Status of global sterile neutrino fits

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Outline

- Introduction to sterile neutrinos
- Introduction to global fit methods
- Recent results from our global fits
- Problems in interpreting global fit results

Neutrino oscillation

- According to the standard model, neutrinos come in three different ‘flavours’:
  - Electron neutrino ν_e

  - Muon neutrino ν_μ

  - Tau neutrino ν_τ

- Neutrinos can change flavour through oscillation.

Neutrino oscillation

- In a model with two flavours, the oscillation probability is:

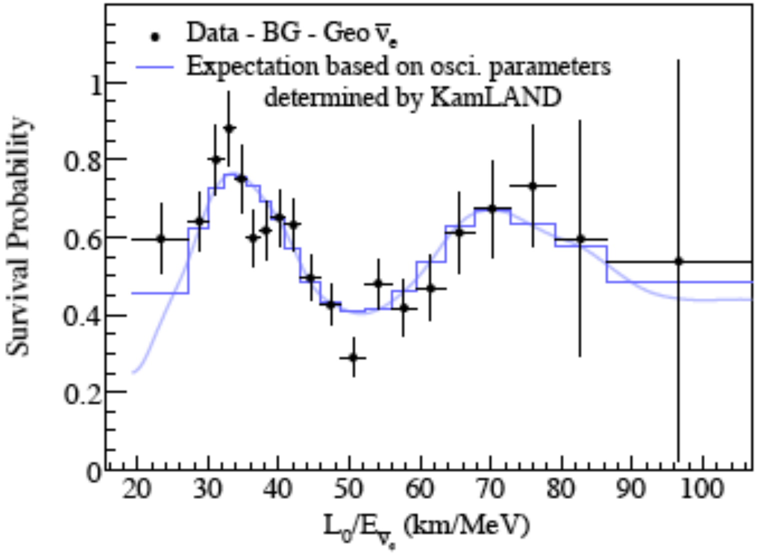
$$P(\nu_a \to \nu_b) = \delta_{ab} - \sin^2(2\theta_{ab}) \sin^2\left([1.27 \text{ GeV eV}^{-2} \text{ km}^{-1}]\Delta m^2 \frac{L}{E}\right)$$

The mixing angle (amplitude)

The mass splitting

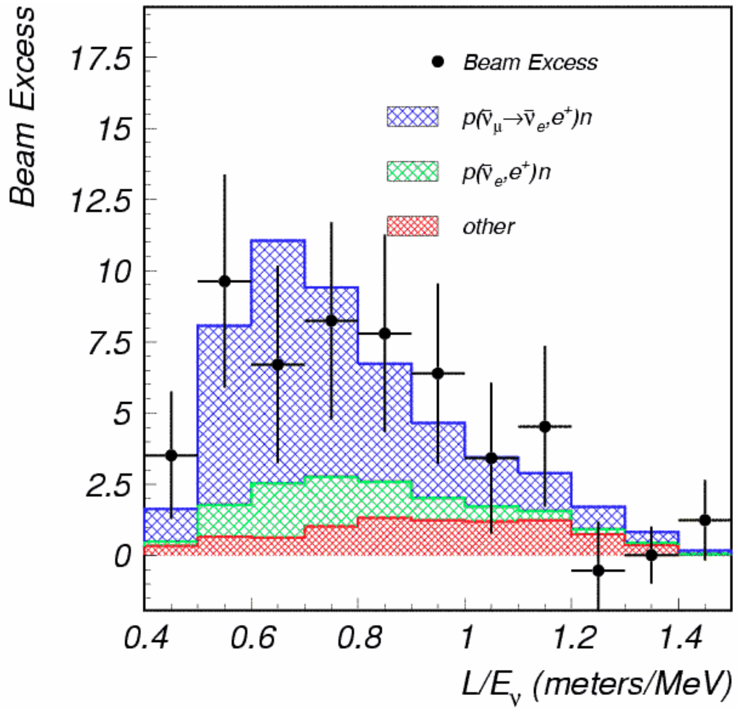
$$\Delta m^2 = |m_1^2 - m_2^2|$$

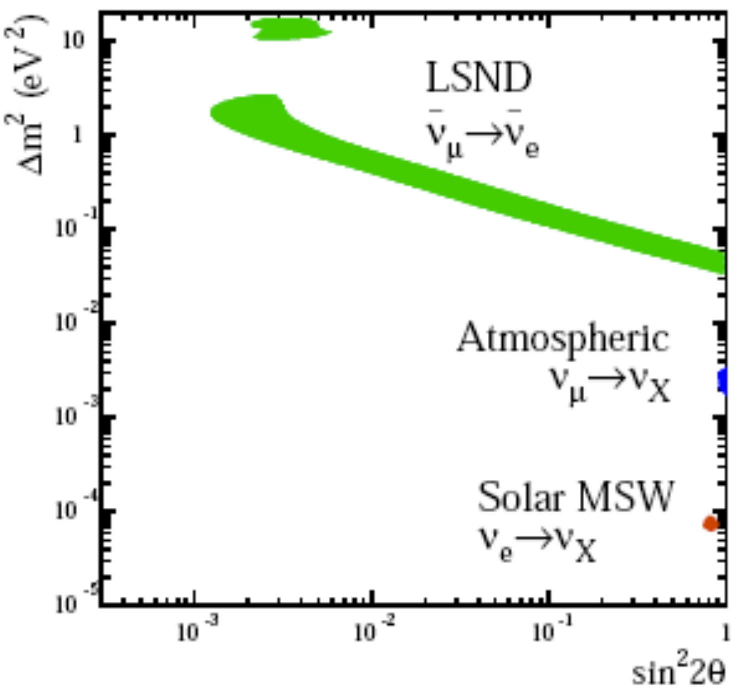
(frequency)



The LSND anomaly

- An excess of $\bar{\nu}_e$ was seen (above background).
- 3.8σ signal.





Simple view of short baseline oscillation

- Standard three flavour oscillation cannot occur over these distances and energies.
  - The oscillation length is much longer.

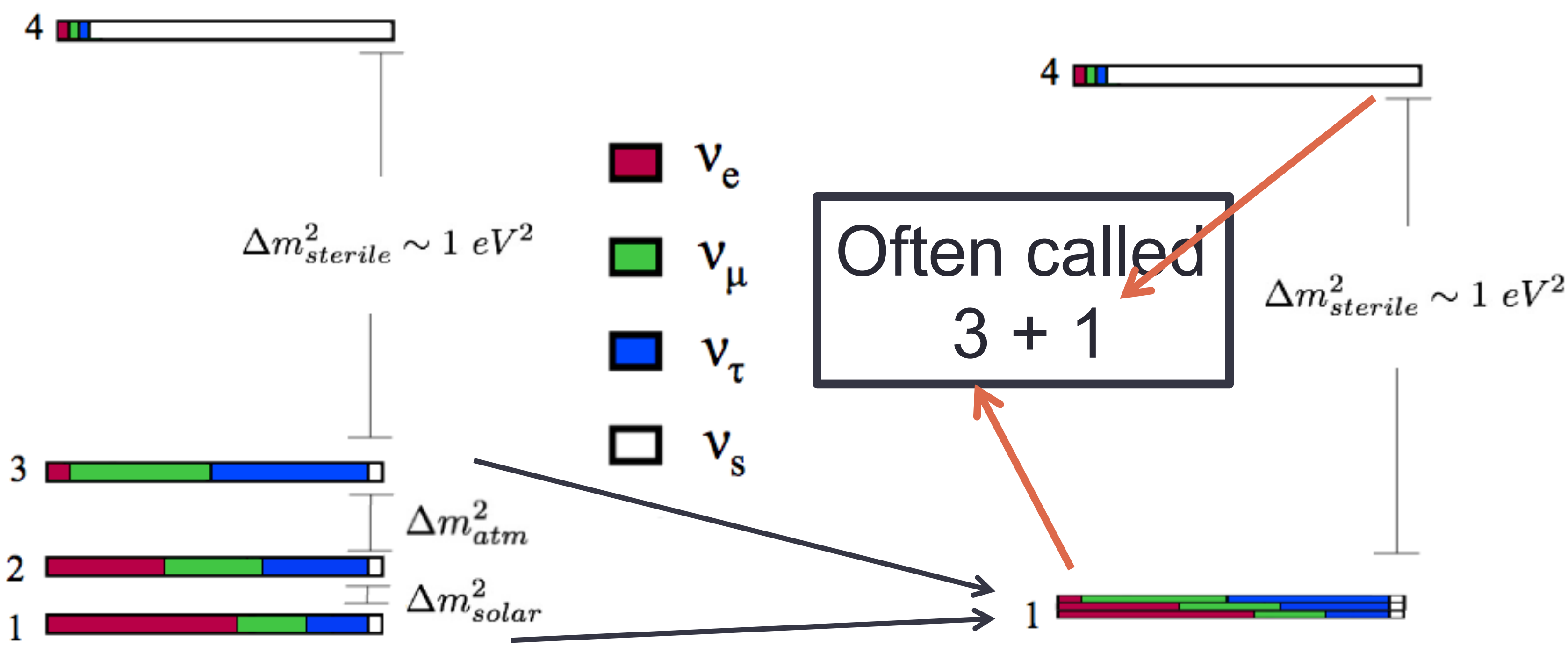
$$\nu_\mu \not\longrightarrow \nu_e$$

- But they can oscillate through a forth kind of neutrino.
  - Which has a shorter oscillation length.

