Holographic Stringy Hadrons

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E. Federovsky, G. Harpaz, N. Katz, U. Kol, Y. Kinar, S. Kuperstein, M. Kruczenski, L. Pando-Zayas, K. Petters, E. Schreiber, S. Seki, D. Vaman, D. Weissman M. Zamaklar

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Introduction

- The holographic duality is an equivalence between certain bulk string theories and boundary field theories.
- Practically most of the applications of holography is based on relating bulk fields (not strings) and operators on the boundary field theories.
- This is based on the usual limit of $\alpha' \rightarrow 0$ with which we go from a closed string theory to a gravity theory for instance.
- However to describe hadrons in reality it seems that we need strings since after all in reality the string tension is not very large (λ of order one)

Introduction

- The main theme of this talk is that there is a wide sector of hadronic physical observables which cannot be faithfully described by bulk fields but rather requires dual stringy phenomena
- It is well known that this is the case for Wilson, ‘t Hooft and Polyakov lines.
- We argue here that in fact also the spectra, decays and other properties of hadrons:

gluballs, mesons and baryons can be recast only by holographic stringy hadrons

Introduction

- The major argument against describing the hadron spectra in terms of fluctuations of fields like bulk fields or modes on probe branes is that they generically do not admit properly the Regge behavior of the spectra.
- For M^2 as a function of J we get from flavor branes only J=0,J=1 mesons and there will be a big gape of order λ in their tension in comparison to high spin stringy mesons.
- The attempts to get the linearity between M^2 and n basically face problems whereas for strings it is an obvious property.

Outline:

- Brief review of Stringy duals of Wilson loops.
- Confining backgrounds.
- Holography and stringy mesons
- The classical spectra of stringy mesons
- On the quantization of the stringy meson
- Fits to mesonic data
- Decay Width of stringy mesons
- Stringy Baryons
- Holographic Regge trajectories of stringy baryons
- Summary

Stringy loops

Confining Wilson Loop

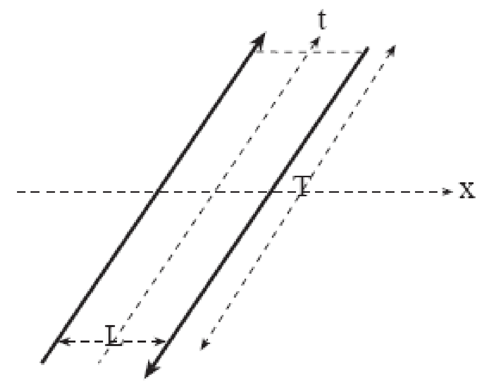
- In SU(N) gauge theories one defines the following gauge invariant operator

$$W(C) = \frac{1}{N} Tr P e^{\oint_C A_\mu \dot{x}^\mu(\tau) d\tau}$$

where C is some contour

- The quark – antiquark potential can be extracted from a strip Wilson line

$$\langle W(C) \rangle = A(L) e^{-TE(L)}.$$



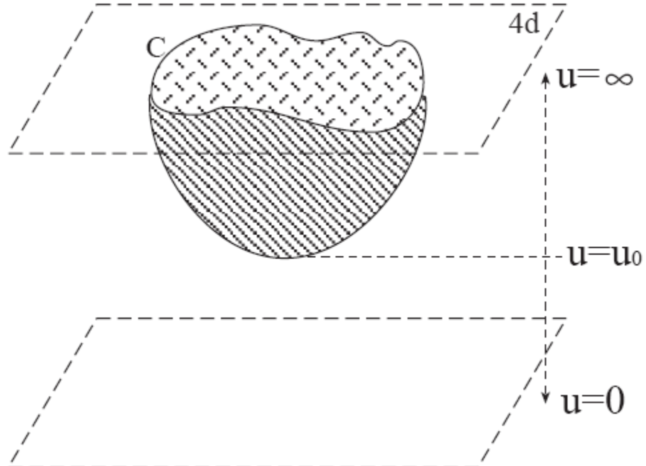
- The signal for confinement is $E \sim T_{st} L$

Stringy Wilson loop

- The natural stringy dual of the Wilson line (which obeys the loop equation) is

$$\langle W(C)\rangle \sim e^{-S_{NG}^{ren}}$$

where S_{NG}^{ren} is the renormalized Nambu Goto action, namely the renormalized world sheet area



Computing the stringy WL in general background

- The basic setup is a d dimensional space time

$$ds^2 = -G_{00}(s)dt^2 + G_{x_{||}x_{||}}(s)dx_{||}^2 + G_{ss}(s)ds^2 + G_{x_T x_T}(s)dx_T^2$$

where x|| are p space coordinates on a Dp brane and s and x_T are radial coordinate and transverse directions.

- The corresponding NG action is

$$S_{NG} = \int d\sigma d\tau \sqrt{det[\partial_\alpha x^\mu \partial_\beta x^\nu G_{\mu\nu}]}$$

- Upon using the gauge $\tau = t$ $\sigma = x$ the NG action reads

$$S_{NG} = T \cdot \int dx \sqrt{f^2(s(x)) + g^2(s(x))(\partial_x s)^2}$$

where

$$f^2(s(x)) \equiv G_{00}(s(x)) G_{x_{||}x_{||}}(s(x))$$

$$g^2(s(x)) \equiv G_{00}(s(x)) G_{ss}(s(x))$$

Sufficient conditions for confinement

- We proved a theorem that sufficient conditions for confinement are if either

(i) f **has a minimum at** s_{min} **and** $f(s_{min}) \neq 0$

(ii) g **diverges at** s_{div} **and** $f(s_{div}) \neq 0$

Confining backgrounds

Confining backgorunds

- There are a handful of backgrounds that admits confining Wilson lines.
- There are bottom-up scenarios. Here we focus on top-down models
- Most of the analysis here is model independent.
- A prototype model of the pure gauge sector is:
- The compactified D4 brane background
- This can be:
- (i) The critical (10d) model
- (ii) The non-critical (6d) model.

Compactified D4 model (Witten's model)

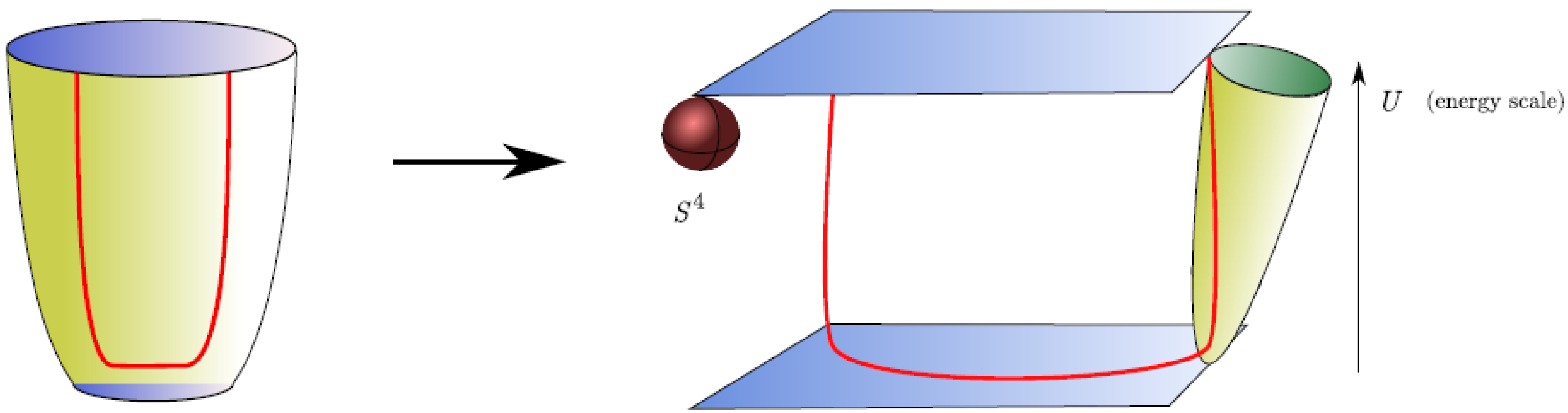
$$ds^2 = \left(\frac{U}{R_{D4}}\right)^{3/2}\left[\eta_{\mu\nu}dX^\mu dX^\nu + f(U)d\theta^2\right] + \left(\frac{R_{D4}}{U}\right)^{3/2}\left[\frac{dU^2}{f(U)} + U^2 d\Omega_4\right]$$

