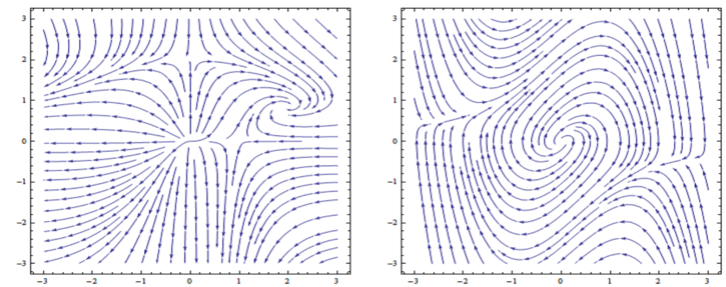
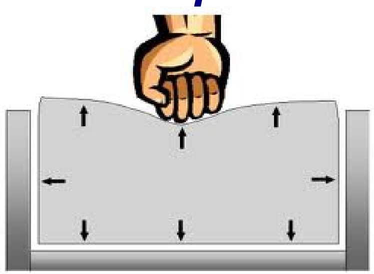
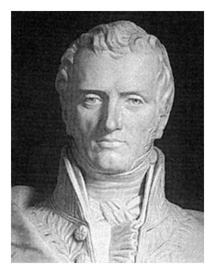
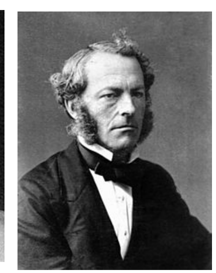
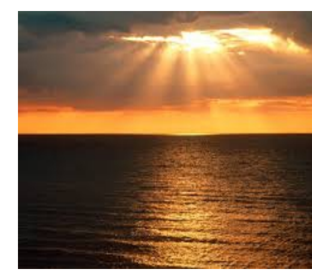
The University of Tokyo, Kavli IPMU

Holography and QCD





Spacetime emergence via holographic RG flow from incompressible Navier-Stokes at the horizon

based on

arXiv:1105.4530; 1307.1367 ; 14**.****

with Ayan Mukhopadhyay

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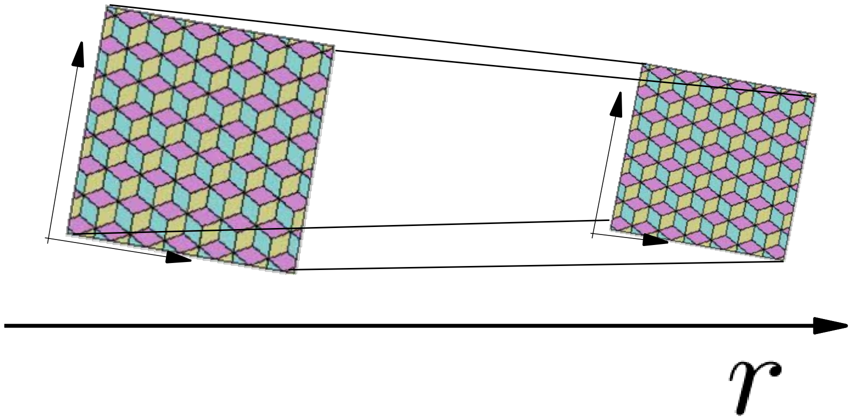
Holography - there is always the radial coordinate



Henningson, Skenderis:Balasubramanian, Kraus;
de Boer, Verlinde,, Verlinde

1. *Is it a renormalization scale?*

Heemskerk, Polchinski; Faulkner, Liu, Rangamani

2. *If yes, why the radial evolution is not a first order ODE?*
3. *Can we build space-time (metric) from a holographic RG flow?*

Too hard!

4. *The fluid/gravity correspondence (boosted black branes in AdS)*

Policastro,Son, Starinetz; Bhattacharyya, Hubeny, Minwalla, Rangamani; Baier, Romatschke, Son, Starinets, Stephanov

Non-relativistic incompressible Navier-Stokes $\qquad r \qquad$ *Conformal fluid*

Horizon —————————————— *Boundary*

Damour; Eling, Fouxon, Oz; Bredberg, Keeler, Lysov, Strominger

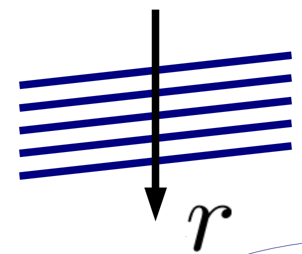
Hydrodynamic RG flow?!

1. Motivation

2. A brief introduction to fluid mechanics

3. The Ansatz - What precisely will flow?

4. The hypersurface foliation



5. Einstein´s equations of motion $R_{\mu\nu} = \ldots$

6. The renormalized energy-momentum tensor $T^{\mathrm{renorm}}_{\mu\nu}$

7. How to eliminate (and reconstract) the metric?

8. RG flow of the thermodynamic data

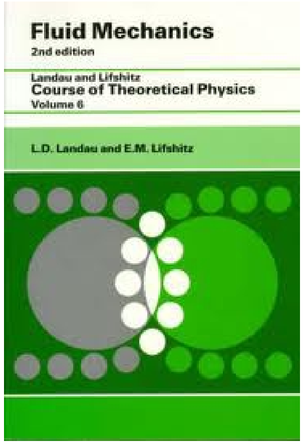
9. Horizon fluid

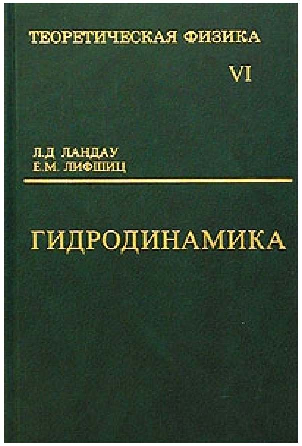
10. Results

A very brief introduction to fluid mechanics I

- *Basic data:* $g_{\mu\nu}(x)\,,\ u_\mu(x)\,,\ T(x)$

weakly curved $\quad u^\mu g_{\mu\nu} u^\nu = -1 \quad \nabla \gg T^{-1}$





- *The energy-momentum tensor:*

$$t_{\mu\nu}(g_\nu, R^\alpha_{\beta\gamma\delta}, \ldots, u_\mu, \nabla_\mu u_\nu, \ldots, T, \nabla_\mu T, \ldots)$$

- *Conservation of the EM tensor is the EoM:* $\nabla^\mu t_{\mu\nu} = 0$

- *Thermodynamics:*

$$\frac{\mathrm{d}\epsilon}{\mathrm{d}s} = T\,, \quad \epsilon + P = Ts\,; \qquad c_s^2 = \frac{\mathrm{d}P}{\mathrm{d}\epsilon} = \frac{\mathrm{d}\ln T}{\mathrm{d}\ln s}$$

The speed of sound:

A very brief introduction to fluid mechanics II

$$\Delta_{\mu\nu} = u_\mu u_\nu + g_{\mu\nu}$$

- *Equilibrium:* $t^{\rm eq}_{\mu\nu} = \epsilon u_\mu u_\nu + p\Delta_{\mu\nu}$

$$u^\mu \nabla_\mu \qquad \Delta_\mu{}^\nu \nabla_\nu$$

- *Euler equations:* $\nabla^\mu t^{\rm eq}_{\mu\nu} = 0 \quad \Rightarrow \quad \begin{cases} Du^\mu = -\nabla_\perp^\mu \ln T & \textbf{- 2nd law} \\ D\ln T = -c_s^2 \nabla\cdot u & \textbf{- continuity} \end{cases}$

