Towards a holographic realization of the quarkyonic phase

1209.5915 [hep-th] with J. de Boer, B. Chowdhury & J. Jankowski

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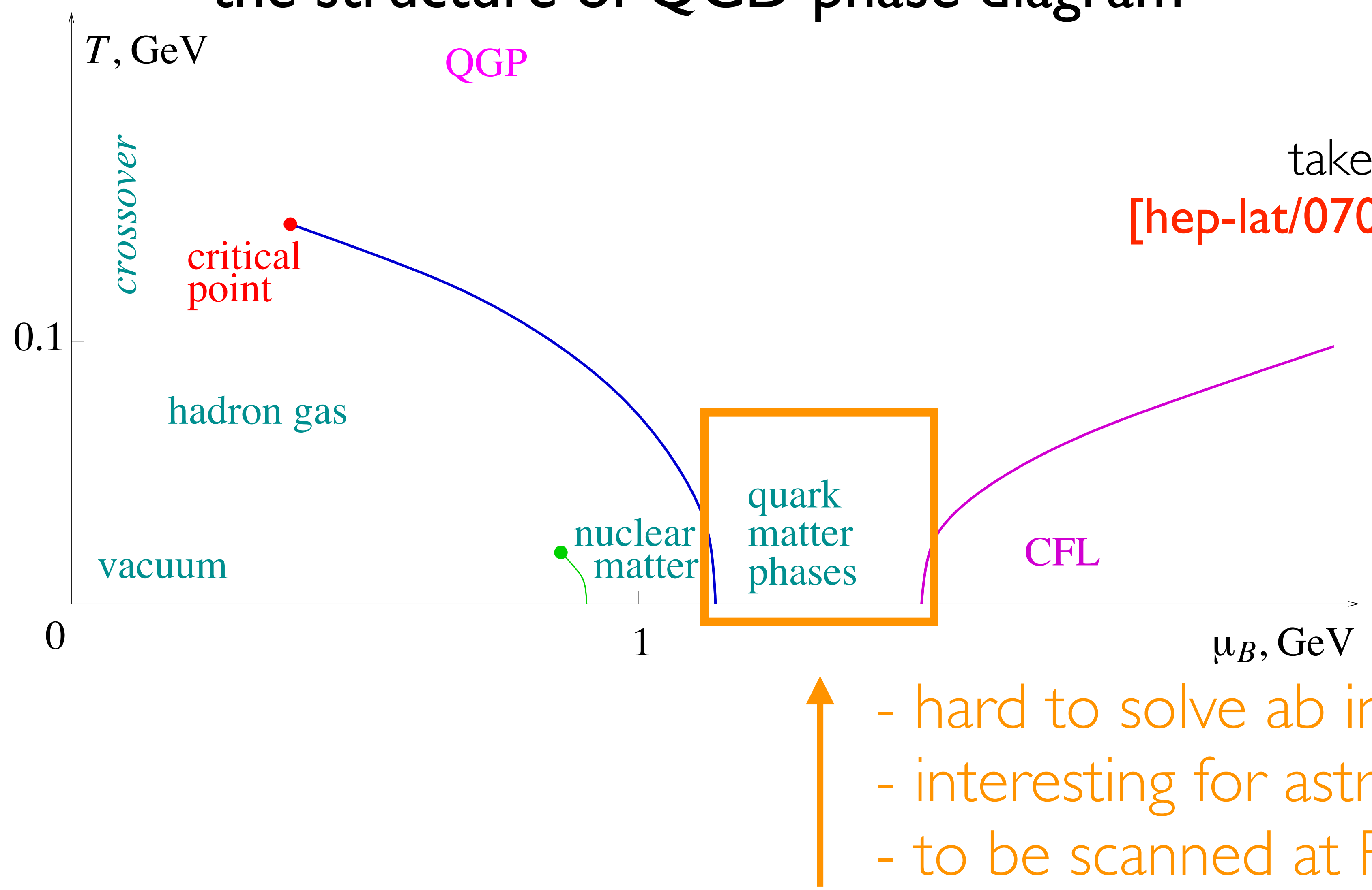
Introduction

Motivation

Two current holographic trends: **non-equilibrium processes**$\Big|_{\lambda\to\infty}$ and **AdS/CMT**

None of them addresses directly one of the biggest challenges in QCD:

the structure of QCD phase diagram



taken from
[hep-lat/0701002] Stephanov

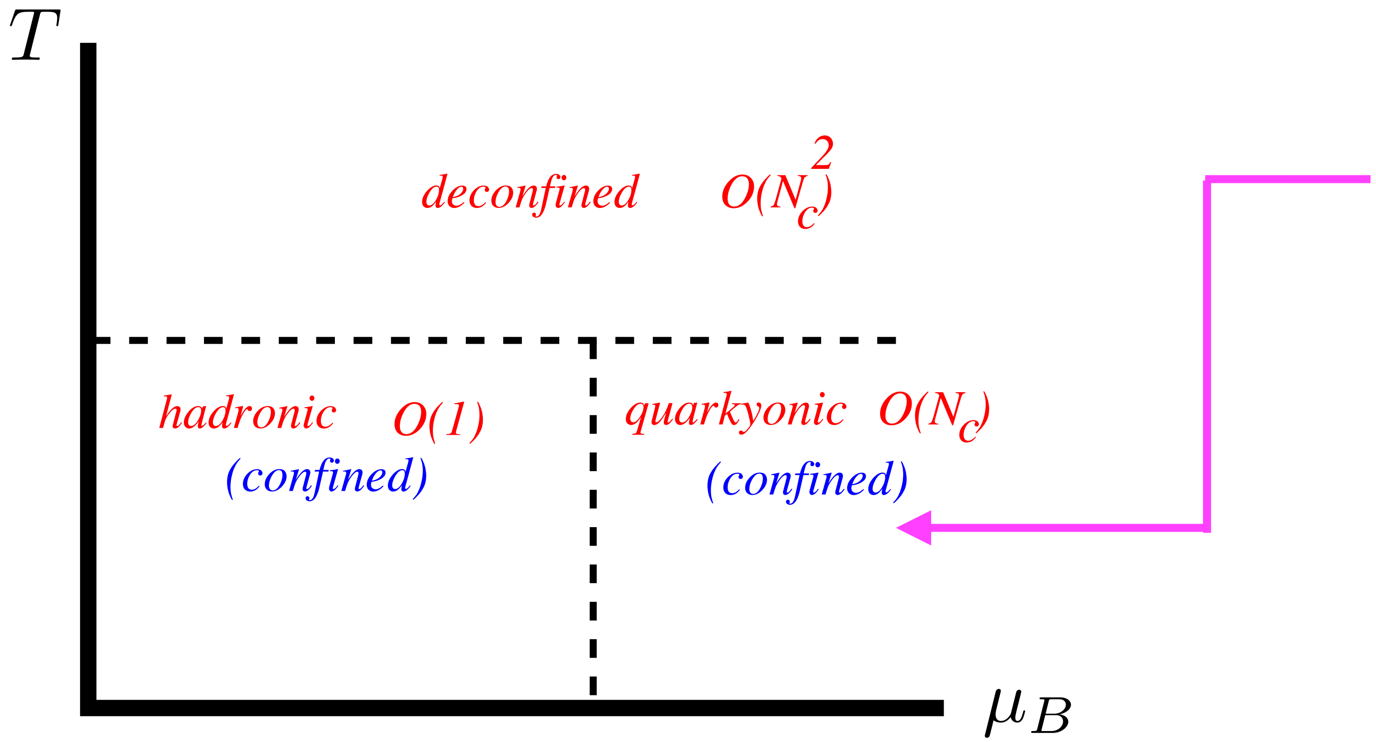
- hard to solve ab initio
- interesting for astrophysics
- to be scanned at FAIR experiment

Why not apply holographic techniques **there** (AdS/CMT of holographic QCD)?