world-volume our 3+1 world

$$f(U) = 1 - \left(\frac{U_\Lambda}{U}\right)^3$$

θ *is a compact Kaluza-Klein circle*

U: radial direction bounded from below $U \geq U_\Lambda$



- The gauge theory and sugra parametrs are related via

$$g_5^2 = (2\pi)^2 g_s l_s, \qquad g_4^2 = \frac{g_5^2}{2\pi R} = 3\sqrt{\pi}\left(\frac{g_s u_\Lambda}{N_c l_s}\right)^{1/2}, \qquad M_{gb} = \frac{1}{R},$$

$$T_{st} = \frac{1}{2\pi l_s^2}\sqrt{g_{tt} g_{xx}}\Big|_{u=u_\Lambda} = \frac{1}{2\pi l_s^2}\left(\frac{u_\Lambda}{R_{D4}}\right)^{3/2} = \frac{2}{27\pi}\frac{g_4^2 N_c}{R^2} = \frac{\lambda_5}{27\pi^2 R^3},$$

5d coupling 4d coupling glueball mass

String tension

- The gravity picture is **valid** only provided that $\lambda_5 >> R$

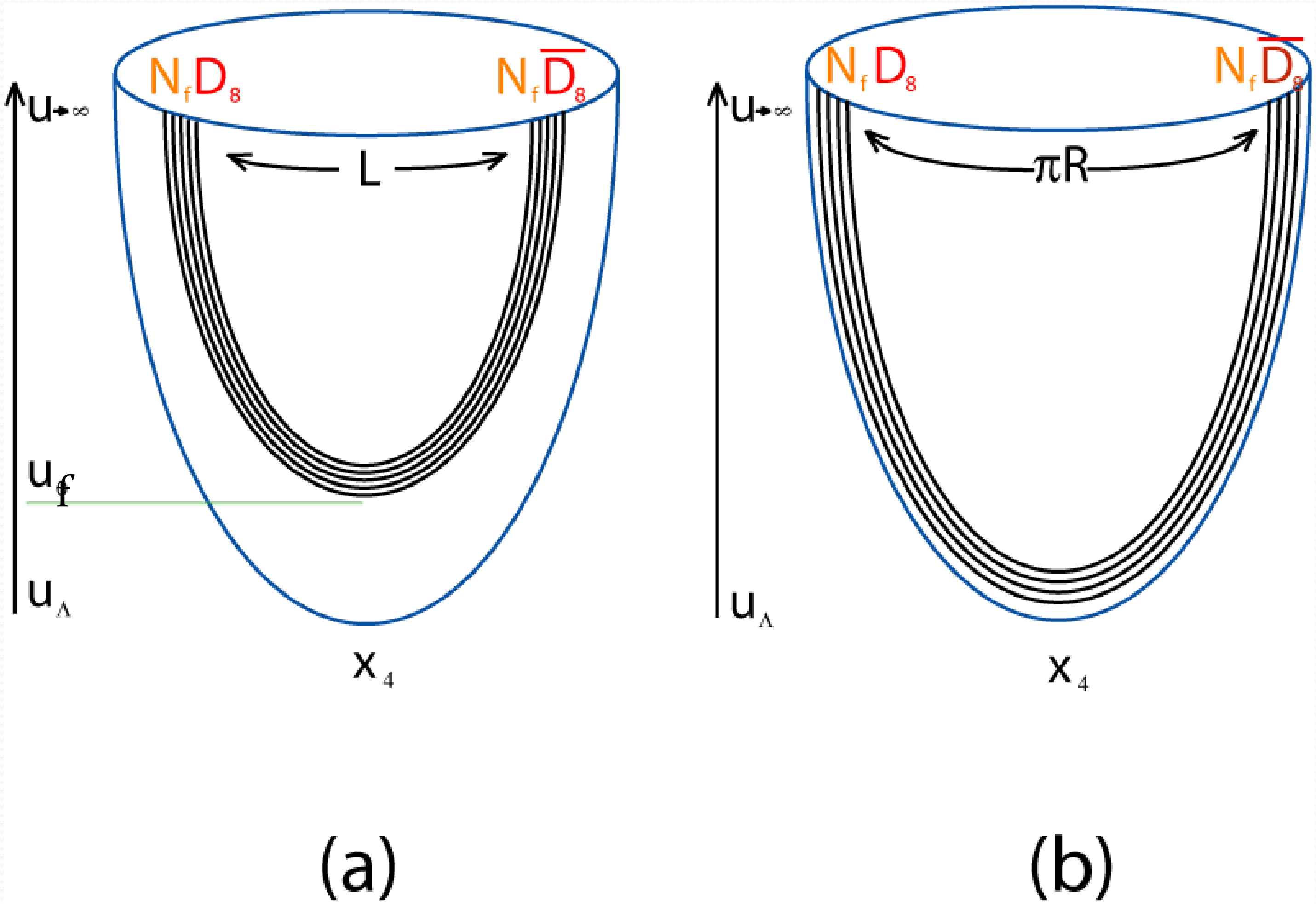
- At energies $E << 1/R$ the theory is **effectively 4d**.

- However it is not really QCD since $M_{gb} \sim M_{KK}$

- In the opposite limit of $\lambda_5 << R$ **we approach QCD**

Adding flavor: The Sakai Sugimoto model

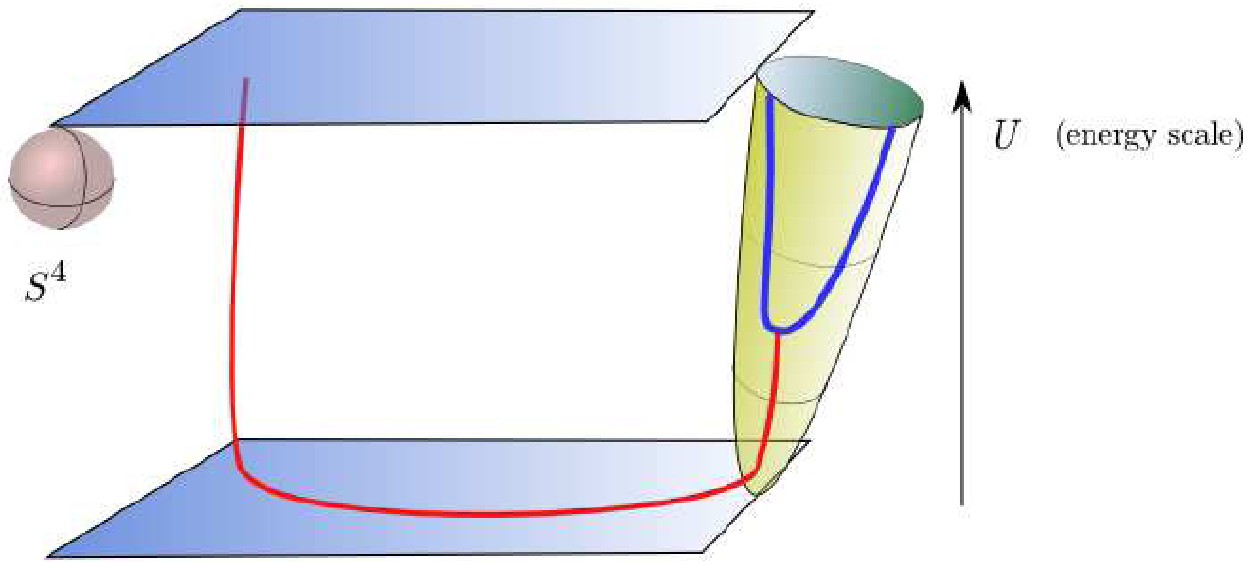
- Adding Nf D8 anti-D8 branes into Witten's model



