Holographic study of magnetically induced ρ meson condensation

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Work in collaboration with David Dudal

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Overview

1. **Introduction**

2. **Holographic set-up**
   - The Sakai-Sugimoto model
   - Introducing the magnetic field

3. **The ρ meson mass**
   - Taking into account constituents
   - Full DBI-action
   - Effect of Chern-Simons action and mixing with pions

Overview

Why study strong magnetic fields?

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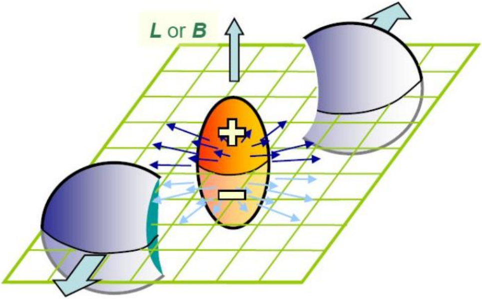
- experimental relevance: appearance in QGP (order $B \sim 15 m_\pi^2$)



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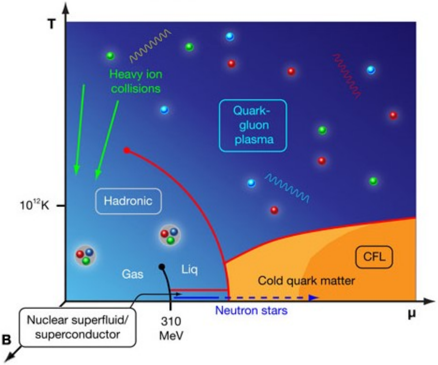
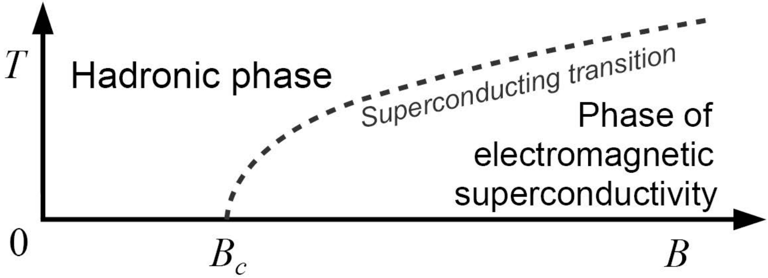
- from a holographic viewpoint: interesting for comparison with lattice
- excellent probe for largely unknown QCD phase diagram



ρ meson condensation

Studied effect: ρ meson condensation (Maxim Chernodub)

QCD vacuum instable towards forming a superconducting state of condensed charged ρ mesons at critical magnetic field B_c



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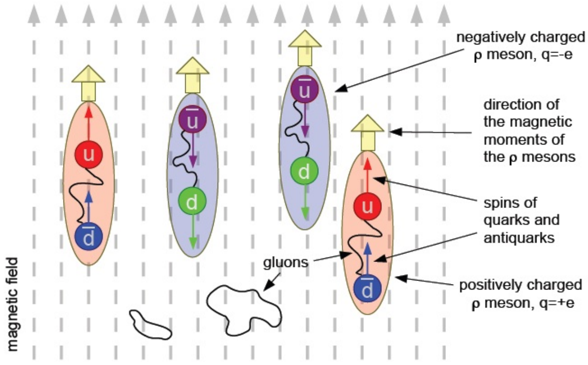

ρ meson condensation: Landau levels

The energy levels ϵ of a free relativistic spin-s particle moving in a background of the external magnetic field $\vec{B} = B\vec{e}_z$ are the Landau levels

Landau levels

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In the lowest energy state $(n = 0,\ p_z = 0)$ their effective mass,

$$m^2_{\rho,eff}(B) = m_\rho^2 - B,$$

can thus become zero if the magnetic field is strong enough.

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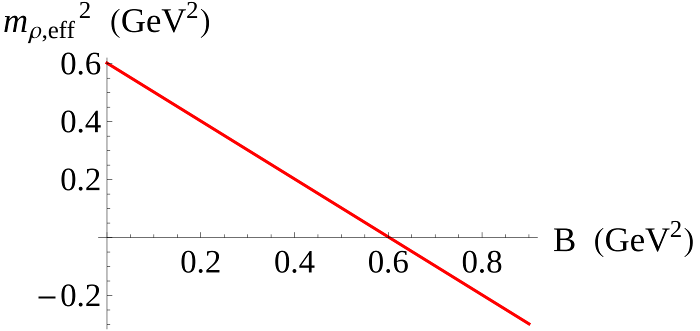