Idea: search for the holographic quarkyonic phase, conjectured within large-N_c QCD.

Large-N_c QCD and the quarkyonic phase

0706.2191 [hep-ph] McLerran & Pisarski



exotic features:

- baryons heavily overlap and produce a **Fermi sea of quarks**;
- this leads to **P ~ $O(N_c)$;**
- the phase is **confining.**

consequences:

- at lower densities, momentum-carrying instabilities of the Fermi surface:

$$\langle \bar{\psi} \exp(2i\mu x_3 \gamma_0 \gamma_3) \psi \rangle$$

0912.3800 [hep-ph] Kojo et al.

(not dual to a massless DBI field!)

0803.3547 [hep-ph] Aharony & Kutasov

- at higher densities*, chiral symmetry restoration due to Pauli blocking
- excitation form then chiral multiplets (disputed)

0709.3080 [hep-ph] Glozman & Wagenbrunn

All this is based on qualitative large-N_c arguments and model studies. Two questions:

- is it all relevant for $N_c = 3$, $N_f = 3$?
- **is there a top-down realization of the quarkyonic phase (e.g. in holography)?**

Why is it interesting holographically?

Existing holographic dictionary ("AdS"/"CFT"):

geometry ——————— adjoint sector

black hole ——————— deconfinement

IR wall ——————— confinement

gauge field ——————— global symmetry

probe D-branes ——————— quenched quarks

connected D-branes ——————— chiral symmetry breaking

D-branes falling into horizon – chiral symmetry restoration

So far, the only known way to restore the chiral symmetry is to deconfine.

Holographic, chirally-symmetric quarkyonic phase requires new ingredients!

Also, hQCD at finite μ_B is interesting from the point of view of AdS/CMT!

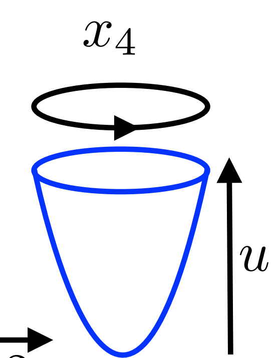
hQCD: vacuum

Holographic QCD model

[hep-th/0412141]
[hep-th/0507073] Sakai & Sugimoto

The microscopics is that of the D4-D8 system.

Confining geometry is generated by N_c D4's:



The tip of the cigar gives the IR cut-off

2-dimensional cigar

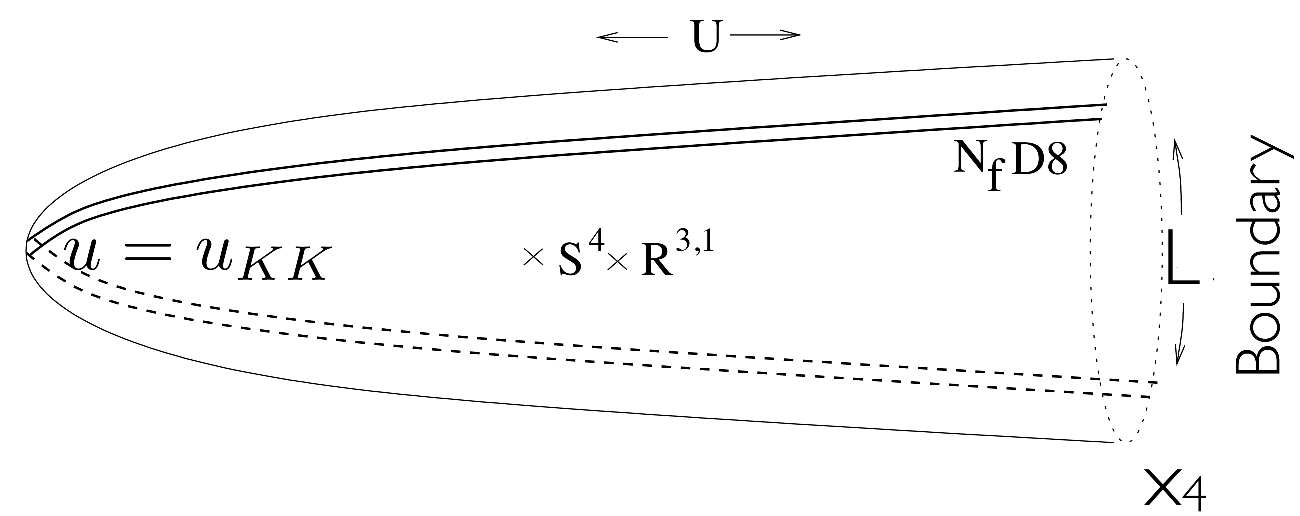
$$f(u) = 1 - \frac{u_{KK}^3}{u^3}$$

$$ds^2/R_4^2 = u^{\frac{3}{2}}(-d\tau^2 + d\vec{x}^2 + f(u)dx_4^2) + u^{-\frac{3}{2}}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right) \quad \text{with} \quad e^\Phi = g_s u^{\frac{3}{4}}, \quad F_4 = \frac{(2\pi)^3 l_s^3 N_c}{\Omega_4}\epsilon_4$$

Field theory directions | Compact direction | Radial direction | 4D sphere

N_f probe D8-branes are localized* in x_4. The flavors are massless.

$$S_{D8} = -\mu_8 \int d^9X \sigma e^{-\phi}\sqrt{-det(g_{ab} + 2\pi l_s^2 F_{ab})}$$
+ Chern-Simons term



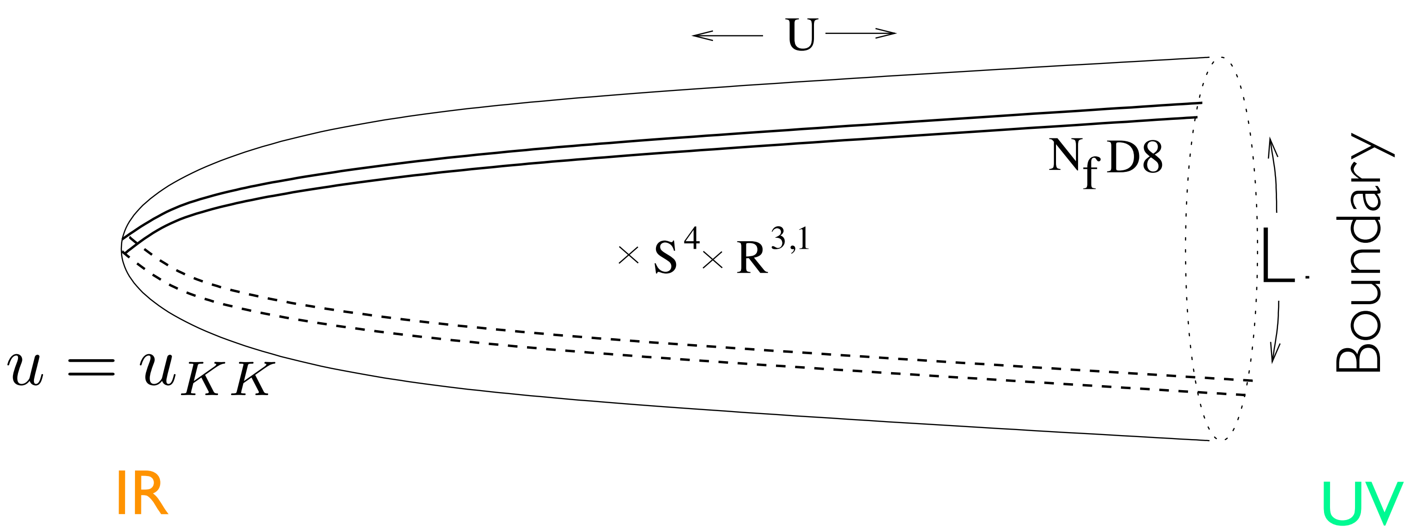
DOFs: $\mathbf{x_4}$ (scalar) & $\mathbf{A_a}$ ($SU(N_f)$ gauge field)

The vacuum is specified by providing L (asymptotic separation in x_4) in units of u_{KK}.