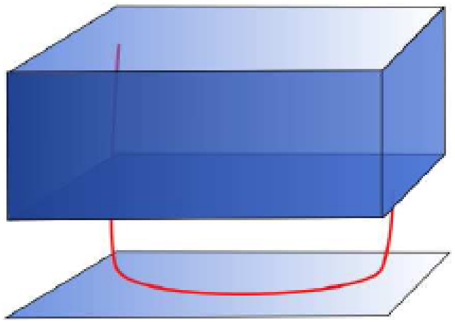
Generalized SS model

Sakai Sugimoto model

Adding flavor: The Sakai Sugimoto model



suppressing everything but U
and our 3+1d world:



The non critical analogous model

- In a similar manner to Witten's model and the Sakai Sugimoto model one can construct a 6d model of compact D4 branes (It is in fact Ads6).
- Instead of stacks of D8 and anti-D8 one can add D4 branes which are transverse to those of the background in one transverse dimension and hence the U shape structure also encodes chiral symmetry breaking with massless pions.

Stringy holographic Mesons

Strings ending on flavor branes

- We have just seen the conditions for a background to admit a confining Wilson lines.
- Again we take a general background and now determine the condition for a Regge-like string meson of

$$x^0 = e\tau, \theta = e\omega\tau, R = R(\sigma), u = u(\sigma), Y^i = Y^i(u_f)$$

- Consider a general background of the form

$$ds^2 = G_{mm} dx^m dx^m = -G_{00} dt^2 + G_{xx} dx^2 + G_{uu} du^2 + G_{yy} dy^2$$

function of u

R^3 coordinate

radial

transverse

- We assume that

$$U > U_f$$

Strings ending on flavor branes

- Define $f \equiv G_{00}$ with $G_{00} = G_{xx}$ and $g^2 = G_{00}G_{uu}$.
- The action than reads

$$S_{NG} = T \int d\tau dR[(fe^2 - f(e\omega R)^2)(f + G_{uu}\dot{u}^2)]^{\frac{1}{2}}$$

- The equation of motion for u(R)

$$\partial_R[g^2 \frac{\epsilon \dot{u}}{G}] = \epsilon(\partial_u G))$$

- Where

$$\epsilon \equiv \sqrt{1 - v^2}$$

$$v = \omega R$$

$$G \equiv \sqrt{f^2 + g^2 \dot{u}^2}$$

Strings ending on flavor branes

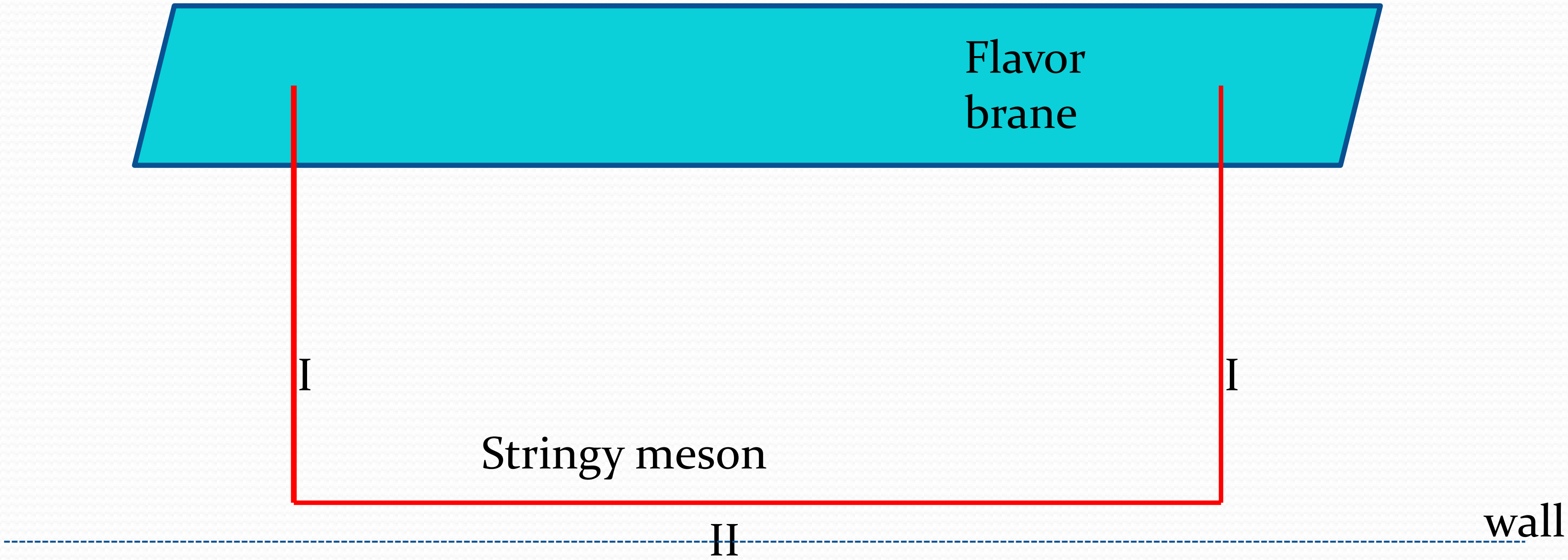
- We now separate the profile into two regions:

- Region (I) vertical $\dot{u} \longrightarrow \infty$

$$\sigma \in (-\pi/2, -\alpha), \sigma \in (\alpha, \pi/2)$$

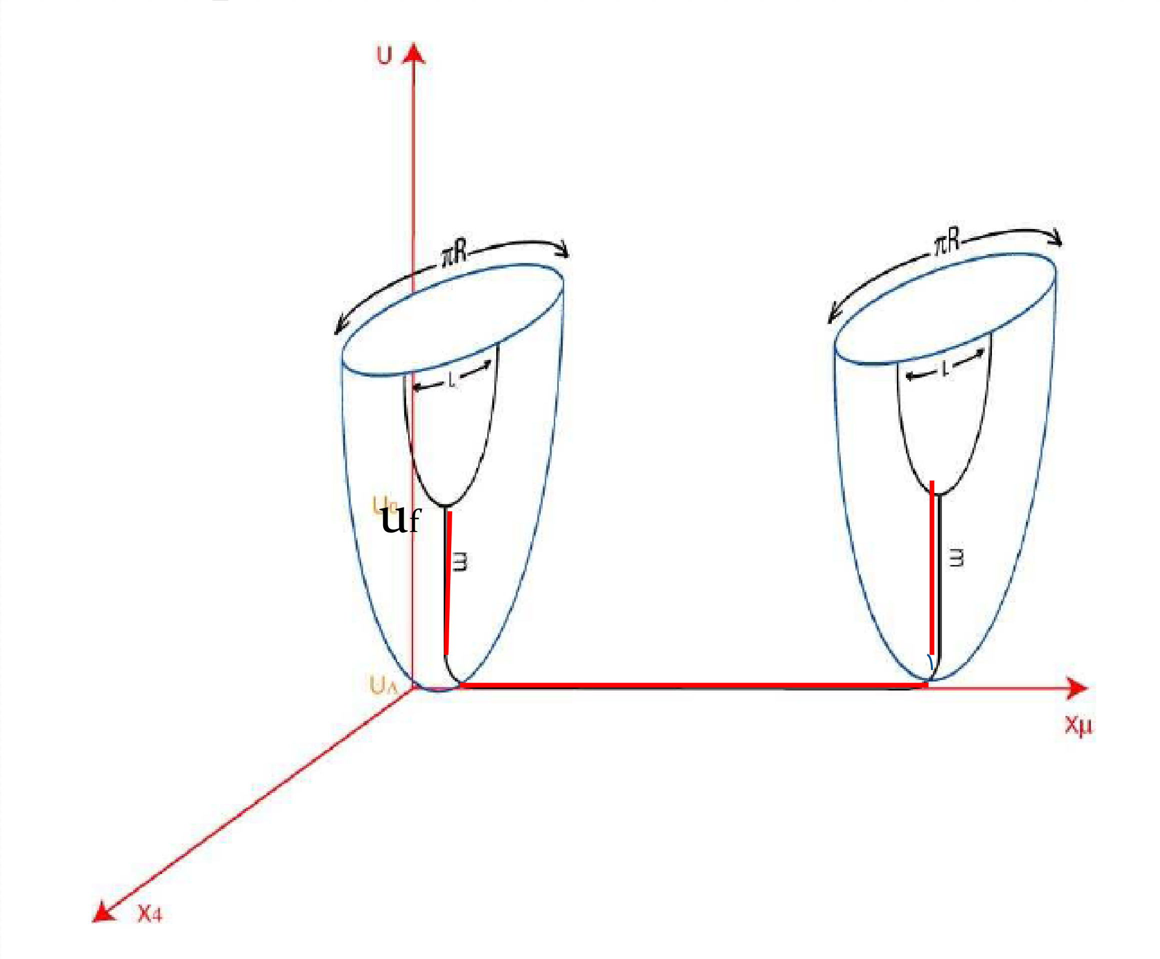
- Region (II) horizontal $\dot{u} \longrightarrow 0,\ u = u_0$

$$\sigma \in (-\alpha, \alpha)$$



Stringy meson in U shape flavor brane setup

- In the generalized Sakai Sugimoto model or its non-critical partner the meson looks like



Necessary conditions for a solution

- To solve the e.o.m in region II we expand in $\dot{u}$

$$\partial_R(\epsilon \dot{u}) = \frac{\epsilon}{2g^2}\partial_u G = \frac{\epsilon}{2g^2}\partial_u f^2 = \frac{\epsilon \partial_u f}{G_{uu}} = 0$$

- Hence to solve the e.o.m in II we need

a) $\partial_u h(u_0) = \partial_u G_{00}(u_0) \rightarrow 0$ or

b) $G_{uu}(u_0) \rightarrow \infty$

Variational analysis

- Since we have decomposed the string profile to two regions, we have to re-investigate the variational analysis

$$\delta S = T\int d^2\sigma \delta R[\frac{\partial L}{\partial R} - \partial_\sigma \frac{\partial L}{\partial R'}] + \int d\tau \delta R \frac{\partial L}{\partial R'}|^{\alpha}_{-\alpha}$$

$$\frac{\partial L}{\partial R} = \frac{-R\dot{\theta}^2\sqrt{f^2R'^2+g^2u'^2}}{\sqrt{\dot{t}-R^2\dot{\theta}^2}}$$

$$\frac{\partial L}{\partial R'} = \frac{h^2R'\sqrt{\dot{t}-R^2\dot{\theta}^2}}{\sqrt{f^2R'^2+g^2u'^2}}$$

- The two regions are

$$\text{I] } R' = 0, u' \neq 0 \; \sigma \in (-\tfrac{\pi}{2}, -\alpha) \cup (\alpha, \tfrac{\pi}{2})$$

$$\text{II] } R' \neq, 0 u' \to 0 \; \sigma \in (-\alpha, \alpha)$$

Variational analysis

- For region II

$$\frac{\partial L}{\partial R} = \frac{-eR\omega^2 f R'}{\sqrt{1-v^2}} \qquad \frac{\partial L}{\partial R'} = ef\sqrt{1-v^2}$$

- Thus, the variation of the action is

$$\delta S = T\left(\int_{-\frac{\pi}{2}}^{-\alpha} + \int_{\alpha}^{\frac{\pi}{2}}\right) d^2\sigma\delta R\frac{-eR_0\omega^2 gu'}{\sqrt{1-v^2}} + \int d\tau\delta Ref\sqrt{1-v^2}|_{-\alpha}^{\alpha}$$

- To have δS=0 we must have

$$ef\sqrt{1-v^2} = \int d\sigma\frac{-eR_0\omega^2 gu'}{\sqrt{1-v^2}} = \frac{eR_0\omega^2}{\sqrt{1-v^2}}\int_{u_0}^{u_f} gdu$$

String end-point mass

- We now define the string end-point quark mass

$$m_q = T\int_{u_0}^{u_f} g dU = T\int_{u_0}^{u_f}\sqrt{G_{00}G_{uu}}du$$

- For δ S=o the system has to obey the condition

$$T_{eff}(1-v^2) = m_q\omega^2 R_0$$

$$T_{eff} = Tf = TG_{00}$$

- This requires that $G_{00}(u_0) > 0$

Condition for a stringy meson

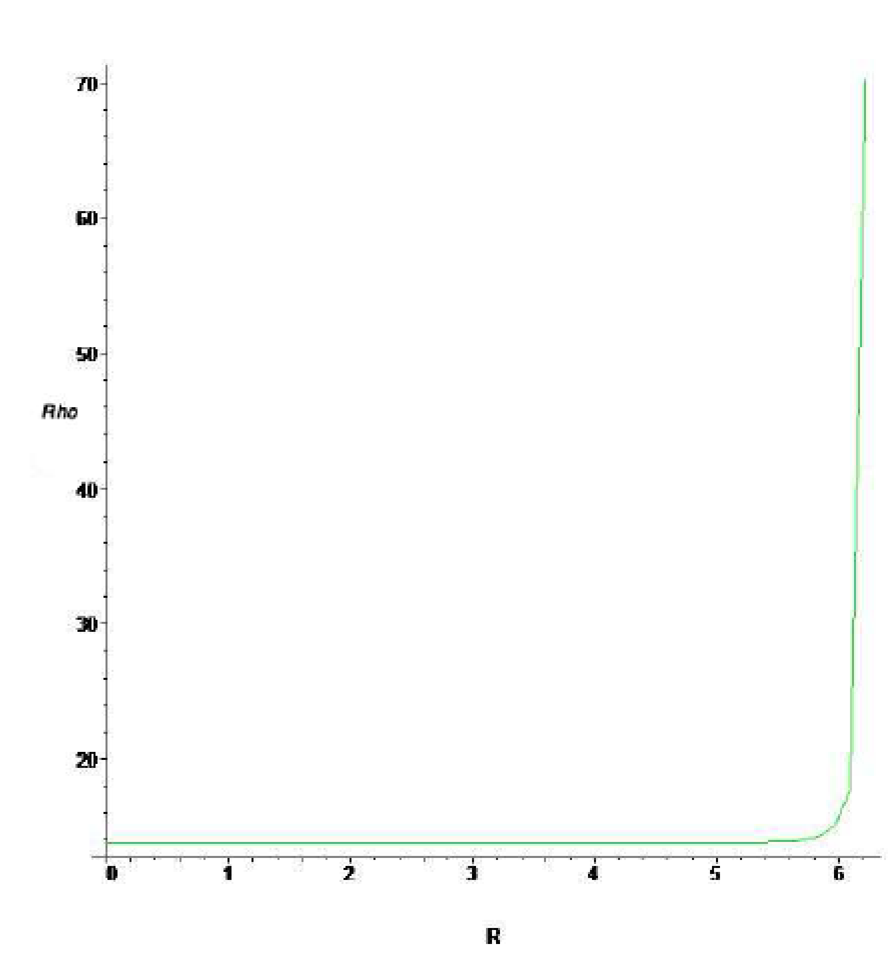
- The conditions to have a solution together with the last constraint read

