Estimation of neutrino oscillation parameters with Markov Chain Monte Carlo (poster)

Leïla Haegel (University of Geneva),
for the T2K collaboration

PhyStat ν
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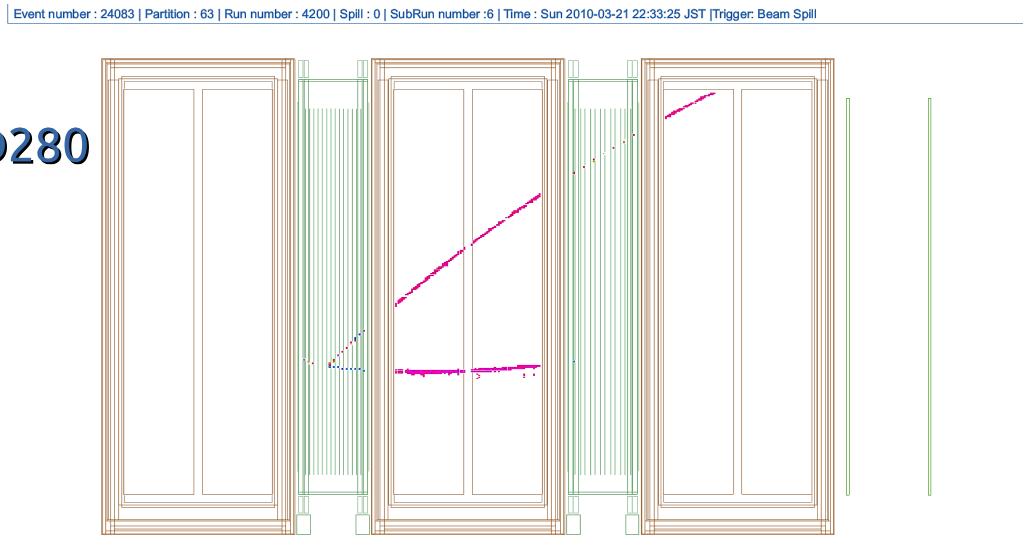
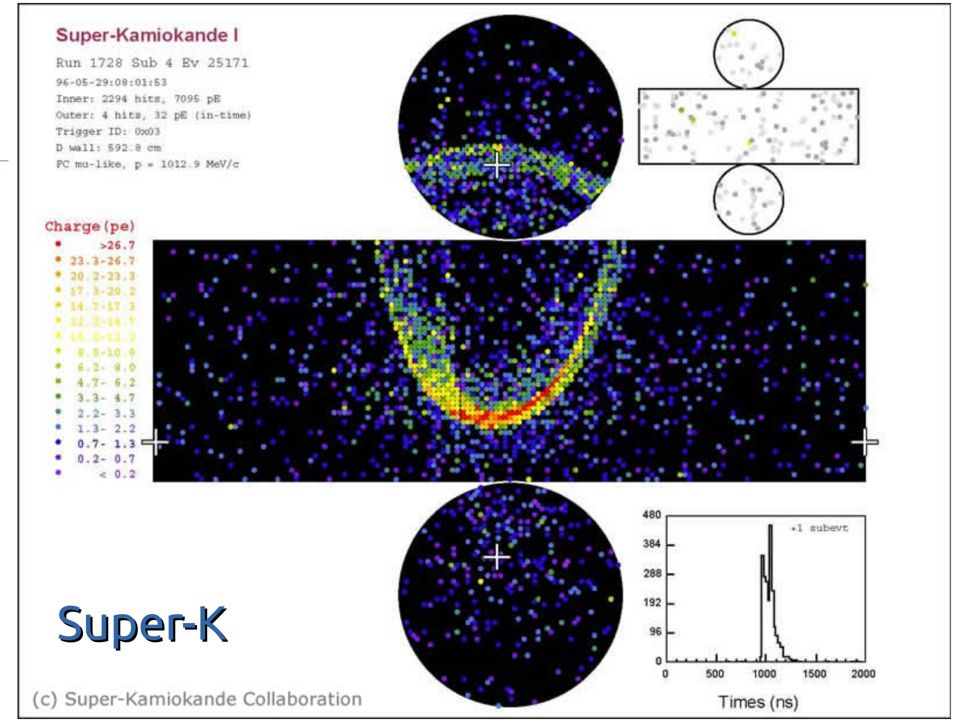
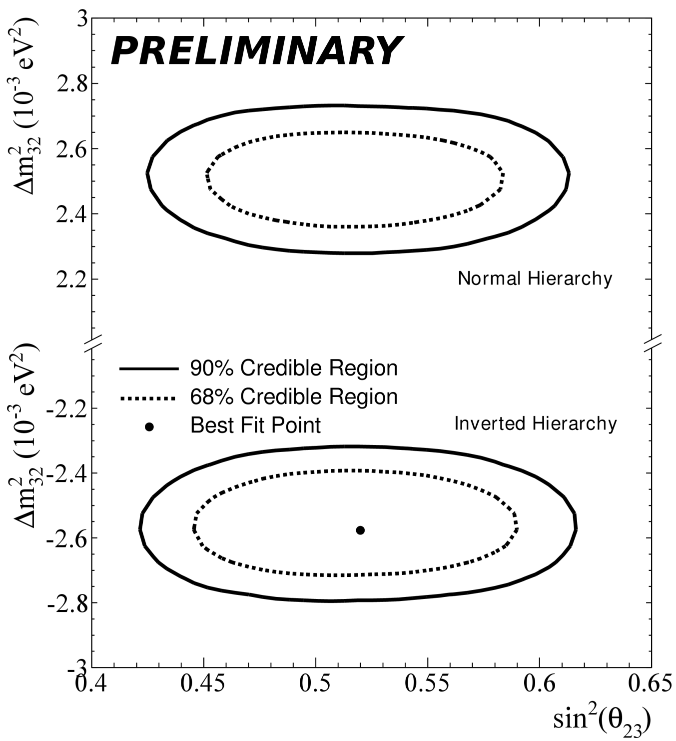
Or how to go from this:

