

Small x Scattering using Gauge/Gravity Duality

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Holography and QCD - Recent progress and challenges
Tokyo, Tuesday, September 24, 2013

Outline

Introduction

Pomeron in AdS

Deep Inelastic Scattering

Deeply Virtual Compton Scattering

Vector Meson Production

Conclusions

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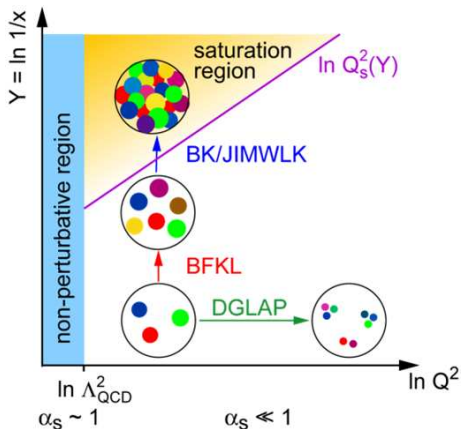
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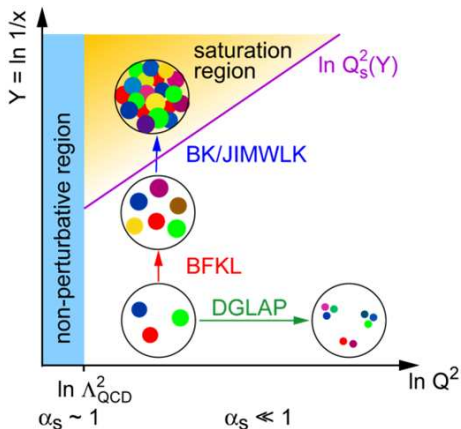
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- ▶ These point to a universal Pomeron exchange as the dominant process.
- ▶ The BFKL equation sums the leading $\log \frac{1}{x}$ diagrams for interaction of gluon on gluon, and leads to power behaviour for the cross section - QCD Pomeron.
- ▶ This perturbative QCD approach works at high Q^2 , and the goal is to extend it as much as possible into the low Q^2 region, typically up to somewhere of the order $Q^2 = 1 - 4\text{GeV}^2$.

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- ▶ Our goal is to apply an alternative method to study the non-perturbative and saturation regions, and also see how much can they be applied to the higher Q^2 region as well.

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- ▶ $A(j, t)$ will have as singularities poles at integer j for fixed t . As we change t , the position of the pole will change, leading to a trajectory

$$j = \alpha(t)$$

- ▶ We can write $A(s, t)$ as a contour integral in the complex plane

$$\begin{aligned} A^\pm(s, t) &= 8\pi \sum_{j=0}^{\infty} (2j+1) A_j^\pm(t) (P_j(z_t) \pm P_j(-z_t)) \\ &= 8\pi i \int_C dj (2j+1) A^\pm(j, t) \frac{P(j, -z_t) \pm P(j, z_t)}{\sin(\pi j)} \end{aligned}$$

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- ▶ $\alpha_i^\pm(t)$ is the position of the pole in the j plane.
- ▶ Take advantage of the asymptotic form of the Legendre polynomials

$$\sqrt{\pi} P(j, z) \sim \frac{\Gamma(j+1/2)}{\Gamma(j+1)} (2z)^j \quad \Re j \geq -1/2$$

- ▶ This will give us a sum in powers of s . At high energy, we can keep just the leading term

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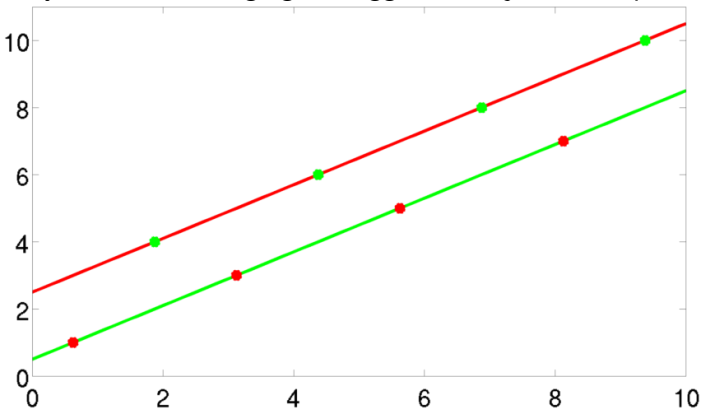
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- ▶ The intercept $\alpha(0) > 1$ leading to non-vanishing

$$\sigma_{tot} \sim s^{\alpha(0)-1}$$

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- ▶ We can expand the exponential to get

$$A(s, -\mathbf{q}_\perp^2) = -2is \int d^2b e^{-i\mathbf{b}_\perp \cdot \mathbf{q}_\perp} \left(i\chi + \frac{(i\chi)^2}{2} + \dots \right).$$

