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Holographic models for QCD in the Veneziano limit

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Holography and QCD – IPMU – 24 September 2013

Outline

1. Introduction and motivation
2. Holographic V-QCD models

   [MJ, Kiritsis arXiv:1112.1261]

3. Finite temperature and chemical potential

   [Alho, MJ, Kajantie, Kiritsis, Tuominen arXiv:1210.4516, arXiv:13nn.xxxx]

4. Spectrum, dilaton and the S-parameter

   [Arean, Iatrakis, MJ, Kiritsis, arXiv:1211.6125, arXiv:1309.2286]

1. Introduction

(Holographic) QCD in the 't Hooft limit

- QCD: $SU(N_c)$ gauge theory with N_f quark flavors (fundamental)
- The 't Hooft limit:
  $N_c \to \infty$, N_f fixed (and $\lambda = g^2 N_c$ fixed)
- Quarks probe the glue dynamics: "quenched" or "probe" approximation, $N_f \ll N_c$
- Many interesting results using probe branes

(Holographic) QCD in the Veneziano limit

- The Veneziano limit:
  $N_c \to \infty$ and $N_f \to \infty$ with $x = N_f/N_c$ fixed
- Both glue and flavor dynamics at leading order: flavor fully backreacts to the glue
- QCD string picture: boundaries of diagrams no longer suppressed
- Need to go beyond probe branes $\Rightarrow$ complications

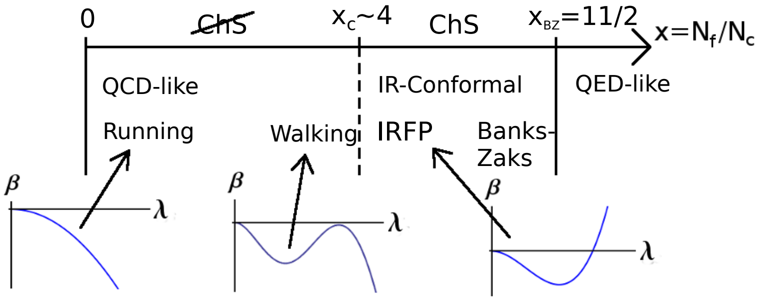
Why holography in Veneziano limit?

- New phases and features expected to appear
  - Phase diagram (at zero temperature, baryon density, and quark mass), varying $x = N_f/N_c$
  - Turning on finite T and μ at finite x
  - Phase with "walking" dynamics $\Rightarrow$ technicolor
- Improved holographic modeling of ordinary QCD?
- Field theory computations tricky at finite N_f/N_c
  - Truncated Dyson-Schwinger equations
  - Inspiration from supersymmetric QCD
  - Lattice: finite size effects

QCD phases in the Veneziano limit

Expected structure at zero T, μ, and quark mass; finite $x = N_f/N_c$

- Phases:
  - $0 < x < x_c$: QCD-like IR, chiral symmetry broken
  - $x_c \leq x < 11/2$: Conformal window, chirally symmetric
- Conformal transition at $x = x_c$
- RG flow of the coupling: running, walking, or fixed point



2. V-QCD

First building block: Glue – 5D dilaton gravity

"Improved holographic QCD" (IHQCD): well-tested string-inspired bottom-up model for pure Yang-Mills

[Gursoy, Kiritsis, Nitti arXiv:0707.1324, 0707.1349]
[Gubser, Nellore arXiv:0804.0434]

Dilaton gravity:

- $\phi \;\leftrightarrow\; \mathrm{Tr}(F^2) \qquad g_{\mu\nu} \;\leftrightarrow\; T_{\mu\nu}$

$$\mathcal{S}_{\mathrm{g}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$

- $\lambda = e^{\phi} \to$ 't Hooft coupling $g^2 N_c$
- Potential $V_g \leftrightarrow$ Yang-Mills β-function

Good agreement with pure YM lattice data, both at zero and finite temperature

[Gursoy, Kiritsis, Mazzanti, Nitti; Panero; . . .]

Second building block: Adding flavor

A recipe for adding quarks (in the fundamental of $SU(N_c)$ and in the probe approximation)

- Space-filling probe $D4 - \bar{D}4$ branes in 5D $\longrightarrow$
  - Tachyon $\quad T \quad \leftrightarrow \bar{q}q$
  - Gauge fields $\quad A^{\mu}_{L/R} \leftrightarrow \bar{q}\gamma^{\mu}(1 \pm \gamma_5)q$
- For the vacuum structure only the tachyon is relevant
- A tachyon action motivated by the Sen action
  - Confining IR asymptotics of the geometry triggers ChSB
  - Gell-Mann-Oakes-Renner relation
  - Linear Regge trajectories for mesons
  - A very good fit of the light meson masses

[Klebanov,Maldacena]

[Bigazzi,Casero,Cotrone,Iatrakis,Kiritsis,Paredes hep-th/0505140,0702155; arXiv:1003.2377,1010.1364]

Holographic V-QCD: the fusion

The fusion:

1. IHQCD: model for glue by using dilaton gravity
2. Adding flavor and chiral symmetry breaking via tachyon brane actions

Consider 1 + 2 in the Veneziano limit with full backreaction
$\Rightarrow$ V-QCD models

[MJ, Kiritsis arXiv:1112.1261]

Defining V-QCD

Degrees of freedom ($T = \tau\mathbb{I}$):

- The tachyon $\tau \leftrightarrow \bar{q}q$, and the dilaton $\lambda \leftrightarrow \mathrm{Tr}F^2$
- $\lambda = e^{\phi}$ is identified as the 't Hooft coupling $g^2 N_c$

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$- N_f N_c M^3 \int d^5x V_f(\lambda, \tau) \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau)}$$

$$V_f(\lambda,\tau) = V_{f0}(\lambda)\exp(-a(\lambda)\tau^2) \; ; \qquad ds^2 = e^{2A(r)}(dr^2 + \eta_{\mu\nu}x^\mu x^\nu)$$

Need to choose V_g, V_{f0}, a, and κ ...

