Multi-disformal invariance of nonlinear primordial perturbations

渡辺悠貴　Yuki Watanabe

National Institute of Technology, Gunma College

with Atsushi Naruko and Misao Sasaki

Published in EPL 111 (2015) 39002 [arXiv:1504.00672]

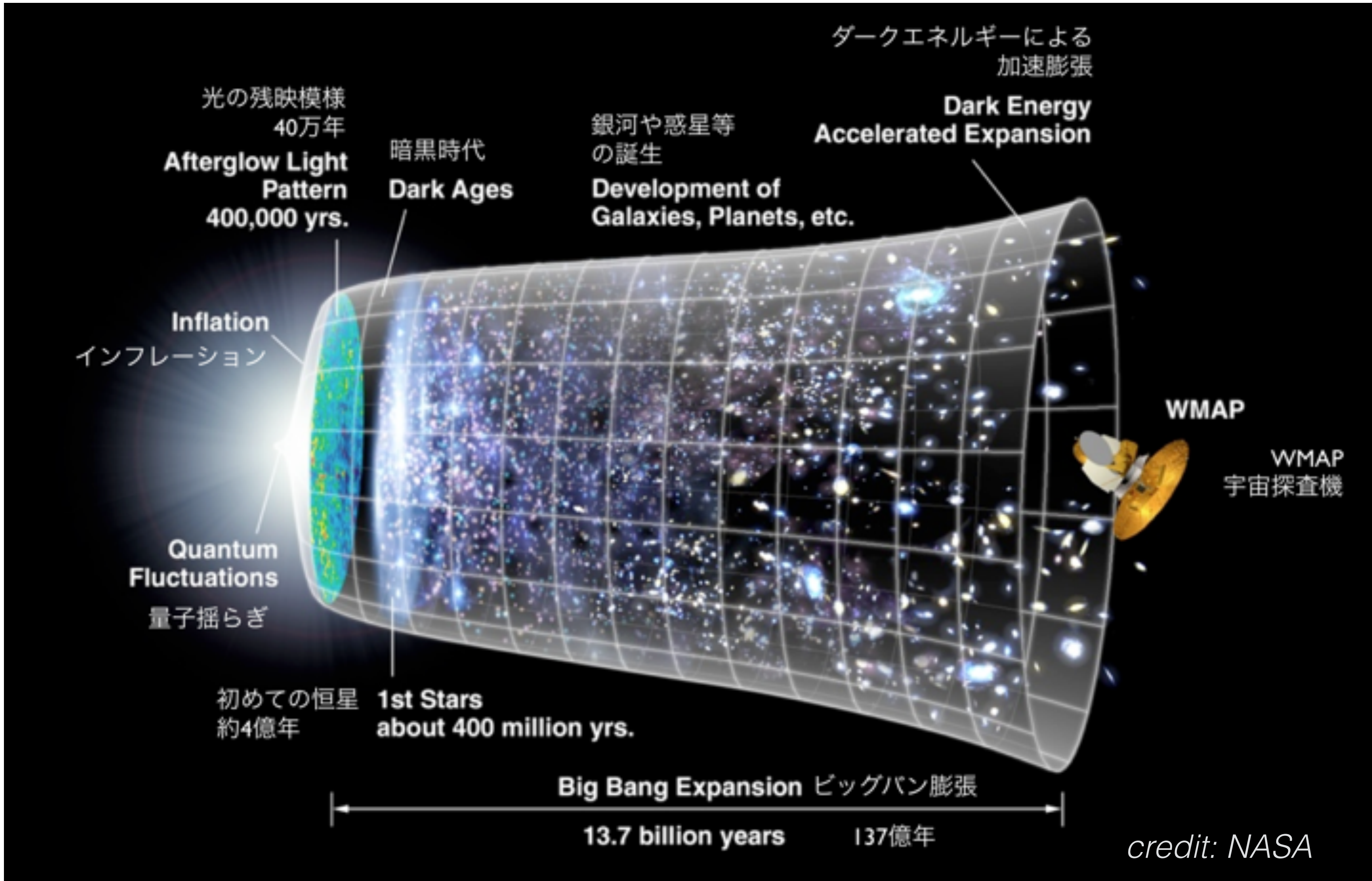
新学術領域　加速宇宙シンポジウム

Kickoff meeting of Kakenhi program “Cosmic Acceleration”

