Spatial Modulation and Topological Current in Holographic QCD Matter

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Outline

p-wave pion condensation in nuclear matter

An old but vital idea of inhomogeneity in nuclear physics, but...

Landau-Migdal interaction in a Fermi liquid

Almost abandoned... and revived recently in quark matter

Interaction controlled by topological current in *B* – *Chiral Magnetic Effect* –

Effect of the topological current induced by the magnetic field

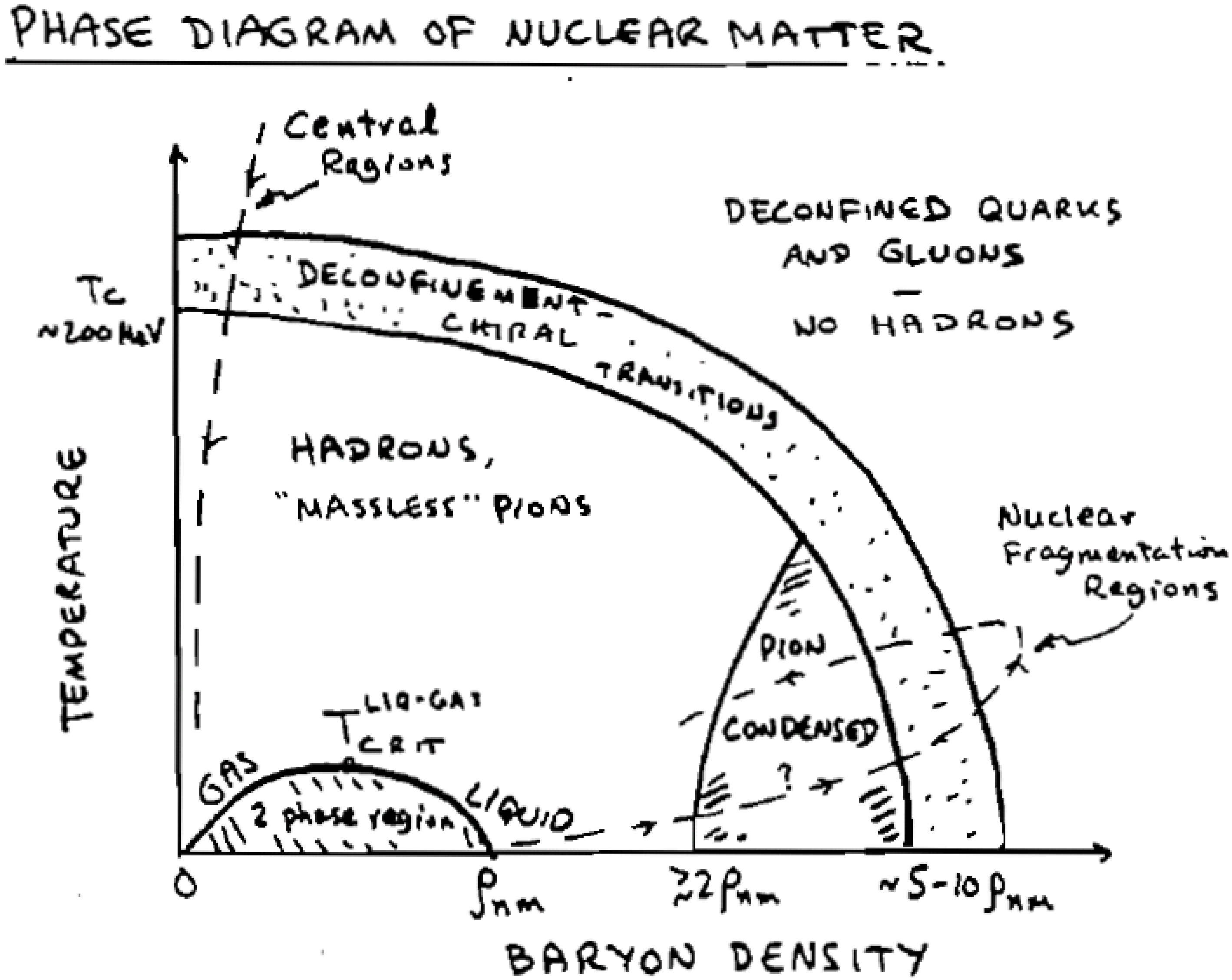
Analysis in the Sakai-Sugimoto model

Inhomogeneous states favored or disfavored by *B*???

Results consistent with what happened to the pion condensation

Historical Phase Diagram of QCD

Baym (1983)



