QCD in strong magnetic fields

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Utrecht University

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Part I: **arXiv:1212.3894 with I. Iatrakis, E. Kiritsis, F. Nitti, A. O' Bannon**
Part II: **ongoing with D. Kharzeev and K. Rajagopal**

QCD under magnetic fields

- Schwinger pair production if $F > m_e^2/e$ for $eB \approx 10^{13}$ G.
- Magnetic de-catalysis: B (de)catalyzes $\langle \bar{q}q \rangle$ is non-monotonic
  Bali et al '12
- rho-meson condensation $\Rightarrow$ superconducting QCD vacuum!
  Chernodub '10 see Nele Callebaut's talk
- Phase diagram of QCD in $\mu - T - B$? depending on $N_f/N_c << 1$ or $\mathcal{O}(1)$

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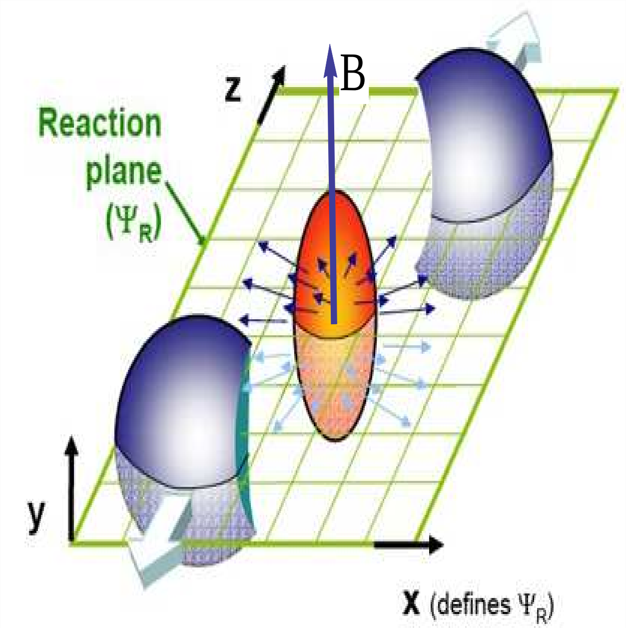
This talk:

- Chiral Magnetic effect Kharzeev, McLerran, Warringa ’07
- Induced currents in the quark-gluon plasma U.G, Kharzeev, Rajagopal,
  ongoing

Heavy ion collisions and magnetic fields



- Initial magnitude of B
- Bio-Savart: $B_0 \sim \gamma Ze \frac{b}{R^3} \Rightarrow \sim 10^{18}(10^{19})$ G at RHIC (LHC).
- $B_0 \sim 10^{10} - 10^{13}$G (neutron stars), 10^{15} (magnetars)
- More relevantly $eB \approx 5 - 15 \times m_\pi^2$ RHIC (LHC).

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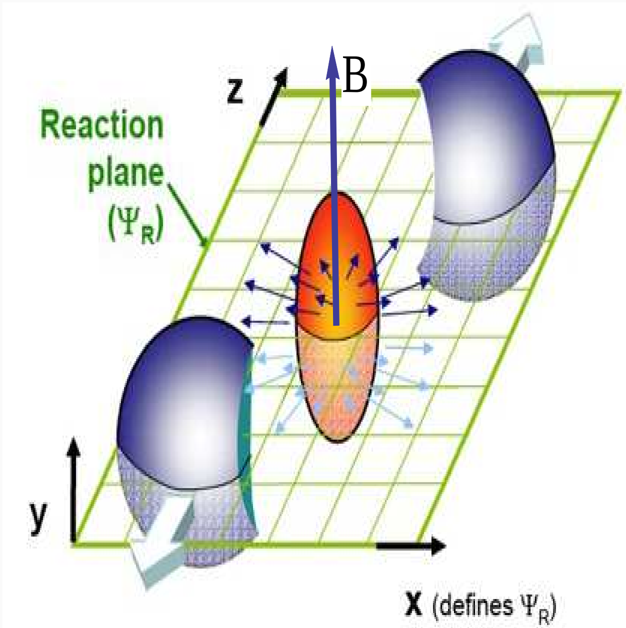

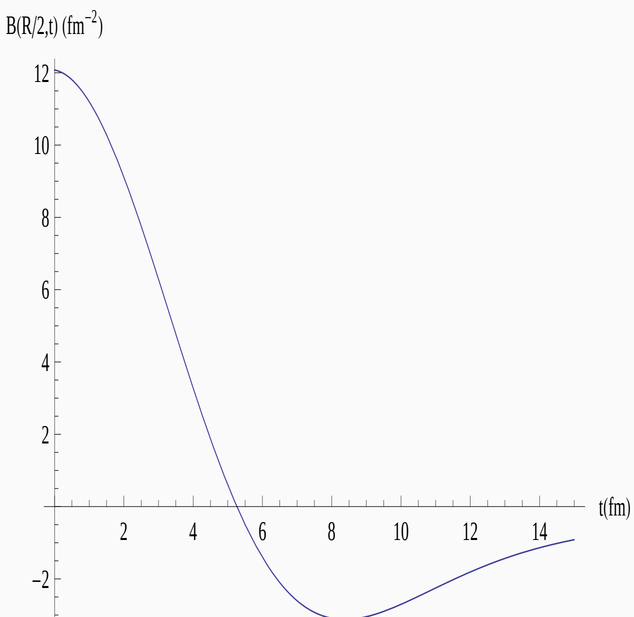

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- Subsequent evolution of B in charged medium
- Solve $\nabla^2 \vec{B} = \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2} + \mu\sigma \frac{\partial \vec{B}}{\partial t}$
- $\vec{B}(t_0) = \hat{y} B_0 e^{-\frac{x^2+y^2}{R^2} - \frac{z^2\gamma^2}{R^2}}$.

Quark-gluon plasma under strong B in the lab!

