Challenges for Holographic (S)QCD

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Introduction

Holographic techniques are most well developed for two classes of gauge theories:

- Theories in which all matter transforms in the adjoint representation of the gauge group, such as N=4 SYM, pure (S)QCD, etc. These theories map via holography to purely closed string theories, which can in some regimes of parameter space be described by (super) gravity.

- Theories with matter in the fundamental representation of the gauge group, but with a number of flavors much smaller than that of colors. Such theories can be described by adding D-branes to a closed string background, and treating them as probes. A famous example is the Sakai-Sugimoto model of QCD.

In these models, open strings ending on the D-branes give rise to mesons, while closed strings in the bulk give glueballs. In the probe approximation, the former are described by a DBI-type action in the gravitational background created by the adjoints.

- Many important questions in gauge theory concern theories with $N_f \sim N_c$. These include applications to phenomenology, and more theoretical issues. Indeed, the physics of non-abelian gauge theories depends at large N on the parameter N_f/N_c.

- QCD is expected to flow in the infrared to a conformal fixed point for $N_f^{\rm crit} < N_f < 11N_c/2$. It would be interesting to study the physics of the conformal phase, and the onset of confinement using holography.

- At present, such studies are done by extrapolating the DBI + gravity description from the probe regime. It is not clear whether this should give a good description of the physics.

- One attitude is to use the gravity + DBI description as a phenomenological one and to try to match its predictions to data.

- To gain confidence in these techniques, it is interesting to see if the holographic description reproduces, at least qualitatively, features that can be derived from the field theory. In this talk I will describe some such properties, which have not been understood from holography yet.

I will actually discuss not QCD but its N=1 supersymmetric analog. This theory exhibits many of the features found in the non-supersymmetric case, and has been studied using holography (Klebanov, Maldacena; Kuperstein, Sonnenschein,…). At the same time, supersymmetry allows for more control, as usual, so we can say more than without it.

I will discuss two issues:

- The behavior of the theory in an external magnetic field for a global symmetry, preserving some of the supersymmetry (J. Lin and DK, to appear).

- Embedding of N=1 SQCD in a larger set of theories, which interpolates between SQCD and theories with known holographic duals (if there is time..).

The basic idea

Given a 4d N=1 SQFT with a global U(1) symmetry, we:

1) Couple the U(1) current supermultiplet to an external vector superfield, which consists of a vector field, gaugino and auxiliary field:

$$(A_\mu, \lambda_\alpha, D)$$

2) Compactify the theory on a two-torus and turn on a magnetic field for the U(1) through the torus:

$$F_{12} = B$$

This breaks supersymmetry completely, but it can be restored by

3) Turning on the auxiliary field D in the U(1) vector superfield. The variation of the background gaugino is

$$\delta\lambda = (F_{\mu\nu}\sigma^{\mu\nu} + iD)\epsilon$$

or

$$\delta\lambda = i\begin{pmatrix} D-B & 0 \\ 0 & D+B \end{pmatrix}\begin{pmatrix} \epsilon_- \\ \epsilon_+ \end{pmatrix}$$

For B=D the background preserves (0,2) SUSY. The purpose of this work is to explore the resulting theories.

(0,2) SUSY in two dimensions

- (0,2) superspace:

  bosonic coordinates: (x^0, x^3)

  fermionic coordinates: $(\theta^+, \bar{\theta}^+)$

  right-moving supercharges:

$$Q_+ = \frac{\partial}{\partial \theta^+} + i\overline{\theta}^+(\partial_0 + \partial_3)\ , \qquad \overline{Q}_+ = -\frac{\partial}{\partial \overline{\theta}^+} - i\theta^+(\partial_0 + \partial_3)$$

superspace covariant derivatives:

$$D_+ = \frac{\partial}{\partial\theta^+} - i\overline{\theta}^+(\partial_0 + \partial_3) \;, \qquad \overline{D}_+ = -\frac{\partial}{\partial\overline{\theta}^+} + i\theta^+(\partial_0 + \partial_3)$$

