Constructing expansion parameters for QCD-type theories

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work done with

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1604.06108

+ work over last ~ 8 years by Unsal, Dunne, Poppitz, Yaffe, Shifman, …

Resurgence for QFT

Belief: QFT observables = resurgent transseries in couplings `λ' and N

$$\mathcal{O}(\lambda) \simeq \sum_n p_n \lambda^n + \sum_c e^{-\frac{S_c}{\lambda}} \sum_k p_{k,c} \lambda^k + \cdots$$

Lots of evidence in special cases:

Integrals with saddles	Stokes, Dingle, Berry, Howls …
matrix models	Marino, Schiappa, Weiss …
topological strings	Aniceto, Hatsuda, Marino, Schiappa, Vonk, …
QM (d=1 QFT)	Basar, Dunne, Kawai, Misumi, Nitta, Sakai, Takei, Sulejmanpasic, Unsal, Zinn-Justin …
some SUSY theories	Aniceto, Dorigoni, Hatsuda, Honda, Russo, Schiappa, …

More generic/realistic d > 1 QFTs, with asymptotic freedom?

QCD-like theories

To write transseries, need some tunably-small expansion parameter ‘λ’.

Power of resurgence:

1. λ exists
2. λ dependence is smooth
3. Transseries representation

→ Can get large λ data from small-λ data

In the simpler examples on last page, suitable ‘λ’ mostly comes for free, so resurgence can be used to squeeze out fascinating physics + mathematics.

In QCD, trouble right at **step 1**. Why should transseries be relevant?

The challenge

QCD coupling runs with energy scale, so which ‘λ’ do we mean?

For ‘high-energy’ observables, can try to view OPE as a transseries…

But what about low-energy observables? They’re the most interesting ones, but no obvious tunably-small coupling.

This talk: summary of work since ~ 2008 on this issue.

2016: finally constructed tunably-small couplings for QCD-like theories.

NB: even with a way to get a small coupling, have to answer:

- What can we say about p_n for large n?
- What are the relevant non-perturbative (NP) saddles?
- How to do reliable semiclassical calculations of NP phenomena?

Have also seen some progress on all of these…

Calculability in asymptotically-free QFTs

To get control over low-energy observables,
need weak coupling in IR as well as UV

If theory has scalar fields, could use the Higgs mechanism; if VEV is large compared to Λ, IR becomes weakly-coupled

Makes electroweak part of SM calculable.

Ability go out onto scalar moduli space is an important ingredient in calculability of most SUSY gauge theories

But we want to study QFTs like QCD, which don't include any scalar fields.

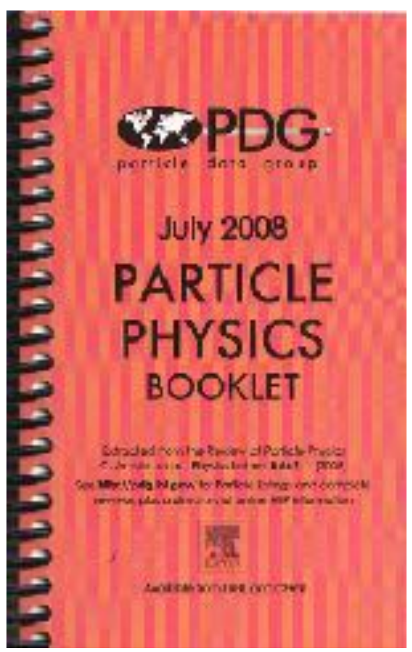
So what is to be done?

Calculability in asymptotically-free QFTs

Key features we need for all value of any putative control parameter:

confinement
spontaneous chiral symmetry breaking

Both vital for QCD phenomenology! Evidence:



actual experiments

numerical experiments

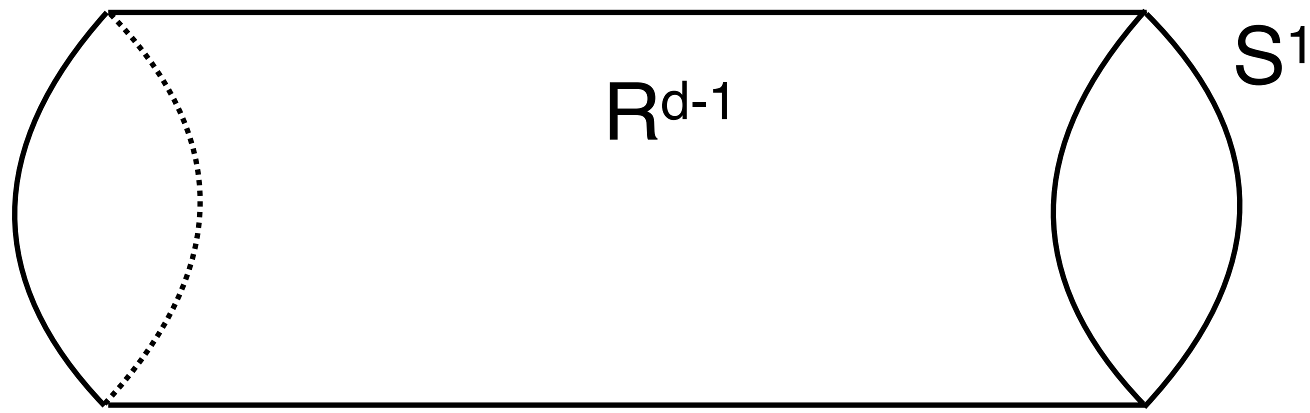
Now we want these features at weak coupling: tall order!