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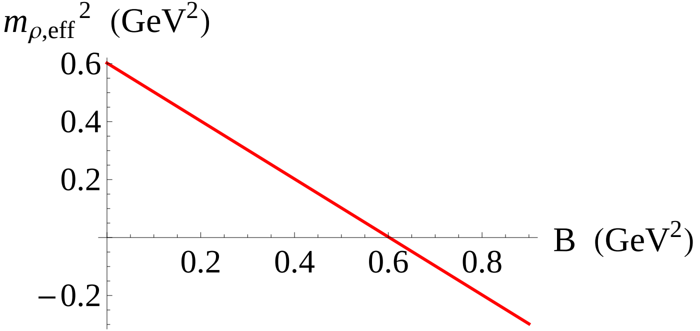

$\Longrightarrow$ The fields ρ and $\rho^\dagger$ condense at the critical magnetic field

$$B_c = m^2_\rho.$$

Abrikosov lattice ground state

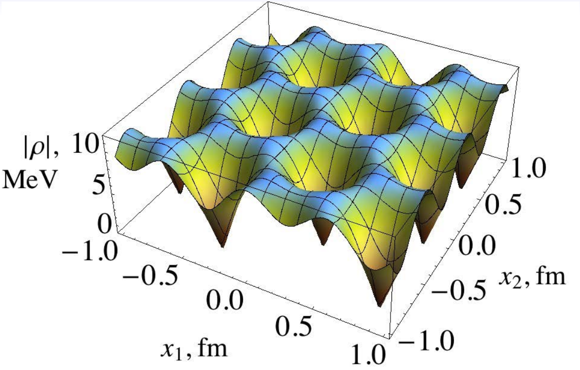

Figure : Absolute value of the superconducting condensate ρ at $B = 1.01 B_c$ in the transversal (x_1, x_2)- plane.
[Chernodub, Van Doorsselaere and Verschelde, 1111.4401]

Similar result in holographic toy model [Bu, Erdmenger, Shock & Strydom, 1210.6669]

ρ meson condensation: different approaches

- phenomenological models: $B_c = m_\rho^2 = 0.6\ \text{GeV}^2$ (bosonic effective model), $B_c \approx 1\ \text{GeV}^2$ (NJL) [1008.1055,1101.0117]

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- lattice simulation: $B_c \approx 0.9$ GeV^2 [1104.3767]
- $\rightsquigarrow$ holographic approach:
  - can the ρ meson condensation be modeled?
  - can this approach deliver new insights? e.g. taking into account constituents, effect on B_c

N.C., Dudal & Verschelde [1105.2217,1309.5042]; Ammon, Erdmenger, Kerner & Strydom [1106.4551]

Overview

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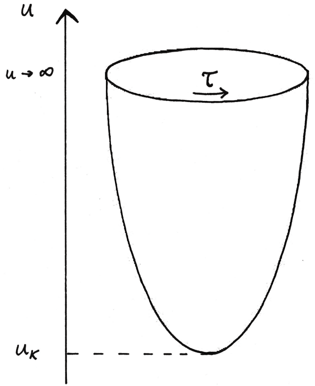
supergravitation in AdS_5 space $\stackrel{dual}{=}$ conformal $\mathcal{N}$=4 SYM theory

$\rightsquigarrow$ Witten: supergravitation in D4-brane background $\stackrel{dual}{=}$ non-conformal non-susy pure QCD-like theory

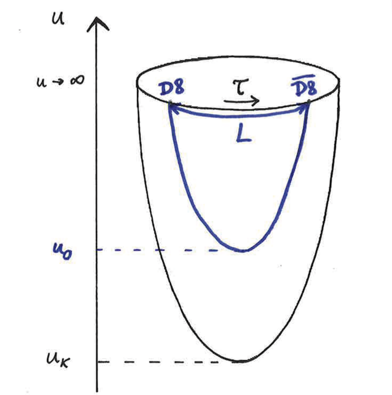
The D4-brane background



$$ds^2 = \left(\frac{u}{R}\right)^{3/2} \left(\eta_{\mu\nu} dx^\mu dx^\nu + f(u) d\tau^2\right) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right),$$

$$e^\phi = g_s \left(\frac{u}{R}\right)^{3/4} \quad , \quad F_4 = \frac{N_c}{V_4} \epsilon_4 \quad , \quad f(u) = 1 - \frac{u_K^3}{u^3} \ ,$$

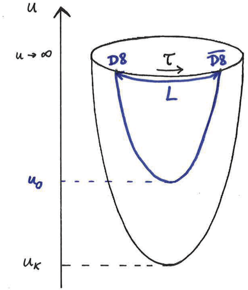
The Sakai-Sugimoto model



- To add flavour degrees of freedom to the theory, add N_f pairs of D8-$\overline{\text{D8}}$ flavour branes [Sakai and Sugimoto, hep-th/0412141].
- Probe approximation $N_f \ll N_c$: backreaction of flavour branes on background is ignored $\sim$ quenched approximation.

The flavour D8-branes

On the stack of N_f coinciding pairs of D8-$\overline{\text{D8}}$ flavour branes lives a $U(N_f)_L \times U(N_f)_R$ theory, to be interpreted as the chiral symmetry in QCD.