In the presence of a gauge field:

$$\mathcal{D}_0 + \mathcal{D}_3 = \partial_0 + \partial_3 + i(A_0 + A_3) \;,$$

$$\mathcal{D}_+ = \frac{\partial}{\partial\theta^+} - i\overline{\theta}^+(\mathcal{D}_0 + \mathcal{D}_3) \;,$$

$$\overline{\mathcal{D}}_+ = -\frac{\partial}{\partial\overline{\theta}^+} + i\theta^+(\mathcal{D}_0 + \mathcal{D}_3) \;,$$

$$\mathcal{D}_0 - \mathcal{D}_3 = \partial_0 - \partial_3 + iV \;,$$

V is a (0,2) vector superfield:

$$V = A_0 - A_3 - 2i\theta^+\overline{\lambda}_- - 2i\overline{\theta}^+\lambda_- + 2\theta^+\overline{\theta}^+ D$$

It is usually described in terms of a field strength:

$$\Upsilon = [\overline{\mathcal{D}}_+, \mathcal{D}_0 - \mathcal{D}_3] = -2\left(\lambda_- - i\theta^+(D - iF_{03}) - i\theta^+\overline{\theta}^+(D_0 + D_3)\lambda_-\right),$$

which satisfies the chirality condition

$$\overline{\mathcal{D}}_+\Upsilon = 0.$$

Two types of matter superfields in a representation r of a gauge group G play a role in our discussion:

◆ Chiral superfield:

$$\Phi = \phi + \sqrt{2}\theta^{+}\psi_{+} - i\theta^{+}\overline{\theta}^{+}(D_0 + D_3)\phi$$

◆ Fermi superfield:

$$\Lambda = \psi_{-} - \sqrt{2}\theta^{+} F - i\theta^{+}\overline{\theta}^{+}(D_0 + D_3)\psi_{-} - \sqrt{2}\overline{\theta}^{+} E$$

which satisfies the chirality constraint

$$\overline{\mathcal{D}}_{+}\Lambda = \sqrt{2}E \ , \qquad \overline{\mathcal{D}}_{+}E = 0$$

- Actions:

$$S_\Upsilon = \frac{1}{8g^2}\mathrm{Tr}\int d^2x d^2\theta \overline{\Upsilon}\Upsilon \ ,$$

$$S_\Phi = -\frac{i}{2}\int d^2x d^2\theta \overline{\Phi}(\mathcal{D}_0 - \mathcal{D}_3)\Phi \ ,$$

$$S_\Lambda = -\frac{1}{2}\int d^2x d^2\theta \overline{\Lambda}\Lambda \ ,$$

In components:

$$S_\Upsilon = \frac{1}{g^2}\mathrm{Tr}\int d^2x \left\{\frac{1}{2}F_{03}^2 + i\overline{\lambda}_-(D_0 + D_3)\lambda_- + \frac{1}{2}D^2\right\} \ ,$$

$$S_\Phi = \int d^2x \left\{-|D_\mu\phi|^2 + i\overline{\psi}_+(D_0 - D_3)\psi_+ - i\sqrt{2}\overline{\phi}T^a\lambda_-^a\psi_+ + i\sqrt{2}\phi T^a\overline{\psi}_+\overline{\lambda}_-^a + \overline{\phi}T^a\phi D^a\right.$$

$$S_\Lambda = \int d^2x \left\{i\overline{\psi}_-(D_0 + D_3)\psi_- + |\mathcal{F}|^2 - |E|^2 - \left(\overline{\psi}_-\frac{\partial E}{\partial\phi_i}\psi_{+i} + \overline{\psi}_{+i}\frac{\partial\overline{E}}{\partial\overline{\phi}_i}\psi_-\right)\right\} \ .$$