The simplest and most reasonable choices do the job!

We have, however, explored generic choices

Matching to QCD

In the UV ($\lambda \to 0$):

- UV expansions of potentials matched with perturbative QCD beta functions $\Rightarrow$

$$\lambda(r) \simeq -\frac{\beta_0}{\log r} \qquad \tau(r) \simeq m(-\log r)^{-\gamma_0/\beta_0}\, r + \sigma(-\log r)^{\gamma_0/\beta_0}\, r^3$$

with $r \sim 1/\mu \to 0$

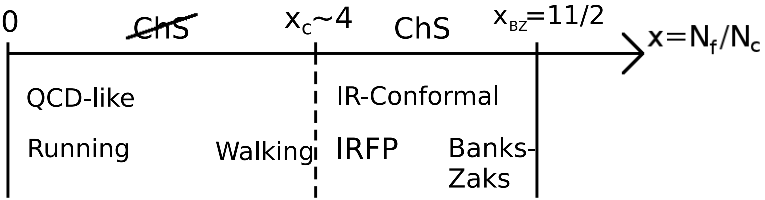
In the IR ($\lambda \to \infty$):

- $V_g(\lambda)$ chosen as for Yang-Mills, so that a "good" IR singularity exists
- $V_{f0}(\lambda)$, $a(\lambda)$, and $\kappa(\lambda)$ chosen to produce tachyon divergence: several possibilities ($\to$ Potentials I and II)
- Extra constraints from the asymptotics of the meson spectra
- Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (!)

Phase diagram of V-QCD

Different phases $\leftrightarrow$ different IR geometries

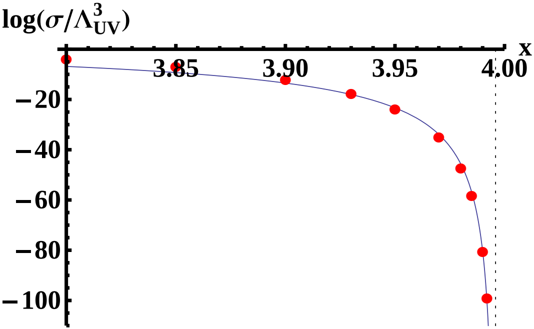
With reasonable potentials, at zero quark mass and temperature, constructing numerically all vacua:



- Meets (standard) expectations from QCD!
- Conformal transition at $x \simeq 4$

[Kaplan,Son,Stephanov;Kutasov,Lin,Parnachev]

Other important features



$$\langle \bar{q}q \rangle \sim \sigma \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

1. Miransky/BKT scaling as $x \to x_c$ from below
   - E.g., The chiral condensate $\langle \bar{q}q \rangle \propto \sigma$
     (From tachyon UV $\tau(r) \sim m_q(\log r)\, r + \sigma(\log r)\, r^3$)
2. Unstable Efimov vacua observed for $x < x_c$
3. Turning on the quark mass possible

3. V-QCD at finite T and μ

Finite T and μ – definitions

Add gauge field

$$\mathcal{S}_{\text{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right]$$
$$- N_f N_c M^3 \int d^5x V_f(\lambda, \tau)$$
$$\times \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)F_{ab})}$$

$$A_0 = \mu - nr^2 + \cdots$$

A more general metric, A and f solved from EoMs

$$ds^2 = e^{2A(r)} \left(\frac{dr^2}{f(r)} - f(r)dt^2 + d\mathbf{x}^2 \right)$$

Various solutions

Two classes of IR geometries:

1. Black hole solutions
   - $f'(r_h) = -4\pi T$; $\quad s = 4\pi M^3 N_c^2 e^{3A(r_h)}$
2. Thermal gas solutions ($f \equiv 1$)
   - Any T, zero s

Two types of tachyon behavior
(quark mass and condensate from UV boundary conditions):

1. Vanishing tachyon – chirally symmetric
2. Nontrivial tachyon – chirally broken

$\Rightarrow$ four possible types of background solutions

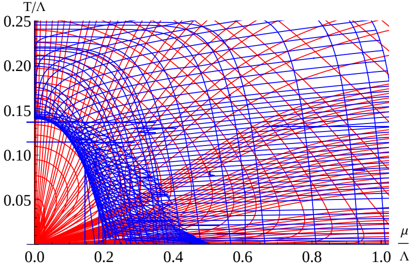
Calculate free energy or pressure in each case, determine the dominant solution

Computation of pressure

Three phases turn out to be relevant

Phases mapped to (μ, T)-plane

- Tachyonic Thermal gas (ChSB), all μ, T (not shown)
- Tachyonless BH (red)
- Tachyonic BH (blue)