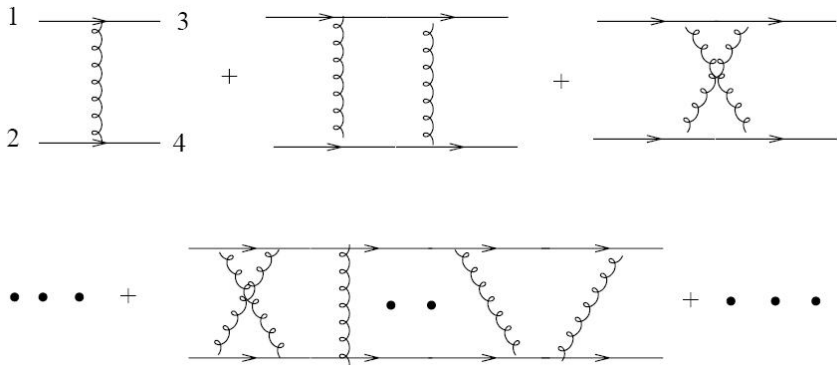
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- ▶ The cutoff position will roughly correspond to

$$z_0 \simeq \frac{1}{\Lambda_{QCD}}.$$

- ▶ Let us write the scattering amplitude [Brower, Polchinski, Strassler, Tan, 2006]

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- ▶ We can show that this would lead to amplitudes

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- ▶ where $\Delta_j = (r/R)^j (\Delta_0) (r/R)^{-j}$. And Δ_0 is the scalar Laplacian in curved space.

- ▶ This will give us for the physical state condition

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- ▶ and for the amplitude we would have

$$\mathcal{T}^{(+)} \sim \int \frac{dj}{2\pi i} \int \frac{d\nu \nu \sinh 2\pi\nu}{\pi} \frac{\Pi(j) s^j}{j - j_0^{(+)} + \mathcal{D}\nu^2}$$

$$\times \langle \mathcal{W}_{R0} \mathcal{V}_P(j, \nu, k, -) \rangle \langle \mathcal{V}_P(j, \nu, k, +) \mathcal{W}_{L0} \rangle$$

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$$\mathcal{V}_P(j, \nu, k, \pm) \sim (\partial X^{\pm} \bar{\partial} X^{\pm})^{\frac{j}{2}} e^{\mp i k \cdot X} e^{(j-2)u} K_{\pm 2i\nu}(|t|^{1/2} e^{-u})$$

- ▶ and for the amplitude we would have

$$\begin{aligned} \mathcal{T}^{(+)} &\sim \int \frac{dj}{2\pi i} \int \frac{d\nu \nu \sinh 2\pi\nu}{\pi} \frac{\Pi(j) s^j}{j - j_0^{(+)} + \mathcal{D}\nu^2} \\ &\times \langle \mathcal{W}_{R0} \mathcal{V}_P(j, \nu, k, -) \rangle \langle \mathcal{V}_P(j, \nu, k, +) \mathcal{W}_{L0} \rangle \end{aligned}$$

- ▶ with $j_0^{(+)}$ given by

$$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda).$$

and $\mathcal{D} = 2/\sqrt{\lambda}$.

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- ▶ Due to conformal invariance, χ is a function of only two variables

$$L = \log(1 + v + \sqrt{v(2+v)})$$

$$\tau = \log\left(\frac{\rho}{2} z z' s\right)$$

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$$\chi_{hw}(\tau, t = 0, z, \bar{z}) = \chi(\tau, 0, z, \bar{z}) + \mathcal{F}(\tau, z, \bar{z}) \chi(\tau, 0, z, z_0^2/\bar{z}).$$

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- ▶ When $t \neq 0$, we will use an approximation

$$\chi_{hw}(\tau, l, z, \bar{z}) = C(\tau, z, \bar{z}) D(\tau, l) \chi_{hw}^{(0)}(\tau, l, z, \bar{z})$$

► The function

$$\mathcal{F}(\tau, z, \bar{z}) = 1 - 4\sqrt{\pi\tau} e^{\eta^2} \operatorname{erfc}(\eta), \quad \eta = \frac{-\log(z\bar{z}/z_0^2) + 4\tau}{\sqrt{4\tau}}$$

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- ▶ Note that these methods can be generalized to Odderon exchange as well [Brower, MD, Tan, 2008].

