Hydrodynamics and beyond...

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M. Heller, RJ, P. Witaszczyk, 1302.0697 [PRL 110, 211602 (2013)]
work in progress and results from 1103.3452, 1203.0755

Outline

What is hydrodynamics? (and higher-order hydrodynamics?)

Some questions

Methods: fluid/gravity duality and boost-invariant flow

The Borel plane and gradient expansion

Borel resummed hydrodynamics

Comparisions with numerics

Conclusions

What is hydrodynamics?

- Universal description of the long wavelength degrees of freedom
- Applies equally well at macroscopic and microscopic scales
- Current most relevant example: quark-gluon-plasma produced at RHIC/LHC

Long wavelength description $\equiv$ gradient expansion $\equiv$ expansion in the number of derivatives

Question: What is the nature of such an expansion?

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2. Amounts to the *assumption* that $T_{\mu\nu}$ is wholly expressed through the flow velocity u^μ, energy density and pressure ($E = 3p$ for conformal fluids)
3. Arrange all possible terms by the number of derivatives of u^μ
4. Coefficients of these terms $\equiv$ transport coefficients characteristic of the underlying microscopic theory
5. Generalized Navier-Stokes equation is just $\partial_\mu T^{\mu\nu} = 0$

$\mathcal{N} = 4$ SYM hydrodynamics:

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

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What is the nature of the gradient expansion?

- Suppose we include terms with more and more derivatives (dissipation)
- Is the resulting series asymptotic (zero radius of convergence)?
- What physics is (quantitatively) responsible for the lack of convergence?

Analogy: perturbative expansion and instanton effects...

Question 2

If the hydrodynamic series is only asymptotic, is it Borel summable?

- What are the singularities on the Borel plane and what is their physical interpretation?
- Can we explicitly construct Borel resummed hydrodynamics?

These questions are very interesting but also quite theoretical...

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- Lublinsky, Shuryak proposed a resummation based on linearized hydrodynamic modes from AdS/CFT with phenomenological motivation
- In our previous work [Heller, RJ, Witaszczyk] we considered the evolution of a spacetime dual to a plasma system evolving from some nonequilibrium initial conditions and its transition to hydrodynamics

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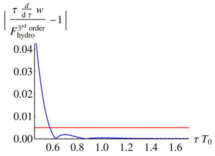

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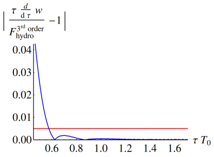

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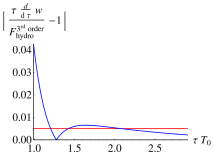

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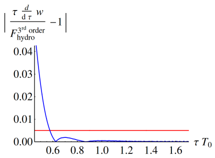

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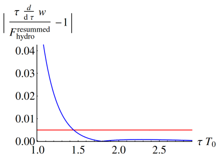

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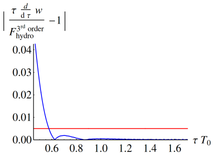

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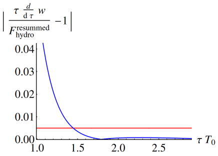

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- Approach pioneered by Bhattacharya, Hubeny, Minwalla, Rangamani
- One starts with a boosted planar black hole representing a plasma system moving with uniform velocity u^μ and with temperature T
- One promotes u^μ and T to slowly varying functions – one has to correct the metric iteratively in an expansion in gradients
- At each order one looks for a (regular) solution of

$$(\text{Linear differential operator})[g_{\mu\nu}^{(n)}] = RHS[\{g_{\mu\nu}^{(j)}\}_{0\leq j\leq n-1}]$$

- Rather complicated to perform the expansion analytically:
  • in general carried out to 2^{nd} order (2^{nd} order viscous hydrodynamics)
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- One starts with a boosted planar black hole representing a plasma system moving with uniform velocity u^μ and with temperature T
- One promotes u^μ and T to slowly varying functions – one has to correct the metric iteratively in an expansion in gradients
- At each order one looks for a (regular) solution of

$$(\text{Linear differential operator})[g^{(n)}_{\mu\nu}] = RHS[\{g^{(j)}_{\mu\nu}\}_{0\leq j\leq n-1}]$$

- Rather complicated to perform the expansion analytically:
  • in general carried out to 2^{nd} order (2^{nd} order viscous hydrodynamics)
  • in boost-invariant case up to 3^{rd} order

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- At each order we have a set of coupled ***linear*** **ODE's**
- Very simple to solve numerically (with very high precision!)
  Much simpler than normal numerical relativity which involves solving nonlinear PDE's!!

Simplicity/our practical motivation $\longrightarrow$ consider the boost-invariant setting

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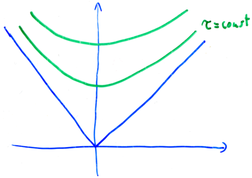
Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- In a conformal theory, $T^\mu_\mu = 0$ and $\partial_\mu T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
- The assumptions of symmetry fix uniquely the flow velocity
- Gradient expansion coincides with an expansion in

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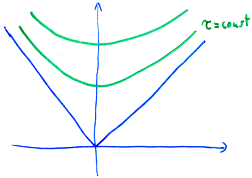

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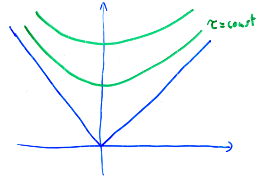

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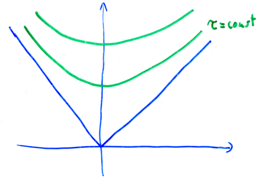

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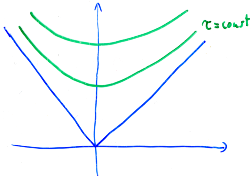

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Large τ behaviour of $\varepsilon(\tau)$