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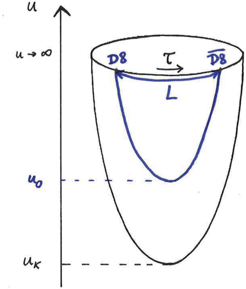

The U-shaped embedding of the flavour branes models **spontaneous chiral symmetry breaking** $U(N_f)_L \times U(N_f)_R \to U(N_f)$.

The flavour gauge field

The $U(N_f)$ **gauge field** $A_\mu(x^\mu, u)$ that lives on the flavour branes describes **a tower of vector mesons** $v_{\mu,n}(x^\mu)$ in the dual QCD-like theory:

$U(N_f)$ gauge field

$$A_\mu(x^\mu, u) = \sum_{n\geq 1} v_{\mu,n}(x^\mu)\psi_n(u)$$

Flavour gauge field and mesons

The way it works:

dynamics of the flavour D8/$\overline{\text{D8}}$-branes: 5D YM theory

$S_{DBI}[A_\mu] = \cdots$, $A_\mu(x^\mu, u) = \sum_{n\geq 1} v_{\mu,n}(x^\mu)\psi_n(u)$

$\downarrow$ integrate out the extra radial dimension u

effective 4D meson theory for $v_\mu^n(x^\mu)$

Approximations of the model

Duality is valid in the limit $N_c \to \infty$ and large 't Hooft coupling $\lambda = g_{YM}^2 N_c \gg 1$, and at low energies (where redundant massive d.o.f. decouple).

Approximations (inherent to the model):

- quenched approximation ($N_f \ll N_c$)
- chiral limit ($m_\pi = 0$, bare quark masses zero)

Choices of parameters:

- $N_c = 3$
- $N_f = 2$ to model charged mesons

How to turn on the magnetic field

A non-zero value of the flavour gauge field $A_m(x^\mu, z)$ on the boundary,

$$A_m(x^\mu, u \to \infty) = \overline{A}_\mu,$$

corresponds to an external gauge field in the boundary field theory that couples to the quarks

$$\overline{\psi} i \gamma_\mu D_\mu \psi \quad \text{with} \quad D_\mu = \partial_\mu + \overline{A}_\mu.$$

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To apply an external electromagnetic field A_μ^{em}, put

$$A_\mu(u \to +\infty) = -iQ_{em} A_\mu^{em} = \overline{A}_\mu$$

[Sakai and Sugimoto hep-th/0507073]

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$$A_2^{em} = x_1 B$$

$$Q_{em} = \begin{pmatrix} 2/3 & 0 \\ 0 & -1/3 \end{pmatrix} = \frac{1}{6}\mathbf{1}_2 + \frac{1}{2}\sigma_3$$

Overview

Plan

- Action:

$$S_{DBI} = -T_8 \int d^4x \, 2 \int_{u_0}^{\infty} du \int \epsilon_4 \, e^{-\phi} \, \mathrm{STr} \sqrt{-\det\left[g_{mn}^{D8} + (2\pi\alpha')iF_{mn}\right]},$$

with

$$\mathrm{STr}(F_1 \cdots F_n) = \frac{1}{n!}\mathrm{Tr}(F_1 \cdots F_n + \text{all permutations})$$

the symmetrized trace,

$$g_{mn}^{D8} = g_{mn} + g_{\tau\tau}(D_m\tau)^2$$

the induced metric on the D8-branes (with covariant derivative $D_m\tau = \partial_m\tau + [A_m, \tau]$),
and

$$F_{mn} = \partial_m A_n - \partial_n A_m + [A_m, A_n] = F_{mn}^a t^a$$

the field strength

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- Gauge field ansatz:

$$\begin{cases} A_m = \overline{A}_m + \tilde{A}_m \\ \tau = \overline{\tau} + \tilde{\tau} \end{cases}$$

1. Determine embedding $\overline{\tau}(u)$ as a function of $\overline{A}_\mu$ (put $\tilde{A}_m = \tilde{\tau} = 0$)
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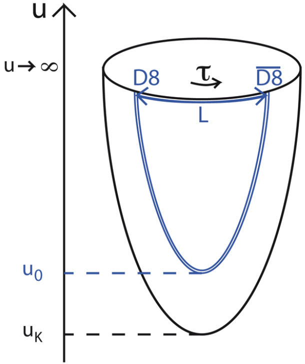
   Expand to order $(2\pi\alpha')^2 \sim \frac{1}{\lambda^2}$ $(\lambda \gg 1)$ vs use full DBI-action

General embedding $u_0 > u_K$



$u_0 > u_K$ to model non-zero constituent quark mass which is related to the distance between u_0 and u_K.