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Weak coupling:

$$\mathcal{K}(k_{\perp}, k'_{\perp}, s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\log s}} e^{-(\log k_{\perp} - \log k'_{\perp})^2 / 4\mathcal{D}\log s}$$

$$j_0 = 1 + \frac{\log 2}{\pi^2} \lambda, \quad \mathcal{D} = \frac{14\zeta(3)}{\pi} \lambda / 4\pi^2$$

Strong coupling:

$$\mathcal{K}(z, z', s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\log s}} e^{-(\log z - \log z')^2 / 4\mathcal{D}\log s}$$

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}, \quad \mathcal{D} = \frac{1}{2\sqrt{\lambda}}$$

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- ▶ $z_0 \simeq \frac{1}{\Lambda_{QCD}}$, the position of the hard wall in AdS space, intuitively should be related to Λ_{QCD}
- ▶ Only three parameters for the conformal model, and 4 for the hard wall.

Outline

Introduction

Pomeron in AdS

Deep Inelastic Scattering

Deeply Virtual Compton Scattering

Vector Meson Production

Conclusions

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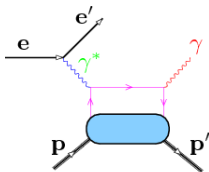


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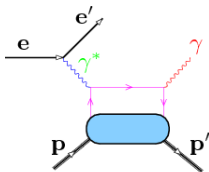


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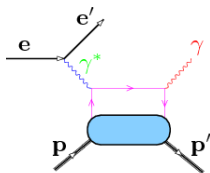


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The basic kinematical variables we need for describing this process are

- ▶ center of mass energy s , the virtuality Q^2 and the scaling variable x

$$\begin{aligned}s &= -(P + k)^2 \\ Q^2 &= -q^\mu q_\mu = -(k - k')^2 > 0 \\ x &\approx \frac{Q^2}{s}\end{aligned}$$

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- ▶ At lower or higher Q^2 there is no experimental data with $x < 0.01$.

As we saw, we are going to calculate F_2 by relating it to the total cross section. This in turn we will calculate using the optical theorem, for which we need the forward scattering amplitude at $t = 0$. Putting it all together, using the eikonal approximation we get [Brower, MD, Sarcevic, Tan, 2010]

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We need to supply the wavefunctions for the photon and the proton. For the photon we will consider an R boson propagating through the bulk that couples to leptons on the boundary (Polchinski, Strassler 2003)

$$P_{13}(z, Q^2) = \frac{1}{z} (Qz)^2 (K_0^2(Qz) + K_1^2(Qz)),$$

We would also need a wavefunction associated to the proton $\phi_p(z)$. For the current analysis, we will assume that the wave function is sharply peaked near the IR boundary z_0 , with $1/Q' \leq z_0$, with Q' of the order of the proton mass. For simplicity, we will simply replace P_{24} by a sharp delta-function

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- ▶ For single Pomeron exchange, the imaginary part is enough due to the optical theorem.
- ▶ The structure function F_2 can be expressed as

$$F_2(x, Q^2) = \frac{g_0^2 \rho^{3/2}}{32\pi^{5/2}} \int dz dz' P_{13}(z, Q^2) P_{24}(z') (zz' Q^2) \times e^{(1-\rho)\tau} \frac{\exp\left(\frac{-(\log z - \log z')^2}{\rho\tau}\right)}{\tau^{1/2}}$$

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Where

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- ▶ This will violate the Froissart bound.
- ▶ The difference between the conformal and confinement depends on the size of the function \mathcal{F} .
- ▶ \mathcal{F} at fixed z, z' , goes to 1 as $\tau \rightarrow 0$ and to -1 as $\tau \rightarrow \infty$. Hence, at small x , $\mathcal{F} \rightarrow -1$ and confinement leads to a partial cancelation for the growth rate. Since \mathcal{F} is continuous, there will be a region over which $\mathcal{F} \sim 0$, and, in this region, there is little difference between the hard-wall and the conformal results.

