The Trailing String in Confining Holographic Theories

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Holography and QCD
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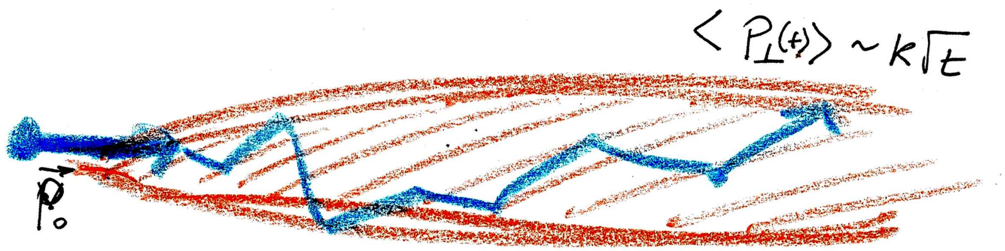
Work with E. Kiritsis, L. Mazzanti, to appear soon

Introduction

A heavy quark can be created out of equilibrium in the quark-gluon plasma produced by a heavy ion collision. It then undergoes a diffusion process governed by the interactions with the medium.



The brownaian-like dynamics causes viscous energy loss and a spread in transverse momentum

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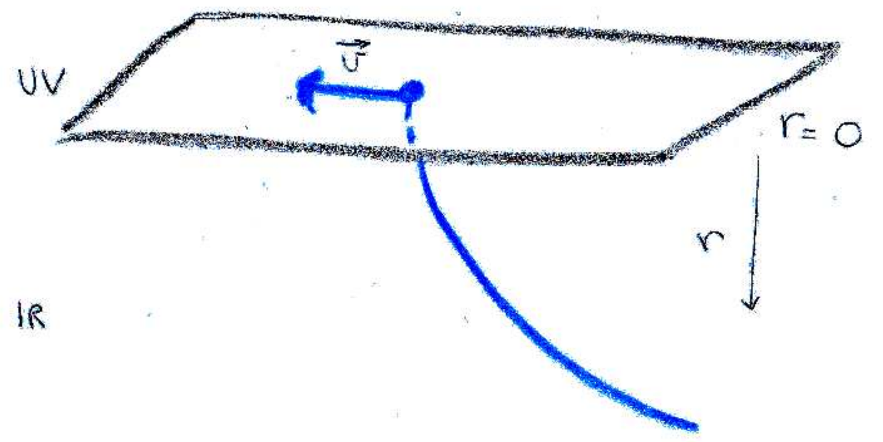

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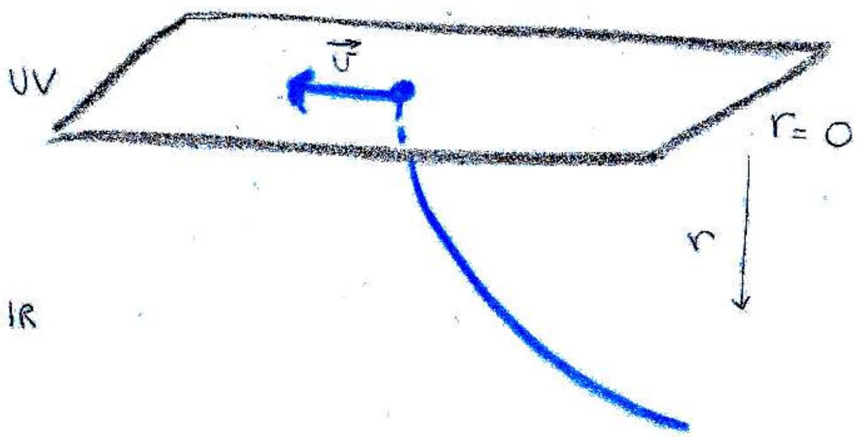

In the gauge/gravity duality context a heavy quark in the gauge-theory side correspond to a trailing fundamental string on the gravity side.

I will discuss the trailing string solution in $T = 0$ vacuum geometries,

in particular those dual to a confining theory.

Motivation

- An important point of contact with heavy ion experiments (jet quenching, heavy flavor suppression).
- From a practical point of view, in order to correctly obtain the dynamics of the probe in the deconfined medium, one needs a subtraction procedure to make basic quantities (Boundary retarded correlators) well defined. The natural way to operate this subtraction is through the vacuum correlator. Whence the need of the vacuum trailing string solution.

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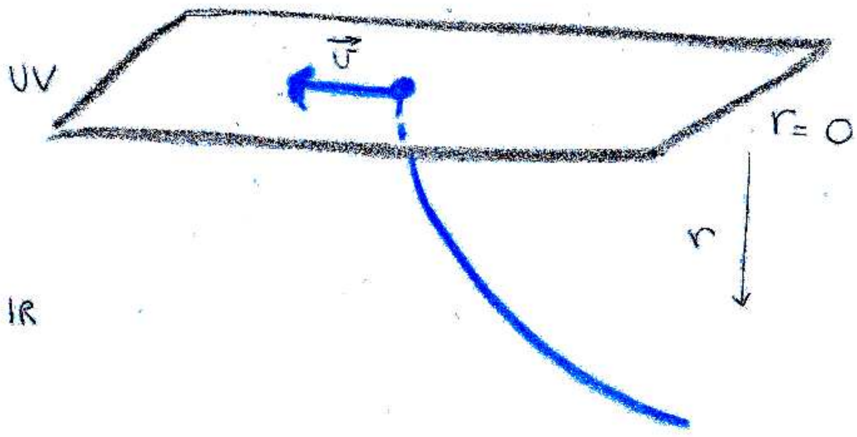
- The vacuum trailing string in confining theories, and its fluctuations, have an interesting and non-trivial structure
  - presence of a confining horizon
  - long distance dissipation effects

The Trailing String



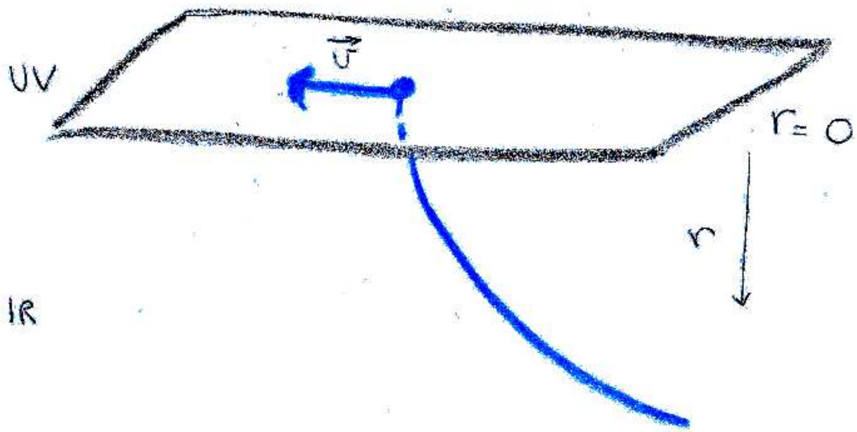
Probe quark on the boundary a 5D asymptotically AdS spacetime

$\Updownarrow$

Classical string attached at the boundary and extending in the interior.