[Aharony et.al. hep-th/0604161]

Numerical fixing of holographic parameters

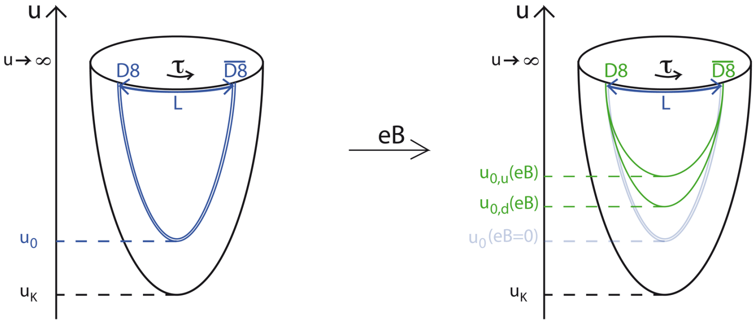
There are three unknown free parameters (u_K, u_0 and $\kappa(\sim \lambda N_c)$). In order to get results in physical units, we fix the free parameters by matching to

- the constituent quark mass $m_q = 0.310$ GeV,
- the pion decay constant $f_\pi = 0.093$ GeV and
- the rho meson mass in absence of magnetic field $m_\rho = 0.776$ GeV.

Results:

$$u_K = 1.39 \text{ GeV}^{-1}, \quad u_0 = 1.92 \text{ GeV}^{-1} \text{ and } \quad \kappa = 0.00678$$

B-dependent embedding for $u_0 > u_K$



Keep L fixed: $u_0(B)$ rises with B. This models **magnetic catalysis of chiral symmetry breaking**

[Bergman 0802.3720; Johnson and Kundu 0803.0038].

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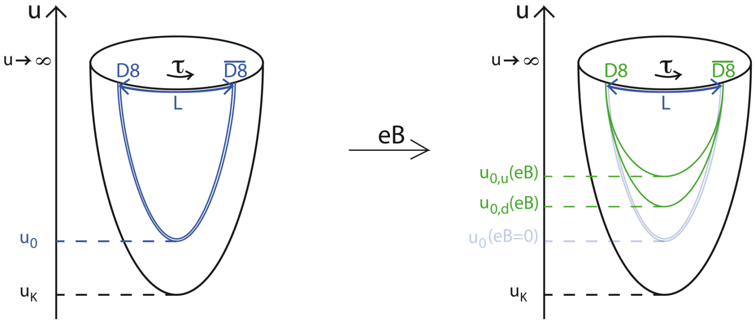

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Non-Abelian: $u_{0,u}(B) > u_{0,d}(B)$! $U(2) \to U(1)_u \times U(1)_d$

B-dependent embedding for $u_0 > u_K$

Change in embedding models:

- chiral magnetic catalysis $\Rightarrow m_u(B)$ and $m_d(B) \nearrow$
- $\vec{B}$ explicitly breaks global $U(2) \rightarrow U(1)_u \times U(1)_d$

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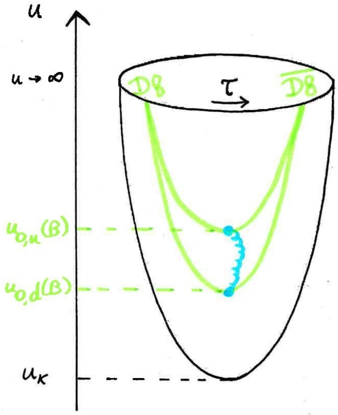
Effect on ρ mass?

- expect $m_\rho(B) \nearrow$ as constituents get heavier
- split between branes generates other mass mechanism: 5D gauge field gains mass through **holographic Higgs mechanism**

B-induced Higgs mechanism



The string associated with a charged ρ meson ($\overline{u}d$, $\overline{d}u$) stretches between the now separated up- and down brane $\Rightarrow$ because a string has tension it gets a mass.

EOM for ρ for $u_0 > u_K$?

Non-trivial embedding

$$\overline{\tau}(u) = \begin{pmatrix} \overline{\tau}_u(u)\theta(u - u_{0,u}) & 0 \\ 0 & \overline{\tau}_d(u)\theta(u - u_{0,d}) \end{pmatrix} \not\sim \mathbf{1},$$

describing the splitting of the branes, severely complicates the analysis.

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$$\mathcal{L}_{5D} = \mathrm{STr}\left\{ ..\left([\tilde{A}_m, \overline{\tau}] + D_m\tilde{\tau}\right)^2 + ..(F_{\mu\nu})^2 + ..(F_{\mu u})^2 + ..\overline{F}_{\mu\nu}[\tilde{A}_\mu, \tilde{A}_\nu] \right.$$
$$\left. + ..(\partial_u\overline{\tau})\overline{F}\left([\tilde{A}, \overline{\tau}] + D\tilde{\tau}\right) F \right\}$$

with all the .. different functions $\mathcal{H}(\partial_u\overline{\tau}, \overline{F}; u)$ of the background fields $\partial_u\overline{\tau}$, $\overline{F}$.