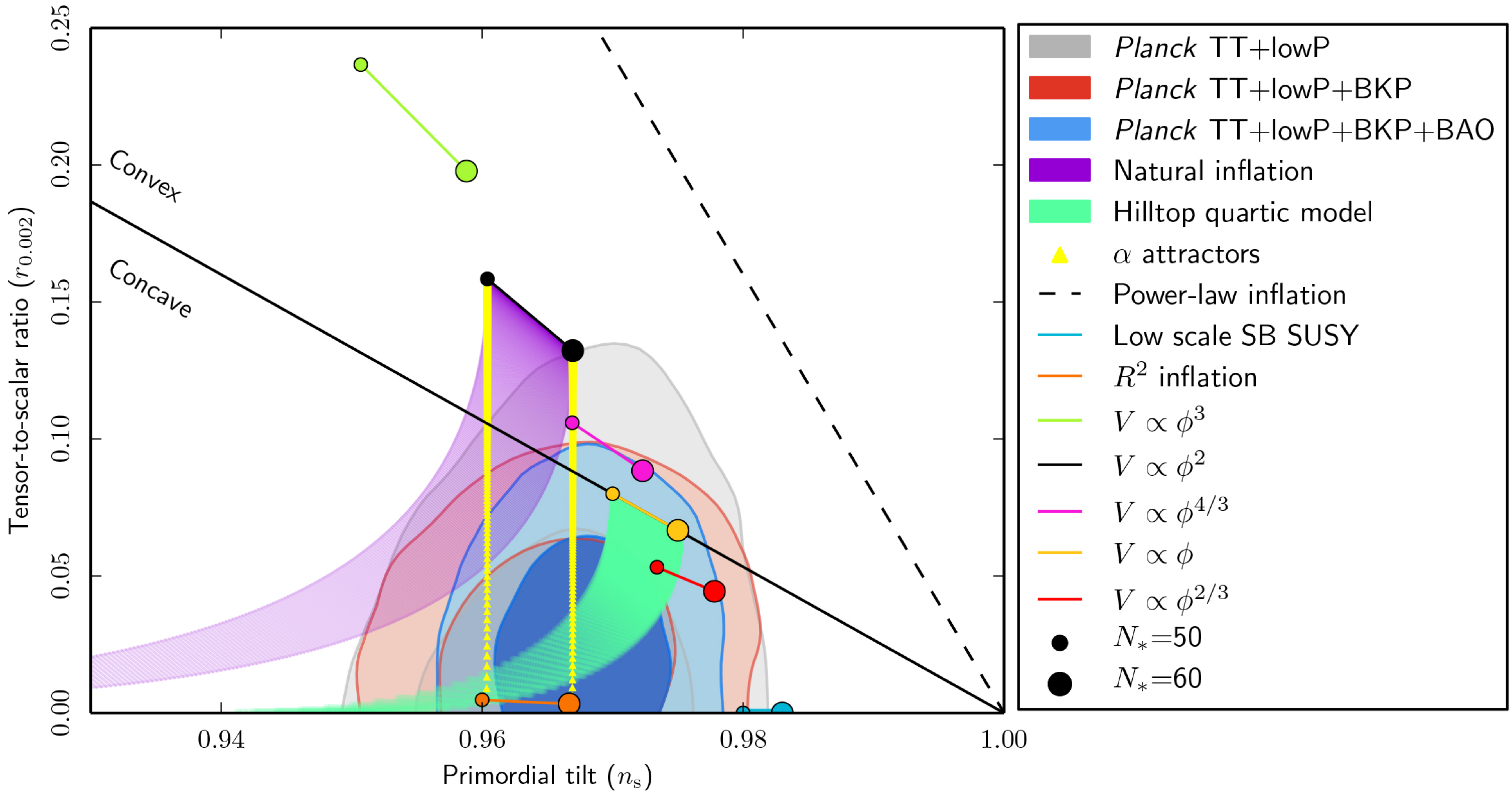
Kavli IPMU, University of Tokyo
Kashiwa, Chiba, Japan
21st September 2015