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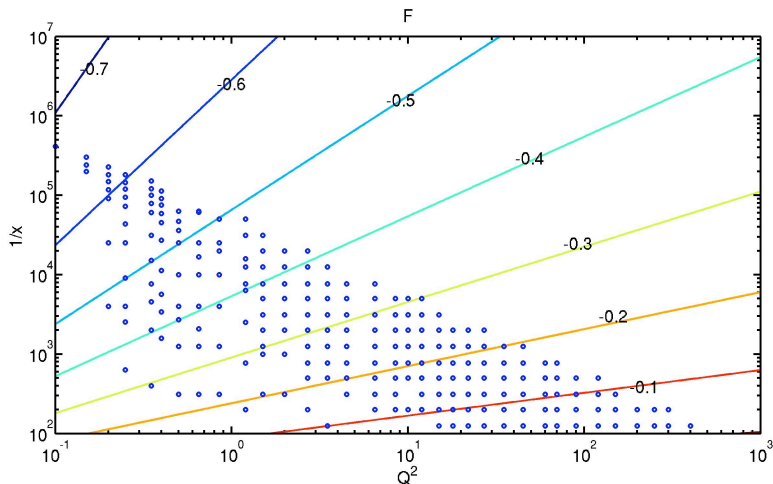


Figure: Contour plot for coefficient function \mathcal{F} as a function of $\log(1/z)$ and $\log(1/x)$, with $z' \simeq z_0$ fixed, $z_0 \sim \Lambda_{QCD}^{-1}$.

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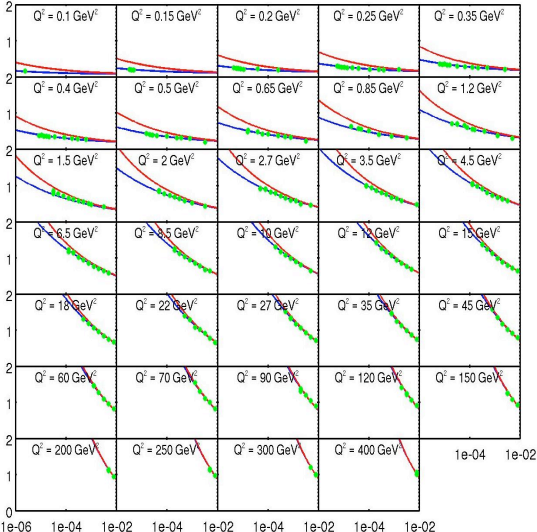
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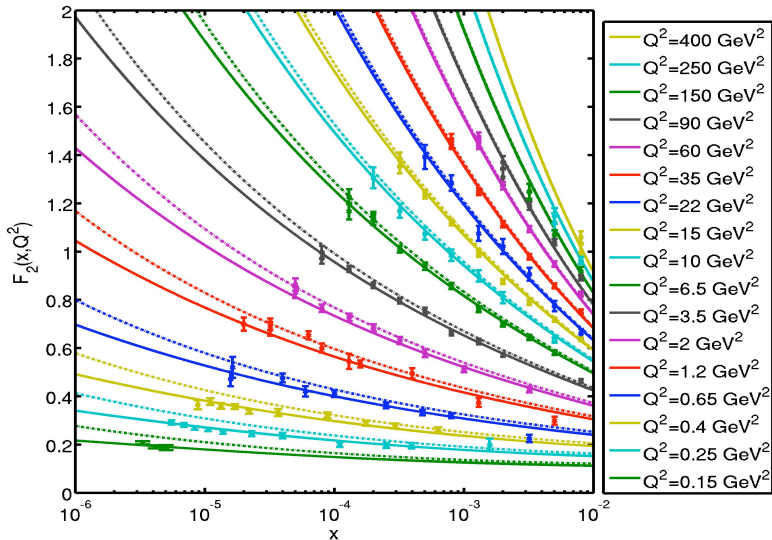
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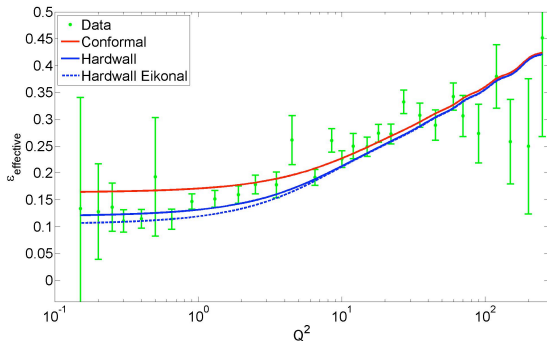


Figure: Q^2 -dependence for effective Pomeron intercept, $\alpha_P = 1 + \epsilon_{eff}$.

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- ▶ Comparable number of parameters and χ^2 (Kowalski et al $\chi^2 \sim 1.2$), but the advantage of our approach is we can go to low Q^2 (the data with lowest Q^2 is at 0.10GeV^2) whereas their approach stops at $Q^2 = 4\text{GeV}^2$.