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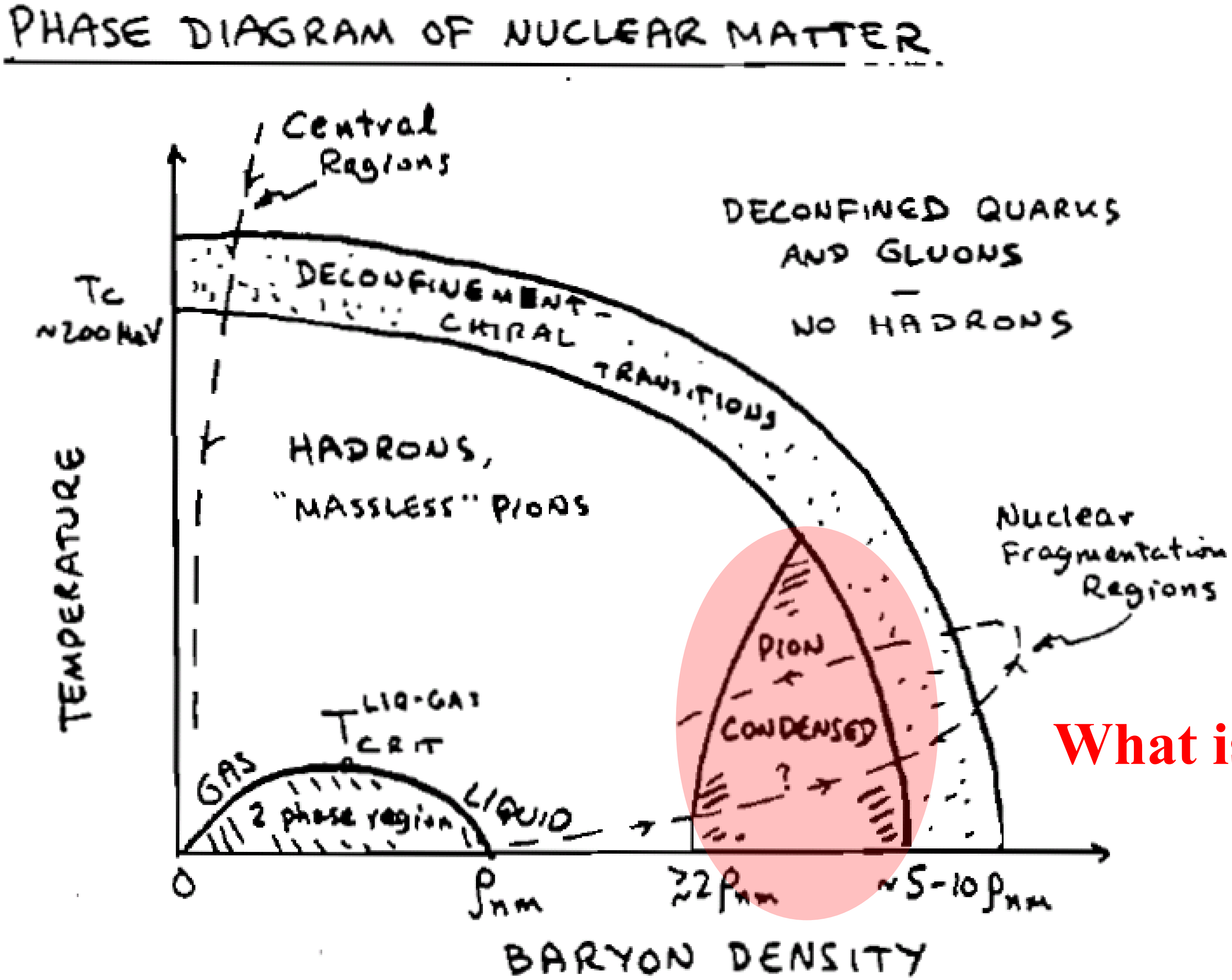
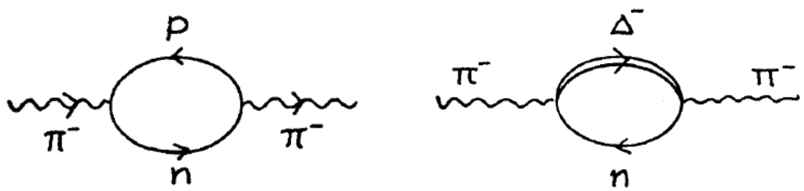


What is this?

p-wave Pion Condensation

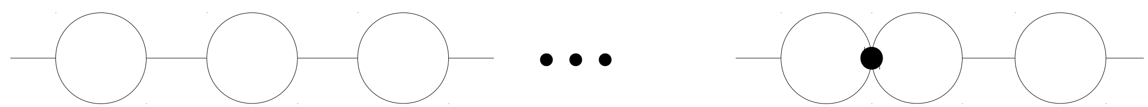
Sawyer-Scalapino, Migdal (1972)

$$\Pi(\omega, k) \rightarrow D^{-1}(\omega=0, k=k_c)=0 \text{ at } \rho=\rho_c$$



+ (short-ranged interactions)

$D \sim$



One Pion Exchange Potential (OPEP)

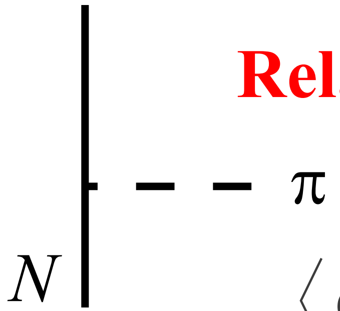
$$V=\frac{m_\pi^2}{3}\frac{g^2}{4\pi}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\frac{e^{-m_\pi r}}{r}+S_{12}\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r}\right]-\frac{g^2}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\delta(r)$$

Landau-Migdal (short-ranged) Interactions

$$f+g\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+f'\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+g'(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)$$

p-wave Pion Condensation

Why *p*-wave?



Relative angular-momentum must be 1 (p-wave)

(negative parity)

$$\langle \sigma \rangle \sim \chi \cos(2qz) \qquad \langle \pi^0 \rangle \sim \chi \sin(2qz)$$

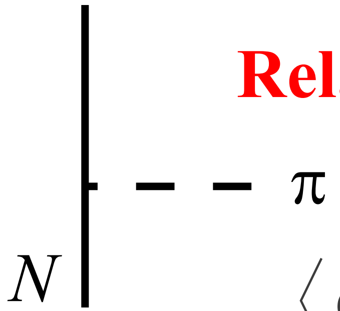
Landau-Migdal Parameter

g' is sensitive to the spin-isospin collective exciation
Gamow-Teller resonance → large g' → **No pion condensation?**

Most of nuclear physicists consider no pion condensation up to a few times normal nuclear density

p-wave Pion Condensation

Why *p*-wave?