- Structure of the analytical result for large τ:

$$\varepsilon(\tau) = \frac{1}{\tau^{\frac{4}{3}}} - \frac{2}{2^{\frac{1}{2}}3^{\frac{3}{4}}}\frac{1}{\tau^{2}} + \frac{1+2\log 2}{12\sqrt{3}}\frac{1}{\tau^{\frac{8}{3}}} + \frac{-3+2\pi^2+24\log 2-24\log^2 2}{324\cdot 2^{\frac{1}{2}}3^{\frac{1}{4}}}\frac{1}{\tau^{\frac{10}{3}}} + \ldots$$

RJ, Peschanski; Nakamura, S-J Sin; RJ; RJ, Heller; Heller

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- By iteratively solving numerically the linear ODE's from fluid/gravity duality, we obtained 240 coefficients in the gradient expansion

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• to get to so high orders we need very high precision computations
• first couple of orders – easy and fast

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Zero radius of convergence
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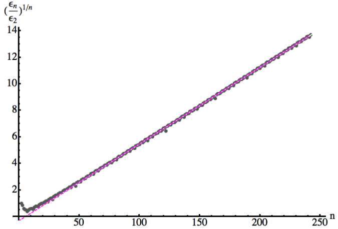

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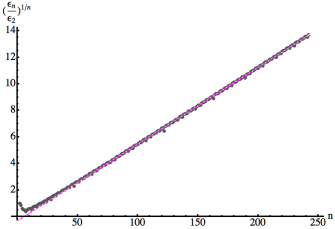

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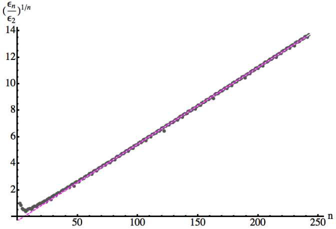

Zero radius of convergence
Asymptotic series...

Borel transform

- Define the Borel transform

$$\tilde{\varepsilon}(u) = \sum_{n=2}^{242} \frac{\varepsilon_n}{n!} u^n$$

- If there are no singularities on the real axis, a Borel resummation of the asymptotic series can be obtained from the integral

$$\varepsilon_{resum}(u) = \int_0^\infty e^{-s} \tilde{\varepsilon}(su)\, ds \qquad \text{where } u = \tau^{-\frac{2}{3}}$$

- $\tilde{\varepsilon}(u)$ has only a finite radius of convergence. In order to locate singularities in the Borel plane, we perform a symmetric Pade approximation...
- Look at poles $\equiv$ zeroes of the denominator of the Pade approximant...

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Singularities in the Borel plane

- The poles on the negative real axis are spurious
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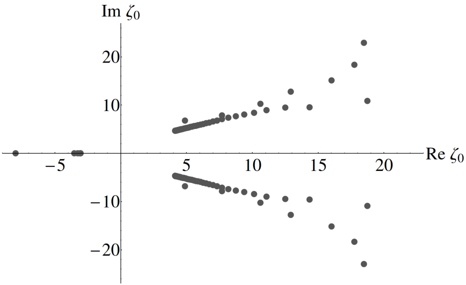

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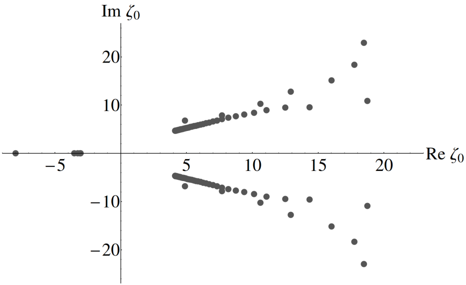

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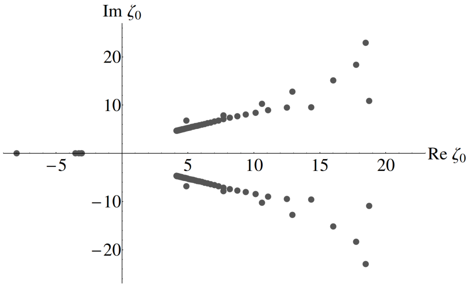

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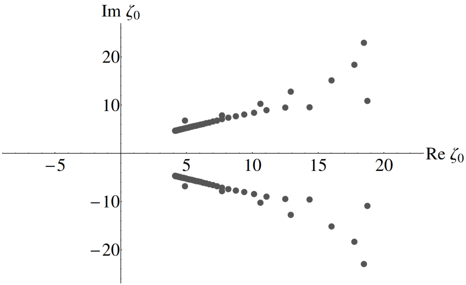

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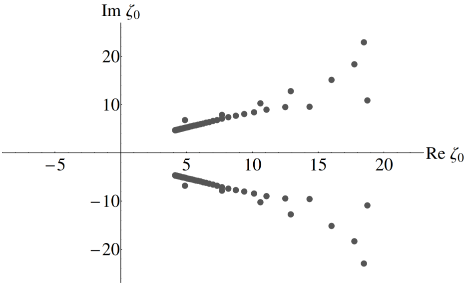

- Branch cuts on the Borel plane
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  Borel resummation should be possible...

Question:

What is the physical interpretation of the branch cut singularities?