Introduction: Inflationary cosmology

- **Quantum fluctuations** are the origin of structures in the Universe
- anti-correlation of CMB TE spectrum -> **super-H curvature perturbations**
- (future) CMB BB spectrum -> **super-H gravitational-wave (GW) perturbations**



Constraint on inflation models [Planck 2015]

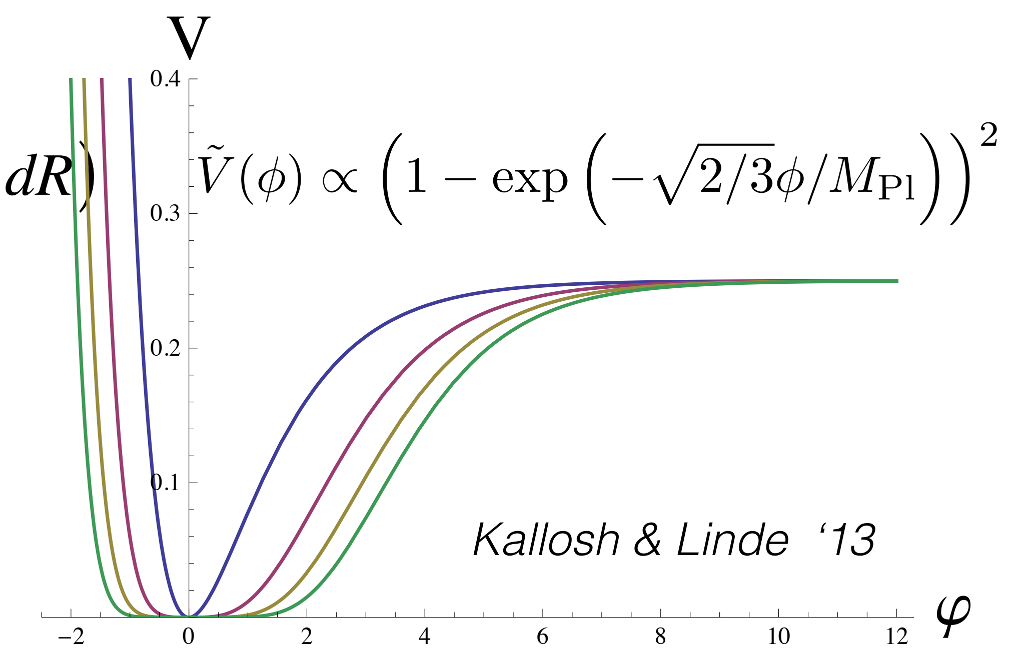


- Large class of models with **non-minimal gravity** are in good agreement with observations, e.g., R2, Higgs, α-attractors

Models with non-minimal coupling (NMC)

- In the context of modified gravity, field theory in curved space-time and higher-dimensional unifying particle physics theories, **NMC** between scalar fields and the Ricci scalar is common
- e.g. $S_{f(R)} = \int d^4x\sqrt{-g}\,f(R) \;\Rightarrow\; \int d^4x\sqrt{-g}\left(\Phi R + \dots\right)$ $\quad (\Phi = df/dR)$

$$S_{Higgs} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(M_{pl}{}^2 + \xi h^2\right)R + \dots\right]$$



- Make **conformal transformation:** $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$