Adiabatic compactification

Unsal, Yaffe, Shifman, … 2008-onward

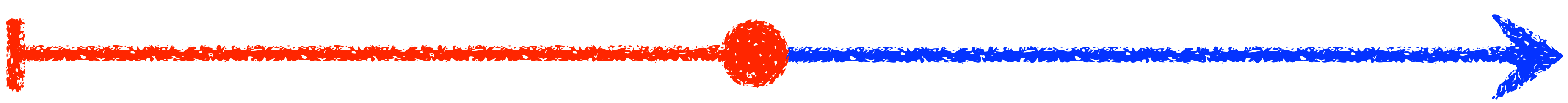
Compactify asymptotically-free 4D QFT to $R^3 \times S^1$

When S^1 size $L \ll \Lambda^{-1}$, theory becomes (sort of) weakly-coupled



With thermal BCs, L = 1/T

Trouble 1: small-L and large-L theories separated by phase transition

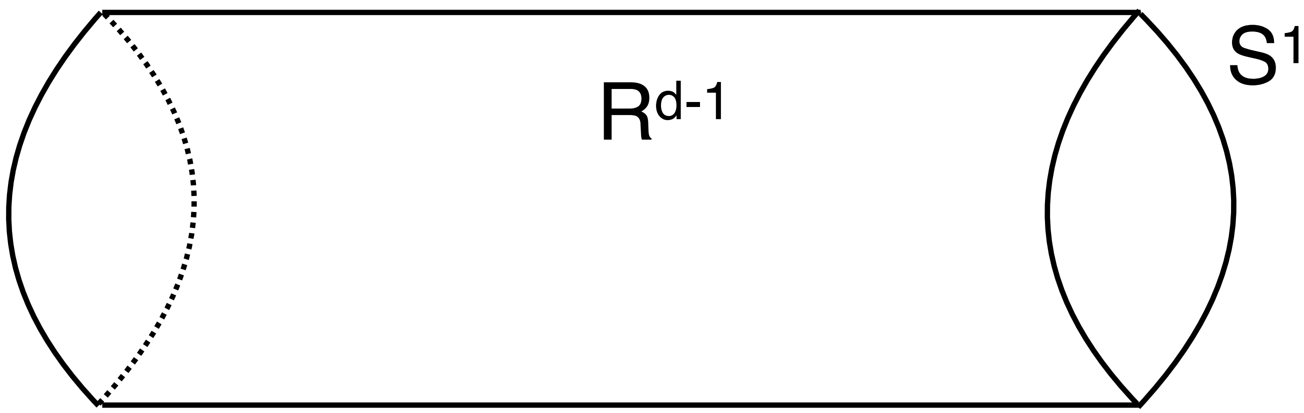


small L	large L
no confinement, no χ-SB	confinement, χ-SB
(Free energy)/$N_c^2 \sim 1$	(Free energy)/$N_c^2 \sim 0$

Compactification

Compactify asymptotically-free 4D QFT to $R^3 \times S^1$

When S^1 size $L \ll \Lambda^{-1}$, theory becomes (sort of) weakly-coupled



With thermal BCs, $L = 1/T$

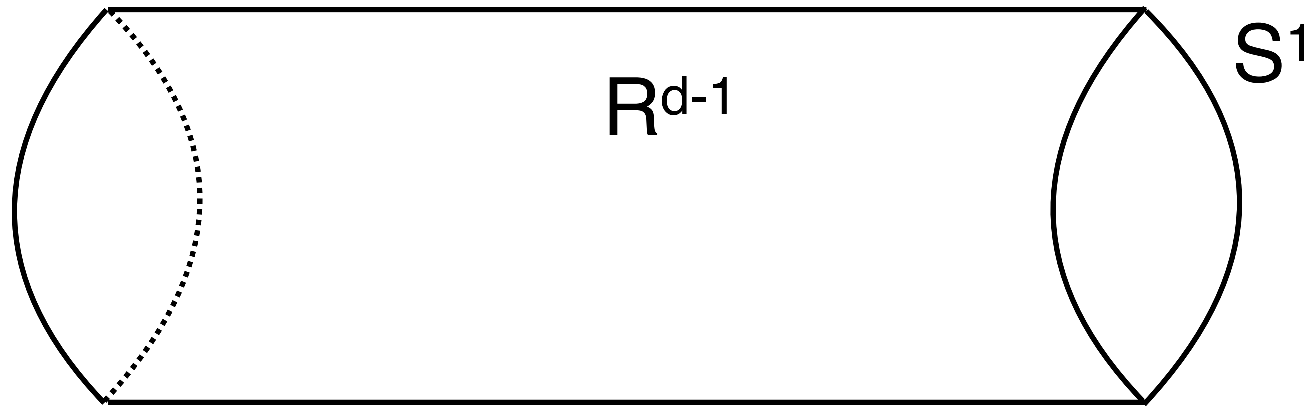
Trouble 1: small-L and large-L theories separated by phase transition

Trouble 2: small-L theory is ~ 3D YM, and is not, in fact, weakly coupled.

So for us thermal compactification is no good at all!

Adiabatic compactification

Unsal, Yaffe, Shifman,
… 2008-onward



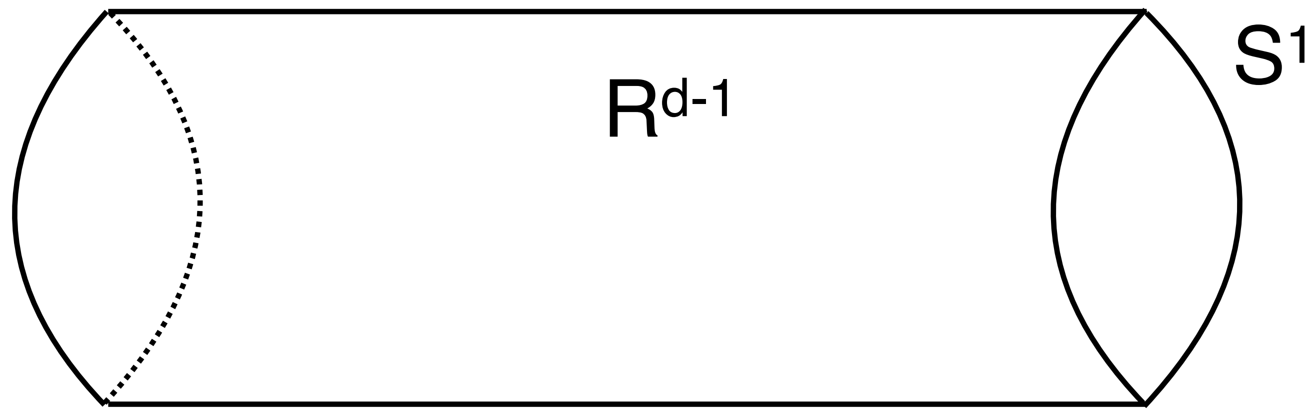
Need to find a situation where instead we get

small L	large L
confinement, χ-SB	confinement, χ-SB
(Free energy)/$N_c^2 \sim 0$	(Free energy)/$N_c^2 \sim 0$

Exploit freedom to choose BCs to make volume dependence milder?

