right)\frac{d^2F(R)}{dR^2} + \left(3G\left(N\left(R\right)\right) + \frac{3}{2}G'\left(N\left(R\right)\right)\right)\frac{dF(R)}{dR} - \frac{F(R)}{2}$$

$$\left(H(N)\right)^2 = G(N)$$

⇨ ***H*(*N*)が与えられた場合、この方程式を解くことにより、*F*(*R*)の表式を得られる。**

線形型 $(H(N))^2 = G_0 N + G_1$

$$F(R) = C_1(6G_0 - 2R)^{3/2}\sqrt{\frac{R}{12G_0} - \frac{1}{4}}\left[1 - \frac{1}{\frac{R}{12G_0} - \frac{1}{4}} - \frac{1}{4\left(\frac{R}{12G_0} - \frac{1}{4}\right)^2}\right]$$

$$+ C_2(6G_0 - 2R)^{3/2} L\left(\frac{1}{2}, \frac{3}{2}; \frac{R}{12G_0} - \frac{1}{4}\right)$$

$C_1,\ C_2$:積分定数

$L(u_1, u_2; y)$:一般化された Laguerre 多項式

$$(N, G_0, G_1) = (50.0, -0.850, 95.0),\ (60.0, -0.950, 115)$$

の場合、

$$(n_{\rm s}, r, \alpha_{\rm s}) = (0.967, 0.121, -5.42\times 10^{-5}),$$

$$(0.967, 0.123, -5.55\times 10^{-5}).$$

指数関数型 $(H(N))^2 = G_2 \mathrm{e}^{\beta N} + G_3$

$$F(R) = C_1 F\left(b_+, b_-, l; y\right)$$

$$+ C_2 \left(12G_3 - R\right)^{(1+1/\beta)} F\left(1 + b_- + \frac{1}{\alpha}, 1 + b_+ + \frac{1}{\alpha}, 2 - d; y\right)$$

$$b_\pm = \frac{-3\beta - 2 \pm \sqrt{\beta^2 - 20\beta + 4}}{4\beta}, \quad d = -\frac{1}{\beta}, \quad y = \frac{12G_3 - R}{12G_3 + 3G_3\beta}$$

$$(N, G_2, G_3) = (50.0, -1.10, 10.0), (60.0, -1.20, 15.0)$$

の場合、

$$(n_\mathrm{s}, r, \alpha_\mathrm{s}) = (0.963, 6.89 \times 10^{-2}, -5.06 \times 10^{-5}),$$

$$(0.965, 5.84 \times 10^{-2}, -4.51 \times 10^{-5}).$$