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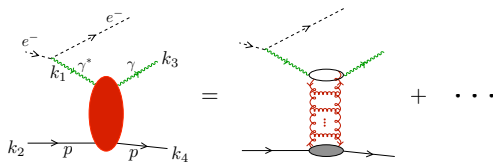
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What is DVCS?

Deeply **V**irtual **C**ompton **S**cattering is the scattering between an offshell photon and a proton.

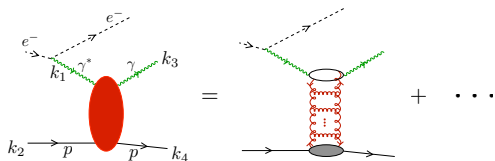
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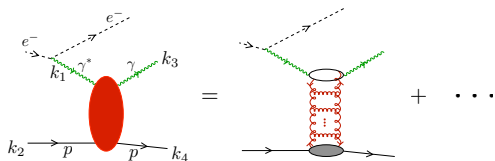
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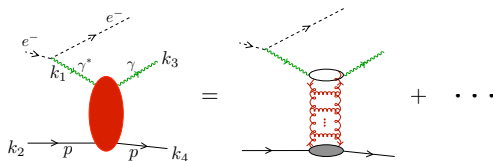
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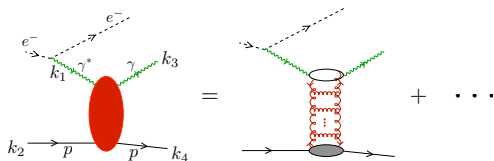
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- ▶ the scaling variable

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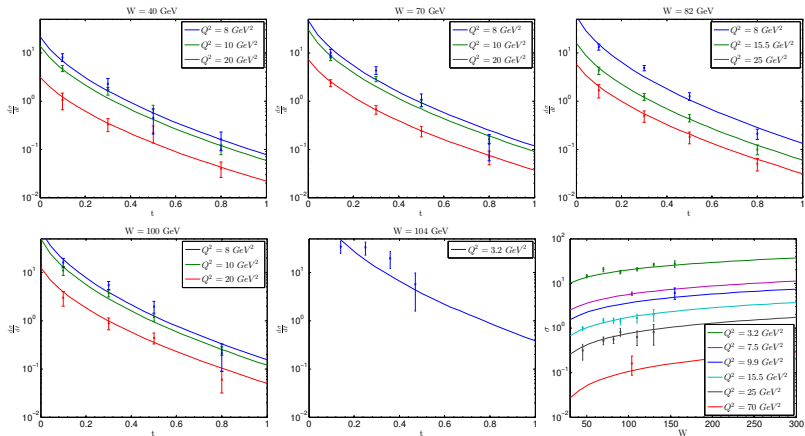
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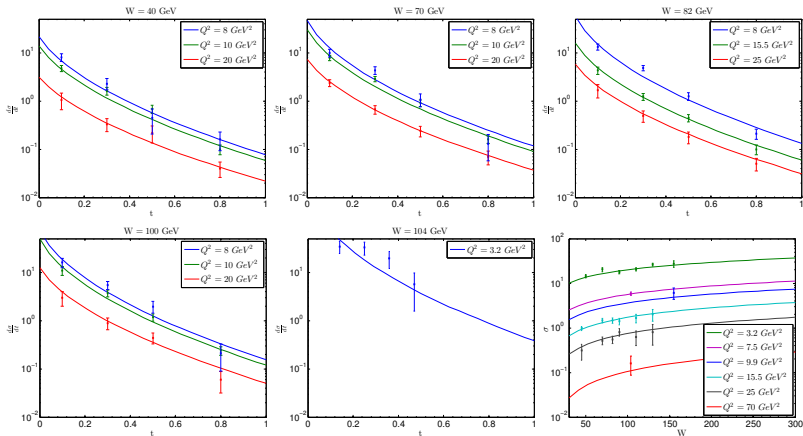
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Running the same fit using the eikonal approximation, instead of just keeping single pomeron exchange, does not improve the fits, due to the fact that the size of χ is small in this kinematical region.