$$\nu_\mu \longrightarrow \nu_s \longrightarrow \nu_e$$

Sterile neutrino models

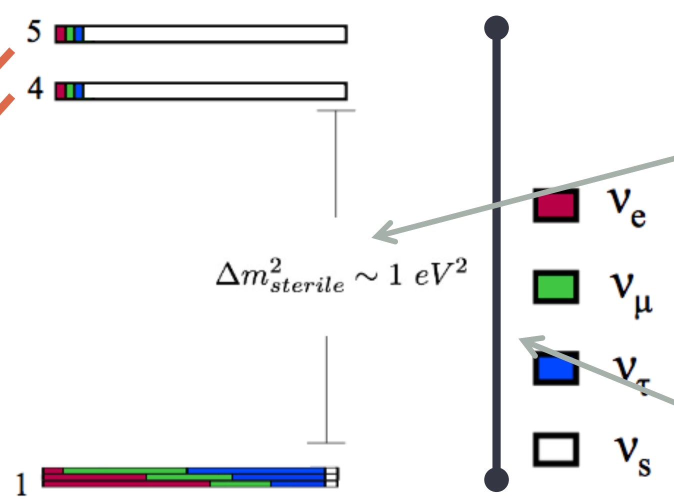
- We can test many different models.
  - 3 + N: 3 degenerate active neutrinos + N heavier sterile neutrinos.
- 3+1 has parameters: $\Delta m^2_{41}, |U_{e4}|, |U_{\mu 4}|$



Sterile neutrino models

- 3+2:
  - Introduces a CP violating phase.
  - Allows differences between neutrino and anti-neutrino oscillation.

Often called
3 + 2



Parameters: Φ_{45},

$\Delta m^2_{41}, |U_{e4}|,$

$|U_{\mu 4}|, |U_{\tau 4}|,$

$\Delta m^2_{51}, |U_{e5}|,$

$|U_{\mu 5}|, |U_{\tau 5}|$

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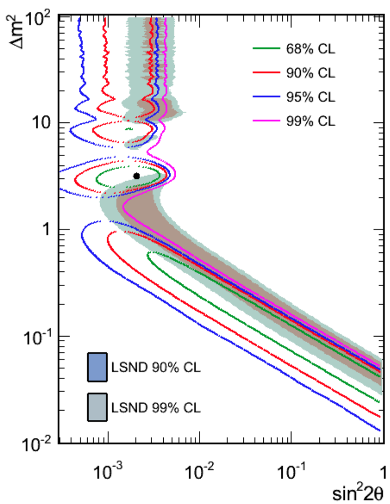
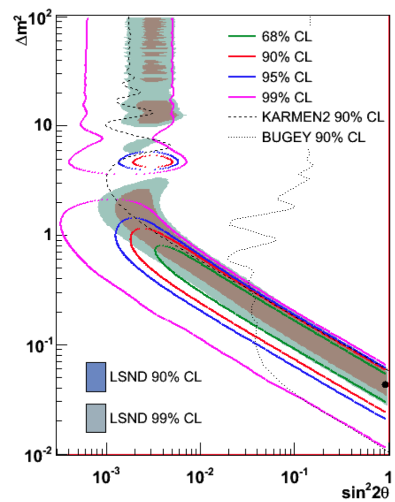
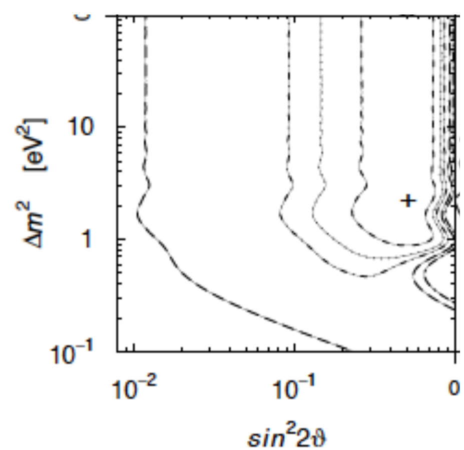
Global data

- LSND was not the only experiment to see an anomaly.
  - Many electron neutrino appearance experiments
    - Which search for muon neutrinos oscillating into electron neutrinos
  - And electron neutrino disappearance experiments
    - Which search for electron neutrinos ‘disappearing’ as they oscillating into other kinds of neutrinos
  - See anomalies.

MiniBooNE



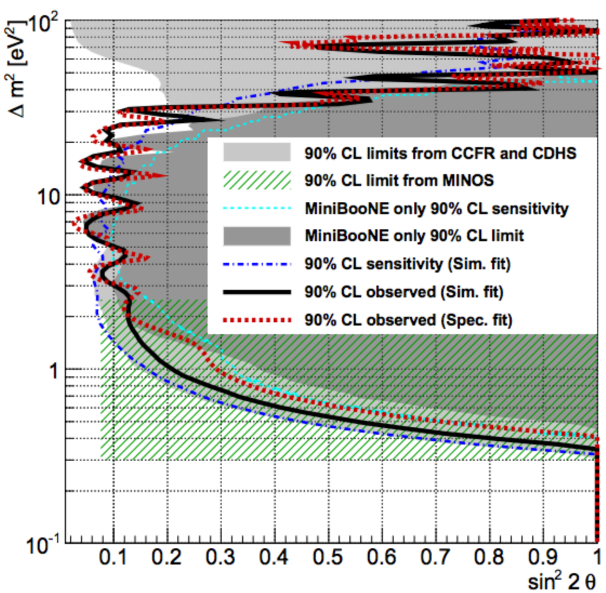


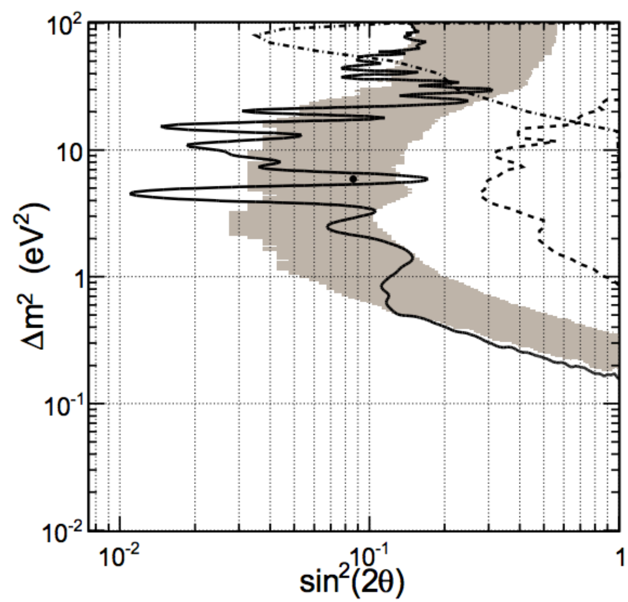


Gallex/SAGE

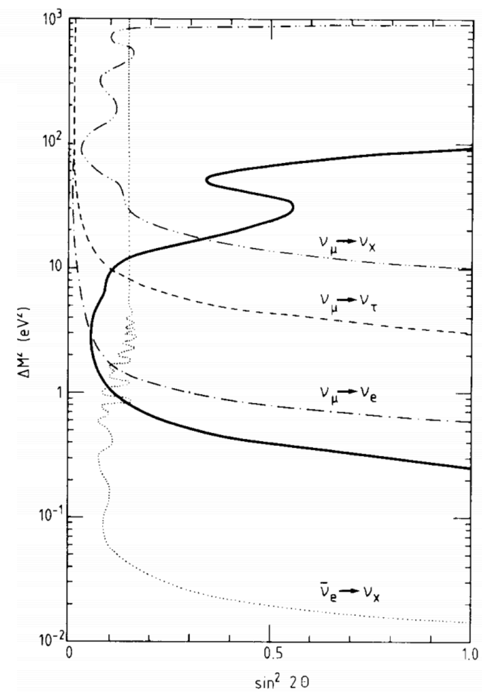
Global data

- However, muon neutrino disappearance experiments have seen no evidence of sterile neutrino anomalies.





MiniBooNE/SciBooNE



CDHS

Global fits

- How do we make sense of all these different datasets?
  - In which some seem to conflict with others.

- We performed a combined fit to all the datasets at once.
  - A so called 'global fit'.