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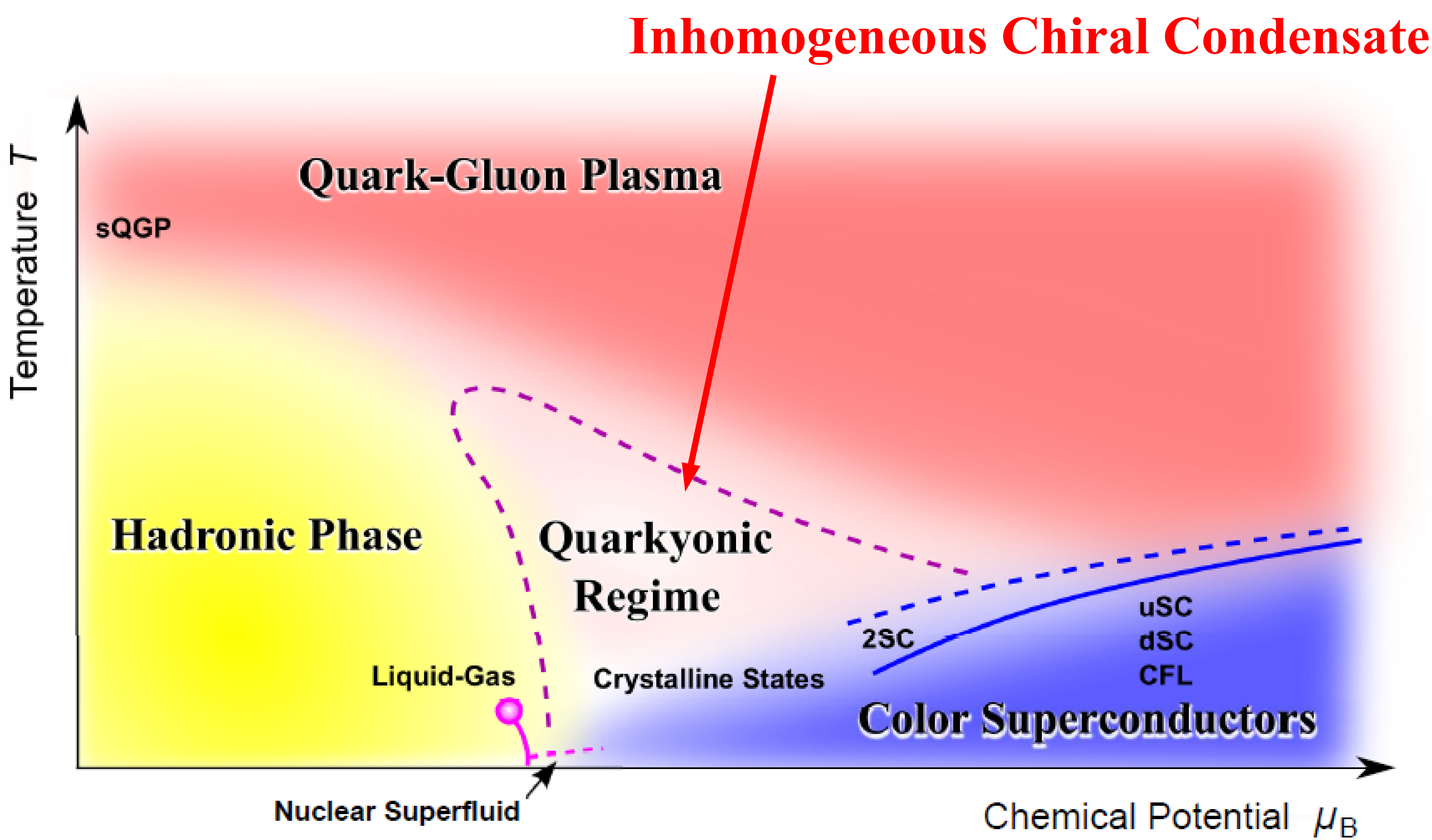
History repeats itself (Nuclear → Quark Matter)

One Possible QCD Phase Diagram

Fukushima-Sasaki (2013)



Energy Gain by Spiral

Chiral spiral (DGR Ansatz)

Deryagin-Grigoriev-Rubakov (1992)

$$\psi(x) = e^{i\gamma_5 \tau_3 q z}\psi'(x) \quad \text{with} \quad \chi = \langle \bar{\psi}'\psi' \rangle$$

$$\langle \sigma \rangle \sim \langle \bar{\psi}\psi \rangle = \chi \cos(2qz)$$

$$\langle \pi^0 \rangle \sim \langle \bar{\psi}\gamma_5\tau_3\psi \rangle = \chi \sin(2qz)$$

Quasi-particle dispersion relation

$$\omega = \sqrt{p_\perp^2 + \left(\sqrt{p_z^2 + M^2} \pm q\right)^2}$$

Vacuum — **favors large *M***

Competitive

High Density — **favors small *M***

Vacuum + High Density
favors large $q \sim M$ most!

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Counterpart of the Landau-Migdal int?
Can be the true ground state with strong *B*

Landau Quantization

Energy dispersion relation in *B*

$$\omega^2 = p_z^2 + \underline{2|g B|(n + 1/2)} + m^2 - 2 s g B$$

Transverse motion = Harmonic Oscillator

- **Light fermions (s=1/2) have zero mode.**
  Pseudo-(1+1) dimensional system of fermions
- **Light vector bosons have (Nielesen-Olesen) instability.**
  Gluons in the chromo-*B* / ρ in a superstrong *B* **(Chernodub)**
- **Charged scalar bosons are all massive.**
  π^+, π^-, … Explicit breaking of isospin symmetry

Etc, etc...

