When instantons do "nothing": the curious case of extended supersymmetry

Erich Poppitz

"Thimble and buttons"

uniform measure/STACK

by Stephen Cruise, "Fashion District", Toronto

main part of talk based on work on N=2 SUSY QM, 1507.04063

with **A. Behtash, T. Sulejmanpasic, M. Unsal**

Motivation:

Instantons play a role in many physical problems.
In QFT, whenever semiclassics "works",
key to understanding important physics, e.g.:

- N=1 SUSY theories: nonperturbative superpotentials.
- N=2 SUSY theories: Seiberg-Witten curves.
- Phenomenological models of chiral symmetry breaking in QCD.
- ...

... see talks by Cherman, Schaefer

Mass gap, confinement & center stability:

QCD(adj)/SYM & deformed Yang-Mills theory on $R^{1,2} \times S^1_L$, at small L

already at weak coupling, a major difficulty:
"How to define & calculate multi-instanton contributions?"

Motivation:

QCD(adj)/SYM & deformed Yang-Mills theory on $R^{1,2} \times S^1_L$, at small L

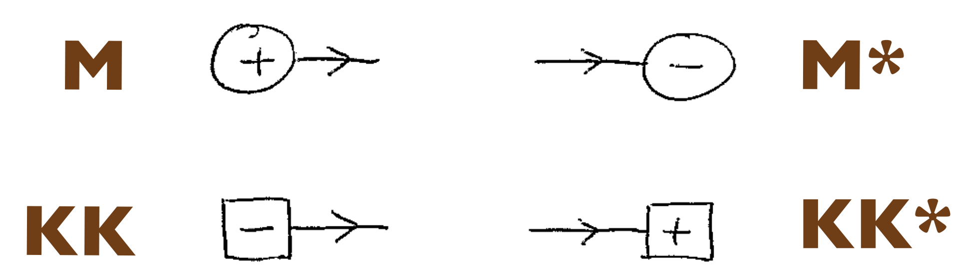
already at weak coupling, a major difficulty:

"How to define & calculate multi-instanton contributions?"

Not merely a question of calculating exponentially suppressed effects.

Instanton—anti-instanton (I-I*), for example, contributions have been found to give the leading effect in many cases.

Ex. 1: SYM, mass gap (confinement) and center stability due to such configurations: vacuum is a dilute gas of "magnetic bions" and "neutral bions." both are different types of I-I* "molecules"



for SU(2)

Motivation:

QCD(adj)/SYM & deformed Yang-Mills theory on $R^{1,2} x S^1_L$, at small L

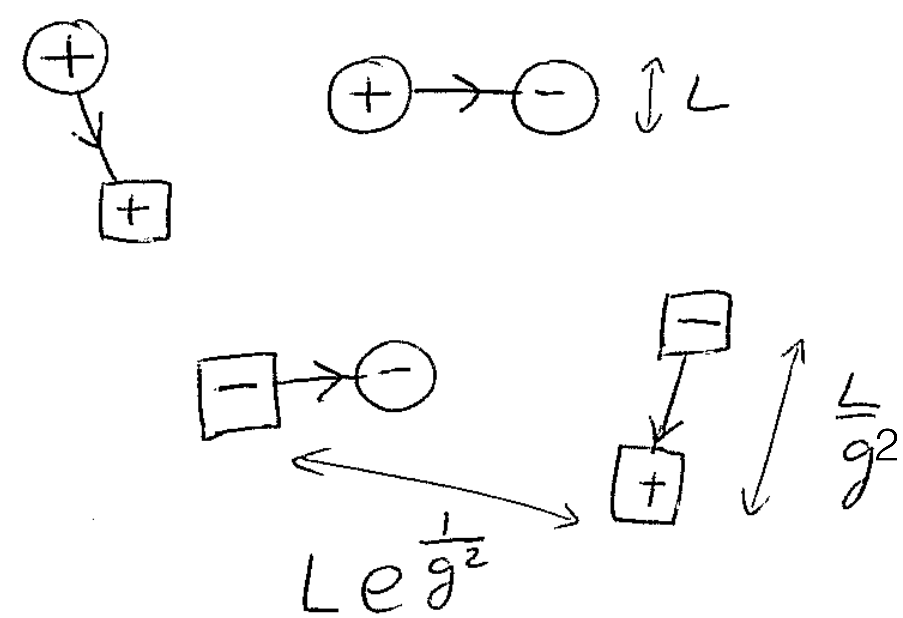
already at weak coupling, a major difficulty:

"How to define & calculate multi-instanton contributions?"

Not merely a question of calculating exponentially suppressed effects.

Instanton—anti-instanton (I-I*), for example, contributions have been found to give the leading effect in many cases.

Ex. 1: SYM, mass gap (confinement) and center stability due to such configurations: vacuum is a dilute gas of "magnetic bions" and "neutral bions." both are different types of I-I* "molecules"



e.g., 2012 work with Schaefer/Unsal

Motivation:

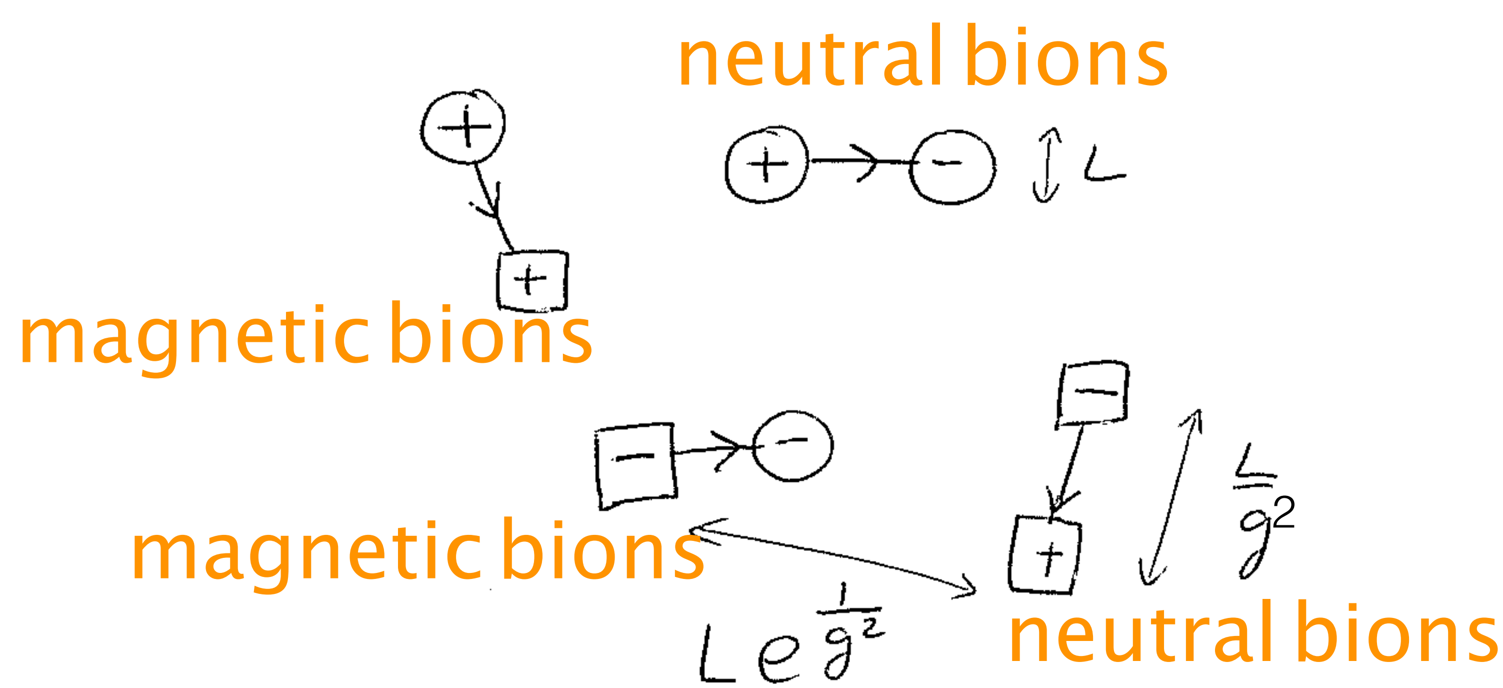
QCD(adj)/SYM & deformed Yang–Mills theory on $R^{1,2} \times S^1_L$, at small L

already at weak coupling, a major difficulty:
"How to define & calculate multi-instanton contributions?"