- FI term for a U(1):

$$S_{\mathrm{FI}} = \frac{t}{4} \int d^2x d\theta^+ \Upsilon \Big|_{\overline{\theta}^+=0} + \mathrm{c.c.} = \frac{it}{2} \int d^2x (D - iF_{01}) + \mathrm{c.c.}$$

$$t = ir + \frac{\theta}{2\pi}$$

- (0,2) superpotential:

$$S_{\mathcal{W}} = -\frac{1}{\sqrt{2}} \int d^2x d\theta^+ \Lambda_a J^a \Big|_{\overline{\theta}^+=0} + \mathrm{c.c.} = -\int d^2x \left\{ \mathcal{F}_a J^a(\phi_i) + \psi_{-a}\psi_{+i} \frac{\partial J^a}{\partial \phi_i} \right\} + \mathrm{c.c.}$$

Free fields in a magnetic field

Consider a free massless scalar field ϕ of charge e under a U(1) gauge field A_μ. We turn on a background magnetic field B,

$$A_2 = Bx_1.$$

The Klein-Gordon equation for ϕ takes the form:

$$(-\partial_0^2 + \partial_3^2 + \partial_1^2 + \widetilde{D}_2^2)\phi = 0$$

where $$\widetilde{D}_2 = \partial_2 + ieBx_1$$

To study the 1+1 dimensional spectrum we separate variables

$$\phi(x^0, x^3; x^1, x^2) = \varphi(x^0, x^3)\chi(x^1, x^2) \ .$$

Taking χ to be an eigenfunction of

$$H = -(\partial_1^2 + \widetilde{D}_2^2) = p_1^2 + (p_2 + eBx_1)^2 \ , \qquad H\chi = m^2\chi,$$

gives rise to a two dimensional scalar field φ of mass m. H is the Hamiltonian of a particle in a magnetic field. The spectrum is:

$$m_n^2 = (2n+1)|eB| \ .$$

To preserve supersymmetry, we need also to turn on a D field. This shifts the spectrum to

$$m_n^2 = (2n+1)|eB| - eD \ .$$

For B=D (the (0,2) supersymmetric case), eB>0 gives a massless particle, while for eB<0 the spectrum is massive.

Can repeat the analysis for free fermions. Get

$$m_+^2 = (2n+1)|eB| - eB, \qquad m_-^2 : (2n+1)|eB| + eB$$

For right (+) and left (-) movers in 2d.

Summary:

A four dimensional free massless chiral superfield Φ with U(1) charge e reduces in a constant B=D>0 field to:

e>0: massless (0,2) chiral superfield.

e<0: massless (0,2) Fermi superfield.

Compactification:

So far we discussed the effects of the magnetic field in non - compact spacetime. We are actually interested in turning on a magnetic field on a torus. This leads to Dirac quantization:

$$eB\mathcal{A} \in 2\pi \mathrm{Z}$$

The states have degeneracy

$$n_e = |e|B\mathcal{A}/2\pi \ .$$

Below, we will normalize the charges such that the degeneracy is |e|.

Four dimensional N=1 theories on a magnetized torus

We start with a four dimensional N=1 supersymmetric gauge theory with gauge group G and matter fields Φ_i in representations r_i. We compactify on a two-torus, and turn on a magnetic field to the U(1) global symmetry under which Φ_i have charges e_i. The symmetry must be non-anomalous:

$$\sum_i e_i T(r_i) = 0 \ ,$$

$$\mathrm{Tr}_r T^a T^b = T(r)\delta^{ab}, \text{ with } a, b = 1, \cdots, \mathrm{dim}\ G$$

The two dimensional matter content is:

1) A gauge superfield Υ.

2) An adjoint chiral superfield Σ.

3) e_i (0,2) chiral superfields Φ_i in the representation r_i for fields with $e_i > 0$.

4) $|e_i|$ Fermi superfields Λ_i in the representation r_i for $e_i < 0$.

5) A chiral superfield Φ_i and a Fermi superfield Λ_i for $e_i = 0$.

Example: N=1 SQCD

Gauge group $G = U(N_c)$, N_f flavors of fundamentals $Q^i, \widetilde{Q}_i$.