Fixing the gauge to disentangle $\tilde{A}$ and $\tilde{\tau}$

Faddeev-Popov gauge fixing:
The functional integral

$$\mathcal{Z} = \int \mathcal{D}A\mathcal{D}\tau \, e^{i\int \mathcal{L}[A,\tau]}$$
$$= C' \int \mathcal{D}A\mathcal{D}\tau \, e^{i\int\left(\mathcal{L}[A,\tau]-\frac{1}{2}\mathcal{G}^2\right)} \det\left(\frac{\delta G[A^\alpha,\tau^\alpha]}{\delta\alpha}\right)$$

is restricted to physically inequivalent field configurations, by imposing the gauge-fixing condition

$$\mathcal{G}[\text{fields}] = 0.$$

Fixing the gauge to disentangle $\tilde{A}$ and $\tilde{\tau}$

We choose the gauge condition on the fields

$$\mathcal{G}^a[\tilde{A},\tilde{\tau}] = \frac{1}{\sqrt{\xi}}\mathcal{H}_m(\partial_u\overline{\tau},\overline{F};u)D_m\tilde{A}^a_m + \sqrt{\xi}\epsilon_{abc}\tilde{\tau}^b\overline{\tau}^c \quad (a=1,2)$$

such that the gauge fixed Lagrangian

$$\mathcal{L}[\tilde{A},\tilde{\tau}] - \frac{1}{2}\mathcal{G}^2$$

no longer contains $\tilde{A}\tilde{\tau}$ mixing terms.
Then we choose $\xi \to \infty$ ("unitary gauge"): $\tilde{\tau}^{1,2}$ decouple.

Remaining gauge freedom in Abelian direction fixed by

$$A^a_u = 0 \quad (a=0,3).$$

Fixing the gauge to disentangle $\tilde{A}$ and $\tilde{\tau}$

In the chosen gauge the Higgs-mechanism is more visible:

- $\tilde{\tau}^{1,2}$ are 'eaten' = Goldstone bosons
- $\tilde{A}_{\mu}^{1,2}$ eating the $\tilde{\tau}^{1,2}$ = massive gauge bosons (mass $\sim \overline{\tau}^2$)
- $\tilde{\tau}^{0,3}$ in the direction of the vev $\overline{\tau}$ = Higgs bosons

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We are left with

$$\mathcal{L}_{5D} = \mathcal{L}[\tilde{\tau}] + \mathcal{L}[\tilde{A}]$$

$\mathcal{L}[\tilde{\tau}]$: Stability of the embedding

$\mathcal{L}[\tilde{\tau}] \rightsquigarrow$ stability of the embedding:
energy density

$$H = \frac{\delta \mathcal{L}}{\delta \partial_0 \tilde{\tau}} \partial_0 \tilde{\tau} - \mathcal{L}$$

associated with fluctuations $\tilde{\tau}^{0,3}$ must fulfill

$$\mathcal{E} = \int_{u_{0,d}}^{\infty} H \; > 0$$

We checked that this is the case.

$\mathcal{L}[\tilde{A}]$: back to the ρ meson EOM

$$\mathcal{L}_{5D} = \mathrm{STr}\left\{..[\tilde{A}_m, \overline{\tau}]^2 + ..(F_{\mu\nu})^2 + ..(F_{\mu u})^2 + ..\overline{F}_{\mu\nu}[\tilde{A}_\mu, \tilde{A}_\nu]\right\}$$

with all the .. different functions $\mathcal{H}(\partial_u\overline{\tau}, \overline{F}; u)$ of the background fields $\partial_u\overline{\tau}$, $\overline{F}$.

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STr-prescription [Myers, Hashimoto and Taylor, Denef et.al.]

$$STr\left(\mathcal{H}(\overline{F})F^2\right) = -\frac{1}{2}\sum_{a=1}^{2} F_a^2 \;\; I(\mathcal{H}) + \sum_{a=0,3} \cdots$$

with

$$I(\mathcal{H}) = \frac{\int_0^1 d\alpha \mathcal{H}(\overline{F}_0 + \alpha\overline{F}_3) + \int_0^1 d\alpha \mathcal{H}(\overline{F}_0 - \alpha\overline{F}_3)}{2}$$

$\mathcal{L}[\tilde{A}]$: back to the ρ meson EOM

$$\mathcal{L}_{5D} = \mathrm{STr}\left\{..[\tilde{A}_m, \overline{\tau}]^2 + ..(F_{\mu\nu})^2 + ..(F_{\mu u})^2 + ..\overline{F}_{\mu\nu}[\tilde{A}_\mu, \tilde{A}_\nu]\right\}$$

with all the .. different functions $\mathcal{H}(\partial_u \overline{\tau}, \overline{F}; u)$ of the background fields $\partial_u \overline{\tau}$, $\overline{F}$.