- LSND $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
- MiniBooNE $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$
  $\nu_\mu \rightarrow \nu_e$
- NuMI in MB $\nu_\mu \rightarrow \nu_e$
- NOMAD $\nu_\mu \rightarrow \nu_e$
- KARMEN $\nu_\mu \rightarrow \nu_e$

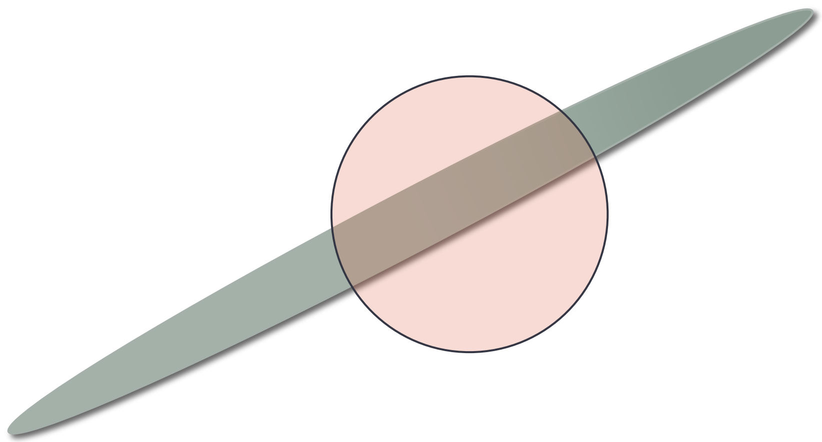
- MINOS CC $\bar{\nu}_\mu \nrightarrow \bar{\nu}_\mu$
- SciBooNE/MiniBooNE $\bar{\nu}_\mu \nrightarrow \bar{\nu}_\mu$
  $\nu_\mu \nrightarrow \nu_\mu$
- CCFR84 $\nu_\mu \nrightarrow \nu_\mu$
- CDHS $\nu_\mu \nrightarrow \nu_\mu$
- Bugey $\bar{\nu}_e \nrightarrow \bar{\nu}_e$
- GALLEX/SAGE $\nu_e \nrightarrow \nu_e$
- KARMEN/LSND x-sec $\nu_e \nrightarrow \nu_e$

Global fit method

- We compute a single objective function, the likelihood.
  - In the frequentist case, this is $\ln \mathcal{L} = -\frac{1}{2}\chi^2$
- This function requires computing expected neutrino fluxes for the model parameters in all experiments, which is expensive.
  - Thus it is not possible to scan over the entire model parameter space.
- Require a more efficient sampling of the parameter space.
  - We use a Markov Chain Monte Carlo (MCMC).

Affine transforms

- Inefficiencies result when actual and proposal distributions do not match.



- This would not be an issue if the algorithm was affine invariant.
- We use the parallel tempering affine invariant algorithm from the Emcee package.
  - arXiv:1202.3665
  - (Re-implemented in C++ for our purposes)

Affine invariant

- Instead of specifying a proposal distribution a priori, draw the new proposal form the current distribution of walkers.
  - For each walker with parameters x:
    - Select a target walker in the same ensemble with parameters x’
    - The new proposed parameters are

$$\vec{x}^{i} = (\vec{x}')^{i-1} + z[\vec{x}^{i-1} - (\vec{x}')^{i-1}]$$

    - Where z is selected from the distribution

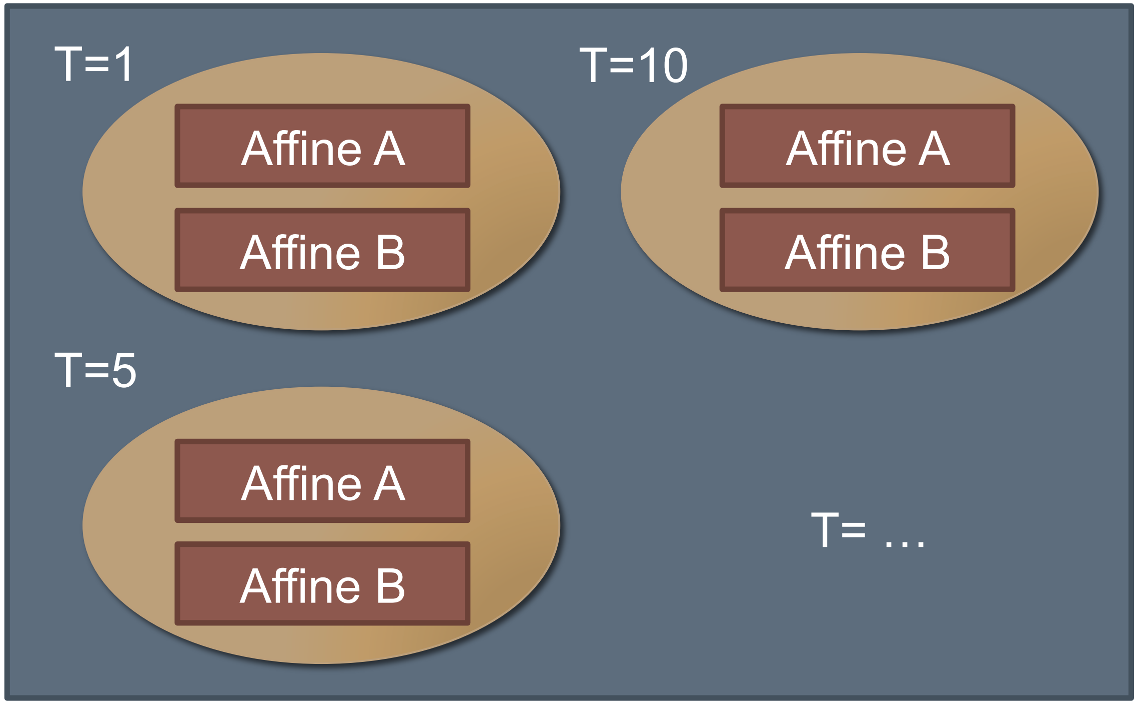
$$Pr(z) = \frac{1}{\sqrt{z}} \qquad z \in \left[\frac{1}{a}, a\right]$$

    - Where a is called the ‘affine parameter’ (usually = 2)
- In the parallel version, where updates happen simultaneously, the is an additional constraint:
  - Each ensemble is divided into two sub-ensembles and walkers from one ensemble select target walkers in the other.
  - Sub-ensembles are then updated sequentially.

Affine invariant parallel tempering MCMC

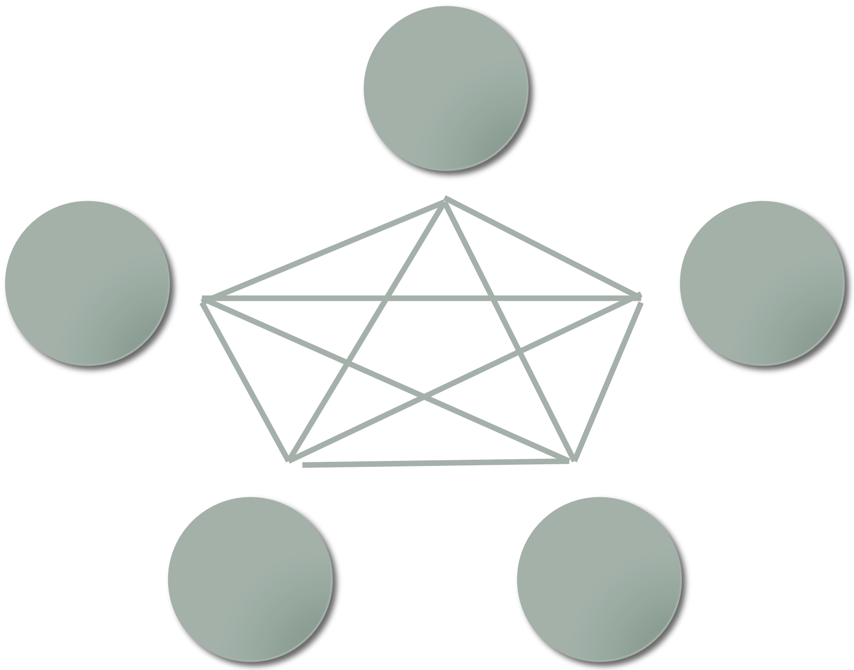
- Multiple ensembles are run in parallel using the standard parallel tempering algorithm.

$$\ln \mathcal{L}(\vec{x})_{\mathrm{PT}} = e^{-\beta \ln \mathcal{L}(\vec{x})_{\mathrm{AI}}} \qquad \beta = 1/T$$



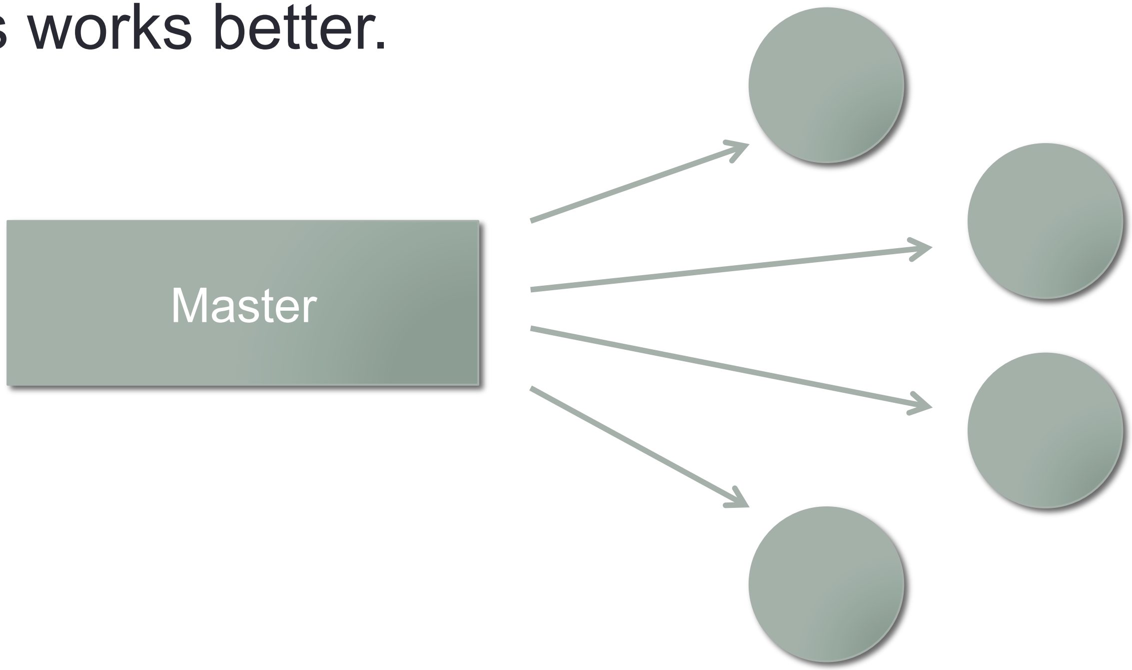
Distributed computing

- MCMC ensembles running in parallel can be distributed amoung computers in a cluster.
- Standard methods of parallelisation (eg: MPI) results in ~N^2 communication problem.