$$\text{a) } \partial_u G_{00}(u_0) = 0 \text{ and } G_{00}(u_0) > 0 \text{ or}$$
$$\text{b) } G_{uu} \to \infty \text{ and } G_{00}(u_0) > 0$$

- These are precisely the condition to have a confining Wilson line

How close is the |_| string to the real holographic one

- This is a numerical calculation of the profile for a string with J=3 rotating in Witten's model background



Energy and Angular momentum

- The Noether charges associated with the shift of t and θ

$$E = T\int d\sigma \frac{\sqrt{f^2+g^2(u')^2}}{\mathcal{E}} = T\int d\sigma\gamma\sqrt{f^2+g^2(u')^2}$$

$$J = T\int d\sigma\omega R^2 \frac{\sqrt{f^2+g^2(u')^2}}{\mathcal{E}} = T\int d\sigma\omega R^2\gamma\sqrt{f^2+g^2(u')^2}$$

- The contribution of the vertical segments

$$E_I = = T\int_{u_\Lambda}^{u_f} du\gamma\sqrt{\frac{f^2}{(\dot{u})^2}+g^2} = 2\gamma_0 T\int_{u_\Lambda}^{u_f} dug(u) \equiv 2\gamma_0 m_{SEP}$$

$$J_I = = T\int d\sigma\omega R^2\gamma\sqrt{f^2+g^2(u')^2} = 2\gamma_0\omega R_0^2 T\int_{u_\Lambda}^{u_f} dug(u) \equiv 2\gamma_0 m_{SEP}\omega R_0^2$$

Energy and Angular momentum

- The string end-point mass is defined as

$$m_{sep} = T\int_{u_\Lambda}^{u_f} du\, g(u)$$

- The horizontal segment contributes

$$E_{II} = = T\int_{-R_0}^{R_0} dR\gamma\sqrt{f^2+g^2(\dot{u})^2} = f(u_\Lambda)T\int_{-R_0}^{R_0}\frac{dR}{\sqrt{1-\omega^2R^2}} = 2\frac{T_{eff}}{\omega}\arcsin(\omega R_0)$$

$$J_{II} = = T\int_{-R_0}^{R_0} dR\gamma\sqrt{f^2+g^2(\dot{u})^2} = f(u_\Lambda)Tw\int_{-R_0}^{R_0}\frac{dRR^2}{\sqrt{1-\omega^2R^2}} = 2T_{eff}[\frac{1}{\omega^2}\arcsin(\omega R_0) - \omega R_0\sqrt{1-\omega^2R_0^2}$$

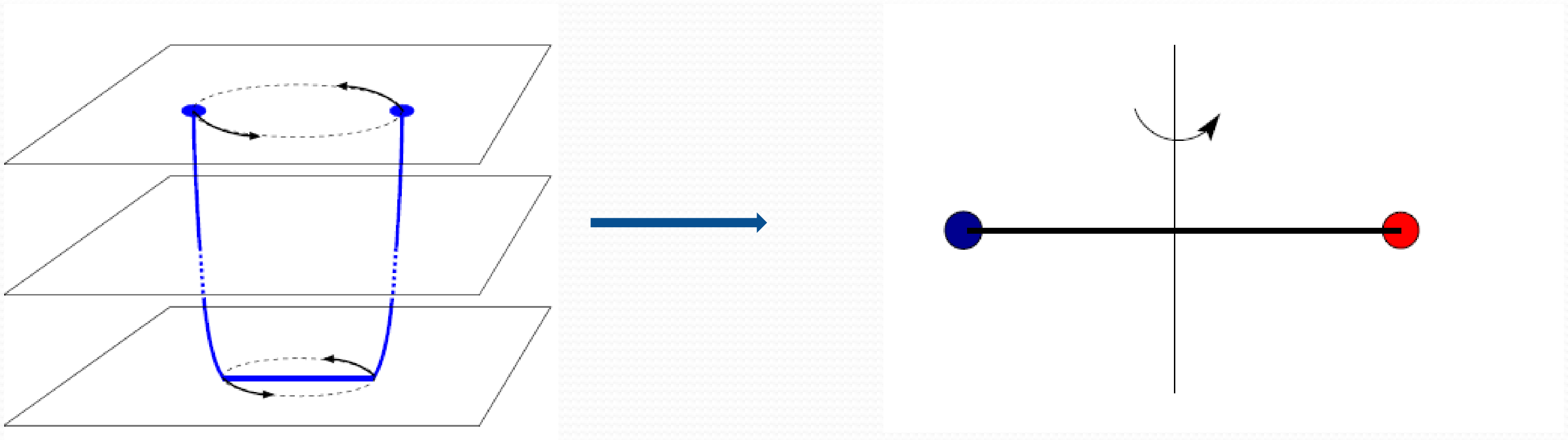
- Combining together all the segments we get

$$E = 2m_{sep}\gamma_0 + 2\frac{T_{effe}}{\omega}\arcsin(\omega R_0)$$

$$J = m_{sep}\gamma_0\omega R_0^2 + \frac{T_{effe}}{\omega^2}\arcsin(\omega R_0)$$

From holographic string to string with massive endpoints

- It is now clear that we can map the energy and angular momentum of the holographic spinning string to those of a string in flat space time with massive endpoints



Energy and Angular momentum

- One can trivially generalize this result to the case of two different string endpoint masses and correspondingly R1 and R2 arms.

$$E = m_{sep_1}\gamma_1 + m_{sep_2}\gamma_2 + \frac{T_{effe}}{\omega}[\arcsin(\omega R_1) + \arcsin(\omega R_2)]$$

$$J = \frac{1}{2} m_{sep_1}\gamma_1 R_1^2 + \frac{1}{2} m_{sep_2}\gamma_2 R_2^2 + \frac{T_{effe}}{2\omega^2}[\arcsin(\omega R_1) + \arcsin(\omega R_2)]$$

- Unlike the original Regge trajectory , now we cannot eliminate w and R and get a direct relation between E and J.

