

「なぜ宇宙は加速するのか? - 徹底的究明と将来への挑戦 -」
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The Origin of Cosmic Objects

Masayuki Umemura

*Center for Computational Sciences
University of Tsukuba*

3D Problems in Cosmology

Dark Matter: How was dark matter generated?

Dark Energy: What accelerates the Universe?

Dark Age: What is the origin of cosmic objects?

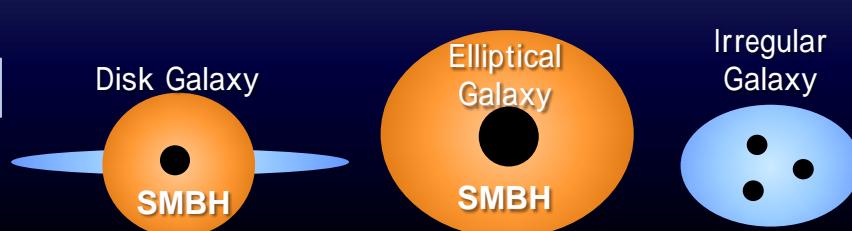
→ **FIRST³:** First Stars, First Galaxies, First BHs

FIRST³: How the First Stars, the First Galaxies, and the First Massive Black Holes Came into Being

$Z=1000$

Cosmic Recombination

	What Happened	What Puzzles	Approach
			Theoretical Observational
Dark Age	Formation of First Stars Formation of First BHs	Mass of First Stars ? Formation of Supermassive Stars ?	Transfer of Ionizing & Dissociating Photons Radiation-dominated Clouds General Relativistic RHD
$z \gg 10$			He II Emission Line Objects Sites of First Galaxies
$z=7$	Formation of First Galaxies Growth of Massive BHs	How First Galaxies Formed ? Supercritical Accretion ? Merger of Massive BHs ? Why SMBH is 1/1000 Bulge ?	Interaction between Gas & Radiation Radiation MHD General Relativistic N-body
$z=0$	Supermassive BH – Bulge Relation	Disk Galaxy Elliptical Galaxy Irregular Galaxy	Survey of High-z AGNs



Supermassive BH-Bulge Relation

Kormendy & Richstone 1995

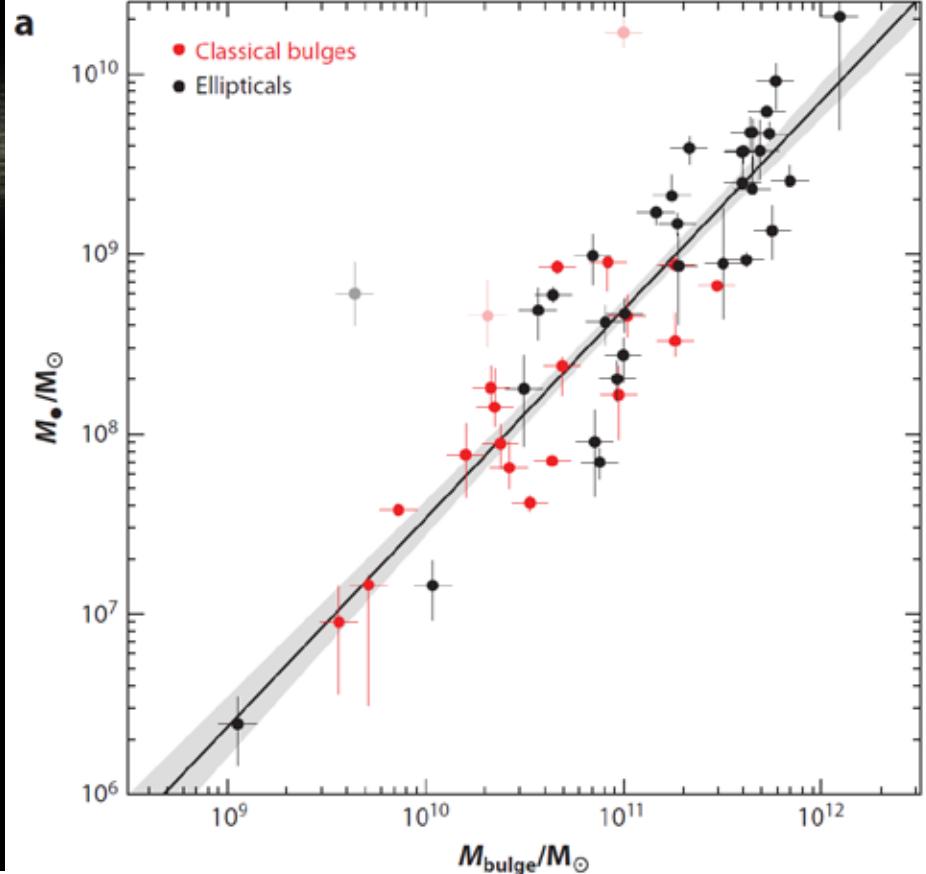
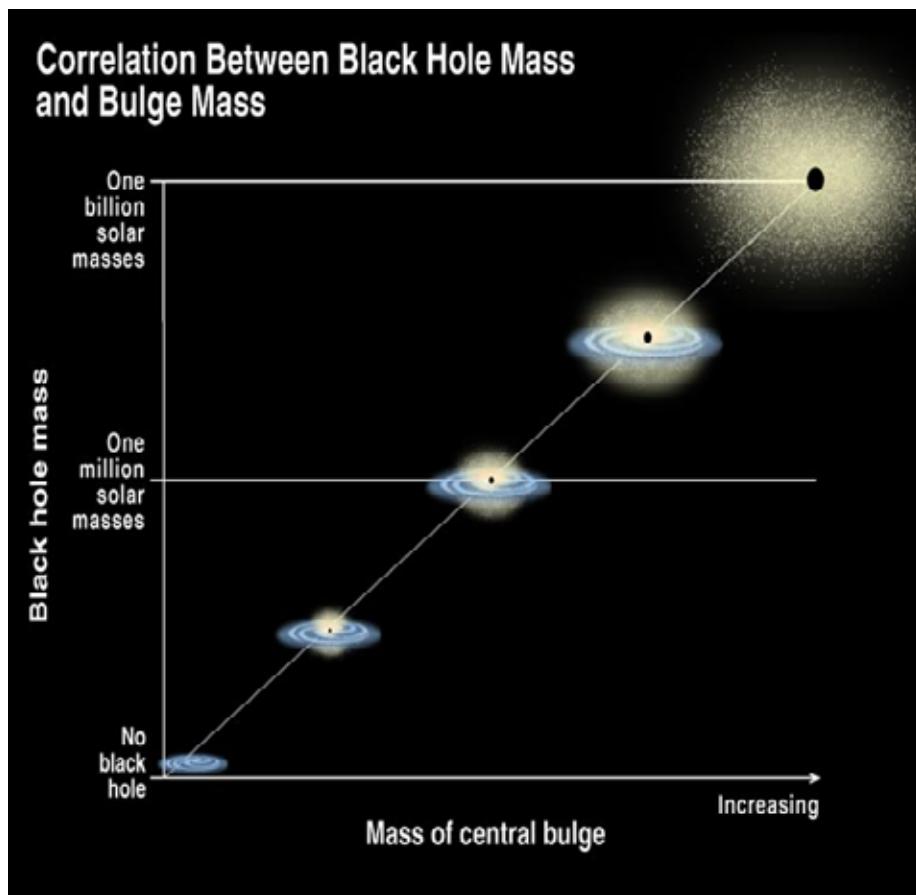
Magorrian et al. 1998

Marconi & Hunt 2003

Kormendy & Ho 2013

$$M_{\text{BH}} / M_{\text{bulge}} \gg 0.001$$

Kormendy & Ho 2013, ARA&A, 51, 511



$$L=6.3' \times 10^{13} L_{\odot}, M_{\text{BH}}=2' \times 10^9 M_{\odot}$$

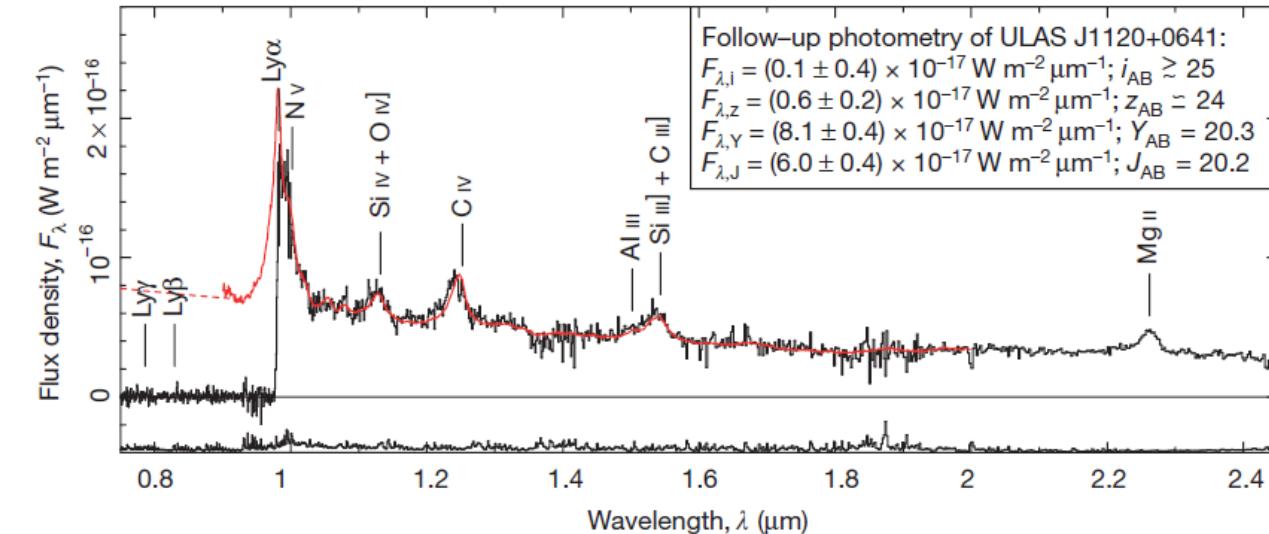


Figure 4 | Rest-frame transmission profile of ULAS J1120+0641 in the region of the Ly α emission line, compared to several damping profiles. The transmission profile of ULAS J1120+0641, obtained by dividing the spectrum by the SDSS composite shown in Fig. 1, is shown in black. The random error spectrum is plotted below the data, also in black. The positive residuals near 0.1230 μm in the transmission profile suggest that the Ly α emission line of ULAS J1120+0641 is actually stronger than average, in which case the absorption would be greater than illustrated. The dispersion in the Ly α equivalent width at a fixed C IV equivalent width of 13% quantifies the uncertainty in the Ly α strength; this systematic uncertainty in the transmission profile is shown in red. The blue curves show the Ly α damping wing of the intergalactic medium for neutral fractions of (from top to bottom) $f_{\text{H}\text{i}} = 0.1$, $f_{\text{H}\text{i}} = 0.5$ and $f_{\text{H}\text{i}} = 1.0$, assuming a sharp ionization front 2.2 Mpc in front of the quasar. The green curve shows the absorption profile of a damped Ly α absorber of column density $N_{\text{H}\text{i}} = 4 \times 10^{20} \text{ cm}^{-2}$ located 2.6 Mpc in front of the quasar. These curves assume that the ionized zone itself is completely transparent; a more realistic model of the H i distribution around the quasar might be sufficient to discriminate between these two models^{25,27}. The wavelength of the Ly α transition is shown as a dashed line; also marked is the N V doublet of the associated absorber referred to in the text.