Chiral symmetry breaking in (N_f=1) hQCD

In the following, for simplicity, $N_f = 1$. Due to large N_c, U(1)$_A$ is not anomalous.

Let's switch from u to z variable: $u = (1+z^2)^{1/3}$, $z = \pm\infty$ correspond to bdry.



At $z = \pm\infty$ there are 2 independent D8's and so 2 U(1)'s.

D-branes connect in the bulk. Where this exactly happens is fixed by extremizing DBI with respect to x_4 at fixed asymptotic separation L .

Dual interpretation: spontaneous symmetry breaking U(1)$_V$ x U(1)$_A$ ⟶ U(1)$_V$.

Evidence: massless pseudoscalar meson in the spectrum (η')

Mesons in the vacuum and the chiral SB

[hep-th/0412141] Sakai & Sugimoto

Mesons: normalized modes of embedding and gauge field perturbations:

- δx_4 : tower of massive scalar and pseudoscalar meson
- δA_a : tower of vector and pseudovector mesons + 1 massless pseudoscalar meson

For the gauge field ansatz $A_\mu(z,t,\vec{x}) = B_\mu \psi(z) e^{-i\omega(k)t + i\vec{k}\cdot\vec{x}}$ one obtains

$$\lambda_n^{CP} = 0.67^{--}\ ,\quad 1.6^{++}\ ,\quad 2.9^{--}\ ,\quad 4.5^{++}\ ,\quad \cdots$$

↑ masses of mesons in units of u_{KK}

($\mathcal{O}(1)$ in N_c and λ)

$$\frac{\lambda_2}{\lambda_1} \simeq \frac{1.6}{0.67} \simeq 2.4 \qquad \text{(hQCD)}\ ,$$

$$\frac{m^2_{a_1(1260)}}{m^2_\rho} \simeq \frac{(1230\ \text{MeV})^2}{(776\ \text{MeV})^2} \simeq 2.51 \quad \text{(experiment)}\ .$$

$$\frac{\lambda_3}{\lambda_1} \simeq \frac{2.9}{0.67} \simeq 4.3 \qquad \text{(hQCD)}\ ,$$

$$\frac{m^2_{\rho(1450)}}{m^2_\rho} \simeq \frac{(1465\ \text{MeV})^2}{(776\ \text{MeV})^2} \simeq 3.56 \quad \text{(experiment)}$$

Mass differences between subsequent mesons come from the chiral SB.

Question I: what happens with the masses at finite baryon density?

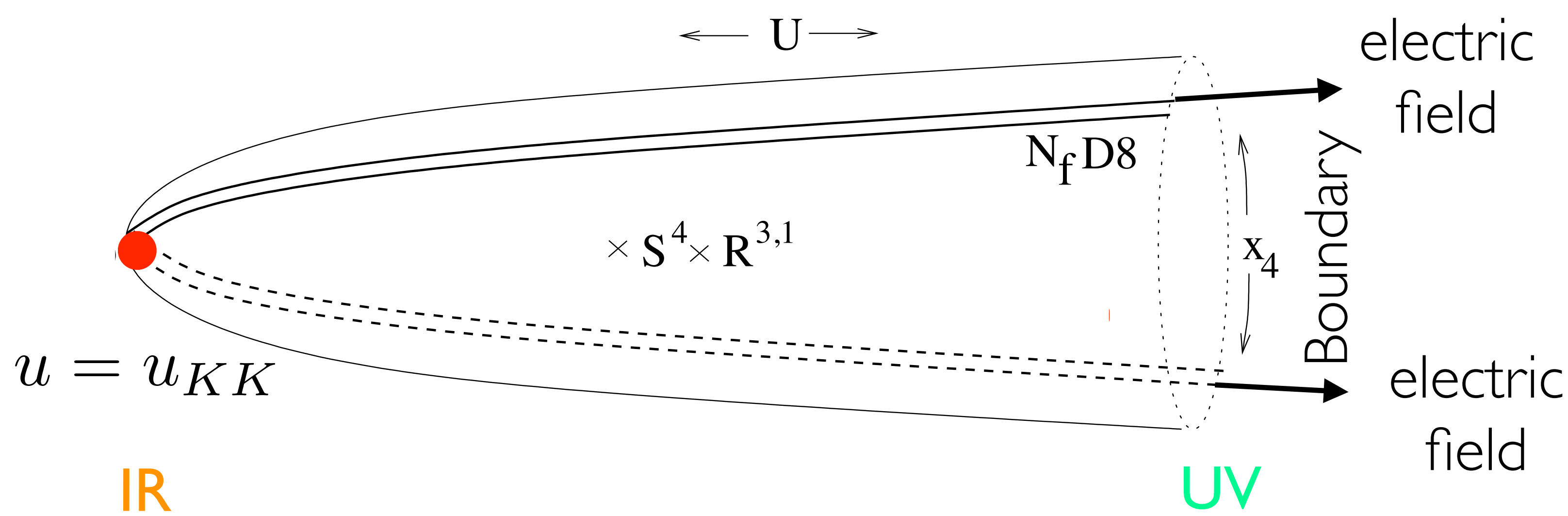
Baryons in N_f=1 hQCD

[hep-th/9805112] Witten

[hep-th/0412141] Sakai & Sugimoto

Baryon chemical potential arises from: $A_0(z) = A_0(-z)$ and $A_0\big|_{z\to\infty} = \mu_B + \frac{1}{z}\rho_B + \ldots$

No horizon, one needs charge carriers (●) to generate radial electric flux.

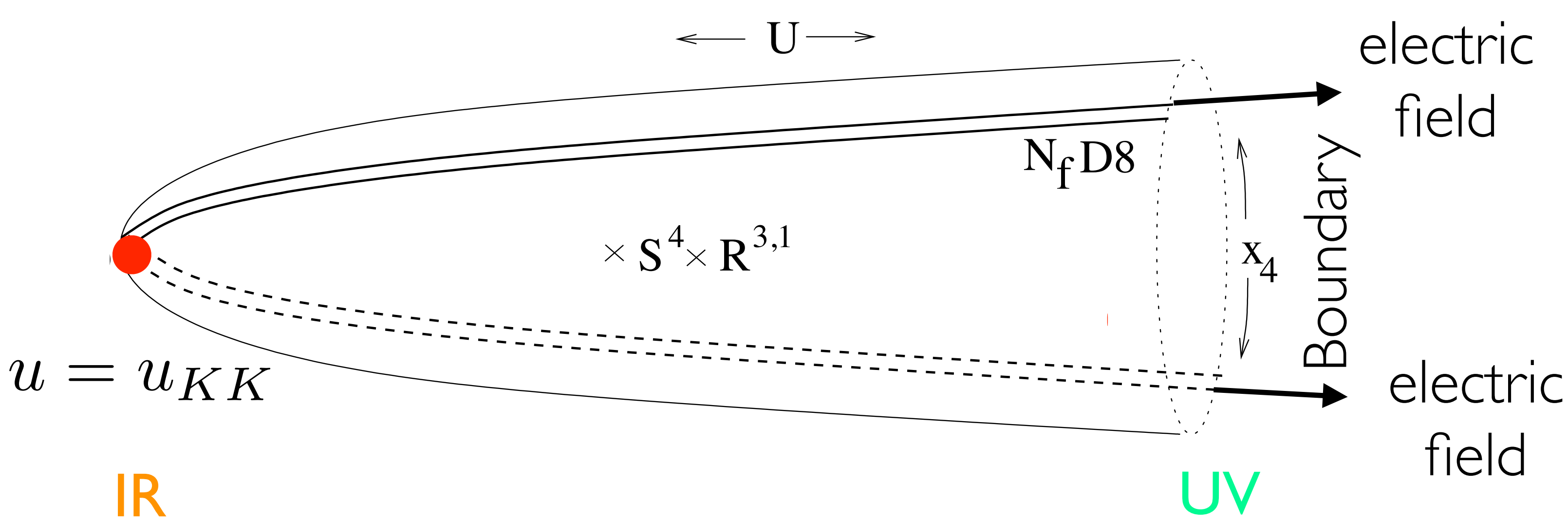


For N_f=1, the charge carriers are the end points of N_c fundamental strings stretched between D4-branes wrapping the S^4 part of the geometry the D8-brane.