PART I: Chiral Magnetic Effect

- *Classically* QGP chiral symmetric: $N_L = N_R$ as $T \approx 500$ MeV $\gg m_u,\ m_d$
- Axial current $\partial_\mu J^{\mu 5} = \partial_\mu \left(\langle \bar{\psi} \gamma^\mu \psi \rangle_L - \langle \bar{\psi} \gamma^\mu \psi \rangle_R \right) = 0$
- Chiral imbalance only due to QM anomaly.
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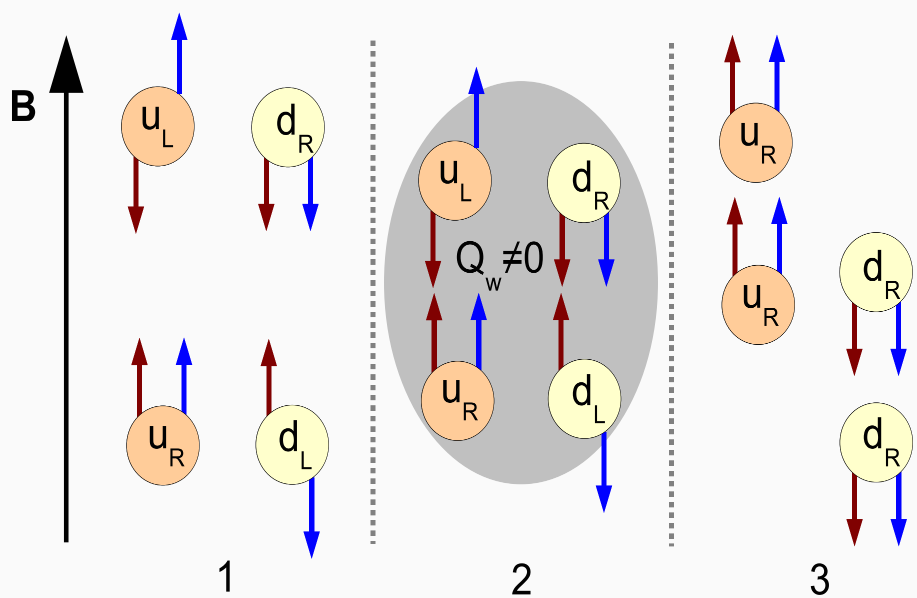

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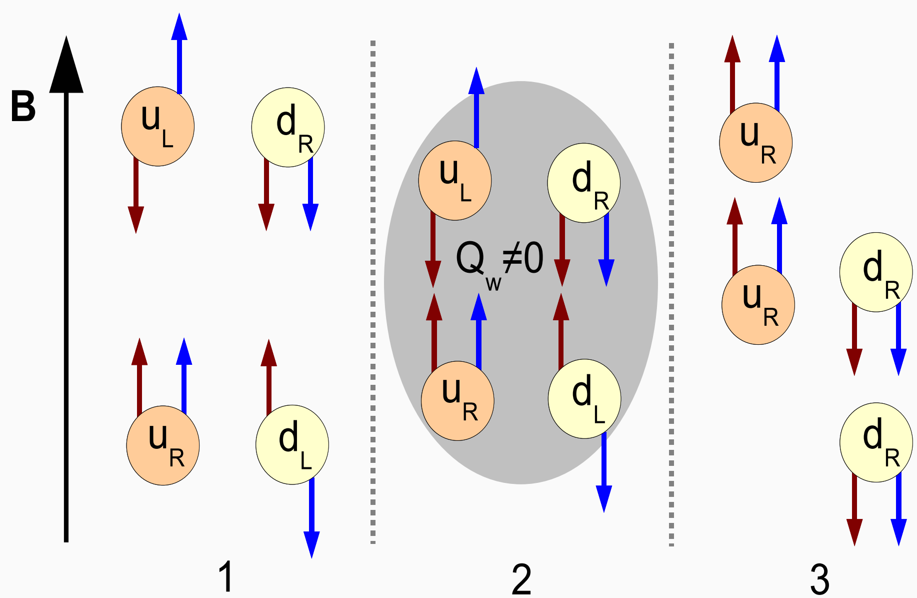

- Macroscopic manifestation of the axial anomaly
- Anomalous magnetohydrodynamics: $\vec{J} = \frac{e^2}{2\pi^2}\,\mu_5\,\vec{B}$

  Kharzeev et al '07, Son, Surowka '09
- μ_5 encodes the imbalance $N_L \neq N_R$

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- Answer: topologically non-trivial gluon configurations
- Gluon winding number: $Q_w = \frac{g^2}{32\pi^2} \int \mathrm{d}^4 x \, F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a \in \mathbb{Z}$.
- Anomaly $\partial_\mu (j^\mu_L - j^\mu_R) = -\frac{N_f g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a \Rightarrow \Delta(N_L - N_R) = 2 N_f Q_w$.

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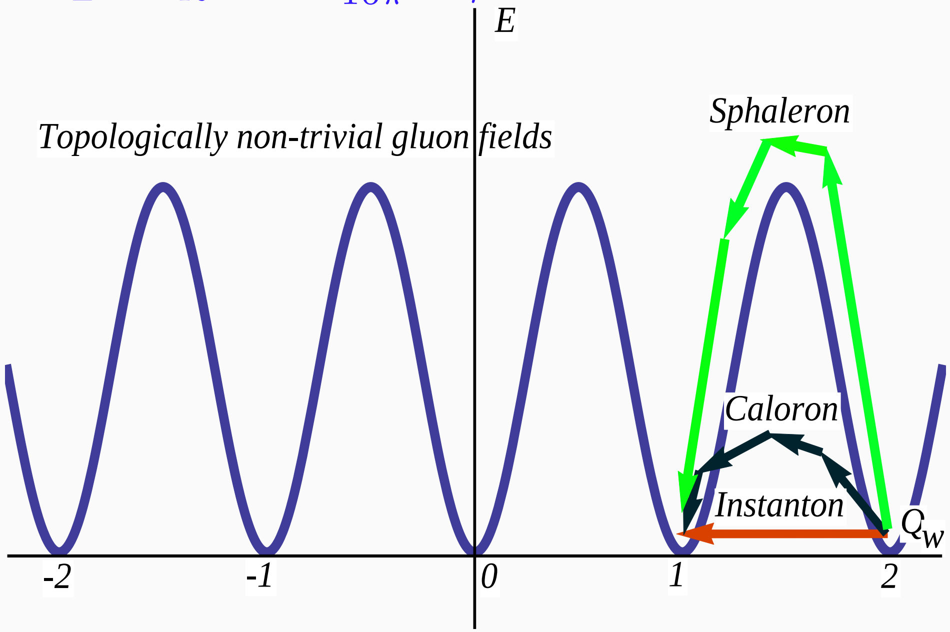

- Sphalerons: thermally induced changes in Q_w
- pQCD: sphalerons the most dominant Q_w decay: Moore et al '97
- Sphaleron decay rate: $\frac{d(N_L - N_R)}{dt d^3x} \approx 192.8 \, \alpha_s^5 \, T^4$