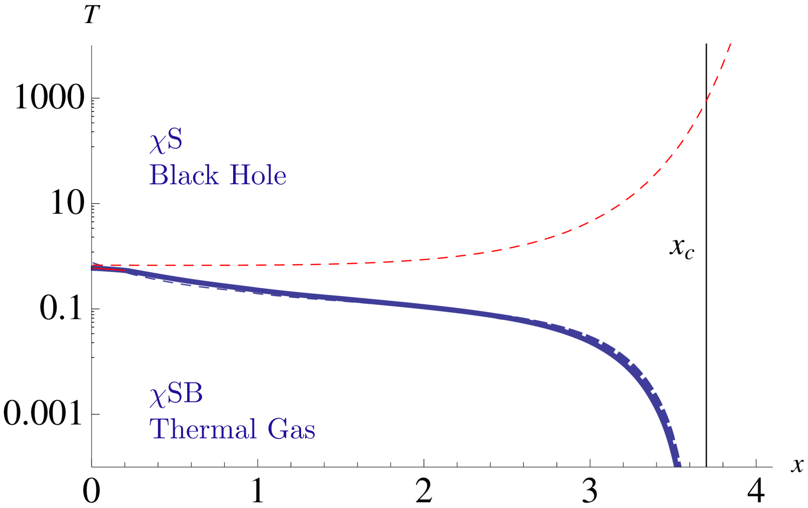
Integrate

$$dp = s\,dT + n\,d\mu$$

along the lines shown

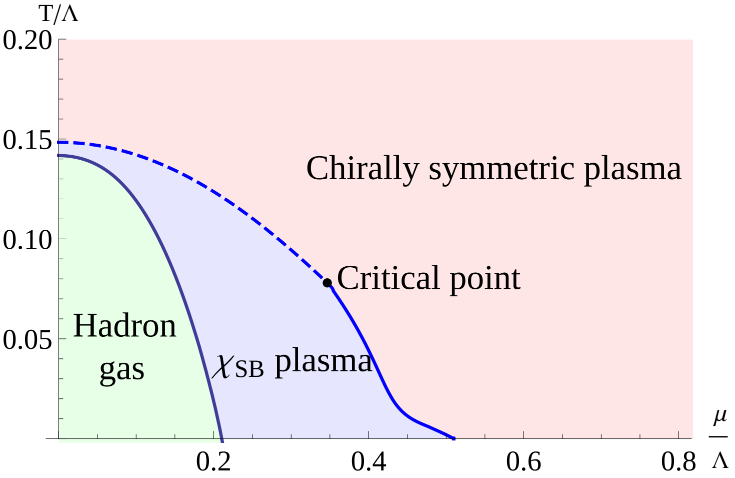
Phase diagram: example at zero μ

Phases on the (x, T)-plane – as expected from QCD



Phase diagram (example)

First attempt: $x = N_f/N_c = 1$, Veneziano limit, zero quark mass



4. Spectrum of V-QCD

Fluctuation analysis

Study at qualitative level:

1. Meson spectra (at zero temperature and quark mass)
   - Four towers: scalars, pseudoscalars, vectors, and axial vectors
   - Flavor singlet and nonsinglet ($SU(N_f)$) states
2. The S-parameter

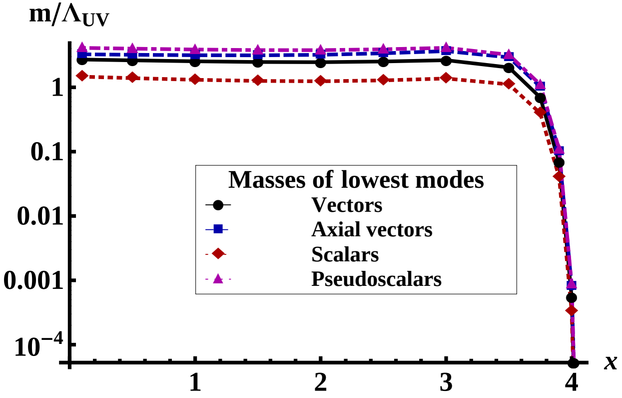
$$S \sim \frac{d}{dq^2} q^2 \left[\Pi_V(q^2) - \Pi_A(q^2)\right]_{q^2=0}$$

Open questions in the region relevant for "walking" technicolor ($x \to x_c$ from below):

- The S-parameter might be reduced
- Possibly a light "dilaton" (flavor singlet scalar): Goldstone mode due to almost unbroken conformal symmetry.
  The 125 GeV state seen at the LHC?

Meson masses

Flavor nonsinglet masses (Example: PotI)

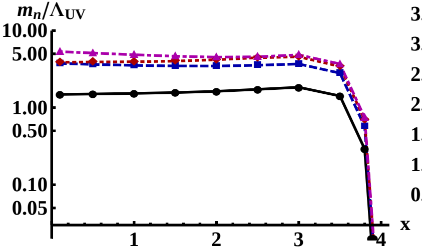


- All masses show Miransky scaling as $x \to x_c$, i.e., $m \sim \exp(-\kappa/\sqrt{x_c - x})$
- Radial trajectories $m_n^2 \sim n$ or $m_n^2 \sim n^2$ depending on potentials

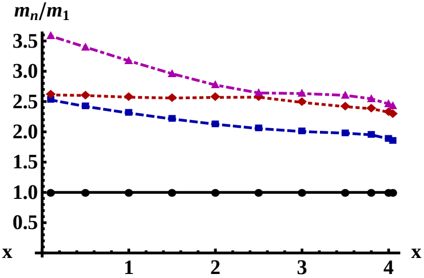
Scalar singlet masses

Scalar singlet spectrum (PotI):

In log scale



Normalized to the lowest state

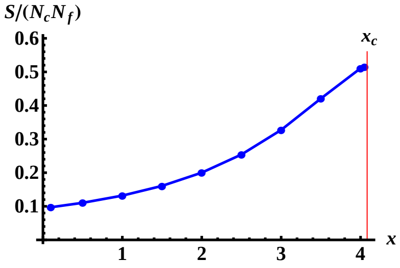


No light dilaton state as $x \to x_c$

S-parameter

$$S \sim \frac{d}{dq^2} q^2 \left[\Pi_V(q^2) - \Pi_A(q^2)\right]_{q^2=0}$$

PotI PotII



The S-parameter increases with x: expected suppression absent

Jumps discontinuously to zero at $x = x_c$

Summary

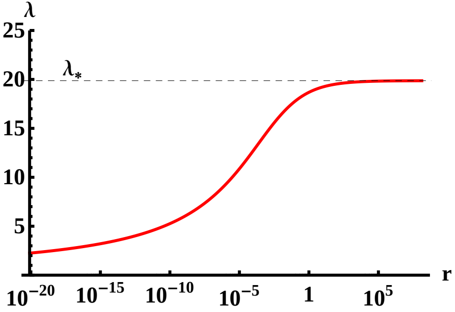
- We explored bottom up models for QCD in the Veneziano limit
- A class of models, V-QCD, was obtained by a fusion of IHQCD with tachyonic brane action
- V-QCD models meet expectations from QCD at qualitative level
- Future work: detailed analysis at finite μ and quantitative fits to QCD