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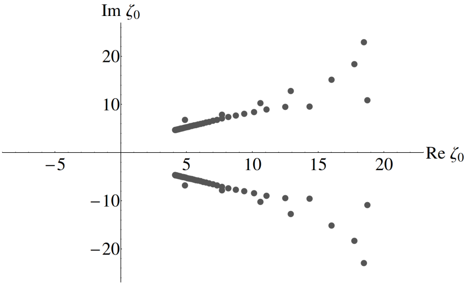

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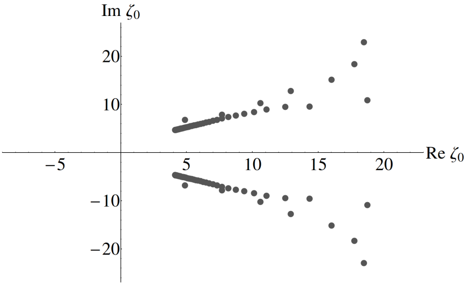

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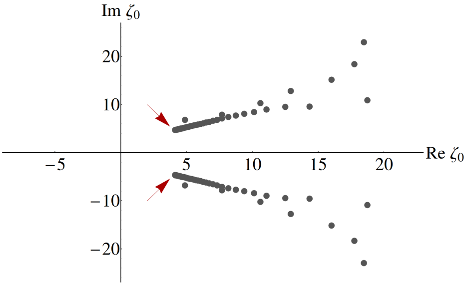

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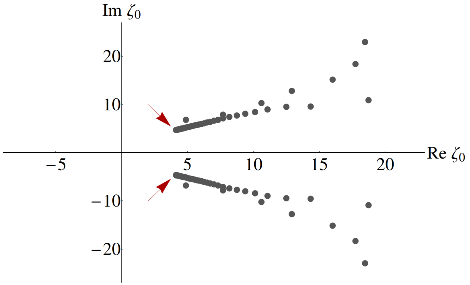

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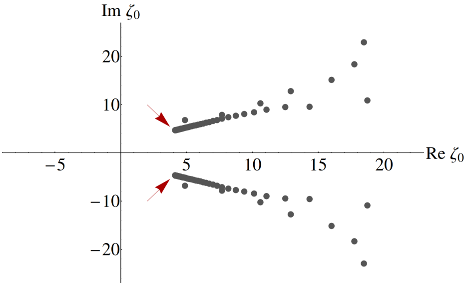

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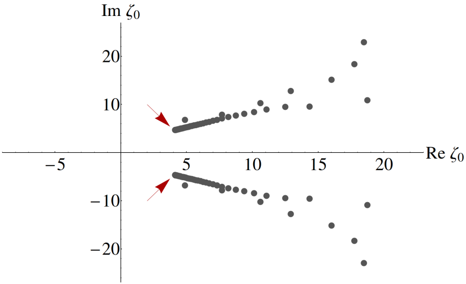

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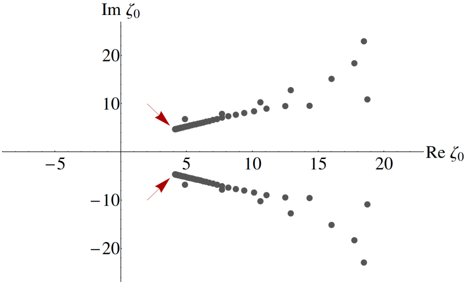

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$$\varepsilon_{resum}(\tau) = \int_0^\infty e^{-\zeta} \tilde{\varepsilon}\left(\zeta/\tau^{\frac{2}{3}}\right) d\zeta$$

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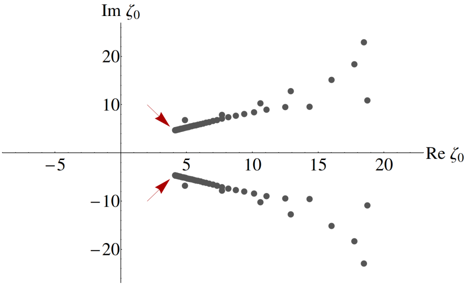

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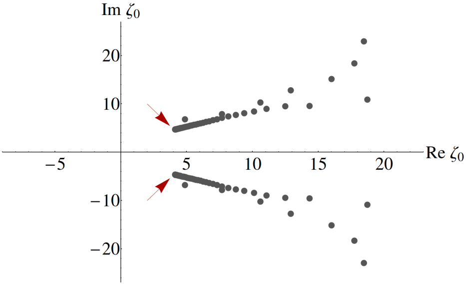

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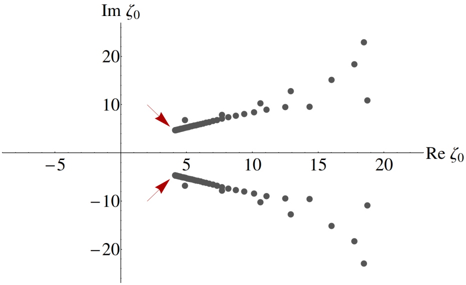

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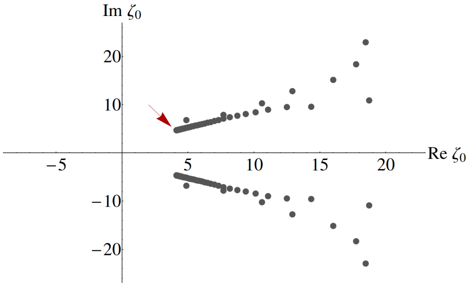

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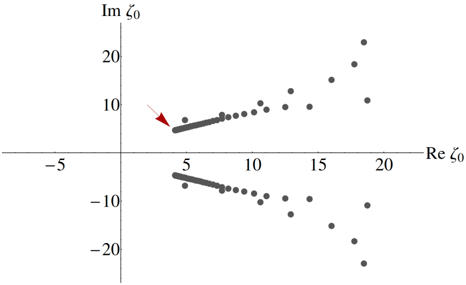

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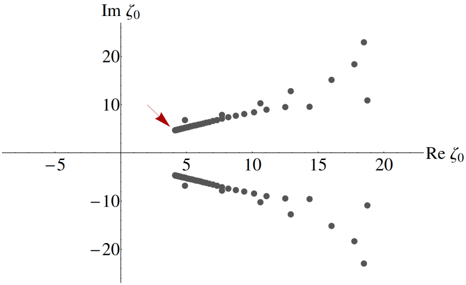

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What is the interpretation of the whole branch cut?

