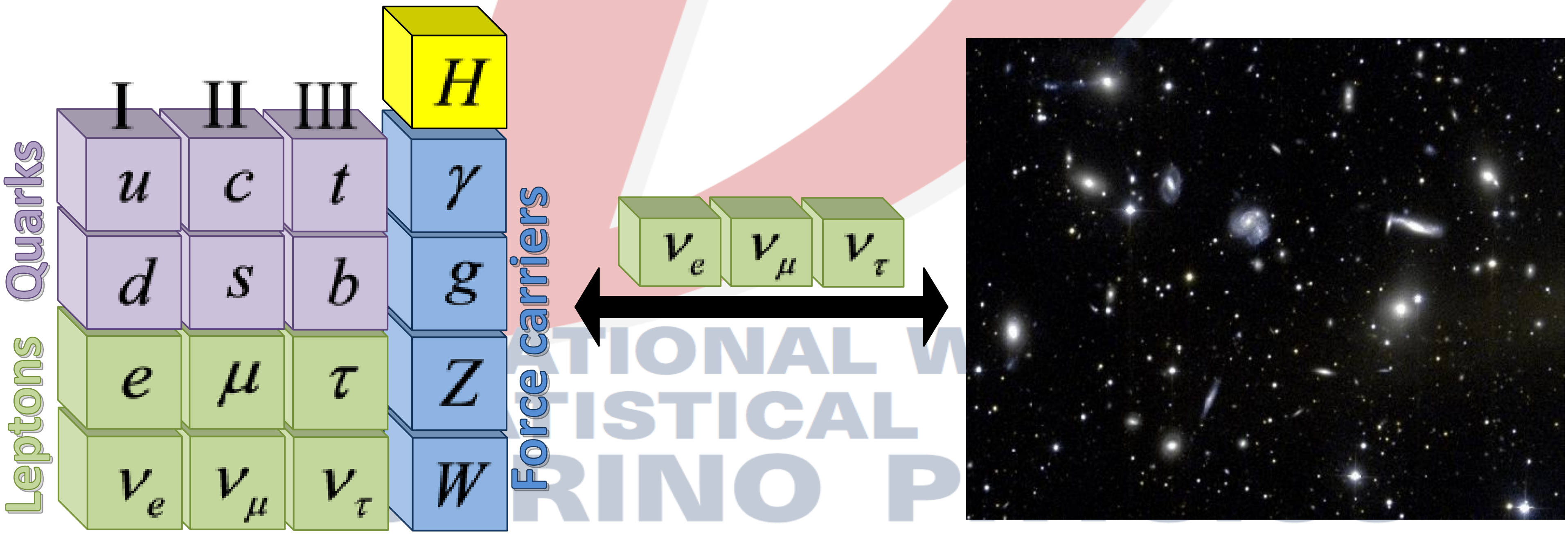
Cosmological Constraints on Neutrino Properties

Francisco Villaescusa-Navarro
OATS/INAF/INFN, Trieste, Italy

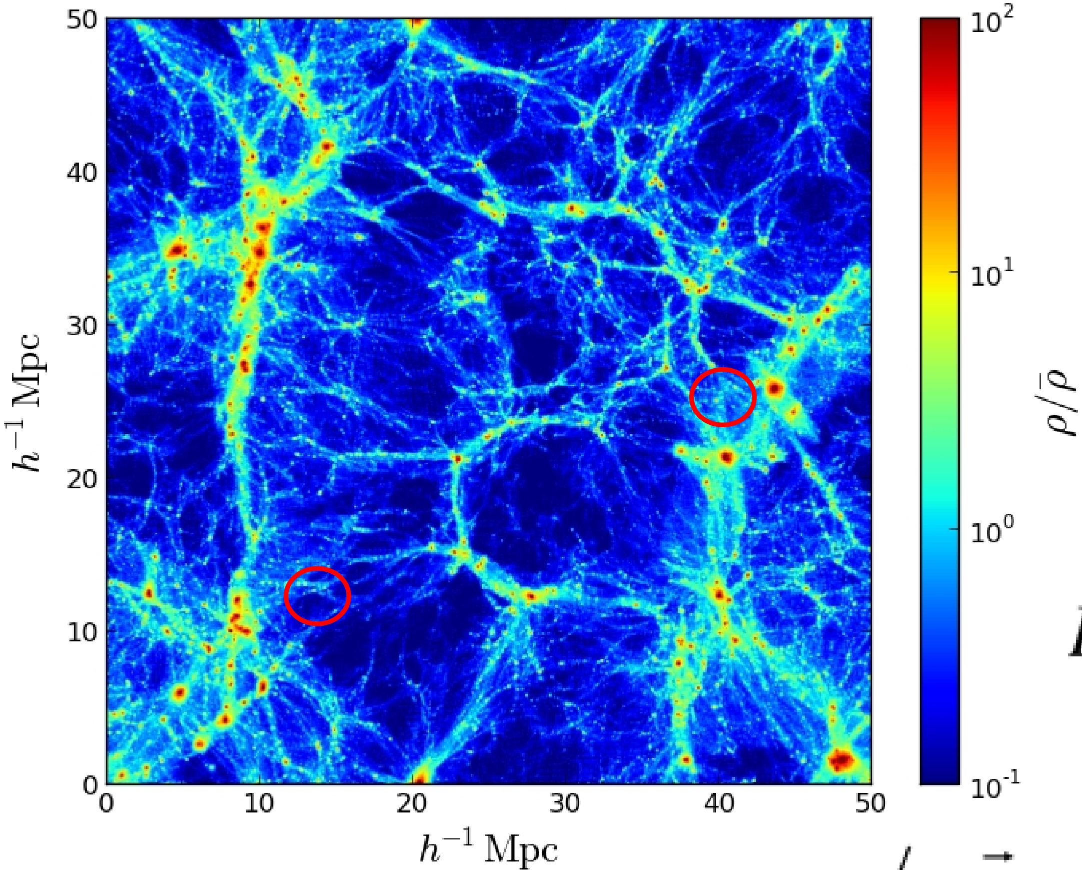


THE KAVLI INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE, KASHIWA, JAPAN
30 MAY–1 JUNE 2016

Outline

- Cosmic neutrino background
- Effects on cosmological observables
- Statistical methods used to constraint neutrino masses: challenges

Statistical estimators in cosmology



$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - \langle\rho\rangle}{\langle\rho\rangle}$$

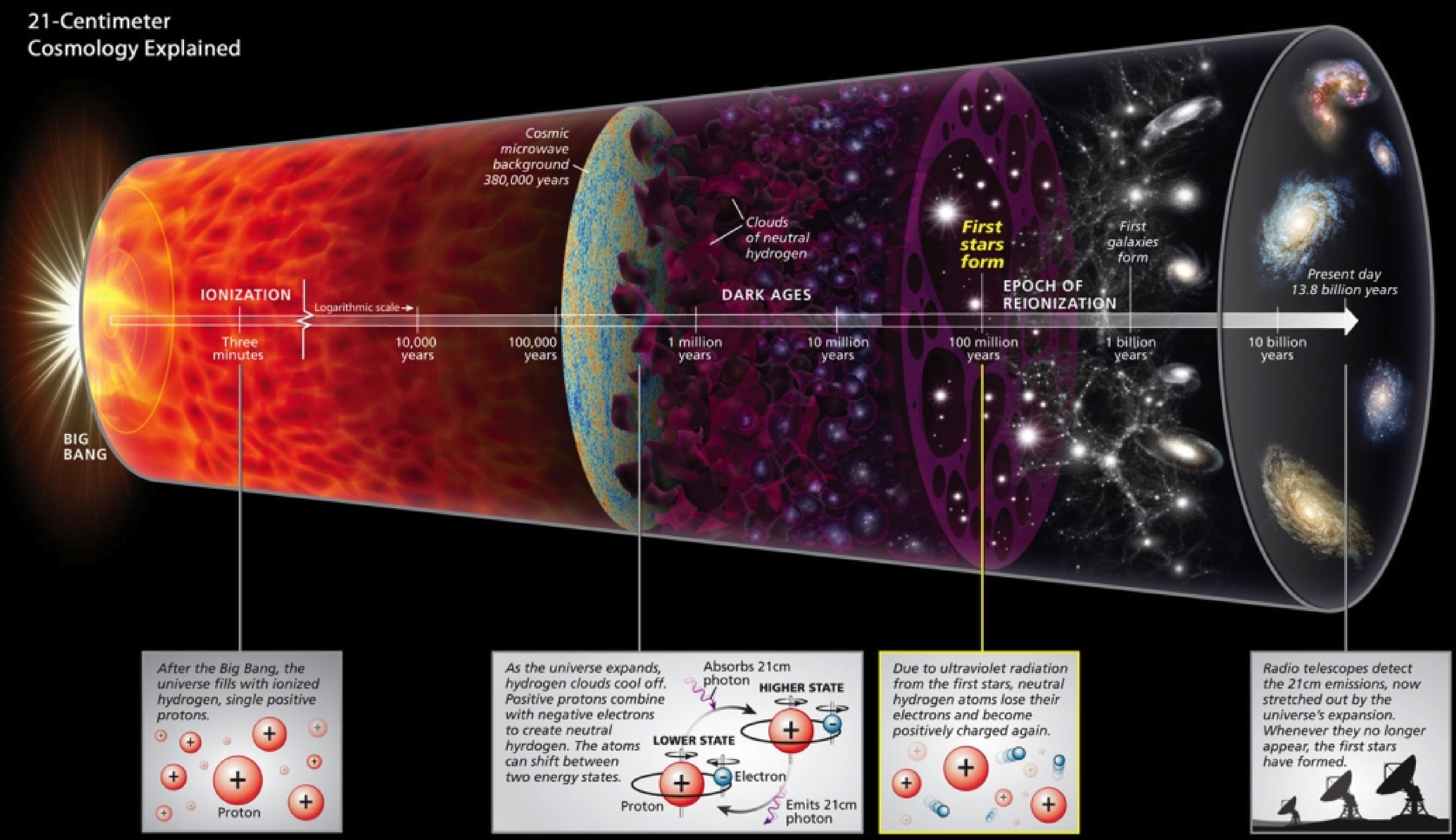
$$\xi(r) = \left\langle \delta(\vec{x})\delta(\vec{x}+\vec{r}) \right\rangle$$

$$P(k) = \frac{1}{(2\pi)^3}\int d^3\vec{r}\; e^{-i\vec{k}\vec{r}}\xi(r)$$

$$\left\langle \delta(\vec{k})\delta(\vec{k}') \right\rangle = (2\pi)^3\delta_D(\vec{k}-\vec{k}')P(k)$$

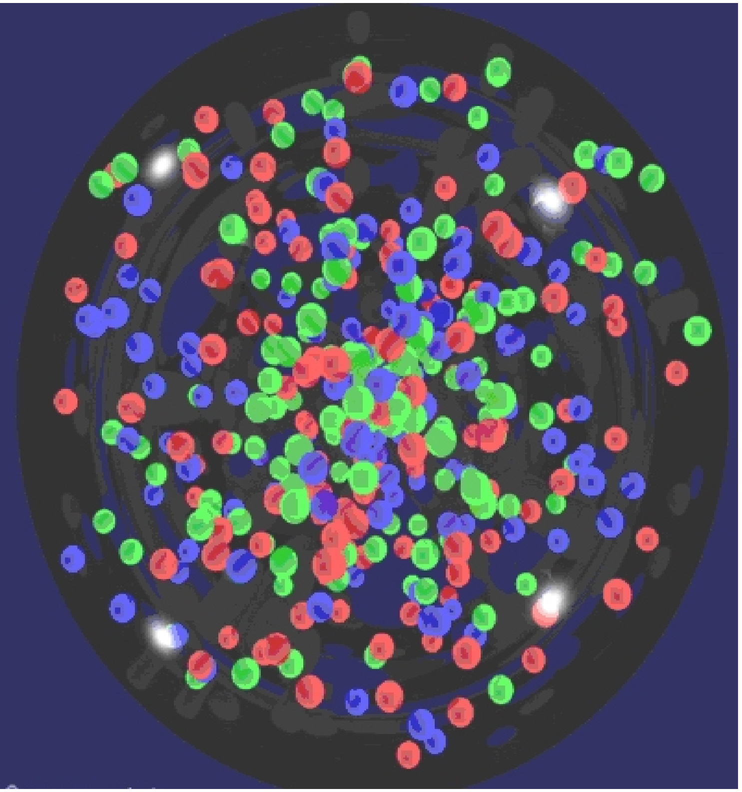
<u>Gaussian density field completely specified by its power spectrum/correlation function!!</u>