$z=6.30$ QSO (0.9Gyr)

Xue-Bing Wu et al. 2015, nature, 518, 512

$$L = 4.29' \cdot 10^{14} L_{\odot}, M_{\text{BH}} = 1.2' \cdot 10^{10} M_{\odot}$$

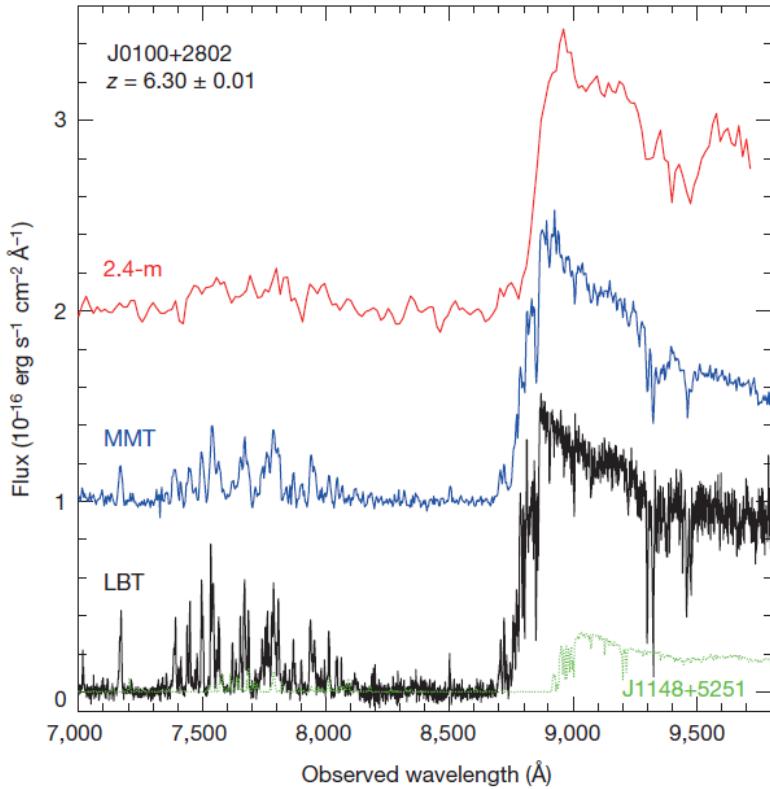


Figure 1 | The optical spectra of J0100+2802. From top to bottom, spectra taken with the Lijiang 2.4-m telescope, the MMT and the LBT (in red, blue and black colours), respectively. For clarity, two spectra are offset upward by one and two vertical units. Although the spectral resolution varies from very low to medium, in all spectra the Ly α emission line, with a rest-frame wavelength of 1,216 Å, is redshifted to around 8,900 Å, giving a redshift of 6.30. J0100+2802 is a weak-line quasar with continuum luminosity about four times higher than that of SDSS J1148+5251 (in green on the same flux scale)¹, which was previously the most luminous high-redshift quasar known at $z = 6.42$.

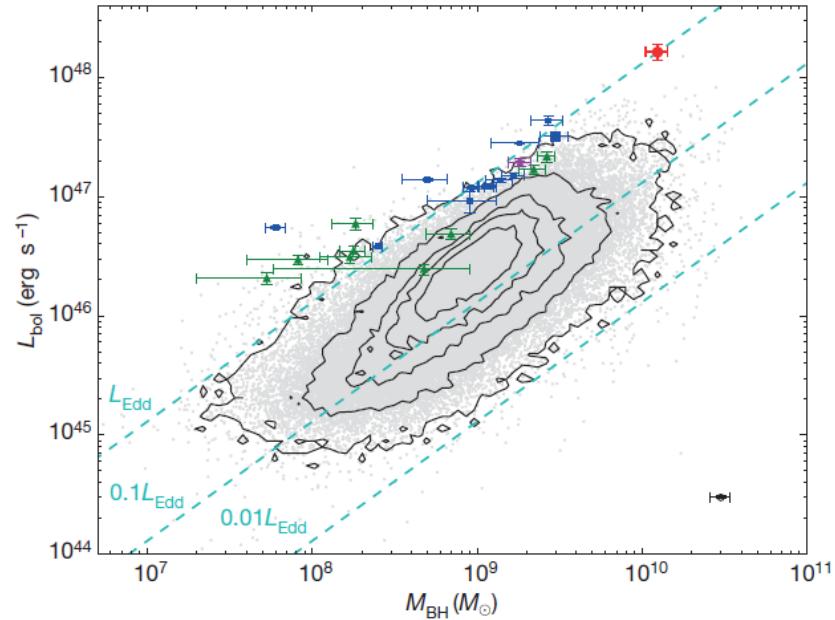
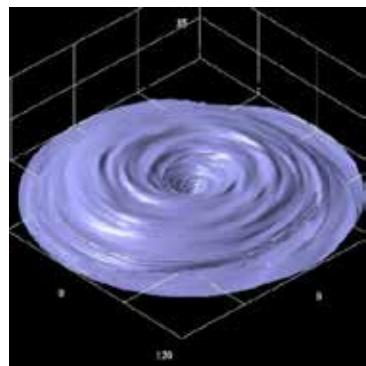


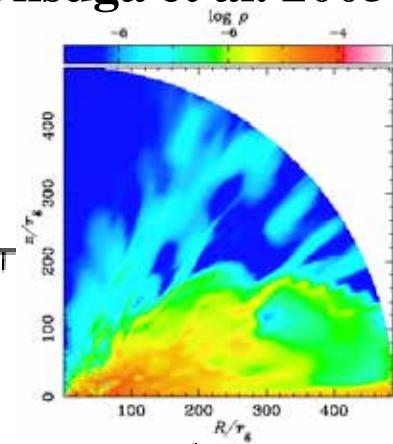
Figure 4 | Distribution of quasar bolometric luminosities, L_{bol} , and black-hole masses, M_{BH} , estimated from the Mg II lines. The red circle at top right represents J0100+2802. The small blue squares denote SDSS high-redshift quasars^{2,10,12}, and the large blue square represents J1148+5251. The green triangles denote CFHQS high-redshift quasars^{11,12}. The purple star denotes ULAS J1120+0641 at $z = 7.085$ (ref. 6). Black contours (which indicate 1 σ to 5 σ significance from inner to outer) and grey dots denote SDSS low-redshift quasars²¹ (with broad absorption line quasars excluded). Error bars represent the 1 σ standard deviation, and the mean error bar for low-redshift quasars is presented in the bottom-right corner. The dashed lines denote the luminosity in different fractions of the Eddington luminosity, L_{Edd} . Note that the black-hole mass and bolometric luminosity are calculated using the same method and the same cosmology model as in the present Letter, and the systematic uncertainties (not included in the error bars) of virial black-hole masses could be up to a factor of three²⁷.

Mass Accretion onto BH

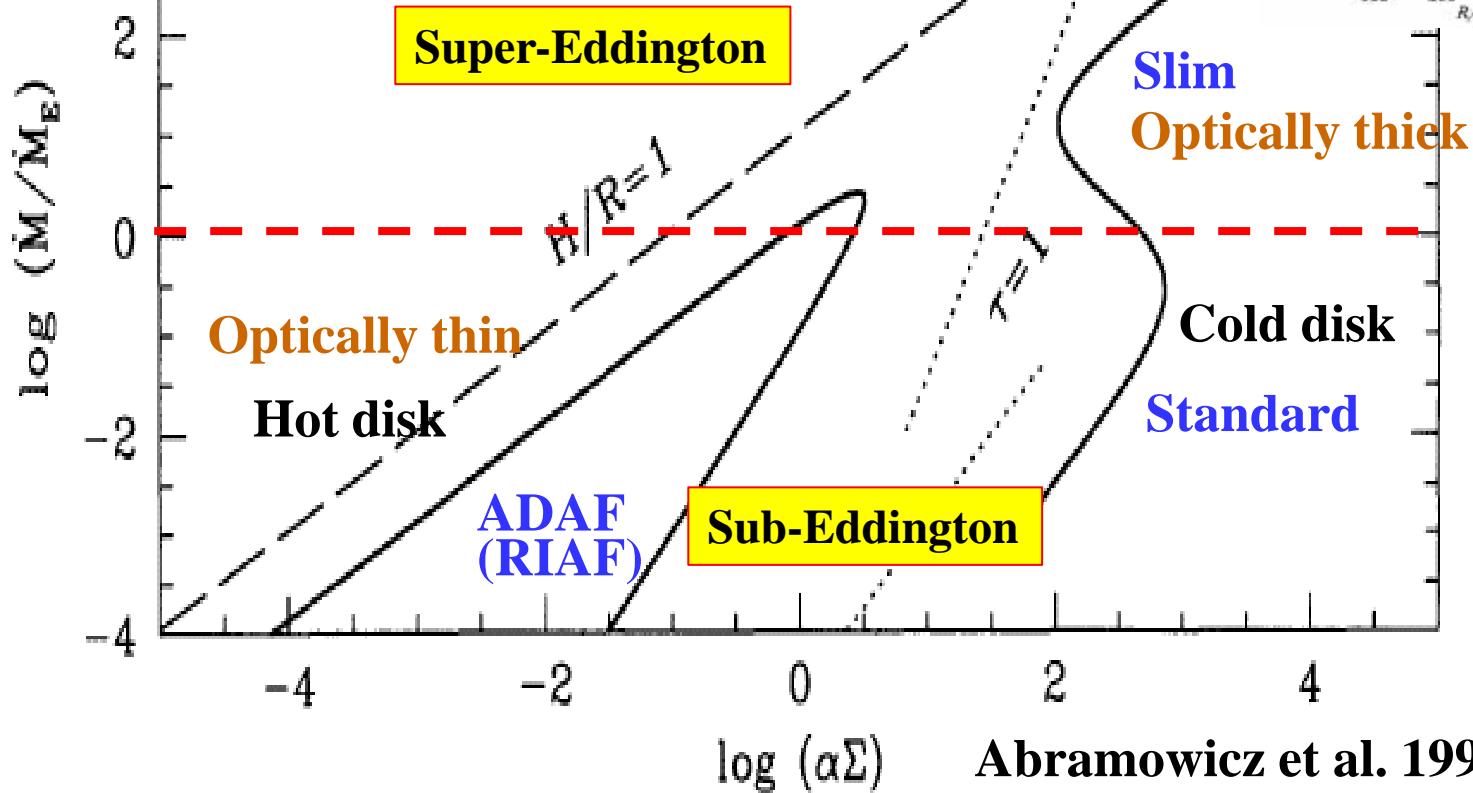
Machida et al. 2004



Ohsuga et al. 2005



1pc ® BH



Abramowicz et al. 1995

Energy Conversion Efficiency in Accretion Flows

$$L = \dot{M} \eta c^2, \quad L_E = \frac{4\pi G c m_p M_{BH}}{s_T} = \dot{M}_E \eta c^2$$

$$t_E \circ \frac{M}{\dot{M}_E} = 4.5 \times 10^7 \frac{\text{days}}{c \frac{M}{\dot{M}_E} \div \phi}$$

Sub-Eddington: RIAF (Radiatively Inefficient Accretion Flow)