Pseudo-(1+1) dimensional System

Dirac Lagrangian in (1+1) dimensions

$$L = \bar{\psi}\left[(\partial_4 + \mu)\gamma^4 + \partial_3\gamma^3\right]\psi \qquad \psi = e^{-\mu\gamma^3\gamma^4 x_3}\psi'$$

$$= \bar{\psi}'\left[\partial^4\gamma_4 + \partial_3\gamma^3\right]\psi' \qquad \bar{\psi} = \bar{\psi}' e^{-\mu\gamma^3\gamma^4 x_3}$$

Thermodynamic potential

$$\Omega/V = -\int_{-\Lambda+\mu}^{\Lambda-\mu}\frac{dp}{2\pi}\frac{|\varepsilon(p)|}{2} - \int_{-\Lambda-\mu}^{\Lambda+\mu}\frac{dp}{2\pi}\frac{|\varepsilon(p)|}{2} + \cdots$$

$$= \Omega(\mu=0)/V - \frac{\mu^2}{2\pi}$$

Surface integral: Anomaly origin
c.f. CS term in Sakai-Sugimoto

No mass suppression as compared to the homogeneous case
Energy gain by spiral maximized in (1+1) dimensions!

Pseudo-(1+1) dimensional System

If the zero-density system has a condensate:

$$\langle \bar{\psi}\psi \rangle = (\text{homogeneous chiral condensate})$$

Rotated system has the same condensate: $\langle \bar{\psi}'\psi' \rangle$

(1+1)-dimensional system forms a "spiral"

$$\langle \bar{\psi}\psi \rangle = \langle \bar{\psi}'\psi' \rangle \cos(2\mu z)$$
$$\langle \bar{\psi}\gamma^3\gamma^4\psi \rangle = \langle \bar{\psi}'\psi' \rangle \sin(2\mu z)$$

Chiral Magnetic Spiral **Basar-Dunne-Kharzeev (2010)**

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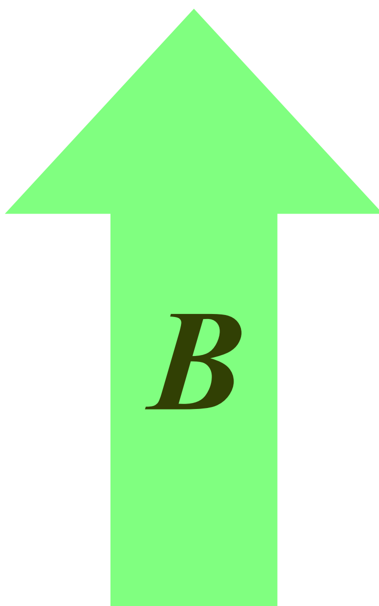
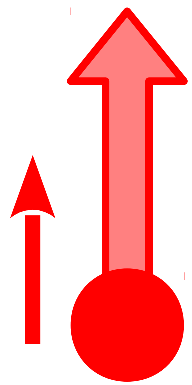
Chiral Magnetic Spiral **Basar-Dunne-Kharzeev (2010)**

B **+ μ causes another interesting phenomenon!**

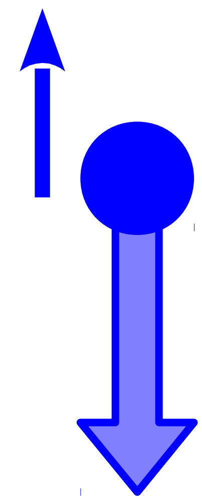
Chiral Magnetic (Separation) Effect

Classical Picture





Left-handed Quarks
= momentum
anti-parallel to
spin



Right-handed Quarks
= momentum
parallel to
spin

$$J_5 \neq 0 \quad \text{if} \quad \mu \text{ or } N \neq 0$$

Kharzeev-McLerran-Warringa (2007)
Fukushima-Kharzeev-Warringa (2008)

Quantum Formula

Chiral Magnetic (Separation) Effect

$$j_5 = \frac{e^2 \mu}{2\pi^2} B \qquad \left(j^\mu = \epsilon^{\mu\nu\rho\sigma} \partial_\nu \varphi F_{\rho\sigma} \right)$$

Vector = Axial-Vector ($\gamma^5 = \gamma^0 \gamma^1$) **in (1+1)**

$$\gamma^\mu \gamma^5 = -\epsilon^{\mu\nu} \gamma_\nu \quad + \quad j_V^\mu = \bar{\psi} \gamma^\mu \psi , \qquad j_A^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$$

$$\longrightarrow \quad j_V^1 = j_A^0 , \qquad j_A^1 = j_V^0$$

Question

At high density

Chiral spiral is expected?

At strong *B*

Chiral spiral is favored by low dimensionality...

Competitive

Axial-current is strengthened leading to –

$$g'(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)$$

that disfavors the chiral spiral...