- *Non-equilibrium EM tensor* $t^{\rm non\text{-}eq}_{\mu\nu}$

number of derivatives

$$\underbrace{\mathcal{S}_i^{(n)} u_\mu u_\nu \qquad \mathcal{S}_i^{(n)} \Delta_{\mu\nu}}_{} \qquad u_\mu \mathcal{V}^{(n)}_{i\nu} + u_\nu \mathcal{V}^{(n)}_{i\mu} \qquad \mathcal{T}^{(n)}_{\mu\nu}$$

scalars *orthogonal vectors* *traceless orthogonal tensors*

- *What can we built on-shell at n=1?*

symmetric and traceless

$$\nabla\cdot u, \quad \nabla_{\perp\mu}\ln T, \quad \sigma_{\mu\nu} = \langle \nabla_{\perp\mu} u_\nu \rangle$$

$$(\text{no} \quad Du^\mu\,!)$$

A very brief introduction to fluid mechanics III

- *Scalars, Vectors and Tensors for n=2:*

$$\mathcal{S}_1^{(2)} = R\,, \quad \mathcal{S}_2^{(2)} = u_\mu R^\mu{}_\nu u^\nu\,, \quad \mathcal{S}_3^{(2)} = (\nabla\cdot u)^2\,,$$

$$\mathcal{S}_4^{(2)} = \nabla_\perp{}^\mu \nabla_{\perp\mu} \ln T\,, \quad \mathcal{S}_5^{(2)} = \nabla_\perp{}^\mu \ln T \nabla_{\perp\mu} \ln T\,, \quad \mathcal{S}_6^{(2)} = \sigma^\mu{}_\nu \sigma^\nu{}_\mu\,, \quad \mathcal{S}_7^{(2)} = \omega^\mu{}_\nu \omega^\nu{}_\mu$$

$$\nabla_{\perp\alpha}\sigma^{\alpha\mu} - u^\mu\sigma^2, \quad \nabla_{\perp\alpha}\omega^{\alpha\mu} - u^\mu\omega^2, \quad \sigma^{\mu\nu}\nabla_{\perp\nu}\ln T, \omega^{\mu\nu}\nabla_{\perp\nu}\ln T, \quad (\nabla\cdot u)\nabla_\perp^\mu \ln T, \quad \Delta^{\mu\alpha}u^\beta R_{\alpha\beta}$$

$$\mathcal{T}_1{}^\mu{}_\nu = \langle R^\mu{}_\nu\rangle\,, \quad \mathcal{T}_2{}^\mu{}_\nu = \left\langle u^\alpha R_\alpha{}^\mu{}_\nu{}^\beta u_\beta\right\rangle\,, \quad \mathcal{T}_3{}^\mu{}_\nu = (\nabla\cdot u)\sigma^\mu{}_\nu, \quad \mathcal{T}_4{}^\mu{}_\nu = \langle \nabla_\perp^\mu \nabla_{\perp\nu} \ln T\rangle\,,$$

$$\mathcal{T}_5{}^\mu{}_\nu = \langle \nabla_\perp{}^\mu \ln T \nabla_{\perp\nu} \ln T\rangle\,, \quad \mathcal{T}_6{}^\mu{}_\nu = \langle \sigma^\mu{}_\tau \sigma^\tau{}_\nu\rangle\,, \quad \mathcal{T}_7{}^\mu{}_\nu = \langle \omega^\mu{}_\tau \omega^\tau{}_\nu\rangle\,, \quad \mathcal{T}_8{}^\mu{}_\nu = \langle \sigma^\mu{}_\tau \omega^\tau{}_\nu\rangle$$

$$\tfrac{1}{2}\left(\nabla_{\perp\mu}u_\nu - \nabla_{\perp\nu}u_\mu\right)$$

7 scalars, 6 vectors and 8 tensors

A very brief introduction to fluid mechanics IV

- Landau-Lifshitz frame:

local velocity of energy transport

$$t^{\mu}{}_{\nu} u^{\nu} = -\epsilon(T) u^{\mu} \quad \text{or} \quad t^{\text{non-eq}}_{\mu\nu} u^{\mu} = 0$$

- The full hydrodynamic EM tensor has **no vector terms**:

$$m_t^{(2)} = 8 \qquad m_s^{(2)} = 7$$

$$t_{\mu\nu} = \epsilon u_\mu u_\nu + p\Delta_{\mu\nu} + \sum_{n=1}^{\infty} \left(\sum_{i=1}^{m_t^{(n)}} \gamma_i^{(n)} \mathcal{T}_{i\mu\nu}^{(n)} + \left(\sum_{i=1}^{m_s^{(n)}} \delta_i^{(n)} \mathcal{S}_i^{(n)} \right) \cdot \Delta_{\mu\nu} \right)$$

$$-\eta\sigma_{\mu\nu} + \ldots \qquad -\zeta(\nabla \cdot u) + \ldots$$

shear

bulk viscosity

A very brief introduction to fluid mechanics V

- *Conformal fluid (Weyl covariance):*

    1. *No scalars* $(\zeta = 0)$

    2. *Only 5 tensors at n=2*

$$\mathcal{T}_2{}^{\mu}{}_{\nu} - \frac{1}{d-2}\mathcal{T}_1{}^{\mu}{}_{\nu}\,, \quad \mathcal{T}_2{}^{\mu}{}_{\nu} - \frac{1}{d-1}\mathcal{T}_3{}^{\mu}{}_{\nu} - \mathcal{T}_4{}^{\mu}{}_{\nu} + \mathcal{T}_5{}^{\mu}{}_{\nu}\,, \quad \mathcal{T}_6{}^{\mu}{}_{\nu}\,, \quad \mathcal{T}_7{}^{\mu}{}_{\nu}\,, \quad \mathcal{T}_8{}^{\mu}{}_{\nu}$$

- *Conformal fluid dual to Einstein gravity:*

$$\begin{aligned} t^{\mu}{}_{\nu} \;&=\; \epsilon_{\rm b} u^{\mu} u_{\nu} + P_{\rm b} \Delta^{\mu}{}_{\nu} - 2\eta_{\rm b}\sigma^{\mu}{}_{\nu} - 2\eta_{\rm b} b \cdot \left[\mathcal{T}_2{}^{\mu}{}_{\nu} - \frac{1}{d-2}\mathcal{T}_1{}^{\mu}{}_{\nu}\right] + \\ &+\; 2\eta_{\rm b}\left(b - \tau_{\omega}\right)\left[\mathcal{T}_2{}^{\mu}{}_{\nu} - \frac{1}{d-1}\mathcal{T}_3{}^{\mu}{}_{\nu} - \mathcal{T}_4{}^{\mu}{}_{\nu} + \mathcal{T}_5{}^{\mu}{}_{\nu}\right] + \\ &+\; 2\eta_{\rm b}\tau_{\omega}\mathcal{T}_6{}^{\mu}{}_{\nu} + 2\eta_{\rm b}\left(\tau_{\omega} - b\right)\eta_{\rm b}\tau_{\omega}\mathcal{T}_7{}^{\mu}{}_{\nu} + 4\eta_{\rm b}\tau_{\omega}\mathcal{T}_8{}^{\mu}{}_{\nu}\,. \end{aligned}$$

Bhattacharyya, Hubeny,Minwalla, Rangamani; Baier, Romatschke, Son, Starinets, Stephanov

The Ansatz

Overall $3m_s^{(n)} + m_v^{(n)} + m_t^{(n)}$ parameteres to fix!