For the antipodal embedding, single baryon will reside at the bottom of the U-shaped D8-brane. The same turns out to hold for the non-antipodal embeddings.

0810.1633 [hep-th] Seki & Sonnenschein

Baryon mass, charge and bdry charge radius



D4-brane wrapping S^4 is a point particle in the radial and field theory dimensions.

Its mass comes from the 5-dimensional DBI action $S = -\mu_4 \int d^5\xi e^{-\phi}\sqrt{-\det(g_{ab})}$

$$S_{D4} = -\frac{\mu_4}{g_s}\frac{8}{3}\pi^2 R_4^3 \int dt U(t) \longrightarrow M_B^0 = \frac{\mu_4}{g_s}\frac{8}{3}\pi^2 R_4^3 U_{KK} = \boxed{\frac{1}{27\pi}\frac{1}{R}\lambda N_c} \quad (x_4 = x_4 + 2\pi R)$$

and it carries N_c units of electric charge w/r D8-brane gauge field.

The bdry charge radius is $\mathcal{O}(1)$ in N_c and λ !

0806.3122 [hep-th] Hashimoto, Sakai & Sugimoto

hQCD: large baryon densities & their dual mean-field description

Scales in the problem: DBI action and the CS term

We are interested in studying hQCD at nonzero baryon density.

To anticipate the interesting parametric regime, let's look at the scales in the problem:

DBI action (schematically)

$$\mathcal{L}_{DBI} \sim N_c \lambda^3 \sqrt{\det(g_{ab} + \lambda^{-1}\partial_a A_b)}$$

Chern-Simons term:

$$\mathcal{L}_{CS} \sim N_c \epsilon^{abcde} A_a \partial_b A_c \partial_d A_e$$

Background radial electric field: possible instabilities towards modulated phase.

0704.1604 [hep-th] Domokos, Harvey **1011.4144 [hep-th]** Ooguri, Park

$$A_a = O(1) \longrightarrow \mathcal{L}_{DBI} \sim N_c \lambda \qquad \mathcal{L}_{CS} \sim N_c \lambda^0$$

$$A_a = O(\lambda) \longrightarrow \mathcal{L}_{DBI} \sim N_c \lambda^3 \qquad \mathcal{L}_{CS} \sim N_c \lambda^3$$

Interesting regime is thus $A_a = O(\lambda)$. This corresponds to $\rho_B \sim \dfrac{1}{N_c}\dfrac{\delta S}{\delta A} = \mathcal{O}(\lambda^2)$

Scales in the problem: holographic baryons

Holographic baryon mass and charge:

$$m_{hB} = \mathcal{O}(N_c \boxed{\lambda}) \qquad q_{hB} = N_c$$

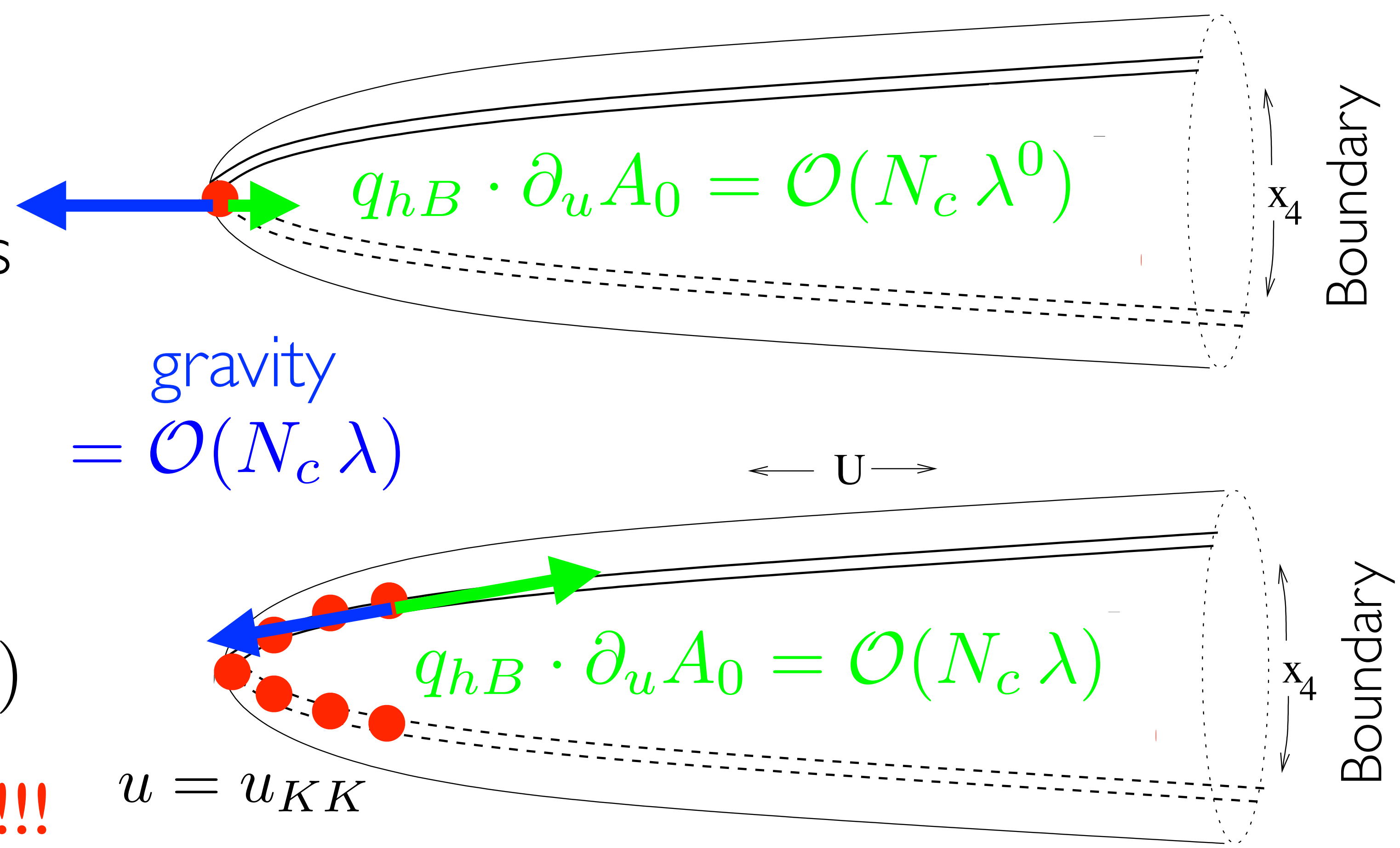
heavy compared to its charge and meson masses

Interactions between holographic baryons

Always repulsive in the $N_f=1$ case (get away by compactifying spatial QFT directions).

small densities

3D bulk charge configs



large densities

$$\rho_B \sim \frac{1}{N_c}\frac{\delta S}{\delta A} = \mathcal{O}(\lambda^2)$$

4D bulk charge configs!!!

Scales in the problem: meson-baryons interactions

Consider gauge field perturbation corresponding to vector or pseudovector meson

$$\partial^2 \delta A = \frac{1}{N_c \lambda} \delta j$$

← current of holographic baryons

↑ generically, the source is suppressed (follows from DBI)

bulk charge density ↓

Let's look now at the current: $\delta j = N_c n \delta u$ ← charge motion

↑ holographic baryon charge

and combine it with Newton's second law* $N_c \lambda \partial \delta u = N_c \partial \delta A \longrightarrow \delta u \sim \frac{1}{\lambda} \delta A$.