Universe history



Cosmic neutrino background

$$T \approx 10^{10}\ \mathrm{K} \approx 1\ \mathrm{MeV}$$



- Photons
- Electrons & Positrons
- Neutrinos & Antineutrinos

$$\nu + e^- \Leftrightarrow \nu + e^- \qquad e^- + e^+ \Leftrightarrow \nu + \bar{\nu}$$

$$n_\nu(p,z)dp \cong \frac{4\pi g_\nu}{(2\pi\hbar c)^3}\left(\frac{p^2 dp}{e^{(p/k_B T_\nu(z))}+1}\right)$$

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma,0} \approx 1.95\ \mathrm{K} \qquad T_\nu(z) = T_{\nu,0}(1+z)$$

S. Weinberg, Cosmology, Oxford University Press, 2008

Cosmic neutrino background

Properties

Redshift; measurement of time in cosmology

z=0 (today) z=∞ (Big Bang)

$$\bar{n}_\nu(z) \cong 113\ (1+z)^3 \frac{(\nu+\bar{\nu})}{\mathrm{cm}^3}$$

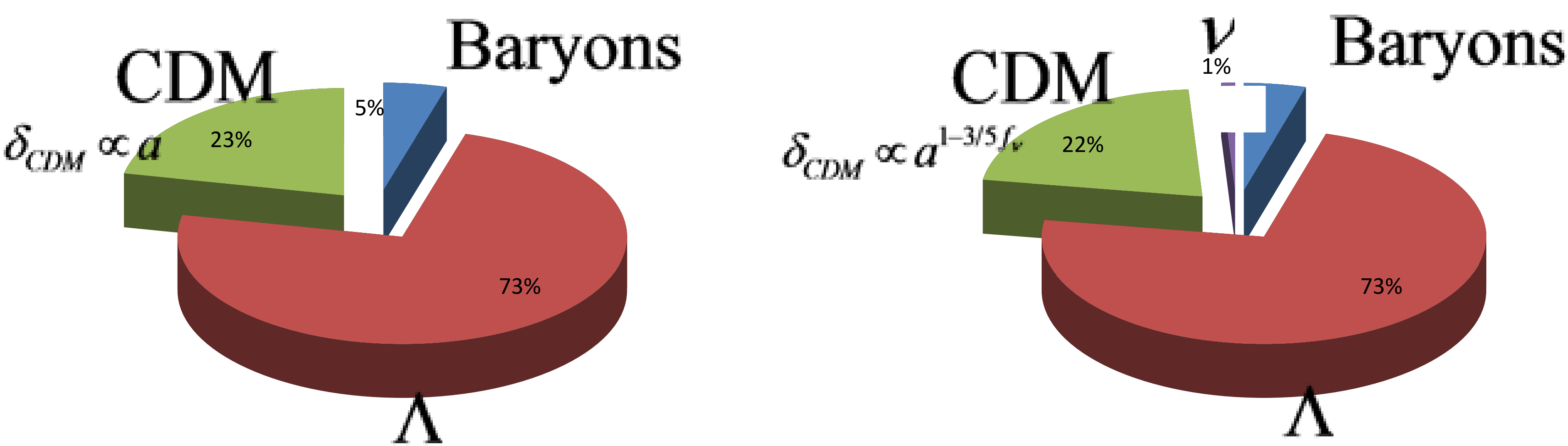
$$\bar{V}_\nu(z) \cong 160(1+z)\left(\frac{\mathrm{eV}}{m_\nu}\right)\ \mathrm{km/s}$$

- Relativistic at high-redshift (early Universe)
- Non-relativistic at low-redshift (late Universe)

Effects at linear order

1. Modification of the Matter-Radiation equality time

1. Slow down the growth of matter perturbations



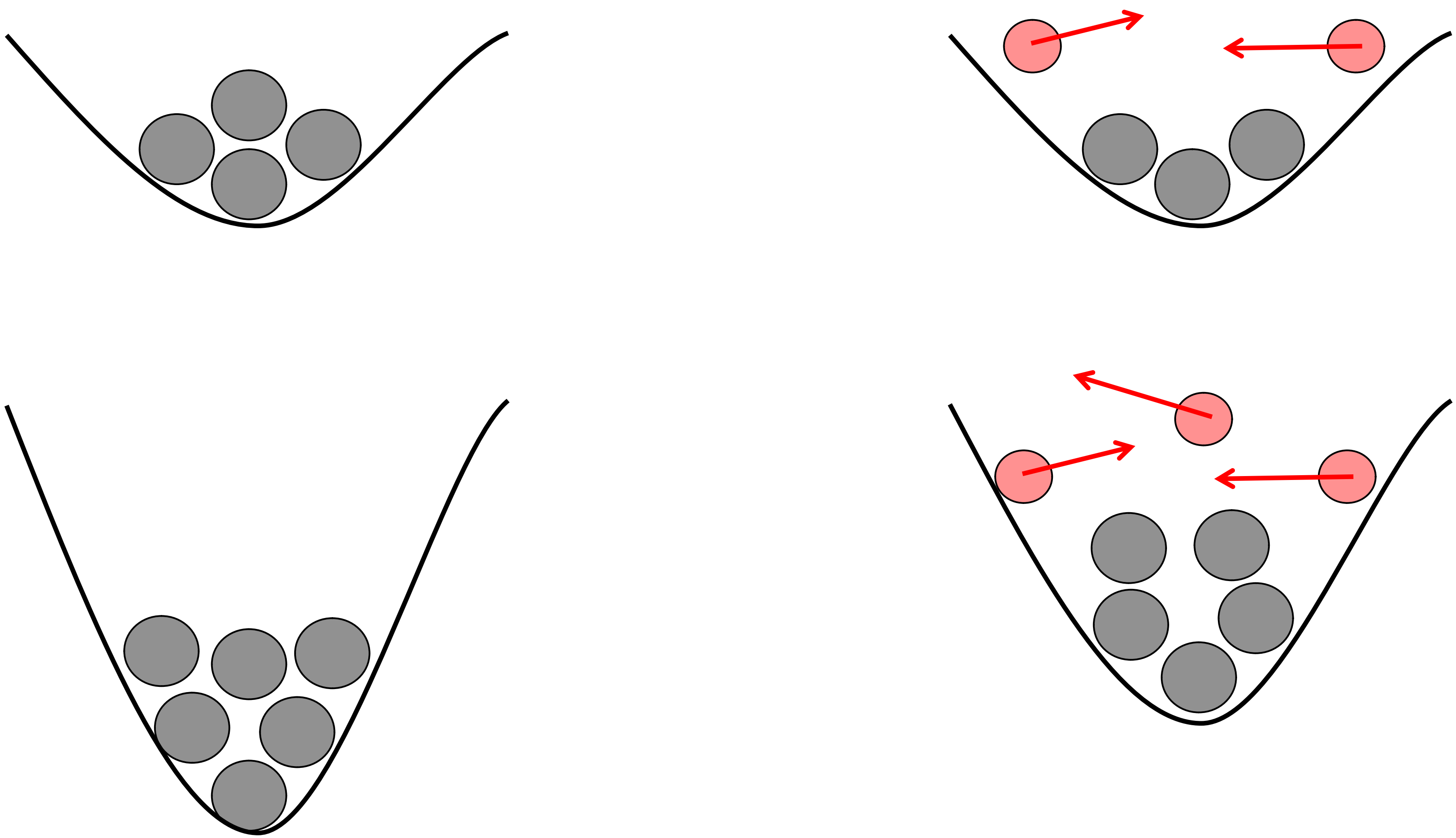
$$\Omega_{\mathrm{DM}} = \Omega_{\mathrm{CDM}} + \Omega_\nu = \text{fixed}$$

