Holographic Estimate of EM mass of hadrons

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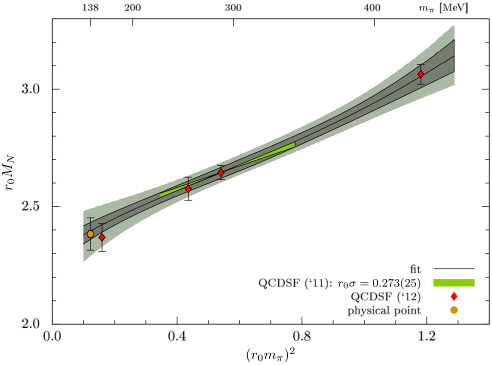
Holography and QCD - Recent Progress and Challenges -

IPMU, Tokyo U., Japan

Introduction

- Solving QCD is hard, since quarks and gluons are not right degrees of freedom at low energy.
- Lattice (QCDSF12)



Introduction and Review

- Holographic QCD is an attempt to solve QCD in terms of hadrons.
- Inspired by AdS/CFT (or open-closed string duality) and holography, hQCD is proposed to be a 5D gravity theory:

$$\mathcal{Z}_{\rm QCD}(G_\mu, q, \bar{q}) = \mathcal{Z}_{\rm hQCD}(g_{MN}, \phi, A_M, \cdots)$$

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Introduction and Review

- The UV values of the bulk fields are postulated as sources for the gauge-invariant operators of boundary theory:

$$e^{iS_{\rm hQCD}(X_0)+\mathcal{O}\left(\frac{1}{N_c},\frac{1}{\lambda}\right)} = \int \mathcal{D}(G,q,\bar{q})\, e^{iS_{\rm QCD}+\left\langle J(G,q,\bar{q})\,\bar{X}\right\rangle}\,.$$

$$(\bar{X} \equiv X_{0{\rm UV}})$$

- The hQCD action then becomes the generating functional for the QCD correlators:

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Previous studies

- Anomalous magnetic moment of muon: (DKH+Kim, 2009)
- For the hadronic LBL we need to calculate 4-point functions of flavor currents:

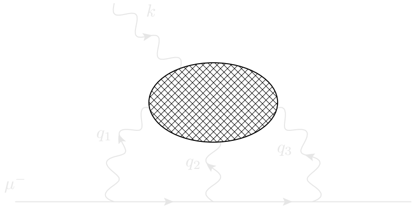

Figure : Light-by-light corrections to muon $g-2$.

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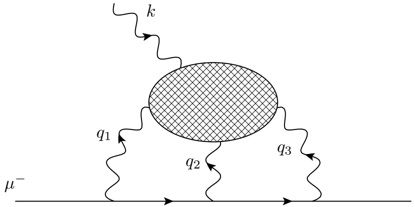

Figure : Light-by-light corrections to muon $g-2$.

Holographic Calculation of Hadronic LBL

- Since there is no quartic term for $A_{Q_{\rm em}}$ ($Q_{\rm em} = 1/2 + I_3$), there is no 1PI 4-point function for the EM currents in hQCD:

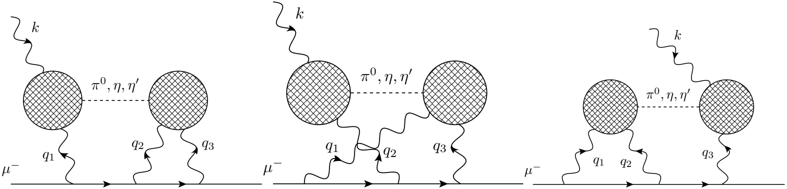

Figure : Light-by-light correction is dominated by the pseudo scalar mesons exchange.

- Higher order terms like F^4 or F^2X^2 terms are suppressed.