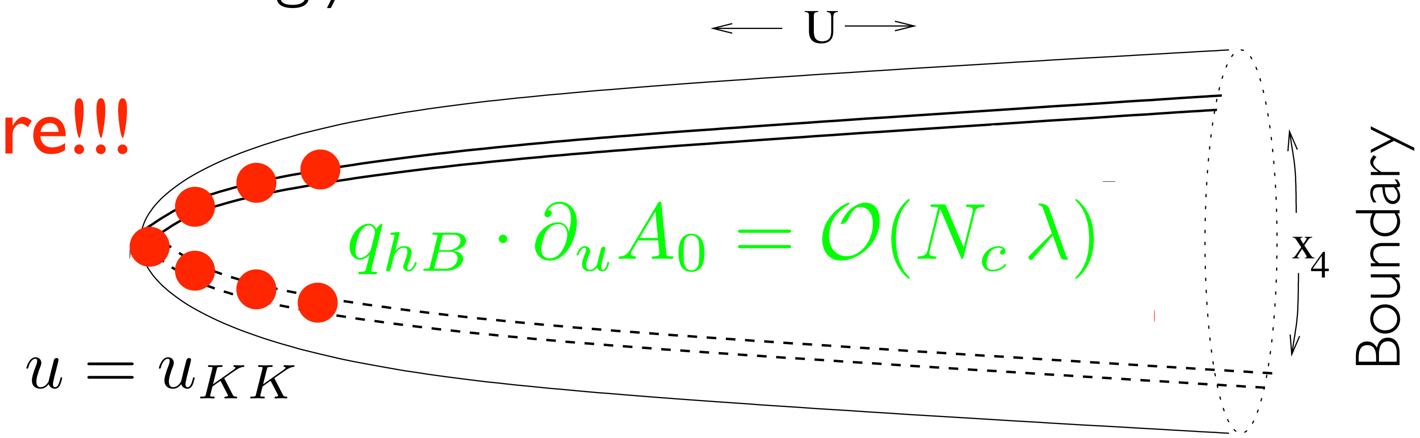
Altogether, we get $\partial^2 \delta A \sim \frac{1}{\lambda^2} n \, \delta A$. RHS non-trivial only for $n \sim \rho_B = \mathcal{O}(\lambda^2)$!

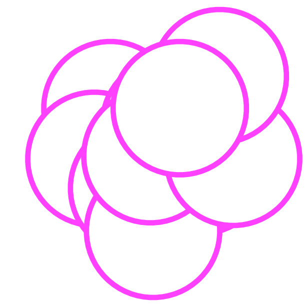
↑ in-medium mass for the bulk gauge field

Why baryons densities $\mathcal{O}(\lambda^2)$ might be quarkyonic?

- Baryons are then strongly correlated.

4D structure!!!





- One might expect CS term driven instabilities breaking translational invariance.

$$A_a = O(\lambda) \longrightarrow \mathcal{L}_{DBI} \sim N_c \lambda^3 \qquad \mathcal{L}_{CS} \sim N_c \lambda^3$$

- Density-enhanced meson-baryon interactions might significantly affect the spectrum:

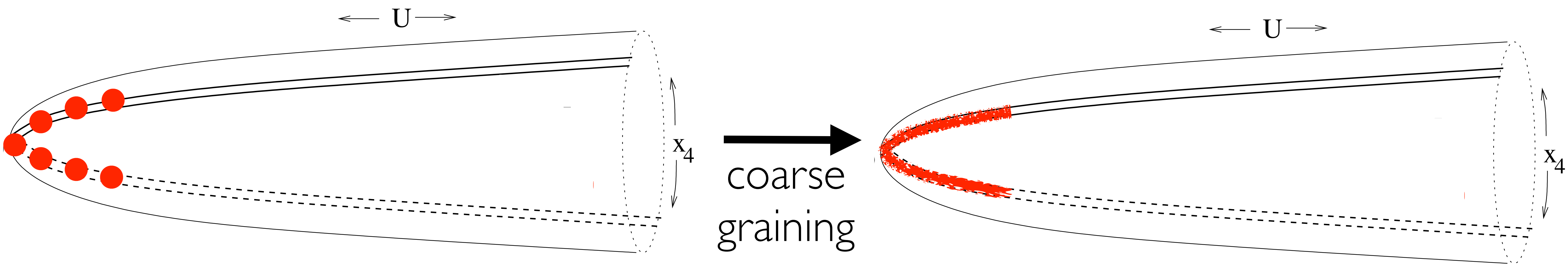
$$\partial^2 \delta A \sim \frac{1}{\lambda^2} n\, \delta A$$

↑

in-medium mass for the bulk gauge field

Towards mean-field approach

We are interested in the mean-field toy model of 4-dimensional charge distribution:



(and the same in the field theory directions)

The first assumption is that the cost of long-wavelength changes of the mean-field gauge field is captured by the original DBI action. This is the first key assumption.

The second assumption is that in any tiny volume we neglect the contribution to the energy density of the medium from the interactions with the microscopic gauge field.

There is also an energy cost of building up 4-dimensional charge distribution that comes from the curvature of the D8 and interactions with the mean-field gauge field.

Mean-field approach: electron star

This leads us to the following effective action $S = S_{DBI} + S_{CS} + S_{dust}$ that bears a striking resemblance to the electron star constructions in AdS/CMT

1008.2828 [hep-th] Hartnoll & Tavanfar

energy density

$$S_{dust}/c = \int d^5\tilde{x}\sqrt{-\det(\tilde{g}_{ab})}\Big\{ -\beta\,\tilde{\xi}^{1/4}(1+\frac{\tilde{z}^2}{\tilde{\xi}^2})^{1/12}\tilde{w}^2 + \gamma\,\tilde{w}^2\tilde{u}^a\left(\tilde{A}_a - \partial_a\tilde{\phi}\right) + \tilde{\lambda}(\tilde{u}_a\tilde{u}^a + 1)\Big\},$$

potential energy

Tildes denote rescalings involving $\lambda\,.\; c \sim N_c\lambda^3$, $\beta = 4\pi^2$, $\gamma = 12\pi^2$, $\frac{u}{u_{KK}} = \tilde{\xi}\left(1+\frac{\tilde{z}^2}{\tilde{\xi}^2}\right)^{\frac{1}{3}}$

