Comments on the universality of strongly coupled field theories at finite temperature

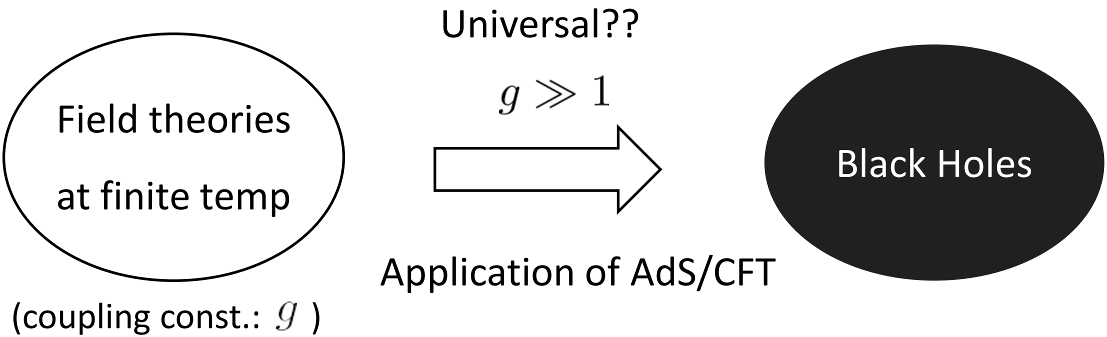
Takeshi Morita

KEK (→ University of Kentucky (from next week.))

Ref) arXiv:1305.0789 (JHEP 07 (2013) 100)
with Shotaro Shiba (KEK)

On going work, with S. Shiba, Toby Wiseman, Benjamin Withers

What is ``the universality of strongly coupled field theories at finite temperature''?



- QCD (AdS/QGP)
- Condensed matter (AdS/CMT)

I will test this conjecture in string theory
by investigating gauge theories.

The original conjecture in string theory

The maximally supersymmetric U(N) gauge theories at finite temp

(coupling const.: g_{YM})

$N \gg 1$

$g_{YM}^2 N \gg 1$

⇒

Black Dp-Branes

gauge/gravity duality

'97 Maldacena

'98 Itzhaki, Maldacena, Sonnenschein, Yankielowicz

How do the black hole natures appear in the supersymmetric gauge theories?

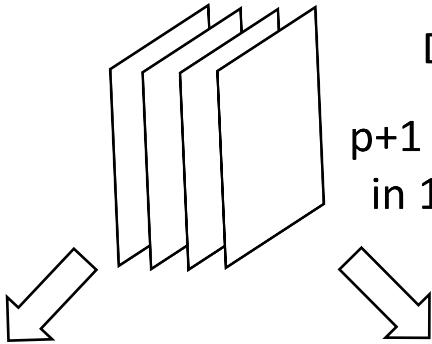
→ It may illuminate what are the essences of the duality.
Strong coupling? Large-N? SUSY?

→ It may be helpful to understand the AdS/QCD and AdS/CMT.

Plan of talk

1. Brief Review of the gauge/gravity correspondence and the SYM theories
2. Thermodynamics of the SYM theories
3. Black brane thermodynamics from the SYM theories
4. Discussions

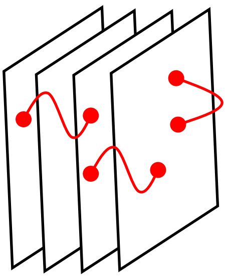
The gauge/gravity proposal was triggered by the discovery of Dp-branes.



Dp-brane

p+1 dimensional objects in 10 dim string theory

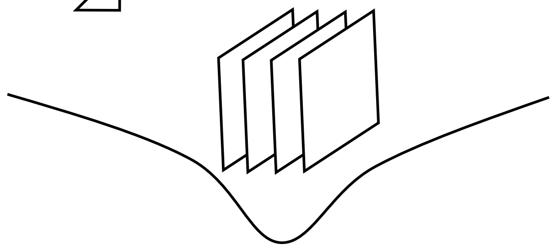
◆ Two aspects



Open string can end on the branes.

open string = gauge field

→ gauge theory



The branes can curve the spacetime.