Small and large mass approximations

- We can get such relations in the limit of
- Small mass

$$J = \alpha' E^2 - \alpha' \frac{4\pi^{1/2}}{3} (m_1^{3/2} + m_2^{3/2}) \sqrt{E}$$

- Large mass

$$J_4 = \frac{2m^{1/2}}{T3\sqrt{3}} (E - 2m)^{3/2} + \frac{7}{\sqrt{1083}m^{1/2}T} (E - 2m)^{5/2} - \frac{1003}{\sqrt{2332803}Tm^{3/2}} (E - 2m)^{7/2}$$

On the quantization of the holographic stringy meson

On the quantization (Briefly)

- The exact quantization of an open string with fixed endpoint is characterized by the relation

$$E_n = \sqrt{(TL)^2 + 2\pi T\left(n - \frac{D-2}{24}\right)}$$

- For a rotating string with no massive endpoints (v=1)

$$E_n = \sqrt{(2\pi TJ) + 2\pi T\left(n - \frac{D-2}{24}\right)}$$

- Which translates to

$$n + J = \alpha' E_n^2 + a \qquad a = \frac{D-2}{24}$$

On the quantization (Briefly)

- For strings with massive endpoints there are two major differences:
- (i) The relation between J T and E is more complicated as we have seen above
- (ii) The eigen-frequencies are not anymore

$$w_n = n$$

- But rather given for the symmetric and anti-symmetric modes respectively

$$w_n^s = -Aarctan(\frac{w_n \pi}{2}) \qquad w_n^s = Aarccot(\frac{w_n \pi}{2})$$

$$A = \frac{2}{\pi}\frac{V}{\sqrt{1-V^2}} sin^{-1}(V$$

- We have developed the relation for (E^2, n, α', a) in the
- WKB approximation and in the large and small masses

Fits of Stringy mesons with massive endpoints to experimental data

Fitting analysis

- Next we confront the theoretical modified Regge relations (M^2, J, n) of the holographic stringy mesons with experimental data.
- It is easier to analyze separately (M^2, J) and (M^2, n)
- For (M^2, J) we use
- (i) The linear original Regge relation

$$J_l = \alpha' E^2 + \alpha_0$$

Fitting analysis: (M^2, J)

- (2) The modified massive Regge relation

$$E_m = 2m\left(\frac{\omega R \arcsin(\omega R) + \sqrt{1-(\omega R)^2}}{1-(\omega R)^2}\right)$$

$$J_m = \frac{m^2}{T}\frac{(\omega R)^2}{(1-(\omega R)^2)^2}\left(\arcsin(\omega R q) + \omega R\sqrt{1-(\omega R)}(\omega R)^2 + \alpha\right.$$

- (3) The small mass limit

$$J_{sm} = \alpha' E^2 - \alpha' \frac{4\pi^{1/2}}{3}(m_1^{3/2} + m_2^{3/2})\sqrt{E} + \alpha_0$$

- (4) The large mass limit

$$J_{lm} = \frac{2m^{1/2}}{T3\sqrt{3}}(E-2m)^{3/2} + \frac{7}{\sqrt{1083}m^{1/2}T}(E-2m)^{5/2} - \frac{1003}{\sqrt{2332803}Tm^{3/2}}(E-2m)^{7/2}$$

Fitting analysis (M^2, n)

- The original linear Regge relation

$$n_l = \alpha' E^2 + \alpha_0$$

- The WKB approximation

$$n_{WKB} = a + \frac{1}{\pi}\int_{x_-}^{x^+} dx \sqrt{(E - V(x; J_s))^2 - m^2 - (J_q/x)^2}$$

Extracted parameters

- The parameters we extract from the fits with the lowest χ^2 are
- (i) α' measured in Gev^{-2} or the string tension $T = \frac{1}{2\pi\alpha'}$
- (ii) α_0 the intercept (dimensionless)
- (iii) (m_1, m_2) the string endpoint masses

The meson trajectories fitted

- For (M^2, J) we compared with the following Regge trajectories

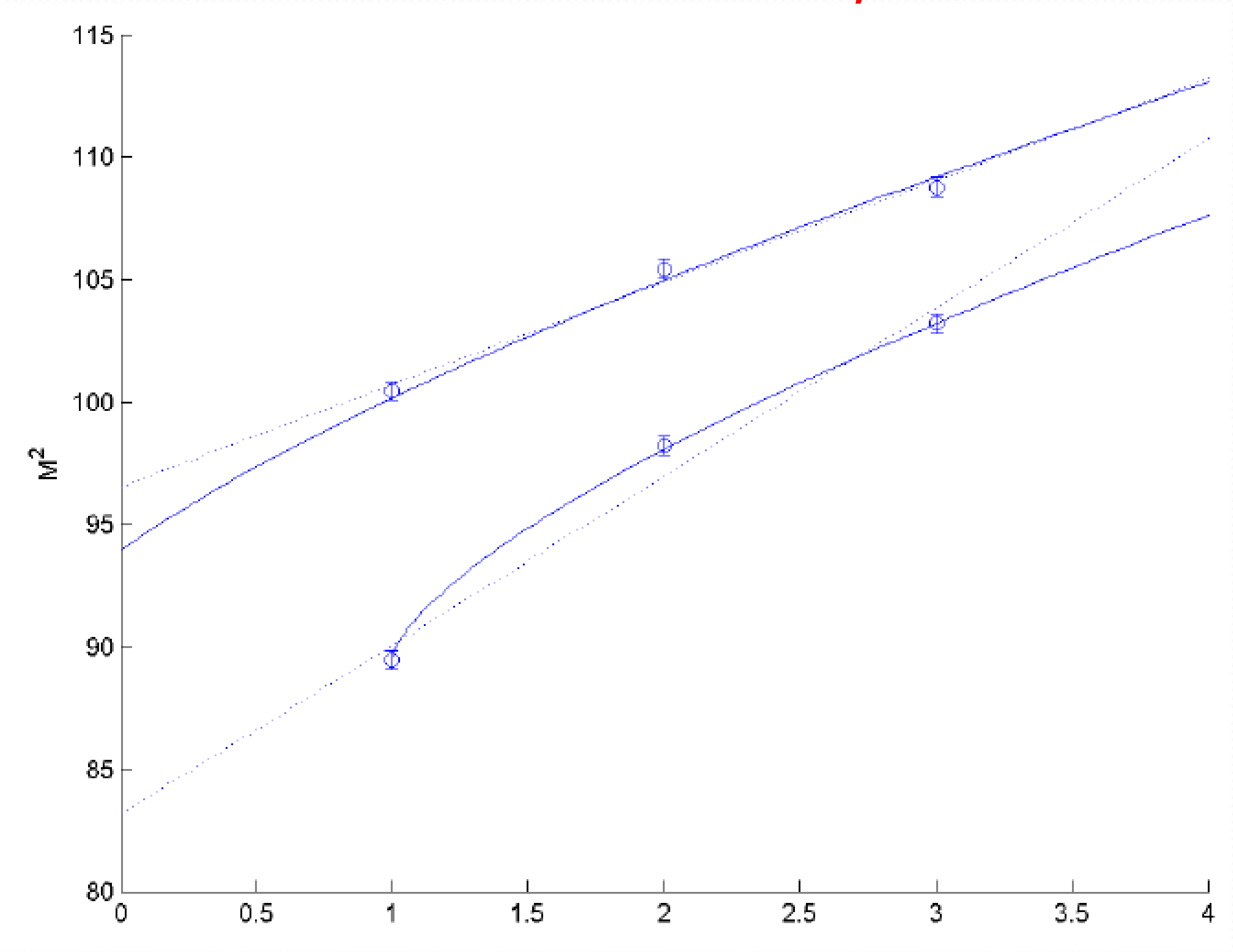
ρ **meson parent** $-\ \rho(775.5 \pm 0.4)(1^{--}), a_2(1318.3 \pm 0.6)(2^{++}), \rho_3(1688.8 \pm 2.1)(3^{--}),$
$a_4(2001 \pm 10)(4^{++}), \rho_5(2350)(5^{--}), a_6(2450 \pm 130)(6^{++})$
ρ **meson daughter** $-\ a_0(984.7 \pm 1.2)1^-(0^{++})\rho(1459 \pm 11)1^+(1^{--}), a_2(1732 \pm 16)1^-(2^{++}), \rho_3(1990)1^+(3^-$
ω **meson** $-\ \omega(782.65 \pm 0.12)(1^{--}), f_2(1275.4 \pm 1.1)(2^{++}),$
$\omega_2(1667 \pm 4)(3^{--}), f_4(2025 \pm 10)(4^{++}), f_6(2465 \pm 50)$
$\mathbf{K}^*$ **meson** $-\ K^*(891.66 \pm 0.26)(1^-), K_2^*(14256 \pm 1.5)(2^+),$
$K_3^*(1776 \pm 7)(3^-), K_4^*(2045 \pm 9)(4^+), K_5^*(2382 \pm 24)(5^-)$
$\mathbf{c\bar{c}}$ **meson** $-\ \Psi(1S)(3.0969), \chi_{c2}(1P)(3.5563), Psi(1D)(3.836)$
$\mathbf{b\bar{b}}$ **meson** $-\ \Upsilon(1s)(1^{--})(9.46), \chi(1P)(2^{++})(9.912), \psi(1D)(3^{--})(10.161)$