- Deform the contour of the inverse Borel transform to encircle the cut and extract the large τ behaviour
- We obtain a preexponential power law factor

$$\tau^{-1.5426+0.5192\,i} \cdot e^{-i\,\frac{3}{2}(3.1193-2.7471\,i)\tau^{\frac{2}{3}}}$$

- The late time geometry is an evolving black hole deformed by viscous corrections (first gradient terms in the fluid/gravity duality)
- This modification of the geometry generates a nontrivial power-law modification of the quasi-normal mode which is

$$\tau^{-1.5422+0.5199\,i} \cdot e^{-i\,\frac{3}{2}(3.1195-2.7467\,i)\tau^{\frac{2}{3}}}$$

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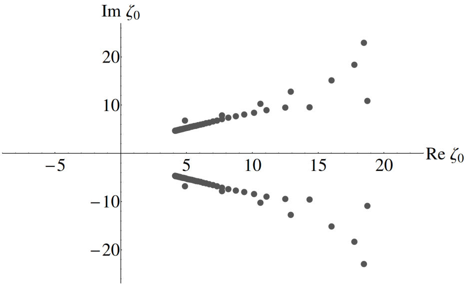

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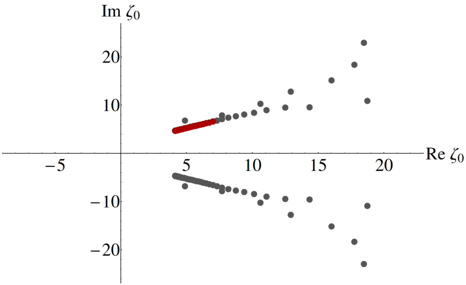

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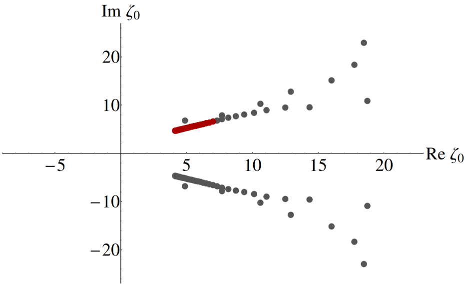

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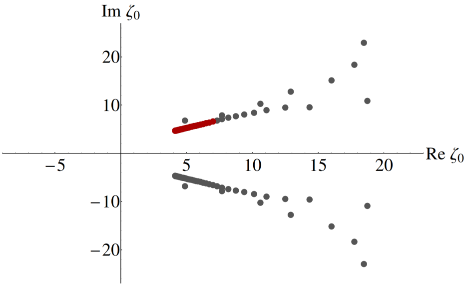

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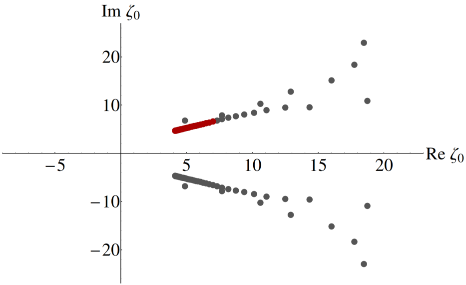

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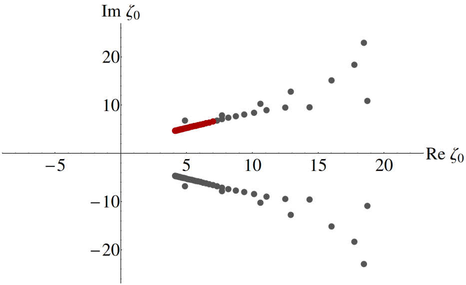

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- Recall the hydrodynamic expression for the temperature $T(\tau)$:

$$\pi T(\tau) = \frac{1}{\tau^{\frac{1}{3}}}\left(1 - \frac{1}{6\tau^{\frac{2}{3}}} + \ldots\right)$$

- Then using the substitution $\pi T t \longrightarrow \int \pi T(\tau) d\tau$ we get

$$\int \pi T(\tau) d\tau \sim \frac{3}{2}\tau^{\frac{2}{3}} - \frac{1}{6}\log\tau + \ldots$$

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- We need to evaluate the integral

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- For this we need to have an analytical continuation of the Borel transform $\tilde{\varepsilon}(..)$ on the *whole* positive real axis $\longrightarrow$ use Pade approximant
- What type of Pade??
- Naively (n, n) would be natural as it would lead to $\varepsilon(\tau) \to const$ as $\tau \to 0$
- However the hydrodynamic series suggests different asymptotics:

$$u \frac{\partial_u \tilde{\varepsilon}(u)}{\tilde{\varepsilon}(u)} \longrightarrow (n, n) \text{ Pade } \longrightarrow \lim_{u \to \infty} u \frac{\partial_u \tilde{\varepsilon}(u)}{\tilde{\varepsilon}(u)} = 3.00000022\ldots$$

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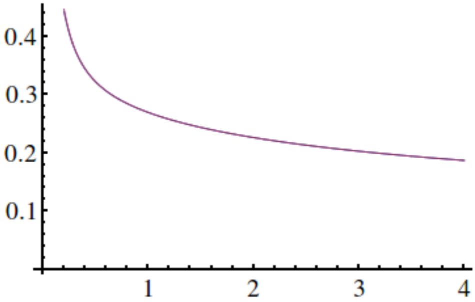

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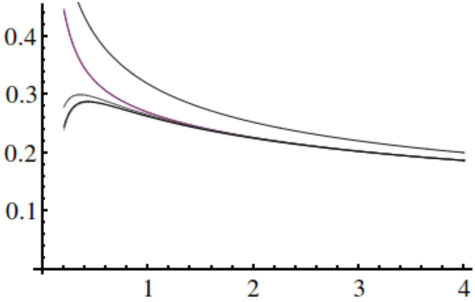