Not merely a question of calculating exponentially suppressed effects.

Instanton—anti-instanton (I-I*), for example, contributions have been found to give the leading effect in many cases.

Ex. 1: SYM, mass gap (confinement) and center stability due to such configurations: vacuum is a dilute gas of "magnetic bions" and "neutral bions." both are different types of I-I* "molecules"

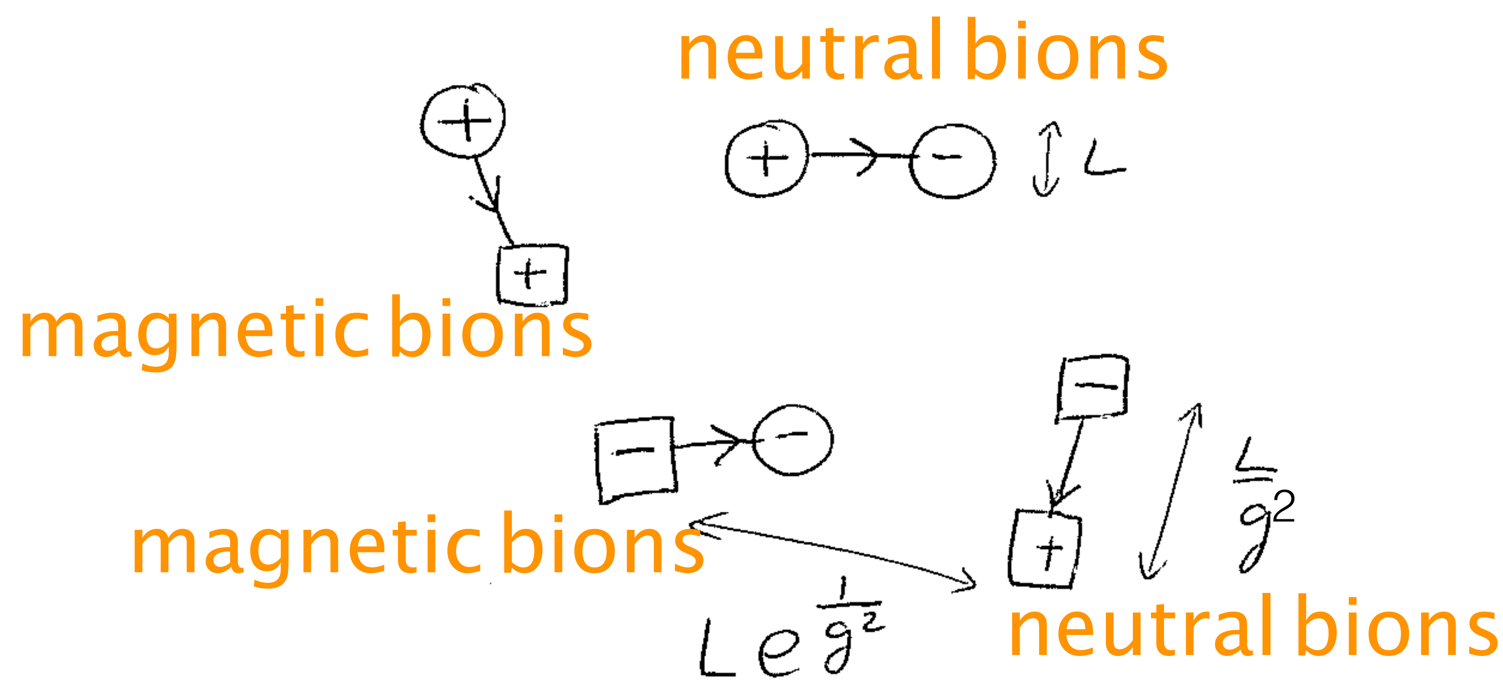


Motivation:

"neutral bions" are particularly bizarre: they are MM* "molecules"

(neutral bions are responsible for center stability and also cancel magnetic bion vacuum energy in SYM)

Ex. 1: SYM, mass gap (confinement) and center stability due to such configurations: vacuum is a dilute gas of "magnetic bions" and "neutral bions." both are different types of I-I* "molecules"



Motivation:

"neutral bions" are particularly bizarre: they are MM* "molecules"

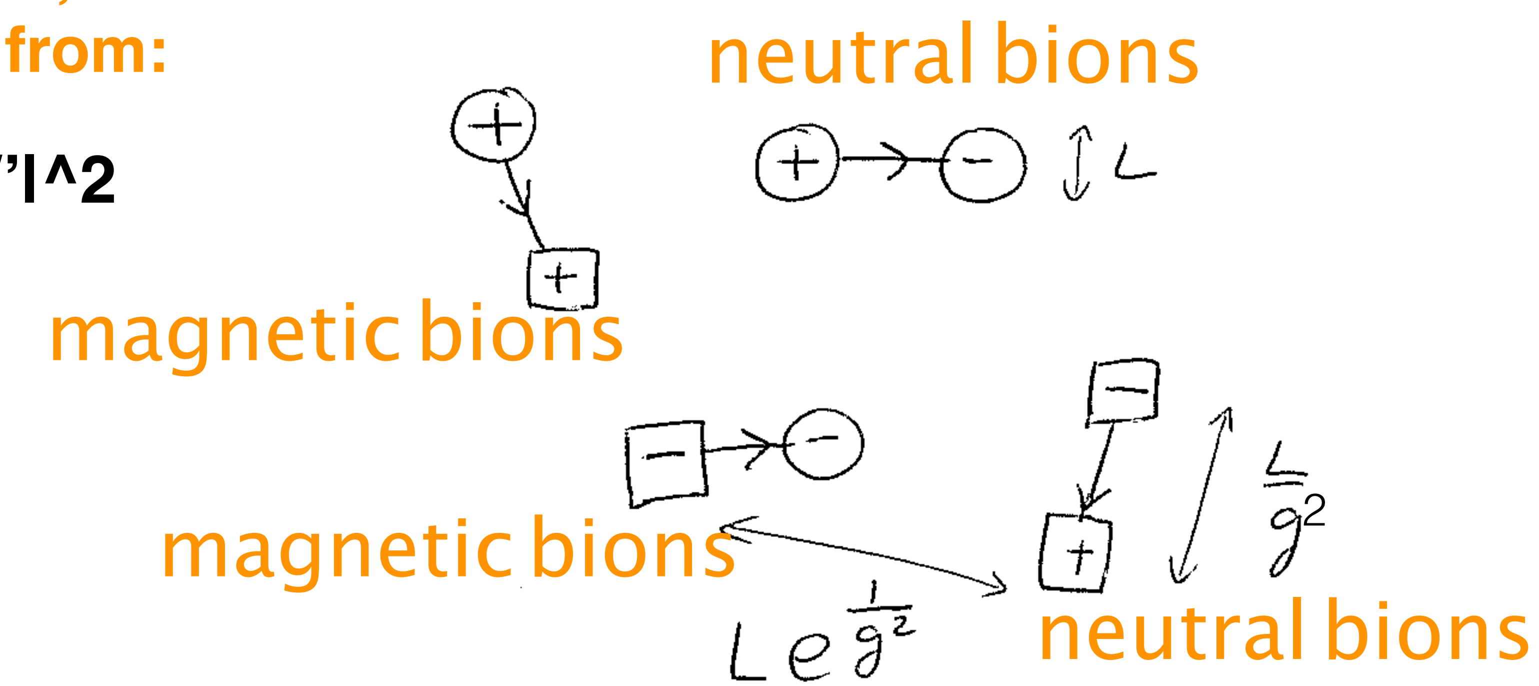
(neutral bions are responsible for center stability and also cancel magnetic bion vacuum energy in SYM)

Turns out, the MM* amplitude makes sense. Despite the attractive-only interactions, a "stable molecule" exists! We know from:

1. supersymmetry, exact W -> V=|W'|^2

2. analytic continuation:
MM* "live" at complex separation

MM* in some sense
"classical" (live in Euclidean)
- no time and no quantum fluctuations
to stabilize, not, e.g. positronium!