Eddington ratio $n_E \circ \frac{\dot{M}}{\dot{M}_E} = 1 \quad \nabla \quad h \gg 0.1 \frac{\dot{M}}{\dot{M}_E}$

Eddington: Standard Disk

$$n_E \circ \frac{\dot{M}}{\dot{M}_E} \gg 1 \quad \nabla \quad h \gg 0.1$$

Super-Eddington: Slim Disk (Photon trapping)

$$n_E \circ \frac{\dot{M}}{\dot{M}_E} > 1 \quad \nabla \quad h \gg 0.1 \frac{\text{days}}{c \frac{\dot{M}}{\dot{M}_E} \div \phi}^{1/2}$$

Eddington Growth of z=7.085 QSO SMBH

$$M_{\text{BH}}(t) = M_0 \exp \left[\frac{\alpha}{c} \eta_E \frac{t - t_E}{t_E} \right], \quad t_E \approx \frac{M}{\dot{M}_E} = 4.5 \times 10^7 \frac{c h}{\alpha} \text{ yr}$$

$$z_{\text{PopIII}} = 20 \quad (t = 1.83 \times 10^8 \text{ yr}), \quad M_0 = 20M_e$$

$$z_{\text{QSO}} = 7.085 \quad (t = 7.83 \times 10^8 \text{ yr}), \quad M_{\text{BH}} = 2 \times 10^9 M_e$$

$$\Delta t = 6 \times 10^8 \text{ yr}$$



$$\eta_E = 1.4 \quad \text{Super-Eddington}$$

Eddington Growth of z=6.30 QSO SMBH

$$M_{\text{BH}}(t) = M_0 \exp \left[\frac{\alpha}{c} n_E \frac{t - t_E}{t_E} \right], \quad t_E \approx \frac{M}{\dot{M}_E} = 4.5 \times 10^7 \frac{c h}{\alpha} \text{ yr}$$

$$z_{\text{PopIII}} = 20 \quad (t = 1.83 \times 10^8 \text{ yr}), \quad M_0 = 20 M_e$$

$$z_{\text{QSO}} = 6.30 \quad (t = 8.84 \times 10^8 \text{ yr}), \quad M_{\text{BH}} = 1.2 \times 10^{10} M_e$$

$$Dt = 7.01 \times 10^8 \text{ yr}$$



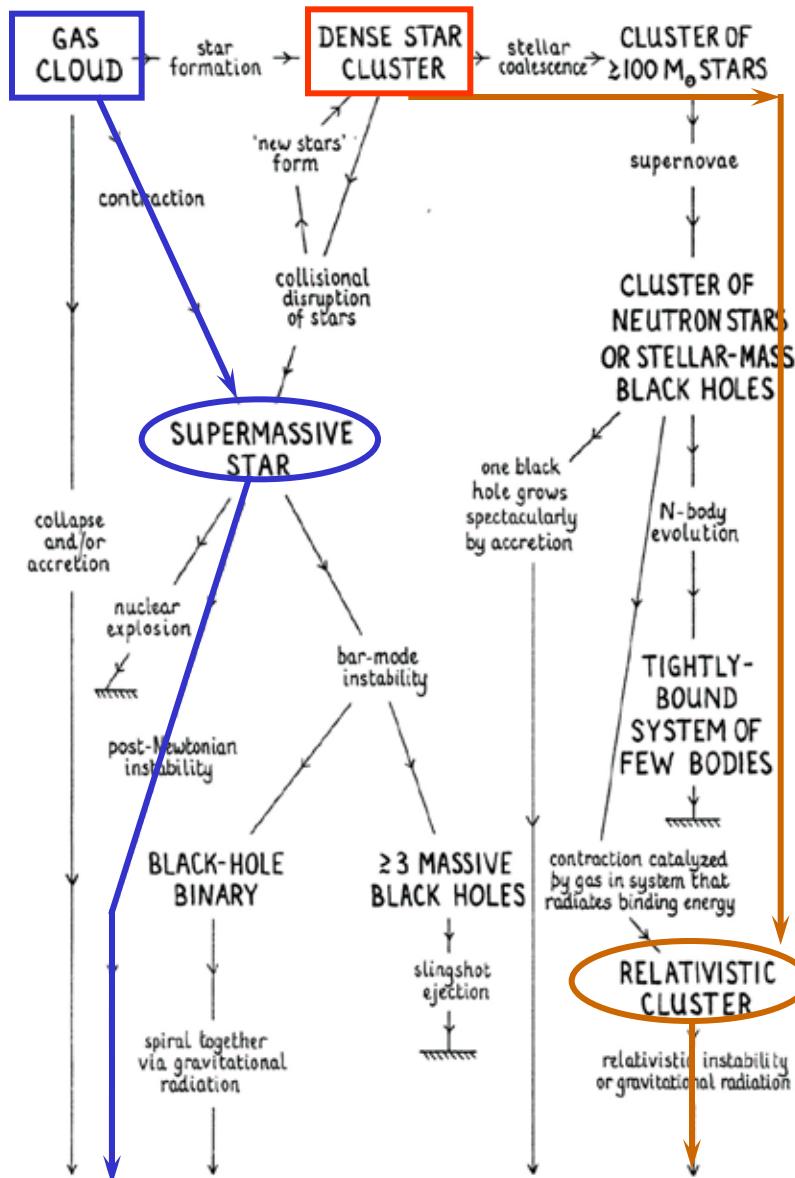
$$n_E = 1.3 \quad \text{Super-Eddington}$$

But, the mass accretion should be intermittent. (Milosavljevic+2009a,b)

When & How First Massive Black Holes Came into being

A black hole with a bright accretion disk. The black hole is a dark, circular void at the center. A bright, glowing ring of orange and yellow light surrounds it, representing the accretion disk where matter is falling onto the black hole. The background is a deep red, suggesting the heat and density of the surrounding space.

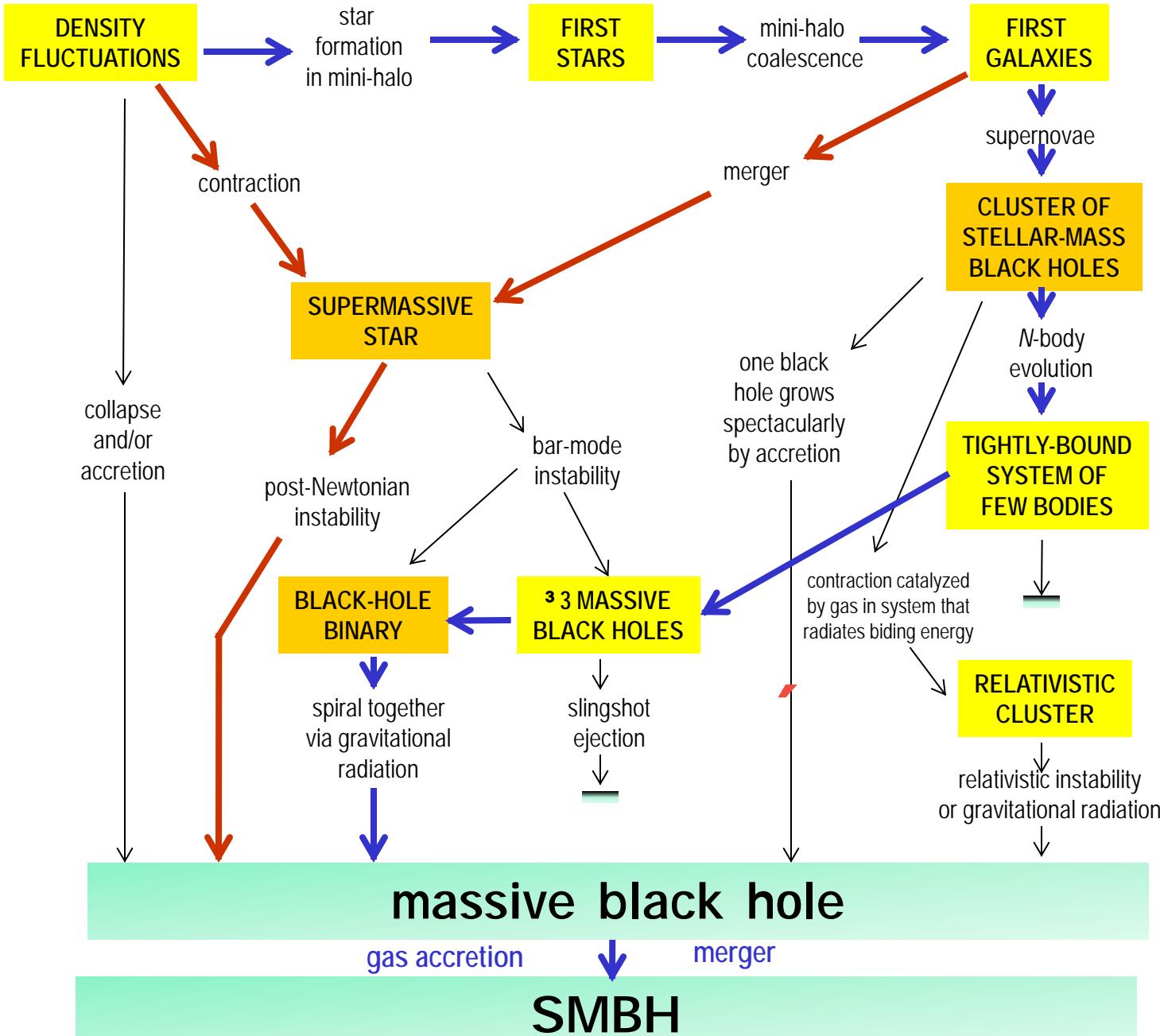
Rees Diagram (1984)