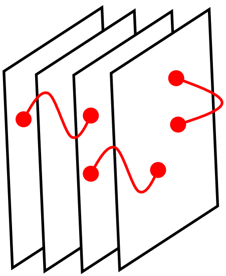
spacetime = gravity = closed string

→ 10 dim supergravity

The gauge/gravity proposal was triggered by the discovery of Dp-branes.

Gauge theory and gravity theory must be related.

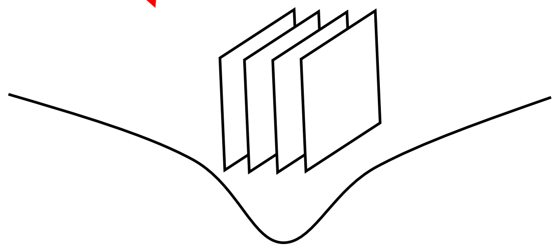
◆ Two aspects



Open string can end on the branes.

open string = gauge field

→ gauge theory



The branes can curve the spacetime.

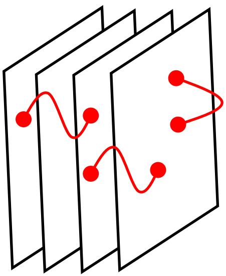
spacetime = gravity = closed string

→ 10 dim supergravity

◆ Review of the p+1 dim gauge theory on N Dp-brane.

Today, I will focus on the gauge analysis.

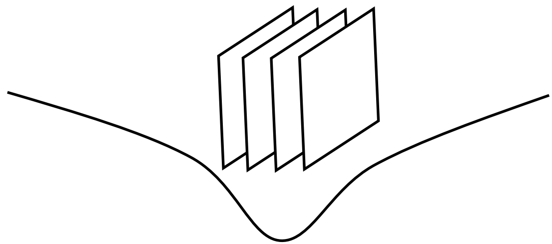
◆ Two aspects



Open string can end on the branes.

open string = gauge field

→ gauge theory



The branes can curve the spacetime.

spacetime = gravity = closed string

→ 10 dim supergravity

◆ Review of the p+1 dim gauge theory on N Dp-brane.

Gauge theory on the N Dp-branes
= p+1 dim U(N) maximally supersymmetric YM theory

◆ Action

$$S = \frac{N}{\lambda_p} \int_0^\beta d\tau \int d^p x \mathrm{Tr} \left[\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 - \frac{1}{4} [\Phi^I, \Phi^J]^2 + \cdots \right]$$

$$\begin{cases} A^\mu_{ij} & (\mu = 0, 1, \cdots, p) \quad p+1 \text{ gauge field} \\ \Phi^I_{ij} & (I = p+1, \cdots, 9) \quad 9-p \text{ adjoint scalar} \\ \Psi^\alpha_{ij} & \text{adjoint fermions} \end{cases} \qquad i, j = 1, \cdots, N$$

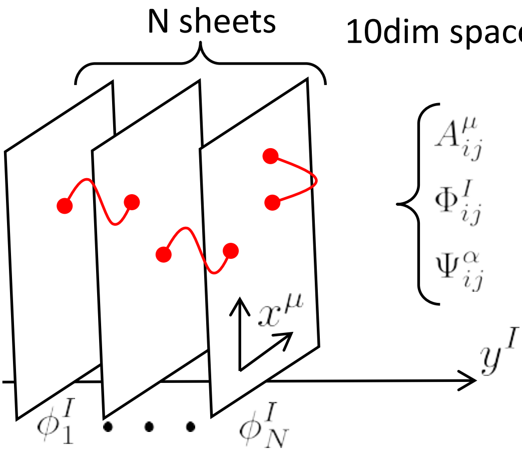
$\lambda_p = g_p^2 N$: 't Hooft coupling of Dp, the mass dimenison is 3-p.

→ A natural dimensionless expansion parameter is λ_p / T^{3-p}.

T : temperature

◆ Review of the p+1 dim gauge theory on N Dp-brane.

◆ Physical meaning:



10dim spacetime = (p+1) + (9-p)

x^μ y^I

$$\begin{cases} A^\mu_{ij} & (\mu = 0, 1, \cdots, p) \quad p+1 \text{ gauge field} \\ \Phi^I_{ij} & (I = p+1, \cdots, 9) \quad 9-p \text{ adjoint scalar} \\ \Psi^\alpha_{ij} & \text{adjoint fermions} \end{cases}$$

$$i, j = 1, \cdots, N$$

IMPORTANT POINT:

The eigenvalues of the scalar Φ^I_{ij} represent the positions of the branes on y^I

$$\Phi^I_{ij} = \begin{pmatrix} \phi^I_1 & & \\ & \ddots & \\ & & \phi^I_N \end{pmatrix}$$

The details of the model are not important but just remember this point in this talk.