Distributed computing

- We found that a single ‘master’ process that runs the MCMC and distributes likelihood evaluations to ‘worker’ nodes works better.



- This also behaves well on clusters where resources are allocated opportunistically.

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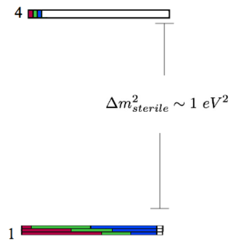
3 + 1



Best fit point:

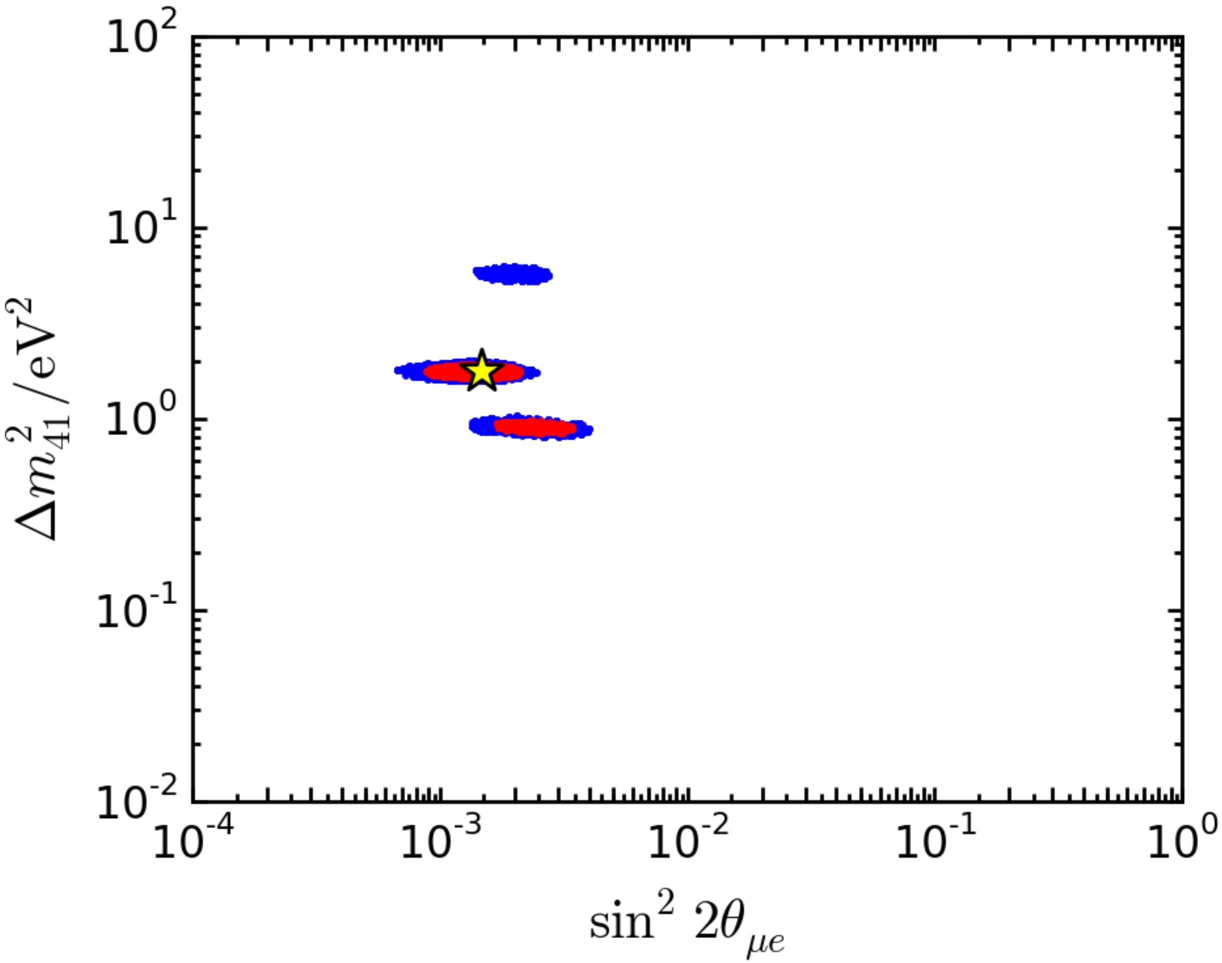
$$\Delta m^2_{41} : 1.75\ \mathrm{eV}^2$$

$$\sin^2 2\theta_{\mu e} : 1.45 \times 10^{-3}$$

$$\chi^2 : 306.81 \quad (312\ \mathrm{dof})$$

$$\chi^2_{\mathrm{null}} : 359.15 \quad (315\ \mathrm{dof})$$

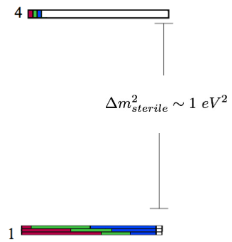
$$\Delta\chi^2 = 52.34 \quad (3\ \mathrm{dof})$$



Red: 90% CL
Blue: 99% CL

arXiv:1602.00671

3 + 1



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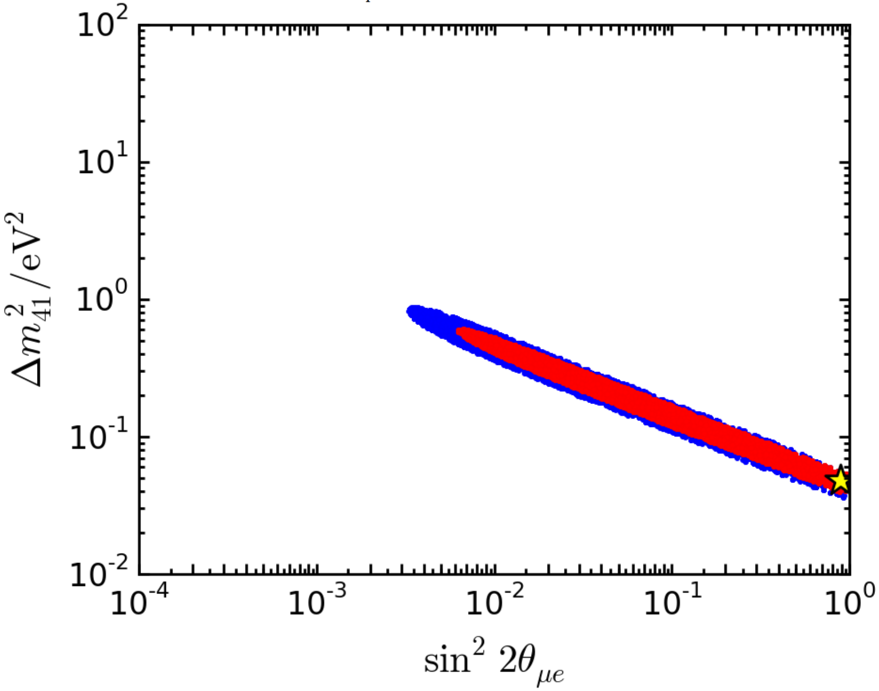
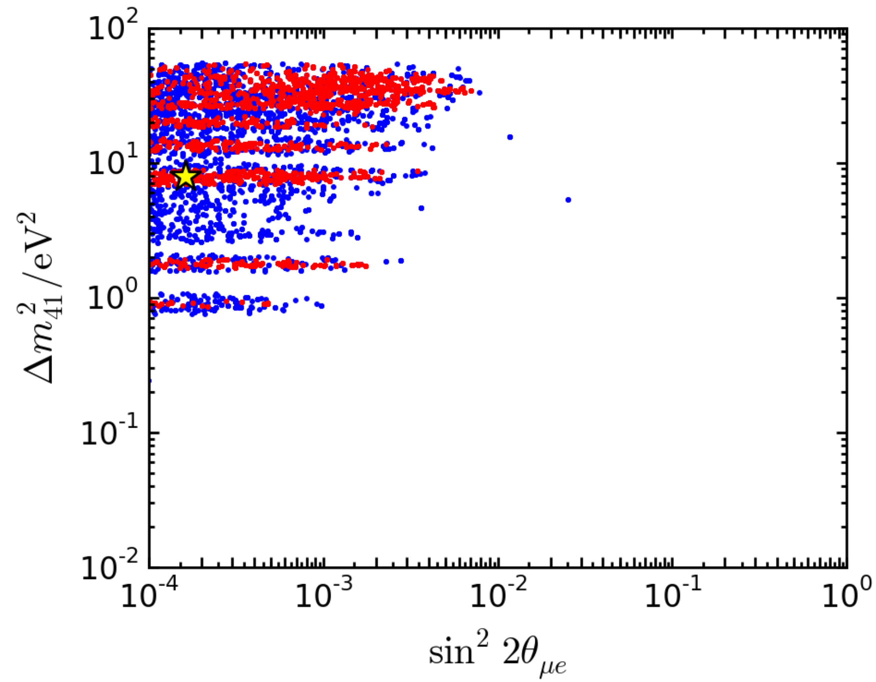