- For (M^2, n) the trajectories used

$\mathbf{c\bar{c}}$ **meson** $-\ \Psi(1s)(3.0969), Psi(2s)(3.686), Psi(3s)(3.7699), Psi(4s)(4.04)$
$\mathbf{b\bar{b}}$ **meson** $-\ \Upsilon(1s)(9.4604), \Upsilon(2s)(10.023), \Upsilon(3s)(10.3533), \Upsilon(4s)(10.580), \Upsilon(5s)(10.865)$

The botomonium trajectories

- To emphasis the deviation from the linearity we start with the botomonium trajectories



The botomonium trajectories

- The parameters extracted from the best fit of the massive modified trajectory are for the parent one

$$a = 1, \alpha' = 0.641, 2m = 9460$$

- For the daughter trajectory the parameters are very similar ($a = 0.62.$)
- The ratio between the χ^2 of the linear and massive trajectories is

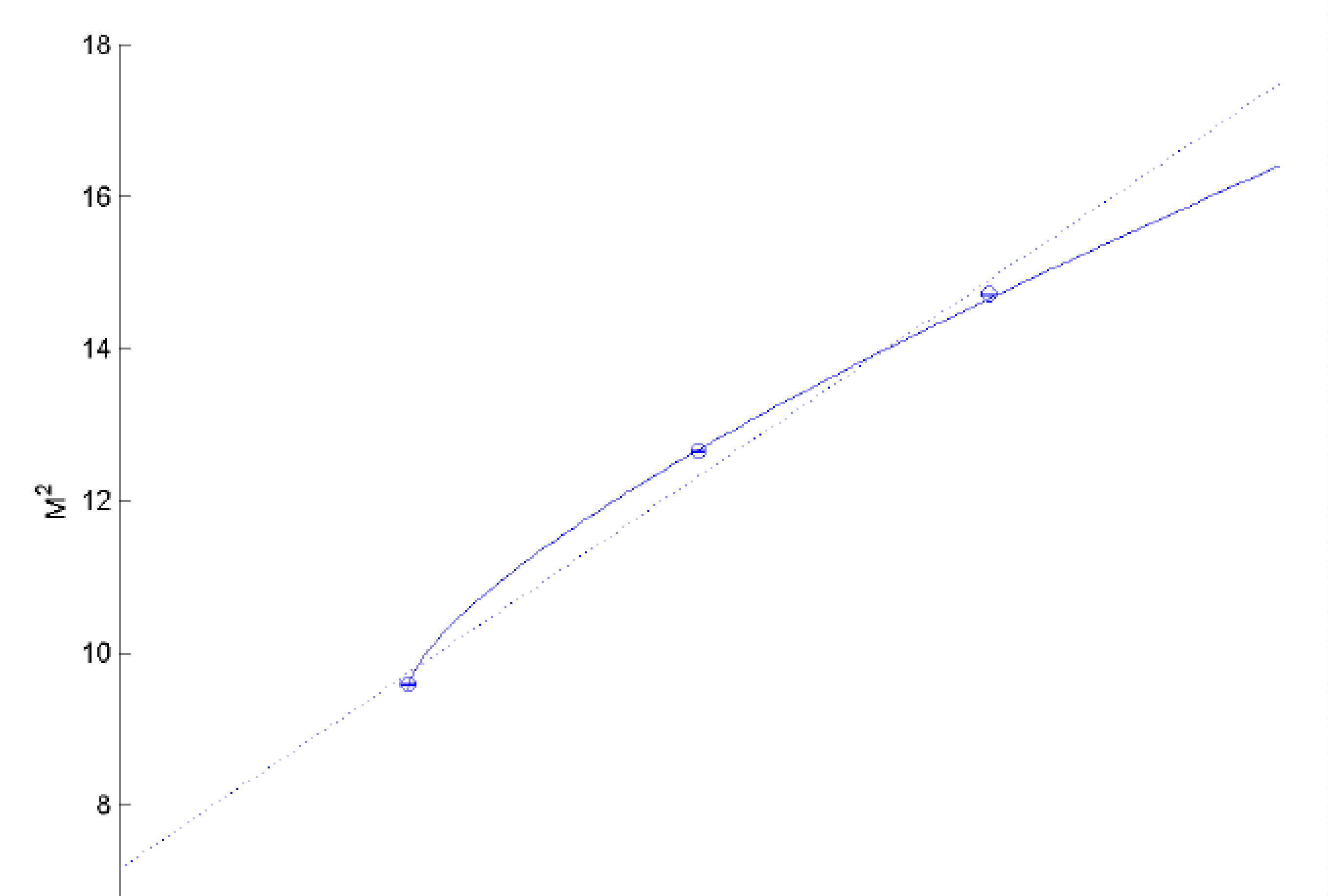
$$\chi_c^2/\chi_l^2 = 0.0001$$

The charmonium trajectories

- For the charmonium trajectory we get

$$a = 1, \alpha' = 0.999, 2m = 3086$$

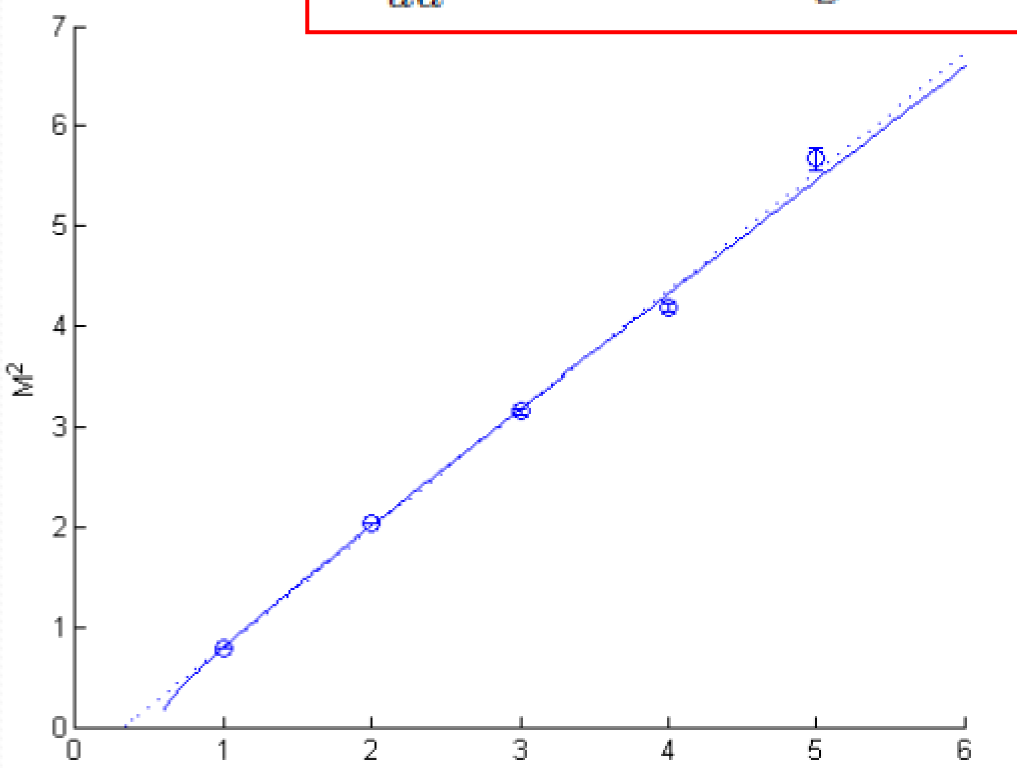