Lesgourgues & Pastor, Physics Reports, 429, 6, 2006

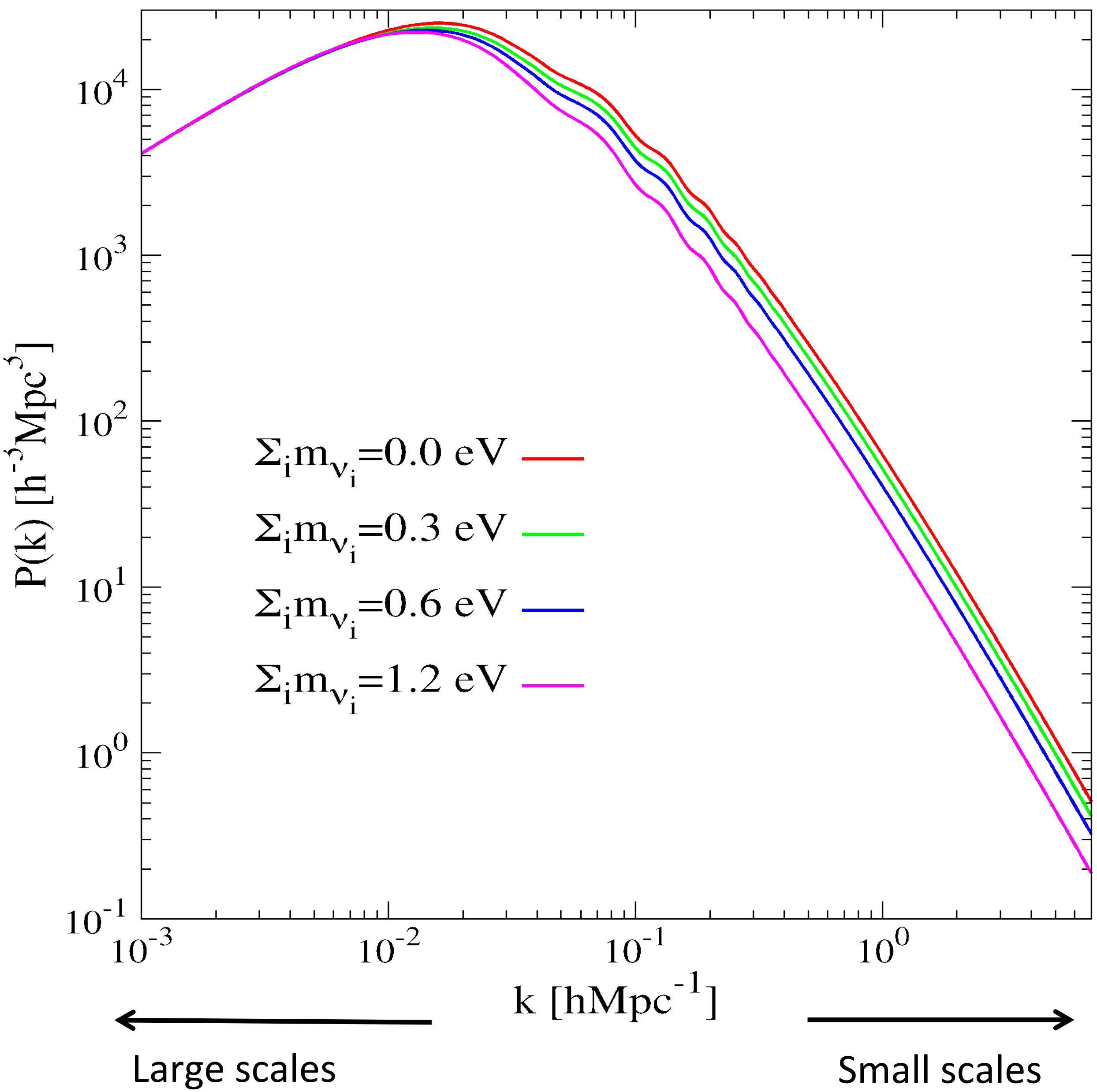
Effects at linear order

Massless neutrinos

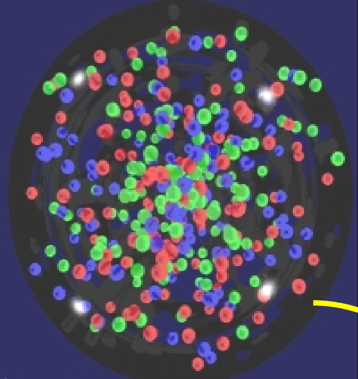
Massive neutrinos

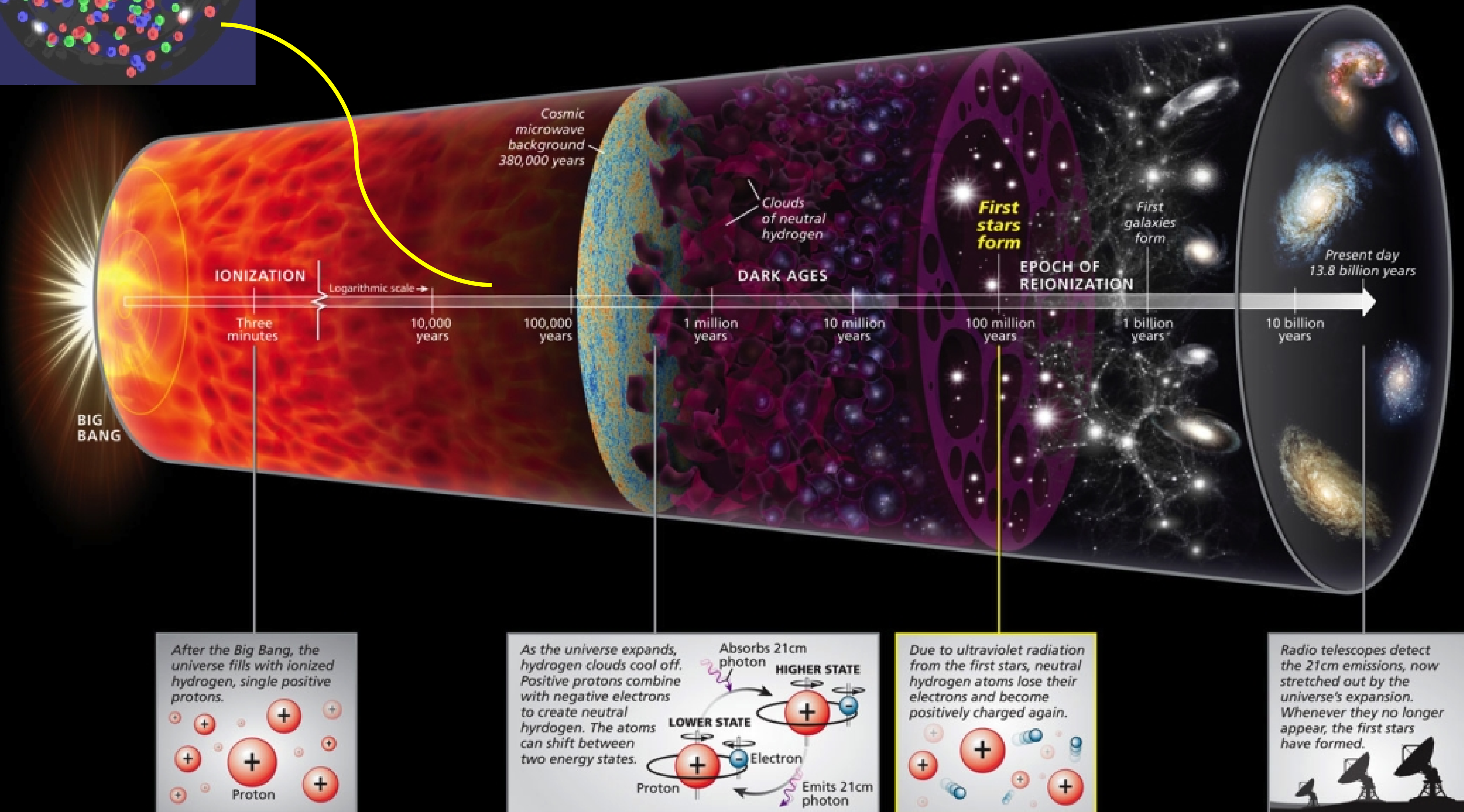


Effects at linear order



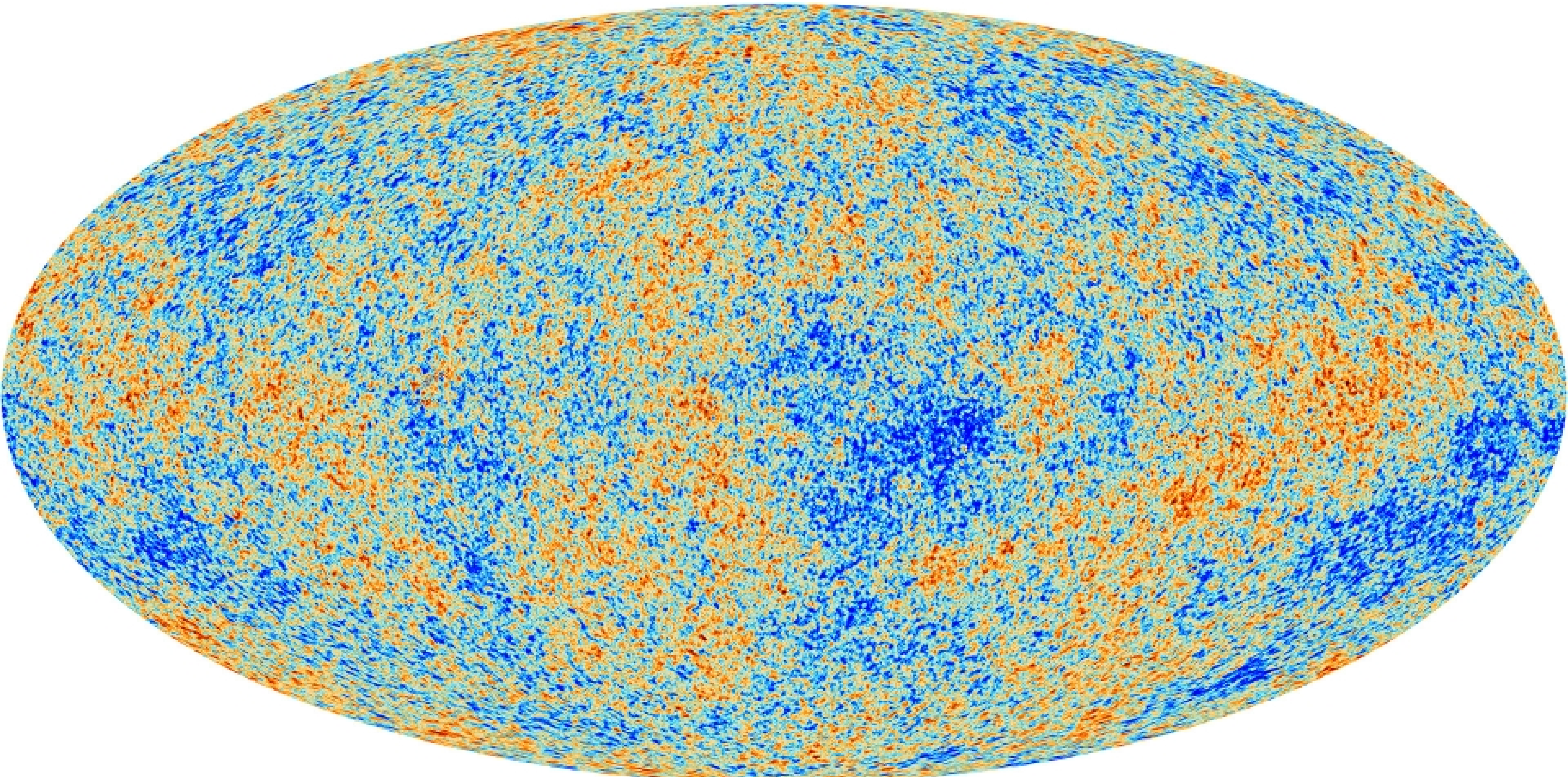
Cosmological observables





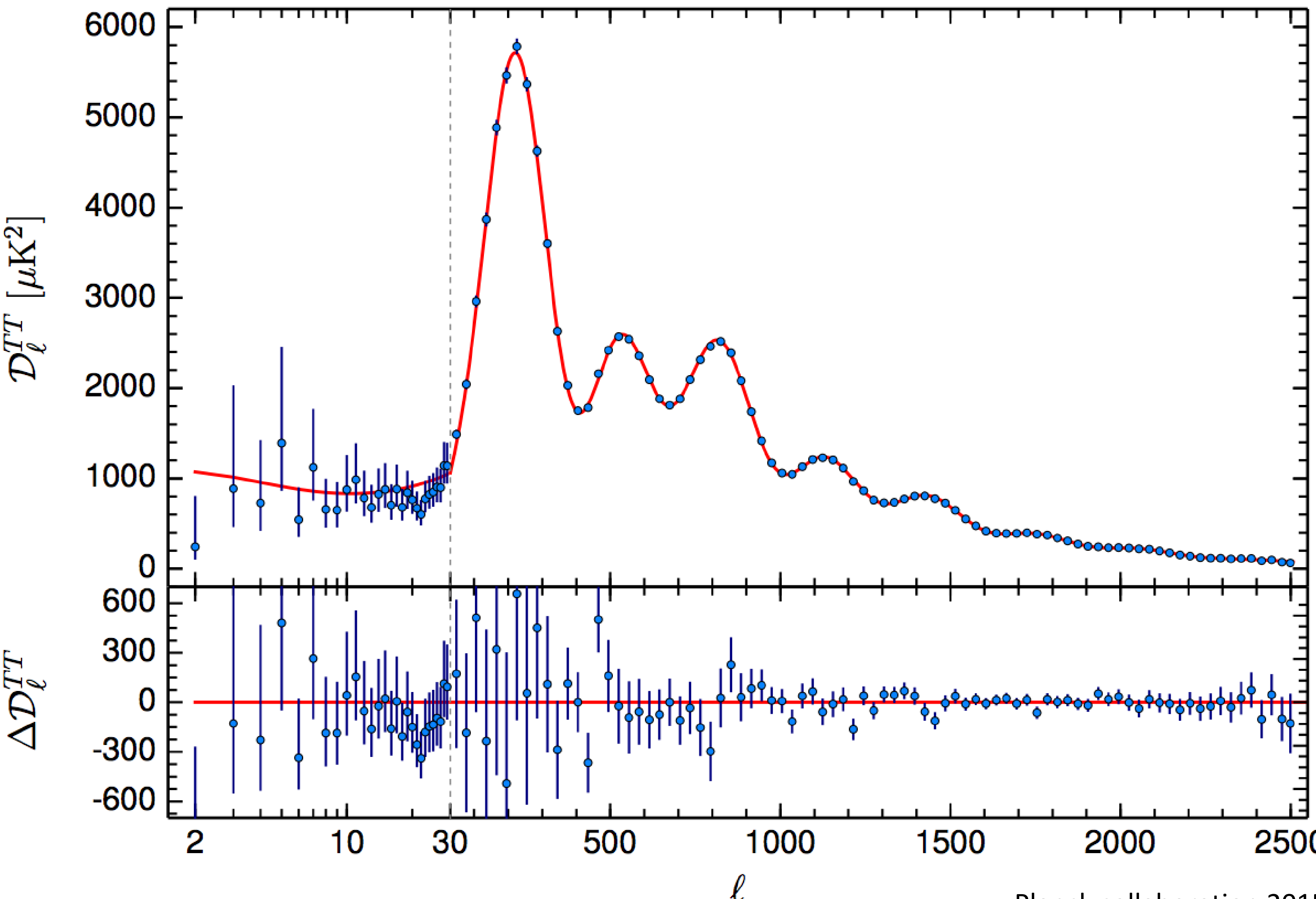
CMB anisotropies

Cosmological constraints are intrinsically statistical!!!