Extra slides

Backgrounds at zero quark mass

Sketch of behavior in the conformal window ($x > x_c$):

- Tachyon vanishes (no ChSB)
- Similar to IHQCD, different potential
  $\Rightarrow$ IR fixed point
- Dilaton flows between UV/IR fixed points

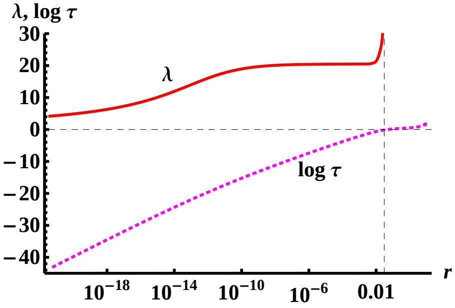


Here UV: $r \to 0$, IR: $r \to \infty$

As x goes below x_c, this solution becomes unstable (tachyon BF bound)

Right below the conformal window ($x < x_c$; $|x - x_c| \ll 1$)

- Dilaton flows very close to the IR fixed point
- "Small" nonzero tachyon induces an IR singularity



Result: "walking"

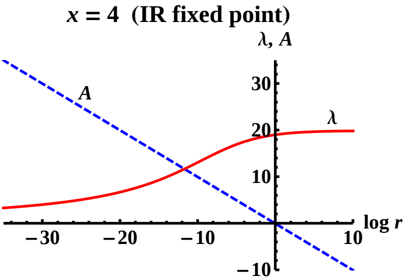
Actual solutions

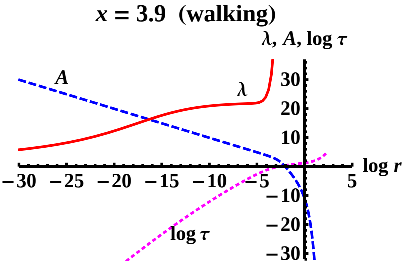
UV: $r = 0$

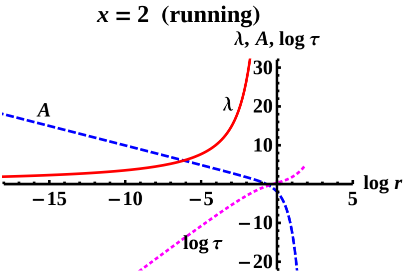
IR: $r = \infty$

$A \sim \log \mu \sim -\log r$

$x_c \simeq 3.9959$







The BF bound and x_c

At an fixed point

$$\tau(r) \sim C_1 r^{\Delta} + C_2 r^{4-\Delta}$$

with

$$-m^2\ell^2 = \Delta(4-\Delta)$$

Requiring real Δ gives the Breitenlohner-Freedman bound for the tachyon (Starinets' lectures)

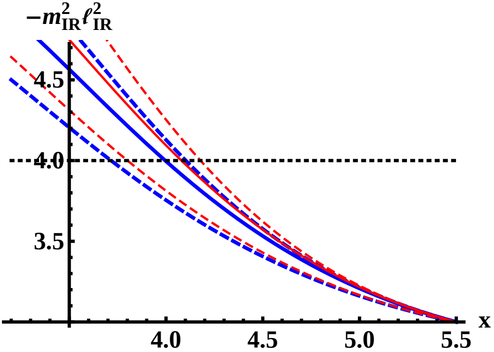
$$-m^2\ell^2 = \Delta(4-\Delta) \leq 4$$

- Saturated for $\Delta = 2$, then $\tau(r) \sim C_1 r^2 + C_2 r^2 \log r$
- Violation of BF bound $\Rightarrow$ instability

Analysis of this instability of the tachyon $\Rightarrow x_c$

Dependence on the UV parameter W_0 and IR choices for the potentials

Resulting variation of the edge of conformal window $x_c = 3.7 \ldots 4.2$



Agrees with most of the other estimates

Potentials I

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x) \\
\kappa(\lambda) &= \frac{1}{\left(1+\frac{115-16x}{288\pi^2}\lambda\right)^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges exponentially:

$$
\tau(r) \sim \tau_0 \exp\left[\frac{81\ 3^{5/6}(115-16x)^{4/3}(11-x)}{812944\ 2^{1/6}}\frac{r}{R}\right]
$$

Potentials II

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x)\frac{1+\frac{115-16x}{216\pi^2}\lambda + \lambda^2/(8\pi^2)^2}{(1+\lambda/(8\pi^2))^{4/3}} \\
\kappa(\lambda) &= \frac{1}{(1+\lambda/(8\pi^2))^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges as

$$
\tau(r) \sim \frac{27\ 2^{3/4}3^{1/4}}{\sqrt{4619}}\sqrt{\frac{r-r_1}{R}}
$$

Effective potential

For solutions with $\tau = \tau_* = \text{const}$

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) - xV_f(\lambda, \tau_*) \right]$$

IHQCD with an effective potential

$$V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_f(\lambda, \tau_*) = V_g(\lambda) - xV_{f0}(\lambda) \exp(-a(\lambda)\tau_*^2)$$

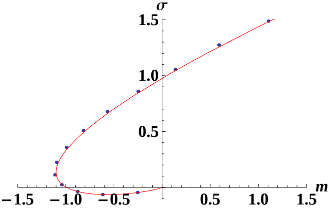
Minimizing for τ_* we obtain $\tau_* = 0$ and $\tau_* = \infty$