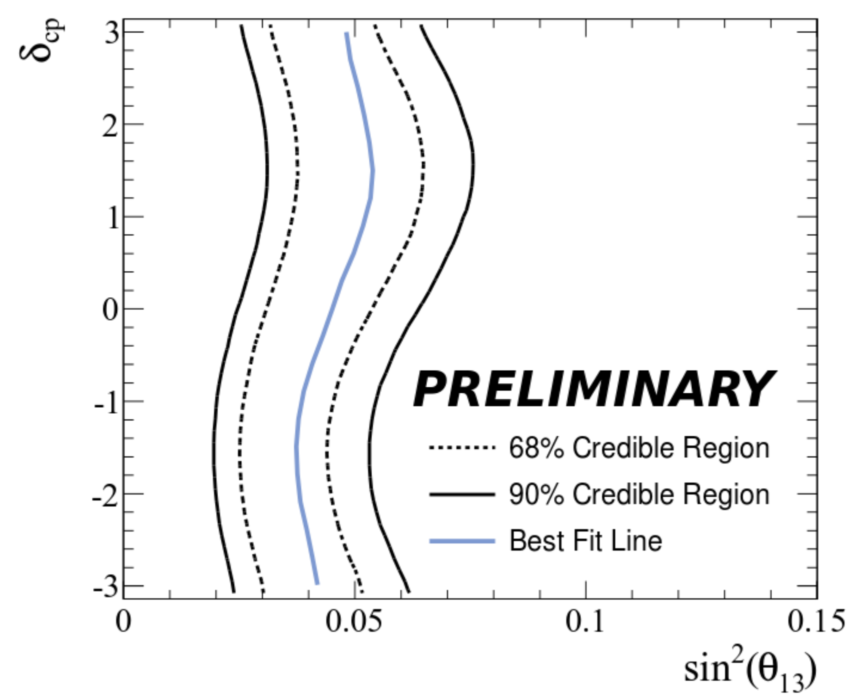
ND280





To this:





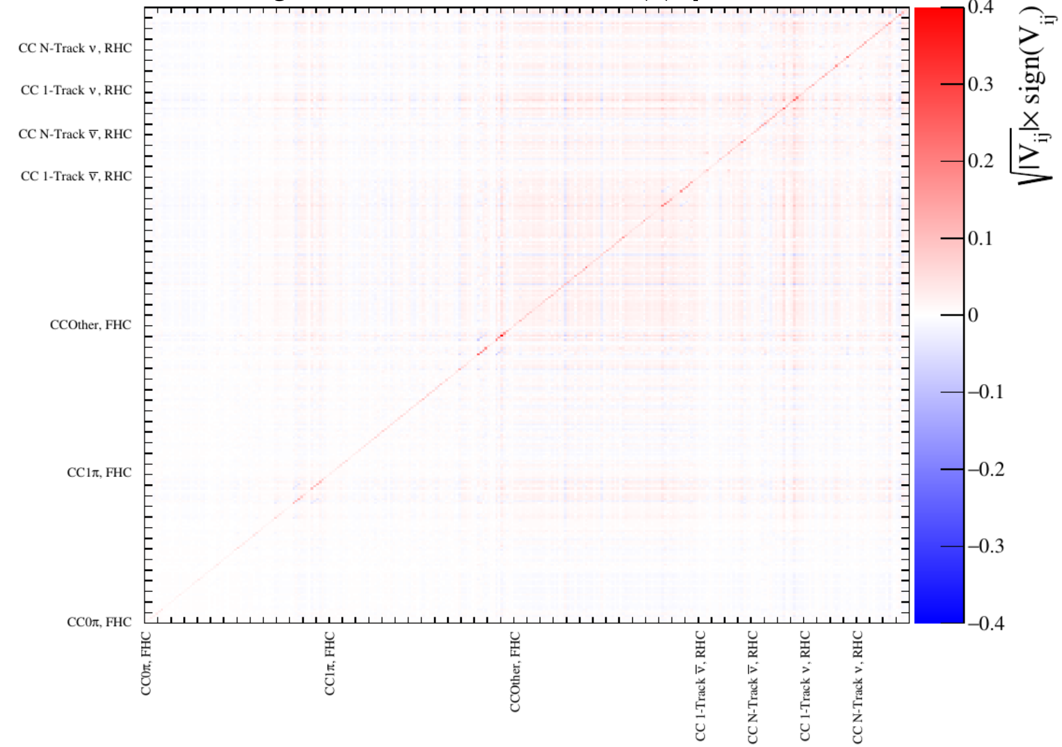
see A. Kaboth talk for more info on T2K

To compare data to MC, one needs to vary the MC until it finds the values of parameters representing best the data.

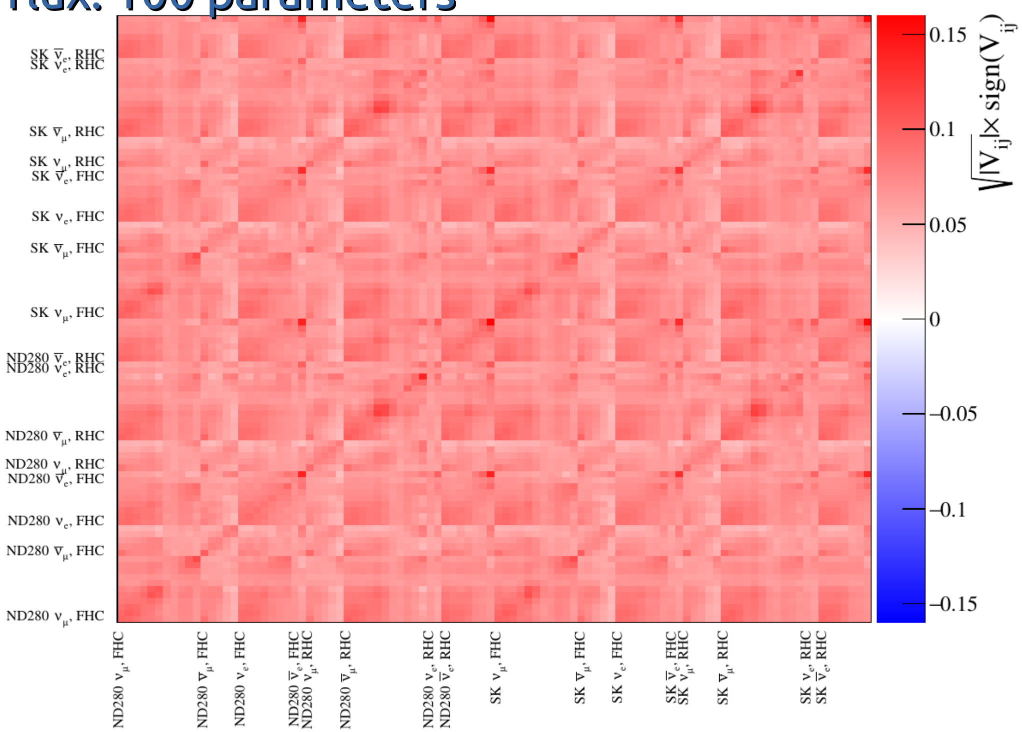
Issue : the parameter space is high dimensional.