ϕ^I_i : the positions of the i-th brane on y^I direction

Plan of talk

1. Brief Review of the gauge/gravity correspondence and the SYM theories
2. Thermodynamics of the SYM theories
3. Black brane thermodynamics from the SYM theories
4. Discussions

◆ Free energy density of N Dp-branes at finite temperatures

(The coefficients are omitted.)

Weak coupling $\lambda_p/T^{3-p} \sim 1$ Strong coupling λ_p/T^{3-p} →

	SYM calculations	Gravity analysis
D0:	$N^2 T$	$N^2 \lambda_0^{-\frac{3}{5}} T^{\frac{14}{5}}$
D1:	$N^2 T^2$	$N^2 \lambda_1^{-\frac{1}{2}} T^3$
D2:	$N^2 T^3$	$N^2 \lambda_2^{-\frac{1}{3}} T^{\frac{10}{3}}$
D3:	$N^2 T^4$	$N^2 T^4$
D4:	$N^2 T^5$	$N^2 \lambda_4 T^6$

Stefan-Boltzmann law

Area law of black brane

$$S_{\text{entropy}} = \frac{\text{Area}}{4G_N}$$

◆ Free energy density of N Dp-branes at finite temperatures

(The coefficients are omitte

Weak coupling

D0: N^2T

D1: N^2T^2

D2: N^2T^3

D3: N^2T^4

D4: N^2T^5

SYM calculations

Stefan-Boltzmann law

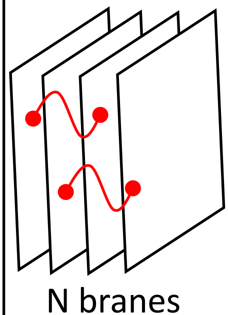
◆ Analysis in the weak coupling

Using the p+1 dim SYM.

$$S = \frac{N}{\lambda_p}\int d^{p+1}x \mathrm{Tr}\left[\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(D_\mu\Phi^I)^2 + \cdots\right]$$

Perturbation around $\Phi^I = 0$.

(Φ^I :positions of the branes.)



The distances between the branes are short.

→ N × N light string modes

(mass of open string ∼length)

According to the Stefan-Boltzmann law

$$F \sim N^2T^{p+1}$$

◆ Free energy density of N Dp-branes at finite temperatures

(The coefficients are omitted.)

Weak coupling $\quad \lambda_p/T^{3-p} \sim 1 \quad$ Strong coupling $\quad \lambda_p/T^{3-p}$ →

	Weak coupling	Strong coupling
D0:	$N^2 T$	$N^2 \lambda_0^{-\frac{3}{5}} T^{\frac{14}{5}}$
D1:	$N^2 T^2$	$N^2 \lambda_1^{-\frac{1}{2}} T^3$
D2:	$N^2 T^3$	$N^2 \lambda_2^{-\frac{1}{3}} T^{\frac{10}{3}}$
D3:	$N^2 T^4$	$N^2 T^4$
D4:	$N^2 T^5$	$N^2 \lambda_4 T^6$
	SYM calculations	Gravity analysis
	Stefan-Boltzmann law	Area law of black brane

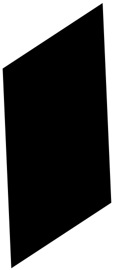
$$S_{\text{entropy}} = \frac{\text{Area}}{4G_N}$$

◆ Free energy density of N Dp-branes at finite temperatures

(The coefficients are omitted.)

◆ Analysis in the strong coupling

Use 10dim SUGRA, which is valid at $\lambda_p/T^{3-p} \gg 1$.