Motivation:

"neutral bions" are particularly bizarre: they are MM* "molecules"

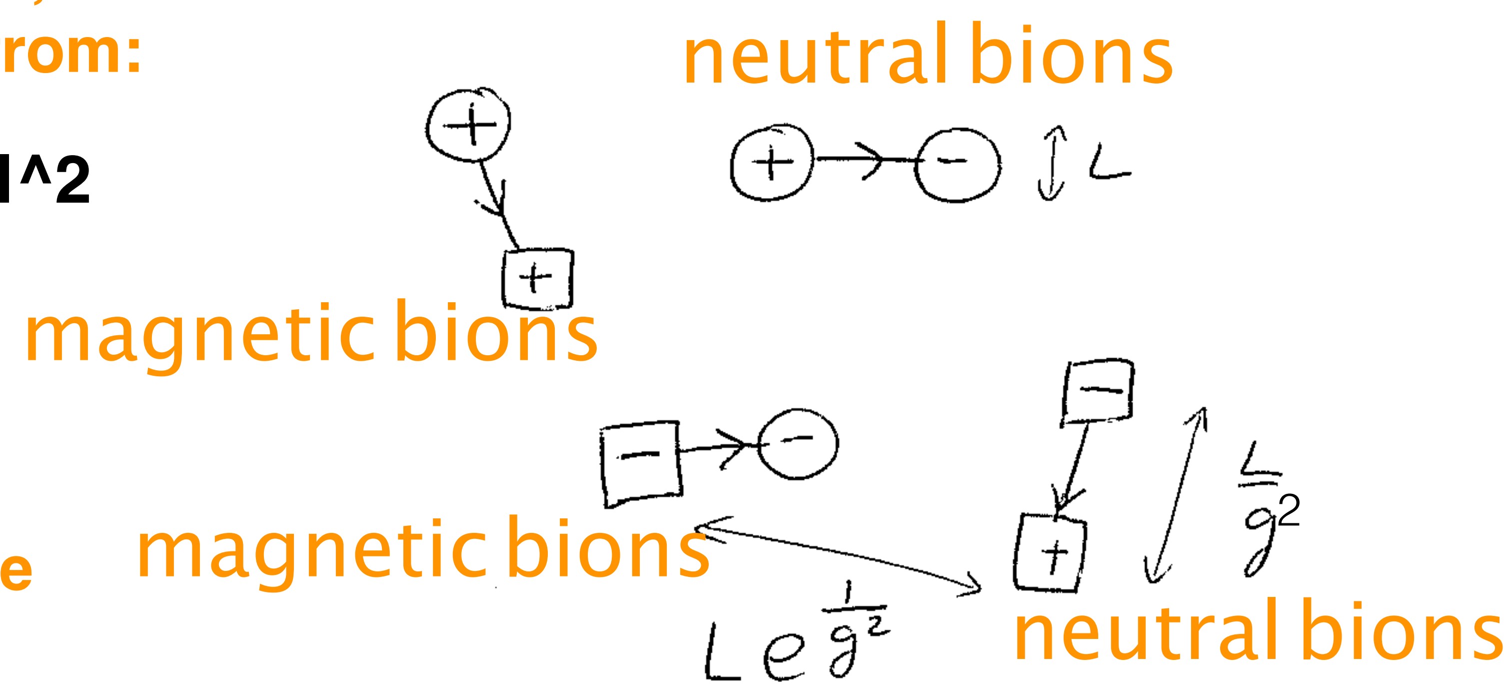
(neutral bions are responsible for center stability and also cancel magnetic bion vacuum energy in SYM)

Turns out, the MM* amplitude makes sense. Despite the attractive-only interactions, a "stable molecule" exists! We know from:

1. supersymmetry, exact W -> V=|W'|^2

2. analytic continuation:
MM* "live" at complex separation

Even more bizzare, the "fugacity" of the MM* objects is <0, ensuring E_vac = 0.



Motivation:

“neutral bions” are particularly bizarre: they are MM* “molecules”

(neutral bions are responsible for center stability and also cancel magnetic bion vacuum energy in SYM)

Turns out, the MM* amplitude makes sense. Despite the attractive-only interactions, a “stable molecule” exists! We know from:

1. supersymmetry, exact W -> V=|W’|^2

2. analytic continuation:
   MM* “live” at complex separation

Even more bizzare, the “fugacity” of the MM* objects is <0, ensuring E_vac = 0.

In semiclassics, any “lump” of positive fugacity lowers vacuum energy (e.g. double well). In SYM, there are “lumps” of both positive and negative fugacity, with equal and opposite contributions to E_vac.

Complexification crucial. Hypothesis that MM* lie on a different “Lefshetz thimble” from the perturbative vacuum - distinguished by a phase (“HTA”)…?

Motivation:

already at weak coupling, a major difficulty:
"How to define & calculate multi-instanton contributions?"

Not merely a question of calculating exponentially suppressed effects.

Instanton—anti-instanton (I-I*), for example, contributions have been found to give the leading effect in many cases.

Ex. 1: SYM, mass gap….

Motivation:

already at weak coupling, a major difficulty:
"How to define & calculate multi-instanton contributions?"

Not merely a question of calculating exponentially suppressed effects.

Instanton—anti-instanton (I-I*), for example, contributions have been found to give the leading effect in many cases.

Ex. 1: SYM, mass gap….

Ex. 2: "Resurgent" cancellations: imaginary parts due to Borel resummation of perturbation theory vs imaginary parts of I-I*

high orders of perturbation theory
double-well QM, non Borel-summable:

ambiguity of Borel sum of pert. series:

$$E_{pert} = -\frac{3}{\pi}\sum_{k=0}^{\infty}(3g)^k k! \longrightarrow \delta E_{Borelsum} = -\frac{3}{\pi}\left(\mp\frac{i\pi}{3g}\right)e^{-\frac{1}{3g}}$$

II* contribution:
requires analytic continuation
Bogomolnyi, Zinn-Justin

$$E_{I\bar{I}} = \frac{1}{\pi g}\left(\mp i\pi + \log\frac{g}{2}\right)e^{-\frac{1}{3g}}$$

Motivation:

Complexification seems crucial. **Hypothesis/dream/ is that MM* lie on a different "Lefshetz thimble" from the perturbative vacuum and are distinguished from it by a phase associated with the thimble... "like" in 1dim integrals:**

$$I(\hbar) = \int_{-\infty}^{\infty} dx\, e^{-\frac{1}{\hbar} f(x)} \underbrace{\longrightarrow}_{\text{steepest descent method}} \sum_{\sigma} n_{\sigma} \int_{\mathcal{J}_{\sigma}} dz\, e^{-\frac{1}{\hbar} f(z)},$$

... see talks by Tanizaki, Dunne, Basar

I will show a "simple," yet not completely trivial, example supporting the need of complexification, in N=2 SUSY QM.

The choice of this example is motivated by QFT: Seiberg-Witten theory on R^3xS^1. In 2011 work with Unsal, we asked "Why don't I-I* molecules on the compact (nonzero holonomy) Coulomb branch of the theory contribute a potential?"

Now, in a supersymmetric theory, every kid knows the answer: potential has to come from W - which is forbidden by N=2 SUSY - or too many zero modes. Nonetheless, without invoking SUSY machinery, we are still not sure of the answer - see end of talk.

The goal of the work I will present is to examine the same question in the simpler context of N=2 SUSY QM, hoping that lessons will be useful... - see end of talk.