massive black hole

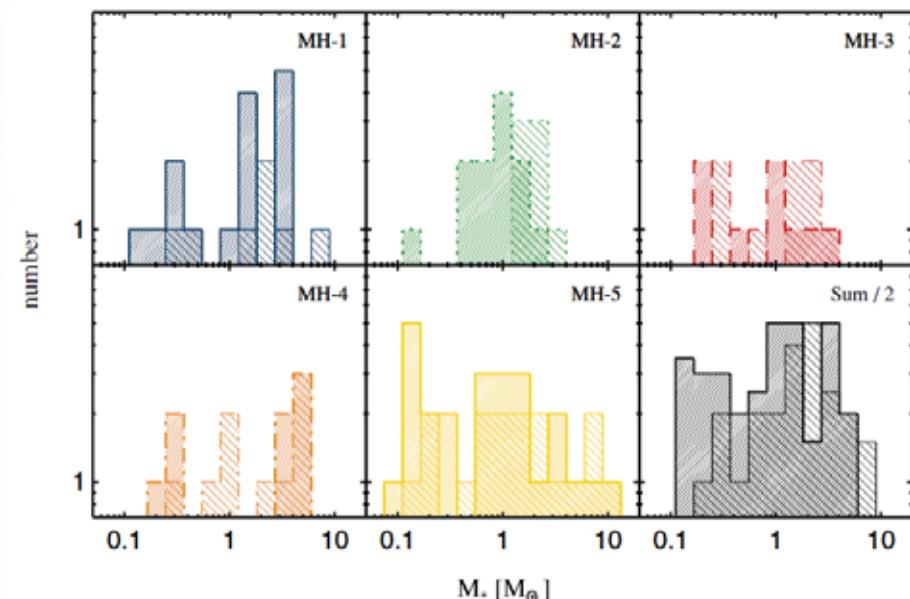
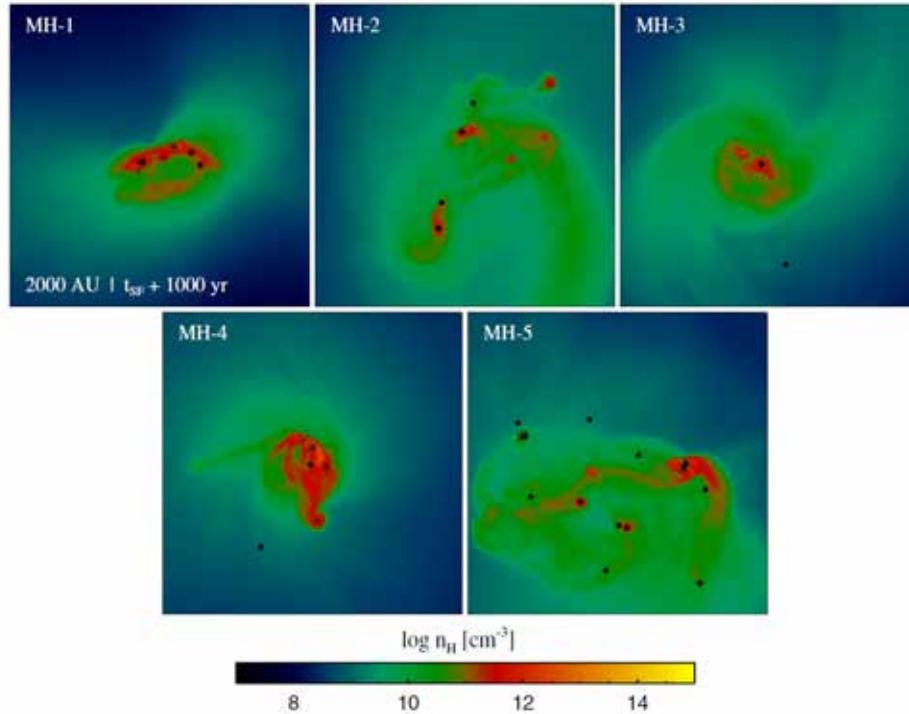
Figure 1 Schematic diagram [reproduced from Rees (106)] showing possible routes for runaway evolution in active galactic nuclei.

Cosmological Rees Diagram



Mass of First Stars

- a few $100M_{\odot}$ (Abel et al. 2000; Bromm et al. 2002; Yoshida et al. 2006)
 $\sim M_{\odot}$ or a few $100M_{\odot}$ (Nakamura & Umemura 2001)
several $10M_{\odot}$ (Clark et al. 2011)
about $40M_{\odot}$ (Hosokawa et al. 2011)
a few – $10M_{\odot}$ (Greif et al. 2011)



Mass of First Stars: Revisited

Susa, Hasegawa, Tominaga, 2014, ApJ, 792, 32

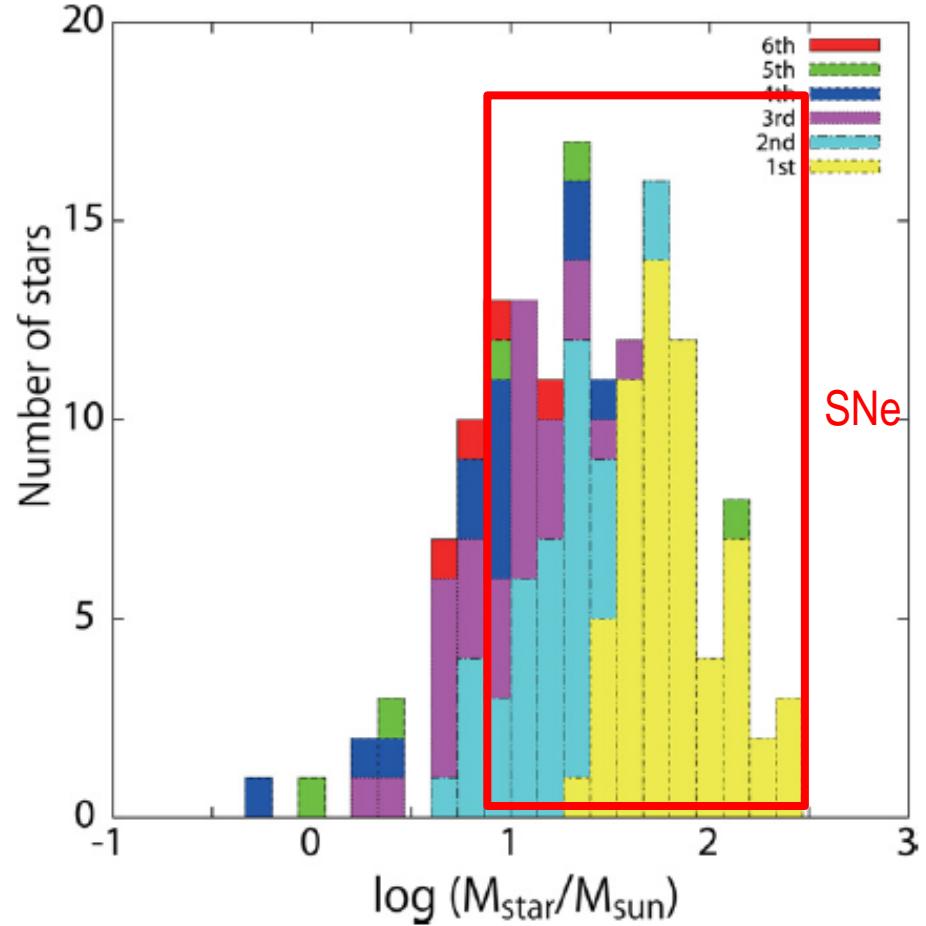
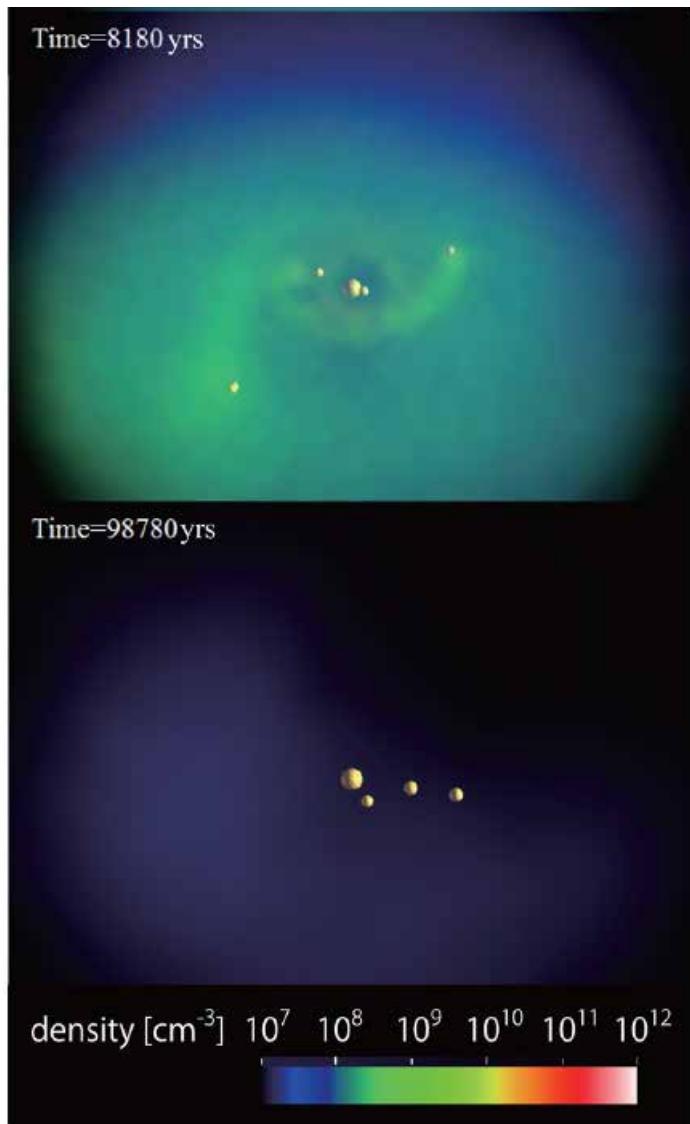
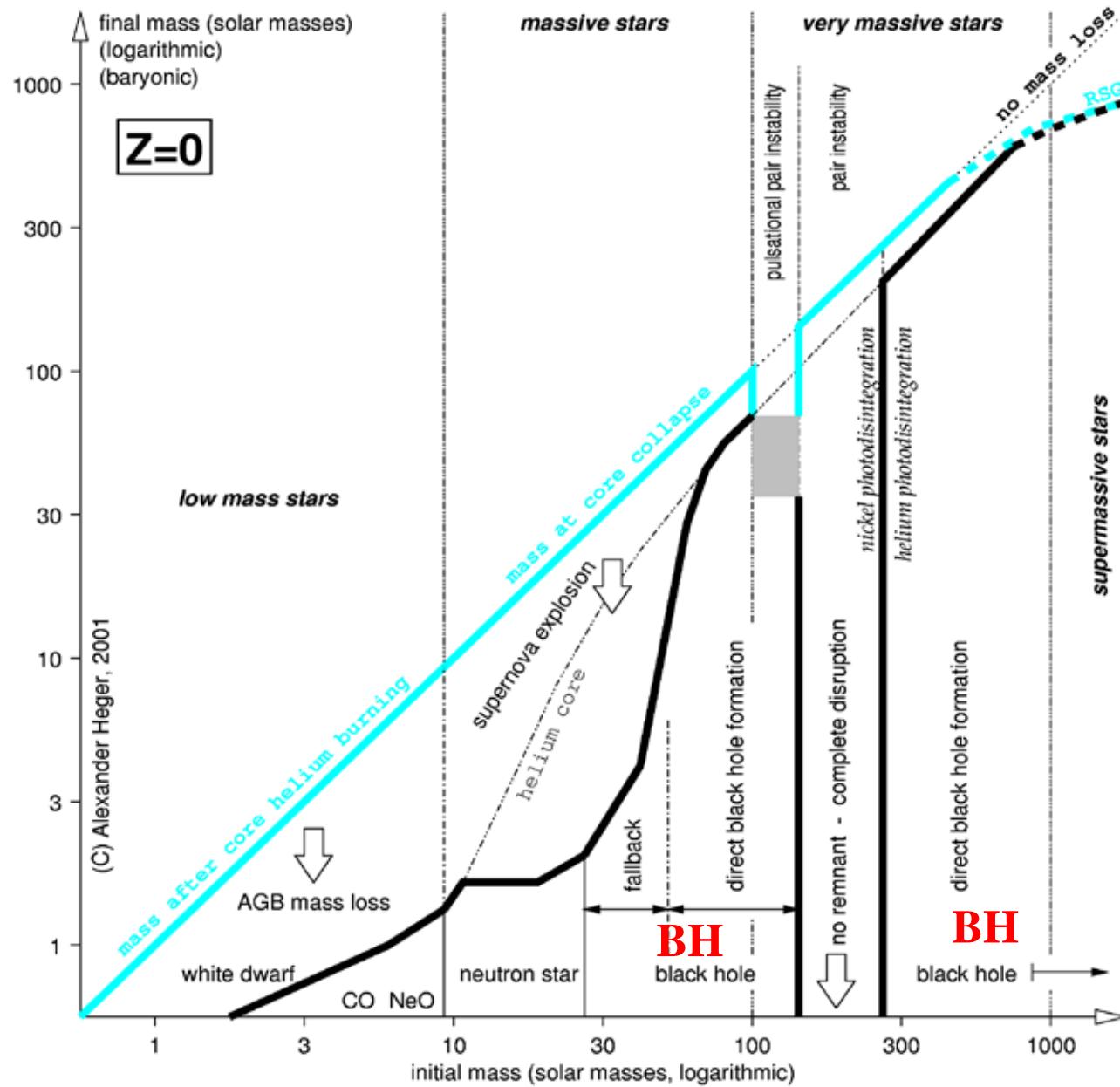


Figure 9. Mass spectrum of first stars is shown. The colors in the histogram correspond to the order of birth of these stars. The color legend in the upper right corner describes the correspondence between the order of birth and the color.