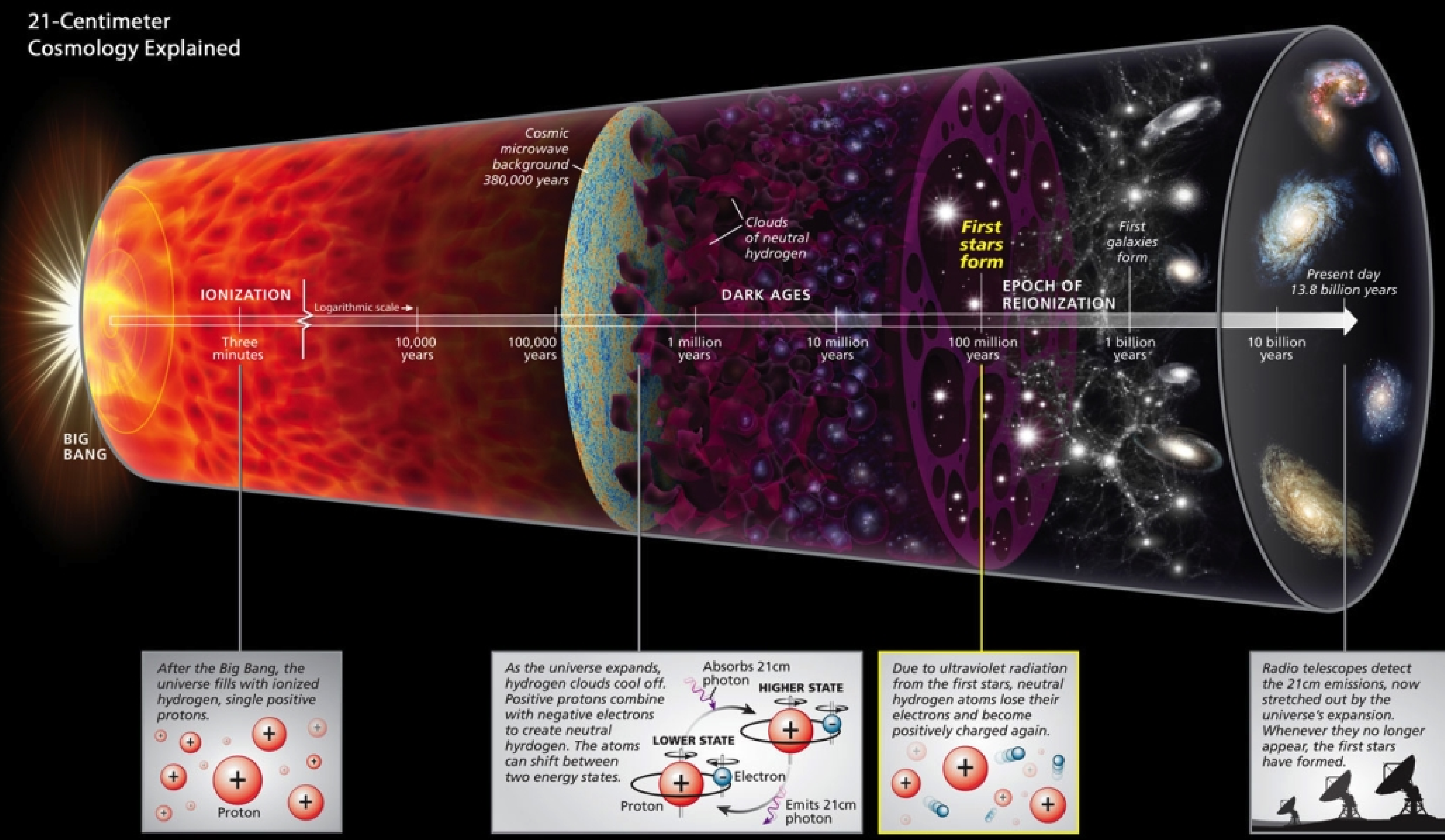
$T_\gamma = 2.7255\ \mathrm{K}$ $\qquad \dfrac{\Delta T_\gamma}{T_\gamma} \sim 10^{-5}$ $\qquad$ Planck collaboration 2015