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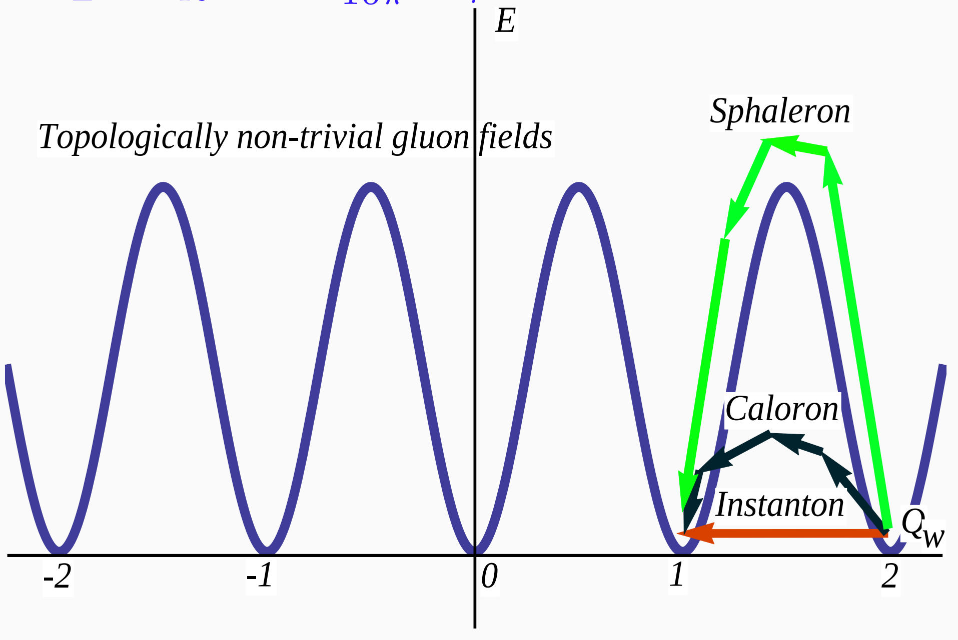

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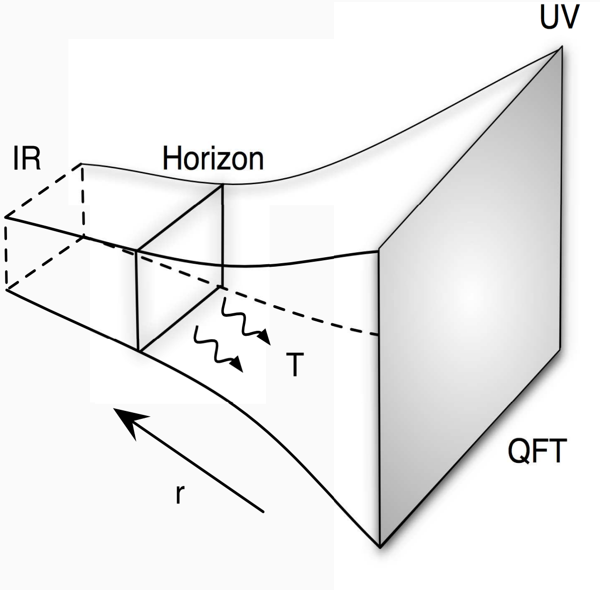
But should we trust pQCD in the Quark-gluon plasma?

Holographic calculation



Finite T, $N_c \gg 1$, $\alpha_s \gg 1$ QFT $\Leftrightarrow$ GR on black holes in 5D

Maldacena '97; Witten; Gubser, Klebanov, Polyakov '98

1. A bulk fluctuation $\phi(x,r) \Leftrightarrow \mathcal{O}(x)$ on the boundary.

$$\exp(-S_{GR}[\phi(x,r) \to \phi_0(x)]) = \langle \exp(-\int dx \mathcal{O}\phi_0) \rangle$$

2. e.g. $\langle \mathcal{O}(x_1)\mathcal{O}(x_2) \rangle$ computed from $\hat{\nabla}^2\phi = m^2\phi$ on the BH.

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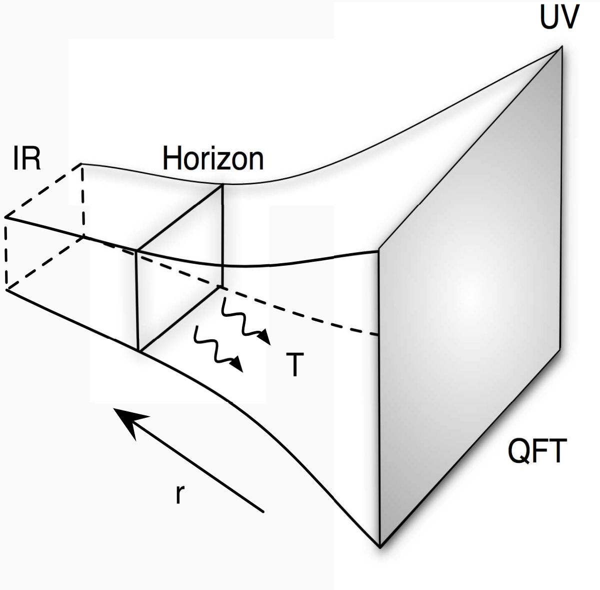

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2. e.g. $\langle \mathcal{O}(x_1) \mathcal{O}(x_2) \rangle$ computed from $\hat{\nabla}^2 \phi = m^2 \phi$ on the BH.
3. Recall $\omega J^{\mu 5}(\omega) \propto \mathrm{Tr} F \tilde{F}(\omega)$. Introduce CP odd axion $a(r, x)$
4. The source term $\int d^4x a_0(x) \mathrm{Tr} F \tilde{F}(x)$ with $a(r, x) \to a_0(x)$ at the boundary.

Holographic calculation of $\Delta(N_L - N_R)$

- Initial excess $N^5 \equiv N_L - N_R$ near thermal equilibrium.
- Described by perturbation $\mathcal{L} \to \mathcal{L} + \epsilon \mathrm{Tr} F\tilde{F}$
- Linear response theory: $\frac{d}{dt} N_5 \to \omega J^{05} \propto \langle \mathrm{Tr} F\tilde{F}\, \mathrm{Tr} F\tilde{F}(\omega)\rangle N_5$
- Should calculate the decay rate $\Gamma_{CS} \sim \langle \mathrm{Tr} F\tilde{F}\, \mathrm{Tr} F\tilde{F}(\omega)\rangle$
- Holography: Study $\hat{\nabla}^2 a(r,x) = 0$ on the 5D BH.