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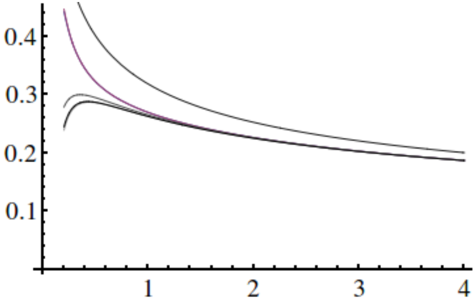

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M. Heller, RJ, P, Witaszczyk, 1103.3452, 1203.0755

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Describe it in terms of lightest degrees of freedom on the AdS side which are relevant at strong coupling

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$$ds^2 = \frac{g_{\mu\nu}(\tau, z) dx^\mu dx^\nu + dz^2}{z^2} \equiv g^{5D}_{\alpha\beta} dx^\alpha dx^\beta$$

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Compute the time-evolution by solving (numerically) 5D Einstein's equations

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- The area of the apparent horizon defines for us the entropy density
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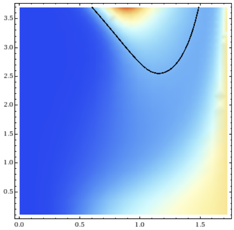

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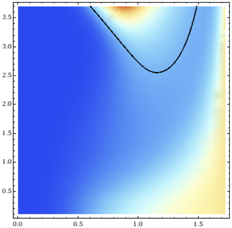

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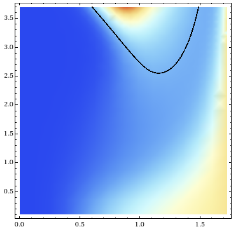

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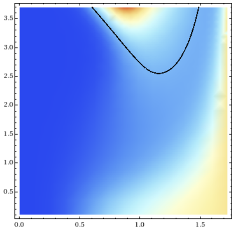

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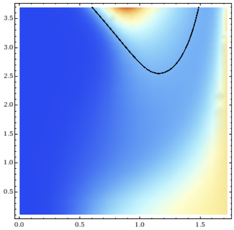

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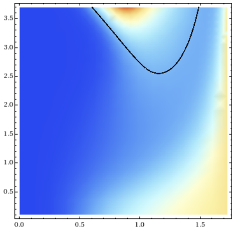

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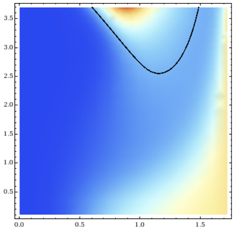

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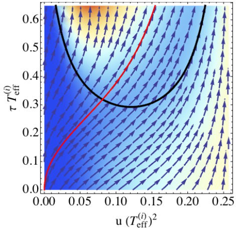

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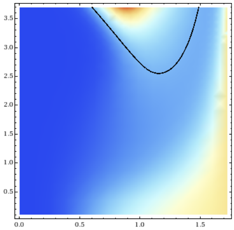

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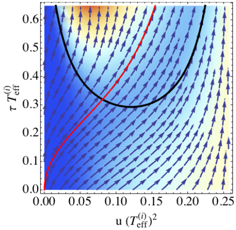

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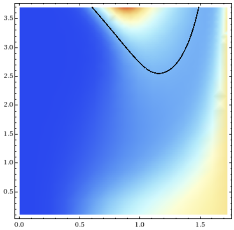

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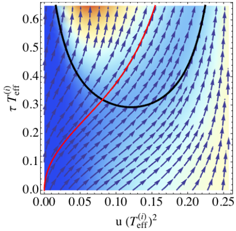

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   (natural values for RHIC: ($\tau_0 = 0.25\, fm$, $T_0 = 500\, MeV$) assumed in [Broniowski, Chojnacki, Florkowski, Kisiel] correspond to $w = 0.63$)
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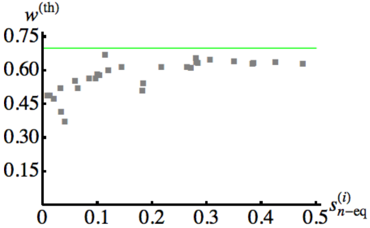

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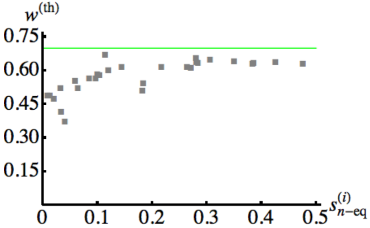

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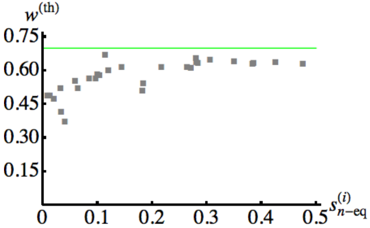

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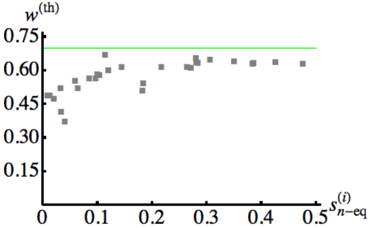

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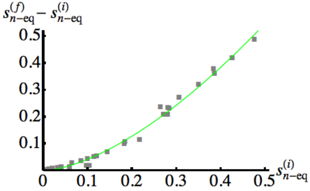

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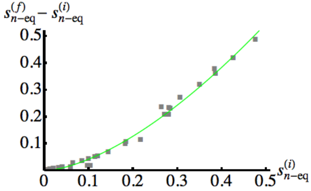

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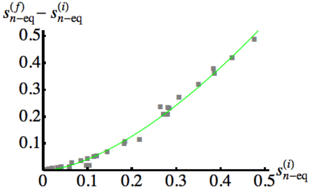

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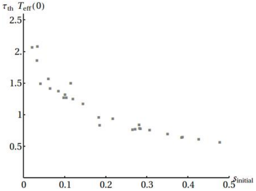

Remarks:

- As mentioned earlier, we get rid of the dependence on the number of degrees of freedom by parametrizing the energy density through an effective temperature given by

$$\varepsilon(\tau) = \frac{3}{8} N_c^2 \pi^2 T_{eff}^4(\tau)$$

- Previously, we normalized our initial data by setting

$$T_{eff}(\tau = 0) = 1$$

  but this is generically unknown in realistic heavy-ion collisions...
- It is much better to fix the normalization through the hydrodynamic tail...

$$\pi T_{eff}(\tau) \sim \frac{1}{\tau^{\frac{1}{3}}} \qquad \text{in the } \tau \to \infty \text{ limit}$$

  The coefficient '1' fixes the units of τ.