1. The hydrodinamical variables have to be redefined at every hypersurface:

$$g_{\mu\nu}(r,x)\,,\;\; u_\mu(r,x)\,,\;\; T(r,x)$$

2. There is no need to find an ***explicit solution.***

3. Instead:

$$u'^{\mu} = \alpha_0 u^\mu + \sum_{n=1}^{\infty}\left(\sum_{i=1}^{m_s^{(n)}} \alpha_i^{(n)} \mathcal{S}_i^{(n)} u^\mu + \sum_{i=1}^{m_v^{(n)}} \beta_i^{(n)} \mathcal{V}^{(n)\,\mu}_{\;\;i}\right)$$

$$\frac{T'}{T} = \lambda_0 + \sum_{n=1}^{\infty}\sum_{i=1}^{m_s^{(n)}} \lambda_i^{(n)} \mathcal{S}_i^{(n)}$$

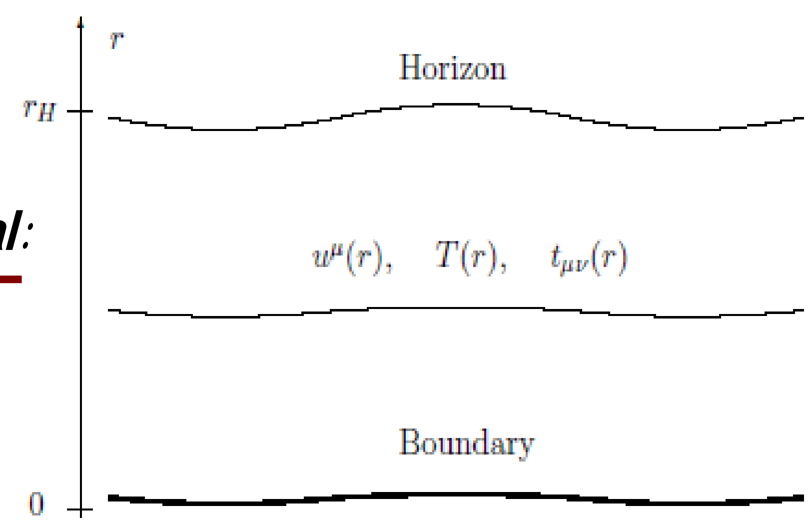
It works!
No metric
needed

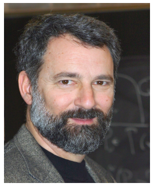
4. All the parameters here are ***auxiliery.***

5. The transport coefficients in
the hydrodynamic EM tensor are ***physical***:

$$\left\{\begin{aligned} \left(\gamma_i^{(n)}\right)' &= \ldots \\ \left(\delta_i^{(n)}\right)' &= \ldots \end{aligned}\right.$$

Beta functions?

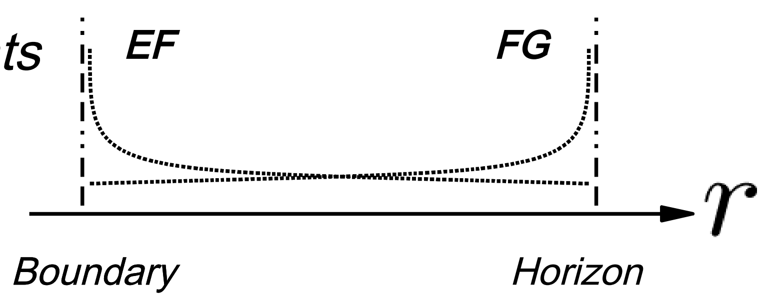




The hypersurface foliation

Fefferman-Graham	Eddington-Finkelstein
$G_{rr} = \left(\frac{l}{r}\right)^2 , \qquad G_{r\mu} = 0$	$G_{rr} = 0 , \quad G_{r\mu} = -u_\mu^{\mathrm{bdy}}(x)$
possible *to eliminate the metric*	***impossible*** *to eliminate the metric*
The radial coordinate corresponds to the RG scale	—
Regular at the boundary, singular at the horizon	*Regular at the horizon, singular at the boundary*

1. *We are interested only in the transport coefficients*
2. *The conformality is more difficult to guarantee*



Einstein´s equations of motion

$$R_{MN} = -\frac{d}{l^2}\, G_{MN}$$

$$ds^2 = \overbrace{\left(\frac{l}{r}\right)^2 \left(dr^2 + g_{\mu\nu}(x,r)dx^\mu dx^\nu\right)}$$

$$z^\mu_\nu = g^{\mu\tau} g'_{\tau\nu}$$

$$z^{\mu\prime}{}_{\nu} - \frac{d-1}{r} z^{\mu}{}_{\nu} + \mathrm{Tr}\, z \left(\frac{z^{\mu}{}_{\nu}}{2} - \frac{\delta^{\mu}{}_{\nu}}{r}\right) = 2R^{\mu}{}_{\nu} \qquad \textit{dynamical}$$

$$\nabla^\mu \mathrm{Tr}\, z - \nabla^\nu z^{\mu}{}_{\nu} = 0 \qquad \textit{constrain} \qquad \nabla^\mu t^{\mathrm{bdy}}_{\mu\nu} = 0$$

$$\mathrm{Tr}\, z' - \frac{1}{r}\mathrm{Tr}\, z + \frac{1}{2}\mathrm{Tr}\left(z^2\right) = 0 \qquad \textit{constrain} \qquad \mathrm{Tr} t^{\mathrm{bdy}} = 0$$

First order Einstein´s equation!

Not yet!

The renormalized energy-momentum tensor

- Dirichlet boundary conditions:

fixed at the boundary

Einstein tensor

$$t^{\mu}{}_{\nu} = \frac{l_{\mathrm{AdS}}^{d-1}}{(16\pi G_N) r^{d-2}} \left(\underbrace{\frac{z^{\mu}{}_{\nu}}{r} - \frac{\mathrm{Tr}\, z}{r}\delta^{\mu}{}_{\nu}}_{\text{Brown-York } K_{\mu\nu} - K\gamma_{\mu\nu}} + \underbrace{\frac{2}{d-2}\left(R^{\mu}{}_{\nu} - \tfrac{1}{2} R\delta^{\mu}{}_{\nu}\right) + \ldots}_{\text{counter-terms}} \right)$$

Q: Can we really use Dirichlet BC?

A1: Yes, because we don't specify the metric …

A2: $\boxed{\text{We can avoid using Dirichlet BC!}}$

See later ...

It is only sufficient to assume that:

1. Universality, the EM tensor is conserved for any solution
2. The <u>bare</u> EM tensor staisfies the junction condition (BY is the only option)
3. The dependence on l_{AdS} comes only via an overall dimensionless factor

How to eliminate the metric?