Why would it help?

Self-Higgsing



When YM compactified on S^1 Polyakov loop becomes an observable

$$\mathrm{Tr}\ \Omega = \mathrm{Tr}\ \mathcal{P} e^{i \oint A_4} \simeq \begin{pmatrix} e^{i\phi_1} & & \\ & e^{i\phi_2} & \\ & & e^{i\phi_{\dots}} \end{pmatrix}$$

Values of eigenvalues ~ classical moduli space

Non-coincident eigenvalues ⇒ breaking SU(N) → $U(1)^{N-1}$

VEV of "A_4" produces a (compact) adjoint Higgs mechanism!

But we don't get to choose eigenvalues: theory picks own vacuum

Adiabatic compactification

Unsal + collaborators, … 2008-onward

At large L have confined phase; tr $\Omega \approx 0$,Z_{Nc} center symmetry.

Pure glue YM (and QCD) at small L
dynamically forces $A_4 = 0 \iff \mathrm{tr}\,\Omega \neq 0$, broken center symmetry.

Idea: add something that leaves large L theory
the “same”, but makes small L limit smooth

Kovtun, Unsal
Jaffe 2008

YM or QCD + 1 massive Dirac adjoint fermion
with *periodic* BCs, $\Lambda \lesssim m \ll 1/L$

Or add appropriate double-trace deformation $\delta S = \int d^4x\, L^{-4} \Sigma_n\, [a(n)\, \mathrm{tr}\, |\Omega^n|^2]$

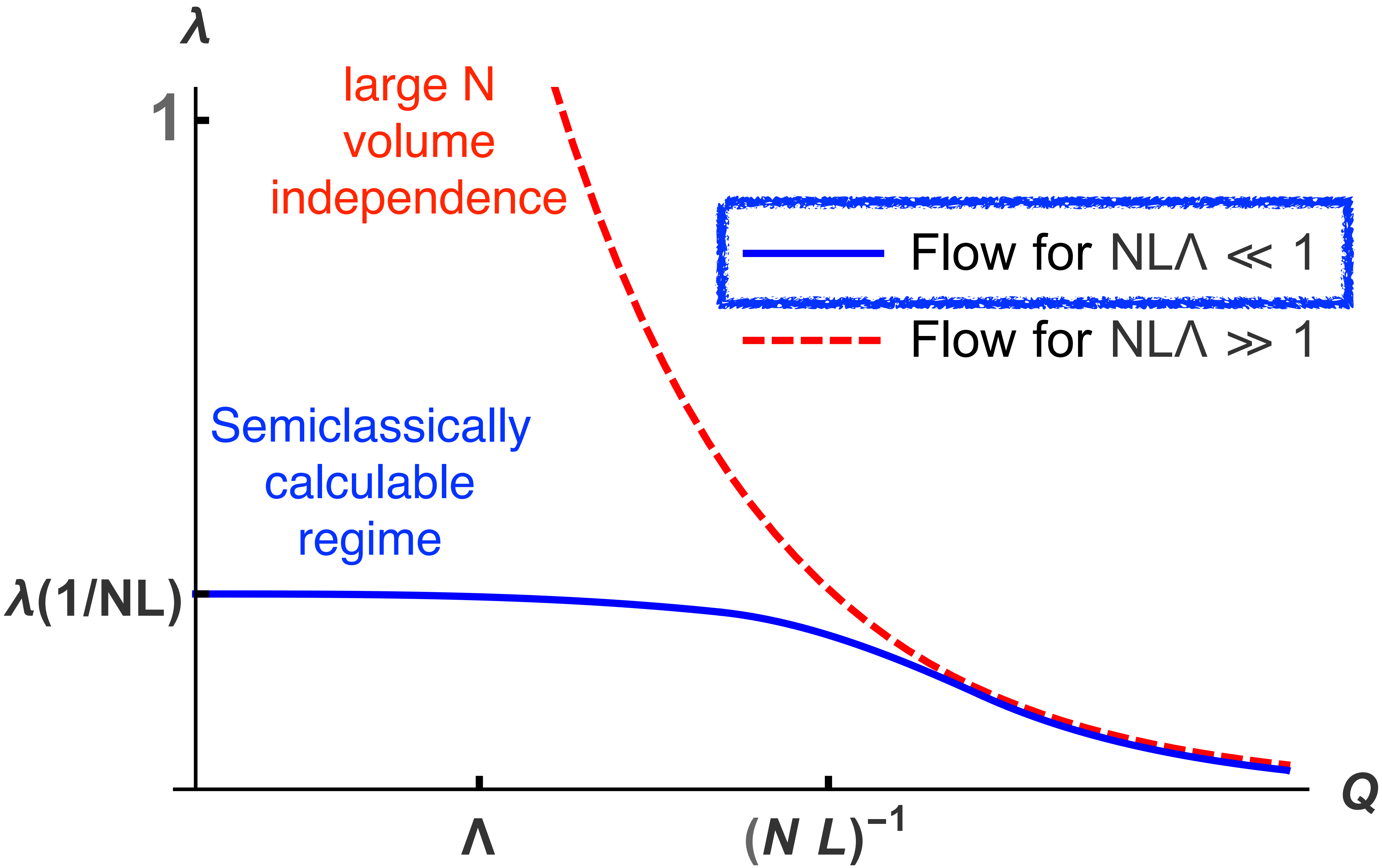
Resulting ‘YM*/QCD*’ theories remains center-symmetric at small L

Non-coincident eigenvalues of $\Omega \Rightarrow$ breaking $SU(N) \rightarrow U(1)^{N-1}$

Adjoint Higgs mechanism drive by VEV of “A_4” !

W-boson mass scale is $m_W = 2\pi/NL$

Coupling flow with adiabatic compactification



The NLΛ << 1 regime gives a weakly-coupled theory at all scales!

Small NLΛ

Unsal + collaborators,
… 2008-onward

Tempting to now interpret NLΛ as the desired control parameter.

Works for pure YM*: perturbative + non-perturbative dynamics under systematic control at small at small NLΛ

YM* develops mass gap, finite string tension, etc, at small NLΛ.
All evidence: observables smooth as a function of NLΛ

What about QCD* ?