What we need is…

Spatial Inhomogeneity

Ooguri-Park, Chuang-Dai-Kawamoto-Lin-Yeh (2010)

Topological Current

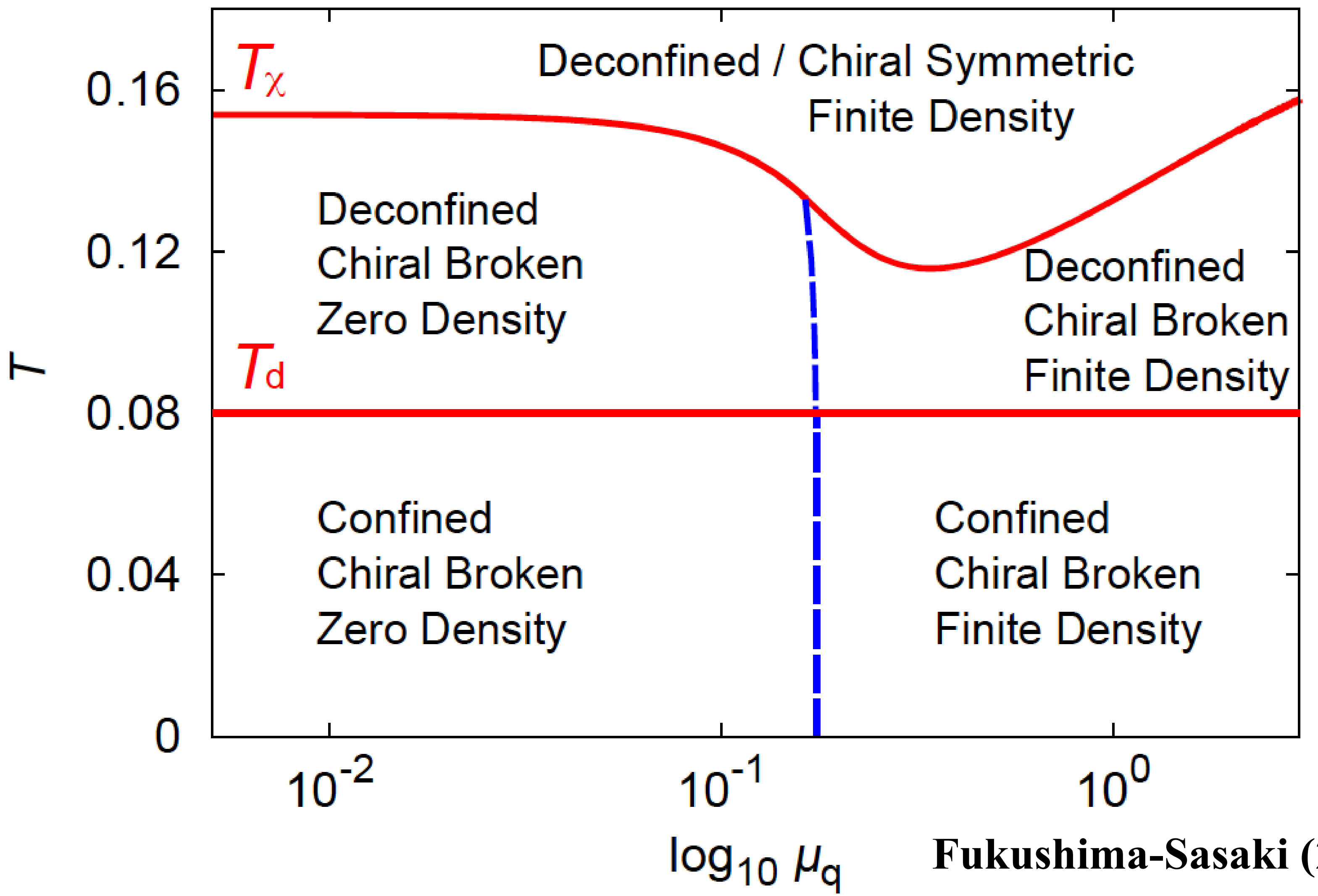
Yee, Rebhan-Schmitt-Stricker (2009)