oscillation: 6 parameters

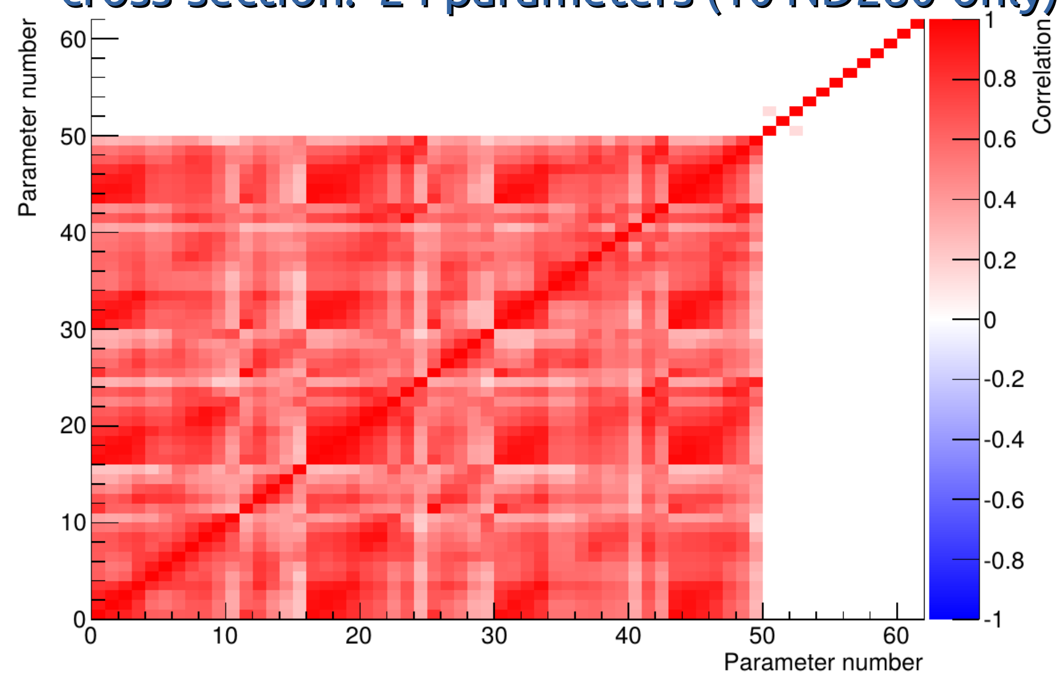
ND280 systematics: 290 (!) parameters



flux: 100 parameters

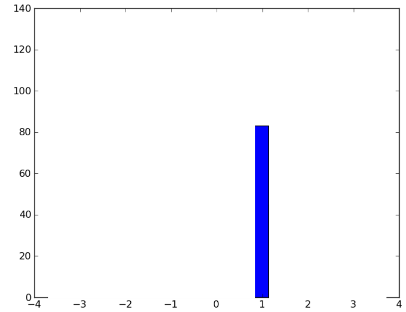


cross-section: 24 parameters (10 ND280-only)



<u>Solution: **Markov Chains.**</u>

$$p(X_0|D) = \frac{p(D|X_0) \times p(X_0)}{p(D)}$$



The chain is made of several steps.
Every step is a new point on the parameter space and only depends on previous one.

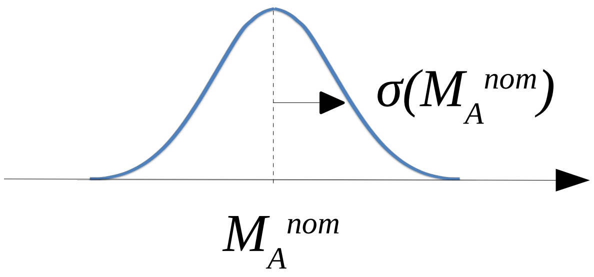
At each step the value of the parameter is changed in the MC and the posterior probability $p(X_0|D)$ of the parameter is computed (Bayesian process).