Large NLΛ: spontaneously broken chiral symmetry
Small NLΛ: unbroken chiral symmetry

Chiral phase transition at NLΛ ~ 1 !

Trouble for program of viewing NLΛ as smooth control parameter for QCD-like theories

Is the idea doomed?

Theoretical understanding of chiral symmetry breaking (χ-SB) mostly based on inspirational phenomenological models:

Nambu-Jona-Lasinio models

Truncated Schwinger-Dyson equation models

Instanton/'dyon' liquid models

…

All constructions: χ-SB happens at **strong coupling**, outside of regime where quantum effects are under systematic control.

Folk belief: χ-SB is fundamentally strongly-coupled, can't happen in weakly-coupled settings. So can't do any better?

Yes, we can. Can make **calculable** χ-SB using adiabatic compactification idea.

Boundary conditions

In theory with quarks, must choose BCs:

$$\psi(x_4 + L, \vec{x}) = \Omega_F \Omega_Q \psi(x_4, \vec{x}),\ \Omega_F \in SU(N_f), \Omega_Q \in U(1)_Q$$

Not important for large L spectrum, but matters at small L!

Experience with 2D sigma models: some choices of BCs allow smoother small L limit than others.

AC+Dorigoni+Unsal; Dunne+Unsal, Sulejmanpasic, …

Can think of Ω_F, Ω_Q as background gauge field holonomies

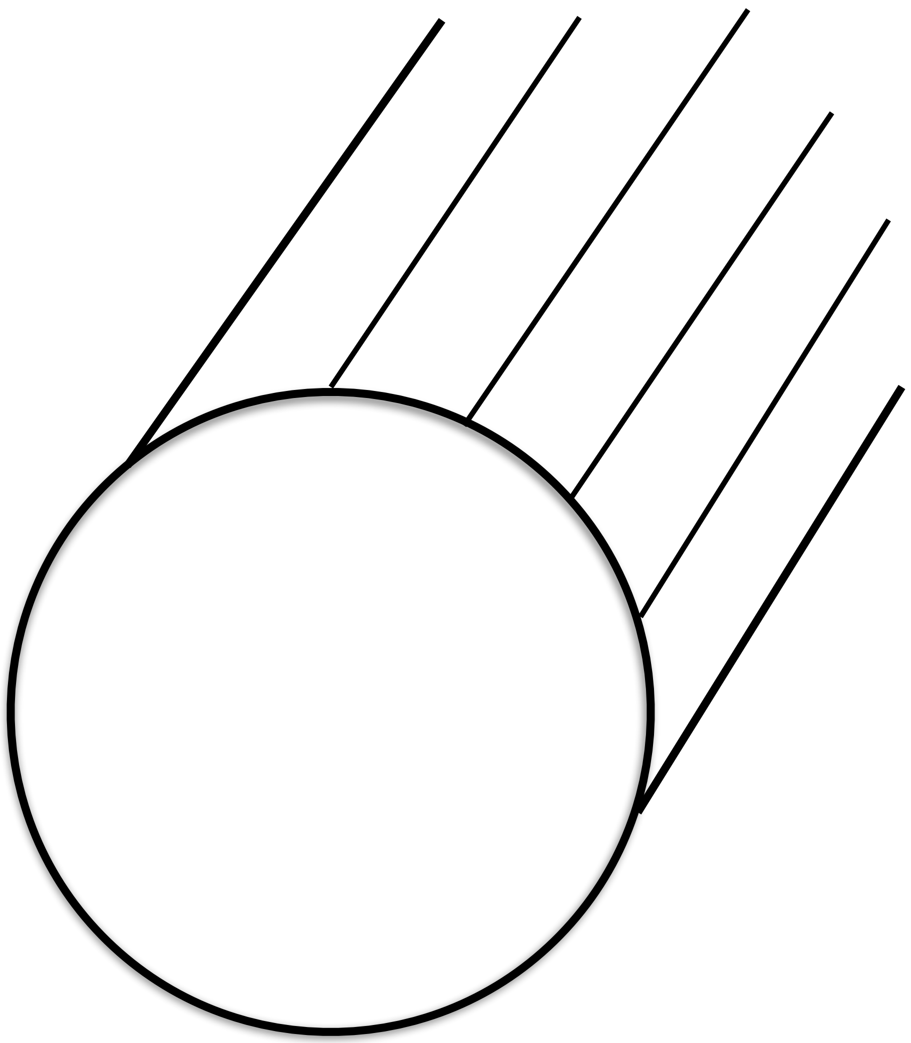
Inspired by 2D examples, explore result of taking flavor-center-symmetric SU(N_F) background holonomies:

$$\Omega_F = \mathrm{diag}(1, e^{2\pi i/N_f}, \ldots, e^{2\pi i(N_f-1)/N_f})$$

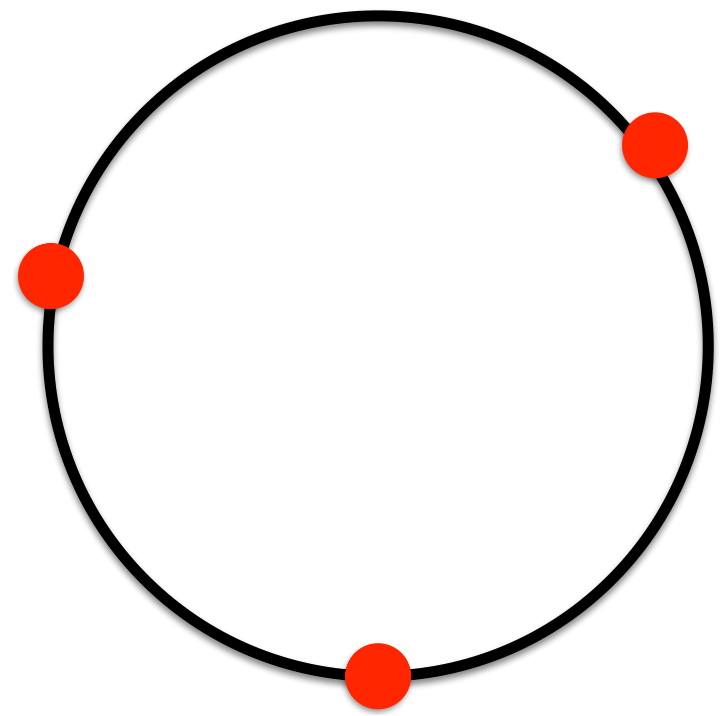
Kouno, Sakai, Yahiro, Sasaki, Makiyama, Iritani, Itou, Misumi, …

$$\text{Preserves } U(1)_L^{N_f-1} \times U(1)_R^{N_f-1} \in SU(N_f)_L \times SU(N_f)_R$$