N Dp-branes at finite temperature

↓

Black Dp-brane solution

According to the Black hole thermodynamics

$$F \sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda_p^{-\frac{3-p}{5-p}}$$

$$\left(S_{\text{entropy}} = -\frac{\partial F}{\partial T} = \frac{\text{Area}}{4G_N} \right)$$

Area law

Strong coupling λ_p/T^{3-p} →

$$N^2 \lambda_0^{-\frac{3}{5}} T^{\frac{14}{5}}$$

$$N^2 \lambda_1^{-\frac{1}{2}} T^3$$

$$N^2 \lambda_2^{-\frac{1}{3}} T^{\frac{10}{3}}$$

$$N^2 T^4$$

$$N^2 \lambda_4 T^6$$

Gravity analysis

Area law of black brane

$$S_{\text{entropy}} = \frac{\text{Area}}{4G_N}$$

◆ Free energy density of N Dp-branes at finite temperatures

(The coefficients are omitted.)

	Weak coupling	$\lambda_p / T^{3-p} \sim 1$	Strong coupling	λ_p / T^{3-p} →
D0:	$N^2 T$		$N^2 \lambda_0^{-\frac{3}{5}} T^{\frac{14}{5}}$	
D1:	$N^2 T^2$		$N^2 \lambda_1^{-\frac{1}{2}} T^3$	
D2:	$N^2 T^3$		$N^2 \lambda_2^{-\frac{1}{3}} T^{\frac{10}{3}}$	
D3:	$N^2 T^4$		$N^2 T^4$	
D4:	$N^2 T^5$		$N^2 \lambda_4 T^6$	
	SYM calculations		Gravity analysis	
	Stefan-Boltzmann law		Area law of black brane	

$$S_{\text{entropy}} = \frac{\text{Area}}{4G_N}$$

◆ Free energy density of N Dp-branes at finite temperatures

But these two regime look quite different...
Can we understand the law of gravity from SYM?

→ Yes! We can explain it through an order estimate.

	SYM calculations	Gravity analysis
D0:	$N^2 T$	$N^2 \lambda_0^{-\frac{3}{5}} T^{\frac{14}{5}}$
D1:	$N^2 T^2$	$N^2 \lambda_1^{-\frac{1}{2}} T^3$
D2:	$N^2 T^3$	$N^2 \lambda_2^{-\frac{1}{3}} T^{\frac{10}{3}}$
D3:	$N^2 T^4$	$N^2 T^4$
D4:	$N^2 T^5$	$N^2 \lambda_4 T^6$

Stefan-Boltzmann law

Area law of black brane

$$S_{\text{entropy}} = \frac{\text{Area}}{4G_N}$$

Plan of talk

1. Brief Review of the gauge/gravity correspondence and the SYM theories

2. Thermodynamics of the SYM theories

3. Black brane thermodynamics from the SYM theories

4. Discussions

◆ Outlines of the SYM calculations

Weak coupling

D0: N^2T

D1: N^2T^2

D2: N^2T^3

D3: N^2T^4

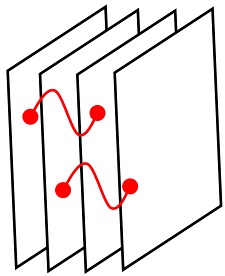
D4: N^2T^5

Stefan-Boltzmann law

◆ Analysis in the weak coupling

$$S=\frac{N}{\lambda_p}\int d^{p+1}x\,\mathrm{Tr}\left[\frac{1}{4}F_{\mu\nu}^2+\frac{1}{2}(D_\mu\Phi^I)^2+\cdots\right]$$

Perturbation around $\Phi^I=0$.
(Φ^I :positions of the branes.)



- Perturbative expansions

$$F\sim N^2T^{p+1}(1+\lambda_p\#\cdots)$$

$$\overset{??}{\underset{\lambda_p/T^{3-p}\gg 1}{\sim}} N^2T^{\frac{2(7-p)}{5-p}}\lambda_p^{-\frac{3-p}{5-p}}$$

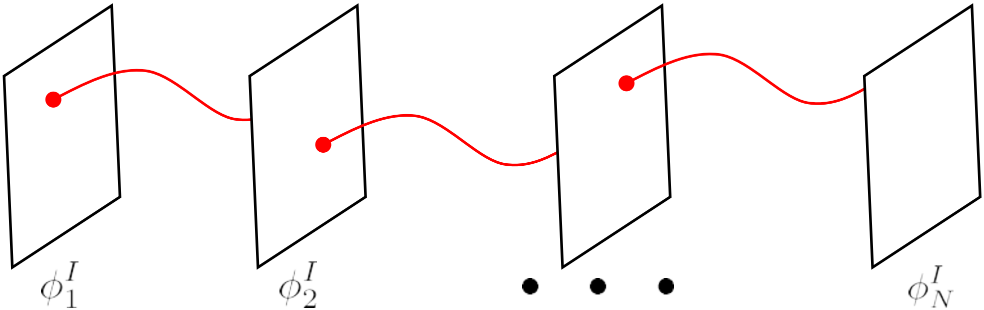
It looks hopeless...