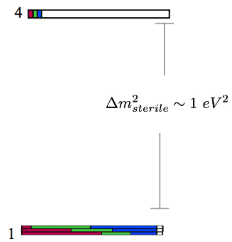
Appearance only datasets



Disappearance only datasets

Disagreement between these datasets.

3 + 1



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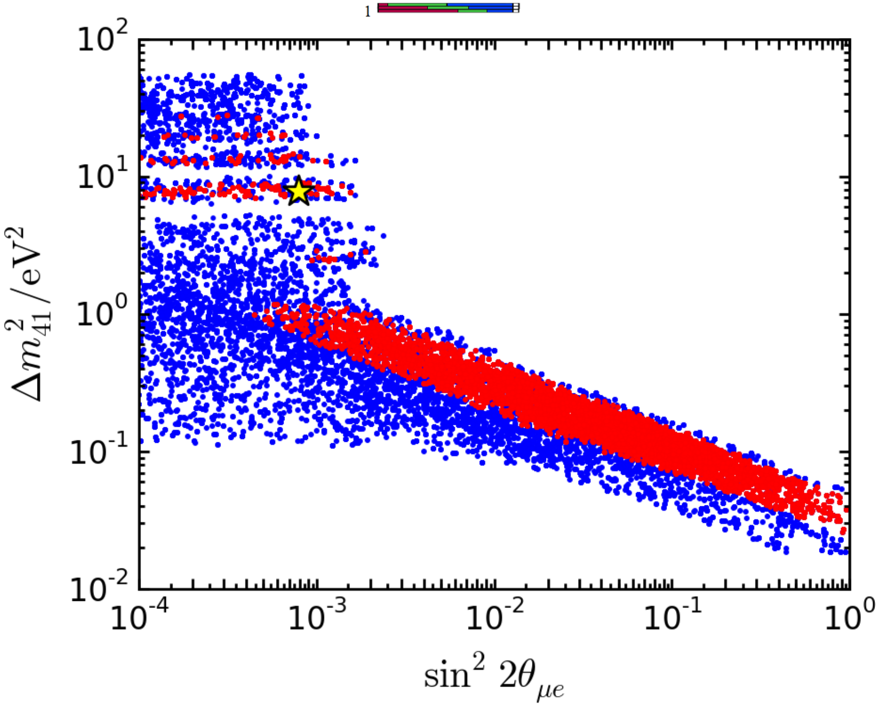
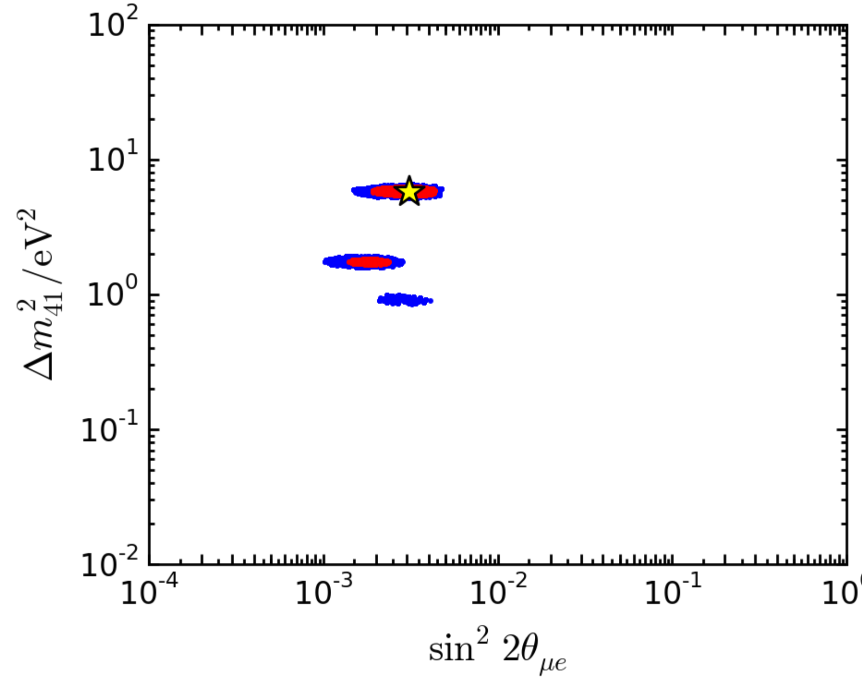


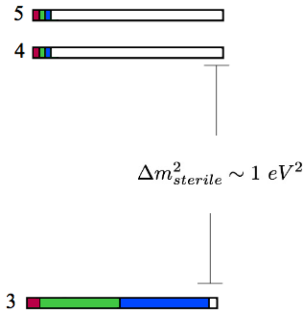
Neutrino only datasets



Anti-neutrino only datasets

Disagreement between these datasets.

3 + 2

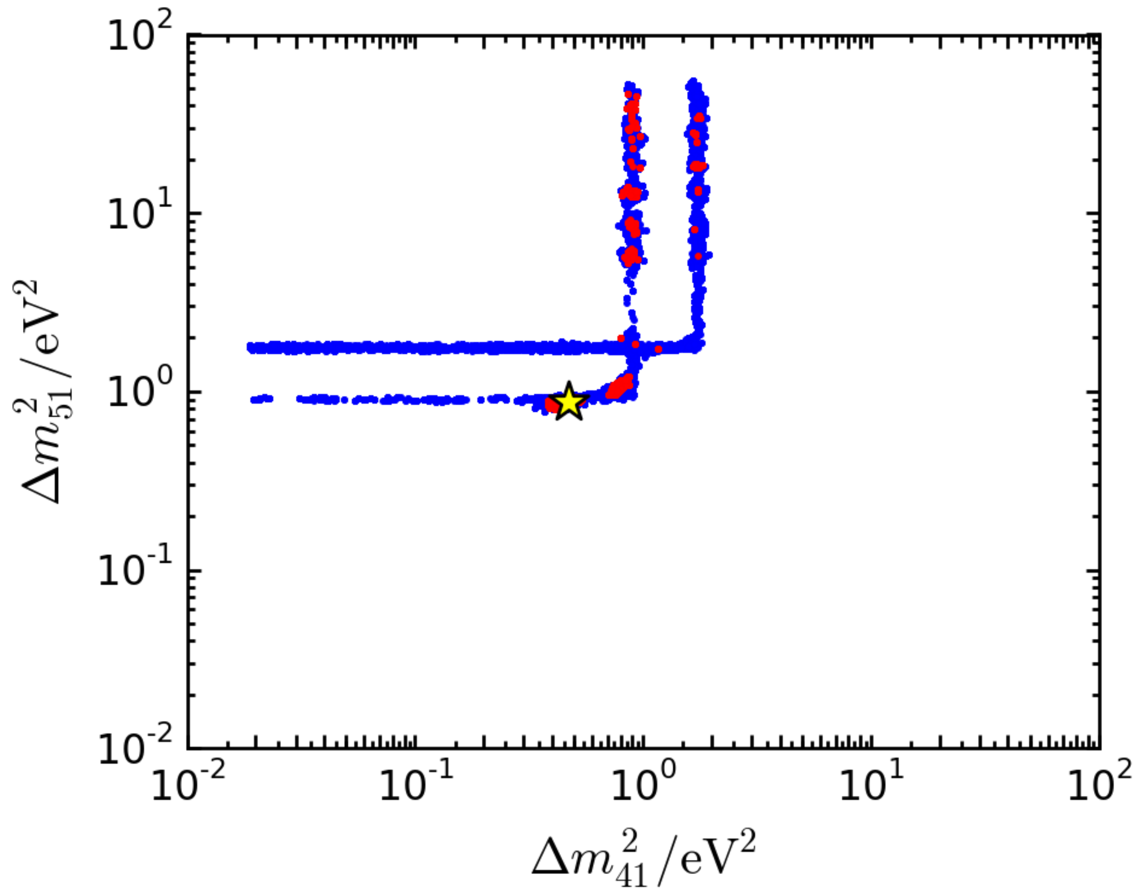


Best fit point:

$$\chi^2 : 302.16 \quad (308 \text{ dof})$$

$$\chi^2_{\text{null}} : 359.15 \quad (315 \text{ dof})$$

$$\Delta\chi^2 = 57 \quad (7 \text{ dof})$$



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arXiv:1602.00671

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Parameter goodness-of-fit (PG) test

arXiv.org > hep-ph > arXiv:hep-ph/0304176

High Energy Physics – Phenomenology

Testing the statistical compatibility of independent data sets

M. Maltoni, T. Schwetz

- Created to address the issue of insensitive bins.
- Is interpreted as a test of the compatibility of two data-sets to the predictions of a neutrino oscillation model.