STr-prescription [Myers, Hashimoto and Taylor, Denef et.al.]

$$STr\left(\mathcal{H}(\overline{F})F^2\right) = -\frac{1}{2}\sum_{a=1}^{2} F_a^2 \;\; I(\mathcal{H}) + \sum_{a=0,3} \cdots$$

with

$$I(\mathcal{H}) = \frac{\int_0^1 d\alpha \mathcal{H}(\overline{F}_0 + \alpha \overline{F}_3) + \int_0^1 d\alpha \mathcal{H}(\overline{F}_0 - \alpha \overline{F}_3)}{2}$$

$$\mathcal{L}_{5D} = -\frac{1}{4} f_1(B)(F^a_{\mu\nu})^2 - \frac{1}{2} f_2(B)(F^a_{\mu u})^2 - \frac{1}{2} f_3(B)\overline{F}^3_{ij}\epsilon_{3ab}\tilde{A}^a_i \tilde{A}^b_j$$
$$-\frac{1}{2} f_4(B)(\tilde{A}^a_\mu)^2(\overline{\tau}^3)^2 - \frac{1}{2} f_5(B)(\tilde{A}^a_i)^2(\overline{\tau}^3)^2$$

EOM for ρ for $u_0 > u_K$

$$S_{5D} = \int d^4x \int du \left\{ -\frac{1}{4} f_1(B) \underbrace{(F^a_{\mu\nu})^2}_{(\mathcal{F}^a_{\mu\nu})^2\psi^2} - \frac{1}{2} f_2(B) \underbrace{(F^a_{\mu u})^2}_{(\rho^a_\mu)^2(\partial_u\psi)^2} - \frac{1}{2} f_3(B) \overline{F}^3_{ij}\epsilon_{3ab} \underbrace{\tilde{A}^a_i \tilde{A}^b_j}_{\rho^a_i\rho^b_j\psi^2} \right.$$

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demand $\int du\, f_1(B)\psi^2 = 1$ and $\int du\, f_2(B)(\partial_u\psi)^2 + f_4(B)(\overline{\tau}^3)^2\psi^2 = m^2_\rho(B)$,

then $\int du\, f_3(B)\psi^2 = k(B) \neq 1$ and $\int du\, f_5(B)(\overline{\tau}^3)^2\psi^2 = m^2_+(B)$

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(with $\mathcal{F}^a_{\mu\nu} = D_\mu \rho^a_\nu - D_\nu \rho^a_\mu$)

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(with $\mathcal{F}^a_{\mu\nu} = D_\mu\rho^a_\nu - D_\nu\rho^a_\mu$)

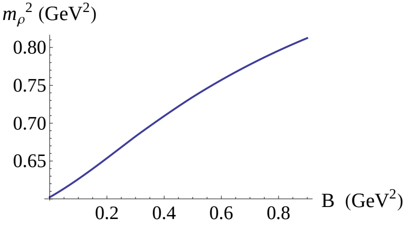
modified 4D Lagrangian for a vector field in an external EM field

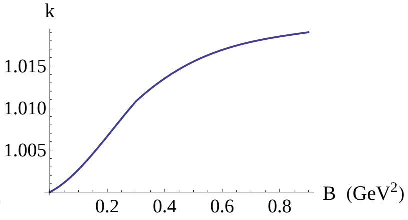
Solve the eigenvalue problem

The normalization condition and mass condition on the ψ combine to the eigenvalue equation

$$f_1^{-1}\partial_u(f_2\partial_u\psi) - f_1^{-1}f_4(\overline{\tau}_3)^2\psi = -m_\rho^2\psi$$

with b.c. $\psi(x=\pm\pi/2)=0, \psi'(x=0)=0$
which we solve with a numerical shooting method to obtain $m_\rho^2(B)$.





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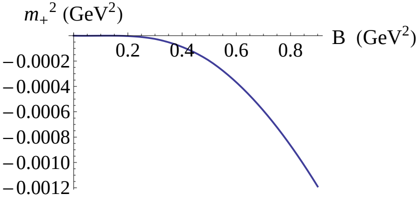

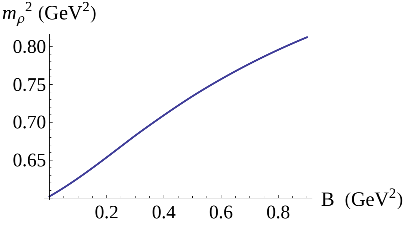

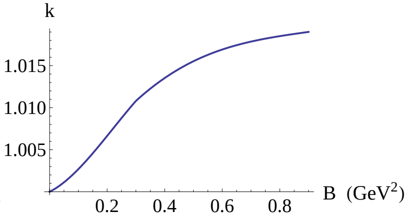

Landau vs Sakai-Sugimoto $u_0 > u_K$

Modified 4D Lagrangian for a vector field in an external EM field with $k(B) \neq 1$