⇨ We should try different initial configurations of the perturbative expansion.

◆ Outlines of the SYM calculations

D0: Smilga ('08)
Dp (p<3): Wiseman ('13)
Dp(p=3,4,6): T.M.-Shiba ('13)

◆ Try a separated brane configuration.



In the SYM theories, it corresponds to large values of the diagonal components.

$$\Phi^I = \begin{pmatrix} \phi_1^I & & \\ & \ddots & \\ & & \phi_N^I \end{pmatrix}$$

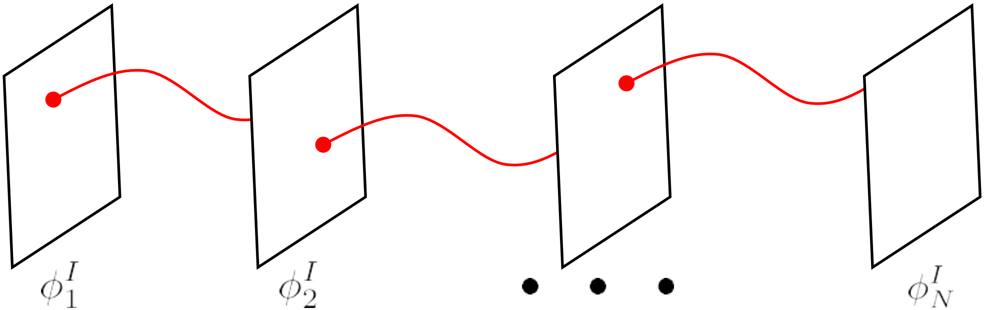
Indeed this configuration is a classical solution of the model.

$$S = \frac{N}{\lambda_p} \int_0^\beta d\tau \int d^p x \mathrm{Tr} \left[\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \Phi^I)^2 - \underline{\frac{1}{4} [\Phi^I, \Phi^J]^2} + \cdots \right]$$

◆ Outlines of the SYM calculations

D0: Smilga ('08)
Dp (p<3): Wiseman ('13)
Dp(p=3,4,6): T.M.-Shiba ('13)

◆ Try a separated brane configuration.



In the SYM theories, it corresponds to
large values of the diagonal components.

$$\Phi^I = \begin{pmatrix} \phi_1^I & & \\ & \ddots & \\ & & \phi_N^I \end{pmatrix}$$

→ The off-diagonal components become massive (i.e. Higgs mechanism).
So we can integrate out them and obtain an effective action
for the diagonal modes.

$$S_{\mathrm{D}p} \sim \frac{N}{\lambda_p} \int d\tau d^p x \sum_{i=1}^{N} \left(\frac{1}{2} \partial^\mu \phi_i^I \partial_\mu \phi_i^I \right) - \sum_{i,j=1}^{N} \frac{(\partial\phi_i - \partial\phi_j)^4}{|\phi_i - \phi_j|^{7-p}} + \cdots ,$$

◆ Outlines of the SYM calculations

D0: Smilga ('08)
Dp (p<3): Wiseman ('13)
Dp(p=3,4,6): T.M.-Shiba ('13)

◆ Try a separated brane configuration.