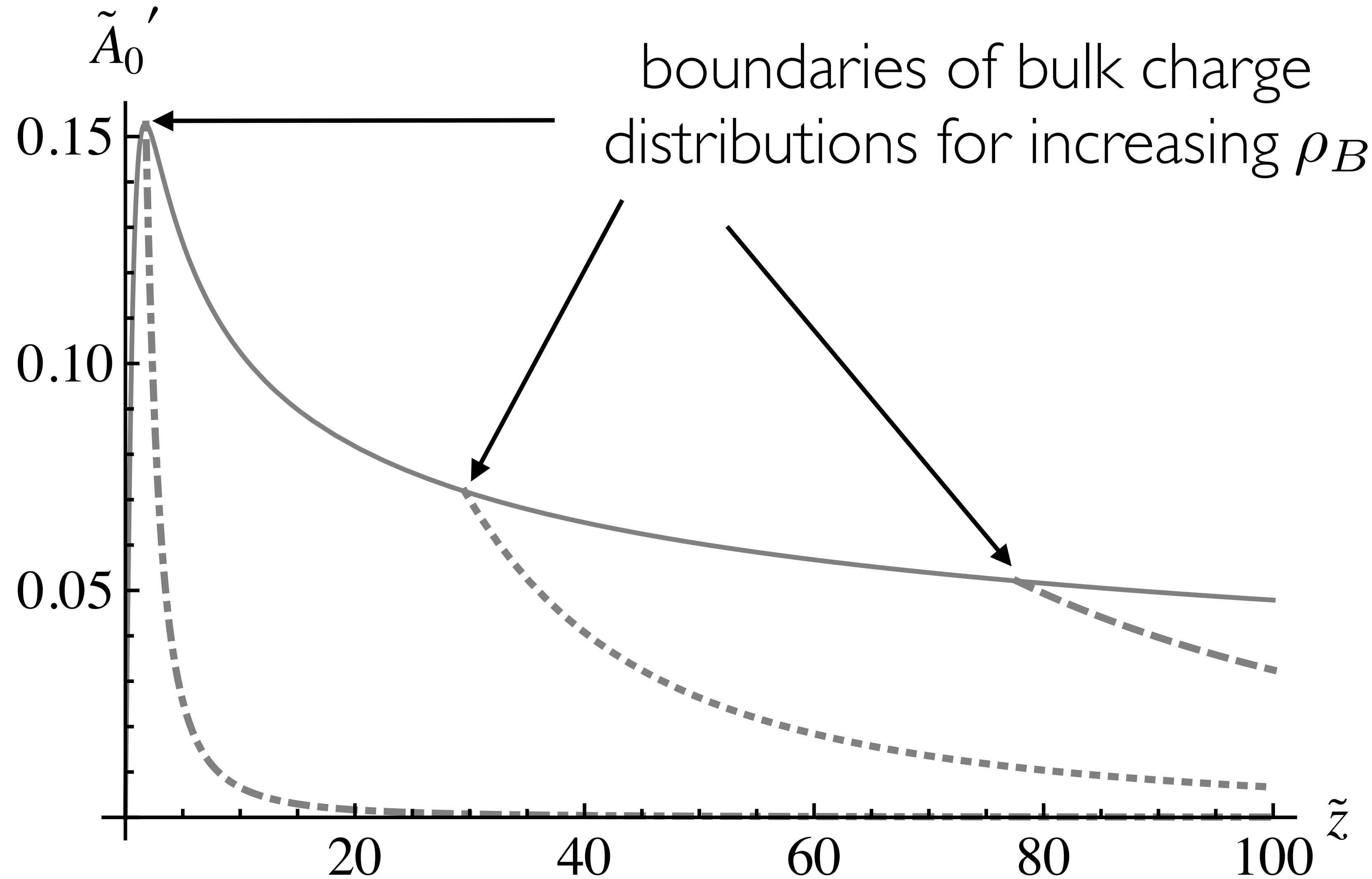
~ mass ~ charge

Equation of state: (energy density) ~ (charge density = $\tilde{w}^2$).

Exactly the same action, up to field redefinitions, was used in 0708.1322 [hep-th] Rozali et al. .

Homogeneous mean-field ground state

Solving EOMs for the coarse-grained fields in the antipodal case gives:



Three observations:

§ the surface of the bulk charge distribution moves towards the bdry as $\rho_B \nearrow$.

§ radial electric field is never large (it turns out also for non-antipodal embeddings).

§ the charge distribution in the core does not depend on external layers!

Spontaneous breaking of translational invariance

The Chern-Simons term and modulated GS

Ooguri et al.
Harvey et al.

Typically, gradients in spacelike dimensions cost energy and so mean field description has a homogeneous ground state.

Although the kinetic term is positive definite, the CS coupling (which is essential in hQCD) is not and it is possible that inhomogeneous configurations win energetically

In holographic systems instabilities towards modulation typically exist if the system has marginally stable normalized modes ($\omega = 0, \vec{k} \neq 0$)

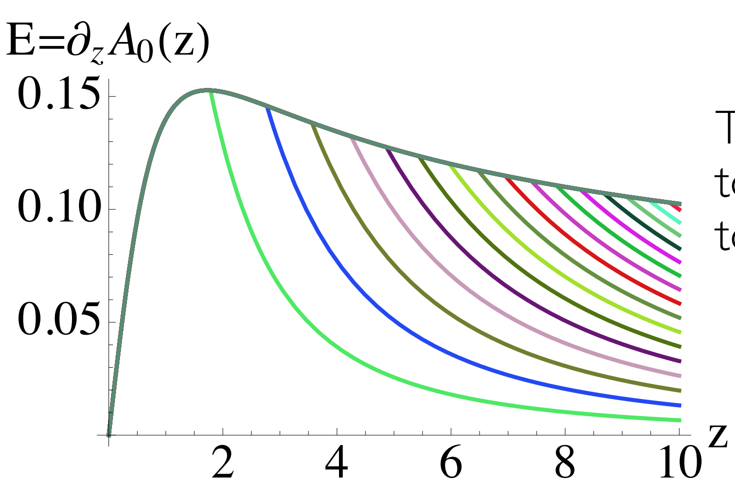
It's been known from earlier works of Harvey et al. and Ooguri et al. that the CS coupling leads to instability at large enough background electric field for the ansatz

$$A_z = 0, \quad A_0 = A_0(z)\,, \quad A_1 = \delta h(z) cos(kx^3), \quad A_2 = -\delta h(z) sin(kx^3), \quad A_3 = 0$$

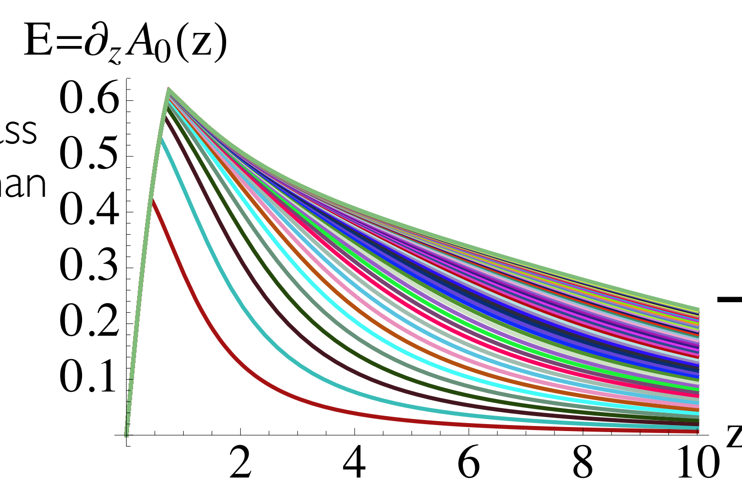
$$\updownarrow \qquad\qquad \updownarrow \qquad\qquad \updownarrow \qquad\qquad \updownarrow$$

$$j^0 = \rho_B \quad \delta j^1_{A/V} = \#_{A/V} cos(kx^3) \quad \delta j^2_{A/V} = -\#_{A/V} sin(kx^3) \quad \delta j^3 = 0$$

Thorough analysis of EOM of the form $\#_2(z)\,\delta h''(z) + \#_1(z)\,\delta h'(z) + (\#_0(z) - E)\,\delta h(z) = 0$ does not reveal unstable modes unless we do the following



Taking baryon mass to be 5x larger than top-down value →



→ **E** becomes large enough to support marginally stable modes leading to inhomogeneous ground state

In-medium excitations

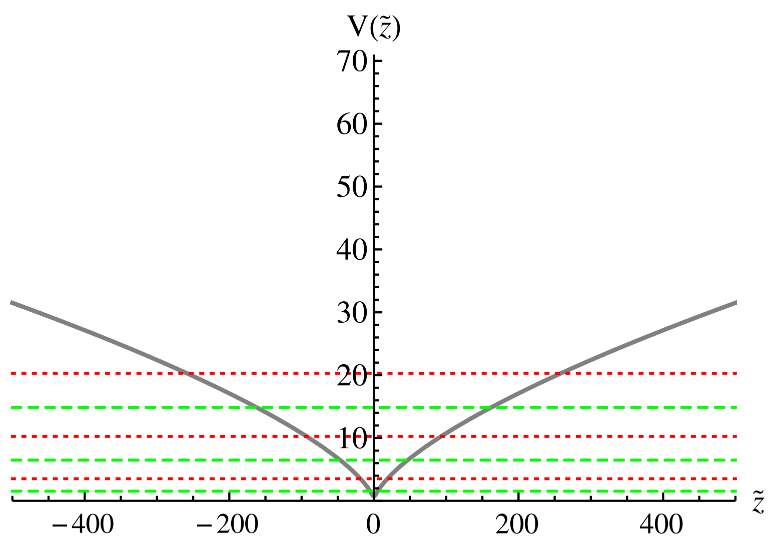
Possible mechanism of „chiral symmetry" restoration