$\Rightarrow$ 1. Flattening of potential $\quad \tilde{V} = V/\Omega^4$

2. Rescaling of field $\quad \varphi = M_{\mathrm{Pl}}\sqrt{3/2}\ln\phi$

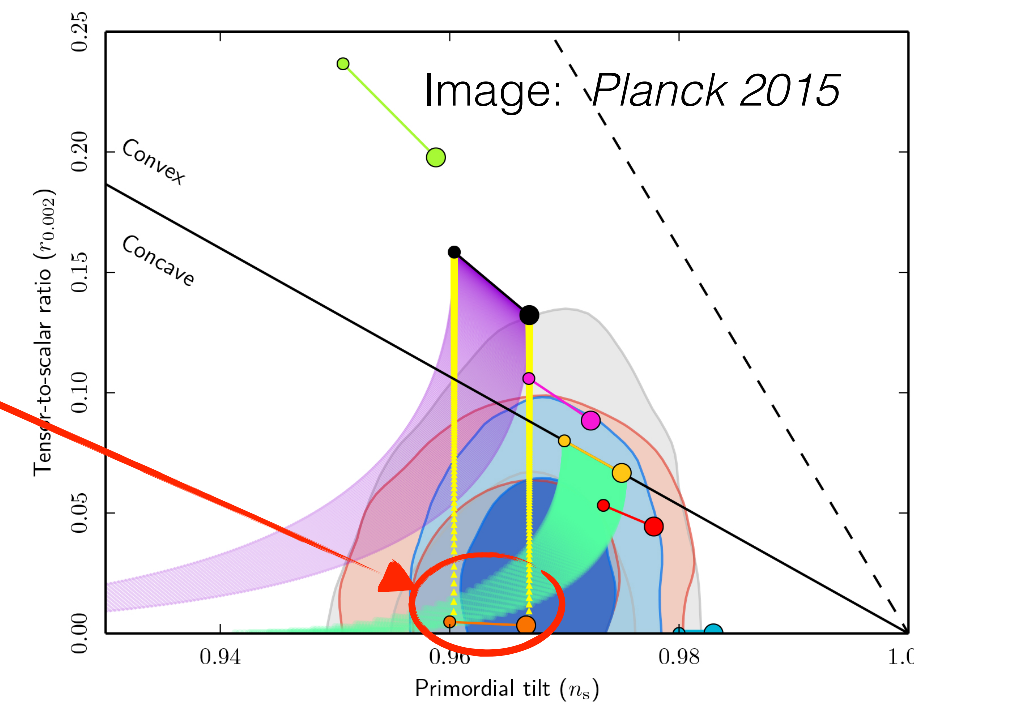
$\Rightarrow$ **Ideal for inflation**

- All predict $\quad n_s = 1 - \dfrac{2}{N_*} \qquad r = \dfrac{12}{N_*^2}$

but N_* depends on reheating, which is different for different models

→Gravitational reheating

[Takeda & YW 1405.3830; YW & White 1503.08430]



Purposes of this study

- More general gravitational theories for inflation? E.g. Horndeski gravity and beyond [Yokoyama-san's talk, Saitou-san's talk]
- The primordial linear tensor power spectrum from inflation can be always cast into the standard form at leading order in derivatives with suitable conformal and *disformal transformations* in EFT of inflation. [Creminelli et al 1407.8439]
- Invariance of the curvature perturbation under *disformal transformations* has been shown at linear order. [Minamitsuji 1409.1566, Tsujikawa 1412.6210]

- We extend the invariance of the curvature and GW perturbations to **fully nonlinear order**. → necessary for non-Gaussianity etc.
- We further show the invariance under a new type of disformal transformation, dubbed **multi-disformal transformation**, generated by multi-component scalar fields.

Decomposing spacetime

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\alpha^2 \mathrm{d}t^2 + \hat{\gamma}_{ij}\left(\mathrm{d}x^i + \beta^i \mathrm{d}t\right)\left(\mathrm{d}x^j + \beta^j \mathrm{d}t\right)$$

α is the lapse function, β^i is the shift vector

$$\hat{\gamma}_{ij} = a^2(t)e^{2\psi}\gamma_{ij}\,, \qquad \det\gamma_{ij} = 1$$

$$\chi \equiv -\frac{3}{4}\triangle^{-1}\left\{\partial^i\left[e^{-3\psi}\partial^j\left(e^{3\psi}(\gamma_{ij} - \delta_{ij})\right)\right]\right\}$$