$\rightsquigarrow$ modified Landau levels and

$$m^2_{\rho,eff}(B) = m^2_\rho(B) + m^2_+(B) - k(B)\,B$$

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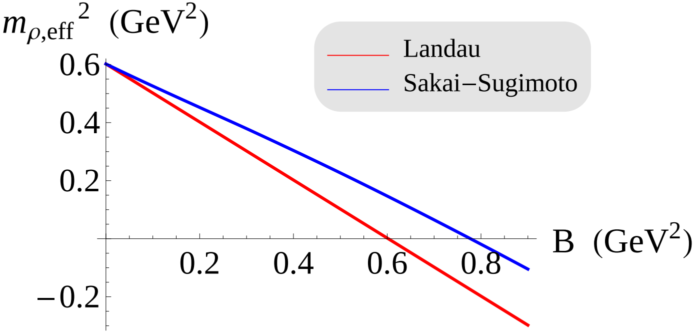
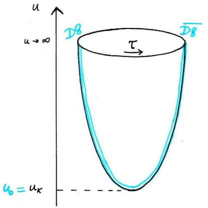
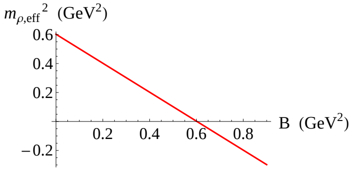


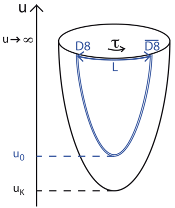
ρ meson condensation in Sakai-Sugimoto

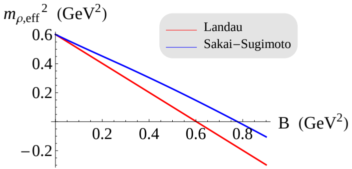
- Antipodal embedding $(u_0 = u_K) \Rightarrow$ Landau levels





- Non-antipodal embedding $(u_0 > u_K) \Rightarrow$ modified Landau levels





Full DBI-action

Reasons for considering full DBI-action:

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- Expansion parameter in action
  $\det(g + iF) = \det g \times \det(1 + g^{-1} iF)$ is $g^{-1} iF$
  $\Rightarrow$ most strict condition
  $eB \ll \frac{3}{2} \left(\frac{u_{0,d}(B=0)}{R} \right)^{3/2} (2\pi\alpha')^{-1} \equiv 0.45\ \mathrm{GeV}^2$

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$$S_{4D} = \int d^4x \left\{ -\frac{1}{4}(\mathcal{F}^a_{\mu\nu})^2 - \frac{1}{2} m_\rho^2(B)(\rho^a_\mu)^2 - \frac{1}{2} b(B)(\mathcal{F}^a_{12})^2 \right.$$
$$\left. -\frac{1}{2} k(B) \overline{F}^3_{ij} \epsilon_{3ab} \rho^a_i \rho^b_j - \frac{1}{2} m_+^2(B)(\rho^a_i)^2 - \frac{1}{2} a(B)\left((\mathcal{F}^a_{i3})^2 + (\mathcal{F}^a_{i0})^2\right) \right\}$$

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Further modified 4D Lagrangian for a vector field in an external EM field

4-dimensional EOM

Standard Proca EOM for charged rho meson $\rho_\mu = (\rho^1_\mu + i\rho^2_\mu)/\sqrt{2}$

$$\begin{aligned} &\mathrm{D}^2_\mu \rho_\nu - 2i\overline{F}^3_{\mu\nu}\rho_\mu - \mathrm{D}_\nu \mathrm{D}_\mu \rho_\mu - m^2_\rho \rho_\nu = 0, \\ &\mathrm{D}_\nu \rho_\nu = 0 \end{aligned}$$

with $\mathrm{D}_\mu = \partial_\mu + i\overline{A}^3_\mu$ and $\mathrm{F}_{\mu\nu} = \mathrm{D}_\mu \rho_\nu - \mathrm{D}_\nu \rho_\mu$

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$$\mathrm{D}_\nu \rho_\nu = 0$$

with $\mathrm{D}_\mu = \partial_\mu + i\overline{A}^3_\mu$ and $\mathrm{F}_{\mu\nu} = \mathrm{D}_\mu \rho_\nu - \mathrm{D}_\nu \rho_\mu$

replaced by

$$(1+a)\mathrm{D}^2_\mu \rho_\nu - i(1+b+k)\overline{F}^3_{\mu\nu}\rho_\mu - (1+a)\mathrm{D}_\nu \mathrm{D}_\mu \rho_\mu$$
$$-(m^2_\rho + m^2_+)\rho_\nu + (b-a)(\mathrm{D}^2_j \rho_\nu - \mathrm{D}_\nu \mathrm{D}_j \rho_j) = 0,$$
$$\mathrm{D}_\nu \rho_\nu = \frac{i}{m^2_\rho}(1+b-k)\overline{F}^3_{\mu\nu}\mathrm{D}_\nu \rho_\mu - \frac{m^2_+}{m^2_\rho}\mathrm{D}_i \rho_i$$