(Gubser '06)

The Trailing String



The string profile is found by extremizing the surface spanned by the string

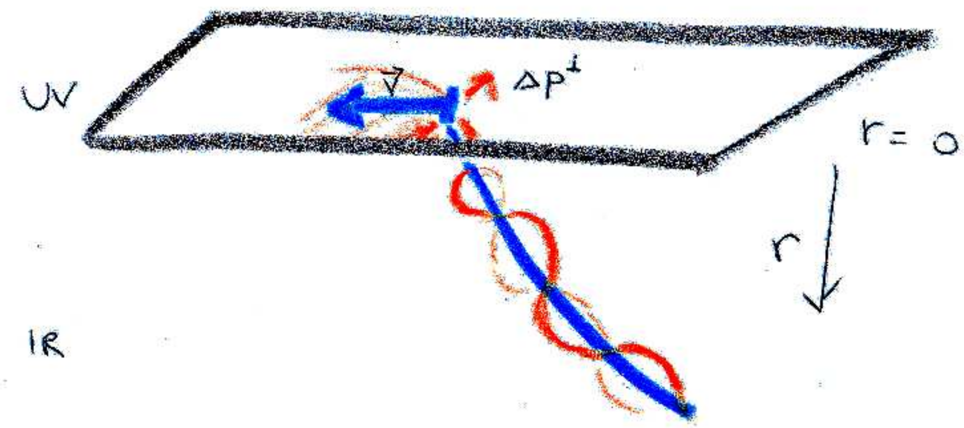
$$S = \frac{1}{2\pi\ell_s^2}\int dtdr\sqrt{-\det g_{ind}}\,,$$

with respect to the embedding coordinates: $\vec{X}(t,r) = \vec{v}t + \vec{\xi}(r)$.

The string exerts a drag force which causes the quark to lose energy:

dual description of in-medium energy loss

The Trailing String Fluctuations



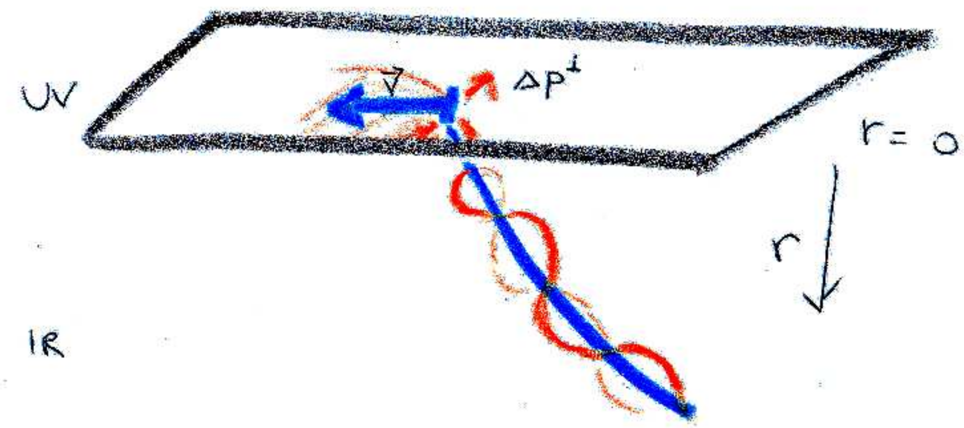
Add small fluctuations along the string:

$$X(t,r) = \xi(r) + \delta X(t,r)$$

they induce a Brownian-like dynamics for the boundary quark, governed by a Langevin equation, and leading to a spread in momentum. (Gubser '05, De Boer *et al* 06, Herzog *et al* 06, Son and Teaney 09) .

Dual description of transverse momentum broadening.

The Trailing String Fluctuations



$$M\delta\ddot{X}(t)+\int dt'\, G_R(t-t')\delta X(t') = \zeta(t), \qquad \langle\zeta(t)\zeta(t')\rangle = G_s(t-t')$$

Generalized Langevin equation.

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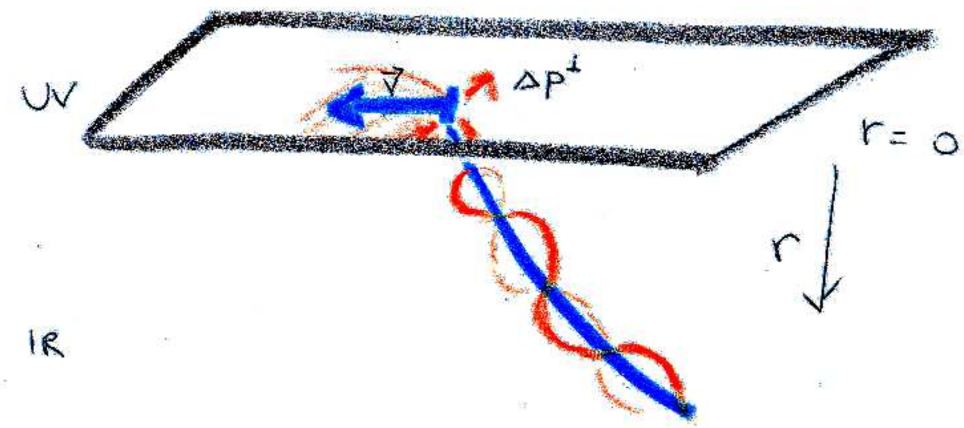

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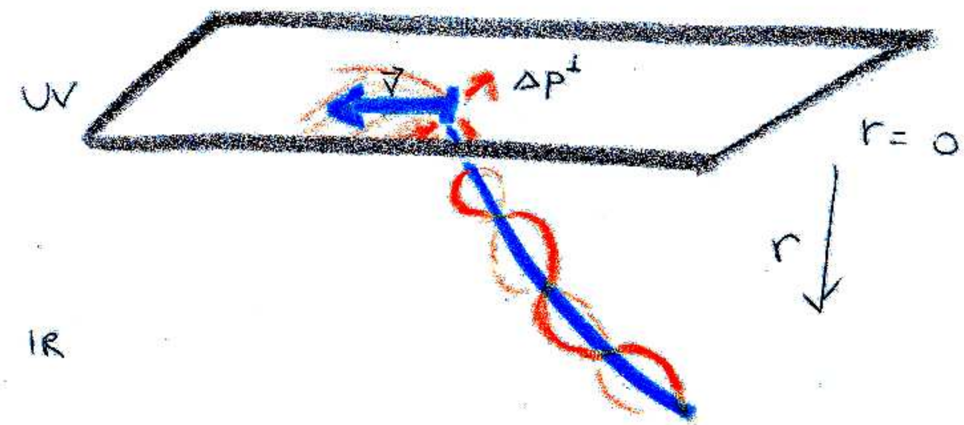
Generalized Langevin equation. Two force terms:

- a classical force with retardation effects;
- a stochastic force with a Gaussian distribution.

Both terms arise from the same underlying physics.