$\triangle$ is the flat 3-dimensional Laplacian and $\partial^i = \delta^{ij}\partial_j$

Decomposing spacetime and curvature pert'n

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\alpha^2 \mathrm{d}t^2 + \hat{\gamma}_{ij}\left(\mathrm{d}x^i + \beta^i \mathrm{d}t\right)\left(\mathrm{d}x^j + \beta^j \mathrm{d}t\right)$$

α is the lapse function, β^i is the shift vector

$$\hat{\gamma}_{ij} = a^2(t)e^{2\psi}\gamma_{ij}\,, \qquad \det\gamma_{ij} = 1$$

$$\chi \equiv -\frac{3}{4}\triangle^{-1}\left\{\partial^i\left[e^{-3\psi}\partial^j\left(e^{3\psi}(\gamma_{ij}-\delta_{ij})\right)\right]\right\}$$

$\triangle$ is the flat 3-dimensional Laplacian and $\partial^i = \delta^{ij}\partial_j$

The uniform ϕ slicing
(comoving slicing if ϕ dominates):

$$\phi = \phi(t)$$

$$\mathfrak{R}_c \equiv \psi_c + \chi_c/3$$

$\mathcal{R}_c$ is the (comoving) curvature perturbation at linear order and $\mathfrak{R}_c$ its non-linear generalization

Decomposing spacetime and GW pert'n

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\alpha^2 \mathrm{d}t^2 + \hat{\gamma}_{ij}\left(\mathrm{d}x^i + \beta^i \mathrm{d}t\right)\left(\mathrm{d}x^j + \beta^j \mathrm{d}t\right)$$

α is the lapse function, β^i is the shift vector

$$\hat{\gamma}_{ij} = a^2(t)e^{2\psi}\gamma_{ij}\,, \qquad \det\gamma_{ij} = 1$$

$$\chi \equiv -\frac{3}{4}\triangle^{-1}\left\{\partial^i\left[e^{-3\psi}\partial^j\left(e^{3\psi}(\gamma_{ij}-\delta_{ij})\right)\right]\right\}$$

$\triangle$ is the flat 3-dimensional Laplacian and $\partial^i = \delta^{ij}\partial_j$

The uniform ϕ slicing: $\phi = \phi(t)$

$$\partial^j \gamma_{ij}^{\mathrm{TT}} = 0$$

$\gamma_{ij}^{\mathrm{TT}}$ is independent of the time-slicing condition at linear order but is slice-dependent at higher orders.

Generalization of conformal transformation
[Bekenstein 1993]

$$\tilde{g}_{\mu\nu} = A(\phi\,, X)\, g_{\mu\nu} + B(\phi\,, X)\, \partial_\mu\phi\partial_\nu\phi\,, \qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

A = 1, B ≠ 0: **Disformal transformation**
A ≠ 1, B = 0: **Conformal transformation**

- Non-linear invariance of curvature and GW perturbations under the conformal transformation *A(ϕ)* has been shown. [Gong, Sasaki et al 2011]

Invariance of nonlinear perturbations

$$\tilde{g}_{\mu\nu} = \quad g_{\mu\nu} + B(\phi\,, X)\,\partial_\mu\phi\partial_\nu\phi\,, \qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

A = 1, B ≠ 0: **Disformal transformation**

The uniform ϕ slicing: $\phi = \phi(t)$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B\,\dot{\phi}^2\,\delta_\mu{}^0\delta_\nu{}^0$$

Only **the lapse function** is affected by the disformal transformation!

$$\tilde{\alpha}^2 = \alpha^2 - B\dot{\phi}^2$$

Thus, the spatial metric (γ_ij) & shift vector (β^i) are invariant to fully nonlinear order → Invariance of curvature and GW perturbations

Invariance of nonlinear perturbations

$$\tilde{g}_{\mu\nu} = A(\phi\,, X)\, g_{\mu\nu} \qquad , \qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