1. We can express ${z^\mu}_\nu$ in terms of ${t^\mu}_\nu$

$$ {z^\mu}_\nu = r^{d-1}\left({t^\mu}_\nu - \frac{\mathrm{Tr}\, t}{d-1}{\delta^\mu}_\nu\right) - \frac{2r}{d-2}\left({R^\mu}_\nu - \frac{R}{2(d-1)}{\delta^\mu}_\nu\right) + ... $$

2. ${z^\mu}_\nu$ can be eliminated from Einstein′s equation

$$ {t^\mu}_\nu{}' - \frac{2r^{2-d}}{d-2}{R^\mu}_\nu{}' + \frac{r^{d-1}}{2(d-1)}\left(\mathrm{Tr}t + r^{2-d}R\right)\left({t^\mu}_\nu - \frac{2r^{2-d}}{d-2}{R^\mu}_\nu\right) + $$
$$ + \frac{1}{d-1}\left(-\mathrm{Tr}t' + \frac{r^{2-d}}{d-2}R' + \frac{\mathrm{Tr}t}{r} + \frac{r^{d-1}}{2(d-1)}\left(\mathrm{Tr}t + r^{2-d}R\right)\left(\mathrm{Tr}t - \frac{r^{2-d}}{d-2}R\right)\right){\delta^\mu}_\nu + \ldots = 0\,. $$

3. What should we do with the r-derivatives of the Ricci tensor?

$$ \Gamma^{\mu\prime}_{\nu\rho} = \tfrac{1}{2}\left(\nabla_\nu {z^\mu}_\rho + \nabla_\rho {z^\mu}_\nu - \nabla^\mu z_{\nu\rho}\right) $$

$$ R'_{\mu\nu} = \tfrac{1}{2}\left(\nabla_\rho\nabla_\mu {z^\rho}_\nu + \nabla_\rho\nabla_\nu {z^\rho}_\mu - \nabla_\alpha\nabla^\alpha z_{\mu\nu} - \nabla_\mu\nabla_\nu \mathrm{Tr}\, z\right) $$

How to eliminate the metric (cont.)?

Just one more example:

$$u^{\mu\prime} = \alpha_0 u^\mu + \ldots$$

$$\begin{aligned}(\nabla\cdot u)' &= \nabla_\mu u^{\mu\prime} + \Gamma^{\mu}_{\nu\mu}{}' u^\nu = \alpha_0(\nabla\cdot u) + \frac{d\alpha_0}{d\ln s} D\ln s + \frac{1}{2} D\,\mathrm{Tr}\, z + \ldots \\ &= \left(\alpha_0 - \frac{d\alpha_0}{d\ln s} - \frac{1}{2}\left(c_s^2 - \frac{1}{d-1}\right)(\epsilon+p)\right)(\nabla\cdot u) + \ldots\end{aligned}$$

Euler equations

$$z^\mu{}_\nu = r^{d-1}\left(p + \tfrac{d-2}{d-1}\epsilon\right) u^\mu u_\nu + \tfrac{\epsilon}{d-1}\Delta^\mu{}_\nu + \ldots$$

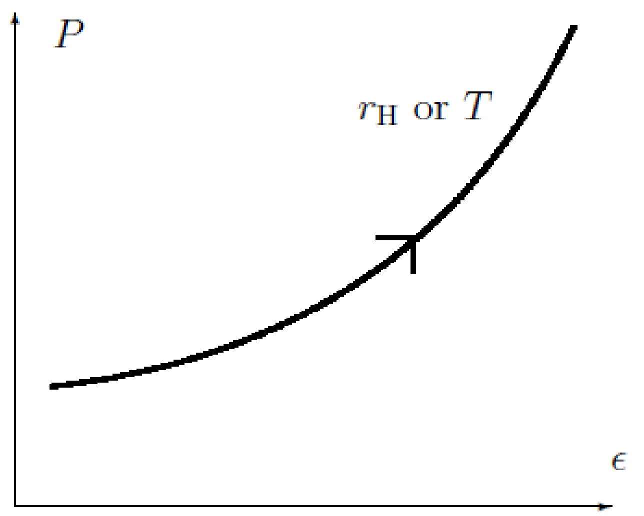
RG flow of the thermodynamic data

$$\begin{cases} \mathrm{Tr}\, t' = \left(\frac{r^{d-1}}{2(d-1)}\mathrm{Tr}\, t + \frac{d}{r}\right)\mathrm{Tr}\, t \\ \epsilon' = \left(\frac{r^{d-1}}{2(d-1)}\epsilon - \frac{1}{r}\right)\mathrm{Tr}\, t \end{cases}$$

Infinite pressure - (late time) horizon

$$\epsilon = \frac{4(d-1)}{r_{\mathrm{H}}^{d} + r^{d}} \qquad p = 4\,\frac{r_{\mathrm{H}}^{d} + (d-1)r^{d}}{r_{\mathrm{H}}^{2d} - r^{2d}}$$

Requiring finite energy at the horizon



Using the thermodynamical equations:

Can be fixed from the Hawking temperature

$$T = \frac{4d}{C} \cdot r_{\mathrm{H}} \frac{\left(r_{\mathrm{H}}^{d} + r^{d}\right)^{1-\frac{2}{d}}}{r_{\mathrm{H}}^{d} - r^{d}}$$

$$\epsilon = \epsilon(T, r) \qquad p = p(T, r)$$

No metric needed!

Reconstruction of the metric

1. *Choose the boundary conditions for the metric, u and T*

2. *Write a new Ansatz for the metric, u and T*

$$
\begin{aligned}
u^\mu(r,x) &= \tilde{A}_0\left(r, T^{(\mathrm{bdy})}(x)\right) u^{\mu(\mathrm{bdy})}(x) + \tilde{A}_1\left(r, T^{(\mathrm{bdy})}(x)\right) (\nabla\cdot u)^{(\mathrm{bdy})}(x) u^{\mu(\mathrm{bdy})}(x) \\
&\quad + \tilde{B}_1\left(r, T^{(\mathrm{bdy})}(x)\right) (\nabla_\perp^\mu \ln T)^{(\mathrm{bdy})}(x) + \ldots. , \\
T(r,x) &= \tilde{L}_0\left(r, T^{(\mathrm{bdy})}(x)\right) T^{(\mathrm{bdy})}(x) + \tilde{L}_1\left(r, T^{(\mathrm{bdy})}(x)\right) (\nabla\cdot u)^{(\mathrm{bdy})}(x) + \ldots
\end{aligned}
$$

3. *Solve the old Ansatz for u' and T'*

4. *And:* $z^\mu_\nu = g^{\mu\tau} g'_{\tau\nu}$

Do we have enough equations?

1a. Traceless orthogonal part of Einstein's equation:

$$\langle t^{\mu}{}_{\nu}{}' \rangle - \frac{2r^{2-d}}{d-2}\langle R^{\mu}{}_{\nu}{}' \rangle - \frac{r^{d-1}}{2(d-1)}\mathrm{Trt}\left(\langle t^{\mu}{}_{\nu} \rangle - \frac{2r^{2-d}}{d-2}\langle R^{\mu}{}_{\nu} \rangle\right) = 0. \quad \longrightarrow \quad \gamma_i^{(n)}$$

1b. Vector part $\longrightarrow$ $\beta_i^{(n)}$

1c. Two scalar components $\longrightarrow$ $\alpha_i^{(n)}$ *and* $\delta_i^{(n)}$

ODEs

2. Norm condition $\longrightarrow$ $\beta_i^{(n)}$

We get the same number of independent equations (algebraic and differential) as the number of variables

The horizon fluid I

- The temperature is infinite on the horizon. Themodynamics breaks down.