CMB anisotropies power spectrum

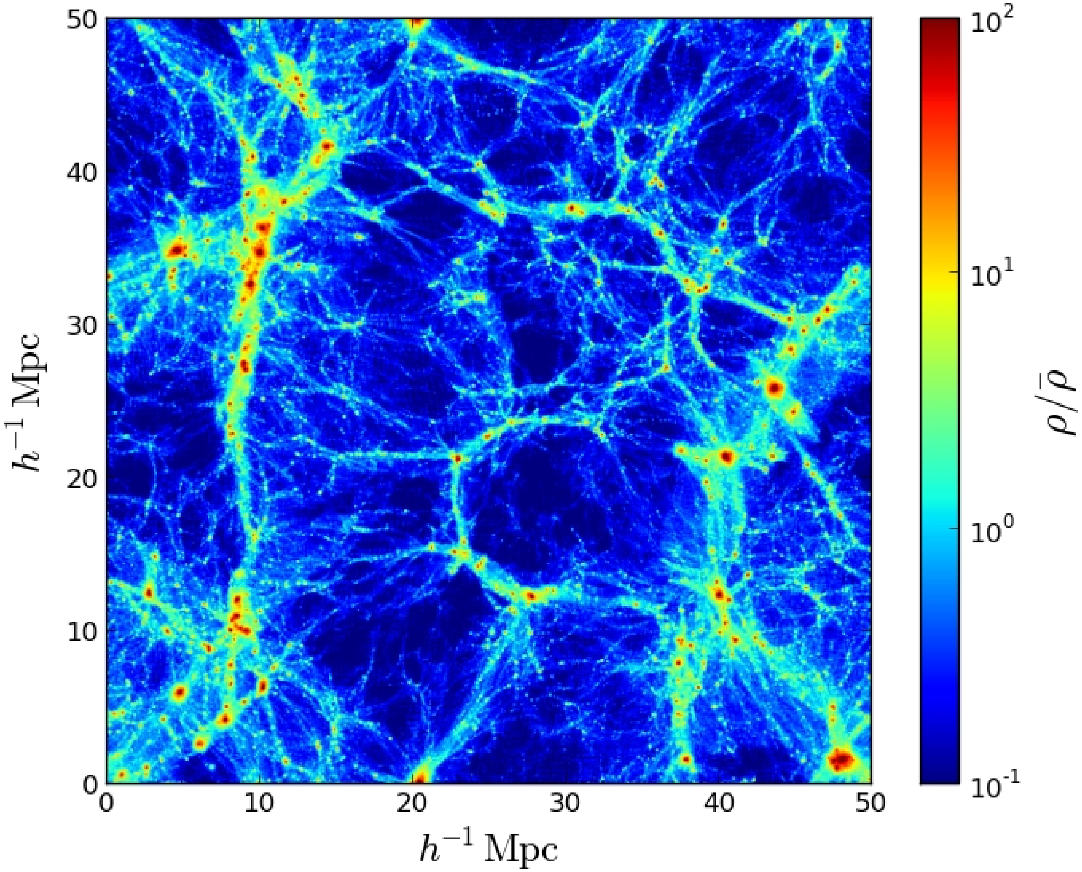


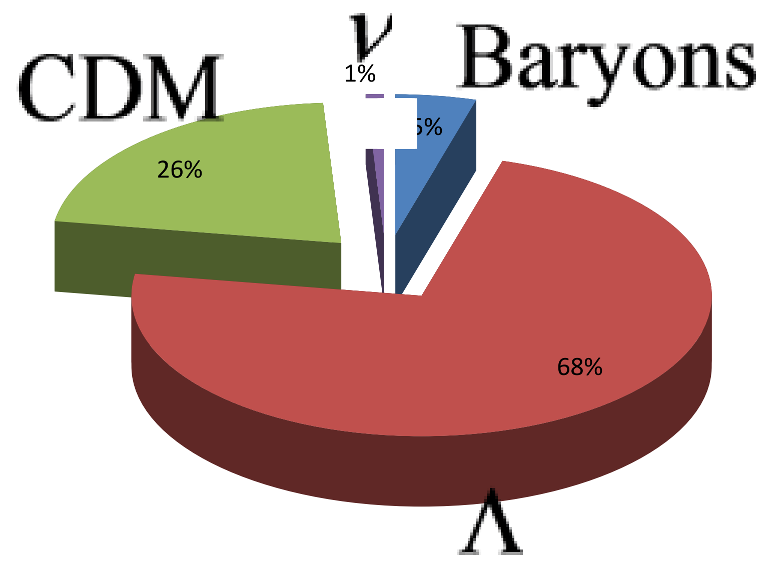
Planck collaboration 2015

Cosmological observables

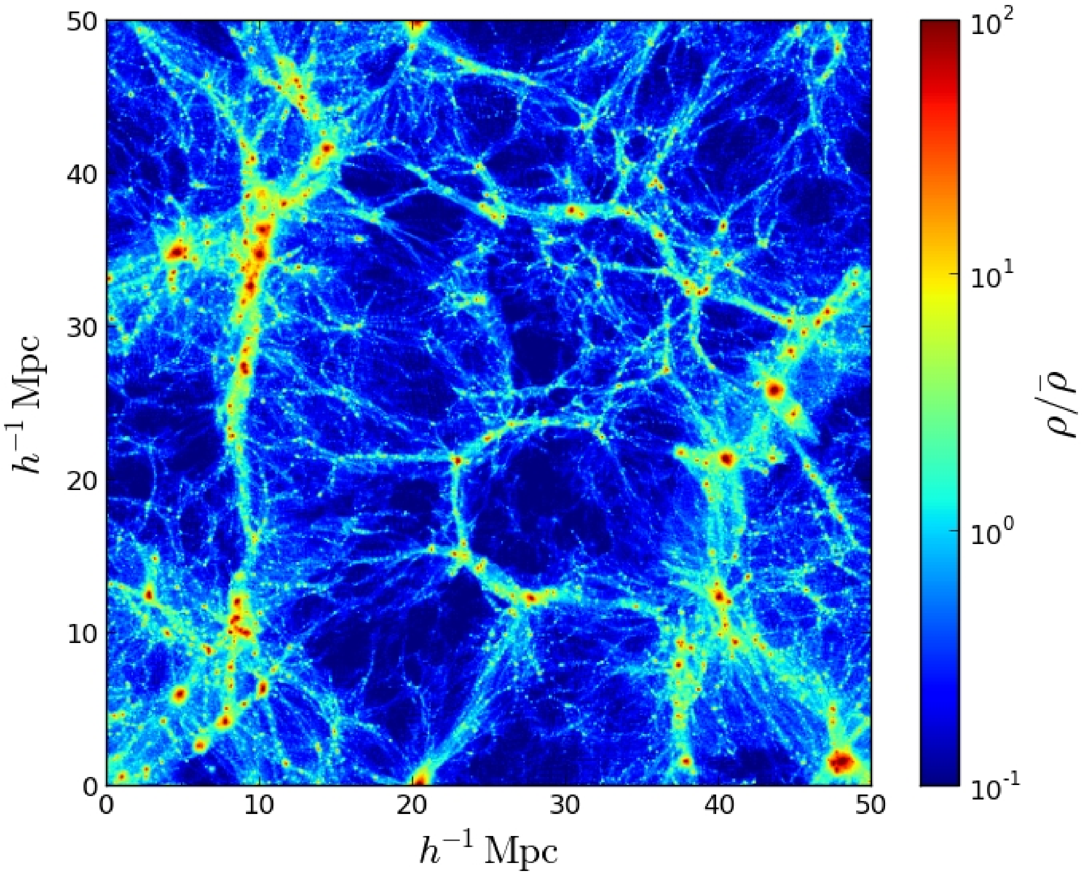


Density field tracers

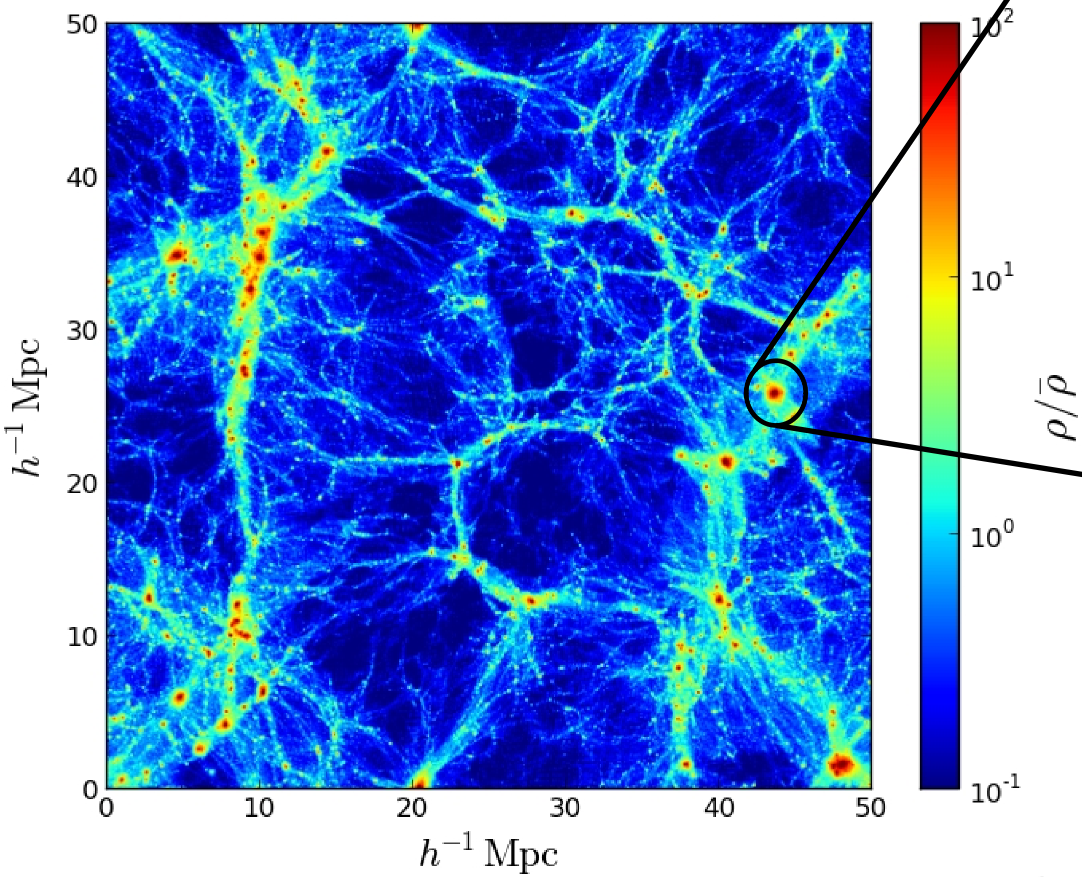


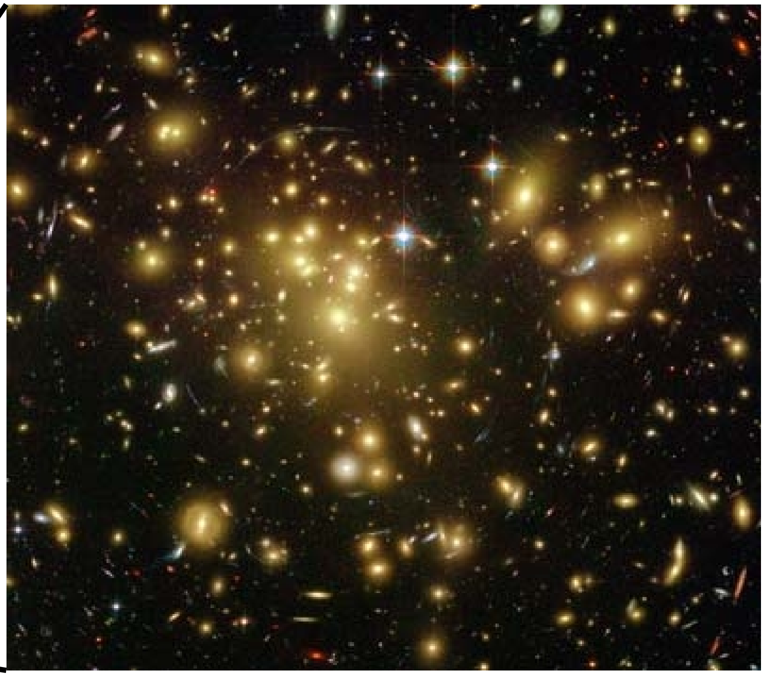


Density field tracers



Density field tracers



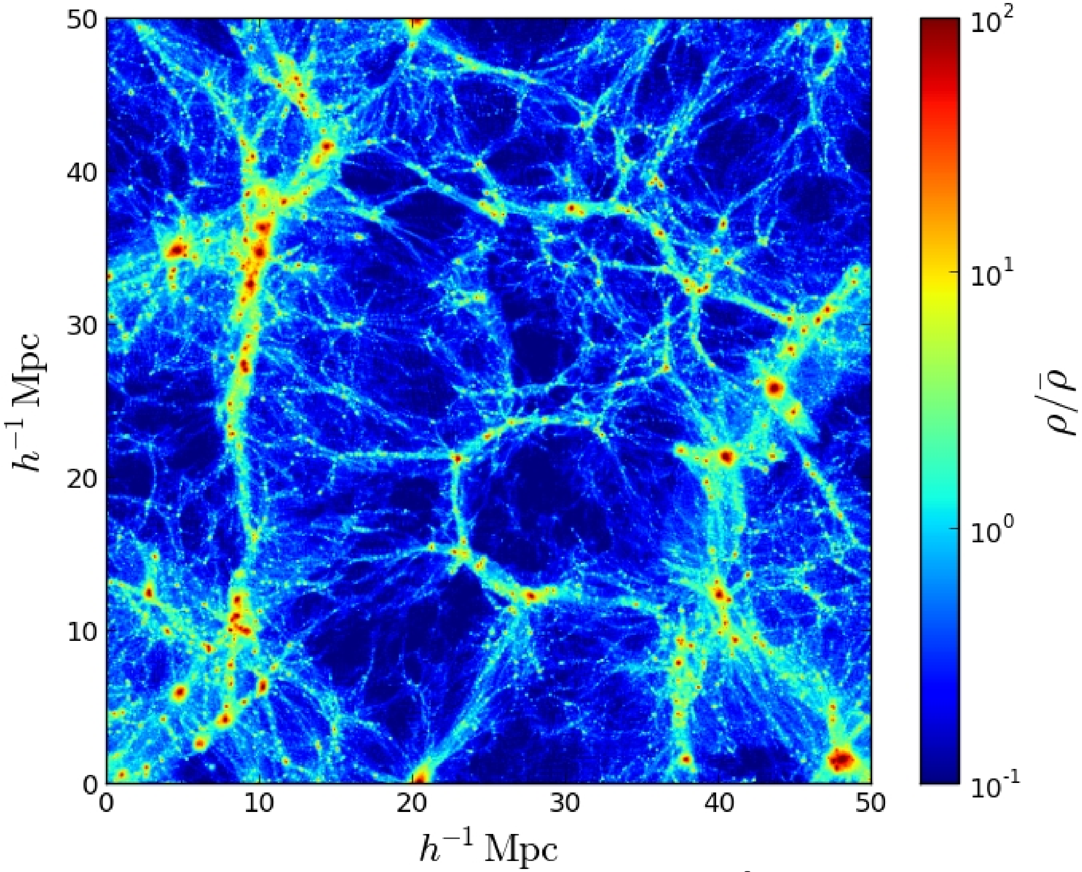


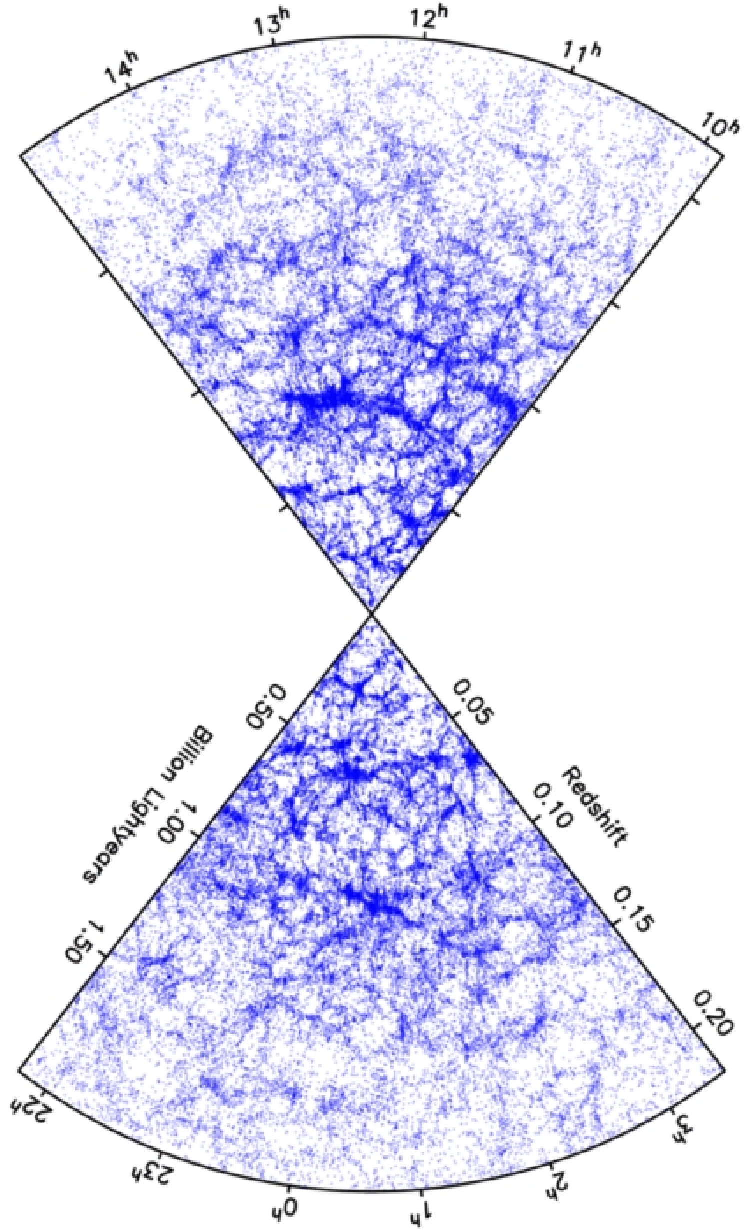
$$\boxed{P_g(k) = b_1^2 P(k)}$$

$$\delta_g(\vec{x}) = f(\delta(\vec{x}))$$

$$\delta_g(\vec{x}) = \frac{1}{k!}\sum_k b_k \delta^k(\vec{x}) \cong b_1 \delta(\vec{x})$$