Pop III Star Evolution

Heger & Woosley 2002, ApJ, 567, 532



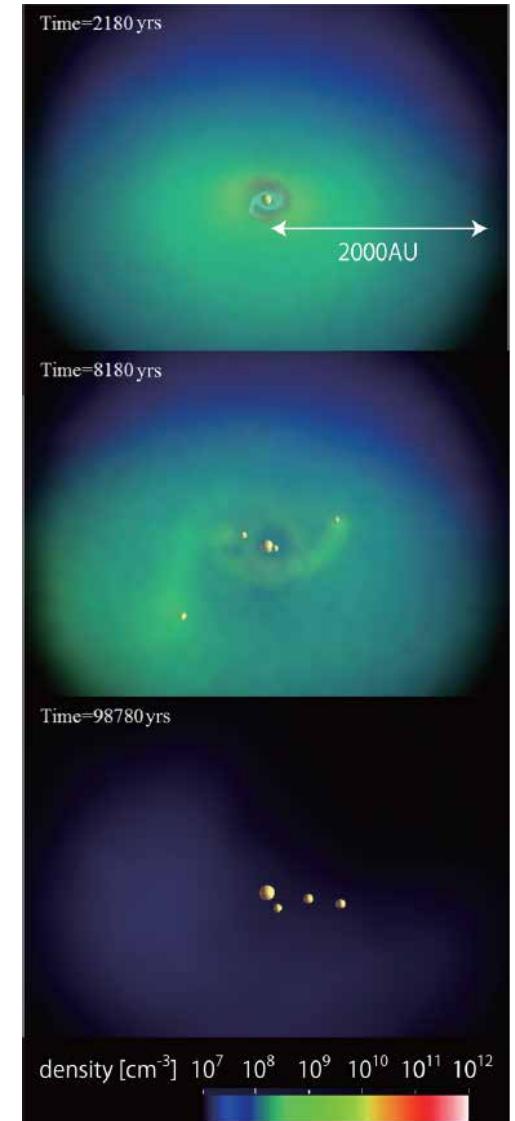
Early Cosmic Merger of Multiple Black Holes

Tagawa, Umemura, Gouda, Yano & Yamai,
2015, MNRAS, 451, 2174

**BH merger via gas dynamical friction
with general relativistic effects**

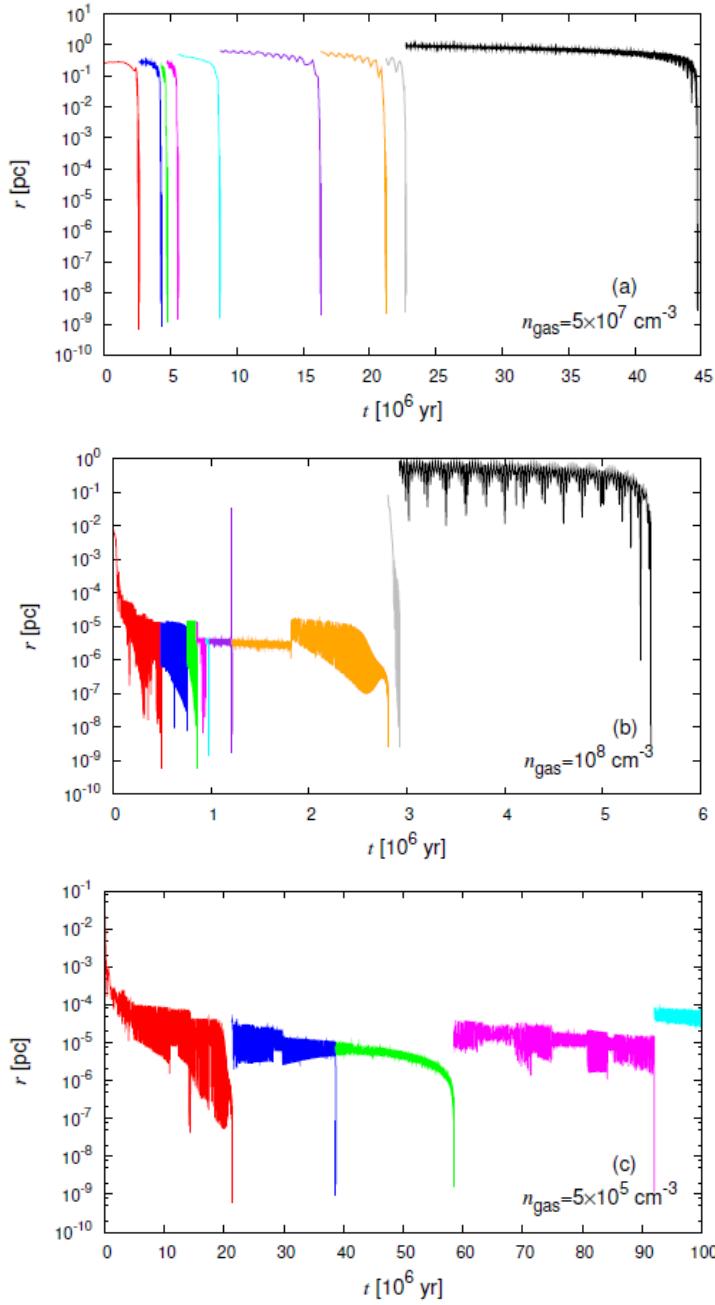
$$\mathbf{a}_{\text{DF},i}^{\text{gas}} = -4\pi G^2 m_i m_{\text{H}} n_{\text{gas}}(r) \frac{\mathbf{v}_i}{v_i^3} \times f(\mathcal{M}_i)$$

$$f(\mathcal{M}_i) = \begin{cases} 0.5 \ln \left(\frac{v_i t}{r_{\min}} \right) \left[\text{erf} \left(\frac{\mathcal{M}_i}{\sqrt{2}} \right) - \sqrt{\frac{2}{\pi}} \mathcal{M}_i \exp(-\frac{\mathcal{M}_i^2}{2}) \right] \\ (0 \leq \mathcal{M}_i \leq 0.8) \\ 1.5 \ln \left(\frac{v_i t}{r_{\min}} \right) \left[\text{erf} \left(\frac{\mathcal{M}_i}{\sqrt{2}} \right) - \sqrt{\frac{2}{\pi}} \mathcal{M}_i \exp(-\frac{\mathcal{M}_i^2}{2}) \right] \\ (0.8 \leq \mathcal{M}_i \leq \mathcal{M}_{eq}) \\ \frac{1}{2} \ln \left(1 - \frac{1}{\mathcal{M}_i^2} \right) + \ln \left(\frac{v_i t}{r_{\min}} \right), \\ (\mathcal{M}_{eq} \leq \mathcal{M}_i) \end{cases}$$



Susa, Hasegawa & Tominaga 2014

The separation of the closest pair



Type A

Gas-drag-driven merger

Type B

Interplay-driven merger
(Three body-driven and thereafter
gas-drag-driven merger)

Type C

Three body-driven merger

All BHs in the First Objects can merger into one in 10^7 yr !

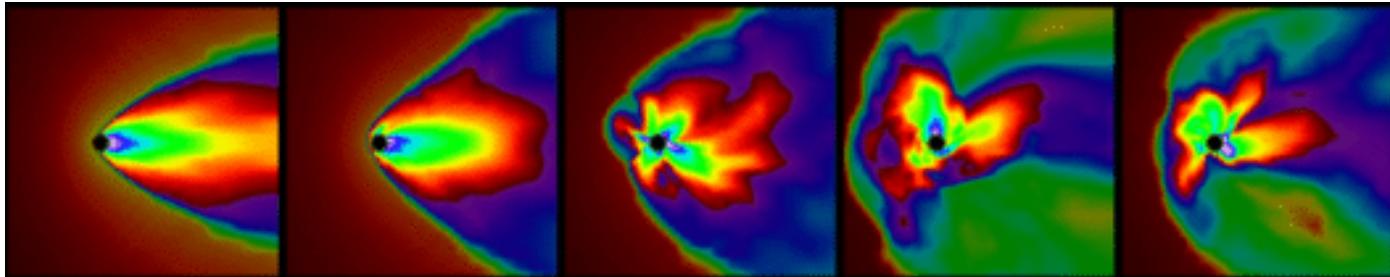
Table 1. The results with $M_{\text{BH}} = 30 M_{\odot}$ and $N_{\text{BH}} = 10$.

$r_{\text{typ}}[\text{pc}]$	1.0	0.464	0.215	0.1	0.0464	0.0215	0.01
$\rho_{\text{BH}}[M_{\odot}\text{pc}^{-3}]$	7.2×10^1	7.2×10^2	7.2×10^3	7.2×10^4	7.2×10^5	7.2×10^6	7.2×10^7
$n_{\text{gas}}[\text{cm}^{-3}]$	N_m $t_{\text{fin}}[\text{yr}]$	type $t_{\text{fin}}[\text{yr}]$	N_m $t_{\text{fin}}[\text{yr}]$	type $t_{\text{fin}}[\text{yr}]$	N_m $t_{\text{fin}}[\text{yr}]$	type $t_{\text{fin}}[\text{yr}]$	N_m $t_{\text{fin}}[\text{yr}]$
10^{12}	0 1.0×10^8	- 1.0×10^8	5 3.6×10^7	A 4.4×10^6	9 1.7×10^6	A 7.0×10^5	9 4.3×10^5
	2 1.0×10^8	A 1.0×10^8	8 1.3×10^7	A 1.8×10^5	9 1.8×10^5	A 2.3×10^4	9 2.8×10^4
10^{11}	5 1.0×10^8	A 4.5×10^7	9 5.1×10^6	A 5.9×10^5	9 7.4×10^4	A 6.3×10^4	9 1.7×10^5
	5 1.0×10^8	A 2.9×10^7	9 3.1×10^6	A 4.5×10^5	9 1.6×10^5	B 3.4×10^5	9 7.7×10^4
10^{10}	8 1.0×10^8	A 2.4×10^7	9 3.5×10^6	A 2.6×10^5	9 4.3×10^5	B 3.5×10^5	9 3.0×10^5
	8 1.0×10^8	A 1.2×10^7	9 1.3×10^6	A 6.5×10^5	9 4.3×10^5	B 5.5×10^5	9 5.1×10^5
5×10^9	9 3.0×10^7	A 5.1×10^6	9 4.0×10^6	B 5.5×10^6	9 3.6×10^6	B 4.2×10^6	9 1.2×10^7
	9 4.5×10^7	A 3.7×10^6	9 2.2×10^7	B 3.2×10^7	9 1.3×10^7	B 4.7×10^6	9 3.6×10^6
10^8	9 3.8×10^7	B 2.3×10^7	9 1.7×10^7	B 3.3×10^7	9 1.8×10^7	B 2.9×10^7	9 1.7×10^7
	9 4.2×10^7	B 3.9×10^7	9 4.2×10^7	B 4.7×10^7	9 6.3×10^7	B 3.5×10^7	9 3.1×10^7
5×10^7	6 1.0×10^8	B 6.0×10^7	6 1.0×10^8	B 1.0×10^8	6 1.0×10^8	C 8.0×10^7	6 1.0×10^8
	2 1.0×10^8	C 6.0×10^8	6 1.0×10^8	C 1.0×10^8	5 1.0×10^8	C 1.0×10^8	4 1.0×10^8
10^6	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	1 1.0×10^8	C 1.0×10^8	0 1.0×10^8
	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8
5×10^5	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8
	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8
10^5	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8
	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8
10^4	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8	- 1.0×10^8	0 1.0×10^8