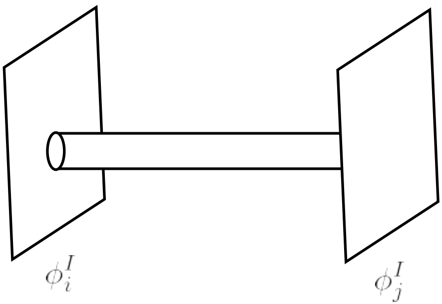
★ The effective action for the diagonal components

$$S_{\mathrm{D}p} \sim \frac{N}{\lambda_p} \int d\tau d^p x \sum_{i=1}^{N} \left(\frac{1}{2} \partial^\mu \phi_i^I \partial_\mu \phi_i^I \right) - \sum_{i,j=1}^{N} \frac{(\partial\phi_i - \partial\phi_j)^4}{|\phi_i - \phi_j|^{7-p}} + \cdots ,$$

Kinetic term of each brane

Attractive forces between the branes at long distance (one-loop effect)

→ They correspond to the classical gravitational force between the branes.



cf) open-closed duality

One-loop of SYM (open sting)

⇕

Tree of supergravity (closed string)

◆ Outlines of the SYM calculations

◆ Try a separated brane configuration.

D0: Smilga ('08)
Dp (p<3): Wiseman ('13)
Dp(p=3,4,6): T.M.-Shiba ('13)

★ The effective action for the diagonal components

$$S_{\mathrm{D}p} \sim \frac{N}{\lambda_p} \int d\tau d^p x \sum_{i=1}^{N} \left(\frac{1}{2} \partial^\mu \phi_i^I \partial_\mu \phi_i^I \right) - \sum_{i,j=1}^{N} \frac{(\partial\phi_i - \partial\phi_j)^4}{|\phi_i - \phi_j|^{7-p}} + \cdots ,$$

Kinetic term of each brane

Attractive forces between the branes at long distance (one-loop effect)

The branes may compose a bound state through the attractive force.

→We can estimate the total energy via virial theorem.

There we use an assumption: $\partial\phi_i^I \sim T\phi_i^I$ (derivative ∼temp.)
OK for free fields

At large-N and strong coupling $(\lambda_p/T^{3-p} \gg 1)$, a solution exists

$$F \sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda_p^{-\frac{3-p}{5-p}}$$

→ Reproduces the gravity result (the area low)!!

◆ Outlines of the SYM calculations

D0: Smilga ('08)
Dp (p<3): Wiseman ('13)
Dp(p=3,4,6): T.M.-Shiba ('13)

SYM theory can explain the supergravity results.

Weak coupling	$\lambda_p/T^{3-p} \sim 1$	Strong coupling	λ_p/T^{3-p}
Stefan-Boltzmann law		Area law of black brane	
(Statistical mechanism)		(Gravity)	

At large-N and strong coupling $(\lambda_p/T^{3-p} \gg 1)$, a solution exists

$$F \sim N^2 T^{\frac{2(7-p)}{5-p}} \lambda_p^{-\frac{3-p}{5-p}}$$

→ Reproduces the gravity result (the area low)!!

◆ Comments on M2 and M5 branes

T.M.-Shiba ('13)

M-brane: Branes in 11 dimensional M-theory

★ The effective action for the Dp-brane

$$S_{\mathrm{D}p} \sim \frac{N}{\lambda_p} \int d\tau d^p x \sum_{i=1}^{N} \left(\frac{1}{2} \partial^\mu \phi_i^I \partial_\mu \phi_i^I \right) - \sum_{i,j=1}^{N} \frac{(\partial\phi_i - \partial\phi_j)^4}{|\phi_i - \phi_j|^{7-p}} + \cdots ,$$

We can derive this long distance potential except the coefficient just by using SUSY and a dimensional analysis.
(The details of the model are not necessary!)

★ The effective action for the M2/M5-brane from the 3dim/6dim SCFT

$$S_{\mathrm{M}p} \sim \int d\tau d^p x \sum_{i=1}^{N} \left(\frac{1}{2} \partial^\mu \phi_i^I \partial_\mu \phi_i^I \right) - \sum_{i,j=1}^{N} \frac{(\partial\phi_i - \partial\phi_j)^4}{|\phi_i - \phi_j|^{8-p}} + \cdots , \quad (p = 2, 5)$$

Through a similar analysis, we estimate their free energies as

$\Rightarrow$ $F_{M2} \sim N^{\frac{3}{2}} T^3$ and $F_{M5} \sim N^3 T^6$

→ These results are consistent with the 11 dim SUGRA including the famous exotic N dependences of the M-branes.