- The pressure and the speed of sound blow up as well.

$$D \ln T = -c_s^2 \nabla \cdot u \ ?$$

Near horizon rescaling!

- The fluid EOMs become regular.
- At the first order the horizon fluid is now an RG flow fixed point, provided the viscosities are finite.
- This fluid is incompressible Navier-Stokes.
- Higher order transport coefficients (TC) do not destroy this fixed point.
- Unique way to solve the first order (beta-)equations for the higher TCs.

The horizon fluid II

Eling,Fouxon, Oz; Bredberg, Keeler, Lysov, Strominger; Bhattacharyya, Minwalla, Wadia

$$r_{\rm H} - r = \xi \cdot \tilde{r}, \quad t = \frac{\tau}{\xi}\,, \qquad \boxed{\xi \to 0}$$

In the local inertial frame of an infalling observer:

We don't have metric!

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}\left(\xi^2\right), \quad u^\mu(r, x_i, t) = \Big(1, \xi v^i\left(r_{\rm H} + \xi\tilde{r}, x_i, \tau/\xi\right)\Big) + \mathcal{O}\left(\xi^2\right)$$

The thermodynamic parameters rescale as follows (different from Refs):

$$\left\{\begin{aligned} T(r, x_i, t) &= \frac{T^{\rm eq}(\tilde{r})}{\xi} + \widetilde{T}\left(r_{\rm H} + \xi\tilde{r}, x_i, \tau/\xi\right). \\ \epsilon(\tilde{r}, x_i, t) &= \epsilon_0 + \xi \cdot \tilde{\epsilon}(\tilde{r}, x_i, \tau/\xi). \\ P(r, x_i, t) &= \frac{\rho_0(\tilde{r})}{\xi} + \xi \cdot P^{\rm non\text{-}rel}(r_{\rm H} + \xi\tilde{r}, x_i, \tau/\xi). \end{aligned}\right.$$

mass density

The horizon fluid III

If the viscosities are finite:

Incompressible …

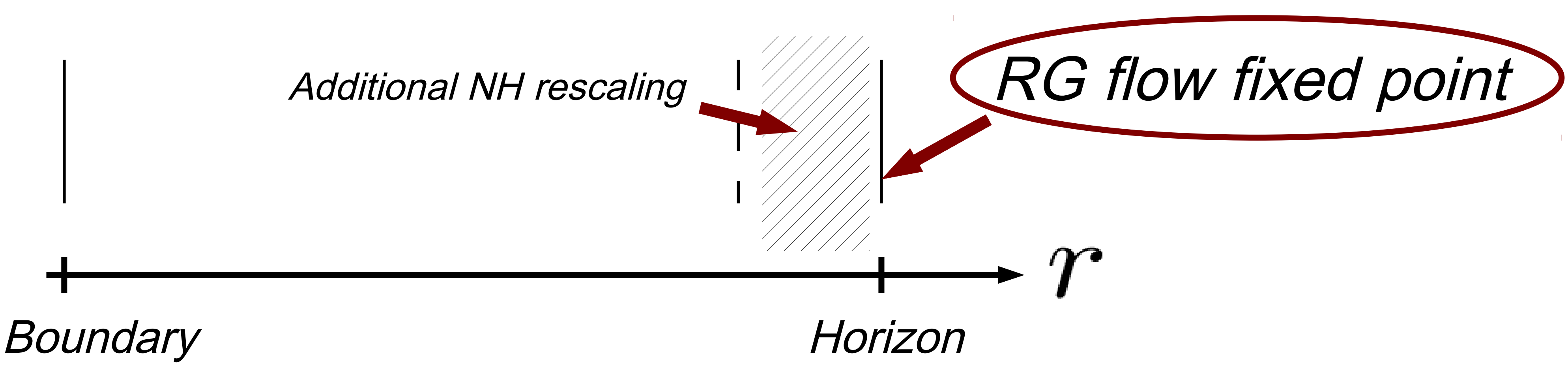
… Navier-Stokes

$$\rho_0\,(\partial_i v_i) + \mathcal{O}(\xi) = 0$$

$$\xi\left(\rho_0(\partial_\tau + v_j\partial_j)v_i + \partial_i P^{\text{n.-r.}} - \eta_{\text{H}}\partial_j(\partial_i v_j + \partial_j v_i) - \left(\zeta_{\text{H}} - \frac{\eta_{\text{H}}}{d-1}\right)\partial_i(\partial_j v_j)\right) + \mathcal{O}(\xi^2) = 0$$

The near horizon rescaling becomes a symmetry :

$$\tilde r \to \xi \tilde r\,, \quad \tau \to \tau/\xi\,, \quad v_i \to \xi v_i\,, \quad \rho_0 \to \rho_0/\xi\,, \quad P^{\text{n.-r.}} \to \xi P^{\text{n.-r.}}$$



The horizon fluid IV

*What near horizon behaviour of the **2nd order** transport coefficient preserves the incompressible Navier-Stokes RG flow fixed point?*

We know how these tensors scale!

Transport coefficient	Corresponding tensor	Near-horizon behaviour will be weaker than
γ_3	$(\nabla \cdot u)\, \sigma^{\mu}_{\nu}$	$(r_{\rm H} - r)^{-1}$
γ_4	$\langle \nabla_{\perp}{}^{\mu} \nabla_{\perp \nu} \ln s \rangle$	$(r_{\rm H} - r)^{-3}$
γ_5	$\langle \nabla_{\perp}{}^{\mu} \ln s \nabla_{\perp \nu} \ln s \rangle$	$(r_{\rm H} - r)^{-7}$
γ_6	$\langle \sigma^{\mu}{}_{\tau} \sigma^{\tau}{}_{\nu} \rangle$	$(r_{\rm H} - r)^{-1}$
γ_7	$\langle \omega^{\mu}{}_{\tau} \omega^{\tau}{}_{\nu} \rangle$	$(r_{\rm H} - r)^{-1}$
γ_8	$\langle \sigma^{\mu}{}_{\tau} \omega^{\tau}{}_{\nu} \rangle$	$(r_{\rm H} - r)^{-1}$

Transport coefficient	Corresponding tensor	Near-horizon behavior should be weaker than
γ_1	$\langle R^{\mu}{}_{\nu} \rangle$	$(r_{\rm H} - r)^{-1}$
γ_2	$\left\langle u^{\alpha} R_{\alpha}{}^{\mu}{}_{\nu}{}^{\beta} u_{\beta} \right\rangle$	$(r_{\rm H} - r)^{-1}$

See later on

The horizon fluid V

… and the scalar TCs:

Transport coefficient	Corresponding scalar	Allowed leading order near-horizon behaviour
δ_3	$(\nabla \cdot u)^2$	$(r_\mathrm{H} - r)^{-2}$
δ_4	$\nabla_\perp{}^\mu \nabla_{\perp\mu} \ln s$	$(r_\mathrm{H} - r)^{-3}$
δ_5	$\nabla_\perp{}^\mu \ln s \nabla_{\perp\mu} \ln s$	$(r_\mathrm{H} - r)^{-7}$
δ_6	$\sigma^\mu{}_\nu \sigma^\nu{}_\mu$	$(r_\mathrm{H} - r)^{-1}$
δ_7	$\omega^\mu{}_\nu \omega^\nu{}_\mu$	$(r_\mathrm{H} - r)^{-1}$

Transport coefficient	Corresponding scalar	Allowed leading order near-horizon behaviour
δ_1	R	$(r_\mathrm{H} - r)^{-2}$
δ_2	$R_{\mu\nu} u^\mu u^\nu$	$(r_\mathrm{H} - r)^{-2}$

Easy to extend to higher orders!