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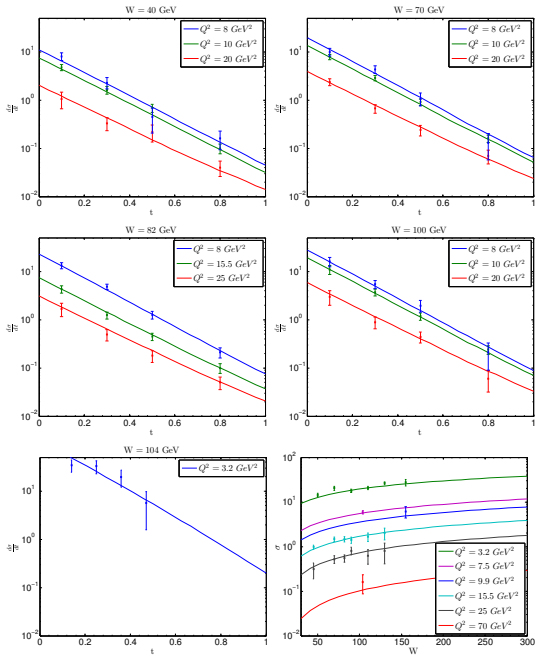
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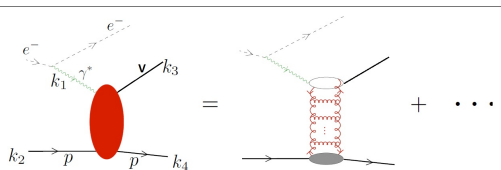
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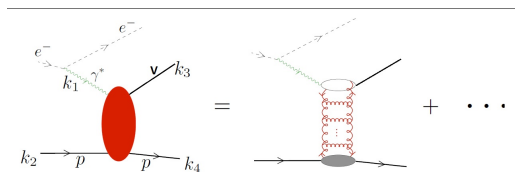
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The vector mesons consist of a quark-antiquark pair, and have the same quantum numbers as the photon, $J^{PC} = 1^{--}$. The production of the ρ^0 , ω , ϕ and J/Ψ was measured at HERA.

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- ▶ In this analysis we use [Costa, MD, Evans, 2013]

$$\Psi_n(z) = -\left(\sqrt{C} \frac{\pi^2}{6} z^2 K_n(Qz)\right) \left(\frac{\sqrt{2}}{\xi J_1(\xi)} z^2 J_n(mz)\right), \quad \Phi(\bar{z}) = \bar{z}^3 \delta(\bar{z} - z_{\star})$$

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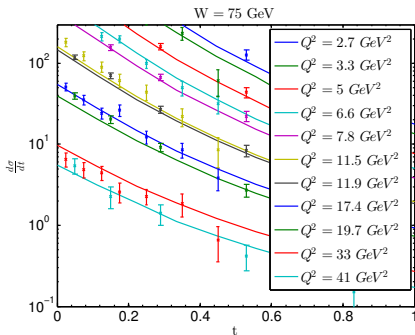
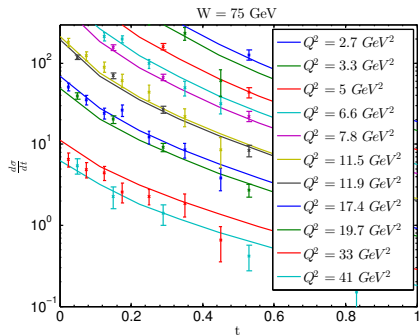
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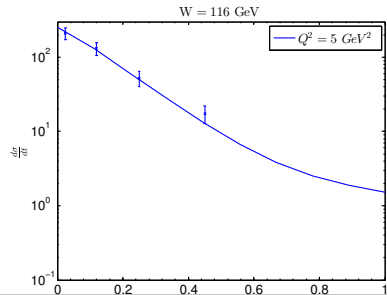
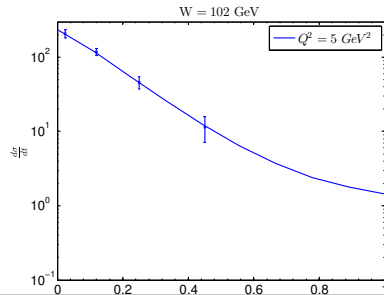
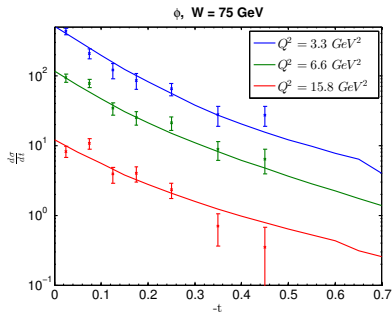
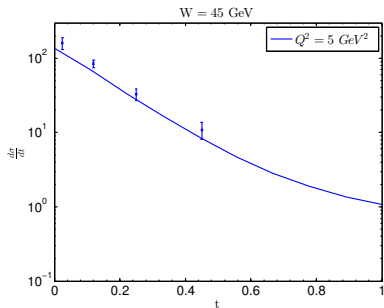
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- ▶ We will look at both the differential and total exclusive cross sections.