Parameter goodness-of-fit (PG) test

$$\chi^2_{PG} = \chi^2_{glob} - (\chi^2_{app} + \chi^2_{dis})$$

$$N_{PG} = (N_{app} + N_{dis}) - N_{glob}$$
$$= 2 \qquad \text{(for 3+1)}$$

- No. of degrees of freedom is set by the model being tested.

2

Parameters:

$$P(\nu_\mu \to \nu_e) \simeq 4|U_{e4}|^2|U_{\mu 4}|^2 \sin^2(1.27\Delta m^2_{41} L/E)$$

$$P(\nu_e \to \nu_e) \simeq 1 - 4(1 - |U_{e4}|^2)|U_{e4}|^2 \sin^2(1.27\Delta m^2_{41} L/E)$$
$$P(\nu_\mu \to \nu_\mu) \simeq 1 - 4(1 - |U_{\mu 4}|^2)|U_{\mu 4}|^2 \sin^2(1.27\Delta m^2_{41} L/E)$$

3

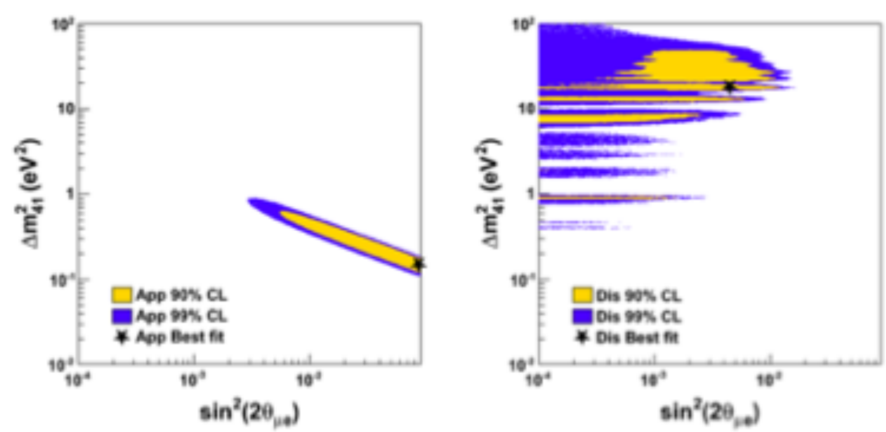
Parameter goodness-of-fit (PG) test

- The PG test observes a severe tension:

arXiv:1207.4765

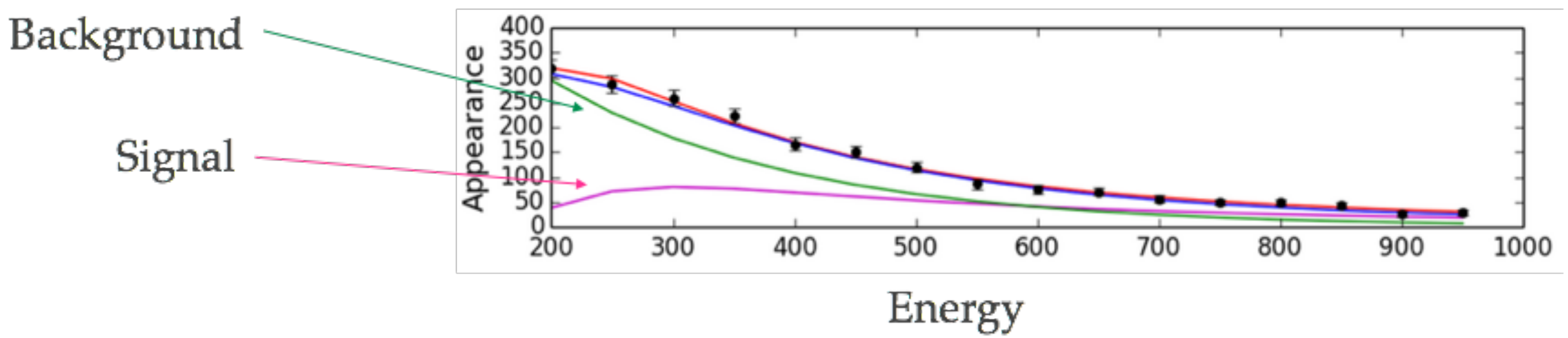
		χ^2_{PG} (dof)	PG(%)
3+1	ν vs. $\bar{\nu}$	15.6 (3)	0.14%
	App vs. Dis	17.8 (2)	0.013%
3+2	ν vs. $\bar{\nu}$	13.9 (7)	5.3%
	App vs. Dis	23.9 (4)	0.0082%
3+3	ν vs. $\bar{\nu}$	10.9 (12)	53%
	App vs. Dis	27.1 (6)	0.014%

What is the effect of nuisance parameters on the PG test?



Nuisance parameters in a toy model

- 3+1 toy model is composed of:
  - Disappearance,
  - Appearance
    - With and without an unexpected background.



Toy model

- Throw many experiments
  - Data points are selected based on random distributions
  - Background is added to data points.
  - Chi^2 fit is performed for Appearance, Disappearance, and Global, with pull parameter (A) for background normalization.
    - For the case of background, $A_{\text{true}} = 0.4, A_{\text{expected}} = 0.0 \pm 0.15 \quad \rightarrow \quad \text{find } A_{\text{fit}}$
  - Calculate the PG

$$\chi^2_{\nu_e app} = \sum_{i=1}^{16} \frac{(d_i^{\nu_e app} - (osc_i^{\nu_e app} + b_i^{\nu_e app}(A_{fit})))^2}{(\sigma_i^{\nu_e app})^2} + \frac{(A_{fit} - A_{exp})^2}{\sigma^2_{A_{exp}}}$$

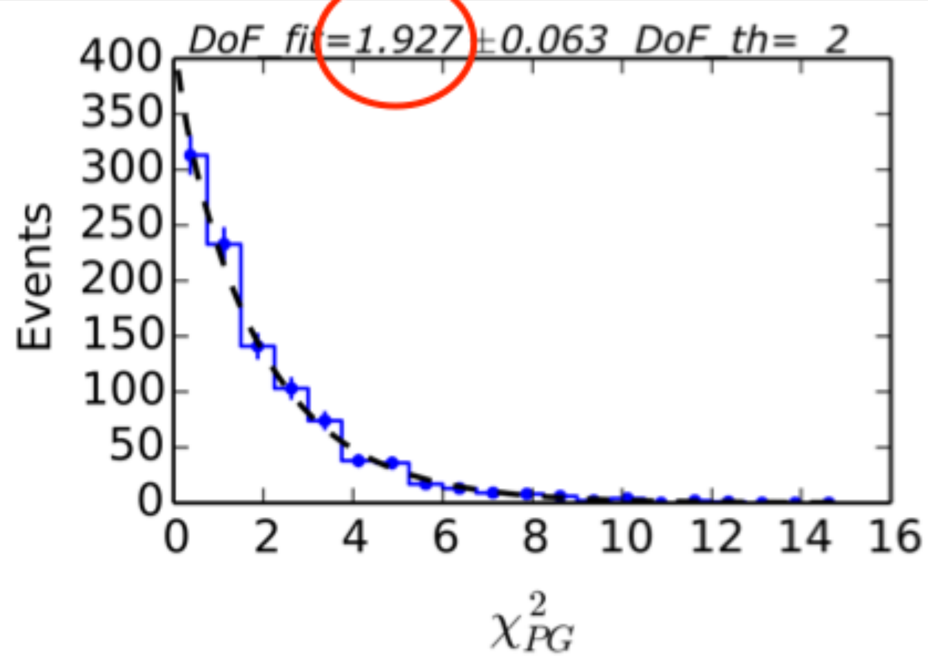
$$\chi^2_{disapp} = \sum_{i=1}^{16} \frac{\left(d_i^{\nu_\mu disapp} - osc_i^{\nu_\mu disapp}\right)^2}{\left(\sigma_i^{\nu_\mu disapp}\right)^2} + \sum_{i=1}^{16} \frac{\left(d_i^{\nu_e disapp} - osc_i^{\nu_e disapp}\right)^2}{\left(\sigma_i^{\nu_e disapp}\right)^2}$$

$$\chi^2_{global} = \sum_{i=1}^{16} \frac{(d_i^{\nu_e app} - (osc_i^{\nu_e app} + b_i^{\nu_e app}(A_{fit})))^2}{(\sigma_i^{\nu_e app})^2} + \frac{(A_{fit} - A_{exp})^2}{\sigma^2_{A_{exp}}} + \sum_{i=1}^{16} \frac{\left(d_i^{\nu_\mu disapp} - osc_i^{\nu_\mu disapp}\right)^2}{\left(\sigma_i^{\nu_\mu disapp}\right)^2} + \sum_{i=1}^{16} \frac{\left(d_i^{\nu_e disapp} - osc_i^{\nu_e disapp}\right)^2}{\left(\sigma_i^{\nu_e disapp}\right)^2}$$

χ^2 PG for many throws is histogrammed

Toy model, no background

- Background normalization (A_{true}) is set to zero.
- χ^2 PG distribution comes out with expected No. of degrees of freedom.
  - For 3+1, we expected 2.