The results: 1st order transport coeff. (TC)

$-2\eta\sigma_{\mu\nu}$ Shear Viscosity

$\epsilon(r_{\rm H}, r)$

$$\eta' = -\frac{r^{d-1}}{2}\epsilon\cdot\eta \qquad \eta(r) = \frac{r_{\rm H}^{2(d-1)}}{\left(r_{\rm H}^d + r^d\right)^{2(1-1/d)}}\cdot\eta_{\rm b}$$

Cannot be fixed at the 1st order!

$\zeta\left(\nabla\cdot u\right)$ Bulk Viscosity

$$\zeta(r) = C_\zeta \cdot \frac{r^d\left(r_{\rm H}^d + r^d\right)^{\frac{2}{d}-1}}{\left(r_{\rm H}^d - r^d\right)^3} \sim \mathcal{O}\left(\left(r_{\rm H} - r\right)^{-3}\right)$$

$C_\zeta = 0$

Otherwise no incompressbile NS at the horizon!

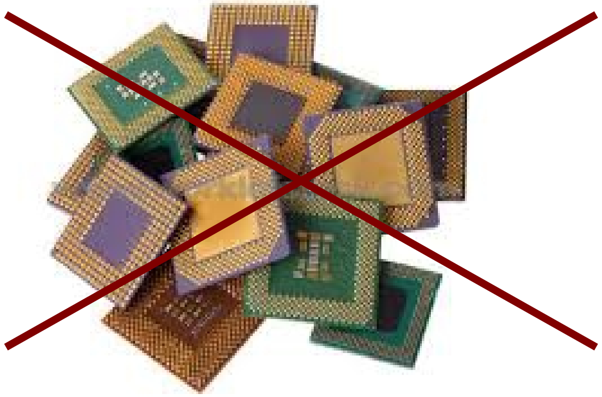
$\zeta(r) = 0$

The results: 2nd order (Tensors and Scalars)

*It can be **easily** shown that:*

$$t_{\mu\nu} = \ldots + \sum_{i=1}^{8} \gamma_i^{(2)} \mathcal{T}_{i\mu\nu}^{(2)} + \left(\sum_{i=1}^{7} \delta_i^{(2)} \mathcal{S}_i^{(2)} \right) \cdot \Delta_{\mu\nu}$$

Working hard ….



1st order ODE,
***one** integration constant*

$$
\boxed{\gamma_1' + \frac{r^{d-1}}{2}\left(-P + \frac{2}{d-1}\epsilon\right)\gamma_1 = \frac{r}{d-2}\left(-P + \frac{2}{d-1}\epsilon\right)}
$$

$$
\begin{aligned}
\gamma_2' + \frac{r^{d-1}}{2}\left(P + \frac{2d-3}{d-1}\epsilon\right)\gamma_2 &= -r^{d-1}\left(2\eta^2 + (P+\epsilon)\left(\gamma_1 - \frac{2r^{2-d}}{d-2}\right)\right)\\
\gamma_3' + \frac{d-2}{2(d-1)}r^{d-1}\epsilon\gamma_3 &= 2\frac{d+1}{d-1}r^{d-1}\eta^2 + \\
&\quad + \frac{r^{d-1}}{2}(P+\epsilon)\cdot\left(\left(c_s^2 + \frac{3}{d-1}\right)\left(\gamma_1 - \frac{2r^{2-d}}{d-2}\right) + \left(c_s^2 + \frac{d}{d-1}\right)\gamma_2\right)\\
\gamma_4' - r^{d-1}\left(P + \frac{d}{2(d-1)}\epsilon\right)\gamma_4 &= 2r^{d-1}c_s^2\eta^2 + \\
&\quad + \frac{r^{d-1}}{2}(P+\epsilon)\cdot\left(\left(c_s^2 - \frac{1}{d-1}\right)\left(\gamma_1 - \frac{2r^{2-d}}{d-2}\right) + \left(c_s^2 - \frac{d-2}{d-1}\right)\gamma_2\right)\\
\gamma_5' - \frac{r^{d-1}}{2}\left(3P + \frac{2d-1}{d-1}\epsilon\right)\gamma_5 &= 2r^{d-1}\left(\frac{\partial c_s^2}{\partial \ln s} - c_s^4\right)\eta^2 - \\
&\quad + \frac{r^{d-1}}{2}(P+\epsilon)\cdot\left(\gamma_1 - \frac{2r^{2-d}}{d-2} + \gamma_2\right)\left(-2c_s^4 + \frac{\partial c_s^2}{\partial \ln s} + (c_s^2+1)\left(c_s^2 - \frac{1}{d-1}\right)\right) + \\
&\quad + r^{d-1}(P+\epsilon)\cdot\left(-\frac{d-3}{2(d-1)}(c_s^2+1)\gamma_2 + \frac{1}{2}\left(c_s^2 + \frac{d+1}{d-1}\right)\gamma_4\right)\\
\gamma_6' + \frac{r^{d-1}}{2}\left(P + \frac{2d-3}{d-1}\epsilon\right)\gamma_6 &= r^{d-1}\left(2\eta^2 + (P+\epsilon)\left(\gamma_1 - \frac{2r^{2-d}}{d-2}\right)\right)\\
\gamma_7' - \frac{r^{d-1}}{2}\left(3P + \frac{2d-1}{d-1}\epsilon\right)\gamma_7 &= r^{d-1}\left(2\eta^2 - (P+\epsilon)\left(\gamma_1 - \frac{2r^{2-d}}{d-2} - 2\gamma_2\right)\right)\\
\gamma_8' - \frac{r^{d-1}}{2}\left(P + \frac{\epsilon}{d-1}\right)\gamma_8 &= 2r^{d-1}\left(2\eta^2 + (P+\epsilon)\left(\gamma_1 - \frac{2r^{2-d}}{d-2} + \gamma_2\right)\right)
\end{aligned}
$$

Sources!