A ≠ 1, B = 0: **Conformal transformation**

The uniform ϕ slicing: $\phi = \phi(t)$

$$\tilde{g}_{\mu\nu} = A\, g_{\mu\nu} = \bar{A}\, g_{\mu\nu} + \delta A\, \bar{g}_{\mu\nu} + \delta A\, \delta g_{\mu\nu}$$

Since the conformal factor *A* does not affect the unimodular part of the spatial metric, GW is invariant to fully nonlinear order

Invariance of nonlinear perturbations

$$\tilde{g}_{\mu\nu} = A(\phi\,, X)\, g_{\mu\nu} \qquad , \qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

A ≠ 1, B = 0: **Conformal transformation**

The uniform φ slicing: $\phi = \phi(t)$

$$\tilde{g}_{\mu\nu} = A\, g_{\mu\nu} = \bar{A}\, g_{\mu\nu} + \delta A\, \bar{g}_{\mu\nu} + \delta A\, \delta g_{\mu\nu}$$

For curvature pert'n: $\mathcal{R} \to \mathcal{R} + \delta A/(2\bar{A}) + \cdots$

δA is sourced by δX, but is vanishing on large scales b/c $\dot{\mathcal{R}}_c \propto H\delta\alpha_c$

$$X_c = \frac{1}{2}\frac{\dot{\phi}^2(t)}{\alpha_c^2(t\,, \boldsymbol{x})} = \frac{1}{2}\dot{\phi}^2(t) - \delta\alpha_c(t\,, \boldsymbol{x})\dot{\phi}^2(t) + \cdots$$

$$\delta\alpha_c = \mathcal{O}(\varepsilon^2)$$

ε represents the terms of 1st order in spatial deriv's.
Curvature pert'n is nonlinearly invariant on super-H scales

Transformation of linear MS equations

GR:

$$\frac{1}{z^2}\frac{1}{\alpha_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{z^2}{\alpha_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{R}_c\right)+c_s^2k^2\mathcal{R}_c=0\,,\qquad z\equiv a\frac{\phi'}{\mathcal{H}}$$

α_0 is the background value of the lapse function and c_s is the sound velocity

Under disformal transformation, $\tilde{\alpha}^2=\alpha^2-B\dot{\phi}^2$, there are **two ways to interpret**:

$$\Rightarrow\quad \frac{1}{z^2}\frac{1}{\tilde{\alpha}_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{z^2}{\tilde{\alpha}_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{R}_c\right)+c_s^2k^2\mathcal{R}_c=0 \qquad \cdots①$$

It takes **the same form** if $\mathrm{d}\tau=\alpha_0\mathrm{d}t$ and $\mathrm{d}\tilde{\tau}=\tilde{\alpha}_0\mathrm{d}t$ $(\mathrm{d}t=a\mathrm{d}\eta)$

On the other hand

$$\Rightarrow\quad \frac{1}{\tilde{z}^2}\frac{1}{\alpha_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\left(\frac{\tilde{z}^2}{\alpha_0}\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{R}_c\right)+\tilde{c}_s^2k^2\mathcal{R}_c=0\,, \qquad \cdots②$$

$$\tilde{c}_s\equiv\frac{\tilde{\alpha}_0}{\alpha_0}c_s\,,\qquad \tilde{z}\equiv\sqrt{\frac{\alpha_0}{\tilde{\alpha}_0}}z$$

It can be interpreted as **the one in modified gravity** with this redefinition of c_s and z.

Transformation of nonlinear equations in spatial gradient expansion

GR [Takamizu et al]:

$$\frac{1}{z^2}\frac{1}{\alpha_0}\frac{\partial}{\partial\eta}\left(\frac{z^2}{\alpha_0}\frac{\partial}{\partial\eta}\mathfrak{R}_c\right)+\frac{c_s^2}{4}{}^{(3)}R\left[e^{2\psi}\gamma_{ij}\right]=\mathcal{O}(\varepsilon^4)$$

$$\psi=\mathfrak{R}_c+\mathcal{O}(\epsilon^2)\ \text{and}\ {}^{(3)}R\ \text{is the spatial scalar curvature}$$