Density field tracers





$$\boxed{P_g(k) = b_1^2 P(k)}$$

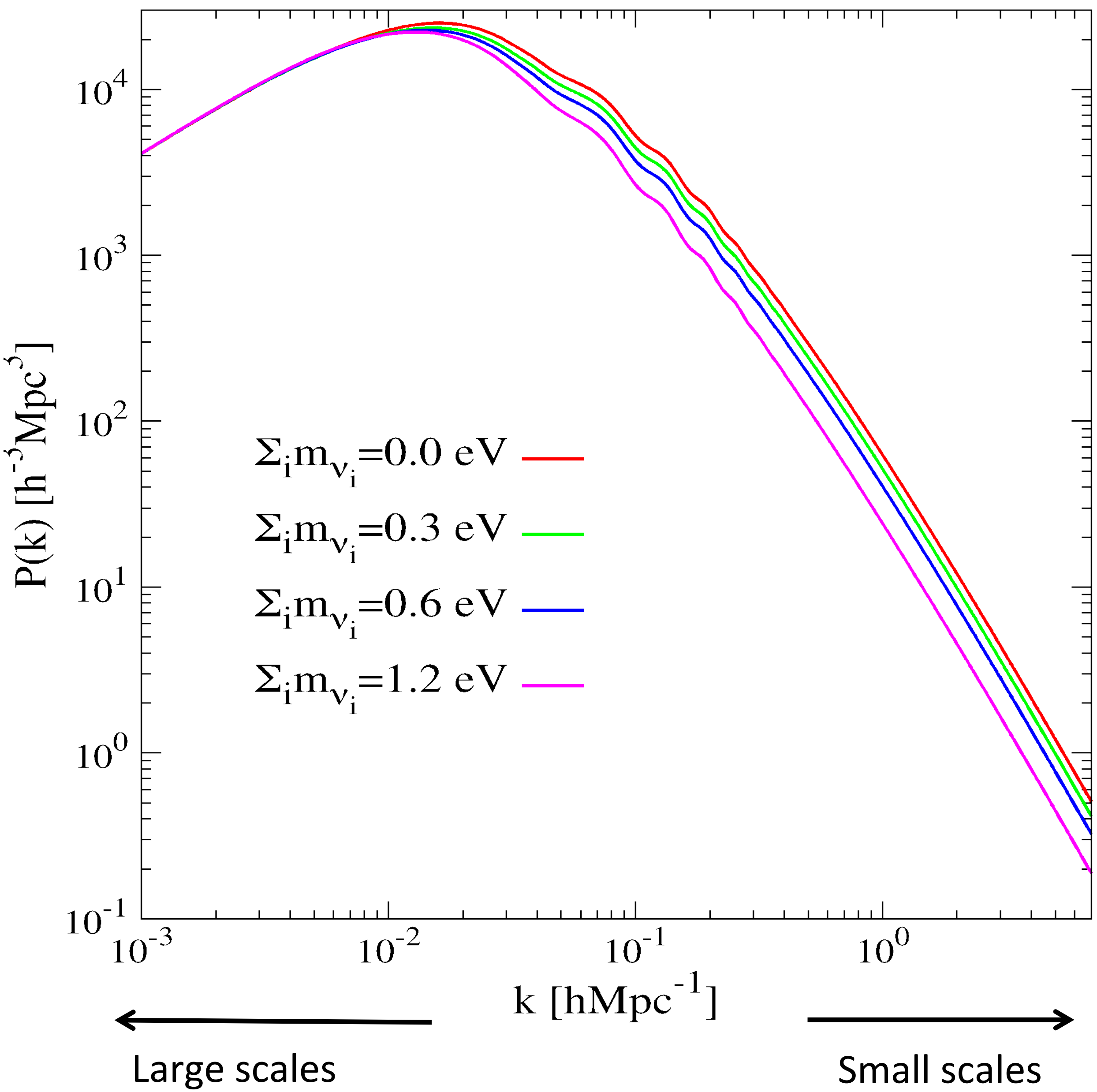
2dF survey

15000 degrees^2

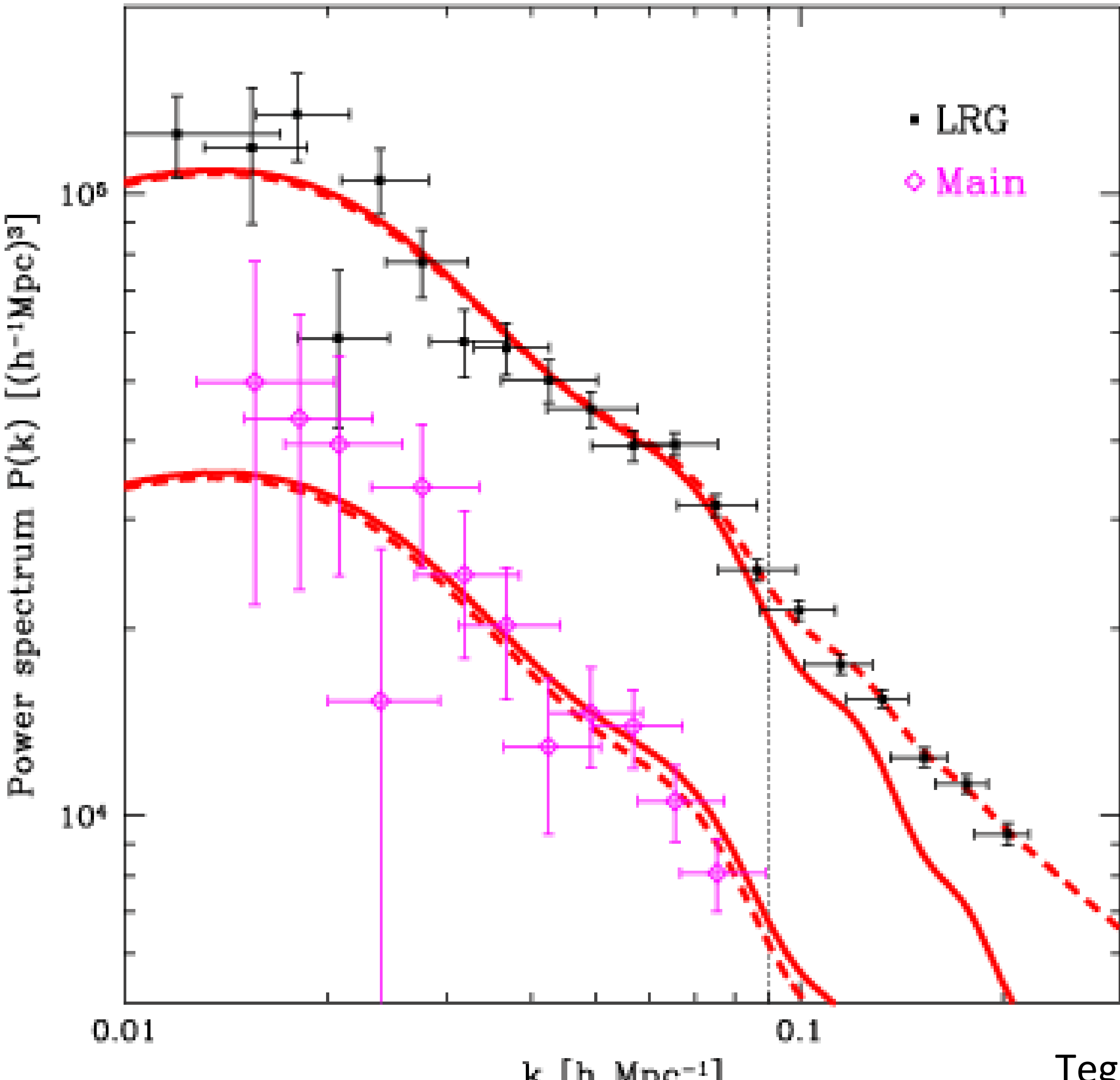
V ~ 0.1 (Gpc/h)^3

~250000 spectroscopic redshifts

Density field tracers

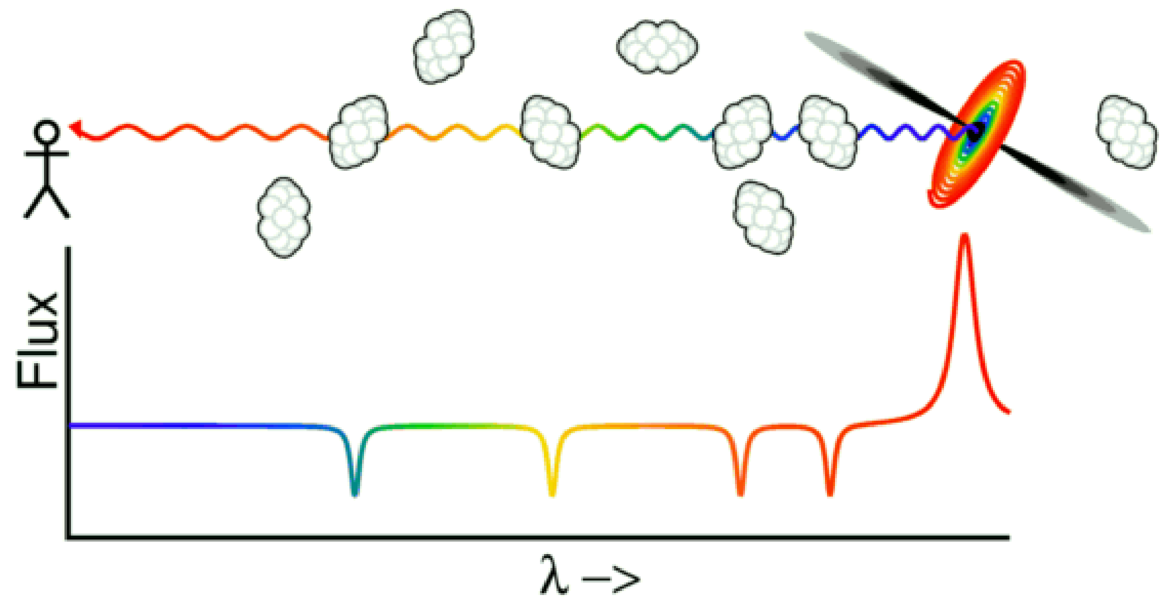


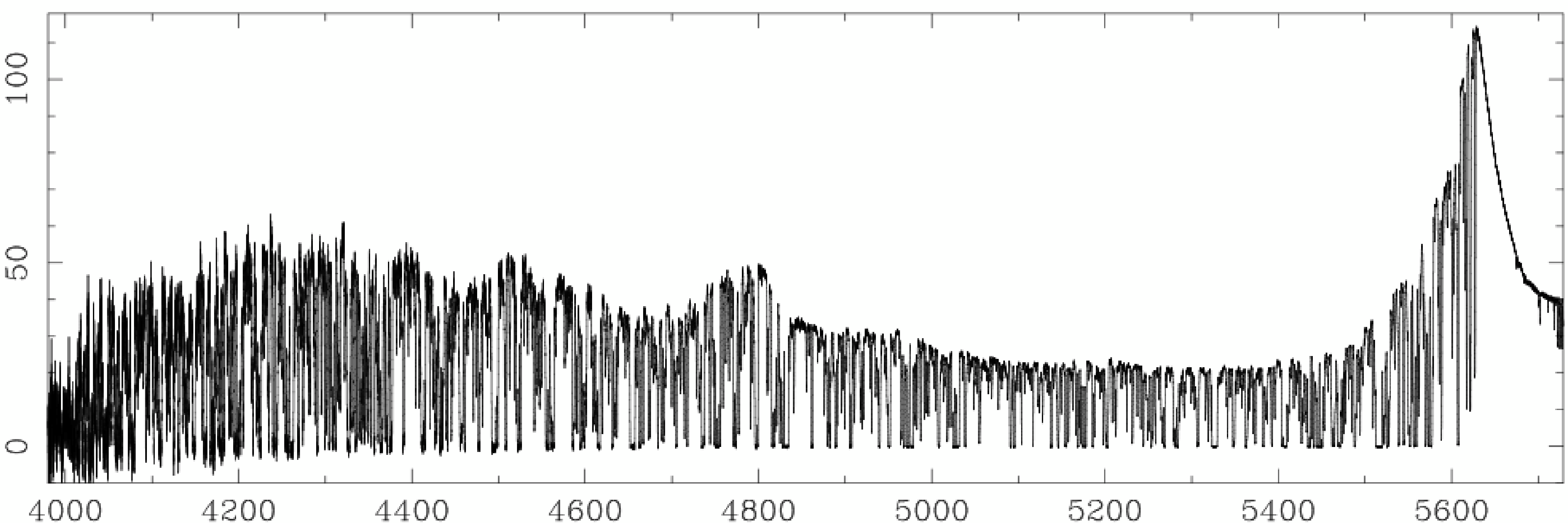
Density field tracers



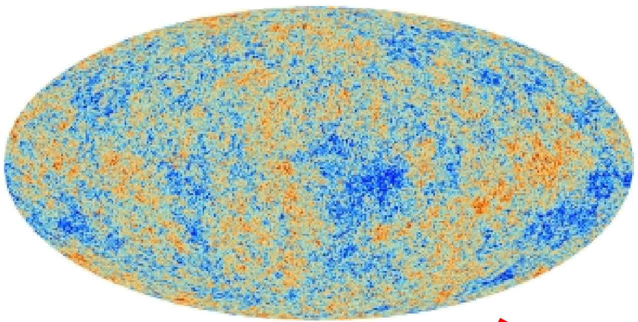
Tegmark et al. 2003

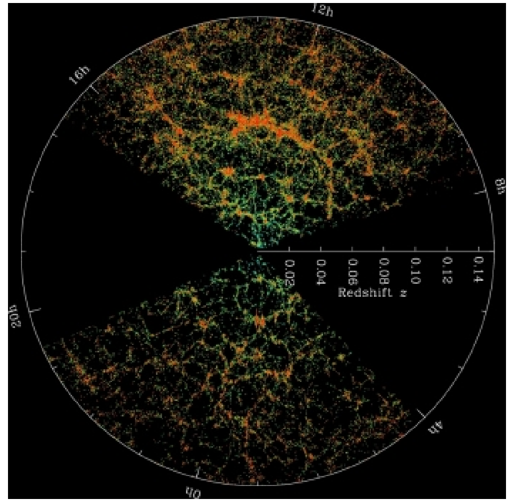
Density field tracers: Lya-forest

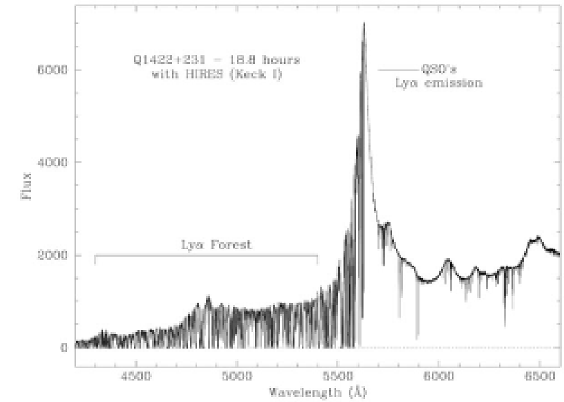




Cosmological constraints







$$\sum m_\nu < 0.72 \text{ eV (95\% C.L.)}$$

Planck TT + lowP
Planck collaboration 2015

$$\sum m_\nu \leq 0.21 \text{ eV (95\% C.L.)}$$

Planck TT + lowP + BAO
Planck collaboration 2015

$$\sum m_\nu \leq 0.13 \text{ eV (95\% C.L.)}$$

Planck TT + lowP + BAO + Lya
Palanque-Delabrouille et al. 2015

Remember: model dependent!!!

Cosmological constraints

$$0.01\ \text{eV} < \sigma\left(\Sigma m_\nu\right) < 0.1\ \text{eV}$$

DESI – LSST – Euclid – Planck,…etc
Font-Ribera et al. 2014

$$\sigma\left(\Sigma m_\nu\right) \approx 0.01\ \text{eV}$$

Euclid clusters + Planck
Sartoris et al. 2015

$$\sigma\left(\Sigma m_\nu\right) \approx 0.03\ \text{eV}$$

SKA + Planck
Villaescusa-Navarro et al. 2015

Parameter estimation

Data $\begin{cases} \vec{x} = \{P(k_1,z_i),P(k_2,z_i),P(k_3,z_i),\ldots,P(k_1,z_j),P(k_2,z_j),P(k_3,z_j),\ldots\} \\ \\ \vec{\theta} = \{\Omega_m,\Omega_b,\Omega_\Lambda,h,\sigma_8,n_s,\sum m_\nu,\ldots,\} \end{cases}$ Model parameters

$$\mathscr{P}(\vec{\theta}\,|\,\vec{x}) = \frac{\mathscr{P}(\vec{\theta})\mathscr{P}(\vec{x}\,|\,\vec{\theta})}{\mathscr{P}(\vec{x})}$$

Assumption!!!

Data are Gaussian distributed

Gaussian likelihood function