2nd order Tensor TC *(continued)*

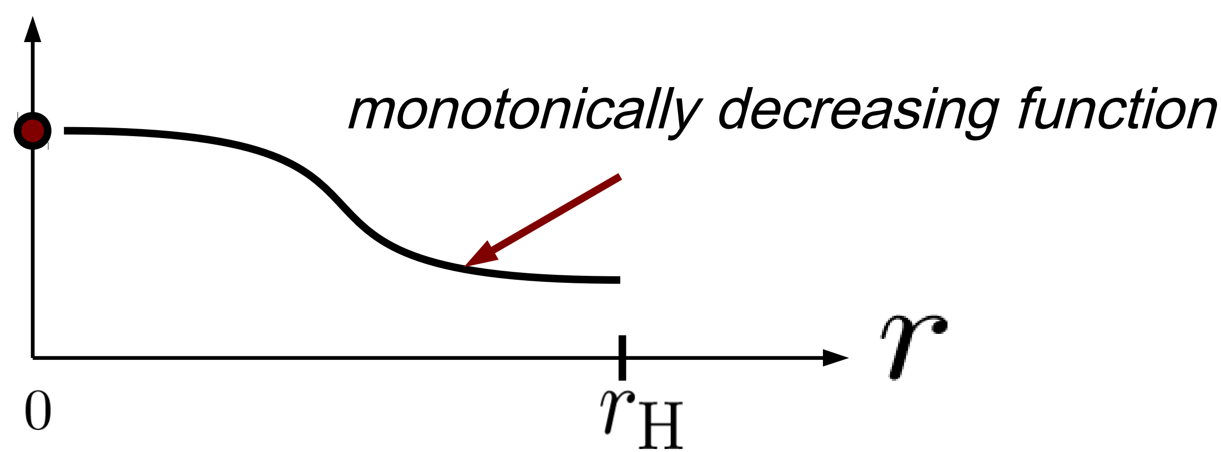
$$\underbrace{\gamma_1' + \frac{r^{d-1}}{2}\left(-P + \frac{2}{d-1}\epsilon\right)\gamma_1} = \frac{r}{d-2}\left(-P + \frac{2}{d-1}\epsilon\right)$$

The ~~«homogeneous»~~ solution behaves like $\mathcal{O}\left(\left(r_{\mathrm{H}} - r\right)^{-1}\right)$

The solution can be determined ***uniquely*** *if we require «regularity»:*

$$\gamma_1(r) = \frac{4}{(d-2)(r_{\mathrm{H}}^d - r^d)}\left(r_{\mathrm{H}}^{d-2}\left(\frac{r_{\mathrm{H}}^d + r^d}{2}\right)^{4/d-1} - r^2\right)$$

$$\gamma_1(0) = \frac{2}{r_{\mathrm{H}}^2}$$



Bhattacharyya, Hubeny, Minwalla, Rangamani; Baier, Romatschke, Son, Starinets, Stephanov

2nd order Tensor TC *(still continued)*

$$\underbrace{\gamma_2' + \frac{r^{d-1}}{2}\left(P + \frac{2d-3}{d-1}\epsilon\right)\gamma_2} \;=\; -r^{d-1}\left(2\overset{\eta_\mathrm{b}}{\eta}{}^2 + (P+\epsilon)\left(\gamma_1 - \frac{2r^{2-d}}{d-2}\right)\right)$$

The «homogeneous» solution is «regular» $\mathcal{O}\left(r_\mathrm{H} - r\right)$

Can we nevertheless fix the integration constant?

$$\eta_\mathrm{b}\,,\;\; C_2$$

2nd order Tensor TC *(just two more slides)*

$$\mathcal{O}\left((r_{\rm H} - r)^{-3}\right)$$

const

$$\gamma_3' + \frac{d-2}{2(d-1)} r^{d-1} \epsilon \gamma_3 = 2\frac{d+1}{d-1} r^{d-1} \eta^2 +$$

$$+ \frac{r^{d-1}}{2}(P+\epsilon) \cdot \left(\left(c_s^2 + \frac{3}{d-1} \right) \left(\gamma_1 - \frac{2r^{2-d}}{d-2} \right) + \left(c_s^2 + \frac{d}{d-1} \right) \gamma_2 \right)$$

$$\mathcal{O}\left((r_{\rm H} - r)^{-1}\right) \qquad \mathcal{O}\left((r_{\rm H} - r)^{-2}\right)$$

Unless …

$$\gamma_1 - \frac{2r^{2-d}}{d-2} + \gamma_2 \sim \mathcal{O}\left((r_{\rm H} - r)^3\right)$$

Very unlikely! Three orders to cancel only with $\eta_{\rm b}$, C_2

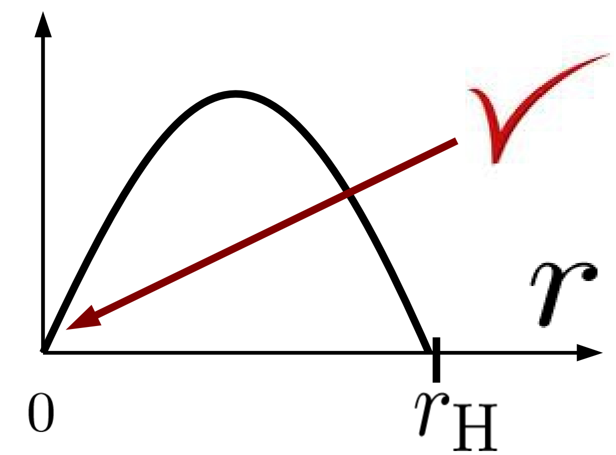
2nd order Tensor TC *(almost the last slide)*

«Magically» it works: $\gamma_1 - \dfrac{2r^{2-d}}{d-2} + \gamma_2 \sim \mathcal{O}\left((r_{\rm H} - r)^3\right)$

$$\gamma_2(r) = 2r_{\rm H}^2 \frac{r_{\rm H}^4 - r^4}{(r_{\rm H}^4 + r^4)^2}\left(\frac{2r_{\rm H}^2 r^2}{(r_{\rm H}^2 + r^2)^2} + \ln\left(\frac{(r_{\rm H}^2 + r^2)^2}{2(r_{\rm H}^4 + r^4)}\right)\right)$$

$$\eta_{\rm b} = \frac{2\sqrt{2}}{r_{\rm H}^3}$$

Fixed only at the 2nd order!



- *Similarly for* $\gamma_7(r)\,,\ \gamma_8(r)$
- *Higher orders needed for* $\gamma_3(r)\,,\ \gamma_4(r)\,,\ \gamma_5(r)\,,\ \gamma_6(r)$
- *Conformality at the boundary* $\gamma_3(r)\,,\ \gamma_4(r)\,,\ \gamma_5(r)$