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AdS/CFT: $\Gamma_{CS} = \frac{(g^2 N_c)^2}{256\pi^3} T^4,$ Son, Starinets '02

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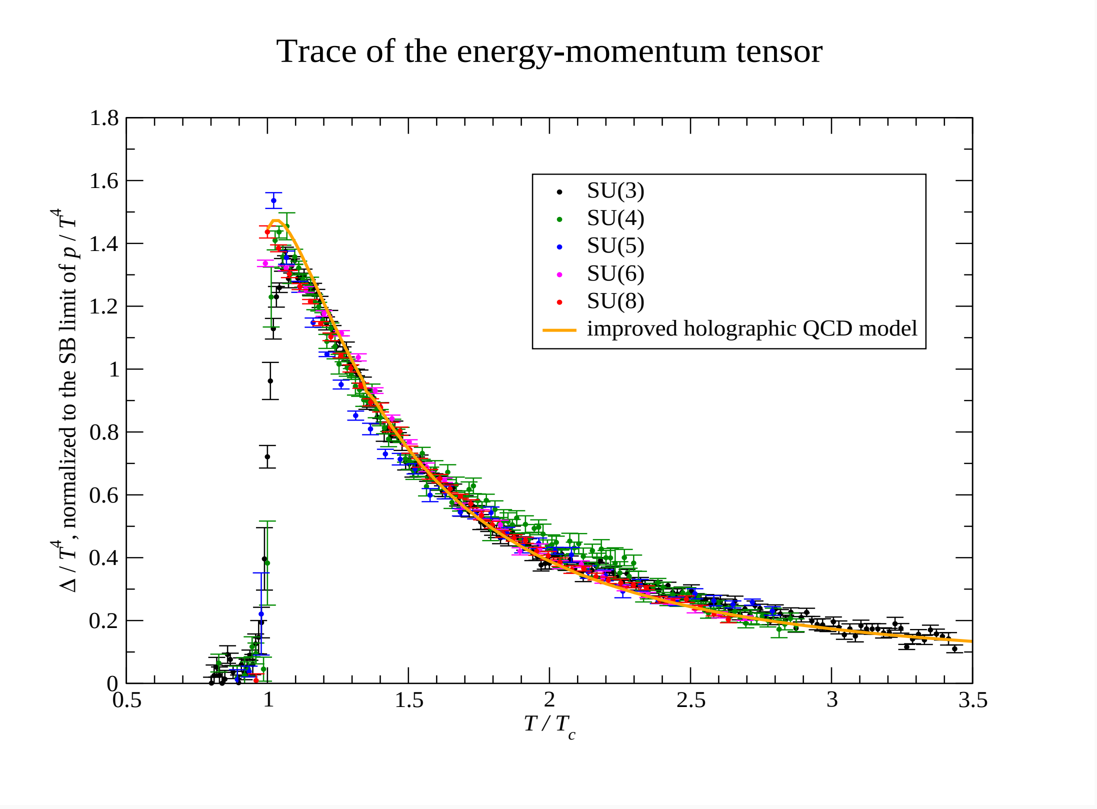
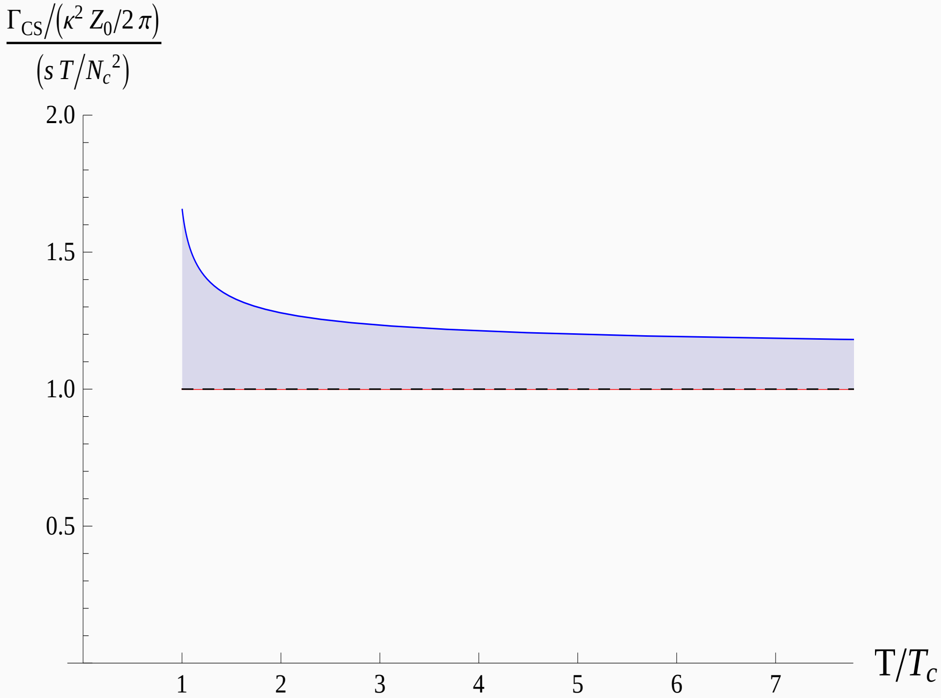
Phenomenologically interesting region $T \approx T_c$ where conformality breaks down:



Improved holographic QCD U.G., Nitti, Kiritsis '07

- $\frac{S_{GR}}{M_p^3 N_c^2} = \int d^5x\sqrt{-g}\left(R - (\partial\Phi)^2 + V(\Phi) - \frac{1}{2N_c^2}\, Z(\Phi)(\partial\alpha)^2\right)$
- Parametrize $Z(\lambda) = Z_0\left(1 + c_1\lambda + c_4\lambda^4\right)$
- Result: $\Gamma_{CS}(T_c) \geq C\, s(T_c)T_c\chi$ U.G, Iatrakis, O'Bannon, Kiritsis, Nitti '12
- where $\chi = \frac{\partial^2\epsilon(\theta)}{\partial\theta^2}$ is the topological susceptibility



for ihQCD to reproduce lattice 0^{+-} glueball spectrum within 1σ.