<u>Example</u> : we want to determine M_A, for which we have a prior value M_A^{nom} and systematic uncertainty $\sigma(M_A^{nom})$.

posterior probability : $p(M_A|D) = \dfrac{p(D|M_A) \times p(M_A)}{p(D)}$

$$p(D|M_A) = likelihood = \frac{(N^{predicted} - N^{data})^2}{\sigma(M_A^{nom})^2}$$

$$p(M_A) = M_A\,prior = Gaussian\ of\ mean\ M_A^{nom}$$
$$and\ width\ \sigma(M_A^{nom})$$



The **Metropolis-Hastings algorithm** ensures that the steps form a random walk sampling the posterior distribution.

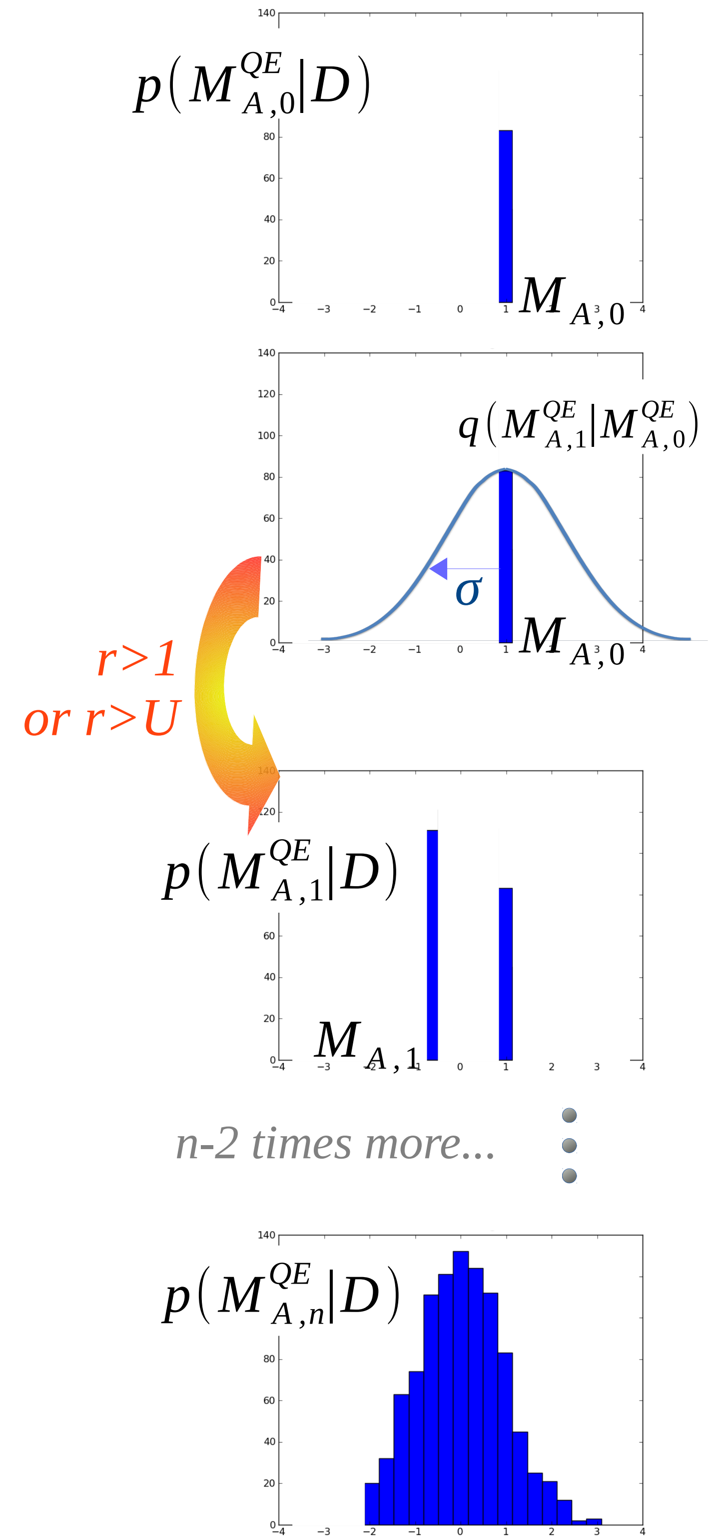
While at a step i, a proposal function $q(X_{i+1}|X_i)$ is used to propose the next step.

MaCh3 uses a Gaussian which width σ is the step size (tuned to maximise the efficiency of the chain).