(appear together in one ***conformal*** *tensor)*

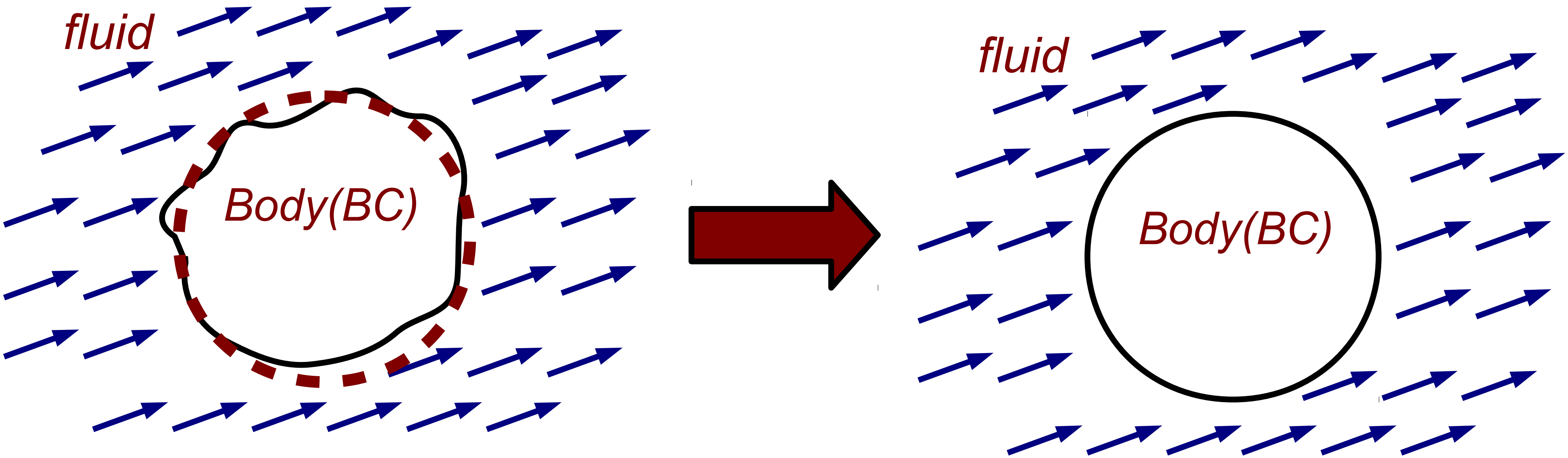
Transport coefficient	Corresponding tensor	Behaviour/Value near the horizon	Value at the boundary	
$\eta(r)$	$-2\sigma^{\mu}{}_{\nu}$	$(\pi T_{\mathrm{b}})^3 / 2\sqrt{2}$	$(\pi T_{\mathrm{b}})^3$	A+
$\gamma_1(r)$	$\langle R^{\mu}{}_{\nu} \rangle$	$\frac{1}{2}(\pi T_{\mathrm{b}})^2$	$(\pi T_{\mathrm{b}})^2$	A+
$\gamma_2(r)$	$\left\langle u^{\alpha} R_{\alpha}{}^{\mu}{}_{\nu}{}^{\beta} u_{\beta} \right\rangle$	$\mathcal{O}\left((r_{\mathrm{H}} - r)\right)$	$-\ln 2\,(\pi T_{\mathrm{b}})^2$	A+
$\gamma_3(r)$	$(\nabla \cdot u)\,\sigma^{\mu}_{\nu}$	$\mathcal{O}\left((r_{\mathrm{H}} - r)\right)$	$-\frac{1}{3}(2 - \ln 2)\,(\pi T_{\mathrm{b}})^2$	
$\gamma_4(r)$	$\langle \nabla_{\perp}{}^{\mu} \nabla_{\perp\nu} \ln s \rangle$	$\mathcal{O}\left((r_{\mathrm{H}} - r)^{-1}\right)$	$-\frac{1}{3}(2 - \ln 2)\,(\pi T_{\mathrm{b}})^2$	3d order?
$\gamma_5(r)$	$\langle \nabla_{\perp}{}^{\mu} \ln s \nabla_{\perp\nu} \ln s \rangle$	$\mathcal{O}\left((r_{\mathrm{H}} - r)^{-3}\right)$	$\frac{1}{9}(2 - \ln 2)\,(\pi T_{\mathrm{b}})^2$	
$\gamma_6(r)$	$\langle \sigma^{\mu}{}_{\tau} \sigma^{\tau}{}_{\nu} \rangle$	$\mathcal{O}\left((r_{\mathrm{H}} - r)\right)$	$(C_6 + \ln 2)\,(\pi T_{\mathrm{b}})^2$	
$\gamma_7(r)$	$\langle \omega^{\mu}{}_{\tau} \omega^{\tau}{}_{\nu} \rangle$	$\mathcal{O}\left((r_{\mathrm{H}} - r)\right)$	$-(2 - \ln 2)\,(\pi T_{\mathrm{b}})^2$	A+
$\gamma_8(r)$	$\langle \sigma^{\mu}{}_{\tau} \omega^{\tau}{}_{\nu} \rangle$	$\mathcal{O}\left((r_{\mathrm{H}} - r)\right)$	$2\ln 2\,(\pi T_{\mathrm{b}})^2$	A+

2nd order Scalar TC

The transport coefficient	The scalar	The homogeneous solution	The full solution behaviour	Boundary value	Solution fixed uniquely
$\delta_1(r)$	R	$r^d \frac{(r_{\rm H}^d + r^d)^{-1+4/d}}{(r_{\rm H}^d - r^d)^3}$	$\mathcal{O}((r_{\rm H} - r)^{-2})$	0	Yes
$\delta_2(r)$	$u_\mu R^\mu_{\ \nu} u^\nu$	$r^d \frac{(r_{\rm H}^d + r^d)^{-2+4/d}}{(r_{\rm H}^d - r^d)^2}$	$\mathcal{O}((r_{\rm H} - r)^{-1})$	0	Yes
$\delta_3(r)$	$(\nabla \cdot u)^2$	$r^d \frac{(r_{\rm H}^d + r^d)}{(r_{\rm H}^d - r^d)^3}$	0	0	Yes
$\delta_4(r)$	$\nabla_\perp{}^\mu \nabla_{\perp\mu} \ln s$	$r^d \frac{(r_{\rm H}^d + r^d)^{-2+8/d}}{(r_{\rm H}^d - r^d)^4}$	$\mathcal{O}((r_{\rm H} - r)^{-1})$	0	Yes
$\delta_5(r)$	$\nabla_\perp{}^\mu \ln s \nabla_{\perp\mu} \ln s$	$r^d \frac{(r_{\rm H}^d + r^d)^{1+4/d}}{(r_{\rm H}^d - r^d)^5}$	$\mathcal{O}((r_{\rm H} - r)^{-5})$	0	No
$\delta_6(r)$	$\sigma^\mu_{\ \nu} \sigma^\nu_{\ \mu}$	$r^d \frac{(r_{\rm H}^d + r^d)^{-3+4/d}}{(r_{\rm H}^d - r^d)}$	$\mathcal{O}((r_{\rm H} - r)^{-1})$	0	No
$\delta_7(r)$	$\omega^\mu_{\ \nu} \omega^\nu_{\ \mu}$	$r^d \frac{(r_{\rm H}^d + r^d)^3}{(r_{\rm H}^d - r^d)^3}$	$\mathcal{O}((r_{\rm H} - r)^{-1})$	0	Yes

So what kind of RG flow is it?

Averaging



$$u^{\mu}, \quad u^{\mu}\eta_{\mu\nu}u^{\nu} = -1 \qquad\qquad \langle u^{\mu}\rangle, \quad \langle u^{\mu}\rangle \eta_{\mu\nu} \langle u^{\nu}\rangle \neq -1$$

$$\boxed{g_{\mu\nu} \neq \eta_{\mu\nu}}$$

See arXiv: 13??.????

Future directions

1. *Higher orders*
2. *Can we determine **all** counterterms from the horizon?*
3. *Stationary black holes (and other setups)*
4. *Other bulk fields (vectors, fermions, …)*
5. *Non-relativistic fluid (on any hypersurface)*
6. *Turbulence*

感謝