The Trailing String Fluctuations



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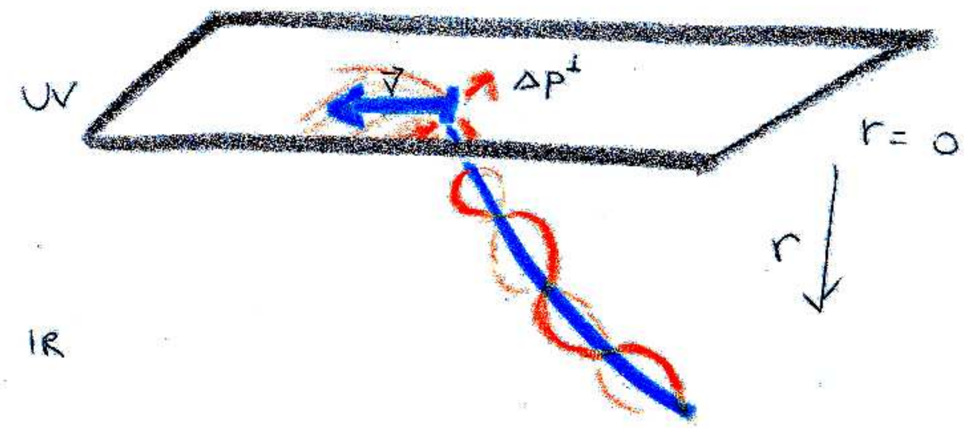
- $G_R(t)$ is the retarded boundary correlator associated to the fluctuations $\delta X(t,r)$ around the classical trailing string.
- $G_s(t)$ is the associated the symmetric correlator, obtained from $G_R(t)$ via a Fluctuation-Dissipation relation, characteristic of the ensemble.

The Trailing String Fluctuations



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long-time limit:

$$M\delta\ddot{X}(t) + \eta\dot{X}(t) = \zeta(t), \qquad \langle\zeta(t)\zeta(t')\rangle = k\delta(t-t')$$

$$\eta = \lim_{\omega\to 0}\frac{\mathrm{Im}\ \mathrm{G_R}(\omega)}{\omega}, \qquad \kappa = \lim_{\omega\to 0} G_s(\omega)$$

Trailing string in 5D black hole

Consider a generic asymptotically AdS 5D black hole:

$$ds^2 = b^2(r)\left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^i dx_i\right]$$

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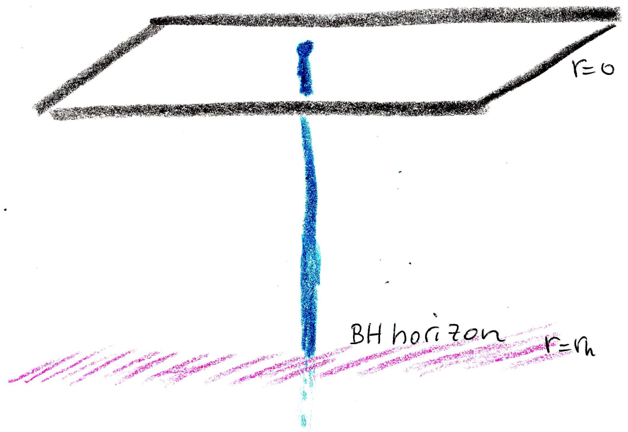
- Dual to a non-conformal gauge theory in thermal equilbrium at a temperature T_h, in a deconfined phase.

Trailing String in a 5D black hole

Let us focus on the static case.



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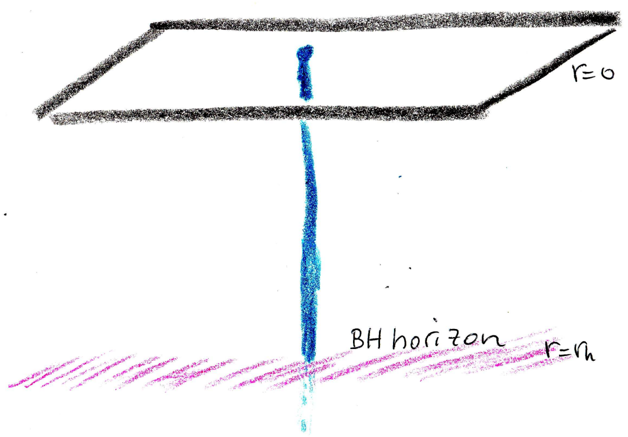

The string falls straight down into the horizon.
The induced 2D worldsheet metric is a 2D black hole with horizon r_h and temperature T_h.

$$ds^2_{ind} = b^2(r)\left[-f(r)dt^2 + f^{-1}(r)dr^2\right],$$

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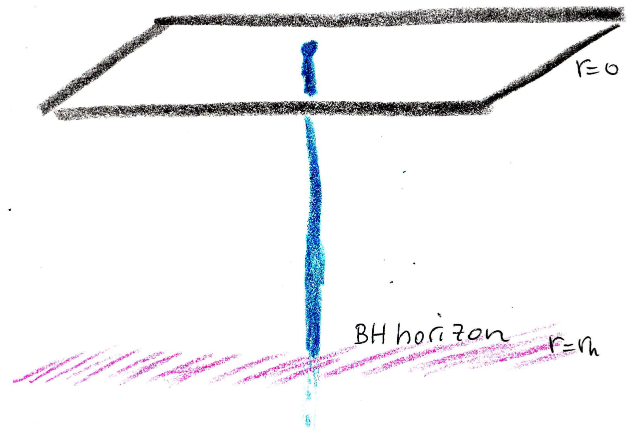

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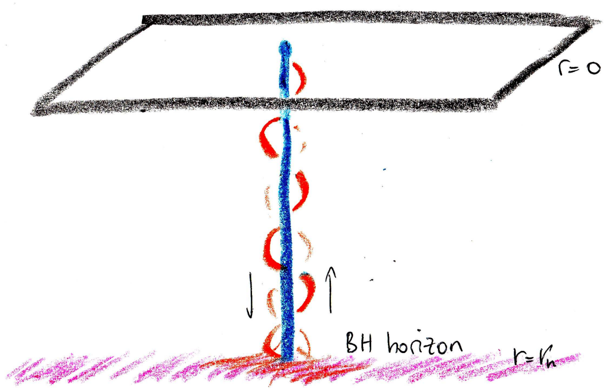
$$ds^2_{ind} = b^2(r)\left[-f(r)dt^2 + f^{-1}(r)dr^2\right], \quad f(r) \simeq 4\pi T_h(r_h - r),\ r \simeq r_h$$

Fluctuations

Add fluctuations: modes going in/out of the horizon.