Subject of talk:

four real supercharges

N=2 SUSY QM = 4d WZ model reduced to 2d

$$g\mathcal{L}_E = |\dot{z}(t)|^2 + |W'(z)|^2 + \begin{pmatrix} \bar{\chi}_1 & \chi_2 \end{pmatrix} \left(-\partial_t + \begin{pmatrix} 0 & \overline{W''(z)} \\ W''(z) & 0 \end{pmatrix} \right) \begin{pmatrix} \chi_1 \\ \bar{\chi}_2 \end{pmatrix}$$

$$W(z) = \prod_{i=1}^{k+1} (z - z_i) \qquad |I_W| = k$$

Witten index=number of critical points of W(z)

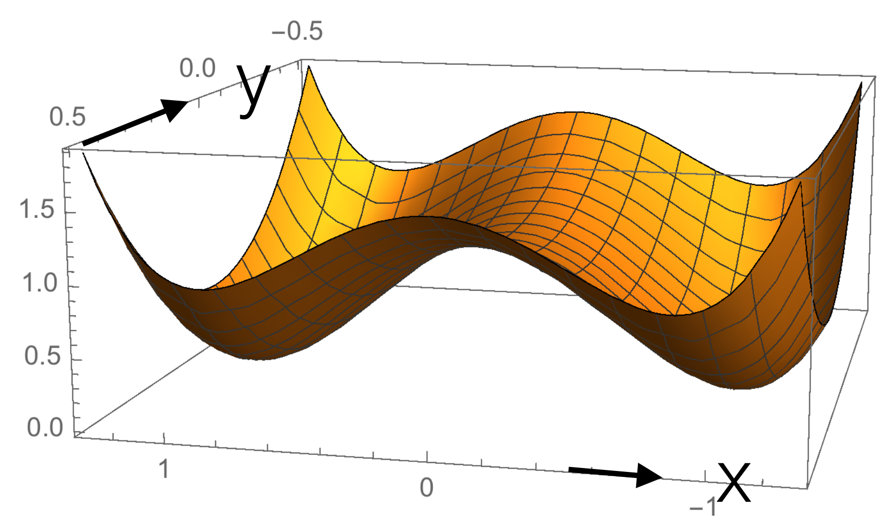
E_vac=0, as opposed to N=1 SUSY QM

$$W(z) = \tfrac{1}{3} z^3 - z a^2$$

number of SUSY ground states = number of critical points

here, ground states all bosonic (ITEP) or all fermionic (IPMU)

potential |W'|^2:



plot for a=1

Subject of talk:

N=2 SUSY QM = 4d WZ model reduced to 2d

$$g\mathcal{L}_E = |\dot{z}(t)|^2 + |W'(z)|^2 + \begin{pmatrix}\bar{\chi}_1 & \chi_2\end{pmatrix}\left(-\partial_t + \begin{pmatrix}0 & \overline{W''(z)} \\ W''(z) & 0\end{pmatrix}\right)\begin{pmatrix}\chi_1 \\ \bar{\chi}_2\end{pmatrix}$$

$$W(z) = \prod_{i=1}^{k+1}(z - z_i) \qquad |I_W| = k$$

Witten index=number of critical points of W(z)

E_vac=0, as opposed to N=1 SUSY QM

$$W(z) = \tfrac{1}{3}z^3 - za^2$$

number of SUSY ground states = number of critical points

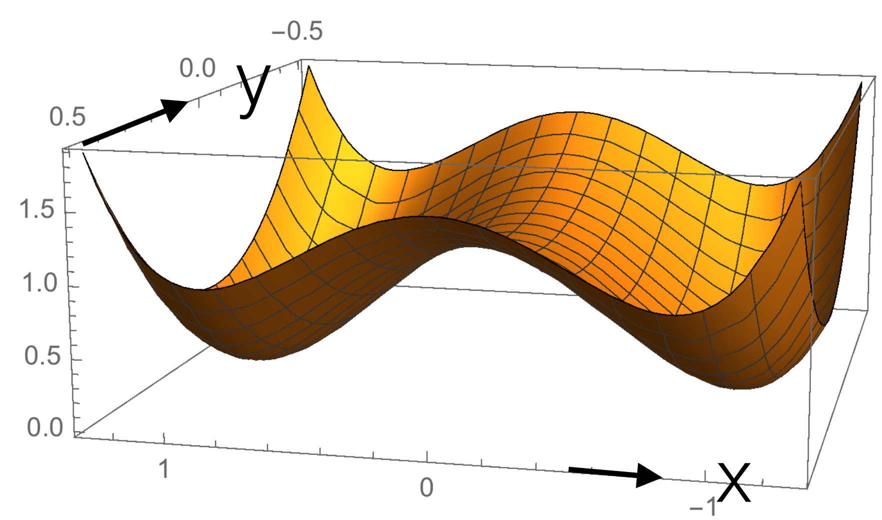
here, ground states all bosonic (ITEP) or all fermionic (IPMU)

BPS (anti)instantons

$$\dot{z} = \pm\overline{W'}$$

$I \rightleftarrows$

$I^* \leftrightarrows$



plot for a=1

$$S_{inst} = \frac{8}{3g}$$

I,I*: tunnelling between minima; two fermion zero modes each

(with opposite “chirality” from 4d p.o.v.)

Subject of talk:

N=2 SUSY QM = 4d WZ model reduced to 2d

$$g\mathcal{L}_E = |\dot{z}(t)|^2 + |W'(z)|^2 + \begin{pmatrix} \bar{\chi}_1 & \chi_2 \end{pmatrix} \left(-\partial_t + \begin{pmatrix} 0 & \overline{W''(z)} \\ W''(z) & 0 \end{pmatrix} \right) \begin{pmatrix} \chi_1 \\ \bar{\chi}_2 \end{pmatrix}$$

$$W(z) = \frac{1}{3} z^3 - z a^2$$ **a=1 from now on!**

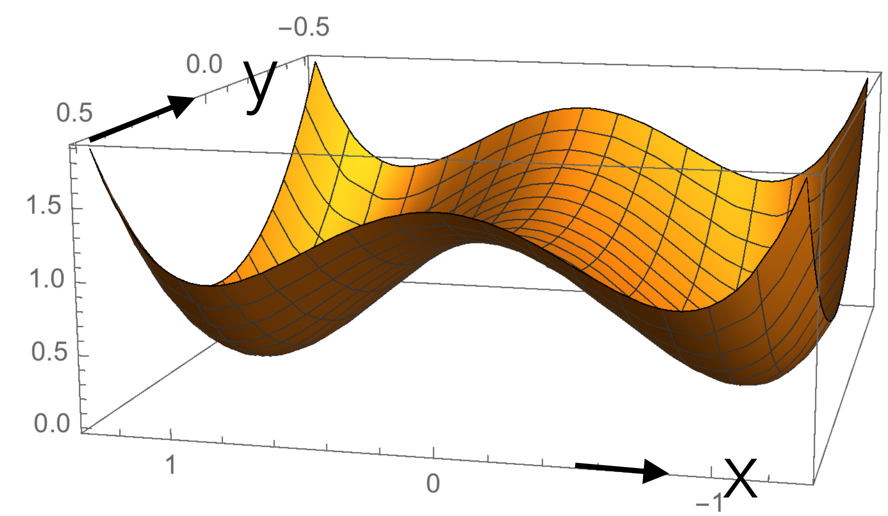
Goal: **Understand E_vac = 0 from next-order semiclassics.**
No supersymmetry, deformation invariance, localization…

(relation to motivation: why no I-I* molecules in SW?)

Upshot: **It's not completely trivial.**

$I \rightleftarrows$ $$x_I = -\tanh t$$

$I^* \leftrightarrows$ $$x_{I^*} = \tanh t$$

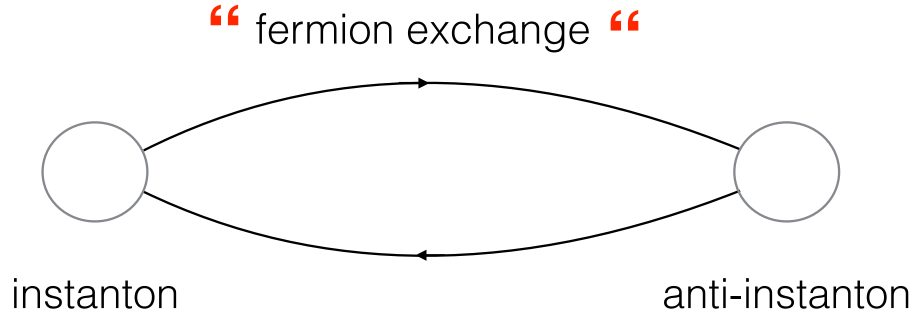


consider an I and an I* very, very,… far apart:

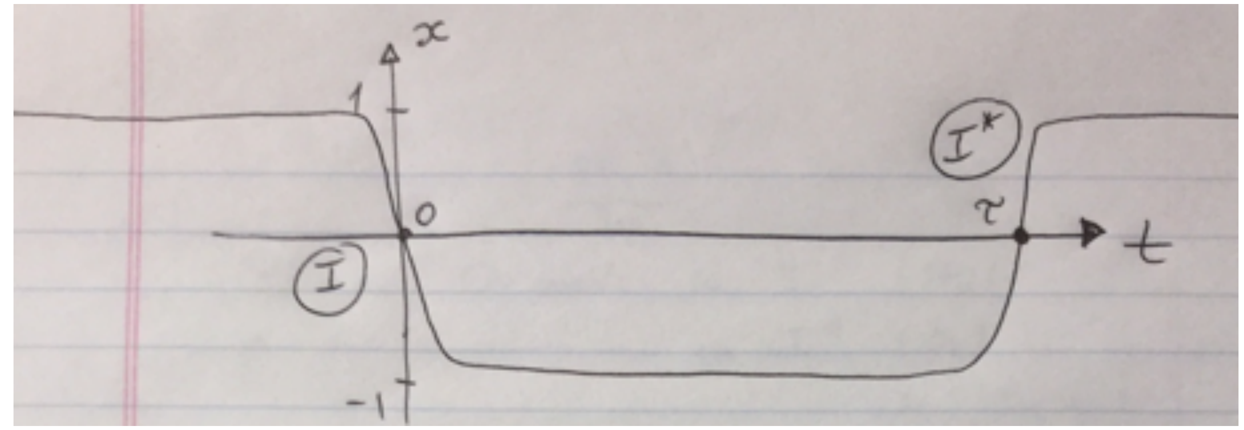
$$x(t,\tau) = x_I(t) + x_{I*}(t-\tau) + 1$$

Main part of talk:

after all, expect that the far away I* will lift the zero modes of I:



$$x(t,\tau) = x_I(t) + x_{I*}(t-\tau) + 1$$



$$(\bar\chi_1\ \chi_2)\begin{pmatrix}-\partial_t & 2x(t,\tau)\\ 2x(t,\tau) & -\partial_t\end{pmatrix}\begin{pmatrix}\chi_1\\ \bar\chi_2\end{pmatrix} = \ldots$$

$$\begin{pmatrix}\chi_1\\ \bar\chi_2\end{pmatrix} = \psi\begin{pmatrix}1\\ 1\end{pmatrix} + \eta\begin{pmatrix}1\\ -1\end{pmatrix}$$

$$(\bar\chi_1\ \chi_2) = \rho\,(1\ \ 1) + \chi\,(1\ \ -1)$$

$$\ldots = 2\rho\underbrace{(-\partial_t + 2x(t,\tau))}_{\bar D}\psi + 2\chi\underbrace{(-\partial_t - 2x(t,\tau))}_{-D}\eta$$

$$= \rho\bar D\psi + \psi(-D)\rho + \eta\bar D\chi + \chi(-D)\eta$$

$$= (\rho\ \ \psi)\begin{pmatrix}0 & \bar D\\ -D & 0\end{pmatrix}\begin{pmatrix}\rho\\ \psi\end{pmatrix} + (\rho\to\eta,\ \psi\to\chi)$$

each term gives Pfaffian, so the product is

$$\det D\bar D = \prod_n \omega_n = \omega_0 \prod_n{}' \omega_n$$

$$\omega_0 \to 0 \quad \text{as } \tau\to\infty$$

e.g.

$$\rho\bar D\psi = \rho\left(-\partial_t - 2\,\mathrm{th}\,t + 2 + 2\,\mathrm{th}(t-\tau)\right)\psi$$

$$(-\partial_t - 2\,\mathrm{th}\,t)\underbrace{\frac{1}{\mathrm{ch}^2 t}}_{\psi_0} = 0$$

ψ has 0-mode in I (ψ_0)

ρ — " —— " — in I* (ρ_0)

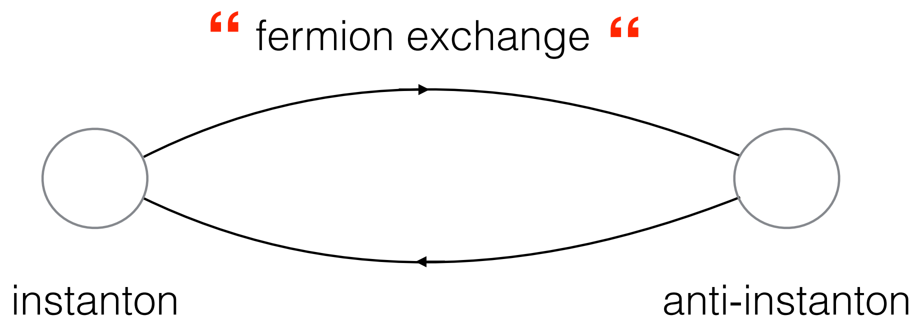
perturbation, small near I,
where I-zero mode localized,
lifts zero eigenvalue by exp. small amount

$$\sqrt{\omega_0} = \int \rho_0(t)\left(2 + 2\,\mathrm{th}(t-\tau)\right)\psi_0(t)\,dt$$

$$= 12e^{-2\tau} \to 0 \quad \text{as } \tau\to\infty$$

Main part of talk:

after all, expect that the far away I* will lift the zero modes of I:



indeed, the contribution to Z from this “fermion exchange graph” is, for fixed separation $\mathcal{T}$:

$$\sim + 144\, e^{-4\tau}\; e^{\frac{32}{g} e^{-2\tau}} \prod_{n\neq 0} \omega_n$$

fermion “exchange”

classical I-I* attraction

factorizes into I-det times I*-det (1+further exp’s)

Major issues:

1. >0 contribution to Z would give <0 E_vac if exponentiated

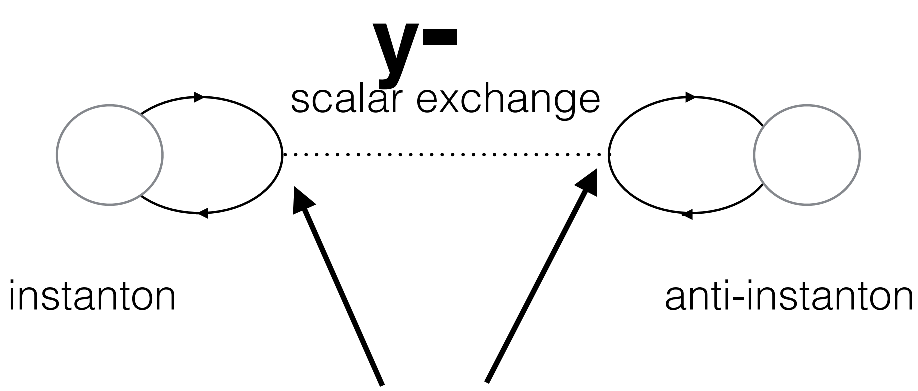
2. we have to integrate over separation - but when $\mathcal{T}$ becomes order one, all of the above is nonsense - as it was derived in the large separation limit

How can this ever make sense? [the index - or Witten - “can’t lie”!?]. Some hints:

- a difference between N=1 and N=2 is the presence of extra scalars
- the Pfaffian became a Det in the x-only background, with y-background ignored

Main part of talk:

$$\chi_1 W'' \chi_2 = \chi_1 (x(t) + i y(t)) \chi_2$$



Yukawa squared = g

the zero eigenvalue of an I at t_1 lifted to Yukawa x square of zero mode wave-function:

$$\frac{3}{2} \int dt \, \frac{y(t)}{\cosh^4(t-t_1)}$$

propagator in I-I*!

classical I-I* attraction, as before

$$\frac{9}{4} \int dt \int dt' \frac{\langle y(t) y(t') \rangle}{\cosh^4(t) \cosh^4(t-\tau)} \; e^{\frac{32}{g} e^{-2\tau}} \prod_{n \neq 0} \omega_n$$

propagator in I-I* is the technically most challenging part of this calculation - not exactly known, only an ~ expression to accuracy $e^{-2\tau}$ $(\tau = t_1 - t_2)$ **:**

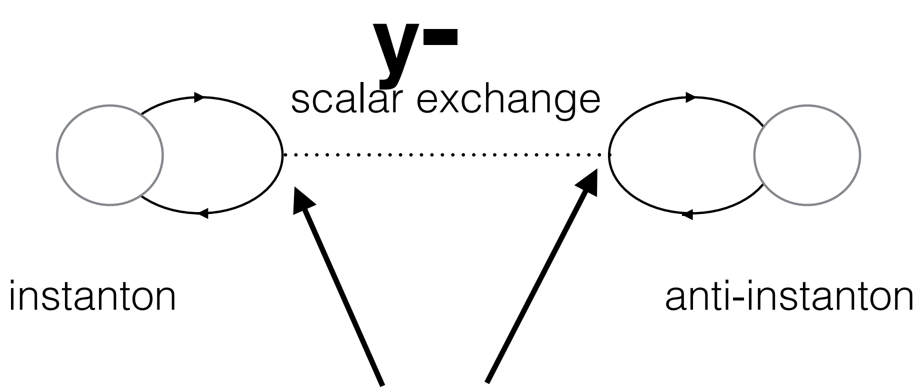
$$\langle y(t) y(t') \rangle = \frac{g}{8a} e^{-2a|t-t'|} \; g(t,t';t_1) \; g(t,t';t_2) \; + \ldots$$

where g(…) is (part of) the propagator in single-I

$$g(t,t';t_0) = -\frac{1}{3} \left\{ 2 \, \mathrm{sign}(t-t') + \tanh[a(t-t_0)] \right\} \times \left\{ -2 \, \mathrm{sign}(t-t') + \tanh[a(t'-t_0)] \right\}$$