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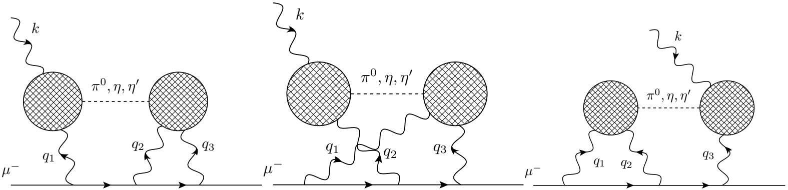

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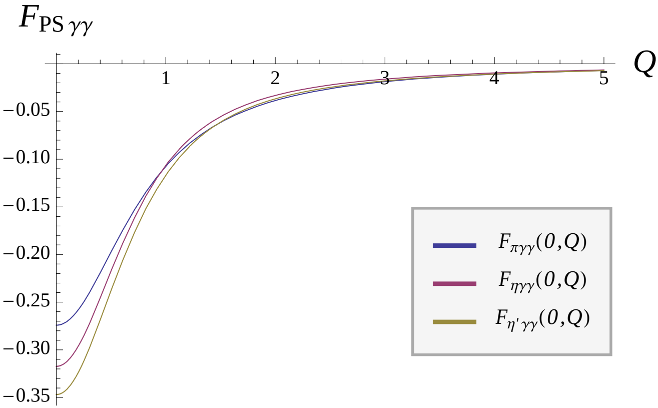
Holographic Calculation of Hadronic LBL

- In hQCD the LBL diagram is dominated by VVA or VVP diagrams, which come from the CS term:

$$F_{\gamma^*\gamma^* P(A)}(q_1, q_2) = \frac{\delta^3}{\delta V(q_1)\delta V(q_2)\delta A(-q_1-q_2)} S_{\mathrm{hQCD}} + \mathcal{O}\left(\frac{1}{N_c}, \frac{1}{\lambda}\right).$$

Anomalous form factors



Hadronic LBL in hQCD

- To calculate the hadronic LBL contribution to a_μ we expand the photon line as

$$J(-iQ, z) = V(q, z) = \sum_\rho \frac{-g_5 f_\rho \psi_\rho(z)}{q^2 - m_\rho^2 + i\epsilon}$$

Table : Muon $g-2$ results from the AdS/QCD in unit of 10^{-10}.

Vector modes	$a_\mu^{\pi^0}$	a_μ^{η}	$a_\mu^{\eta'}$	$a_\mu^{\rm PS}$
4	7.5	2.1	1.0	10.6
6	7.1	2.5	0.9	10.5
8	6.9	2.7	1.1	10.7

- In the LMD+V model by Nyffeler

$$a_\mu^{\rm PS} = 9.9(1.6) \times 10^{-10}$$

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EM mass of Pions - Vacuum Alignment

- In QCD the chiral symmetry is spontaneously broken,

$$G \longrightarrow H$$

- Pions and Kaons are massless in the chiral limit, because they are fluctuations on the vacuum manifold, G/H, along the flat directions of broken generators. (Goldstone 1961)
- When the chiral symmetry is approximate, however, the vacuum degeneracy is lifted and they become massive, the pseudo-Nambu-Golstone bosons. (Weinberg 1972)

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EM mass of Pions - Vacuum Alignment

- Current quark mass gives pion mass, in the isospin limit,

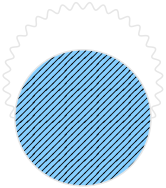
$$F_\pi^2 m_\pi^2 = 2 m_q \langle q\bar{q} \rangle \ . \quad \mathrm{(PCAC)}$$

- In hQCD (Aharony+Kutasov; Hashimoto et al 2008)

$$m_\pi^2 = \frac{1}{TV} \frac{\delta^2}{\delta\pi^2} S_{\mathrm{hQCD}} \Big|_{\pi=0}$$

EM mass of Pions - Vacuum Alignment

- EM interaction breaks the isospin symmetry and contributes to the vacuum energy, lifting its degeneracy.
- The corrections to the vacuum energy is



$$\Delta E_{\rm vac} = -\frac{e^2}{2}\int \mathrm{d}^4x\, \Delta^{\mu\nu}(x)\, \langle 0|\, U^\dagger\, \mathrm{T} J_\mu^{Q_{\rm em}}(x) J_\nu^{Q_{\rm em}}(0) U\, |0\rangle\ .$$

with $U = \exp\left(2i\pi/F_\pi\right)$.