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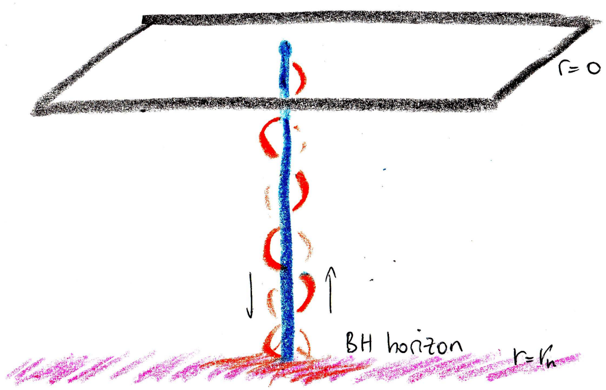

The fluctuation equation close to the horizon is

$$\delta X'' - \frac{1}{(r_h - r)}\delta X' + \frac{\hat{\omega}^2}{(r_h - r)^2}\delta X = 0, \qquad \hat{\omega} \equiv \frac{\omega}{4\pi T_h}$$

Correlators

The retarded correlator is found by the Policastro-Son-Starinets prescription

$$G_R(\omega) = \left[\mathcal{G}(r)\,\delta X_R'(\omega,r)\right]_{r\to 0}, \quad \delta X_R(\omega,r) \to \begin{cases} 1 & r \to 0 \\ (r-r_h)^{-i\hat{\omega}} & r \to r_h \end{cases}$$

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Gursoy, Mazzanti, Kiritsis, FN 1006.3261

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At long times, the dynamics is encoded by the Langevin coefficient

$$\kappa = \lim_{\omega\to 0} G_s(\omega) = 2T_h\eta$$

Green's functions: High frequency limit

The large ω limit obtained via WKB approximation

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This diverges too fast for G_R to be physical:

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  require $G(\omega) \sim 1/\omega$

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- Remark: the leading behavior is temperature-independent.

Dressed spectral density

UV-safe spectral densities can be defined: Subtract the correlator obtained from the vacuum background. Mazzanti, Kiritsis, FN, 1111.1008:

$$G_R^{(ph)}(\omega) = G_R(\omega) - G_R^{(vac)}(\omega)$$

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- This prescription can be obtained with a change of variable in the quark path integral.

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Two qualitatively very different cases:

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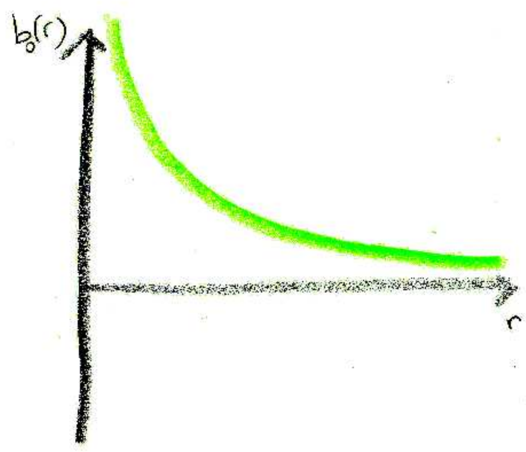
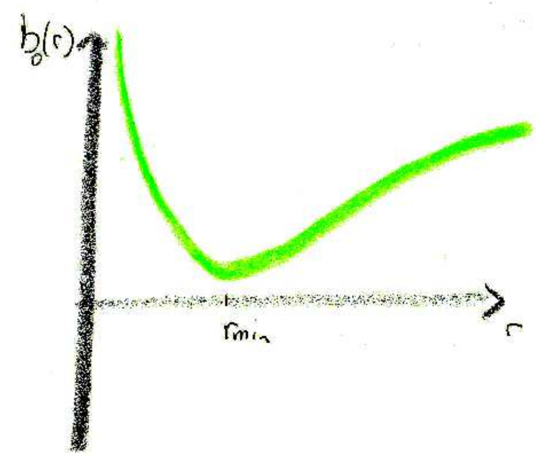
Confinement is essentially equivalent to the presence of a minimum of the bulk scale factor $b(r)$ (cfr. J .Sonnenschein's talk)



non-confining



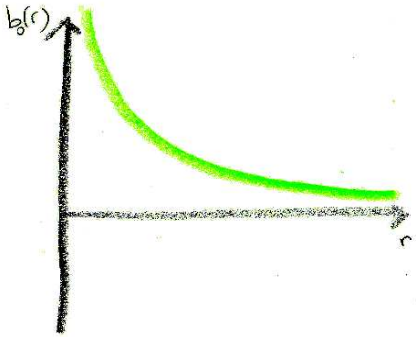
confining $\sigma_c = b^2(r_m)$

Non-confining case

$$ds^2 = b^2(r)\left[dr^2 + dx^\mu dx_\mu\right],$$

the string profile satisfies:

$$\xi'(r) = \frac{C}{\sqrt{b^4(r) - C^2}}$$



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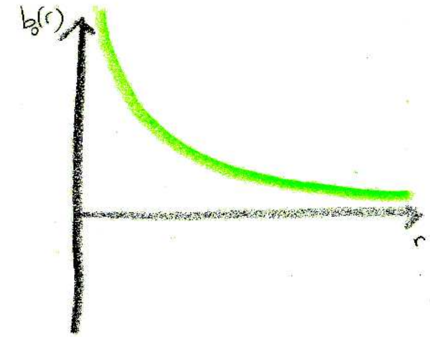
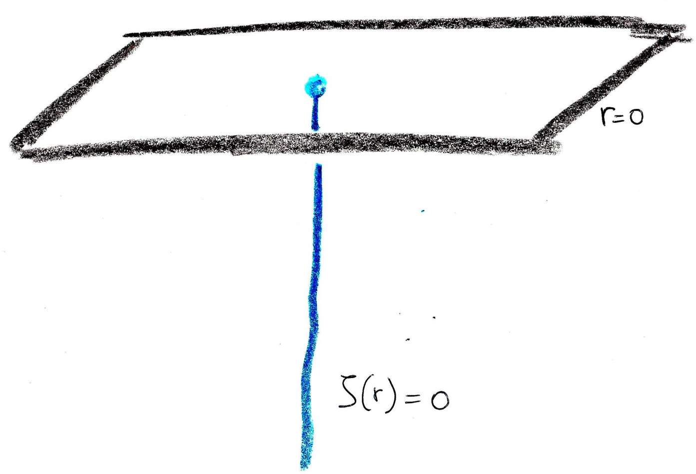


As $b \to 0$, regularity requires $C = 0$: the embedding is trivial, $\xi = 0$



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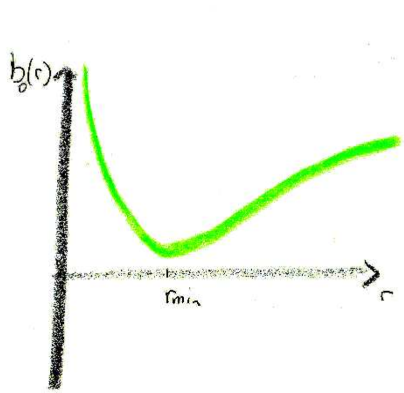

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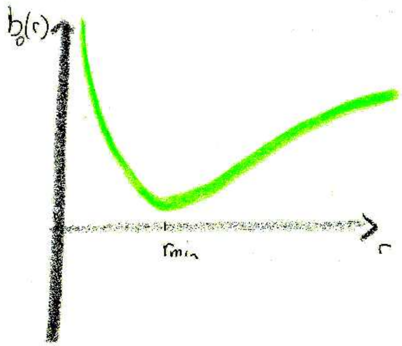