Non-perturbative Method → Sakai-Sugimoto Model

Holographic QCD Model with $D4+D8+\overline{D8}$ **Sakai-Sugimoto (2004)**

Phase Diagram in SSM

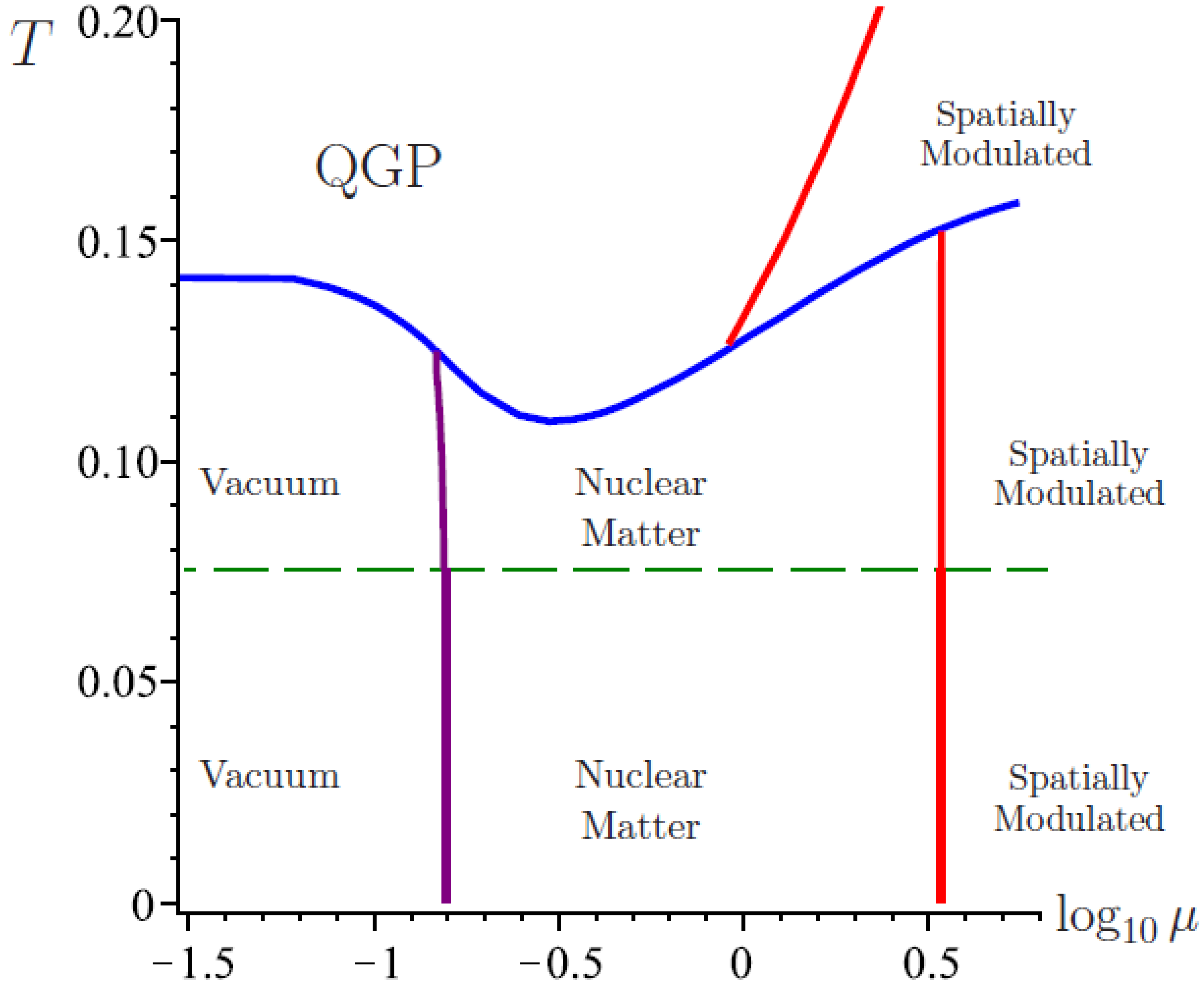
Bergman-Lifschytz-Lippert (2007)



Fukushima-Sasaki (2013)

Inhomogeneous States in SSM

Chuang-Dai-Kawamoto-Lin-Yeh (2010)



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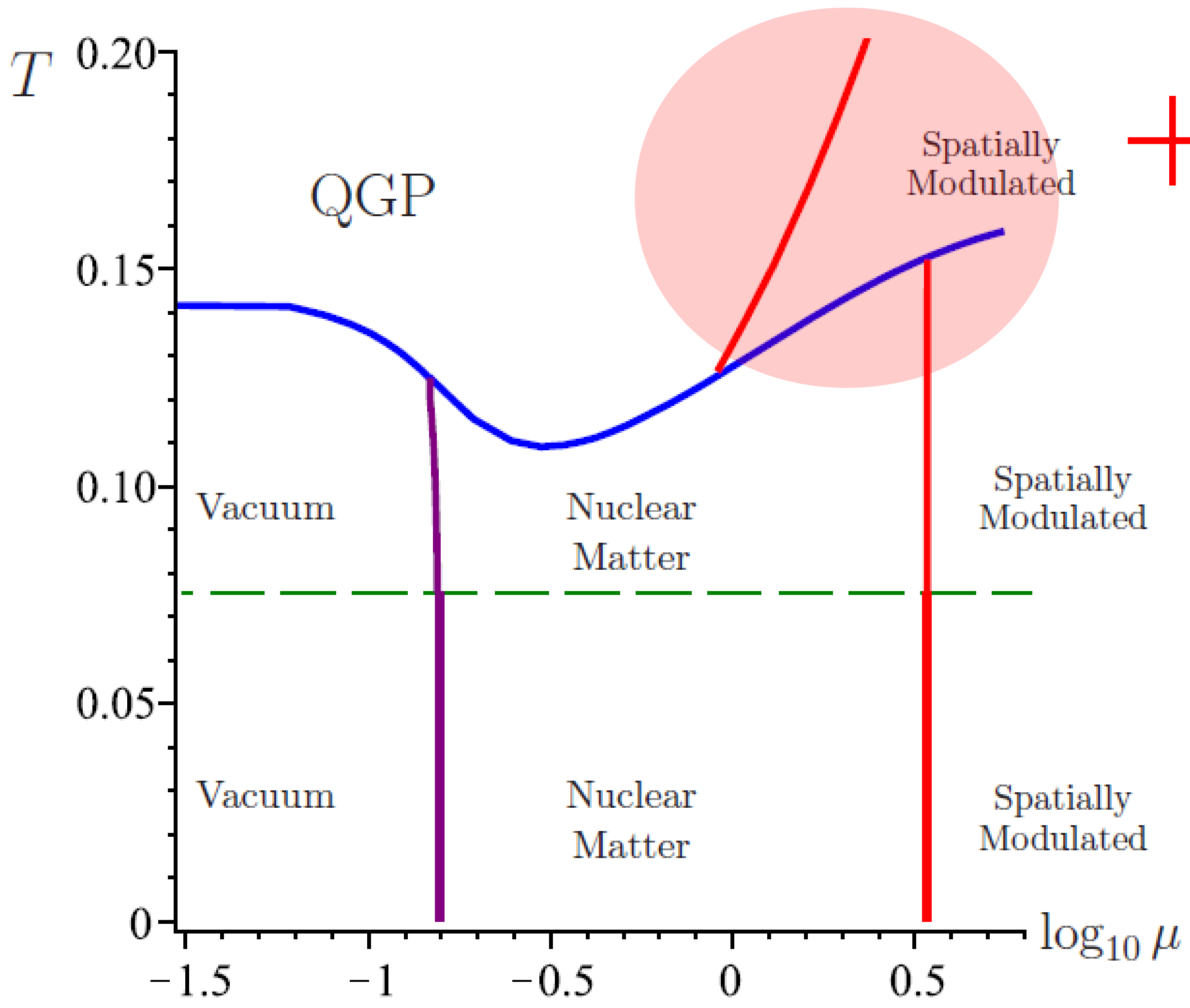