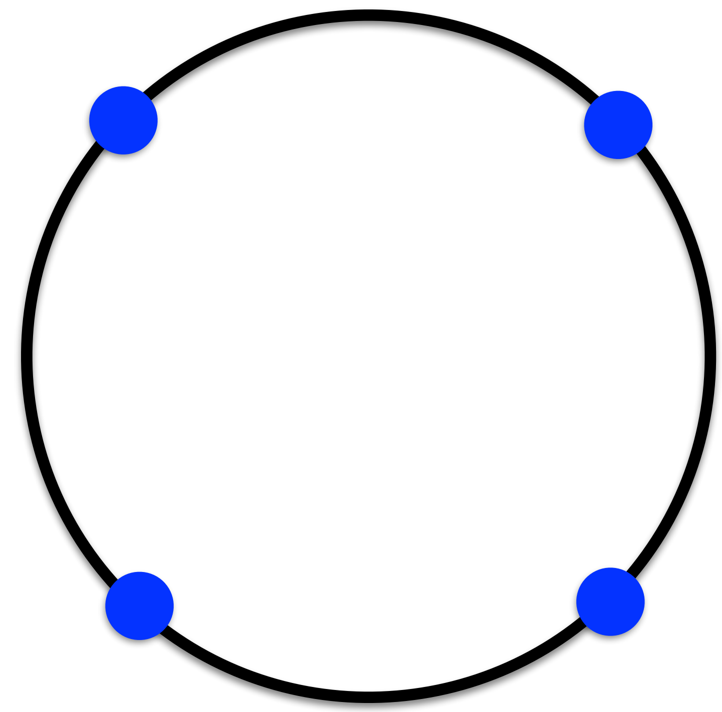
Three circles



Compactification circle



Eigenvalue circle for background flavor holonomy Ω_F



Eigenvalue circle for dynamical color holonomy Ω

Large L expectations

Background holonomies/twisted BCs are equivalent to **imaginary** \`isospin' chemical potentials $\tilde{\mu} \sim 1/L$

Large L low-energy dynamics captured by chiral perturbation theory

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\, \partial_\mu U \partial^\mu U^\dagger \to \mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\, D_\mu U D^\mu U^\dagger$$

$$D_\mu = \partial_\mu + i[\mu_I \tau_3, \cdot]$$

$N_F = 2$ example

$$m_{\pi^0}^2 = m_\pi^2$$

$$m_{\pi^\pm}^2 = m_\pi^2 - \mu_I^2 \;\to\; m_{\pi^\pm}^2 = m_\pi^2 + \tilde{\mu}_I^2$$

N_F-1 'pions' remain gapless, all others pick up positive gaps $E^2 \gtrsim 1/L^2$

If small L limit is smooth, should get N_F -1 gapless NGBs.

Small L limit in perturbation theory

At long distances ℓ >> N_c L ~ 1/m_W

$$SU(N_c) \to U(1)^{N_c - 1}$$

due to the center-symmetric background holonomy.

The light fields are N_c - 1 “Cartan gluons”

$$F_{\mu\nu,k} = \frac{1}{N} \sum_{p=0}^{N-1} e^{-2\pi i k p / N_c} \mathrm{Tr}\,\left(\Omega^p F_{\mu\nu}\right)$$

Small-L physics easiest to describe using 3D Abelian duality

Small L limit in perturbation theory

N_c - 1 Cartan gluons are gapless to all orders in perturbation theory

$$F^i_{\mu\nu} = g^2/(2\pi L)\epsilon_{\mu\nu\alpha}\partial^\alpha\sigma^i$$

$$S_\sigma = \int d^3x \, \frac{g^2}{8\pi^2 L}(\partial_\mu\vec{\sigma})^2.$$

Noether current for $[U(1)_J]^{N_c-1}$ shift symmetry conserved so long as there are no magnetic monopoles in theory.

So the glue DOFs produce a light sector with N_c-1 gapless 3D scalars, before considering NP effects.

Beyond perturbation theory

Unsal,
Yaffe, Shifman,
Poppitz,
Sulejmanpasic,
Zhitnitsky
...

Thanks to dynamical Abelianization of SU(N_c) gauge symmetry, BPST instanton fractionalizes into N_c constituents

$$\mathcal{M}_i \sim e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_i \cdot \vec{\sigma}}$$

assuming no massless fermions.

Interactions induced by the magnetic-charge-carrying M_i events produce a potential — and hence a mass gap — for σ