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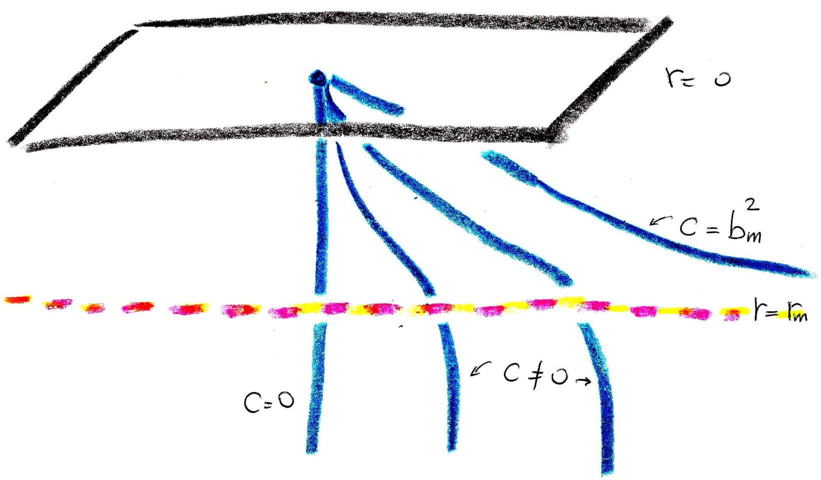
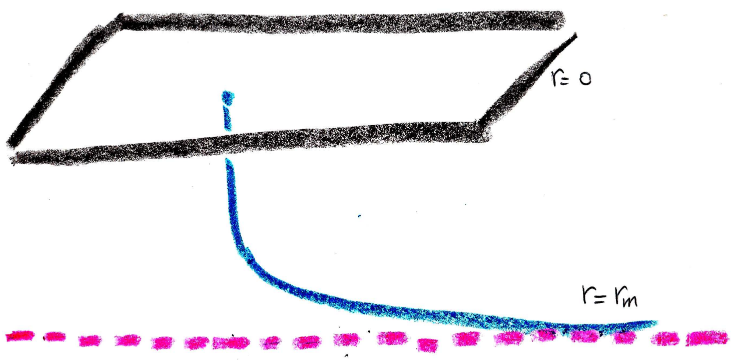
One-parameter family of solutions with $0 \le C \le b^2(r_m)$



Confining Trailing String

The extremal $C = b_m^2$ string is the one with lowest action. It does not extend beyond the *confining horizon* $r = r_m$, and it extend to infinity along one of the spatial directions.



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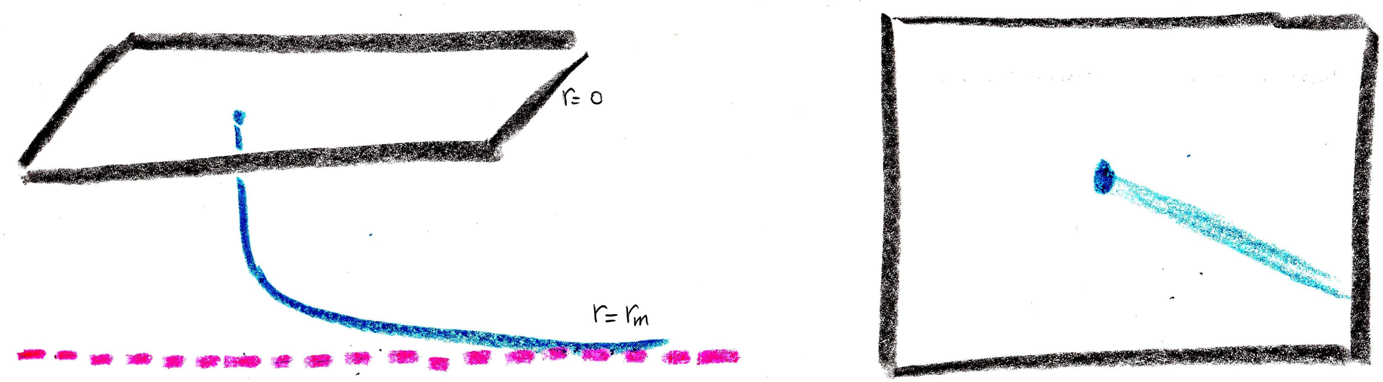

Asymptotically it looks like a straight string with fixed tension b_m^2 i.e. the confining string tension of the dual theory: it is the QCD flux-tube.

Confining string geometry

Worldsheet induced metric:

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- close to the confining horizon r_m it reduces to

$$ds^2 \sim b_m^2\left[-dt^2 + (4\pi T_m)^2\frac{dr^2}{(r_m - r)^2}\right] \quad T_m = (4\pi)^{-1}\sqrt{b_m''/b_m}$$

Confining horizon geometry

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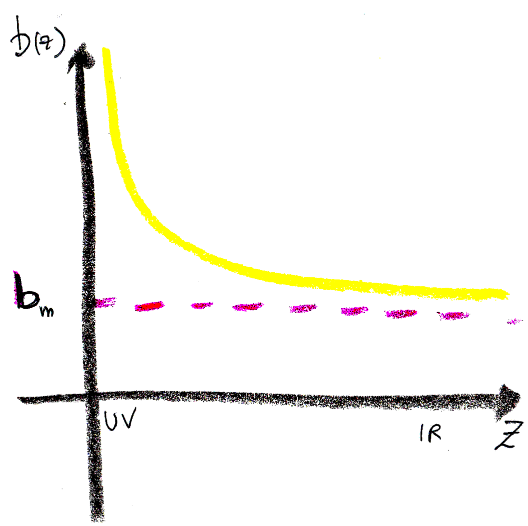
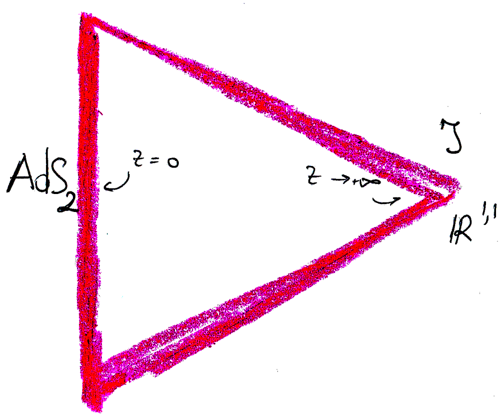
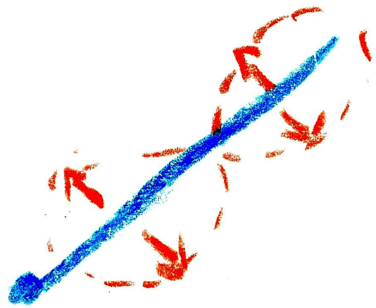




Transverse vs. longitudinal fluctuations

We can distinguish between fluctuations longitudinal and transverse to the boundary direction of the string:



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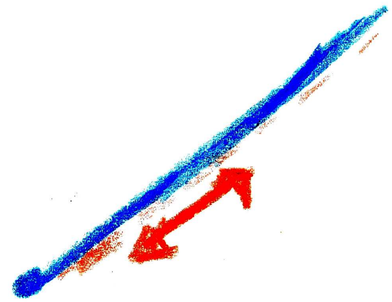