By the same reasoning as in the linear case, it takes **the same form if the proper time is rescaled**, or is interpreted as **the one in modified gravity with rescaled c_s and z**.

$$\tilde{c}_s\equiv\frac{\tilde{\alpha}_0}{\alpha_0}c_s\,,\qquad\qquad \tilde{z}\equiv\sqrt{\frac{\alpha_0}{\tilde{\alpha}_0}}z$$

Transformation of nonlinear equations in spatial gradient expansion

GR:

$$\frac{1}{z_t^2}\frac{1}{\alpha_0}\frac{\partial}{\partial\eta}\left(\frac{z_t^2}{\alpha_0}\frac{\partial}{\partial\eta}\gamma_{ij}^{\mathrm{TT}}\right)+\frac{1}{4}\left(e^{-2\psi\,(3)}R_{ij}\left[e^{2\psi}\gamma_{ij}\right]\right)^{\mathrm{TT}}=\mathcal{O}(\varepsilon^4)$$

$$z_t \equiv a \text{ and } (\cdots)^{\mathrm{TT}} \text{ denotes the transverse-traceless projection}$$

By the same reasoning as in the linear case, it takes **the same form if the proper time is rescaled**, or is interpreted as **the one in modified gravity with rescaled c_s and z**.

$$\tilde{c}_t \equiv \frac{\tilde{\alpha}_0}{\alpha_0}\,, \qquad \tilde{z}_t \equiv \sqrt{\frac{\alpha_0}{\tilde{\alpha}_0}}z_t = \sqrt{\frac{\alpha_0}{\tilde{\alpha}_0}}a$$

$$\frac{1}{\tilde{z}_t^2}\frac{1}{\alpha_0}\frac{\partial}{\partial\eta}\left(\frac{\tilde{z}_t^2}{\alpha_0}\frac{\partial}{\partial\eta}\gamma_{ij}^{\mathrm{TT}}\right)+\frac{\tilde{c}_t^2}{4}\left(e^{-2\psi\,(3)}R_{ij}\left[e^{2\psi}\gamma_{ij}\right]\right)^{\mathrm{TT}}=\mathcal{O}(\varepsilon^4)$$

Multi-disformal transformation

Suppose there are $\mathcal{N}$ component scalar field, ϕ^I $(I = 1, \cdots, \mathcal{N})$.

$$\tilde{g}_{\mu\nu} = A(\phi^I, X^{IJ})\, g_{\mu\nu} + B_{KL}(\phi^I, X^{IJ})\, \partial_\mu \phi^K \partial_\nu \phi^L ,$$

$$X^{IJ} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi^I \partial_\nu \phi^J$$

Adiabatic limit: $\phi^I = \phi^I(\varphi)$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_{KL}\Big[\phi^I(\varphi), X^{IJ}(\varphi, \partial\varphi)\Big] (\phi^K)'(\phi^L)' \partial_\mu \varphi \partial_\nu \varphi , \qquad (\phi^I)' \equiv \frac{\mathrm{d}\phi^I}{\mathrm{d}\varphi}$$

The uniform φ slicing:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + B_{KL}\Big[\phi^I(\varphi), X^{IJ}(\varphi, \dot{\varphi}/\alpha)\Big] (\phi^K)'(\phi^L)' \dot{\varphi}^2 \delta_\mu{}^0 \delta_\nu{}^0$$

Since the multi-disformal transformation only affects **the lapse function**, we can apply the same argument as before!

Summary

- The curvature and tensor perturbations on the uniform ϕ slicing are **fully nonlinearly invariant** under the disformal transformation.
- The same conclusion can be drawn for a multi-component extension of the disformal transformation, dubbed **multi-disformal transformation**, on the uniform φ slicing in the adiabatic limit.
- Once a 2nd order differential eq. is obtained in **modified gravity or EFT**, one can map it into **the same form** as the one in GR by a suitable disformal transformation, linearly and non-linearly (the next leading order in gradient expansion).