We define the ratio : $r = \dfrac{p(X_1|D)\,q(X_0|X_1)}{p(X_0|D)\,q(X_1|X_0)}$

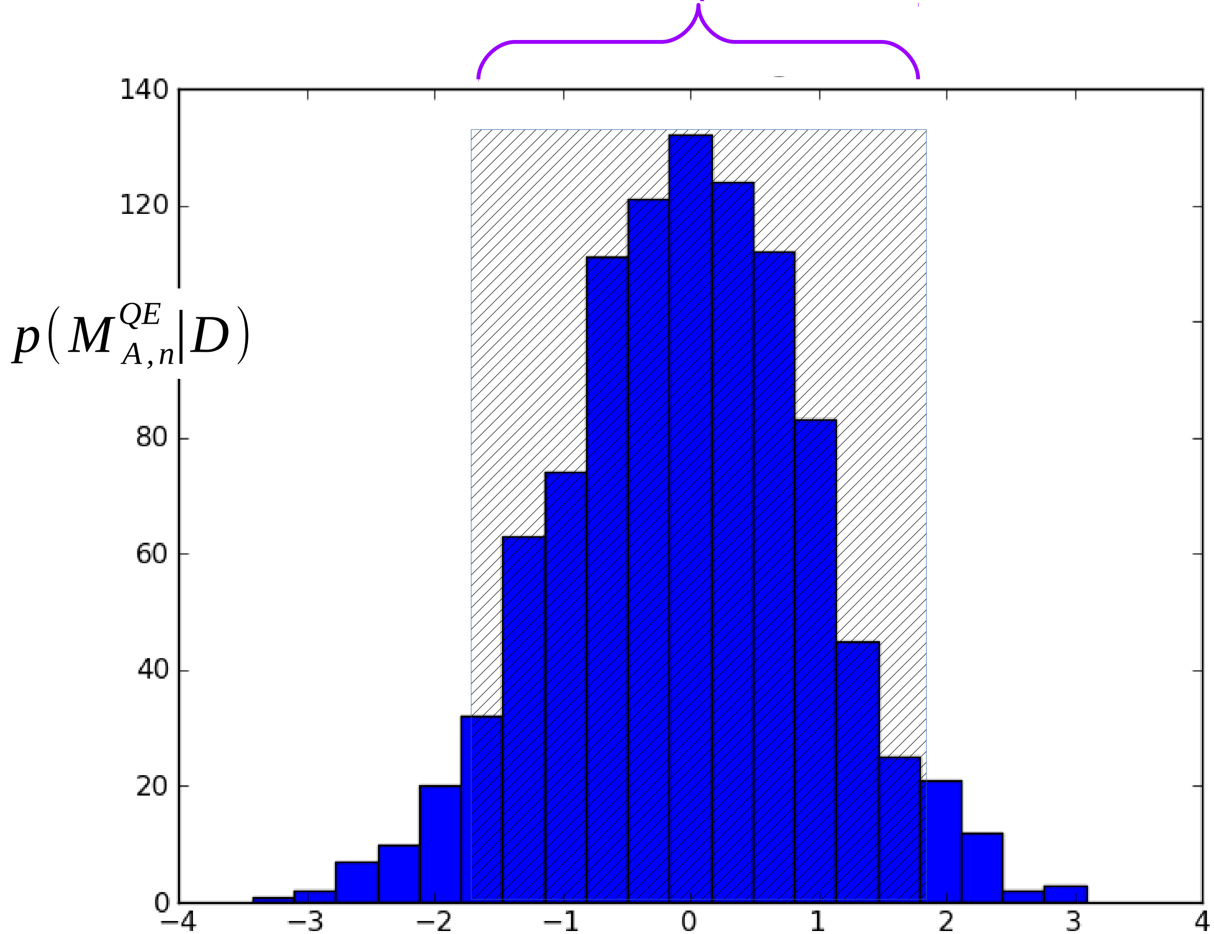
- if $r \geqslant 1$ the step is accepted ;
- if $r < 1$ a random value U between [0,1] is thrown and the step is accepted if $r > U$

Distribution of steps is proportionnal to posterior probability of the parameter.



When the posterior probability density is found,
sum the bins with most content until integral = 68.3% for 1σ
95% for 3σ
... to give the desired credible interval

M_A^{QE} uncertainty range



come to see the poster
for T2K results !