Summary - part I

- Calculated the Γ_{CS} in non-conformal holography
- CME is proportional to Γ_{CS}
- Comparison of AdS/CFT with non-AdS/non-CFT at T_c:
  $\Gamma_{CS}^{CFT} \approx 0.045 T_c^4$ vs. $\Gamma_{CS} > 1.64 T_c^4$
- Precise value at T_c ambiguous but a lower limit exists.
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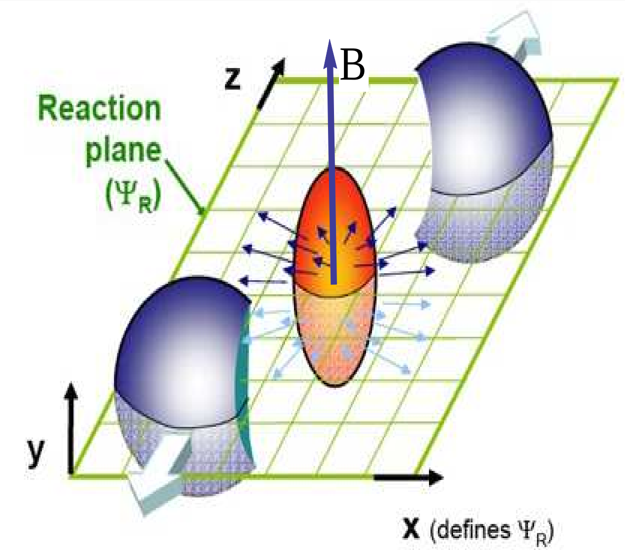
Outlook:

- To fix the ambiguity, determine $Z(\phi) \Rightarrow$ compare $\mathrm{Tr} F \wedge F$ Euclidean correlators with lattice
- Determine $\Gamma_{CS}(B, T)$
- How about other non-conformal models? e.g. Craps et al '12 for Sakai-Sugimoto

PART II: Induced currents in the QGP

ongoing work with D. Kharzeev and K. Rajagopal

Biggest problem in search for magnetic effects in QGP: how to distinguish effects of B from effect of anisotropy ?



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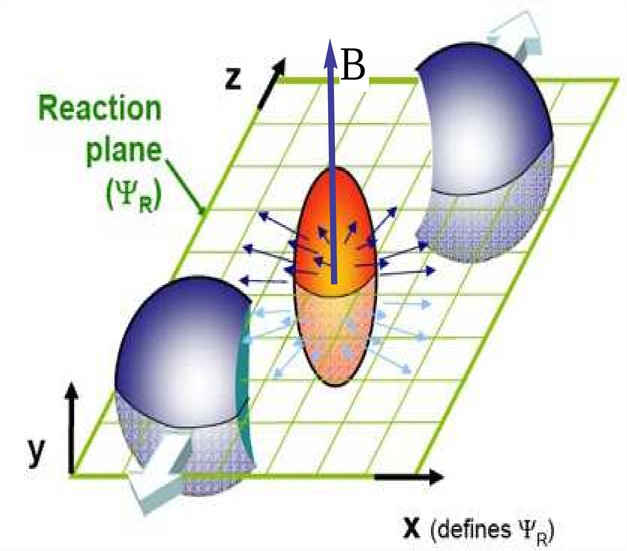
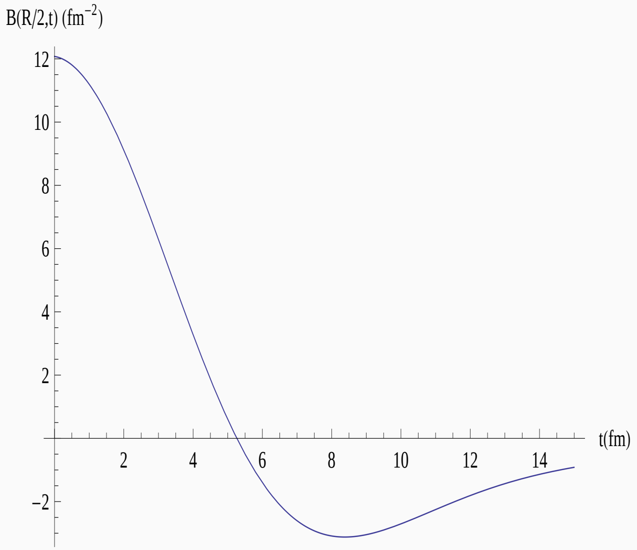
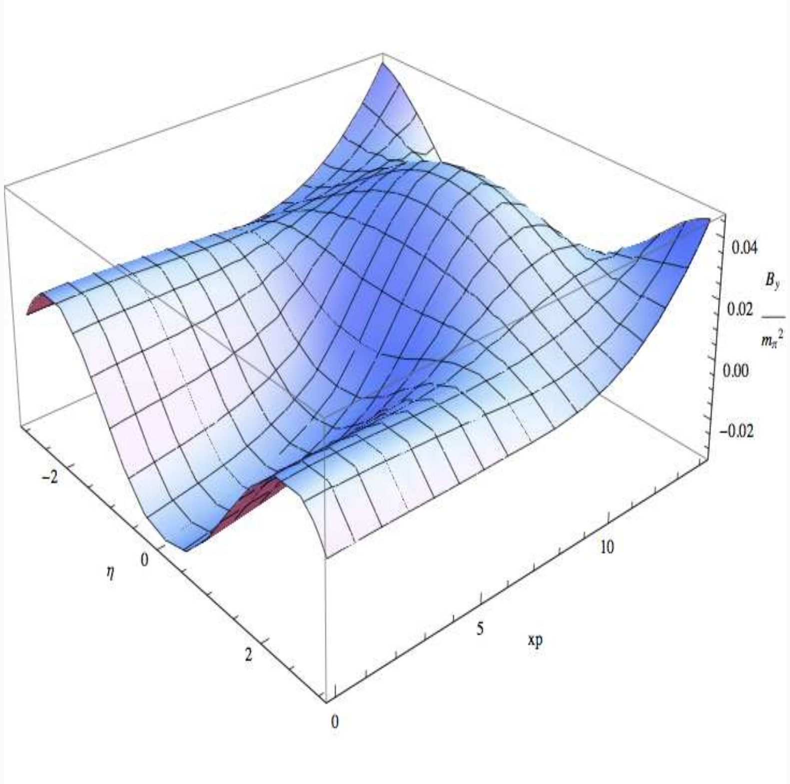




Charged medium:

- t-dependence of B induces Faraday currents $\vec{J}_F \sim \sigma \vec{E}_F$ with $\nabla \times \vec{E}_F = -\frac{\partial \vec{B}}{\partial t}$
- Expanding medium induces Hall currents $\vec{J}_H \sim \sigma \vec{E}_H$ with $\vec{E}_H = -\vec{u} \times \vec{B}$
- We ignore anomalous currents in this talk.