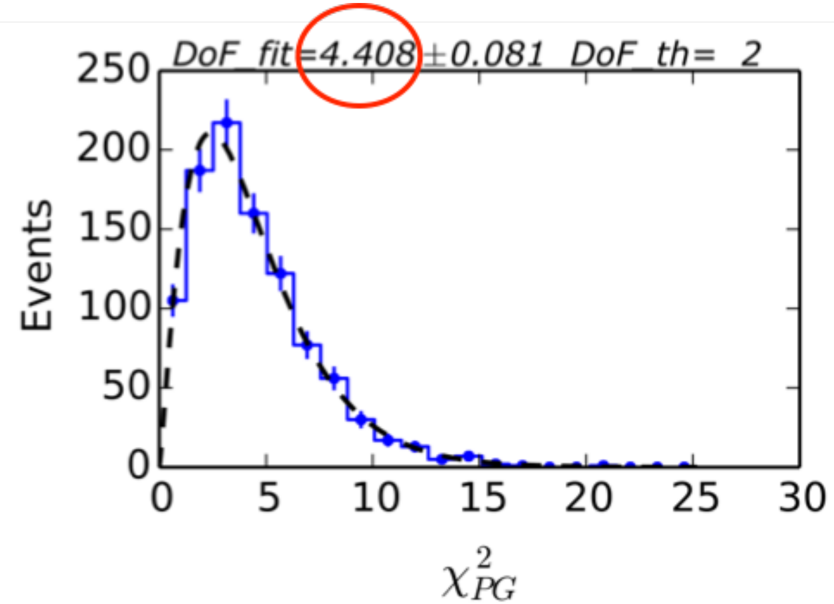


PG test works in this case

Toy model, with background

- Now,
  - $A_{\mathrm{true}} = 0.4$
  - $A_{\mathrm{expected}} = 0.0 \pm 0.15$

- χ^2 PG distribution now has an incorrect No. of degrees of freedom.



PG test fails in this case

Toy model conclusions

- Changing the scaling of the background model changes the d.o.f of the underlying χ^2_{PG} distribution.

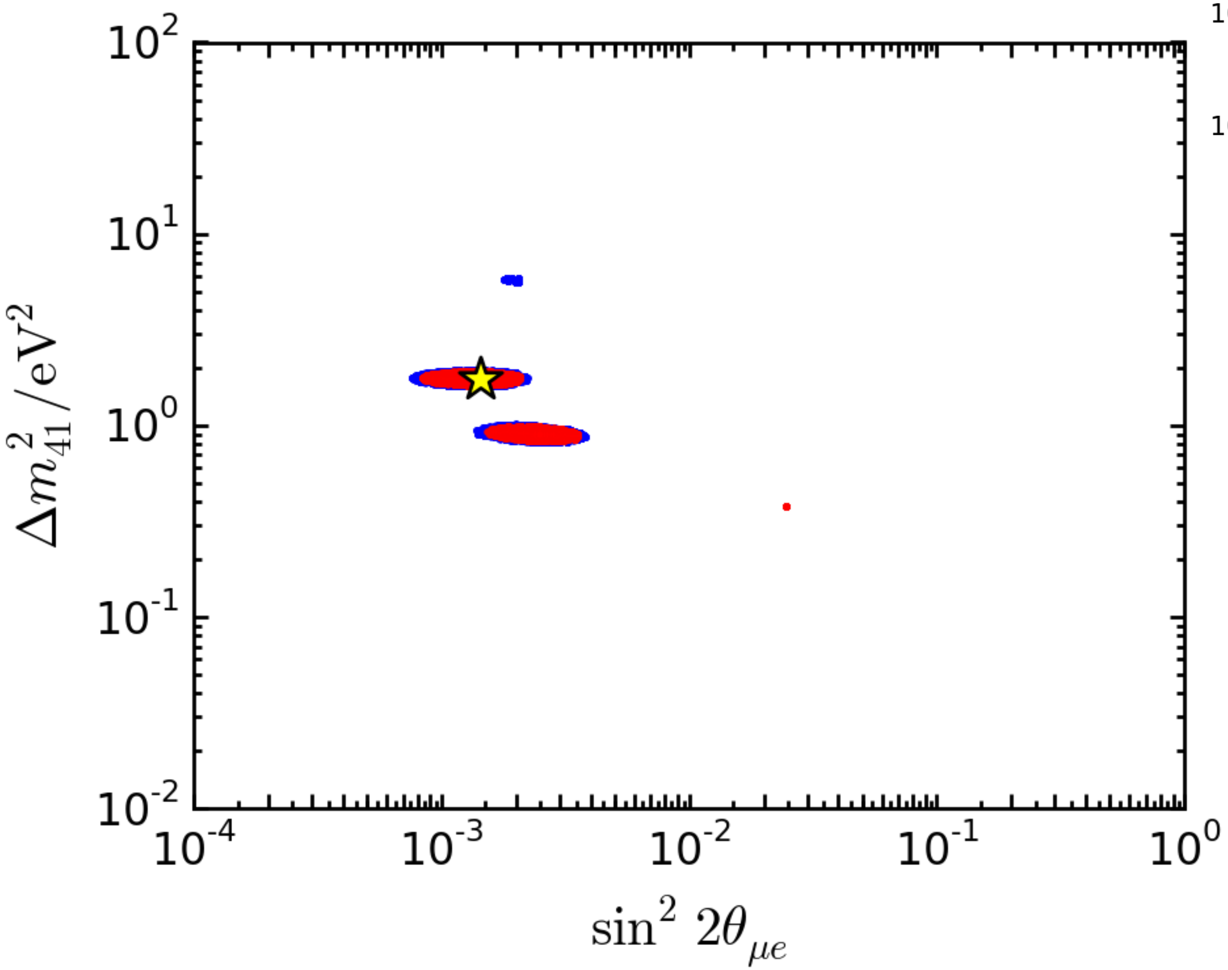
- Some experiments (eg: MiniBooNE) have backgrounds that can look like a signal.

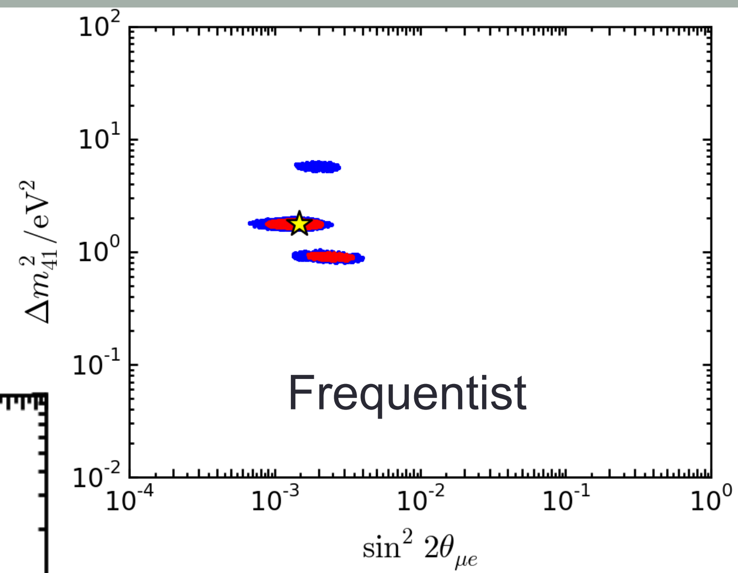
3 + 1 Bayesian

- Confidence regions are based on a $\Delta\chi^2$
- This assumes that the global fit χ^2 is distributed according to the χ^2 distribution.
- This may not be true, as the global fit is a function of sinusoids.
- Feldman-Cousins suggest generating the distribution of the fit metric numerically.
  - Not computationally feasible for the global fit.

- Instead, we can drawn Bayesian credible intervals.
  - Depends on the density of the MCMC fit points, rather than directly on the fit metric.

3 + 1 Bayesian





Red: 90% CI
Blue: 99% CI

A few other issues

- The Global fit is not a perfect description of the statistics or physics.
- It assumes that experiments are independent,
  - But experiments have correlations in their nuisance parameters (such as cross sections)
- Nuisance parameters are not included in the MCMC
  - All published analyses used in the global fit maximise their fit metric over the nuisance parameters, rather than marginalising them.
- Some analyses make assumptions about the oscillation model that are not true in a global fit.
  - Such as, assuming there is no electron neutrino appearance in a muon neutrino disappearance analysis.