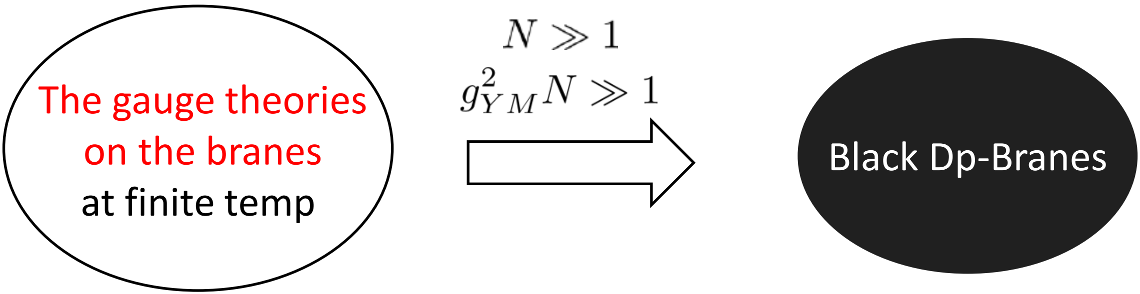
Plan of talk

1. Brief Review of the gauge/gravity correspondence and the SYM theories
2. Thermodynamics of the SYM theories
3. Black brane thermodynamics from the SYM theories
4. Discussions

◆ Summary & Discussions

The gauge theories on Dp, M2 and M5 branes can explain the black brane thermodynamics at strong coupling.

In string theory, this result is important for the BH micro states (c.f. D1D5P system) and M-theory dynamics.

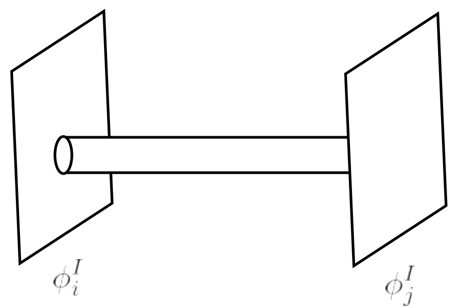


◆ Summary & Discussions

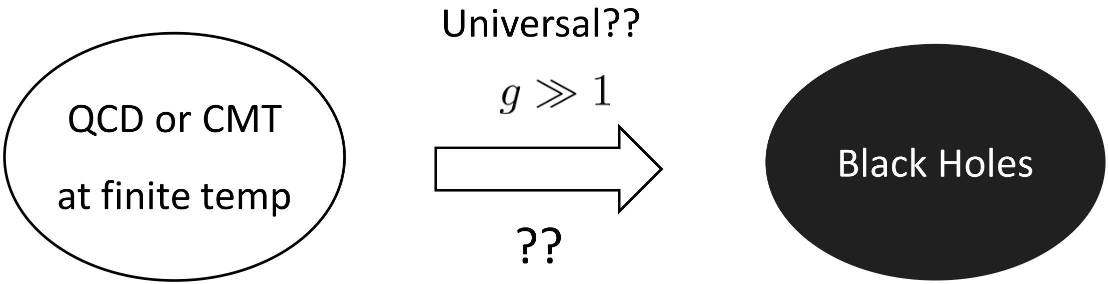
The gauge theories on Dp, M2 and M5 branes can explain the black brane thermodynamics at strong coupling.

◆ The following ingredients are essential for the appearance of the black hole natures in the SYM theories.

- Dynamics of the moduli field ϕ_i^I
  (Dynamics of the brane
  in the transverse directions.)
- maximal supersymmetry
  → The appearance of the gravitational potential at long distance.
- Large-N limit
- Strong coupling



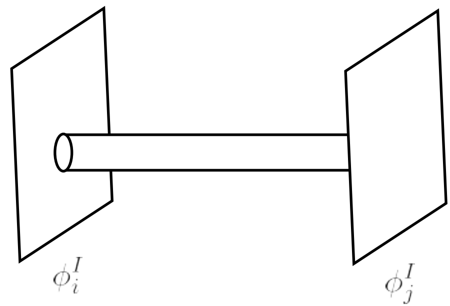
◆ Go back to the Original Question



(coupling const.: g)

QCD and CMT have neither moduli nor supersymmetry...

- Dynamics of the moduli field ϕ_i^I
  (Dynamics of the brane
  in the transverse directions.)
- maximal supersymmetry
  → The appearance of the gravitational potential at long distance.
- Large-N limit
- Strong coupling



◆ Go back to the Original Question