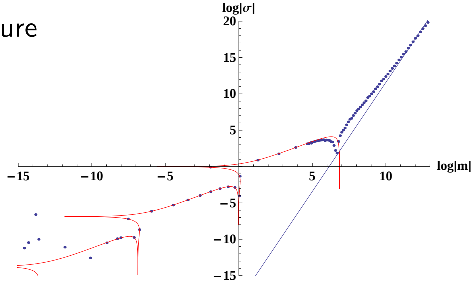
- $\tau_* = 0$: $V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$;
  fixed point with $V'_{\text{eff}}(\lambda_*) = 0$
- $\tau_* \to \infty$: $V_{\text{eff}}(\lambda) = V_g(\lambda)$ (like YM, no fixed points)

Efimov spiral

Ongoing work: the dependence $\sigma(m)$ of the chiral condensate on the quark mass

- For $x < x_c$ spiral structure



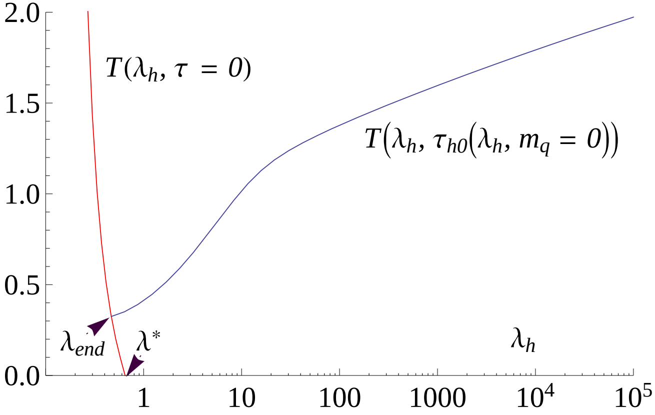


- Dots: numerical data
- Continuous line: (semi-)analytic prediction

Allows to study the effect of double-trace deformations

Black hole branches

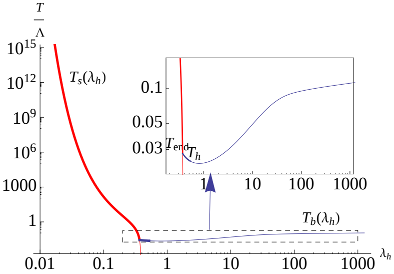
Example: PotII at $x = 3$, $W_0 = 12/11$



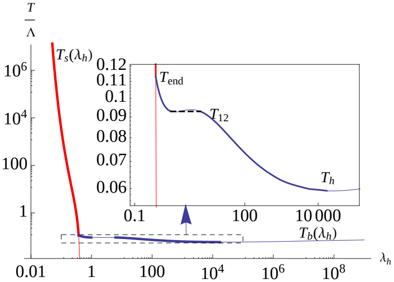
Simple phase structure: 1st order transition at $T = T_h$ from thermal gas to (chirally symmetric) BH

More complicated cases:

PotII at $x = 3$, W_0 SB



PotI at $x = 3.5$, $W_0 = 12/11$

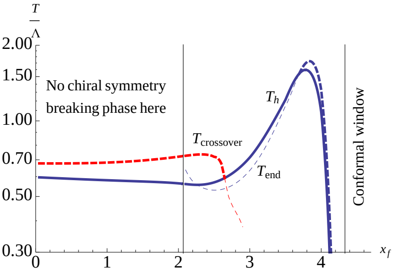


- Left: chiral symmetry restored at 2nd order transition with $T = T_{\text{end}} > T_h$
- Right: Additional first order transition between BH phases with broken chiral symmetry

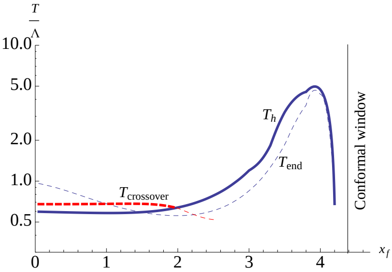
Also other cases ...

Phase diagrams on the (x, T)-plane

PotI$_*$ W_0 SB

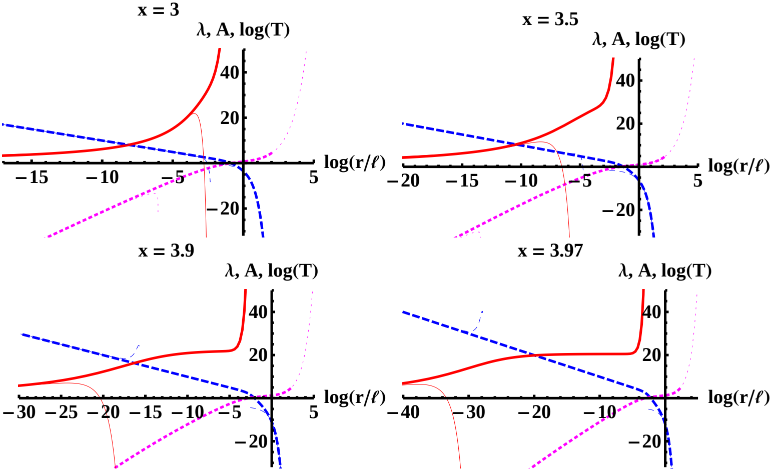


PotII$_*$ W_0 SB

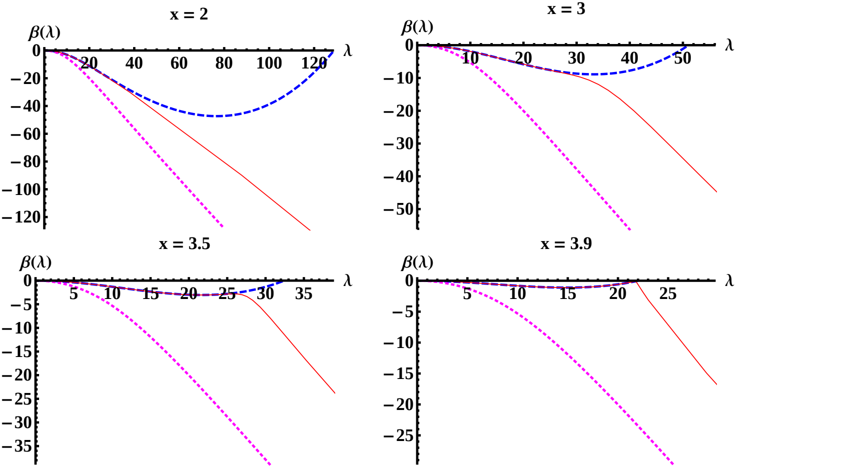


Backgrounds in the walking region

Backgrounds with zero quark mass, $x < x_c \simeq 3.9959$ (λ, A, τ)