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Effective temperature T_{eff} as a function of τ

green line: 3^{rd} order hydrodynamics
red line: Borel resummed hydrodynamics
• The point $\tau_0 = 0.25\,fm$, $T_0 = 500\,MeV$ ($w \equiv T\tau = 0.63$) corresponds here to $\tau \sim 3.14$ (our thermalization criterion was very strict)

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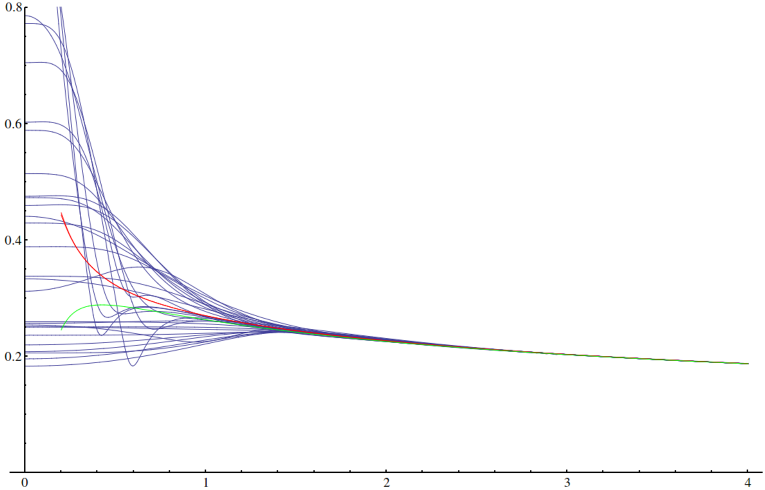

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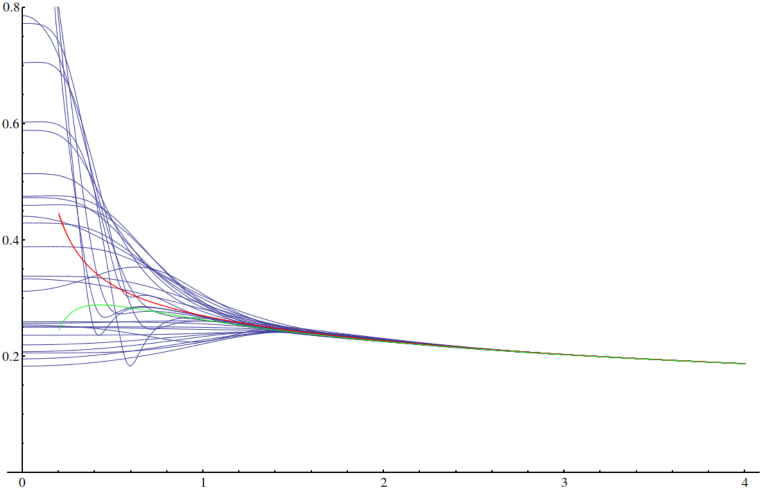

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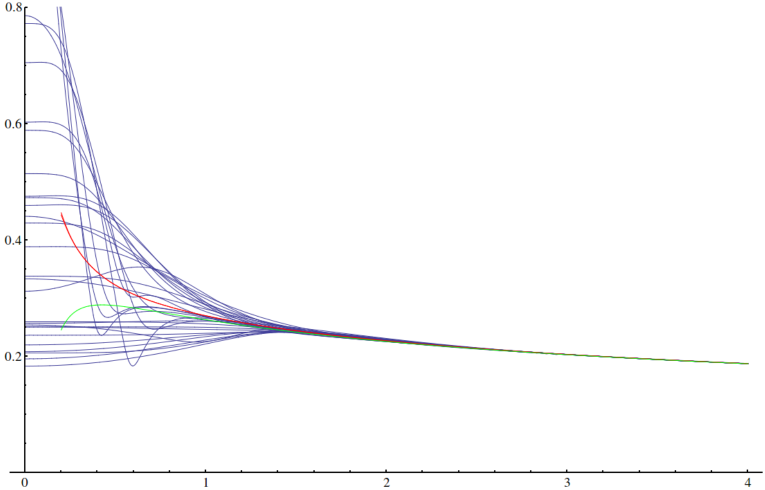

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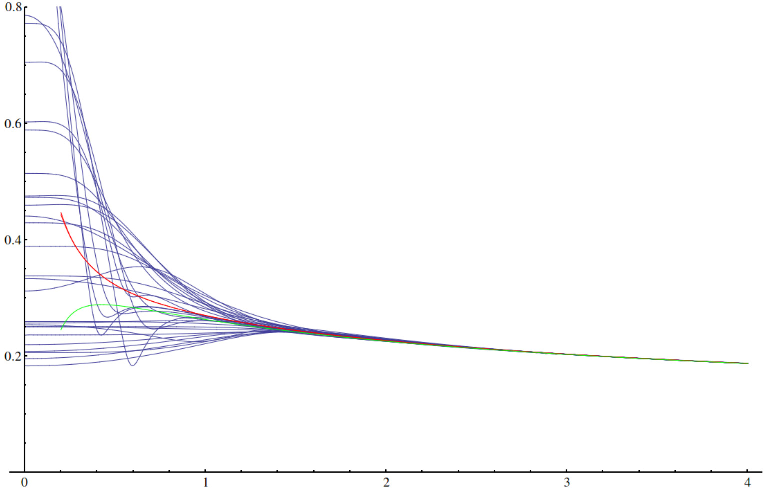