- For initial $\vec{B}(t_0) = \hat{y} B_0 e^{-\frac{x^2+y^2}{R^2} - \frac{z^2\gamma^2}{R^2}}$ straightforward to calculate:



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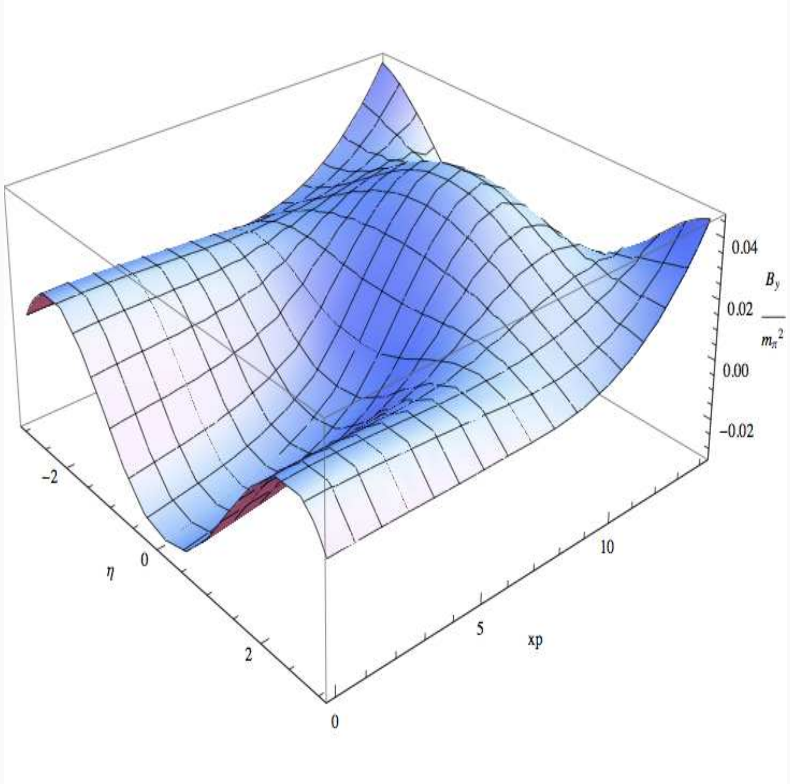
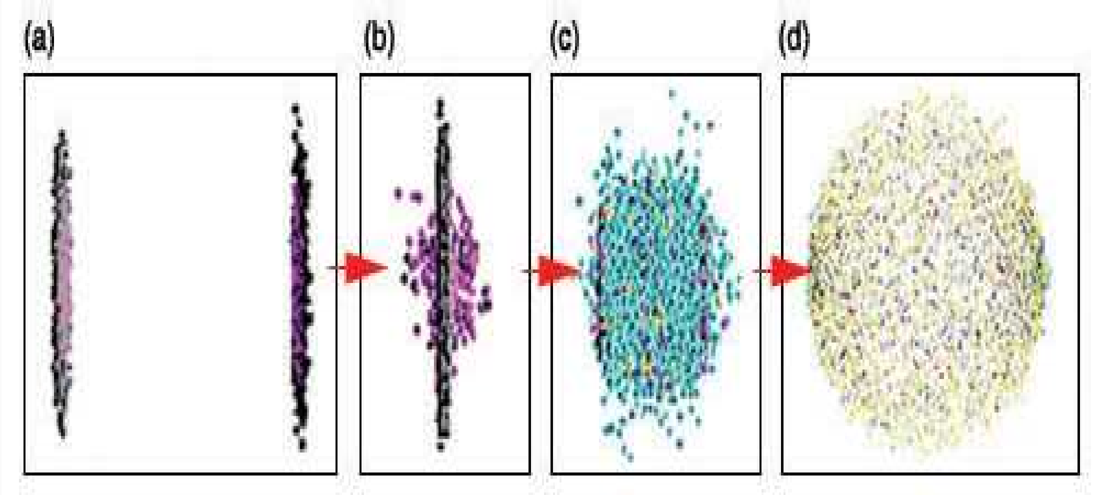


- More realistic profiles include particle fluctuations but Gaussian is sufficient for us Tuchin ’11



- However QGP is an expanding fluid with 4-velocity $u^\mu(x)$
- One has to add the induced velocity v_B to $u^\mu(x)$
- Suppose we know u^μ, assume $|\vec{v}_B| \ll |\vec{u}|$
- Treat v_B as perturbation, ignore backreaction on expanding fluid profile u:
- Construct the total 4-velocity $V^\mu \sim u^\mu + v_B^\mu$: contains all observable information on time varying B.

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- As a first step, assume: Bjorken '83
  1. Boost invariance along z: $\xi = z\partial_t + t\partial_x$
  2. Rotation around z: $\xi = x\partial_y - y\partial_x$
  3. Translations in transverse plane: $\xi = \partial_x$ and $\xi = \partial_y$
- Solution to $[\xi, u] = 0$ is $u = \partial_\tau$ $(ds^2 = -d\tau^2 + \tau^2 d\eta^2 + dx_\perp^2 + x_\perp^2 d\phi^2)$
- Hydrodynamics: $\nabla_\mu T^{\mu\nu} = 0$ with
  $T_{\mu\nu} = \epsilon u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu)$ + visc.
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- Gubser's flow Gubser '10
- Replace $\xi = \partial_x,\ \partial_y$ with $\xi_i = \partial_i + q^2\left[2x^i x^\mu \partial_\mu - x^\mu x_\mu \partial_i\right]$
- Solution to $[\xi, u] = 0$ is $u = \cosh\kappa\partial_\tau + \sinh\kappa\partial_\perp$ with $\kappa = \frac{2q^2\tau x_\perp}{1+q^2\tau^2+q^2x_\perp^2}$
- Solution to Hydrodynamics: $\nabla_\mu T^{\mu\nu} = 0$ with $\epsilon = \frac{\hat{\epsilon}_0}{\tau^{4/3}} \frac{(2q)^{8/3}}{\left[1+2q^2(\tau^2+x_\perp^2)+q^4(\tau^2-x_\perp^2)^2\right]^{4/3}}$

How to test Gubser's flow?

- Hadron spectrum from hydrodynamic flow: Cooper-Frye:

$$S_i = p^0 \frac{dN_i}{dp^3} = -\frac{g_i}{(2\pi)^3} \int d\Sigma_\mu p^\mu F\left(\frac{p^\mu V_\mu}{T_f}\right)$$

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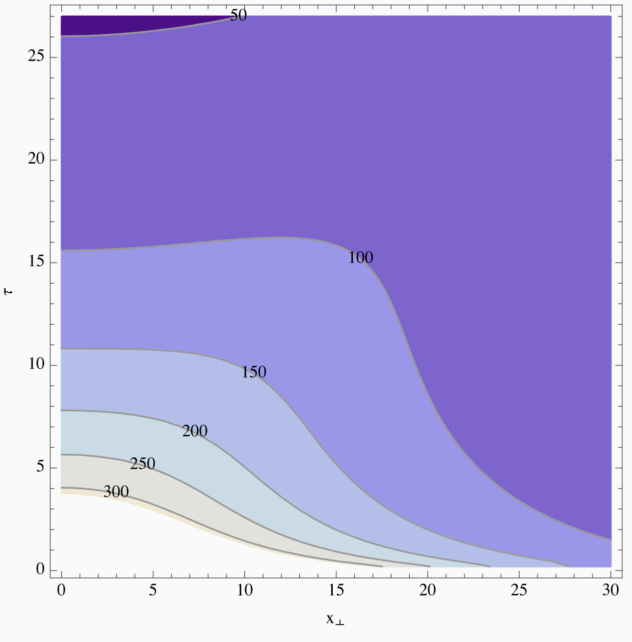