$+B$

Sketch of the Calculations

Action for the flavor sector

$$S = N \int d^4x\, du\, u^{1/4} \sqrt{-\det(g_{\alpha\beta} + F_{\alpha\beta})}$$

DBI action

$$+ \frac{\alpha}{3!} N \int d^4x\, du\, \epsilon^{\mu_1\mu_2\mu_3\mu_4\mu_5} A_{\mu_1} F_{\mu_2\mu_3} F_{\mu_4\mu_5}$$

Chern-Simons action

Density $\bar{a}_0(u)$ $\bar{a}_0(\infty) = \mu$

Current $\bar{a}_z(u)$

Magnetic field $\bar{F}_{xy} = B(u) = B$

$$S \sim \int du\, u^{5/2} \sqrt{(1 - a_0'^2 + f a_z'^2)(1 + B^2 u^{-3})} + 4\alpha B \int du\, \bar{a}_z \bar{a}_0'$$

Sketch of the Calculations

Equations of motion (w.r.t. a_0 and a_z)

$$\rho = u\bar{a}_0'\sqrt{\frac{u^3+B^2}{1-\bar{a}_0'^2+f\bar{a}_z'^2}} - 4\alpha B\bar{a}_z \qquad 0 = uf\bar{a}_z'\sqrt{\frac{u^3+B^2}{1-\bar{a}_0'^2+f\bar{a}_3'^2}} - 4\alpha B\bar{a}_0$$

Asymptotic solutions:

$$\bar{a}_z(u\to\infty) \simeq -\frac{8\alpha}{3}B\mu u^{-3/2} \qquad \bar{a}_0(u\to\infty)\simeq \mu - \frac{3\alpha}{2}\rho u^{-3/2}$$

$\sim j_5$

Introducing spatial modulations:

$$F_{\alpha\beta} \to F_{\alpha\beta} + f_{\alpha\beta}(\omega, k)$$

$$f_{\alpha\beta} = \partial_\alpha a_\beta - \partial_\beta a_\alpha$$

$\longrightarrow$ $D^{-1}(\omega=0, k=k_c)=0$

Sketch of the Calculations

Equations of motion (w.r.t. a_x , a_y , a_z)

From the condition that they are normalizable or

$$a_x(u=\infty)=a_y(u=\infty)=a_z(u=\infty)\to 0$$

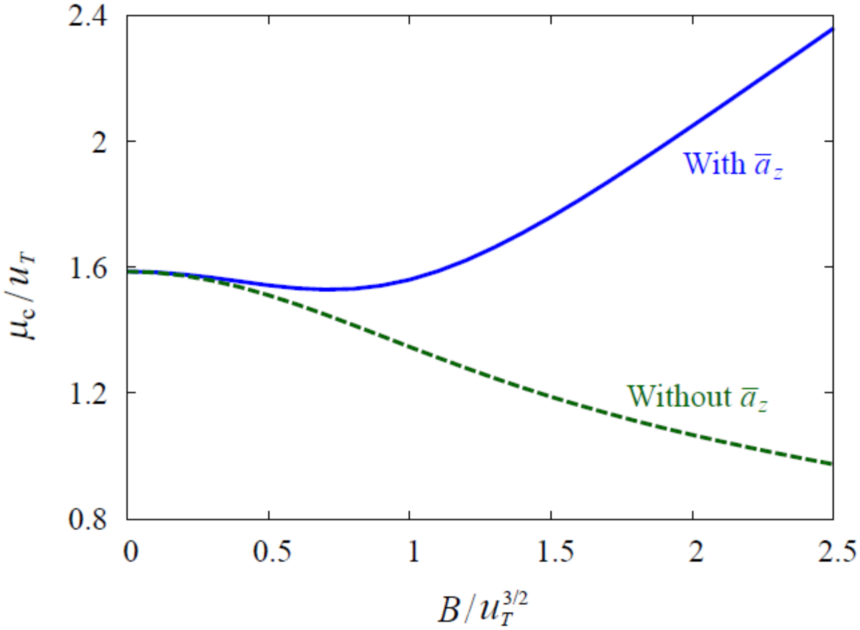
→ k_x , k_y , k_z are found for $\rho > \rho_c$

Without *B*

$$\rho_c = 3.715\, u_T^{5/2}$$

(Ooguri-Park 2010)