All
BHs
merge

First
Objects

Bondi-Hoyle-Lyttleton Accretion



<http://wwwmpa.mpa-garching.mpg.de/~mor/bhla.html>

$$\dot{M}_{\text{BHL}} = 4\pi r \frac{G^2 M^2}{(v^2 + c_s^2)^{3/2}} \quad \backslash \quad M = \frac{1}{M_0^{-1} - at}$$

$$t_* = 1.7 \cdot 10^6 \text{ yr} \approx \frac{v}{c} \frac{\ddot{r}^3}{\ddot{r}^3} \approx \frac{n}{\rho} \frac{\ddot{r}^{-1}}{\ddot{r}^{-1}} \approx \frac{M_0}{c} \frac{\ddot{r}^{-1}}{\ddot{r}}$$

$$\dot{M}_{\text{BHL}} = 0.6 M_e \text{ yr}^{-1} \approx \frac{v}{c} \frac{\ddot{r}^{-3}}{\ddot{r}^{-3}} \approx \frac{n}{\rho} \frac{\ddot{r}^{-2}}{\ddot{r}^{-2}} \approx \frac{M_0}{c} \frac{\ddot{r}^{-2}}{\ddot{r}}$$

$$\text{cf} \quad \dot{M}_E = 2 \cdot 10^{-4} M_e \text{ yr}^{-1} \approx \frac{M_0}{c} \frac{\ddot{r}^{-2}}{\ddot{r}}$$

Accretion vs Merger

Tagawa, Umemura, Gouda, 2015, to be submitted

Critical mass accretion rate (below which the merger precedes)

$$\dot{M}_{\text{crit}} / \dot{M}_{\text{BHL}} = 0.34 \frac{\propto n_{\text{gas}}}{\propto 10^7 \text{ cm}^{-3}} \div \frac{\propto r_{\text{BH}}^{-1}}{\propto 10^7 M_{\odot} \text{ pc}^{-3}} \div \dots \gg 10^{3-4} \dot{M}_{\text{E}}$$

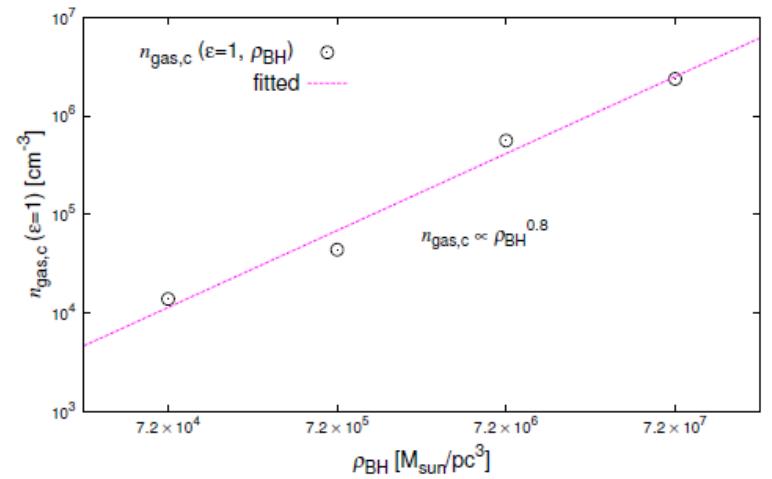
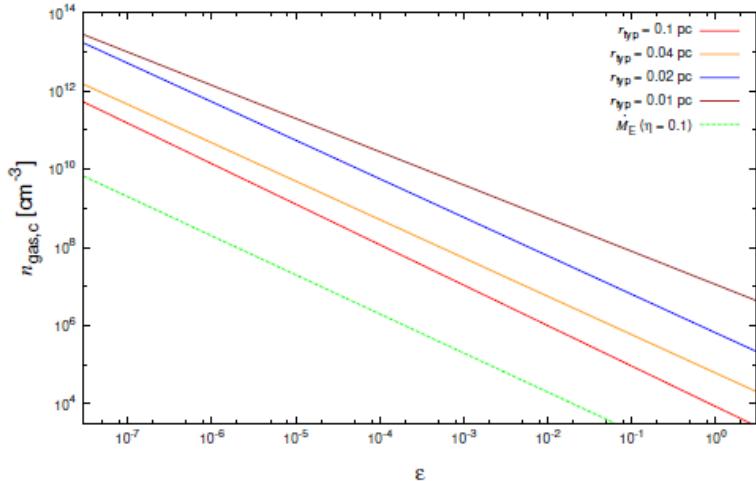
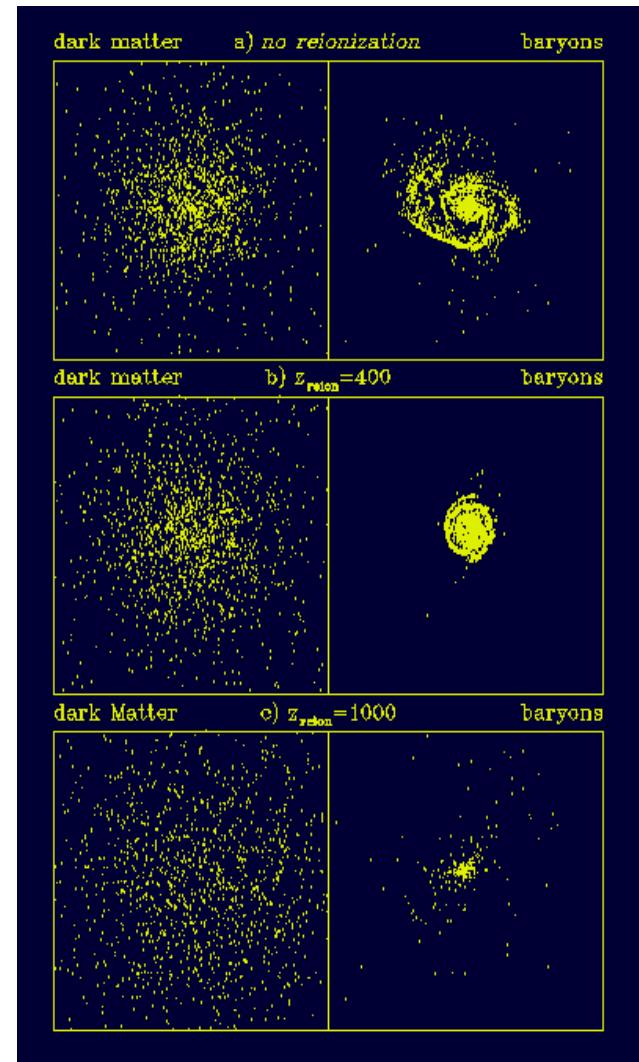


Figure 6. The plots represent the critical gas number density as a function of the BH density. The line is the curve fitted by $\left(\frac{n_{\text{gas},c}}{1 \text{ cm}^{-3}} \right) = a \left(\frac{\rho_{\text{BH}}}{1.0 M_{\odot}/\text{pc}^3} \right)^b$.

Direct Collapse to Supermassive Stars with $10^{4-6} M_\odot$

- “ Compton drag by CMB radiation
Umemura, Loeb, Turner 1993
- “ Supression of H₂ Cooling
Bromm & Loeb 2003
Begelman, Volonteri & Rees 2006
Inayoshi & Omukai 2012
- “ Heating by DM annihilation
Spolyar, Freese, Gondolo 2008



General Relativistic Instability

Baumgarte & Shapiro 1999, ApJ, 526, 941

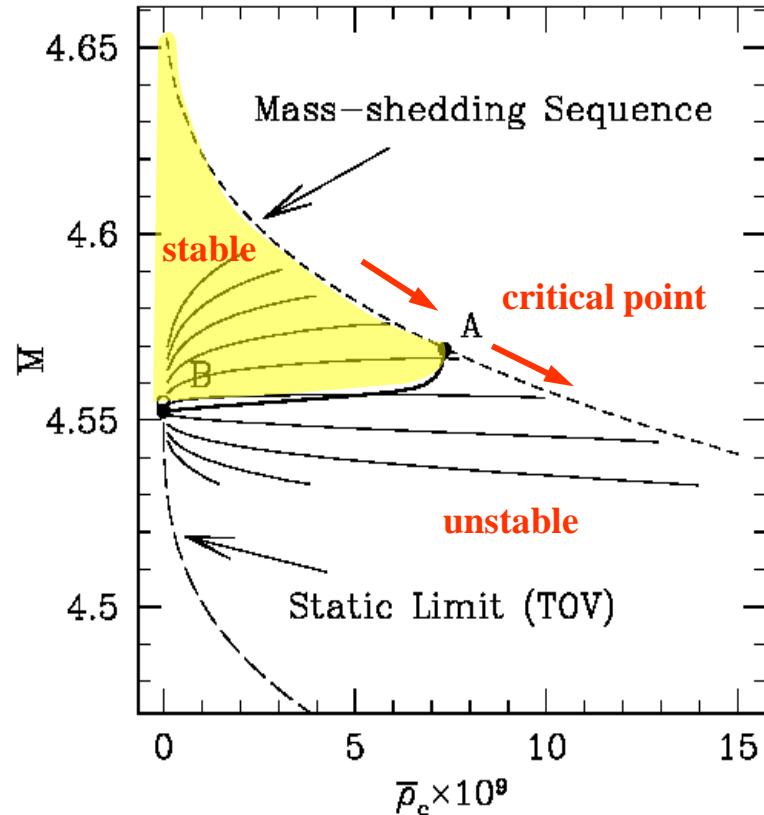
Rapidly rotating supermassive star
in equilibrium



Post-Newtonian

Saijo, Baumgarte, Shapiro & Shibata
(2002, ApJ, 569, 349)

Rigid rotating SMS P collapse

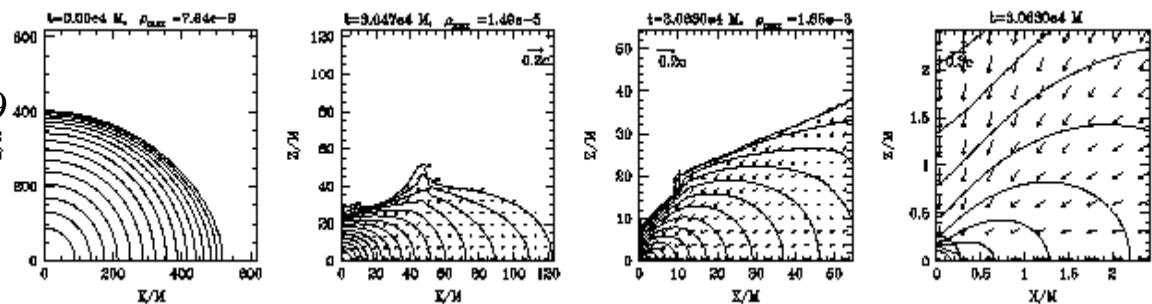


Numerical Relativity

Shibata & Shapiro 2002, ApJ, 572, L39

$g=4/3$ Polytrope (radiation-dominated)

Kerr BH with $a \gg 0.75$



But, no General Relativistic Radiation Hydrodynamic (GR-RHD) simulations !