Main part of talk:

$$\chi_1 W'' \chi_2 = \chi_1 (x(t) + i y(t)) \chi_2$$



Yukawa squared = g

the zero eigenvalue of an I at t_1 lifted to Yukawa x square of zero mode wave-function:

$$\frac{3}{2} \int dt \, \frac{y(t)}{\cosh^4(t - t_1)}$$

propagator in I-I*!

classical I-I* attraction, as before

$$\frac{9}{4} \int dt \int dt' \frac{\langle y(t) y(t') \rangle}{\cosh^4(t) \cosh^4(t - \tau)} \; e^{\frac{32}{g} e^{-2\tau}} \prod_{n \neq 0} \omega_n$$

After dust settles, left with (up to common tau-independent factors) two positive contributions to Z:

scalar exchange

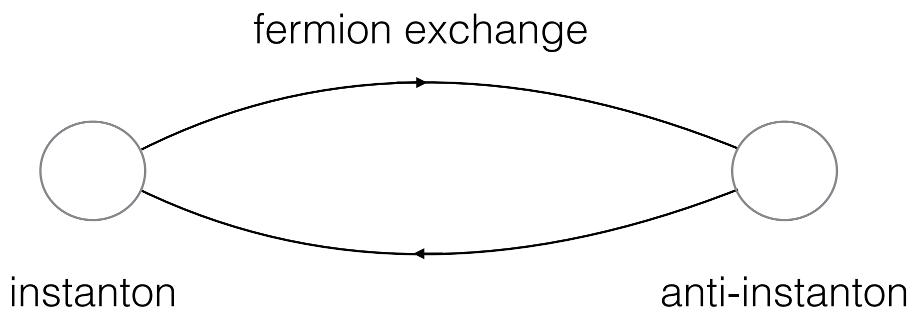
$$\sim + \frac{9}{2} g \, e^{-2\tau} \quad e^{\frac{32}{g} e^{-2\tau}}$$

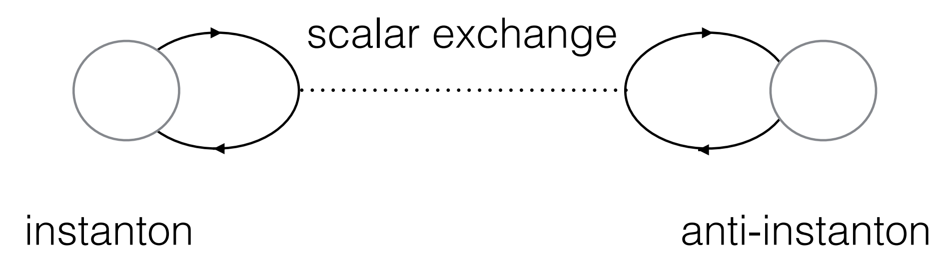
fermion exchange (from two slides ago)

$$\sim + 144 \, e^{-4\tau} \; e^{\frac{32}{g} e^{-2\tau}}$$

If exponentiated, we'd still have E_vac < 0, not having solved our problem…?

Main part of talk:





$$+\ 144\, e^{-4\tau}\ e^{\frac{32}{g} e^{-2\tau}}$$

$$+\ \frac{9}{2}\, g\, e^{-2\tau}\ \ e^{\frac{32}{g} e^{-2\tau}}$$

same sign (up to common factors, not dependent on tau)

different order in the coupling "g"

different order in $e^{-2\tau}$

valid at asymptotically large τ **only**

proposed reconciliation:

integrate each contribution on steepest descent paths in the tau-plane *(=one-dimensional projection of Lefshetz thimbles path integral?)*

Main part of talk:

F: $0 = -4 - 2 \times \frac{32}{g} e^{-2\tau}$

$$e^{-2\tau_*^F} = e^{-i\pi + \log\frac{16}{g}} = -\frac{16}{g}$$

$$+\ 144\, e^{-4\tau}\ e^{\frac{32}{g} e^{-2\tau}}$$

S: $0 = -2 - 2 \times \frac{32}{g} e^{-2\tau}$

$$e^{-2\tau_*^S} = e^{-i\pi + \log\frac{32}{g}} = -\frac{32}{g}$$

$$+\ \frac{9}{2}\, g\, e^{-2\tau}\ e^{\frac{32}{g} e^{-2\tau}}$$

same sign (up to common factors, not dependent on tau)

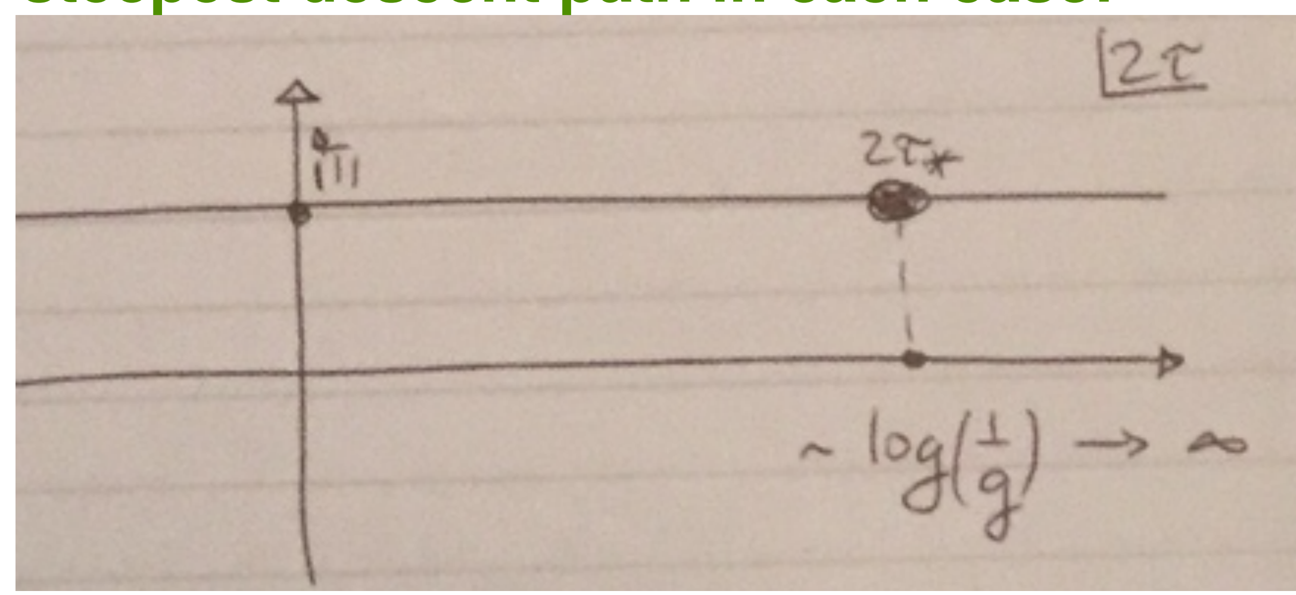
different order in the coupling "g"

different order in $e^{-2\tau}$

valid at asymptotically large τ **only**

↑

steepest descent path in each case:

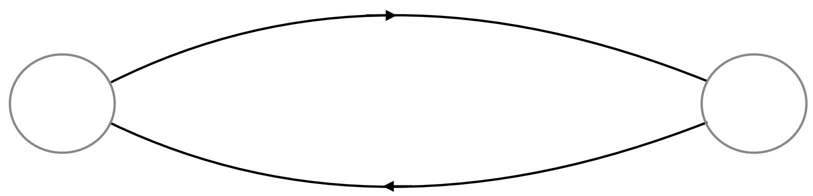


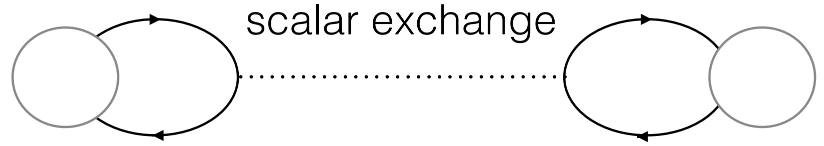
notice how all these issues are taken care of!