Beta functions along the RG flow (evaluated on the background),
zero tachyon, YM $x_c \simeq 3.9959$



Holographic beta functions

Generalization of the holographic RG flow of IHQCD

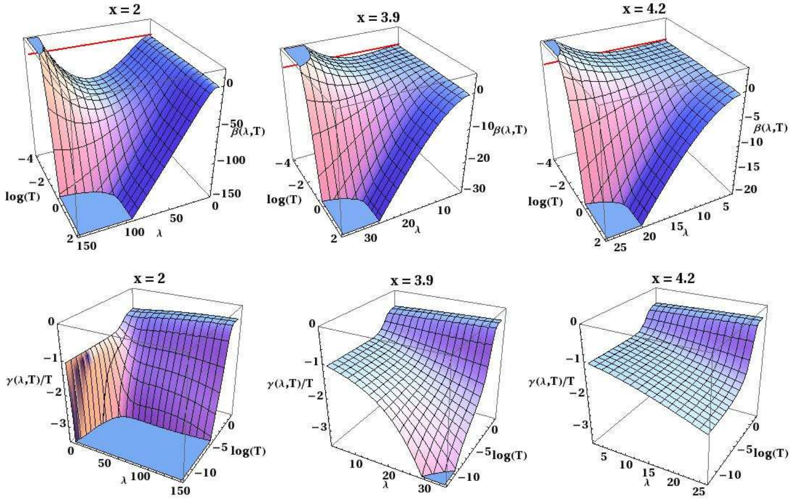
$$\beta(\lambda,\tau) \equiv \frac{d\lambda}{dA}\ ; \qquad \gamma(\lambda,\tau) \equiv \frac{d\tau}{dA}$$

linked to

$$\frac{dg_{\mathrm{QCD}}}{d\log\mu}\ ; \qquad \frac{dm}{d\log\mu}$$

The full equations of motion boil down to two first order partial non-linear differential equations for β and γ

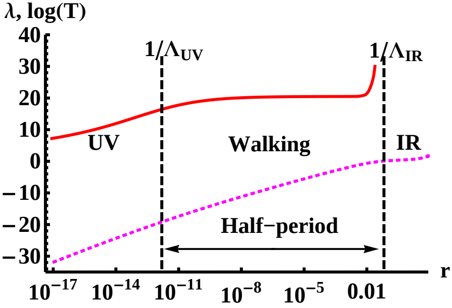
“Good” solutions numerically (unique)



Miransky/BKT scaling

As $x \to x_c$ from below: walking, dominant solution

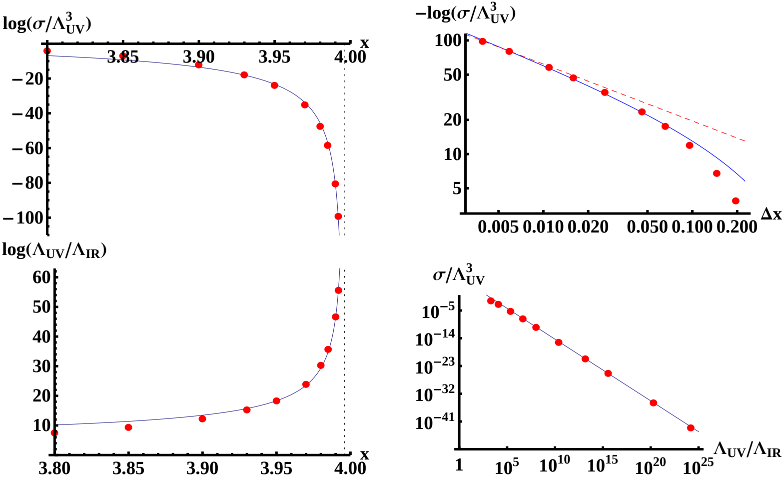
- BF-bound for the tachyon violated at the IRFP
- x_c fixed by the BF bound:
  $\Delta = 2$ & $\gamma_* = 1$
  at the edge of the conformal window



- $\tau(r) \sim r^2 \sin(\kappa\sqrt{x_c - x}\log r + \phi)$ in the walking region
- "0.5 oscillations" $\Rightarrow$ Miransky/BKT scaling, amount of walking $\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$

As $x \to x_c$ with known κ

$$\langle \bar{q} q \rangle \sim \sigma \sim \exp(-2\pi/(\kappa\sqrt{x_c - x}))$$
$$\Lambda_{UV}/\Lambda_{IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$$

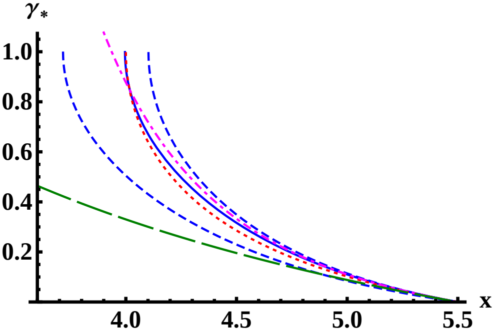


γ_* in the conformal window

Comparison to other guesses

V-QCD (dashed: variation due to W_0)
Dyson-Schwinger
2-loop PQCD
All-orders β

[Pica, Sannino arXiv:1011.3832]