Difficulties in GR-RHD

A) In relativistic motion, the steady state of radiation fields cannot be assumed.

[Time-dependent transfer equation should be solved.]

C) Light bending, frame-dragging, and gravitational redshifts should be included.

[Transfer should be solved along the geodesics.]

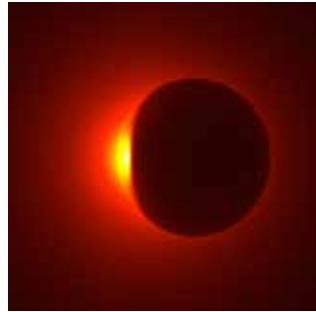
B) Causality should be retained.

[We should solve the propagation of wave fronts in proper time.]

D) GR energy-momentum tensor of radiation should be obtained.

[LNRF (locally non-rotating reference frame) should be transformed to the curved space.]

We have overcome all these difficulties !



Vermeer

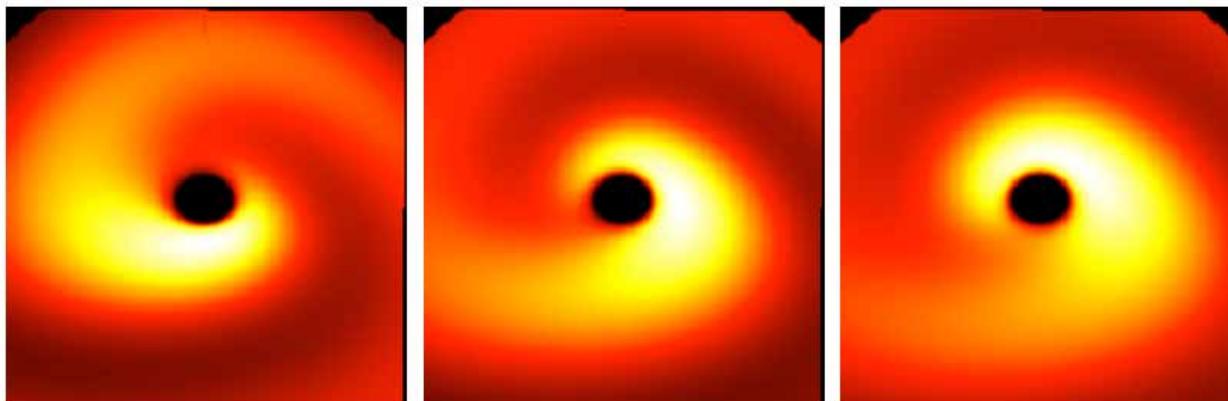
Master of light



Variable Eddington-tensor Radiation-hydrodynamics with Metric Enchained Ray-tracing

General Relativistic Radiation Transfer
Radiation Hydrodynamics in Curved Space

Rhota Takahashi & Masayuki Umemura



GR Radiation Transfer

General Relativistic Boltzmann Equation of Photons

$$\frac{dI_n}{d\ell} = E_n - A_n I_n$$

$I_n \circ \frac{I_n}{n^3}$: Invariant specific intensity

$E_n \circ \frac{h_n}{n^2}$: Invariant emissivity

$A_n \circ nc_n$: Invariant extinction

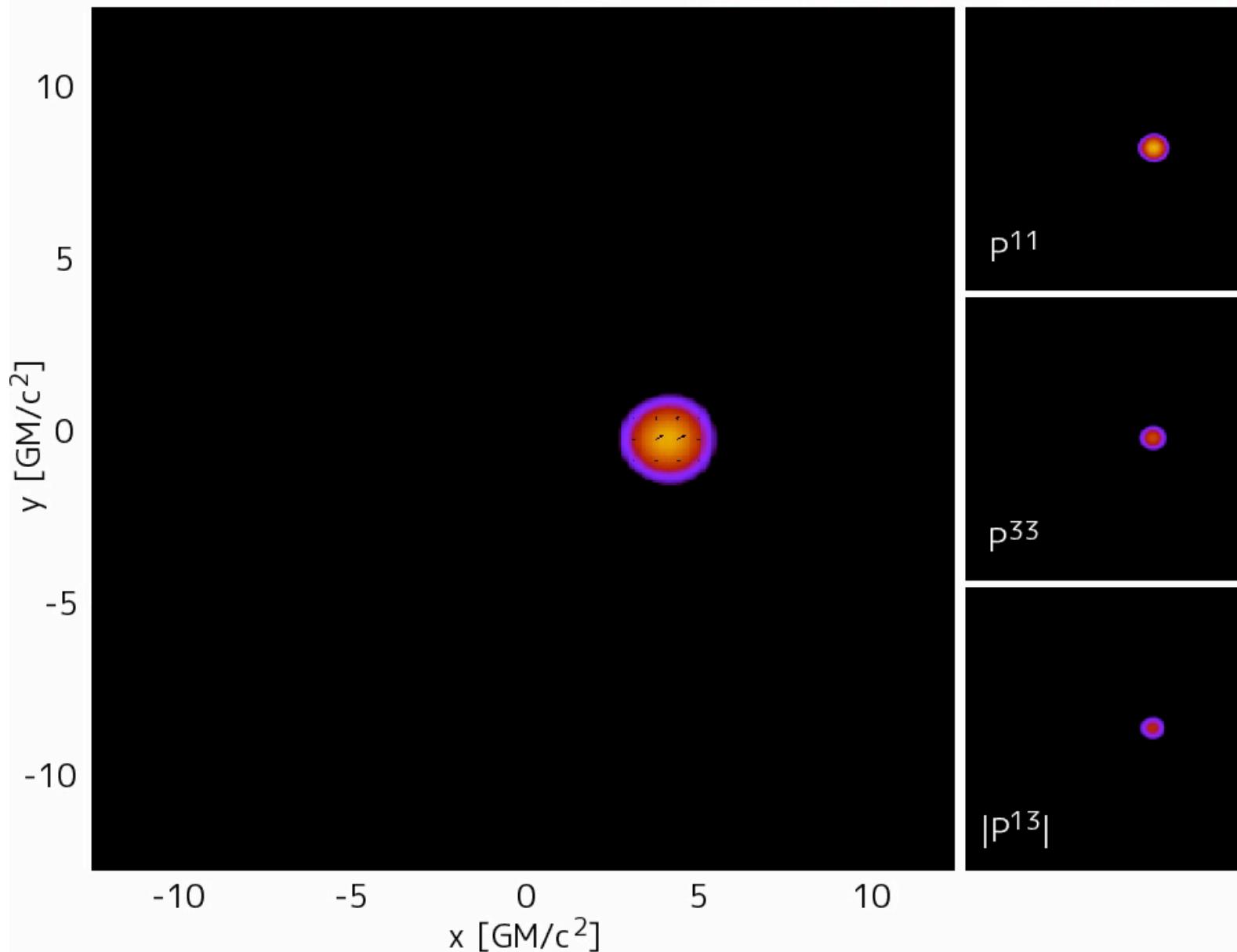
∅ Solve GR radiative transfer along geodesics

∅ Obtain invariant specific intensity in 6D phase space

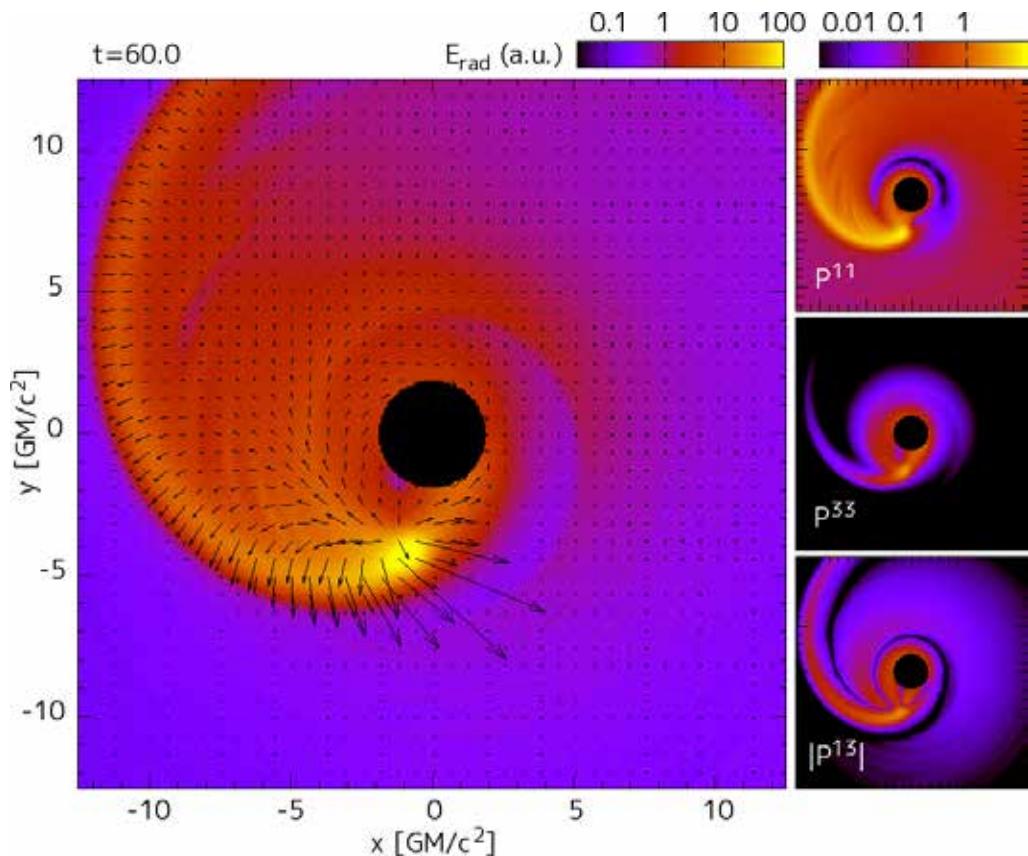
Dynamical Test 2: Photon wave front from a rotating hot spot