		σ [nb]				$d\sigma/dt$ [nb/GeV ²]		
		ρ	ϕ	Ω	J/ ψ	ρ	ϕ	J/ ψ
m [GeV]		0.77549	1.019445	0.78265	3.096916	0.77549	1.019445	3.096916
N		48	27	6	38	35	21	84
C o n f o r m a l	χ^2	0.92	0.60	0.0099	0.28	1.7	1.3	2.9
	g_0^2	4.6	1.8	0.53	0.62	1.6	0.25	0.56
	ρ	0.76	0.73	0.64	0.70	0.65	0.54	0.72
	z^* [GeV ⁻¹]	3.4	3.0	1.8	0.98	2.1	2.5	2.2
	χ^2	0.88	0.61	0.015	0.30	1.7	1.4	1.8
H a r d w a l l	g_0^2	4.1	1.8	0.67	0.75	2.2	0.38	0.69
	ρ	0.76	0.73	0.66	0.71	0.69	0.59	0.75
	z^* [GeV ⁻¹]	3.6	3.6	1.5	0.87	2.2	2.5	2.4
	z_0 [GeV ⁻¹]	4.8	4.4	7.3	5.3	7.7	8.6	4.6

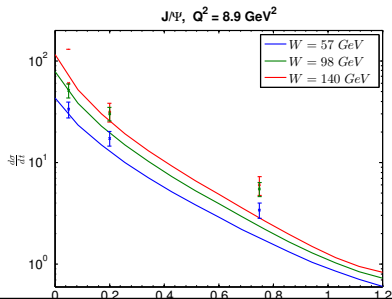
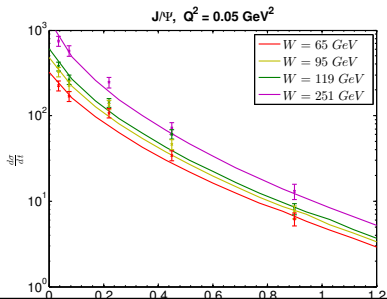
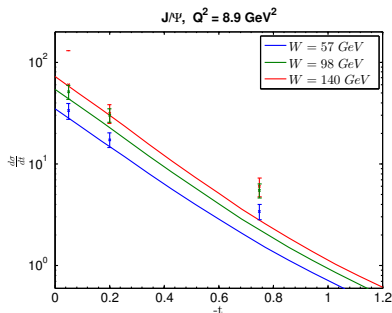
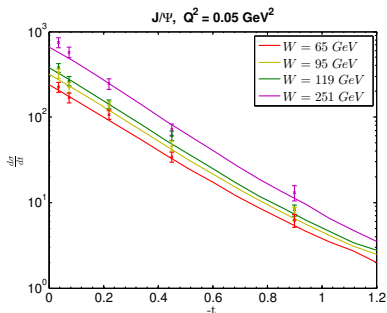
Differential cross section for the ρ meson:



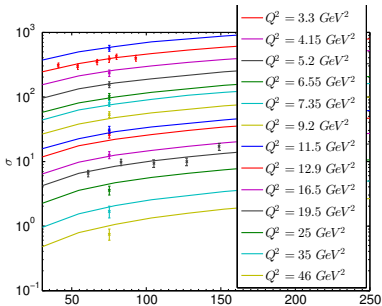
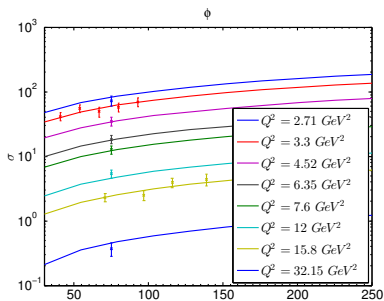
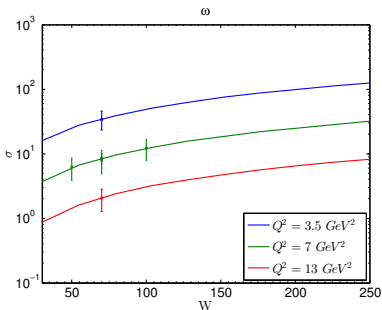
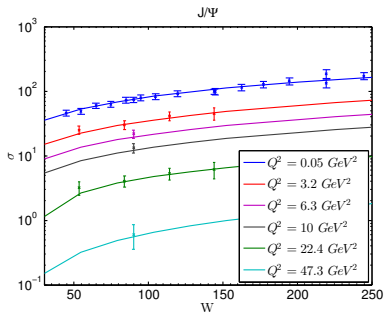
Differential cross section for the ϕ meson (hardwall model):