(by contour integration - in, I think, a rather unusual and intricate manner)

- only remains to do the integral E_vac?=0

Main part of talk:





$$+\ 144\, e^{-4\tau}\ e^{\frac{32}{g}e^{-2\tau}} \qquad\qquad +\ \frac{9}{2}\, g\, e^{-2\tau}\ e^{\frac{32}{g}e^{-2\tau}}$$

Indeed, after shifting the contour, $2\tau \to i\pi + 2\tau$**, we obtain integral**

$$\int d\tau e^{-\frac{32}{g}e^{-2\tau}} \left(-\frac{9g}{2}e^{-2\tau} + 144 e^{-4\tau}\right)$$ **= 0 if integrated over the entire thimble**

Reminder:

Goal: **Understand E_vac = 0 from plain next-order semiclassics … no localization, no deformation invariance…**

Upshot: **It's not completely trivial.**

Results: we have shown this vanishing to next to leading semiclassics: order e^(- 2 S_inst)

Main part of talk/summary:

Goal: Understand E_vac = 0 from plain next-order semiclassics
… no localization, no deformation invariance…

Upshot: It’s not completely trivial.

Salient points:

- Complexifying the quasi-zeromode was crucial
- I and I* seem to “live” a complex & large (i.e. consistent semiclassically) separation apart
- Extra-scalar exchange; interplay between higher orders in g and saddle point integral.
- None of the luxury of N=1 SUSY QM: no local effective fermion-less theory where I-I* ’molecule’ is an exact saddle *(in that way ‘closer’ to QFT?)*

Main part of talk/discussion:

Goal: **Understand E_vac = 0 from plain next-order semiclassics … no localization, no deformation invariance…**

Upshot: **It's not completely trivial.**

Compare other points of view:

A vs Yung's point of view- I am putting words in his mouth, taken from his N=1 4d SQCD work:

our integrand is, in fact, a double total derivative! $\frac{9\ g^2}{256}\ \partial_\tau^2\ e^{\frac{32}{g}e^{-2\tau}} = \left(\frac{9}{2}\ ge^{-2\tau} + 144\ e^{-4\tau}\right) e^{\frac{32}{g}e^{-2\tau}}$

if integrated over the naive 0->infinity contour on real axis, would give 0 at infinity; at the origin, advocate "picture" that at tau=0 I+I* is = perturbative contribution, set to zero by SUSY

- cf. Yung's derivation of superpotential in SQCD

in contrast, in the thimble case, the main contribution comes from a region near saddle point, where semiclassical calculation is believable

Main part of talk/discussion:

Goal: **Understand E_vac = 0 from plain next-order semiclassics … no localization, no deformation invariance…**

Upshot: **It's not completely trivial.**

Compare other points of view:

B vs "BZJ prescription": integrate over $\tau \in \mathbb{R}^+$ with $g \to -g$; then $-g \to e^{i\pi}(-g)$

follow BZJ word for word, we obtain an exponentially small result at small (negative) g

$$E_0(-g) \sim -e^{2S_{inst}} \int_0^\infty e^{-\frac{32}{g}e^{-2\tau}} (144 e^{-4\tau} - \frac{9g}{2} e^{-2\tau}) = +\frac{9g}{4}\, e^{2S_{inst} - \frac{32}{g}} = +\frac{9g}{4} e^{-\frac{80}{3g}}$$

- BZJ throw it out *before* continuing back (without discussion… generic there)

if we don't, we are left with $E_0(g) \sim -\frac{9g}{4}\, e^{+\frac{80}{3g}}$ - exp. large & in conflict with SUSY…

in the thimble calculation no such issue arises…

Main part of talk/discussion:

Goal: **Understand E_vac = 0 from plain next-order semiclassics … no localization, no deformation invariance…**

Upshot: **It's not completely trivial.**

Compare other points of view:

B vs "BZJ prescription": integrate over $\tau \in \mathbb{R}^+$ with $g \to -g$; then $-g \to e^{i\pi}(-g)$

follow BZJ word for word, we obtain an exponentially small result at small (negative) g

$$E_0(-g) \sim -e^{2S_{inst}} \int_0^\infty e^{-\frac{32}{g}e^{-2\tau}}(144e^{-4\tau} - \frac{9g}{2}e^{-2\tau}) = +\frac{9g}{4}\, e^{2S_{inst}-\frac{32}{g}} = +\frac{9g}{4}e^{-\frac{80}{3g}}$$

- BZJ throw it out *before* continuing back (without discussion… generic there)

if we don't, we are left with $E_0(g) \sim -\frac{9g}{4}\, e^{+\frac{80}{3g}}$ - exp. large & in conflict with SUSY…

in the thimble calculation no such issue arises…

So, it looks like thimble is the way to go… **still, have we "really understood" it all?**

Final comments etc. - **mostly things I, *not my collaborators!*, am confused or don't know about…**

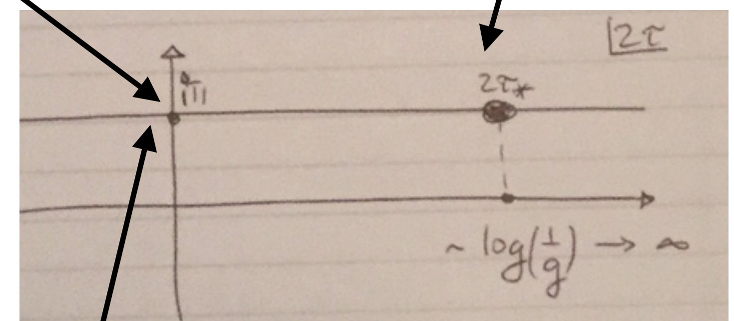
1

recall we integrated over entire contour - which comes close to origin

perhaps, one can deform contour far enough away from the origin (so that unit absolute values of tau are never approached) to justify integration over entire contour - but the exact vanishing over the thimble begs an explanation (why not vanish only up to e^(-2S_inst)?) - is it an accident?

but not here

integrand believable here



if we stop integration here, we get ~ e^(-12S_inst), well below next e^(-4S_inst) order - but order depends sensitively on cutoff

2

not unrelated: technology we used is *very 1960's* (e.g. Langer's paper on the condensation point)

the structures lurking probably beg for more…

otherwise, imagine how one could do higher orders??

Final comments etc. - **mostly things I, *not my collaborators!*, am confused or don't know about…**

(my grandmother)

Final comments etc. - **mostly things I, *not my collaborators!*, am confused or don't know about…**

(my grandmother)

Well, my young gentleman, what it is exactly that you think you are doing?

Final comments etc. - mostly things I, *not my collaborators!*, am confused or don't know about…

(my grandmother)

Well, my young gentleman, what it is exactly that you think you are doing?

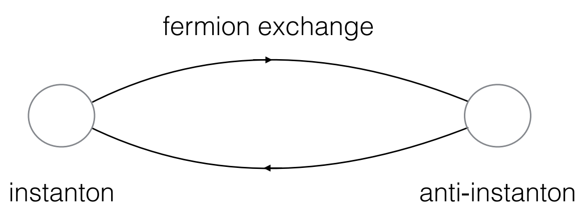
(**experimental "mathematics"?**
handicraft? *кустарничество…?*)

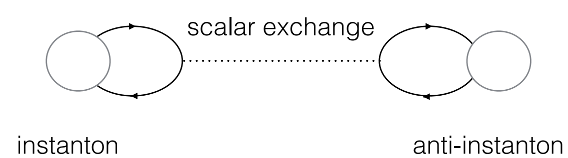
Final comments etc. - **mostly things I, *not my collaborators!*, am confused or don't know about…**

3 what lessons can we draw for QFT - the original motivation, SW theory on R^3 x S^1? does the SUSY QM calculation allow itself to be 'bootstrapped' to QFT?

To this end, consider first the heuristic argument about role of thimbles in N=2 SUSY QM.