Boltzmann calculations

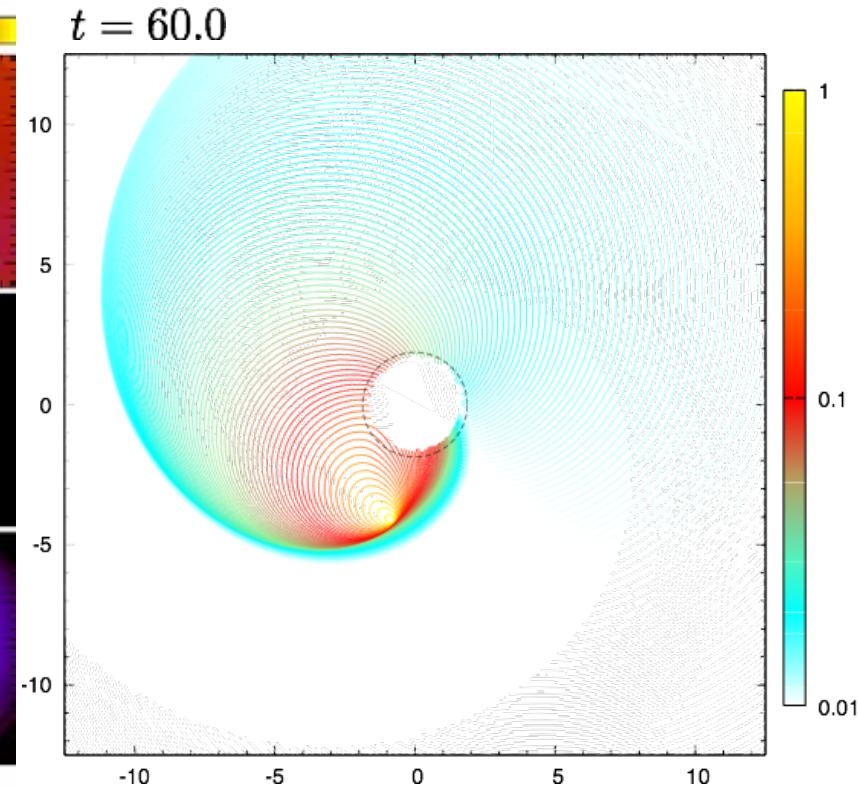
t=0.0



Boltzmann calculations



Ray-tracing calculations



$$N_i(r) = 90, N_j(f) = 256$$

Geodesics = 4608

Primordial BHs in Inflation

Carr & Lidsey, Phys. Rev. D 48, 543 (1993)

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Kawasaki, Sugiyama, Yanagida, Phys. Rev. D 57, 6050 (1998)

Bringmann, Kiefer, & Polarski, Phys. Rev. D 65, 024008 (2002)

Kawaguchi et al. MNRAS, 388, 1426 (2008)

Drees & Erfani, JCAP, 4, 5 (2011); JCAP, 1, 35 (2012)

Kohri, Lin & Matsuda, Phys. Rev. D 87, 103527 (2013)

Erfani, Phys. Rev. D 89, 083511 (2014)

Clesse & Garcia-Bellido, Phys. Rev. D 92, 023524 (2015)

Massive Primordial BHs from Hybrid Inflation

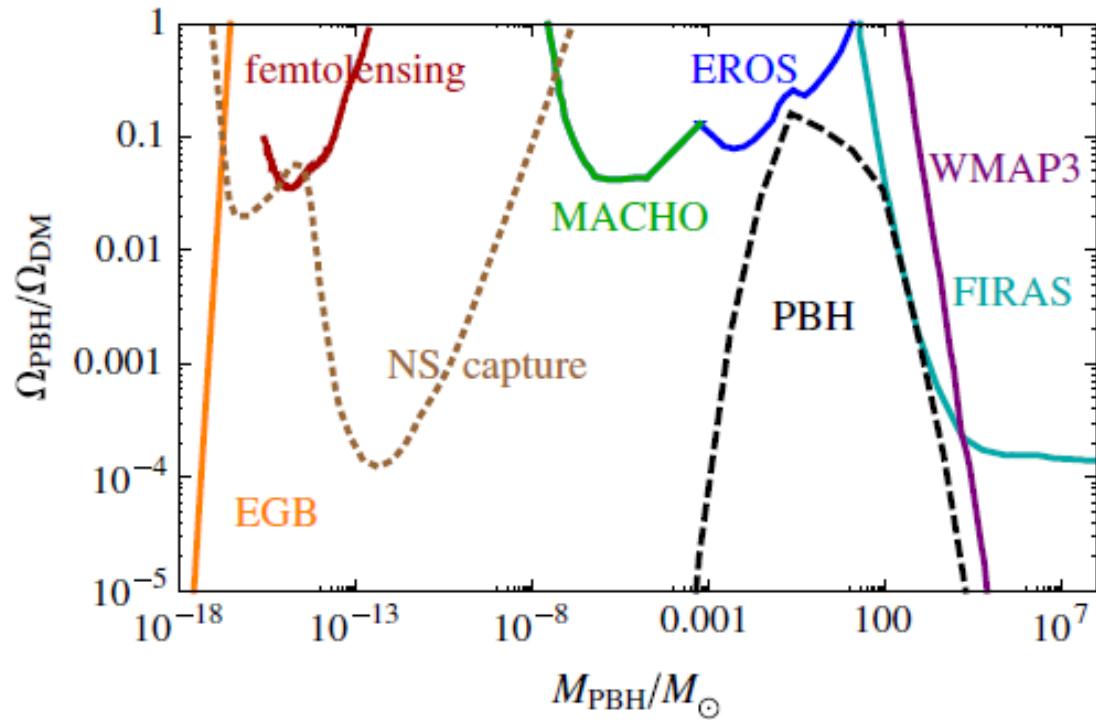
Clesse & Garcia-Bellido, Phys. Rev. D 92, 023524 (2015)

PBHs produced from the collapse of large curvature perturbations generated during a mild-waterfall phase of hybrid inflation

Dark matter candidates

Swiss-cheese-like structure of the Universe (leading to apparent accelerated cosmic expansion)

1. Lifetime of primordial black holes
2. Light element abundances
3. Extragalactic photon background
4. Galactic background radiation
5. Femtolensing of gamma-ray bursts
6. Capture of PBHs by neutron stars
7. Microlensing surveys
8. CMB spectral distortions



Constraint from Cosmic Reionization Epoch

Planck 2015 results $Z_{\text{reion}} = 8.8^{+1.7}_{-1.4}$

Minimum photon number rate (Madau, Haardt, Rees 1999):

$$\dot{n}_{\text{rec}} = \frac{n(z=0)}{t_{\text{rec}}} = 4.1 \cdot 10^{-23} \frac{\alpha + z}{10} \frac{\alpha W h^2}{0.04} \frac{\odot}{\odot} C_5 \text{ s}^{-1} \text{ cm}^{-3} \text{ <comoving>}$$

Ionizing photon number rate from accreting BHs

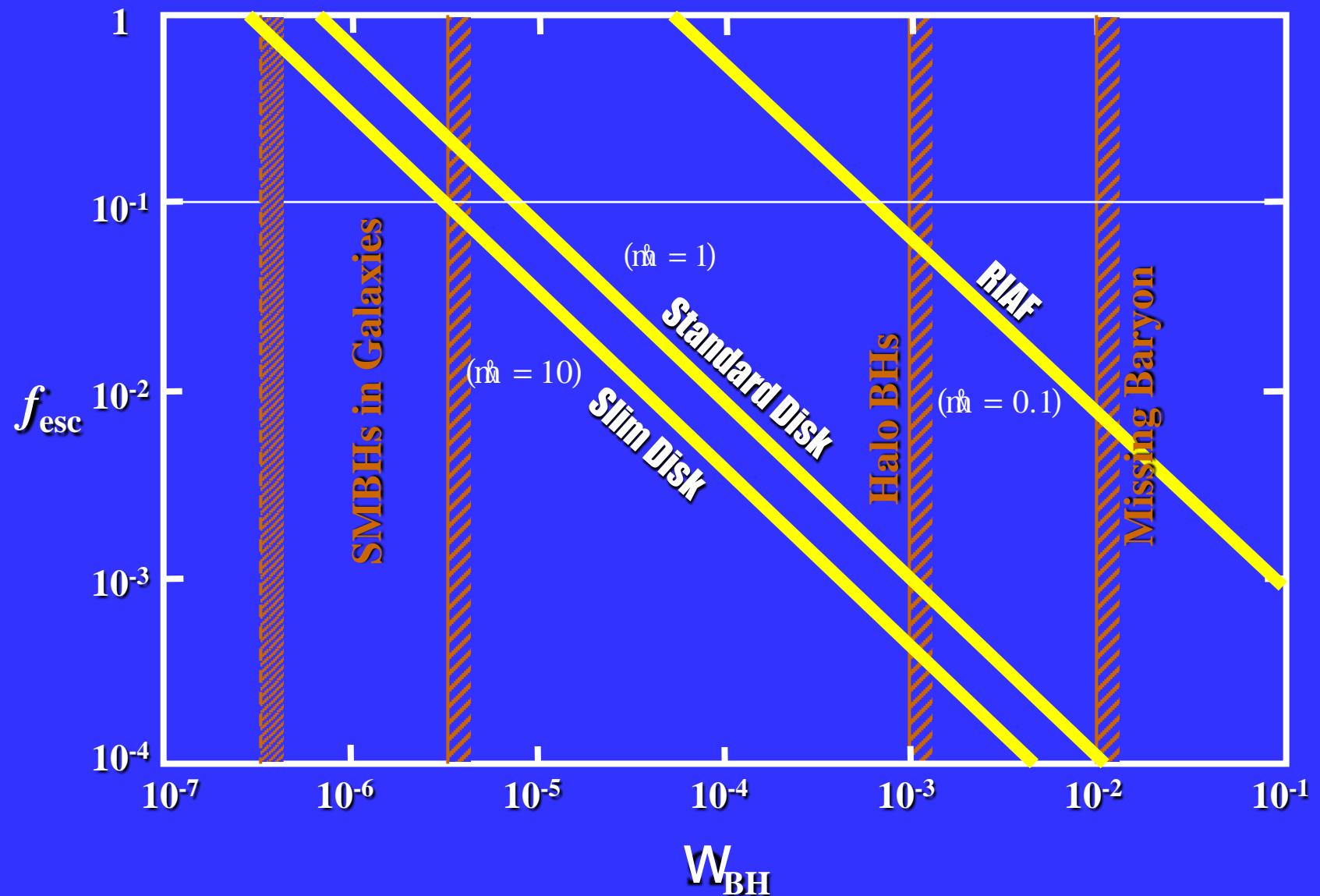
Eddington luminosity density:

$$L_E = \frac{4\rho G c m_p}{s_T} W_{\text{BH}} r_c = 2.4 \cdot 10^{-26} W_{\text{BH}} \frac{\alpha W h^2}{0.04} \frac{\odot}{\odot} \text{ erg s}^{-1} \text{ cm}^{-3}$$

$$\dot{n}_g = \frac{L_E}{h n_L} f_{\text{UV}} \times f_{\text{esc}}$$

$$= 5.3 \cdot 10^{-22} \frac{\alpha W_{\text{BH}}}{10^{-4}} \frac{\odot}{\odot} \frac{\alpha f_{\text{UV}}}{0.1} \frac{\odot}{\odot} \frac{\alpha f_{\text{esc}}}{0.1} \frac{\odot}{\odot} \frac{\alpha W h^2}{0.04} \frac{\odot}{\odot} \text{ s}^{-1} \text{ cm}^{-3} \text{ <comoving>}$$

Constraint from Planck 2015



Conclusions

- ∅ Formation of SMBHs is one of greatest mysteries in astrophysics.
- ∅ All BHs in the first objects can merge in 10^7 yr.
- ∅ BH Merger is faster than accretion, if mass accretion rate is less than 10% of the Bondi-Hoyle-Lyttleton accretion rate.
- ∅ GR-RHD simulations on SMS are coming soon (in two years?)
- ∅ Formation of PBHs is a piece of the puzzle.
- ∅ Constraint on PBHs from cosmic reionization (Planck 2015) is
$$10^{-4} < f_{\text{esc}} W_{\text{BH}} < 10^{-7}$$

Thank you for your attention