Generalized Landau levels

Landau levels

$$\epsilon^2_{n,s_z}(p_z) = p_z^2 + m_\rho^2 + (2n - 2s_z + 1)B$$

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replaced by

$$\epsilon_n^2(p_z) = \mathcal{B}p_z^2 + \frac{m_\rho^2 + m_+^2}{1+a} + (2n+1)B(\mathcal{B} - \frac{\mathcal{M}}{2}) + \frac{(1+b-k)}{2}\frac{B^2}{m_\rho^2}$$
$$\pm B\left\{\mathcal{M}\left(\frac{(2n+1)^2}{4} + \mathcal{K} - 2\mathcal{B}\right) + (\mathcal{K} - 2\mathcal{B})^2\right.$$
$$\left. -(1+b-k)(2n+1)\xi(\mathcal{K} - 2\mathcal{B} + \frac{\mathcal{M}}{2}) + \frac{(1+b-k)^2}{4}\xi^2\right\}^{1/2}$$

with

$$\mathcal{B} = \frac{1+b}{1+a}, \quad \mathcal{K} = \frac{1+b+k}{1+a}, \quad \mathcal{M} = \frac{b-a}{1+a} - \frac{m_+^2}{m_\rho^2} \quad \text{and} \quad \xi = \frac{B}{m_\rho^2}$$

Effective ρ meson mass from full DBI-action

Condensing solution $n = 0$, $p_z = 0$ for transverse charged ρ mesons $\rho = (\rho_x^- - i\rho_y^-)$ and $\rho^\dagger = (\rho_x^+ + i\rho_y^+)$

$$m^2_{\rho,eff}(B) = m^2_\rho - B$$

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$$m_{\rho,eff}^2(B) = m_\rho^2 - B$$

becomes

$$m_{\rho,eff}^2(B) = \frac{m_\rho^2(B) + m_+^2(B)}{1 + a(B)} - \frac{k(B)}{1 + a(B)} B$$

ρ meson condensation in Sakai-Sugimoto

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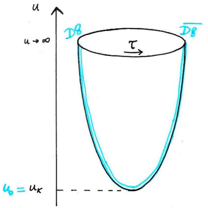

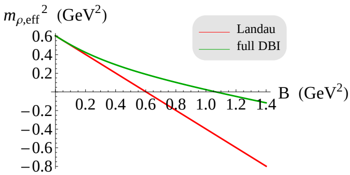

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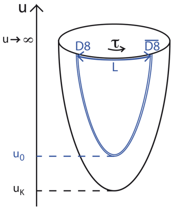

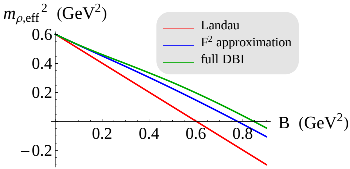

Effect of Chern-Simons action and mixing with pions

- $S = S_{DBI} + S_{CS}$ with

$$S_{CS} \sim \int \mathrm{Tr}\left(\epsilon^{mnpqr} A_m F_{np} F_{qr} + \mathcal{O}(\tilde{A}^3)\right)$$

- $\rho\pi B$ mixing terms in the Chern-Simons action:

$$S_{CS} \sim B \int \left\{ \partial_{[0}\pi^0 \rho^0_{3]} + \frac{1}{2}\left(\partial_{[0}\pi^+ \rho^-_{3]} + \partial_{[0}\pi^- \rho^+_{3]}\right)\right\} + \cdots ,$$

  but only between pions and longitudinal ρ meson components

- so no influence of pions on condensation of transversal ρ meson components (in order $\tilde{A}^2$ analysis)

Conclusion: back to objectives

Studied effect: ρ meson condensation

- phenomenological models: $B_c = m_\rho^2 = 0.6$ GeV^2
- lattice simulation: slightly higher value of $B_c \approx 0.9$ GeV^2
- $\rightsquigarrow$ holographic approach:
  - can the ρ meson condensation be modeled? yes
  - can this approach deliver new insights? e.g. taking into account constituents, effect on B_c
    Up and down quark constituents of the ρ meson can be modeled as separate branes, each responding to the magnetic field by changing their embedding. This is a modeling of the chiral magnetic catalysis effect. We take this into account and find also a string effect on the mass, leading to a $B_c \approx 0.8$ GeV^2. Effect of full DBI is further increase of B_c.

Thank you for your attention!

Questions?