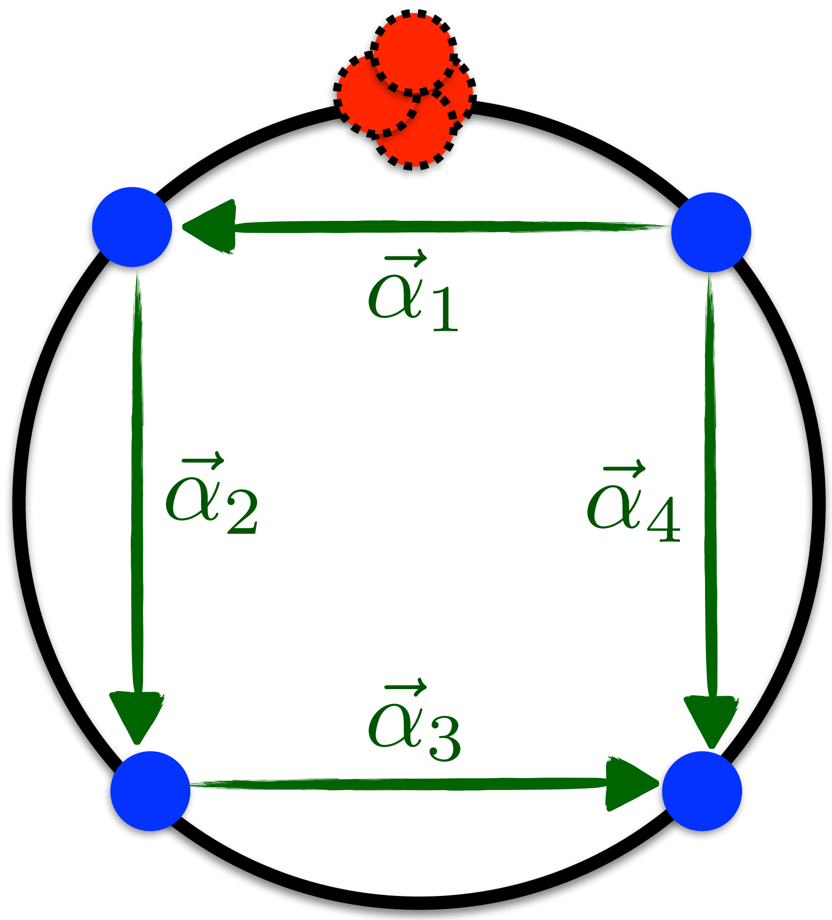
$$V(\vec{\sigma}) = m_W^3 e^{-\frac{8\pi^2}{g^2 N_c}} \sum_i \cos(\vec{\alpha}_i \cdot \vec{\sigma})$$

Massless fermions make things more subtle due to fermion zero modes on monopole-instantons

Fermion zero modes

van Baal + collaborators, 1999

Without Z_{Nf} twist, collective hopping phenomenon:



$N_c = N_F = 4$

$$\mathcal{M}_1 \sim e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_1 \cdot \vec{\sigma}} \det_{a,b}\left[\bar{\psi}_{L,a}\psi_{R,b}\right]$$

Invariant under $SU(N_F)_L x SU(N_F)_R x U(1)_Q$, but not $U(1)_A$

All $2N_f$ 'instanton' zero modes stick to a single monopole-instanton

Fermion zero modes

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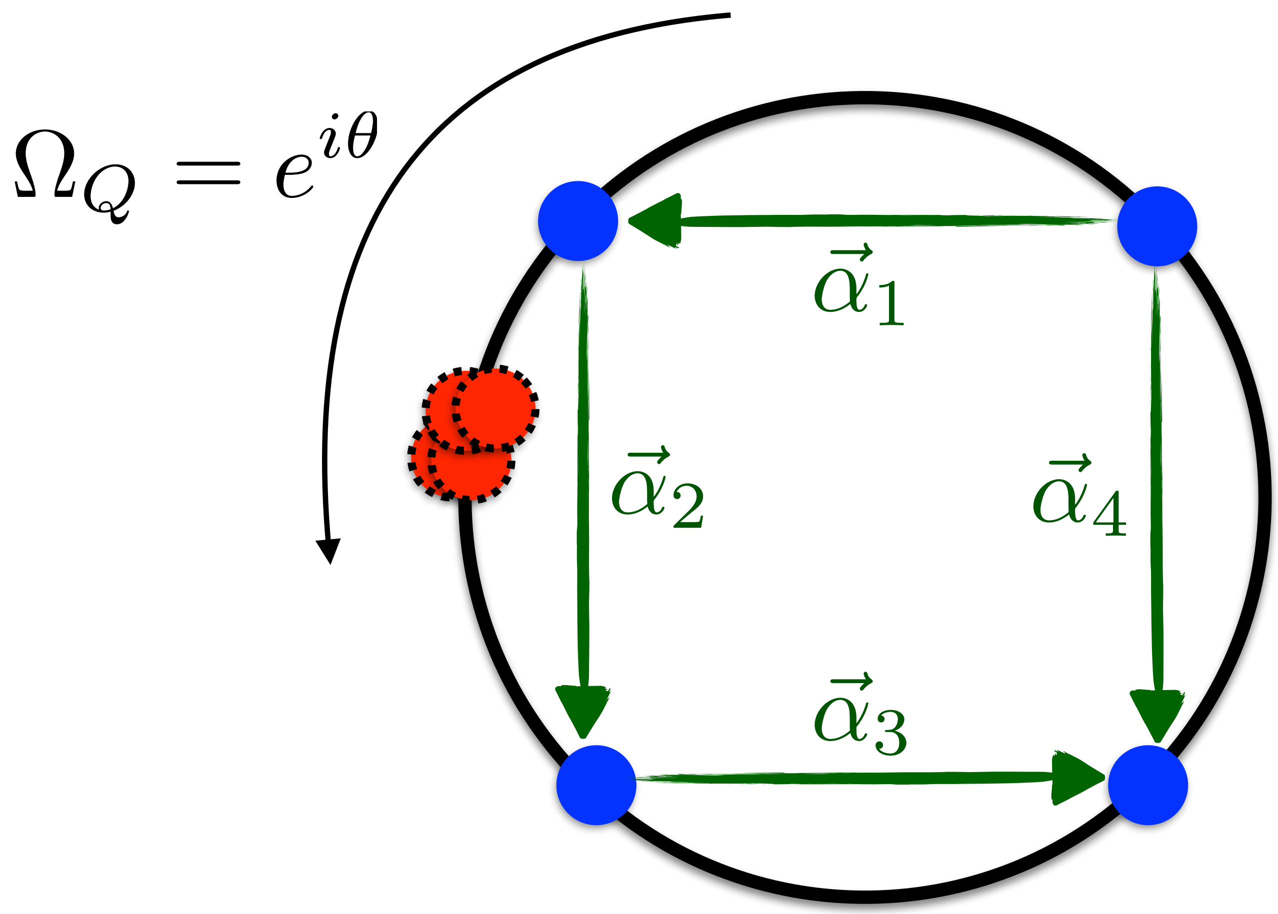
Without Z_{N_f} twist, collective hopping phenomenon:



$N_c = N_F = 4$

$$\mathcal{M}_2 \sim e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_2\cdot\vec{\sigma}} \det_{a,b}\left[\bar{\psi}_{L,a}\psi_{R,b}\right]$$

Invariant under $SU(N_F)_L x SU(N_F)_R x U(1)_Q$, but not $U(1)_A$

All $2N_f$ 'instanton' zero modes stick to a single monopole-instanton

Fermion zero modes

Localization of all $2N_F$ fermion zero modes means
3D EFT is a sort of weakly-coupled 3D NJL model

$$S \sim \int d^3x \left(\sum_a \bar{\psi}_a \gamma^\mu \partial_\mu \psi_a + m_W e^{-8\pi^2/\lambda} e^{i\vec{\alpha}_1 \cdot \vec{\sigma}} \det_{a,b} \bar{\psi}_a \psi_b \right)$$

Known NOT to produce χ-SB, except at strong coupling,
where it's out of systematic control.