$$\mathscr{L} = \frac{1}{(2\pi)^{n/2}\,|C\,|^{1/2}}\exp\left(-\frac{1}{2}\sum_{ij}(\vec{x}-\vec{x}^{th})_i C_{ij}^{-1}(\vec{x}-\vec{x}^{th})_j\right)$$

Best fit and confident intervals typically found through MCMC

Parameter estimation

Assumption!!!

Gaussian likelihood function

$$\mathcal{L} = \frac{1}{(2\pi)^{n/2}\,|C|^{1/2}} \exp\left(-\frac{1}{2} \sum_{ij} (\vec{x} - \vec{x}^{\,th})_i C^{-1}_{ij} (\vec{x} - \vec{x}^{\,th})_j \right)$$

Most of the work in cosmology has been to accurately predict

Importance of the covariance?

- Not known theoretically
- Need to be estimated somehow

Parameter estimation

Assumption!!!

Gaussian likelihood function

$$\mathcal{L} = \frac{1}{(2\pi)^{n/2}\,|C|^{1/2}} \exp\left(-\frac{1}{2} \sum_{ij} (\vec{x} - \vec{x}^{th})_i C^{-1}_{ij} (\vec{x} - \vec{x}^{th})_j \right)$$

- ~~Internal methods~~
  - ~~Jackknife~~
  - ~~Bootstrap~~
- Mocks

Norberg et al. 2009

Parameter estimation

Assumption!!!

Gaussian likelihood function

$$\mathcal{L} = \frac{1}{(2\pi)^{n/2} |C|^{1/2}} \exp\left(-\frac{1}{2} \sum_{ij} (\vec{x} - \vec{x}^{th})_i C_{ij}^{-1} (\vec{x} - \vec{x}^{th})_j \right)$$

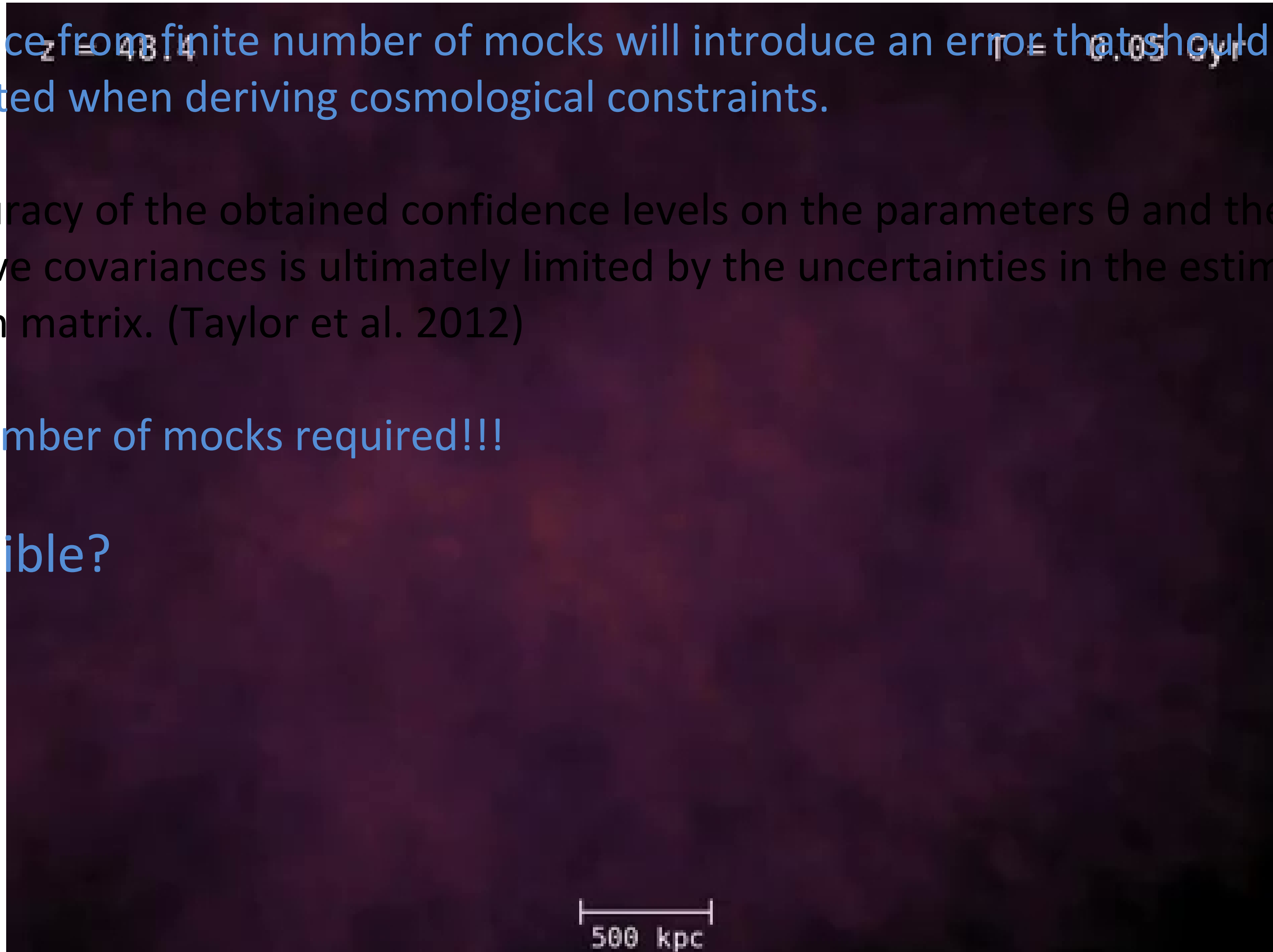
$$\hat{C} = \frac{1}{N-1} \sum_{i=1}^{N} \left(P(k_i) - \langle P(k_i) \rangle \right) \left(P(k_i) - \langle P(k_i) \rangle \right)^T$$

$$\hat{C}^{-1} = \qquad \hat{C}^{-1}$$

Hartlap et al. 2007

Underestimation of the size of the confidence regions

Parameter estimation



Covariance from finite number of mocks will introduce an error that should be propagated when deriving cosmological constraints.

The accuracy of the obtained confidence levels on the parameters θ and their respective covariances is ultimately limited by the uncertainties in the estimated precision matrix. (Taylor et al. 2012)

Large number of mocks required!!!

- Feasible?