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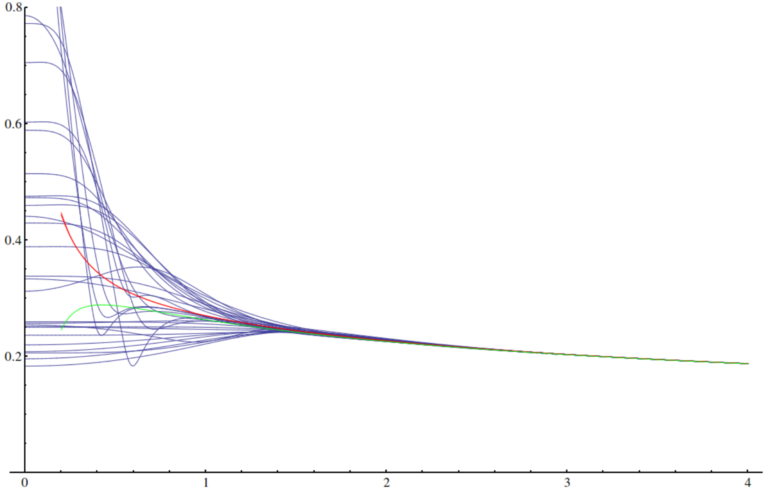

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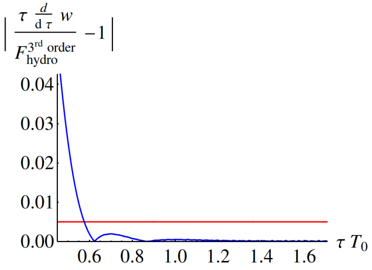

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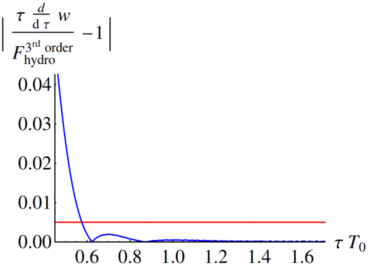

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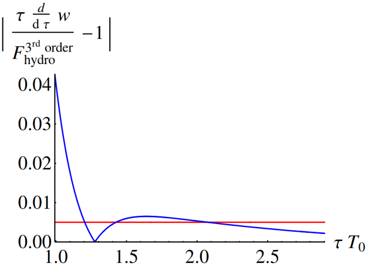

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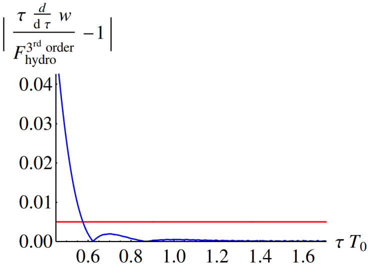

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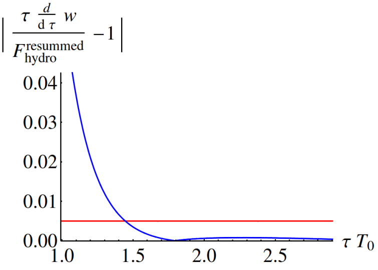

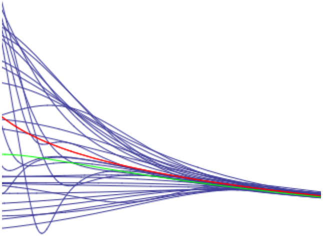

How to model deviations from (all-order) hydrodynamics?

- Is there a simple phenomenological model simpler than 5D Einstein's equations??

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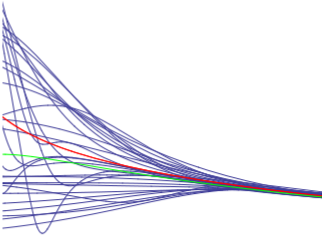

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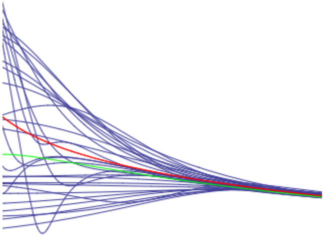

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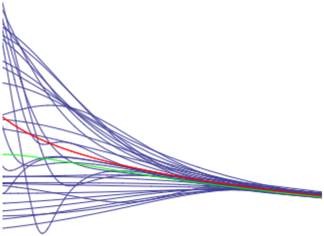

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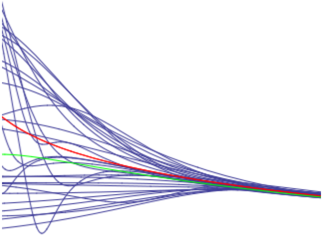

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- Each quasinormal mode represents an independent degree of freedom from the 4D perspective...
- Recall the generic structure of QNM modes for a boost-invariant flow (including first viscous corrections)