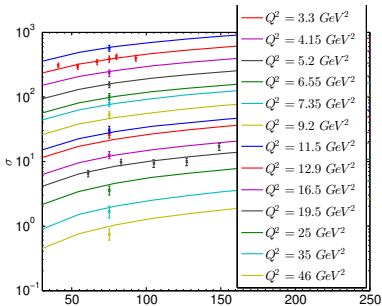
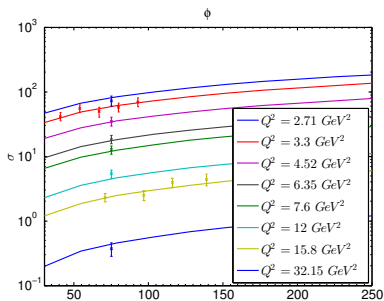
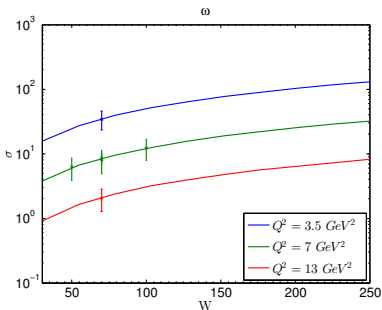
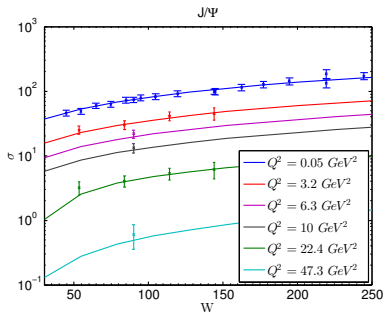
Differential cross section for the J/Ψ meson (hardwall model):



Cross sections for the conformal model:



Cross sections for the hardwall model:



Outline

Introduction

Pomeron in AdS

Deep Inelastic Scattering

Deeply Virtual Compton Scattering

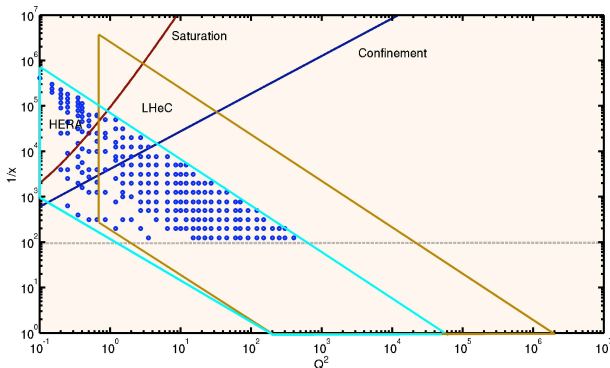
Vector Meson Production

Conclusions

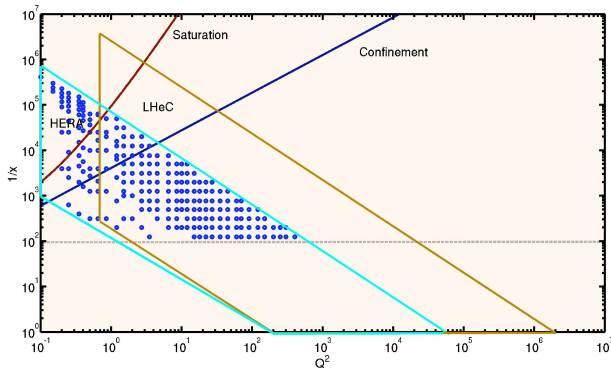
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The above order of lines is the opposite of what is generally thought. Is it an artifact of our model?

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- ▶ It might therefore be possible to extend some of the insights we gain even into the weak coupling regime.
- ▶ The hard wall model, although a simple modification of AdS, seems to capture effects of confinement well. Interesting to repeat some of the calculations using a different confinement model to identify precisely what features are model independent.

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- ▶ Eventually it would be good to have a single set of parameters that fits several different processes.
- ▶ We can also try to use a different AdS model of confinement (for example the soft wall model [Brower, MD, Raben, Tan, in preparation]) and combine our methods with work by others (for example on the vector meson wavefunctions).

Thank you!