Global symmetry:

$$SU(N_f)_L \times SU(N_f)_R \ .$$

The U(1) we use assigns charges e_i to Q^i and $\widetilde{e}_i$ to $\widetilde{Q}_i$.

The charges satisfy:

$$\sum_i e_i = \sum_i \widetilde{e}_i = 0$$

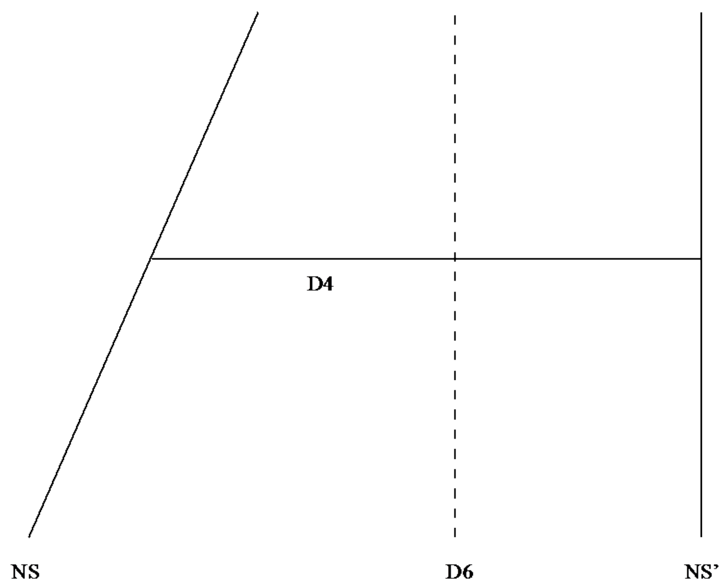
E.g. can take half of the e_i to be +1 and half to be -1; same for $\widetilde{e}_i$.

Three dimensional description

Since we are planning to compactify to two dimensions, we can view the construction by starting with a 3d theory with N=2 SUSY, obtained by compactifying N=1 SQCD on a circle, and place it on an extra circle. The magnetic field in the 4d (0123) becomes in the three dimensional theory in (013) an expectation value of a scalar field $\phi_2 = Bx^1$. This can be thought of as a position-dependent real mass term for the chiral superfields.

This three dimensional description is useful for analyzing the dynamics.

The gauge theories in question have a useful string theory description as theories on systems of intersecting D-branes and NS5-branes. Four dimensional N=1 SQCD is described by:



$$N_c\ D4 : (01236);\quad N_f\ D6 : (0123789)$$

$$NS :\ (012345);\quad NS' :\ (012389)$$

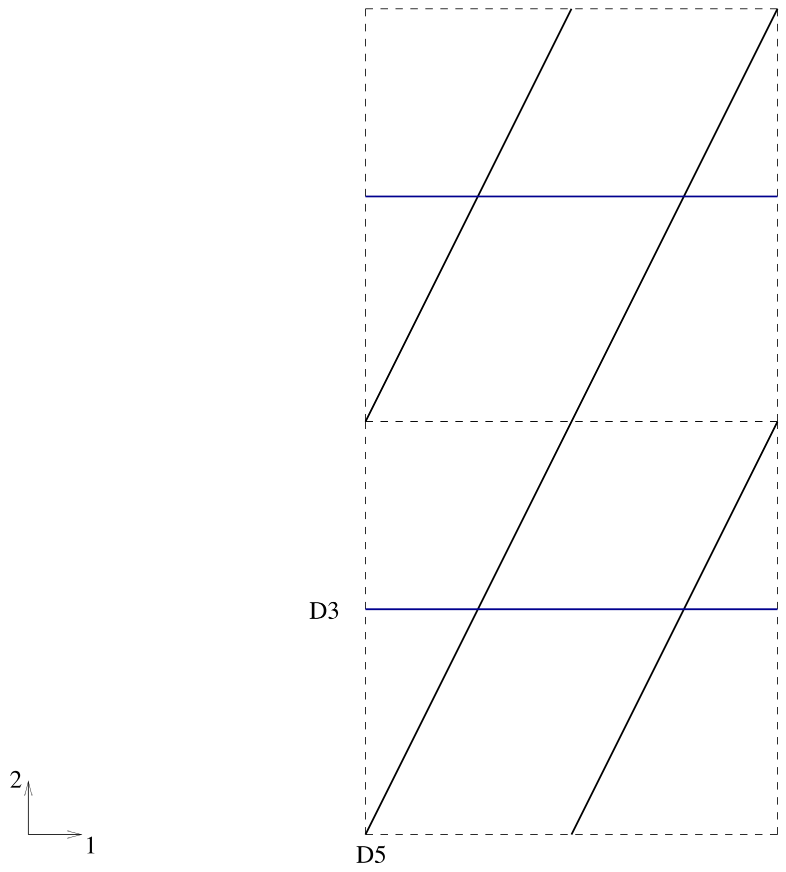
The three dimensional theory is obtained by replacing:

D4 (01236) --> D3 (0136)

D6 (0123789) --> D5 (013789)

We want to compactify on a circle and turn on the background fields B and D.

Turning on the B field corresponds in the brane system to rotating the D5-branes in the (12) plane:



- The D field corresponds to a rotation of the D5-branes in the (67) plane.

- Quantization of B, localization of the fundamentals on the torus, and the degeneracy |e| are manifest in the brane picture.

- The fact that the configuration preserves (0,2) SUSY in (03) is the familiar fact that a rotation in two complex planes preserves ½ of SUSY. Define $z_1 = x^1 + ix^7,\ z_2 = x^2 + ix^6$ and rotate $z \to \Omega z$.

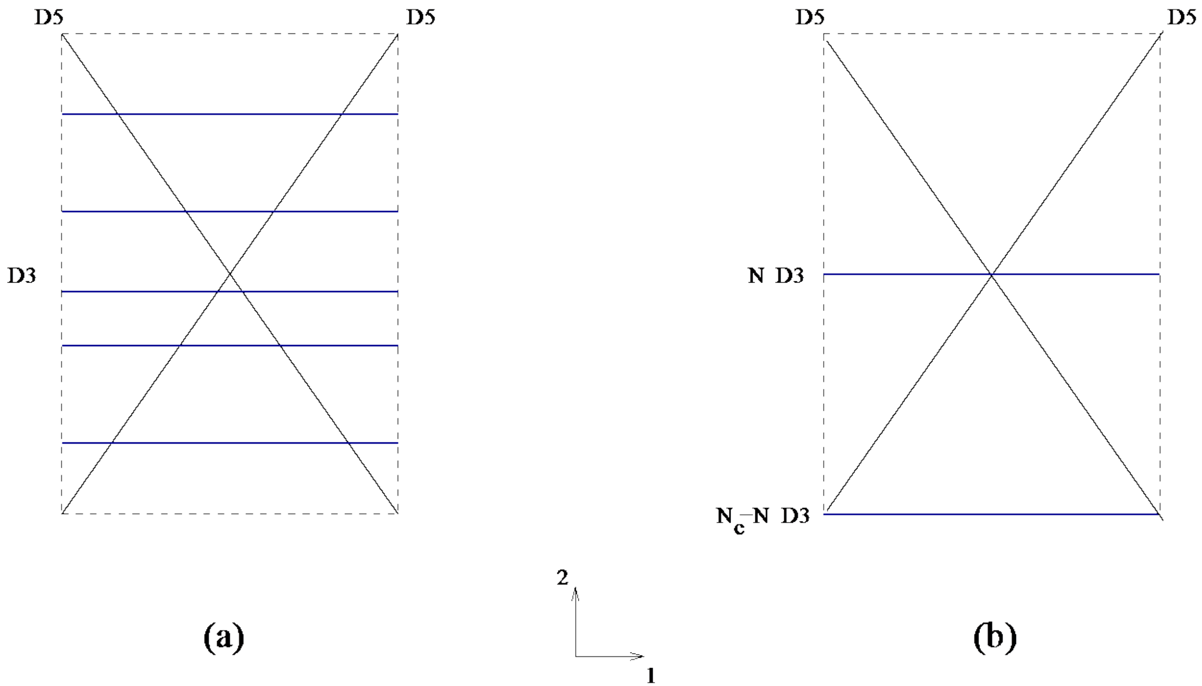
The quantum theory

The theory in which half of the U(1) charges are +1 and half -1 gives rise after compactification to two dimensions to the spectrum