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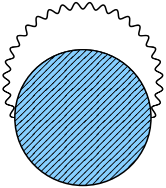

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EM mass of Pions - Vacuum Alignment

- The EM mass is now

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = \frac{\partial^2}{\partial\pi_+\partial\pi_-}\Delta E[U]\Big|_{U=1} = e^2 M^2\,,$$

where

$$M^2 = \frac{1}{F_\pi^2}\int \mathrm{d}^4x\Delta^{\mu\nu}(x)\left\langle 0|T\left[V_\mu^3(x)V_\nu^3(0) - A_\mu^3(x)A_\nu^3(0)\right]|0\right\rangle\ .$$

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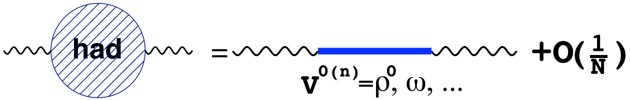
- Since the current-current correlators are given in hQCD as



the EM mass becomes (Das et al 1967)

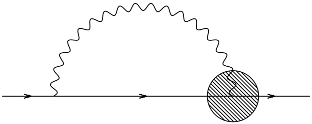
$$e^2M^2 = \frac{3e^2}{F_\pi^2}\sum_n \int_p \left(\frac{f_{V_n}^2}{\left(p^2 - m_{V_n}^2\right) m_{V_n}^2} - \frac{f_{A_n}^2}{\left(p^2 - m_{A_n}^2\right) m_{A_n}^2} \right) .$$

$$= \frac{3e^2}{8\pi^2 F_\pi^2}\sum_n \left(f_{V_n}^2 \ln\frac{\Lambda}{m_{V_n}} - f_{A_n}^2 \ln\frac{\Lambda}{m_{A_n}} \right) .$$

- $m_{\pi^\pm} - m_{\pi^0} \simeq 7.0\ \mathrm{MeV}$ for the lowest: $4.6\ \mathrm{MeV}$ (exp.)

EM mass of baryons - Radiative corrections

- The EM mass of nucleon is given as



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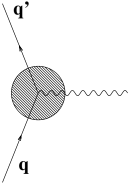

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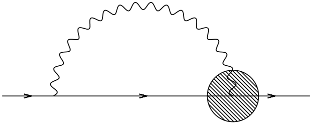

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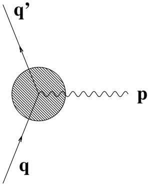

- In hQCD the form factor can be written as

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EM mass of baryons - Radiative corrections

- The nucleon form factors are defined as

$$\langle p' | J^\mu(x) | p \rangle = e^{iqx} \bar{u}(p') \mathcal{O}^\mu(p, p') u(p) ,$$

By the Lorentz invariance and the current conservation we get

$$\mathcal{O}^\mu(p, p') = \gamma^\mu F_1(Q^2) + i \frac{\sigma^{\mu\nu}}{2m_N} q_\nu F_2(Q^2) .$$

- The hQCD action for baryons has been constructed. (HIY 2007, HRYY 2008)

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EM mass of baryons - Radiative corrections

- For the Sakai-Sugimoto model it is found as

$$S_{\mathrm{hQCD}} = \int_{x,w} \left[-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - i m_b(w) \bar{\mathcal{B}}\mathcal{B} + \kappa(w) \bar{\mathcal{B}}\gamma^{mn} F_{mn}^{SU(2)_I} \mathcal{B} + \cdots \right]$$

  where

$$\kappa(w) \simeq \frac{0.18 N_c}{M_{KK}}$$

  ($M_{KK} = 0.94\ \mathrm{GeV}$ is the ultraviolet cutoff of the SS model.)