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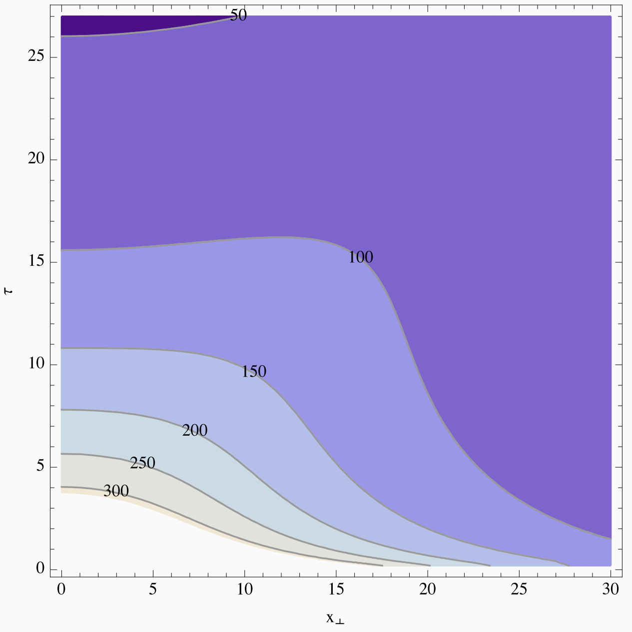

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- $S_i(p_T) =$

$$\frac{g_i}{2\pi^2} \int dx_\perp x_\perp \tau_f \left\{ K_1\left(\frac{m_T u^\tau}{T_f}\right) I_0\left(\frac{p_T u^\perp}{T_f}\right) - \tau_f' p_T K_0\left(\frac{m_T u^\tau}{T_f}\right) I_1\left(\frac{p_T u^\perp}{T_f}\right) \right\}$$

- Gubser's flow is independent of Φ_p and Y

Comparison to data

Need to fix parameters q and $\hat{\epsilon}_0$. Some tension between realistic spectrum and hydronization temperature $T_h \approx 400 - 550$ MeV $\Rightarrow$ Optimal solution $q^{-1} = 6.5$ fm and $\hat{\epsilon}_0 = (8.7)^4$

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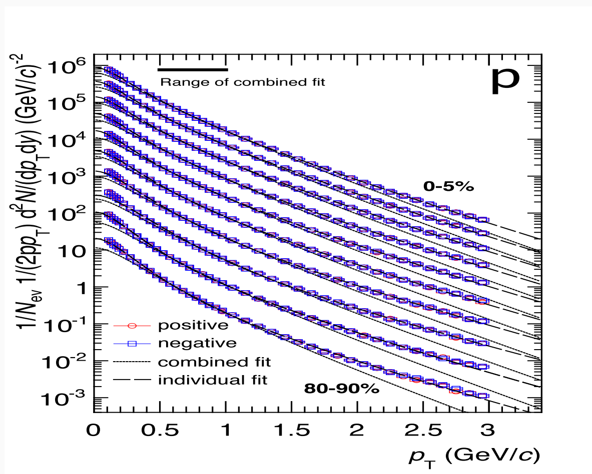

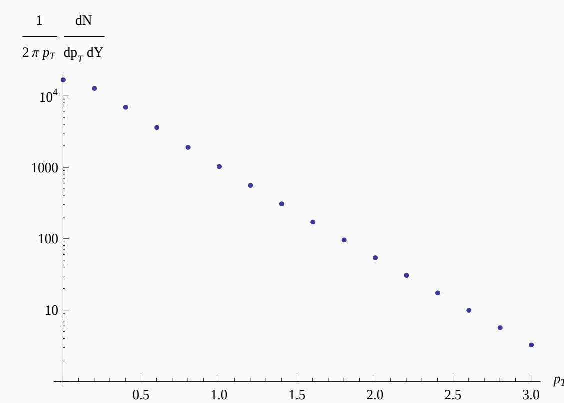

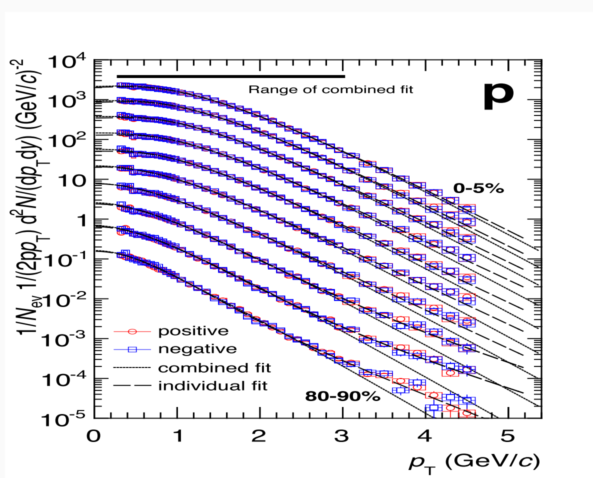

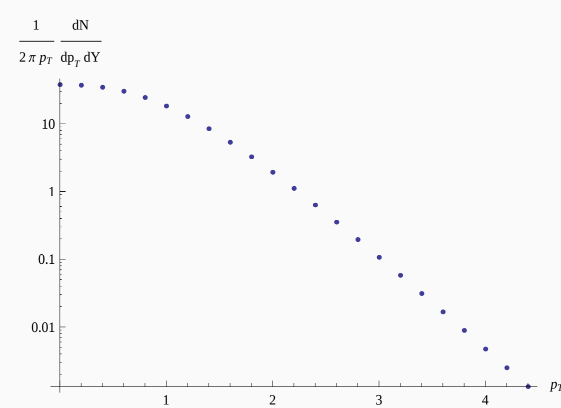

Effect of induced currents on spectra

- Decompose spectrum in flow parameters:
  $S_i = v_0\left(1 + v_1(p_T, Y)\cos(\phi_p) + \cdots\right)$
- Effects of magnetically induced currents most clearly seen in " charge identified directed flow parameter" v_1:
- Directed flow only from π^+ or π^- or p.
- $v_1|_{\text{Gubser}} = 0$ but $v_1|_{\text{Gubser}+B} \neq 0$, solely due to B!
- Partial results at RHIC, no charge identified results at LHC yet.