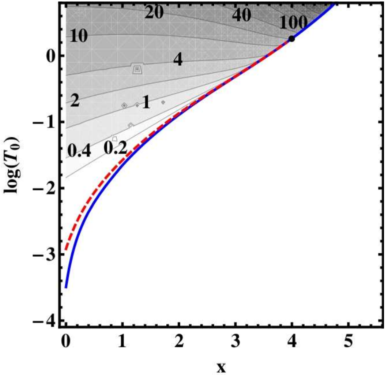
Parameters

Understanding the solutions for generic quark masses requires discussing parameters

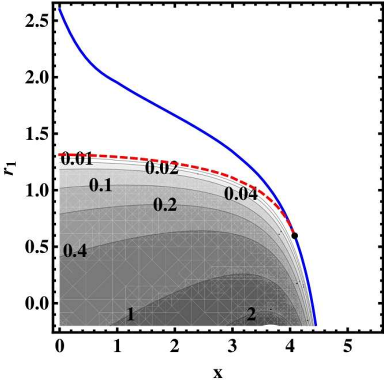
- YM or QCD with massless quarks: no parameters
- QCD with flavor-independent mass m: a single (dimensionless) parameter m/Λ_{QCD}
- In this model, after rescalings, this parameter can be mapped to a parameter (τ_0 or r_1) that controls the diverging tachyon in the IR
- x has become continuous in the Veneziano limit

Map of all solutions

All "good" solutions ($\tau \neq 0$) obtained varying x and τ_0 or r_1
Contouring: quark mass (zero mass is the red contour)

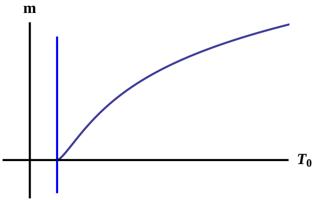


"Potentials I" $\leftrightarrow$ T_0



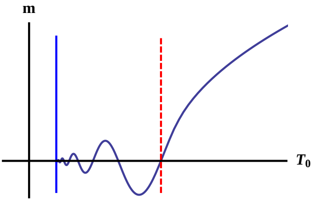
"Potentials II" $\leftrightarrow$ r_1

Mass dependence and Efimov vacua



Conformal window ($x > x_c$)

- For $m = 0$, unique solution with $\tau \equiv 0$
- For $m > 0$, unique "standard" solution with $\tau \neq 0$

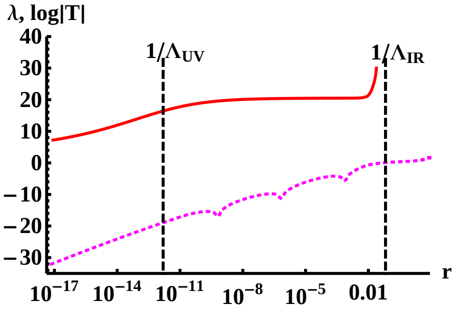


Low $0 < x < x_c$: Efimov vacua

- Unstable solution with $\tau \equiv 0$ and $m = 0$
- "Standard" stable solution, with $\tau \neq 0$, for all $m \geq 0$
- Tower of unstable Efimov vacua (small $|m|$)

Efimov solutions

- Tachyon oscillates over the walking regime
- $\Lambda_{UV}/\Lambda_{IR}$ increased wrt. "standard" solution

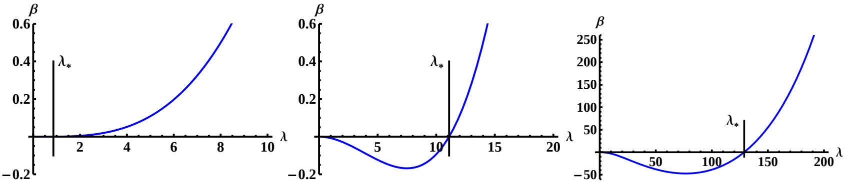


Effective potential: zero tachyon

Start from Banks-Zaks region, $\tau_* = 0$, chiral symmetry conserved ($\tau \leftrightarrow \bar{q}q$), $V_{\rm eff}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$

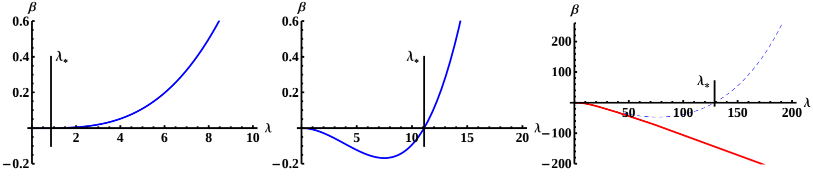
- $V_{\rm eff}$ defines a β-function as in IHQCD – Fixed point guaranteed in the BZ region, moves to higher λ with decreasing x
- Fixed point λ_* runs to ∞ either at finite $x(<x_c)$ or as $x \to 0$

Banks-Zaks	Conformal Window	
$x \to 11/2$	$x > x_c$	$x < x_c$??



Effective potential: what actually happens

Banks-Zaks	Conformal Window	
$x \to 11/2$	$x > x_c$	$x < x_c$



$\tau \equiv 0$	$\tau \equiv 0$	$\tau \neq 0$

- For $x < x_c$ vacuum has nonzero tachyon (checked by calculating free energies)
- The tachyon screens the fixed point
- In the deep IR τ diverges, $V_{\text{eff}} \to V_g \Rightarrow$ dynamics is YM-like

Where is x_c?