So if we set $\Omega_F = 1$, there must be a chiral transition
between small L and large L regimes of deformed QCD!

Then for $N_F > 1$, small and large L regimes not smoothly connected.

Shifman+Unsal
2009

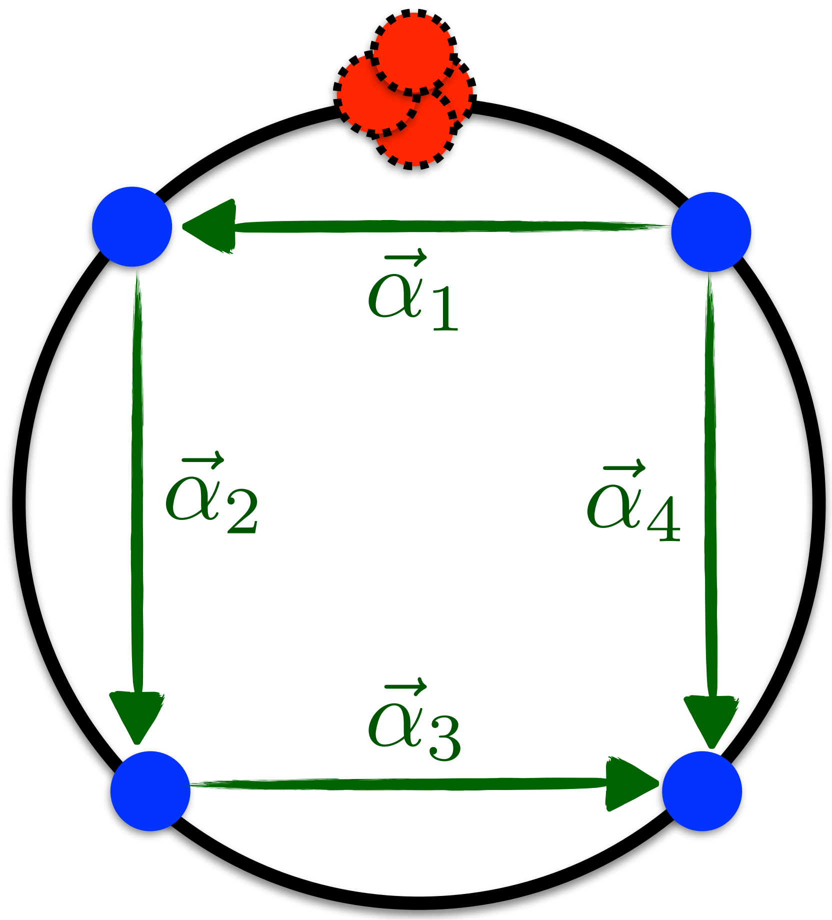
Fermion zero modes

With $\mathbb{Z}_{N_f}$ twist

AC, Schafer, Unsal, 2016

using index theorem of Poppitz+Unsal 2008

$N_C = N_F = 4$



Fermion zero modes

$$\text{With } \mathbb{Z}_{N_f} \text{ twist}$$

AC, Schafer, Unsal, 2016

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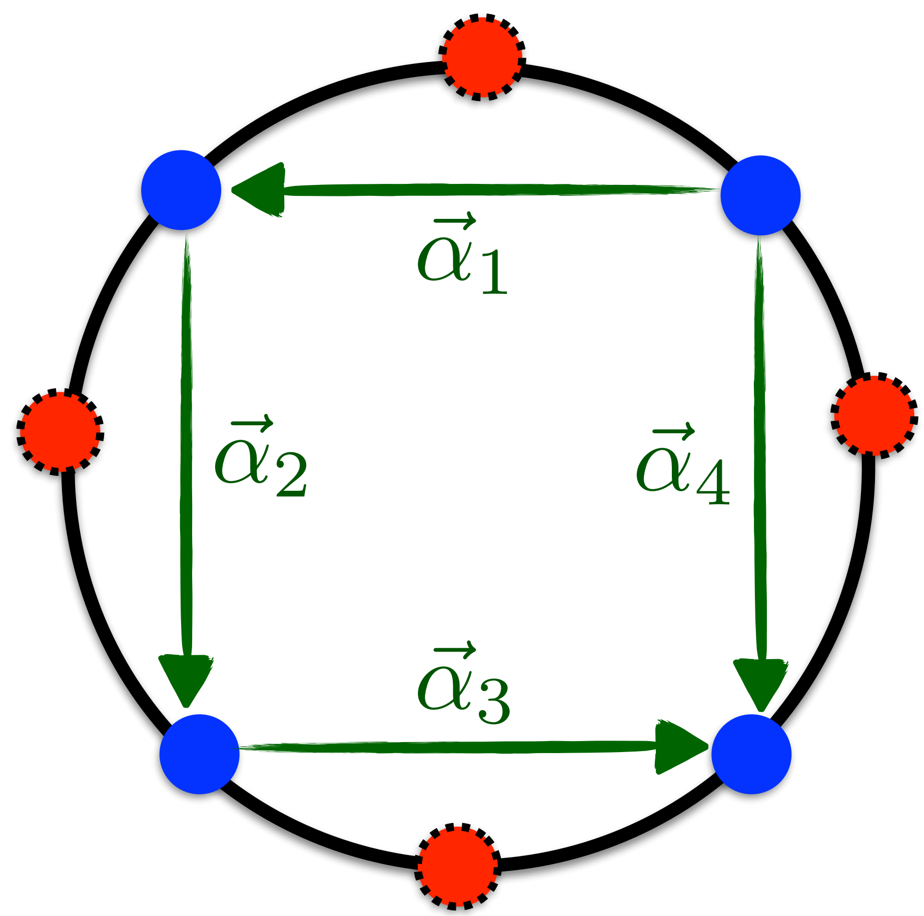
$N_c = N_F = 4$



$$\mathcal{M}_i = e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_i \cdot \vec{\sigma}} (\bar{\psi}_{L,i} \psi_{R,i}), \qquad i = 1, \ldots N_f$$

In fact this drives chiral symmetry breaking!