field	$SU(N_f/2)_1$	$SU(N_f/2)_2$	$SU(N_f/2)_3$	$SU(N_f/2)_4$	$U(1)_e$
Q^1	$N_f/2$	1	1	1	$+1$
Λ^2	1	$N_f/2$	1	1	-1
$\widetilde{\Lambda}_1$	1	1	$\overline{N_f/2}$	1	-1
$\widetilde{Q}_2$	1	1	1	$\overline{N_f/2}$	$+1$

In addition there are the adjoint fields Υ, Σ.

The classical theory has a Coulomb branch, parametrized by the eigenvalues of Σ. From the brane perspective it corresponds to (a):



(a) (b)

Quantum mechanically, it is lifted and replaced by the discrete set of vacua (b).

Thus the theory splits at low energies into decoupled theories living at the two intersections. Stability of the vacuum requires that the number of flavors is larger than the number of colors at each intersection. This means that the integer N labeling the vacua runs over the range

$$\max(0, N_c - \frac{1}{2}N_f) \leq N \leq \min(N_c, \frac{1}{2}N_f)$$

In a given vacuum, the theory at a given intersection is a sigma model on the Higgs branch of the U(N) theory with $N_f/2$ flavors which has central charge

$$c_R = c_L = 3(N_f N - N^2)$$

In four dimensions, N=1 SQCD exhibits Seiberg duality. The magnetic theory has gauge group $U(N_f - N_c)$, magnetic quarks $q, \tilde{q}$ and singlet mesons M, coupled to the quarks via the superpotential $W = Mq\tilde{q}$. After reduction to two dimensions one finds the fields

field	$SU(N_f/2)_1$	$SU(N_f/2)_2$	$SU(N_f/2)_3$	$SU(N_f/2)_4$	$U(1)_e$
λ_1	$\overline{N_f/2}$	1	1	1	-1
q_2	1	$\overline{N_f/2}$	1	1	$+1$
$\tilde{q}^1$	1	1	$N_f/2$	1	$+1$
$\tilde{\lambda}^2$	1	1	1	$N_f/2$	-1
M_1^1, Λ_1^1	$N_f/2$	1	$\overline{N_f/2}$	1	0
M_2^2, Λ_2^2	1	$N_f/2$	1	$\overline{N_f/2}$	0
M_2^1	$N_f/2$	1	1	$\overline{N_f/2}$	$+2$
Λ_1^2	1	$N_f/2$	$\overline{N_f/2}$	1	-2

and superpotential $W = M_1^{\ 1}\lambda_1\tilde{q}^1 + M_2^{\ 2}q_2\tilde{\lambda}^2 + \Lambda_1^2 q_2\tilde{q}^1$.

One again finds that the classical Coulomb branch is lifted, replaced by a discrete set of theories labeled by an integer

$$\max(0, \frac{1}{2}N_f - N_c) \leq \hat{N} \leq \min(N_f - N_c, \frac{1}{2}N_f) \ .$$

Seiberg duality in four dimensions leads to a duality in two dimensions between the electric and magnetic models, with the map

$$\hat{N} = \frac{N_f}{2} - N \ .$$

If the gravity + DBI description provides a good holographic description for N=1 SQCD, it should be able to reproduce our results. Turning on the magnetic field for the bulk gauge field corresponding to the U(1) global symmetry we discussed should lead to a three dimensional AdS vacuum with the properties found in the field theory above. This is a challenge for the holographic description.