How is the edge of the conformal window stabilized?
Tachyon IR mass at $\lambda = \lambda_*$ $\leftrightarrow$ quark mass dimension

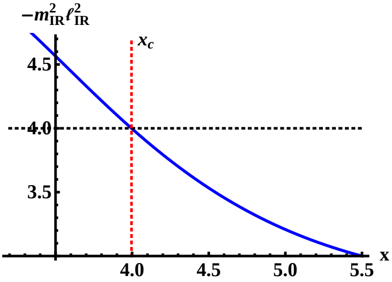
$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = \Delta_{\rm IR}(4 - \Delta_{\rm IR}) = \frac{24 a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - x V_0(\lambda_*))}$$

$$\gamma_* = \Delta_{\rm IR} - 1$$

Breitenlohner-Freedman (BF) bound (horizontal line)

$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = 4 \;\Leftrightarrow\; \gamma_* = 1$$

defines x_c



Why $\gamma_* = 1$ at $x = x_c$?

No time to go into details . . . the question boils down to the linearized tachyon solution at the fixed point

- For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) < 4 \qquad (x > x_c)$:

$$\tau(r) \sim m_q r^{\Delta_{\rm IR}} + \sigma r^{4-\Delta_{\rm IR}}$$

- For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) > 4 \qquad (x < x_c)$:

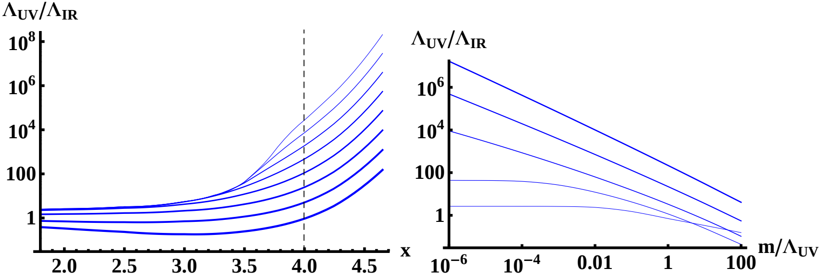
$$\tau(r) \sim C r^2 \sin\left[({\rm Im}\Delta_{\rm IR}) \log r + \phi\right]$$

Rough analogy:
Tachyon EoM $\leftrightarrow$ Gap equation in Dyson-Schwinger approach
Similar observations have been made in other holographic frameworks

[Kutasov, Lin, Parnachev arXiv:1107.2324, 1201.4123]

Mass dependence

For $m > 0$ the conformal transition disappears
The ratio of typical UV/IR scales $\Lambda_{UV}/\Lambda_{IR}$ varies in a natural way
$m/\Lambda_{UV} = 10^{-6}, 10^{-5}, \ldots, 10$ $x = 2, 3.5, 3.9, 4.25, 4.5$



sQCD phases

The case of $\mathcal{N} = 1$ $SU(N_c)$ superQCD with N_f quark multiplets is known and provides an interesting (and more complex) example for the nonsupersymmetric case. From Seiberg we have learned that:

- $x = 0$ the theory has confinement, a mass gap and N_c distinct vacua associated with a spontaneous breaking of the leftover R symmetry Z_{N_c}.
- At $0 < x < 1$, the theory has a runaway ground state.
- At $x = 1$, the theory has a quantum moduli space with no singularity. This reflects confinement with ChSB.
- At $x = 1 + 1/N_c$, the moduli space is classical (and singular). The theory confines, but there is no ChSB.
- At $1 + 2/N_c < x < 3/2$ the theory is in the non-abelian magnetic IR-free phase, with the magnetic gauge group $SU(N_f - N_c)$ IR free.
- At $3/2 < x < 3$, the theory flows to a CFT in the IR. Near $x = 3$ this is the Banks-Zaks region where the original theory has an IR fixed point at weak coupling. Moving to lower values, the coupling of the IR $SU(N_c)$ gauge theory grows. However near $x = 3/2$ the dual magnetic $SU(N_f - N_c)$ is in its Banks-Zaks region, and provides a weakly coupled description of the IR fixed point theory.
- At $x > 3$, the theory is IR free.

Saturating the BF bound (sketch)

Why is the BF bound saturated at the phase transition (massless quarks)??

$$\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

- For $\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) < 4$:
  $\tau(r) \sim m_q r^{4-\Delta_{\mathrm{IR}}} + \sigma r^{\Delta_{\mathrm{IR}}}$
- For $\Delta_{\mathrm{IR}}(4-\Delta_{\mathrm{IR}}) > 4$:
  $\tau(r) \sim C r^2 \sin\left[(\mathrm{Im}\Delta_{\mathrm{IR}}) \log r + \phi\right]$
- Saturating the BF bound, the tachyon solutions will engtangle $\rightarrow$ required to satisfy boundary conditions
- Nodes in the solution appear trough UV $\rightarrow$ massless solution

Saturating the BF bound (sketch)

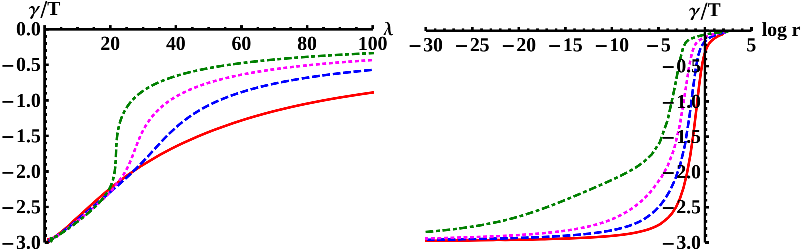
Does the nontrivial (ChSB) massless tachyon solution exist?
Two possibilities:

- $x > x_c$: BF bound satisfied at the fixed point $\Rightarrow$ only trivial massless solution ($\tau \equiv 0$, ChS intact, fixed point hit)
- $x < x_c$: BF bound violated at the fixed point $\Rightarrow$ a nontrivial massless solution exist, which drives the system away from the fixed point

Conclusion: phase transition at $x = x_c$
As $x \to x_c$ from below, need to approach the fixed point to satisfy the boundary conditions $\Rightarrow$ nearly conformal, "walking" dynamics

Gamma functions

Massless backgrounds: gamma functions $\frac{\gamma}{\tau} = \frac{d \log \tau}{dA}$



$x = 2, 3, 3.5, 3.9$