Broken and unbroken symmetries

Before taking into account NP effects,
symmetry for gluons and fermions is

$$[U(1)_J]^{N_c-1} \times U(1)_V^{N_F-1} \times U(1)_A^{N_F-1} \times U(1)_Q$$

Symmetry only in perturbation theory, not sacred.

Subgroup of anomaly-free symmetry

Must be respected by all effective vertices in theory

Broken and unbroken symmetries

AC, Schafer, Unsal, 2016

At NP level, must understand symmetries preserved by

$$e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_k\cdot\vec{\sigma}}(\bar{\psi}_{L,k}\psi_{R,k})$$

$[U(1)_V]^{Nf-1} x U(1)_Q$ is obvious. What about axial transformations?

$$U(1)_A^{N_F-1}: \quad (\bar{\psi}_{L,k}\psi_{R,k}) \to e^{i\epsilon_k}(\bar{\psi}_{L,k}\psi_{R,k})$$

[ϵ_k have single linear constraint to account for $U(N_F)_A \rightarrow SU(N_F)_A$]

Monopole-instanton vertex naively not invariant?!

Broken and unbroken symmetries

AC, Schafer, Unsal, 2016

At NP level, must understand symmetries preserved by

$$e^{-\frac{8\pi^2}{g^2 N_c}} e^{i\vec{\alpha}_k \cdot \vec{\sigma}} (\bar{\psi}_{L,k} \psi_{R,k})$$

$[U(1)_V]^{Nf-1} x U(1)_Q$ is obvious. What about axial transformations?

$$U(1)_A^{N_F - 1} : \quad (\bar{\psi}_{L,k} \psi_{R,k}) \to e^{i\epsilon_k} (\bar{\psi}_{L,k} \psi_{R,k})$$

Monopole-instanton vertex invariance **requires**

$$\begin{aligned} (\bar{\psi}_{L,k} \psi_{R,k}) \quad &\to \quad e^{i\epsilon_k} (\bar{\psi}_{L,k} \psi_{R,k}), \\ e^{i\vec{\alpha}_k \cdot \vec{\sigma}} \quad &\to \quad e^{-i\epsilon_k} e^{i\vec{\alpha}_k \cdot \vec{\sigma}}. \end{aligned}$$

Broken and unbroken symmetries

AC, Schafer, Unsal, 2016

So monopole-instanton operators are indeed invariant under

$$U(1)_V^{N_F-1} \times U(1)_A^{N_F-1} \times U(1)_Q$$

The “cost” is that N_F - 1 ‘dual photons’ pick up an **exact** shift symmetry, coming from intertwining of topological and axial symmetries

They remain exactly massless, even at non-perturbative level.

All topological molecules have uncompensated fermi zero modes. No “magnetic bions” exist here.

Where is the promised chiral symmetry breaking?

Chiral symmetry breaking

AC, Schafer, Unsal, 2016

Gapless 'dual photons' **are** precisely the "pions"

They transform under $[U(1)_A]^{N_f-1}$.
Giving them any VEV - including zero - breaks chiral symmetry.

Immediately gives non-perturbative chiral-symmetry-breaking constituent quark masses, as expected from models:

$$S \sim \int d^3x \sum_a \left(\bar{\psi}_a \gamma^\mu \partial_\mu \psi_a + m_W e^{-8\pi^2/\lambda} e^{i\vec{\alpha}_a \cdot \vec{\sigma}} \bar{\psi}_a \psi_a \right)$$

$$\Downarrow$$

$$S \sim \int d^3x \sum_a \left(\bar{\psi}_a \gamma^\mu \partial_\mu \psi_a + m_W e^{-8\pi^2/\lambda} \bar{\psi}_a \psi_a \right)$$

First systematic derivation of constituent quark mass we're aware of.

Chiral Lagrangian

AC, Schafer, Unsal, 2016

Turning on a small quark mass m_q gives $m_\pi \sim m_q^{1/2}$, since soaking up zero modes with quark mass insertion gives

$$S_{m_q} \sim \int d^3x \left[m_W^2 m_q e^{-\frac{8\pi^2}{\lambda}} e^{i\vec{\alpha}_k \cdot \vec{\sigma}} + \text{h.c.} \right]$$

Theory satisfies expected GMOR relation $m_\pi^2 f_\pi^2 = m_q \langle \bar{\psi}\psi \rangle$

In fact dual photon action can be written as

$$S_\sigma = L \int d^3x \left[\frac{f_\pi^2}{4} \text{Tr}\, \partial_\mu \Sigma' \partial^\mu \Sigma'^\dagger - c\, \text{Tr}\, (M_q^\dagger \Sigma' + \text{h.c.}) \right]$$

Σ’ is usual chiral field restricted to maximal torus, as expected from large L. But at small L, f_π is **calculable**:

$$f_\pi^2 = \left(\frac{g}{\pi L \sqrt{6}} \right)^2 = \frac{N_c \lambda m_W^2}{24 \pi^4}$$

So what did we learn?

There exists a small L limit of QCD with systematically calculable χ-SB, at weak coupling and low monopole-instanton density

χ-SB driven by “condensation” of monopole-instantons, which induces a chiral condensate.

Pions mapped to dual photons, constituent quark masses come for free.

Supports continuity between large and small L

What about resurgence?

Resurgence and QCD

Found a control parameter, $\eta = N_c L \Lambda$, for QCD-like theories

So QCD observables = resurgent transseries in couplings `λ' and N

$$\mathcal{O} \simeq \sum_n p_n(N_c)\lambda^n + \sum_m e^{-S_m/\lambda} \sum_k p_{k,m}(N_c)\lambda^k$$

$$\lambda \sim \frac{8\pi^2}{\beta_0 \log(1/\eta)}$$

To use resurgence theory, need to compute the sums

For progress so far, see talks by Erich and Thomas!

The end.