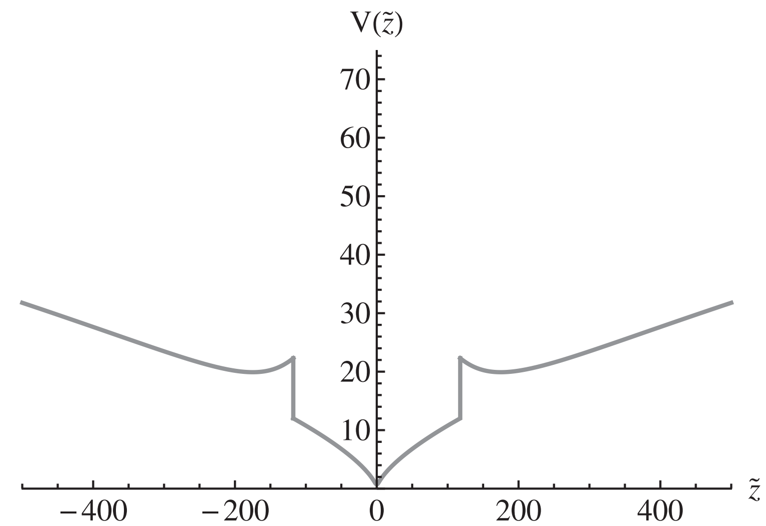
EOM for axial and vector meson perturbations can be rewritten as a Schr. eqn.

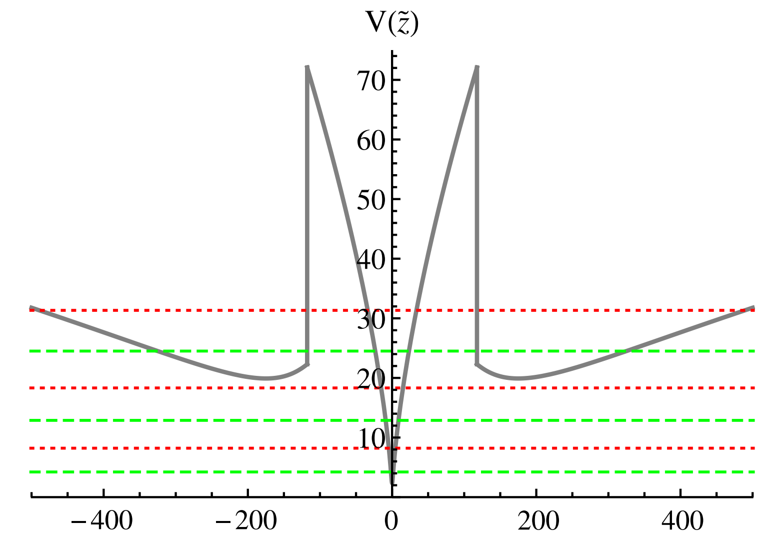
antipodal case: 1) vacuum 2) dense medium (~~BMin~~) 3) dense medium

ω_V^2

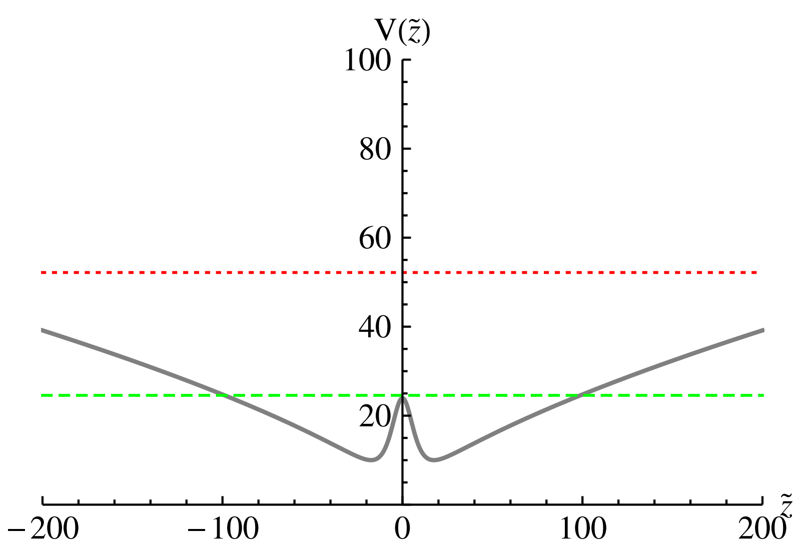
ω_A^2

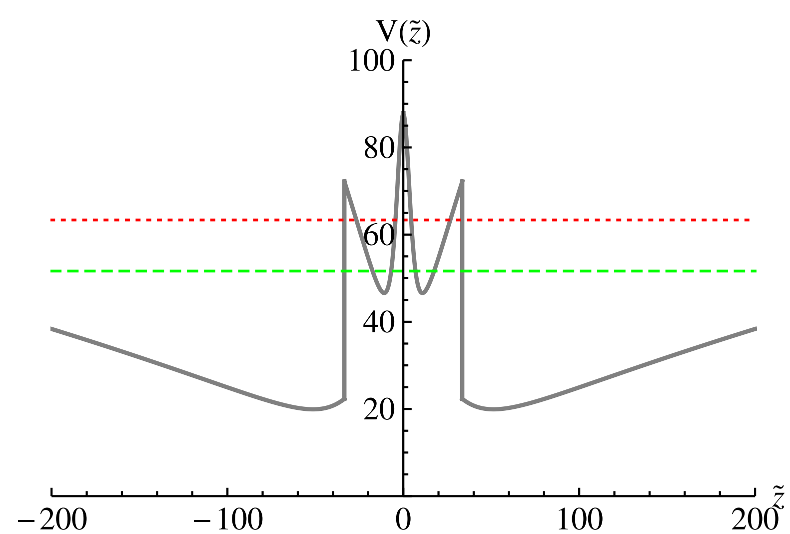






v. nonantipodal case: a) vacuum b) dense medium





Approximate chiral symmetry restoration for the lowest vector and axial mesons!

Relation to other works

Relation to other works and possible extensions

§ Our mean-field description is the same* as **0708.1322 [hep-th]** by Rozali et al. and in many ways resembles the electron star from **1008.2828 [hep-th]** by Hartnoll & Tavanfar.

§ It has to be contrasted with 3-dimensional charge distributions considered first by **0708.0326 [hep-th]** by Bergman et al. which apply only at $\rho_B \ll \mathcal{O}(\lambda^2)$!

§ More microscopic justification for 4-dimensional lattices at large enough densities comes from **1201.1331 [hep-th]** and **1304.7540 [hep-th]** by Sonnenschein et al..

§ It would be interesting to repeat the calculation from **1304.7097 [hep-th]** by Seki & Sin for 4-dimensional charge densities relevant for $\mathcal{O}(\lambda^2)$ baryon densities.