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EM mass of baryons - Radiative corrections

- In hQCD the photon field can be expanded in the basis of vector mesons:

$$A(q,w) = \sum_n \frac{g_{v^{(n)}}\psi_n(w)}{Q^2 + m_n^2} \, ,$$

where the decay constant of the *n*-th vector mesons is given as $g_{v^{(n)}} = m_n^2 \zeta_n$ with

$$\zeta_n = \frac{\lambda N_c}{108\pi^3} M_{KK} \int_{-w_{max}}^{w_{max}} dw \frac{U(w)}{U_{KK}} \, \psi_{(n)}(w) \, .$$

EM mass of baryons - Radiative corrections

- The resulting EM form factors then take the form

$$
\begin{aligned}
F_1(Q^2) &= \sum_{n=1}^{\infty} \left(g_{V,min}^{(n)} Q_{\rm em} + g_{V,mag}^{(n)} \, \tau^3 \right) \frac{\zeta_n m_n^2}{Q^2 + m_n^2} \,, \\
F_2(Q^2) &= F_2^3(Q^2) \, \tau^3 = \tau^3 \sum_{n=1}^{\infty} \frac{g_2^{(n)} \zeta_n m_n^2}{Q^2 + m_{2n}^2} \,,
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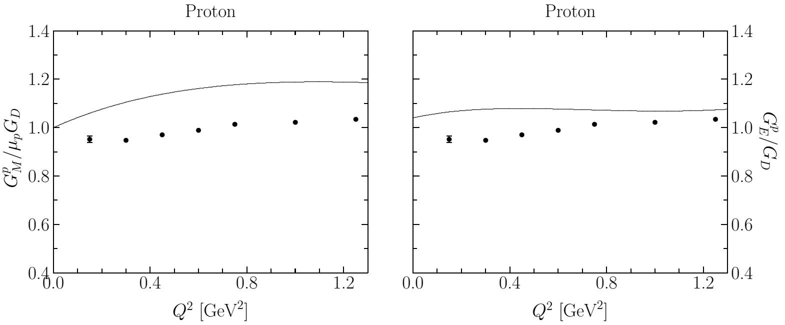
g_{V,min}^{(n)} &= \int_{-w_{max}}^{w_{max}} dw \, |f_L(w)|^2 \, \psi_{(n)}(w) \\
g_{V,mag}^{(n)} &= 2 \int_{-w_{max}}^{w_{max}} dw \, \kappa(w) \, |f_L(w)|^2 \, \partial_w \psi_{(n)}(w) \,, \\
g_2^{(n)} &= 4 m_N \int_{-w_{max}}^{w_{max}} dw \, \kappa(w) f_L^*(w) f_R(w) \psi_{(n)}(w) \,.
\end{aligned}
$$

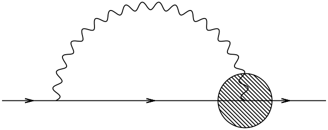
EM mass of baryons - Radiative corrections

- The EM form factors of nucleons for SS model (HRYY 2009): $G_D = 1/(1+Q^2/0.71)^2$, $G_M^p(Q^2) = F_1^p(Q^2) + F_2^p(Q^2)$, $G_E^p(Q^2) = F_1^p(Q^2) - \frac{Q^2}{4m_N^2} F_2^p(Q^2)$:



EM mass of baryons - Radiative corrections

- The EM mass of nucleons becomes



$$\delta M = e^2 \sum_n \int_p \frac{4M}{p^2+M^2} \frac{g_n}{p^2+m_n^2} \frac{1}{p^2}$$

$$= e^2 \sum_n \frac{g_n}{4\pi^2} \frac{M}{m_n^2} \int_0^1 \mathrm{d}x \ \ln\left[\frac{xM^2+(1-x)m_n^2}{x\,M^2}\right] .$$

($\delta M = -2.65\ \mathrm{MeV}$ for the lowest two vector mesons. Preliminary!)

$\delta M^{\mathrm{exp}} = -1.3\ \mathrm{MeV}$

Corrections to vector mesons masses

- The EM mass to vector mesons can be calculated similarly.
- The current quark mass contributions to nucleons and vector mesons can also be calculated by the radiated corrections due to pion loops.

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Conclusion and Outlook

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