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- One can estimate that $s = \int_0^\tau \pi T(\tau) d\tau$ at the transition to hydrodynamics would set the scale for how many QNM would be relevant there...
- It turns out that $s = 1.6 ... 3$ and $Im\ \omega_{QNM} = 2.75, 4.76, 6.77, ...$
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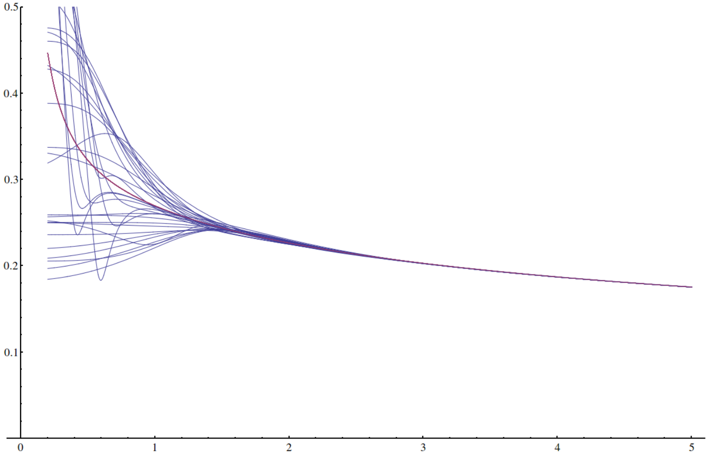

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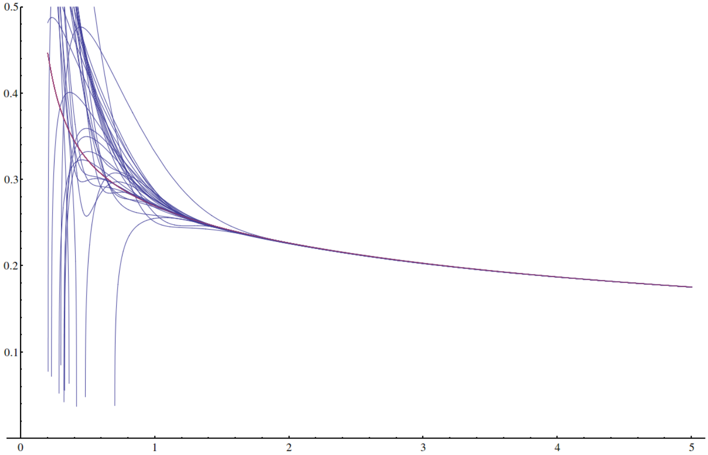

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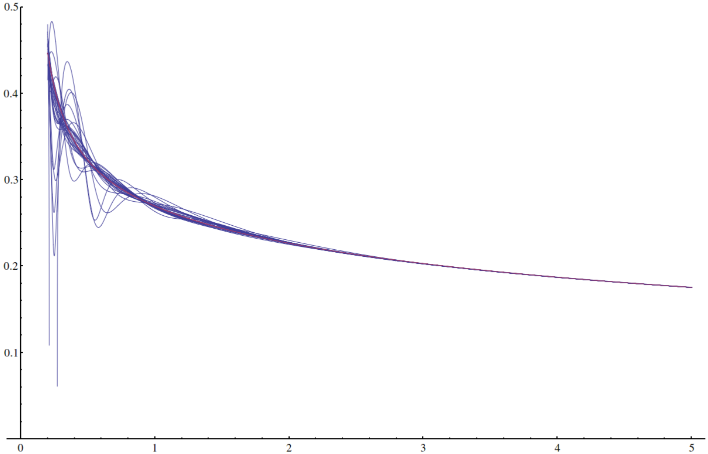

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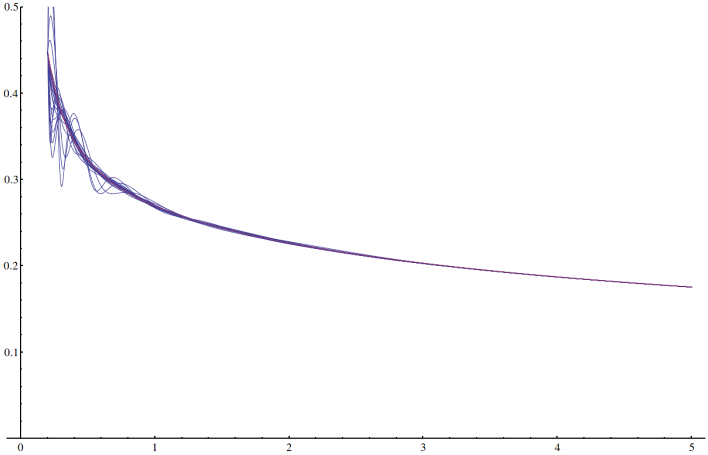

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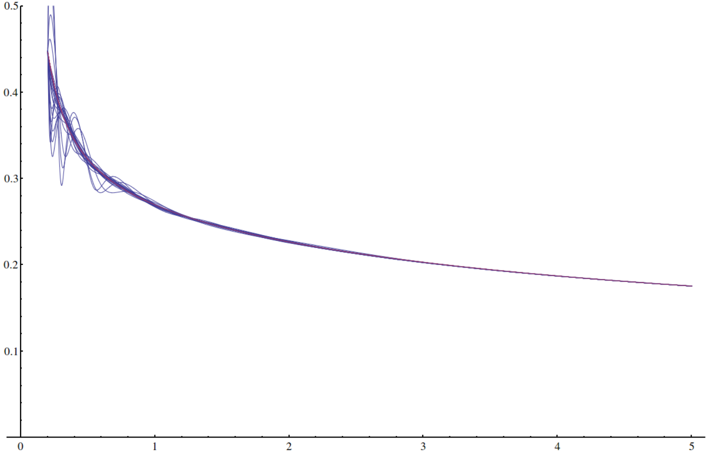

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- The singularities in the Borel plane have a clear physical origin — they correspond to the lowest **non-hydrodynamic** modes/degrees of freedom
- Analogy with perturbative expansion in QFT and instanton effects
- Hydrodynamic expansion captures *quantitatively* fine details of these leading nonhydrodynamic modes
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- We do not find poles on the positive real axis suggesting the existence of a Borel resummation, which can be constructed using Pade approximants
- Higher order hydrodynamics seems relevant for 'small' initial data...
- 4D perspective on deviations from hydro???
- QNM frequencies for QCD??? (lattice?)