§ Things are similar to AdS/CMT. Below idea from Sean Hartnoll:

"fully fractionalized"

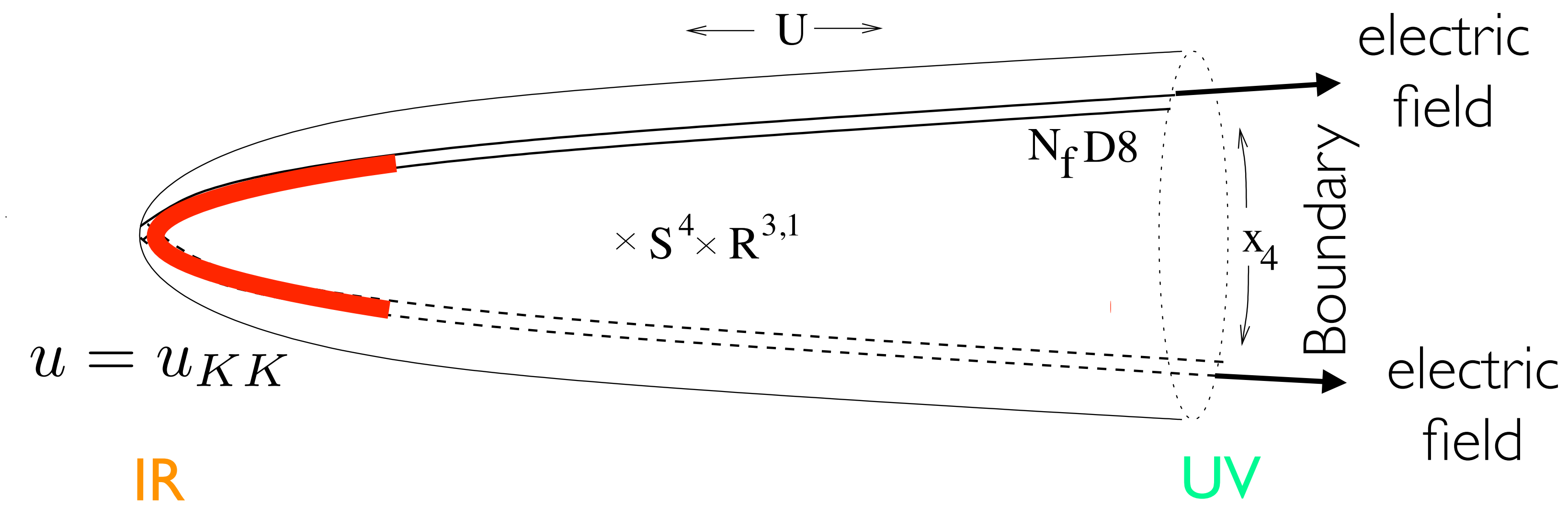
"mesonic"

1111.2606 [hep-th]
Hartnoll & Huijse

Summary

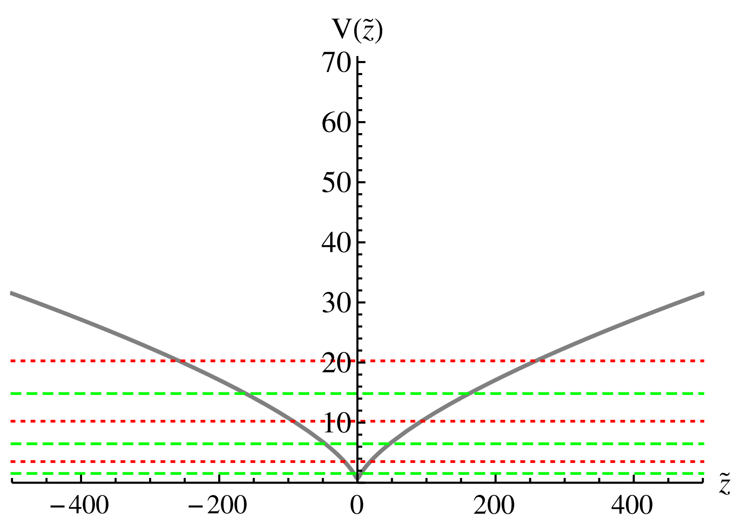
Summary

§ Main idea: focus on $\mathcal{O}(\lambda^2)$ baryon densities:

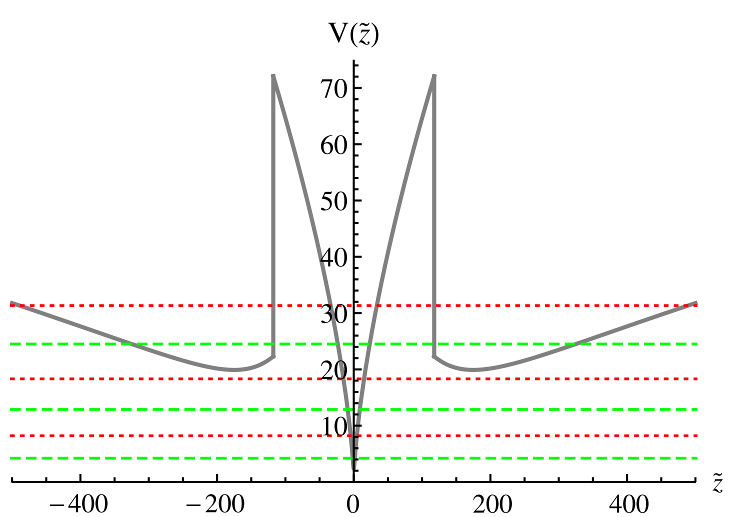


§ Possible mechanism for chiral symmetry restoration:
strong interactions with the thick layer of holographic baryons

§ General theme:
condensed matter physics of holographic QCD



vs.



Almost final thoughts

§ I don't think our approximations lead to the model having the quarkyonic phase.

§ Whether it appears in a truly microscopic hQCD construction is an open problem.

§ One thing is certain: $\rho_B = \mathcal{O}(\lambda^2)$ leads to something (?) interesting that is not 3D.

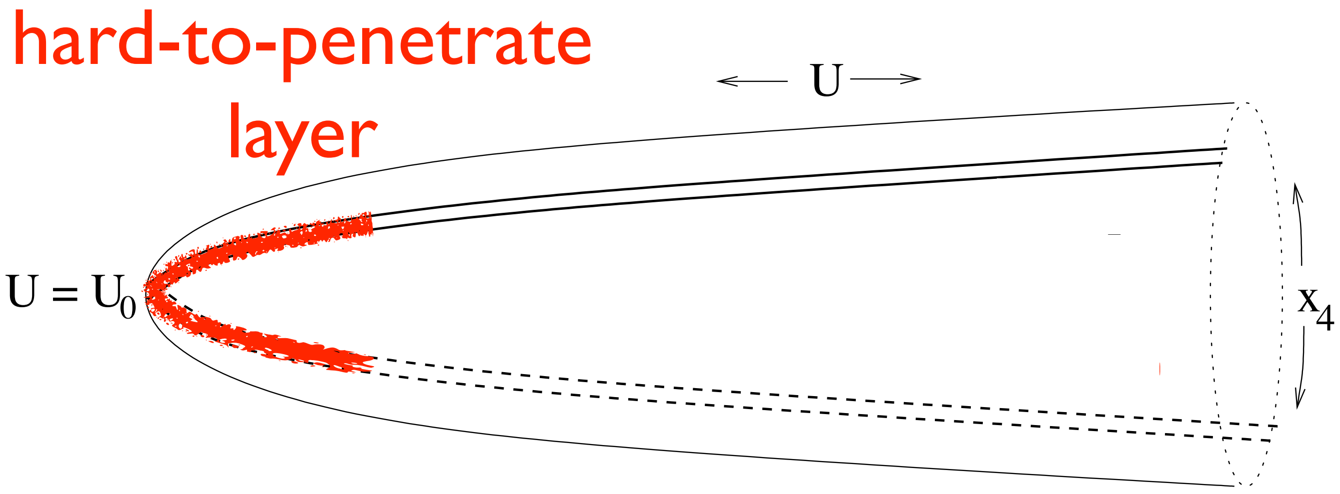
§ What I would like to see is a model in which the charge distribution gets significantly denser in the core as we add more and more layers.

§ Then interesting many-body effects following from the nonlinearities of DBI might start playing role, changing completely the picture presented here and other works.

§ The CS term might then still lead to interesting macroscopic modulation patterns, possibly visible also in other observables, e.g. the chiral condensate.

Wishful thinking

(Approximate) chiral symmetry restoration at large densities:



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