1. That these two contributions are present requires no calculation:





2. Estimating scaling with tau also does not require calculation:

classical II* attraction

$$e^{\frac{32}{g}e^{-2\tau}} \quad e^{-4\tau} \qquad\qquad e^{\frac{32}{g}e^{-2\tau}} \quad g e^{-2\tau}$$

fermion exchange:
two 1d massive particles, mass=2

scalar exchange:
one 1d massive particle

3. The fact that these have a chance of cancelling if done by saddle point is now obvious.
   The hard work is to get the coefficients - which we showed were right on!

Natural question: do MM 'molecules' in SW theory have a similar chance to cancel?*

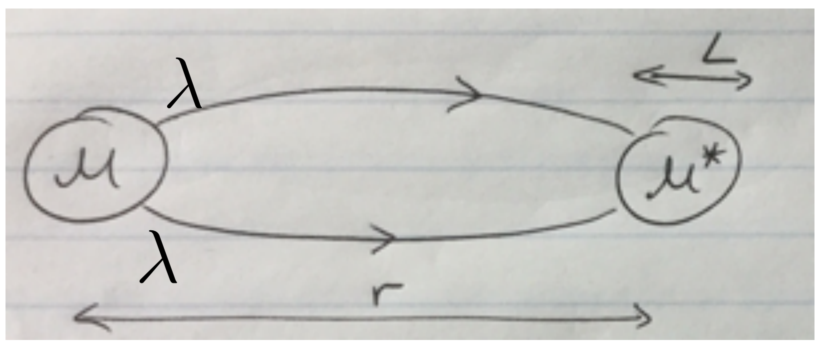
Final comments etc. - **mostly things I, *not my collaborators!*, am confused or don't know about…**

3 what lessons can we draw for QFT - the original motivation, SW theory on R^3 x S^1? does the SUSY QM calculation allow itself to be 'bootstrapped' to QFT?

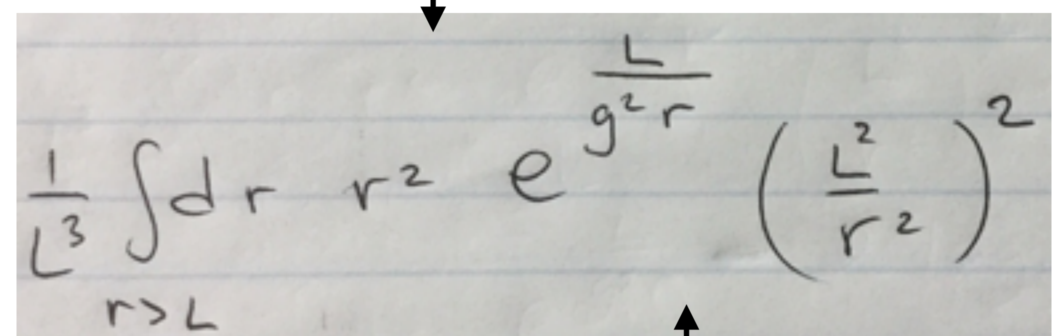
Natural question: do MM 'molecules' in SW theory* have a similar chance *to cancel?*

N=1 SYM, first, on R^3 x S^1 (size L, small):

SU(2)



magnetic Coulomb+scalar (holonomy) attraction



two massless fermion propagators

all interactions attractive - use SUSY, or BZJ, or HTA — nonzero contribution, physics…

HTA = BSSU = Behtash, Schaefer, Sulejmanpasic, Unsal

N=2 SYM on the compact Coulomb branch:

- no classical coupling of M/M* to extra adjoint scalar ϕ
- two more fermion zero modes of M/M* (another adjoint Weyl fermion ψ)
- Yukawa coupling to adjoint scalar: $\sim \lambda\, \psi\, \phi$

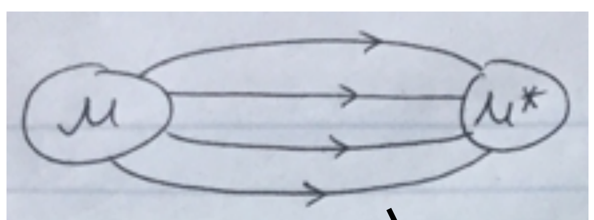
Final comments etc. - **mostly things I, *not my collaborators!*, am confused or don't know about…**

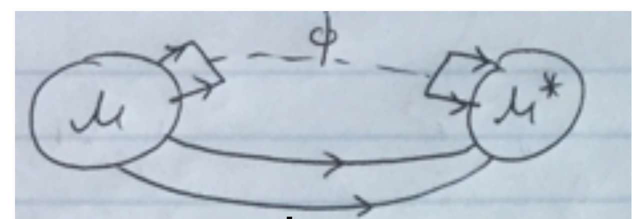
3 what lessons can we draw for QFT - the original motivation, SW theory on R^3 x S^1? does the SUSY QM calculation allow itself to be 'bootstrapped' to QFT?

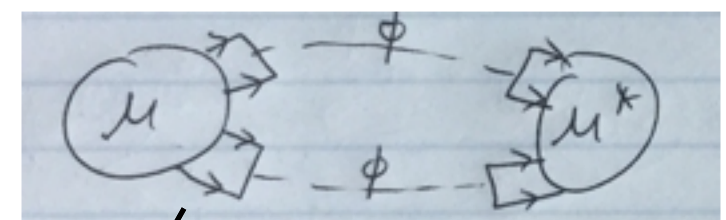
Natural question: do MM 'molecules' in SW theory* have a similar chance *to cancel?*

N=2 SYM on the compact Coulomb branch:

- no classical coupling of M/M* to extra adjoint scalar ϕ
- two more fermion zero modes of M/M* (another adjoint Weyl fermion ψ)
- Yukawa coupling to adjoint scalar: $\sim \lambda\,\psi\,\phi$







L=1 $$\int_{r>1} dr\, r^2\, e^{\frac{1}{g^2 r}} \left(\left(\frac{1}{r^2}\right)^4 + g^2\left(\frac{1}{r^2}\right)^2 \frac{1}{r} + g^4 \frac{1}{r^2} \right)$$

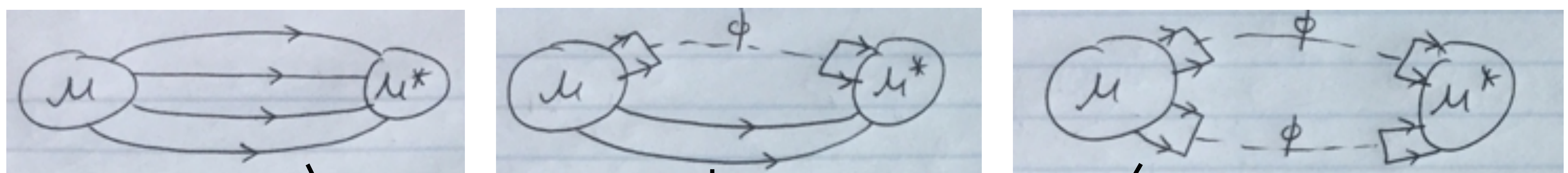
Final comments etc. - **mostly things I, *not my collaborators!*, am confused or don’t know about…**

3 what lessons can we draw for QFT - the original motivation, SW theory on R^3 x S^1? does the SUSY QM calculation allow itself to be ‘bootstrapped’ to QFT?

Natural question: do MM ‘molecules’ in SW theory* have a similar chance *to cancel?*

N=2 SYM on the compact Coulomb branch:

- no classical coupling of M/M* to extra adjoint scalar ϕ
- two more fermion zero modes of M/M* (another adjoint Weyl fermion ψ)
- Yukawa coupling to adjoint scalar: $\sim \lambda \, \psi \, \phi$



L=1

$$\int_{r>1} dr \, r^2 \, e^{\frac{1}{g^2 r}} \left(\left(\frac{1}{r^2}\right)^4 + g^2 \left(\frac{1}{r^2}\right)^2 \frac{1}{r} + g^4 \frac{1}{r^2} \right)$$

$\frac{1}{r_*} = -6g^2$ $\quad$ $\frac{1}{r_*} = -3g^2$ $\quad$ $\frac{1}{r_*} = 0$

saddle point values, including measure r^2: $\sim (g^2)^6$ $\quad$ $\sim -(g^2)^4$ $\quad$ $\sim (g^2)^2$

The relative minus sign between the first two terms is encouraging!

But the g^2-power counting suggests that this “calculation” is lacking… ***WHAT?***