With *B*



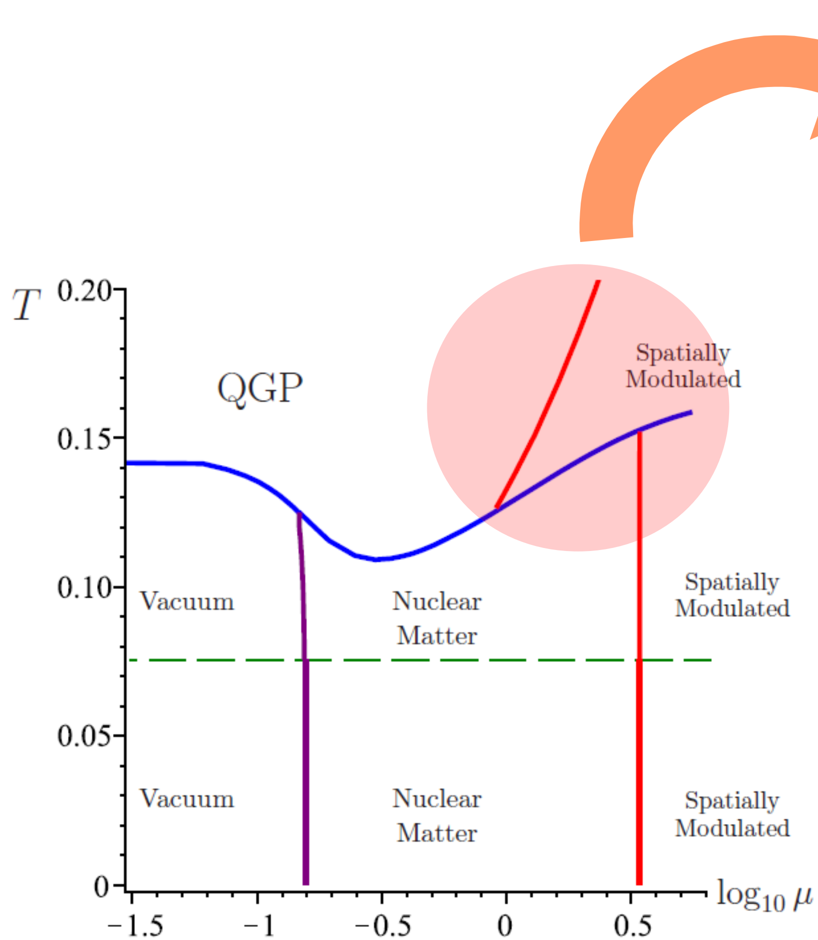
The presence of j_5 completely changes the results!

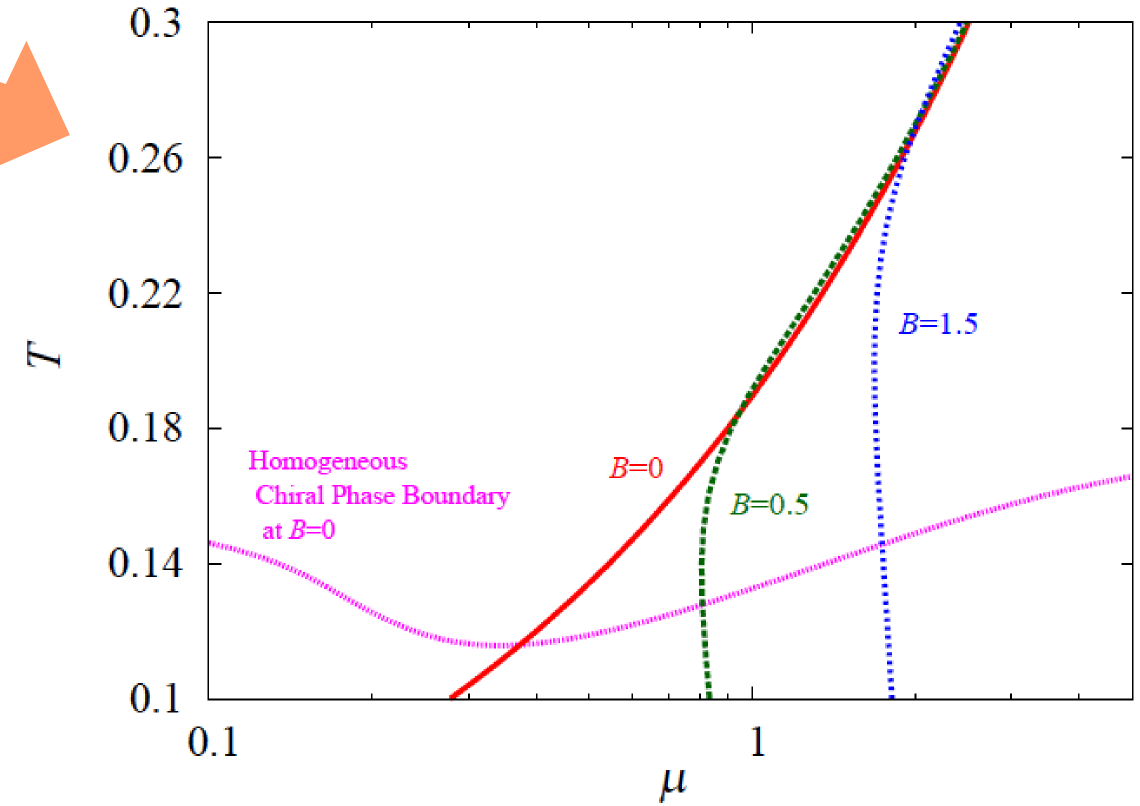
(Fukushima-Morales 2013)

Phase Diagram with B

Smaller with increasing *B*

Fukushima-Morales (2013)





Same tendency as what happened to the *p*-wave pion condensation

Summary and...

Sakai-Sugimoto model is a powerful tool to investigate the *chiral sector of large-N_c QCD*.

→ Phase Diagram ??

Axial-vector interaction disfavors the spatial modulations in the same way as discussed by nuclear physicists long long time ago.

Full structure of the phase diagram with an extra axis of strong B ??

Earlier chiral phase transition? (Magnetic Inhibition)

(Fukushima-Hidaka 2012)

Deconfinement?