Calculation of v_1

- Need to calculate the total current $V_{Gubser+B}$
- In lab frame: u^μ, B, $E_{Faraday}$
- Go to fluid rest frame by $\Lambda(-\vec{u}) \Rightarrow B'$ and E' include both Faraday and Hall
- Solve for stationary current:
$$m\frac{d\langle \vec{v'_B}\rangle}{dt} = q\langle \vec{v'_B}\rangle \times \vec{B}' + q\vec{E}' - \mu m \langle \vec{v'_B}\rangle = 0\,,$$
- Go back to lab frame $\Lambda(-\vec{u}) \Rightarrow V$ includes both $\langle v_B \rangle$ and u.

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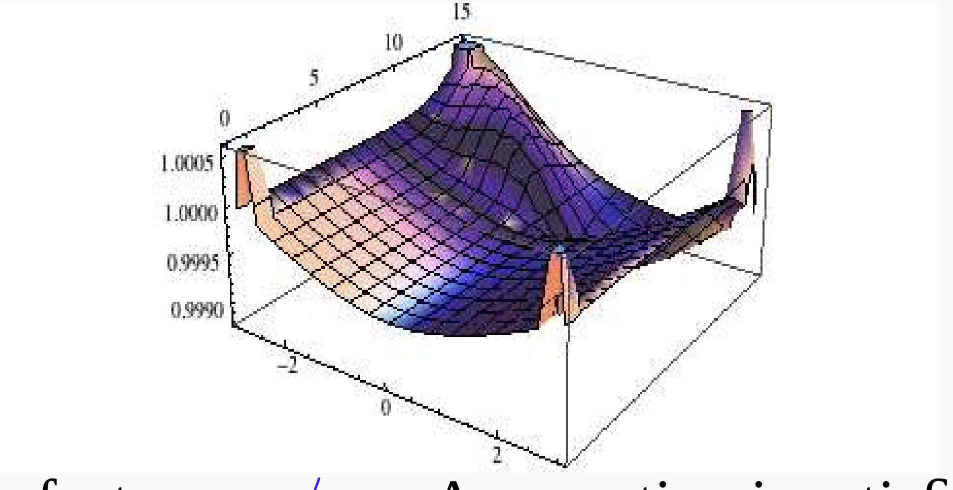
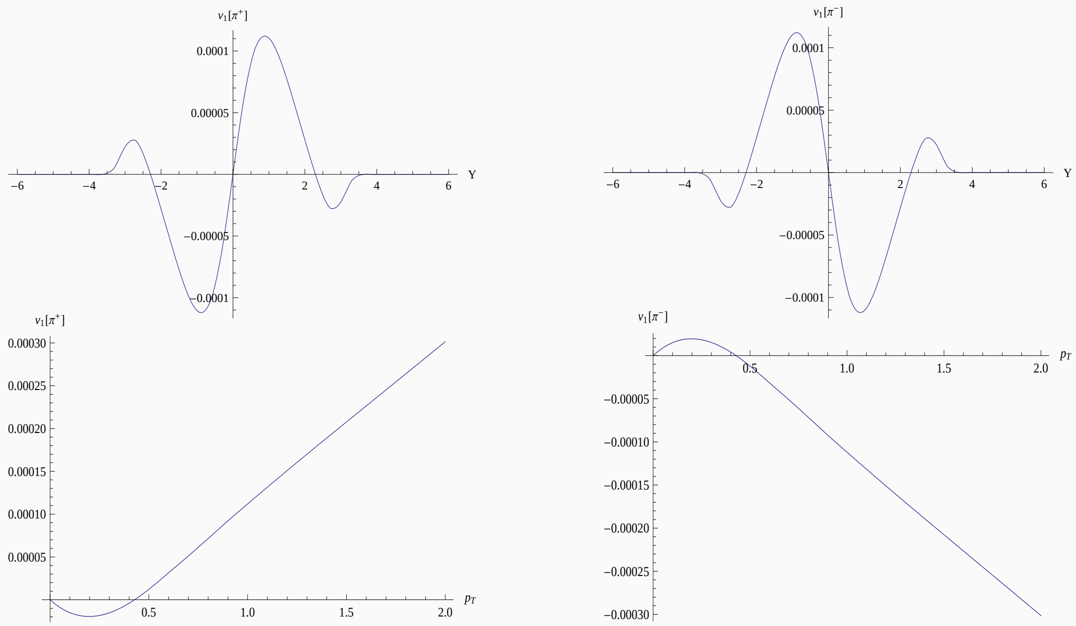
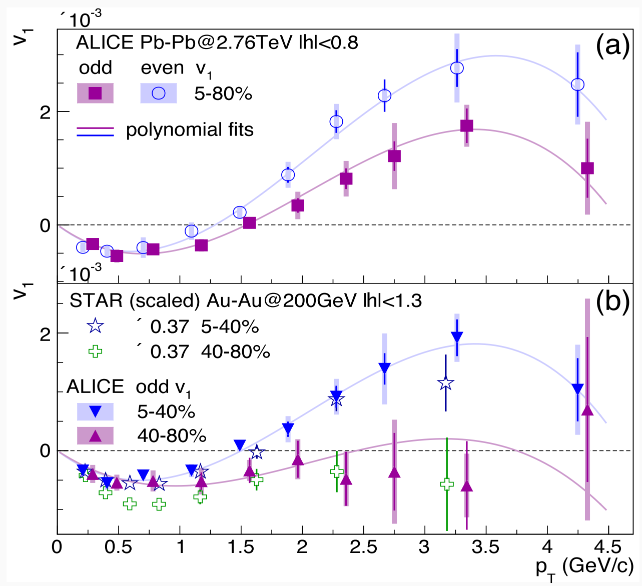
  $$m\frac{d\langle\vec{v'_B}\rangle}{dt} = q\langle\vec{v'_B}\rangle \times \vec{B}' + q\vec{E}' - \mu m\langle\vec{v'_B}\rangle = 0\,,$$
- Go back to lab frame $\Lambda(-\vec{u}) \Rightarrow V$ includes both $\langle v_B \rangle$ and u.
- Calculation is trustable only when $\langle|\vec{v}_B|\rangle \ll |\vec{u}|$



Ratio of Lorentz factors γ_V/γ_G: Assumption is satisfied.

Predictions for charge identified v_1





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  - Anomalous transport
  - Analogies with cosmology?

THANK YOU !