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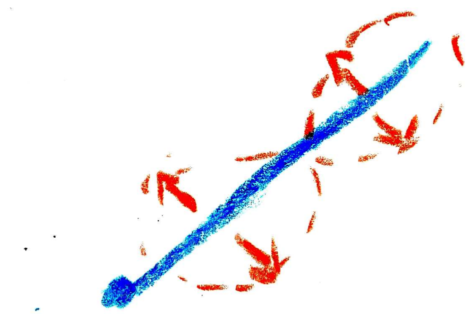

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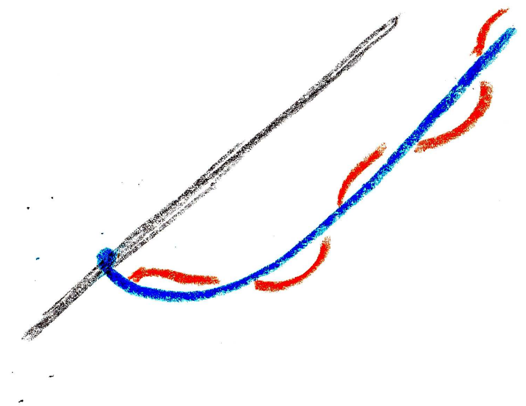

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Transverse vs. longitudinal fluctuations

The $\perp$ and $\|$ fluctuations behave differently:

$$\partial_r \left[R\, \partial_r \left(\delta X^{\perp} \right) \right] + \frac{\omega^2 b^4}{R} \delta X^{\perp} = 0,$$

$$\partial_r \left[\frac{R^3}{b^4}\, \partial_r \left(\delta X^{\|} \right) \right] + \omega^2 R \delta X^{\|} = 0$$

$$R(r) = \sqrt{(b^4(r) - b_m^2)}$$

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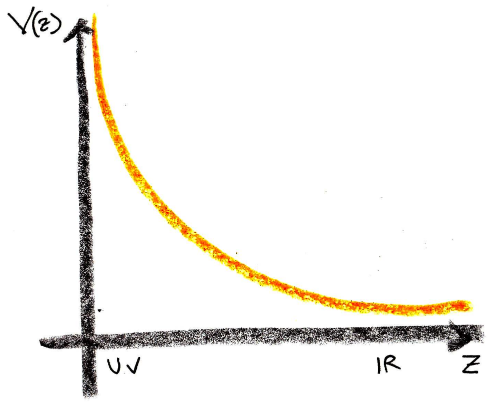
$$R(r) = \sqrt{(b^4(r) - b_m^2)} \sim b_m^2 \, 4\pi T_m \, (r_m - r), \qquad r \simeq r_m.$$

Angular dependence

The fluctuations $\perp$ and $\parallel$ to the direction of the string behave differently.
One can transform the equation into the form of a Schrödinger problem:



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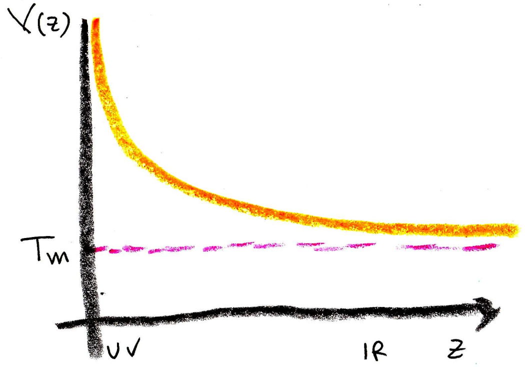

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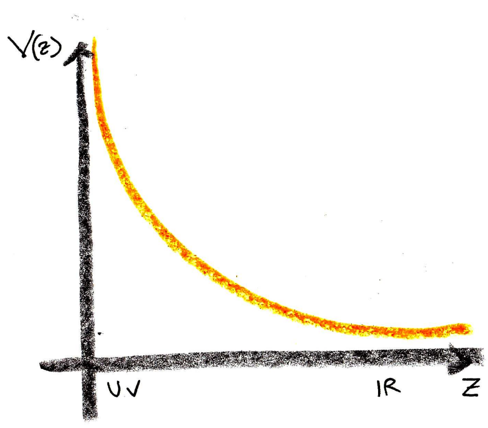

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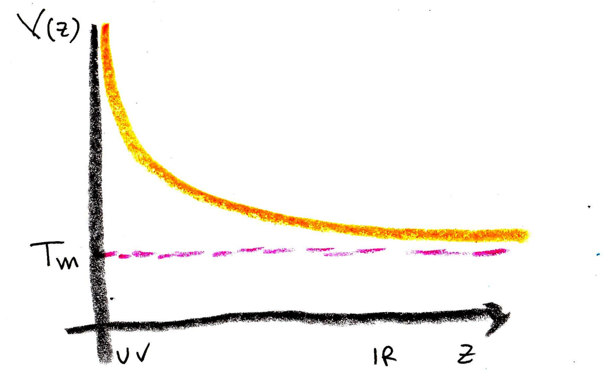

Longitudinal

The transverse modes have a continuous spectrum starting at $\omega = 0$,

the longitudinal modes are gaped and start at $\omega = 4\pi T_m$

An Effective Temperature?

The equation for transverse fluctuations close to r_m is:

$$\delta X'' - \frac{1}{(r_m - r)}\delta X' + \frac{\hat{\omega}^2}{(r_m - r)^2}\delta X = 0, \qquad \hat{\omega} \equiv \frac{\omega}{4\pi T_m}$$

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Are we seeing the emergence of an *effective temperature set by the confinement scale* .

The fluctuations are not in a thermal state because the metric is asymptotically Minkowski at infinity.

If we choose infalling boundary conditions,

$\delta X(\omega, r) \simeq (r_h - r)^{-i\hat{\omega}} \sim e^{i\omega z}$

the retarded correlator is governed by the characteristic scale T_m

which plays the same role as T in the BH.

Dissipation at zero temperature

The transverse boundary correlator at small frequency behaves as:

$$G_R^T(\omega) \simeq i b_m^2 \omega + O(\omega^2), \quad b_m^2 = \sigma_c \quad the\, confining\, string\, tension$$

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The confining vacuum is dissipative for the fluctuations of a single quark and the dissipation time scale is again set by the confinement scale.

Dissipation at zero temperature

Because of the gap in the longitudinal mode, the imaginary part of the longitufinal boundary correlator vanishes identically at small frequency:

$$G_R^T(\omega) = 0 \qquad \omega < 4\pi T_m$$

$$G_R^T(\omega) \sim \sqrt{\left(\frac{\omega}{4\pi T_m}\right)^2 - 1} \qquad \omega \gtrsim 4\pi T_m$$

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$[ImG_R/\omega]_{\omega\to 0} = 0$ and there is no viscous friction for longitudinal modes.