Conclusion

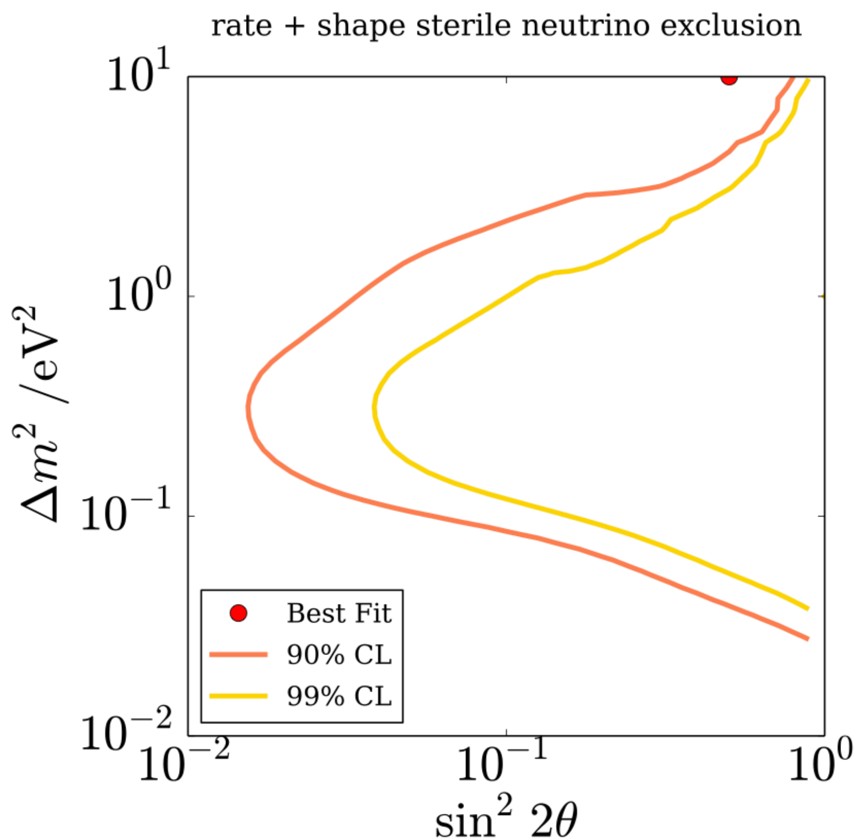
- Anomalies have been observed in short baseline experiments.
- These can be explained with the addition of one or more sterile neutrinos.
  - However, there is unexplained tensions between the datasets.
- PG test can be used to quantify the tension between datasets.
  - But it does not appear to be correctly distributed in certain circumstances.
- We are in the process of evaluating sterile neutrino models in a Bayesian context.
  - Is there an equivalent construction to the PG test?

THANK YOU

BACKUP

IceCube sterile neutrino search

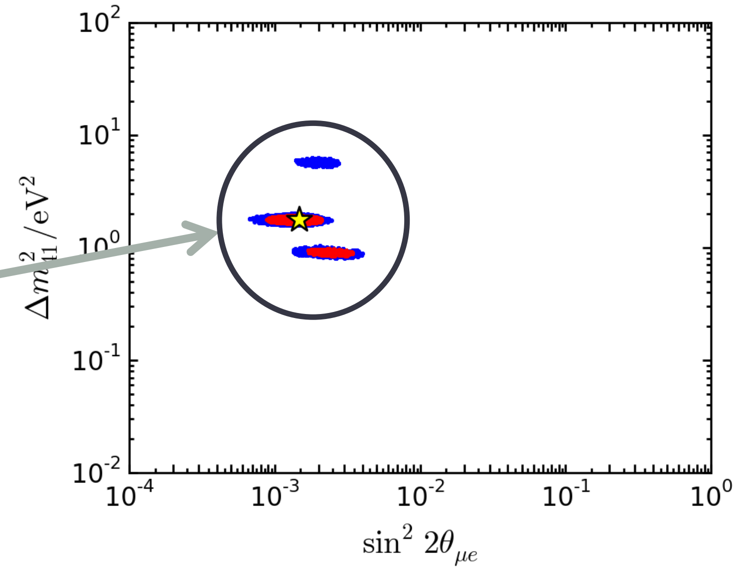
- Gigaton ice Cherenkov detector at the south pole.
- Atmospheric neutrinos are used in the sterile neutrino search.
- These travel through the earth before they reach IceCube.
  - Base-line too long for normal oscillations to be resolved.
- The MSW matter effect causes a resonant depletion in the flux of $\bar{\nu}_\mu$
- IceCube finds no evidence of muon neutrino disappearance in 1 year of data.



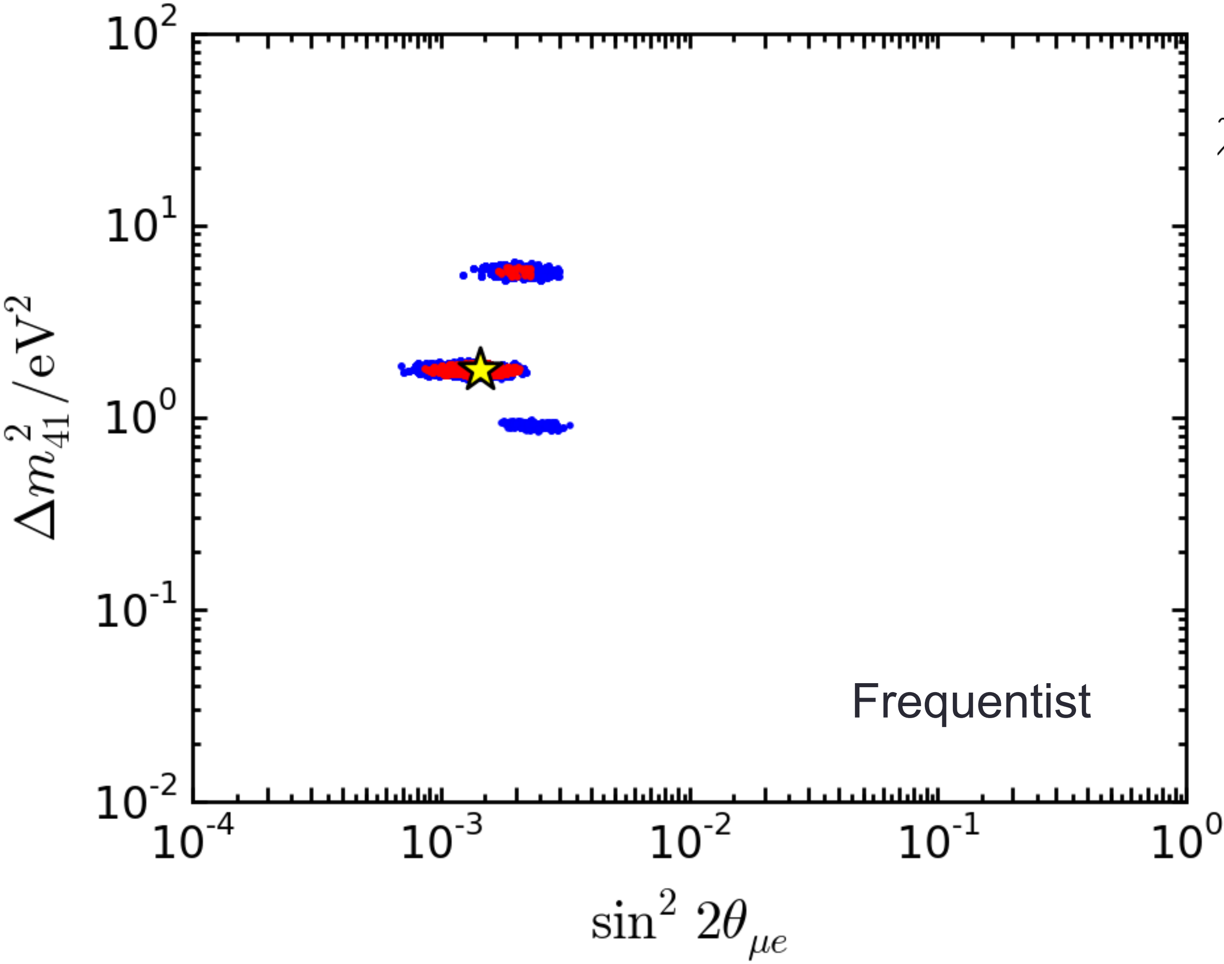
Adding IceCube data to the fit

- The full IceCube sterile analysis fitting procedure is too computationally expensive to include directly into the global fit at present.

- Instead, we use the global fit to find a set of most likely points in parameter space.
  - The IceCube analysis likelihood was evaluated at these points, and the resulting χ^2 combined with the global fit.



Adding recent IceCube data to the fit



$\chi^2 : 518.59 \quad (520 \text{ dof})$

$\chi^2_{\text{null}} : 568.84 \quad (524 \text{ dof})$

Red: 90% CL
Blue: 99% CL

Model parameters

- The mixing matrix then has only 3 parameters to fit:

Known, from long base-line/solar experiments

$$U_{3+1} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{bmatrix}$$

Unconstrained, non-interacting

- Along with the unitary matrix constraint, this makes 4 parameters: $\Delta m^2, |U_{e4}|, |U_{\mu 4}|, |U_{\tau 4}|$