Parameter estimation

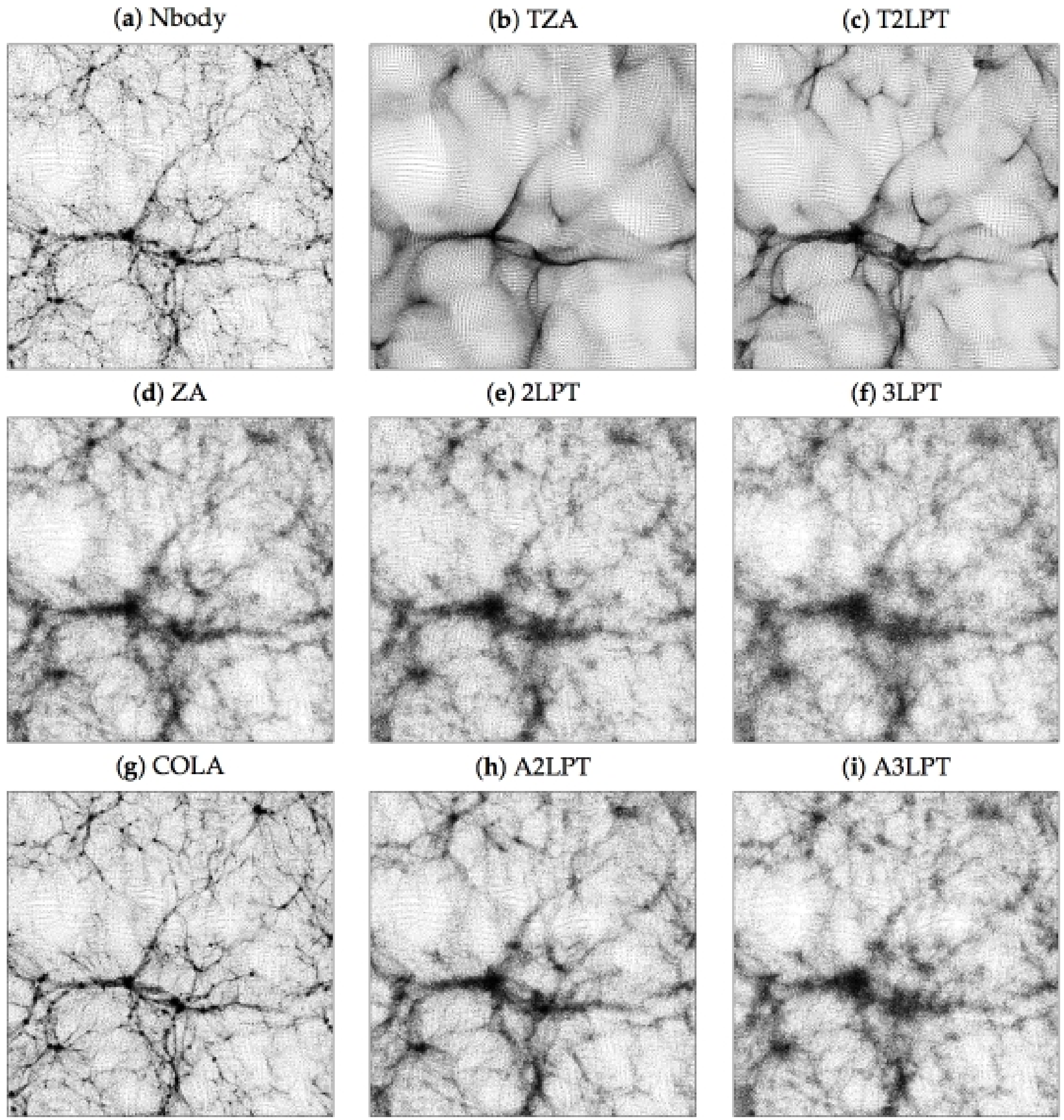


Figure 2. Large-scale structure in a 20 h^{-1} Mpc thick slice of a square of 128 h^{-1} Mpc, as recovered by different methods: **(a)** N-body simulation; **(b)** truncated ZA; **(c)** truncated 2LPT; **(d)** ZA; **(e)** 2LPT; **(f)** 3LPT; **(g)** COLA; **(h)** augmented 2LPT; **(i)** augmented 3LPT.

Monaco 2016

Parameter estimation

Assumption!!!

Gaussian likelihood function

$$\mathcal{L} = \frac{1}{(2\pi)^{n/2} |C|^{1/2}} \exp\left(-\frac{1}{2} \sum_{ij} (\vec{x} - \vec{x}^{th})_i C_{ij}^{-1} (\vec{x} - \vec{x}^{th})_j \right)$$

$$\hat{C} = \frac{1}{N-1} \sum_{i=1}^{N} \left(P(k_i) - \langle P(k_i) \rangle \right) \left(P(k_i) - \langle P(k_i) \rangle \right)^T$$

$$\hat{C}^{-1} = \frac{N-p-2}{N-1} \hat{C}^{-1}$$

Hartlap et al. 2007

Underestimation of the size of the confidence regions

$$\mathcal{L} = \frac{\bar{c}_p |C|^{-1/2}}{\left[1 + \frac{\sum_{ij} (\vec{x} - \vec{x}^{th})_i C_{ij}^{-1} (\vec{x} - \vec{x}^{th})_j}{N-1} \right]^{N/2}}$$

Estimated covariance matrix follows a Wishart distribution. Compute the likelihood by marginalising over the inverse-Wishart distribution of the true covariance matrix Σ, conditioned on C. This gives a modified multivariate t-distribution

Sellentin & Heavens 2016

Parameter estimation

$$\mathcal{L} = \frac{1}{(2\pi)^{n/2} |C|^{1/2}} \exp\left(-\frac{1}{2} \sum_{ij} (\vec{x} - \vec{x}^{th})_i C_{ij}^{-1} (\vec{x} - \vec{x}^{th})_j \right)$$

- We expect that the covariance matrix depends on the parameters!!!
- That dependence is neglected in current analysis
- It is feasible to estimate it on the whole parameter space via MCMC?

Parameter estimation

$$\mathcal{L} = \frac{1}{(2\pi)^{n/2} |C|^{1/2}} \exp\left(-\frac{1}{2} \sum_{ij} (\vec{x} - \vec{x}^{th})_i C_{ij}^{-1} (\vec{x} - \vec{x}^{th})_j \right)$$

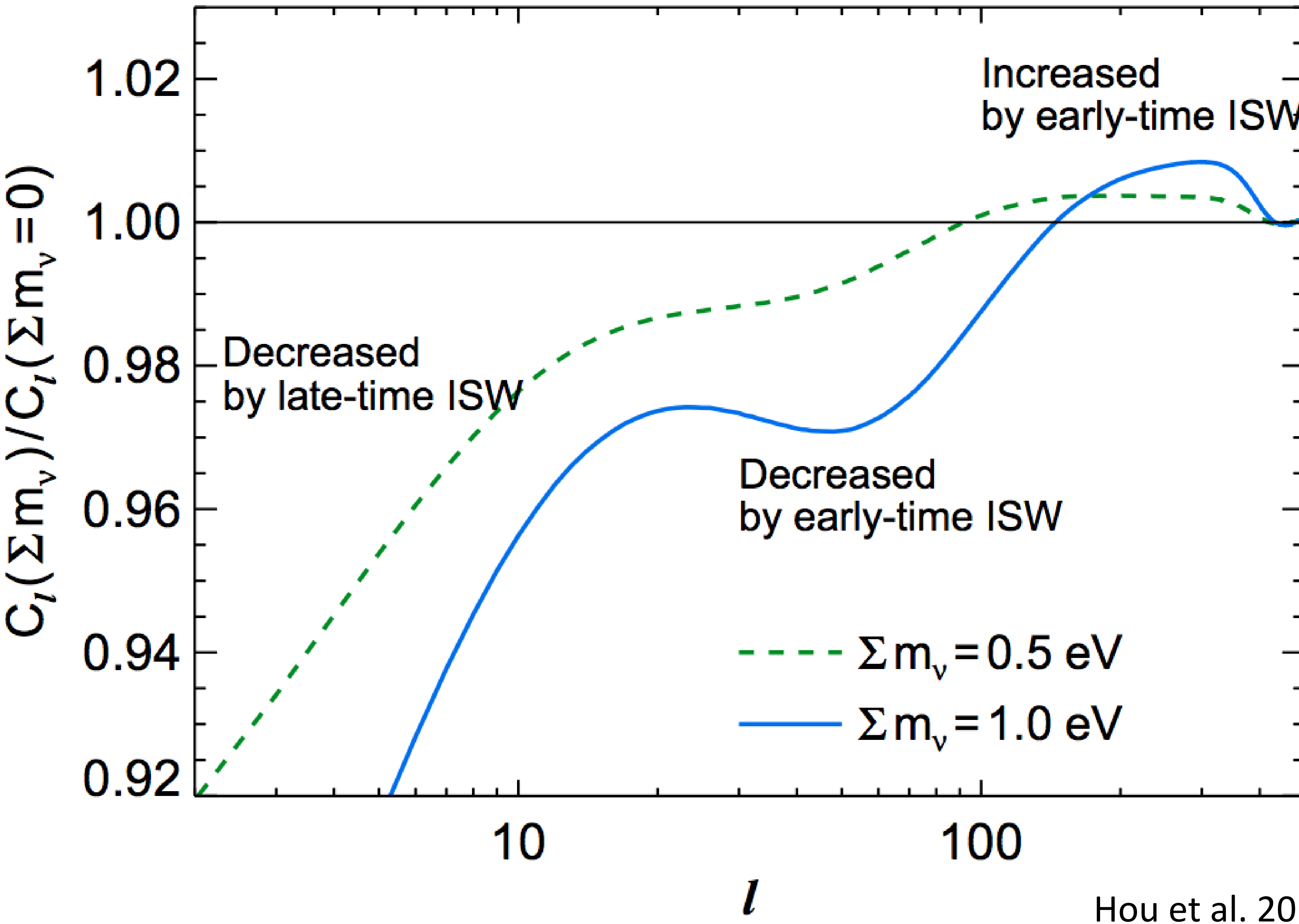
1. Can we keep describing the likelihood as a multivariate Gaussian?
2. Can we compute covariance matrices to percent accuracy on non-diagonal elements?
3. Can we neglect the dependence of covariance on parameters?

Conclusions

- Big Bang theory predicts existence of a CvB
- Massive neutrino effects well understood at linear order
- Use cosmological observables to weigh neutrinos
- Challenges for subpercent cosmology:

  covariance matrices

Neutrino effects on CMB



Hou et al. 2014

$$\mathcal{G}(\vec{x}|\vec{\mu}(\vec{p}), C) = \frac{1}{\sqrt{|2\pi C|}} \exp\left(-\frac{1}{2}(\vec{x} - \vec{\mu})^T C^{-1} (\vec{x} - \vec{\mu})\right)$$

$$S = \frac{1}{N-1} \sum_{i=1}^{N} (\vec{x}_i - \bar{\vec{x}})(\vec{x}_i - \bar{\vec{x}})^T$$

→ C is not a random object – the estimator S is

→ $C^{-1} = \alpha \langle S^{-1} \rangle$ where $\alpha = \frac{N - p - 2}{N - 1}$

→ Hartlap et al. (2007): Gaussian with $C^{-1} \to \alpha S^{-1}$

But: Removing <> means ignoring the randomness of S!

From Elena Sellentin Talk
Sellentin & Heavens 2016

Hartlap et al.'s method from (2007):

$$C^{-1} \to \alpha S^{-1}$$

$$L(\vec{x}|\vec{\mu}, S) = \int \mathcal{G}(\vec{x}|\vec{\mu}, C)\underline{\underline{\mathcal{P}(C|S, N)}}\mathrm{d}C$$ ← matrix integration

matrix-valued prob. dens. func.

- $\mathcal{P}(S|C) = \text{Wishart distribution}$
- $\pi(C) \propto |C|^{-(p+1)/2}$

$$\to \mathcal{T}(\vec{x}|\vec{\mu}, S) \propto \frac{|S|^{-1/2}}{\left[1 + \frac{(\vec{x}-\vec{\mu})^T S^{-1} (\vec{x}-\vec{\mu})}{N-1}\right]^{\frac{N}{2}}}$$

Eq. (12) in arXiv 1511:05969 (changes 3 lines of codes)

From Elena Sellentin Talk
Sellentin & Heavens 2016