Averaging over angles

The string solution breaks spontaneously spatial rotations $SO(3) \to SO(2)$. For a given solution, the correlator will be anisotropic:

$$G^{ij} = G^L(\omega) n^i n^j + G^T(\omega)(\delta^{ij} - n^i n^j) \qquad \vec{n} = \vec{n}(\theta, \varphi)$$

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To quadratic order in the fluctuations:

$$Z = \int \frac{d\Omega}{4\pi} \exp i \int d\omega\, \delta X^i(\omega) G_{ij}(\omega, \Omega) \delta X^j(-\omega)$$

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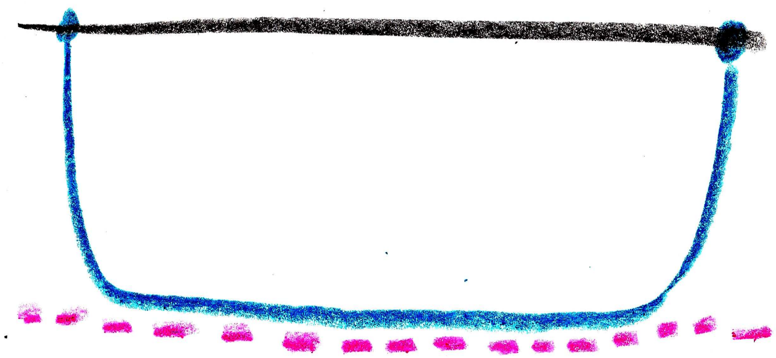
Physical picture: the Shadow Quark

The friction coefficient arises because of infalling boundary condition. This has a simple physical interpretation:



look at the trailing string as half of the confining string connecting two quarks, one of which is observed, the other (shadow quark) infinitely far.

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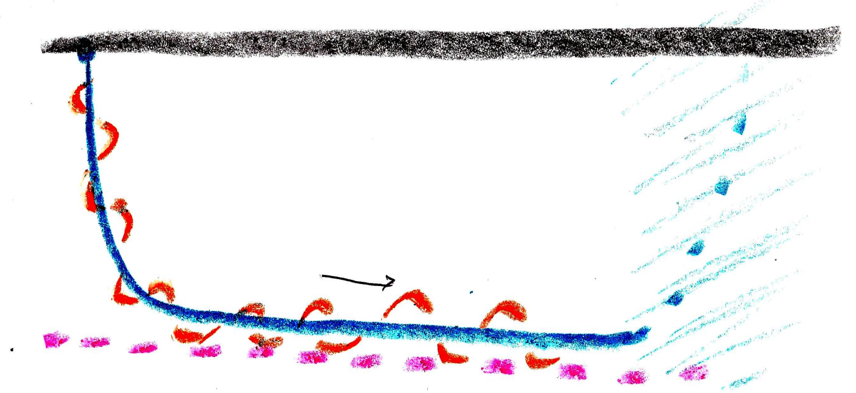

All calculation done on a single (observed) quark should be done by assuming that no information is available or comes from the shadow quark. E.g. the infalling wave condition at $z \longrightarrow \infty$

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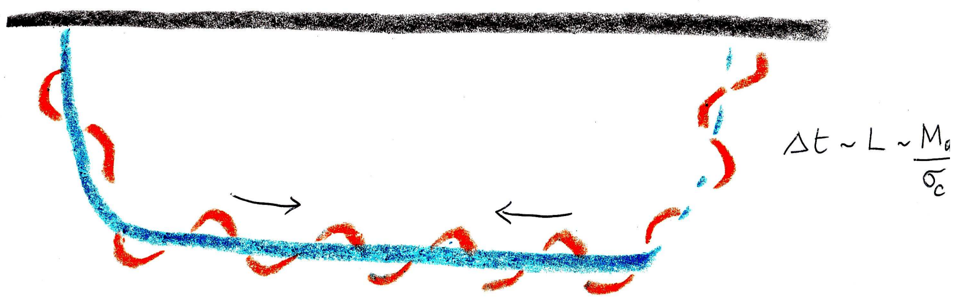
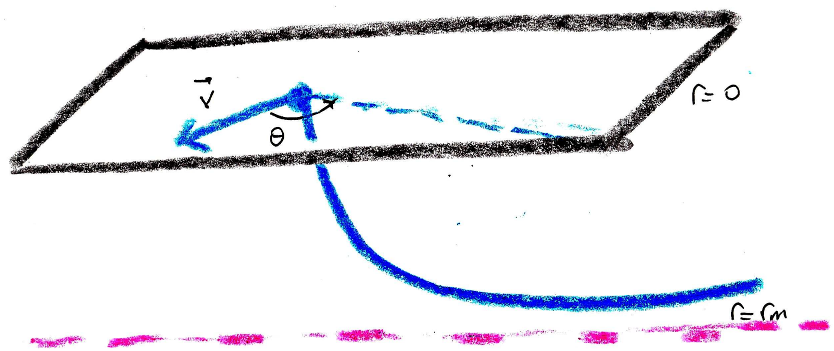


In practice, for any finite string, sooner or later information will come back and the system will become non-dissipative: a finite length of the string destroys the small-ω linear term in ImG_R. A finite quark mass M will introduce an IR cutoff to the string length.

Trailing String at finite velocity

Suppose the boundary quark has a constant velocity $\vec{v}$. The more general solution is now:

$$X(r,t) = \vec{v}t + \vec{\xi}(r), \quad \xi'(r) = \frac{\vec{c}}{\sqrt{b^4 - C^2}}, \quad |\vec{c}|^2 + \frac{(\vec{v}\cdot\vec{c})^2}{1-v^2} = C^2$$



Again, the action does not depend on the string direction, and the preferred solution is the one with $C = b_m^2$.

Drag Force

From the classical solution we can compute the net force exherted by the string, as the worldsheet momentum. This correspodns to the vev of the force operator dual to δX:

$$\langle \vec{\mathcal{F}} \rangle = \frac{\sigma_c}{(1 - v^2 \sin^2\theta)^{3/2}} \begin{pmatrix} \cos\theta \\ (1 - v^2)\sin\theta\cos\varphi \\ (1 - v^2)\sin\theta\sin\varphi \end{pmatrix}$$

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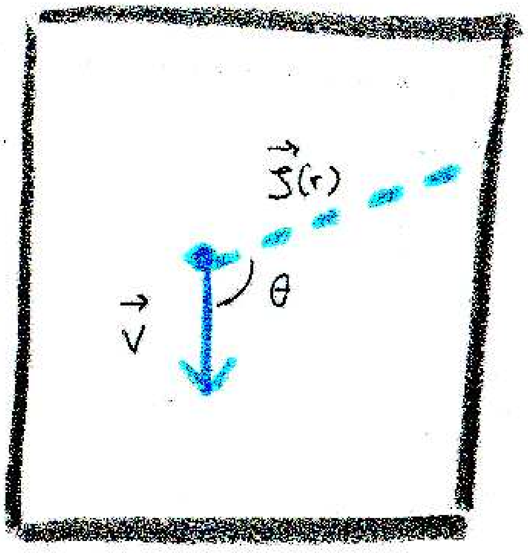
For $v \ll 1$ it is directed along the string, and it represents the constant force applied by a string with tension σ_c on its endpoint.

Fluctuations

Now we have two ways of defining transverse and longitudinal modes:

- With respect to $\vec{v}$
- With respect to the string direction.



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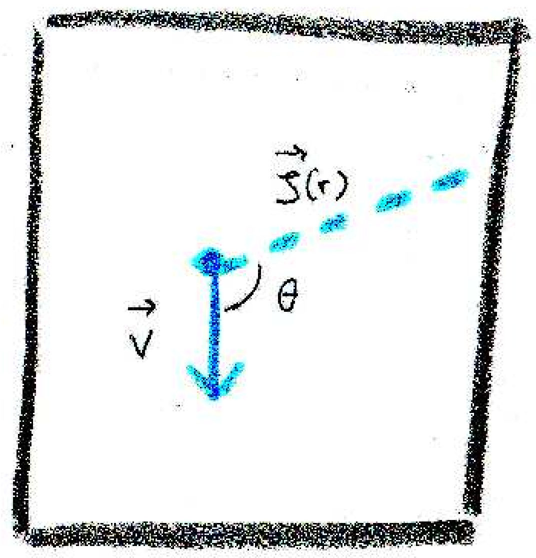
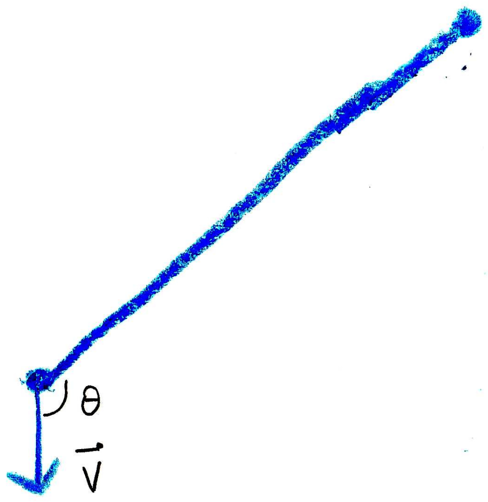


Only the first distinction has a meaning on the boundary.

Integrating over angles

The way to perfom the angular average is not unique.

- We can average with the usual meausre on the unit sphere. This means assuming that for each choice of the angle, the shadow-quark has the same velocity as the observed quark.



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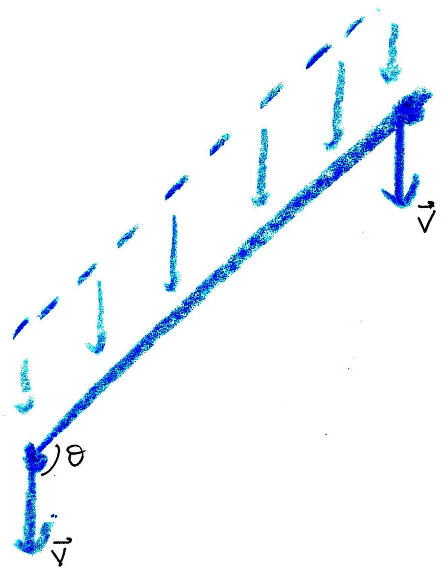

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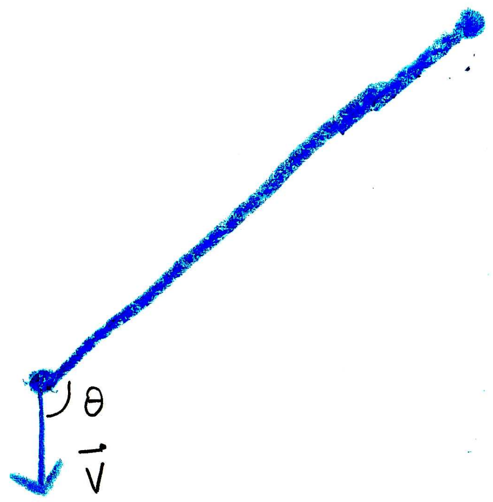
The way to perfom the angular average is not unique.

- We can assume the shadow quark has an arbitrary constant velocity $\vec{w}$ and average over that. As $t \to \infty$, the angle θ will be constant and function of $\vec{w}$ and $\vec{v}$:



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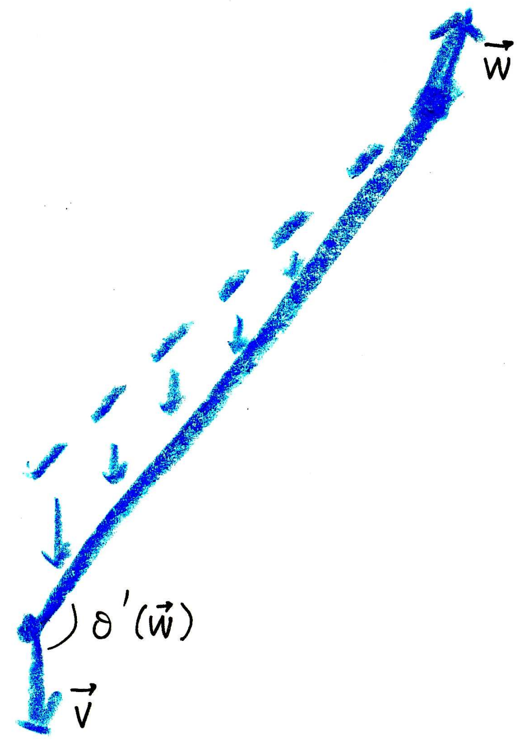

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$$cos\theta = \frac{\vec{v}}{|v|} \frac{\vec{w} - \vec{v}}{|\vec{w} - \vec{v}|}$$

Averaged correlators

Both averaging procedures can be carried out analytically. The result is always in the form:

$$\langle G^{||} \rangle = A(v) G^L(\omega) + B(v) G^T(\omega), \quad \langle G^{\perp} \rangle = C(v) G^T(\omega) + D(v) G^L(\omega)$$

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- $G^{T,L}(\omega)$: (essentially) the same correlators we found in the static case;
- A, B, C, D : simple functions of v which depend on the kind of average.
- In the static limit we obtain the expected isotropic result:

$$\langle G^{\|,\perp}\rangle \to \frac{1}{3}G^L + \frac{2}{3}G^T \qquad v \to 0$$

- The large-ω behavior is universal.

Consistency condition

How to decide which average is the correct one?

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Under the same general assumptions that give the Generalized Langevin equation as the effective description of the quark fluctuations, a low-frequency Ward identity must be obeyed:

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- the simple angular average over the string direction DOES NOT satisfy this relation;
- the average over the velocity of the shadow quark $\vec{w}$ DOES.
- This is a consistency check that the shadow-quark picture (complete ignorance about the string boundary at infinity) is the correct one if we want to describe a consistent ensemble

Conclusion

- We computed the trailing string solution and corresponding fluctuation in confining holographic backgrounds.
- The solution exhibits new features (e.g. spontanous breaking of isotropy), and interesting dynamics (viscous drag and low frequency modes at zero temperaure)
- Integration over moduli gives correlators which can be used to define physical Langevin correlators for the diffusion problem at finite temperature.
- The consistent interpratation of the vacuum correlator is in terms of a quark pair, one of which is very far and unobserved.
- Next: finite mass quarks, comparison with experimental results.