- Now the improvement over the linear is $\chi_c^2/\chi_l^2 = 0.041$



The K* trajectories

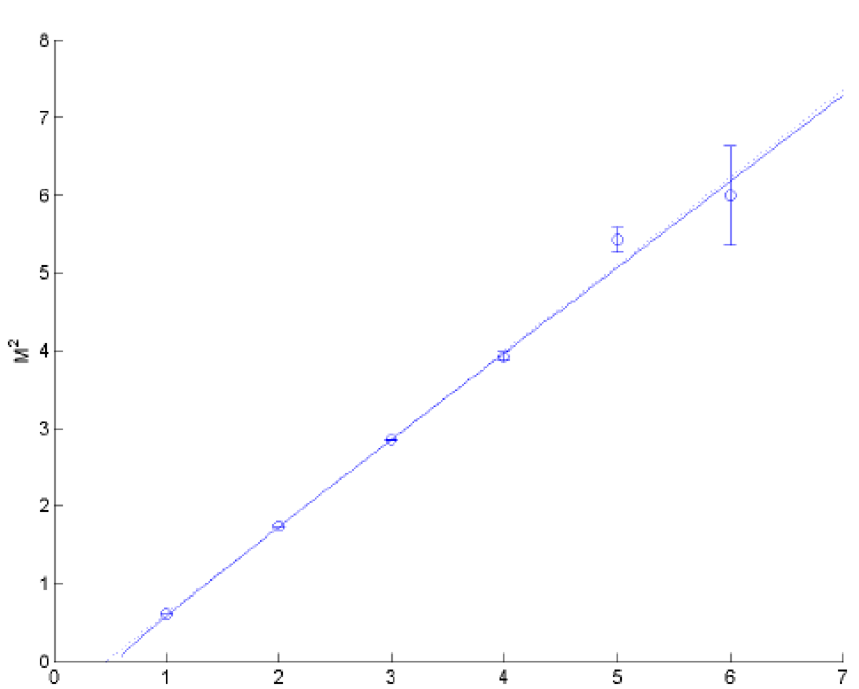
- The K* mesons $K^*(892), K_2^*(1430), K_3^*(1780), K_4^*(2045), K_5^*(2380)$ are constructed from $d\bar{s},\ u\bar{s},\ \bar{u}s,\ \text{or}\ \bar{d}s$ and have S=1
- The best fitted tension and intercept are $a = 0.6, \alpha' = 0.91$
- The best fitted masses are

$$m_{ud} = 115 \quad m_s = 410$$



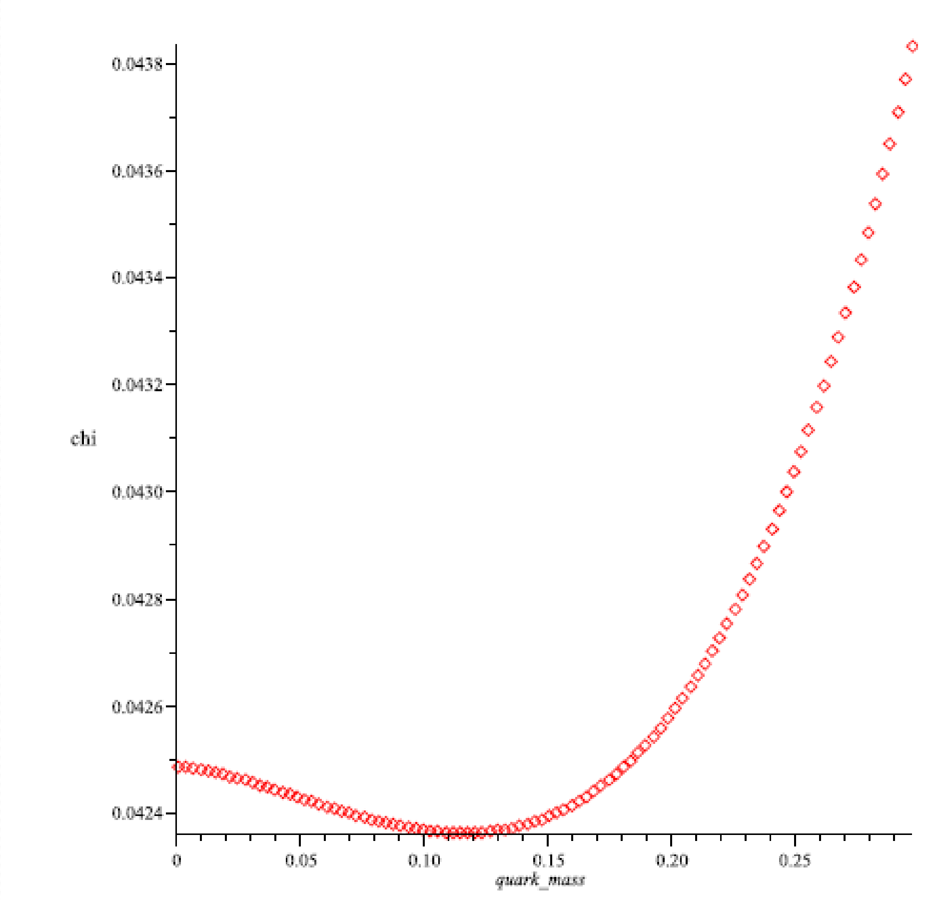
The ρ trajectories

- For the ρ mesons trajectories the difference between the original linear and modified trajectories is the smallest
- For the linear $a = 0.45.\ \alpha' = 0.888$
- For the massive $a = 0.6, \alpha' = 0.916, 2m = 229$
- The ratio is $\chi_c^2/\chi_l^2 = 0.810.$



Extracting the optimal mass of the string endpoint

- To see the optimal value of the mass of the endpoint we computed as a function of m_{sep}.
- It is clear that the massive endpoint fits better than the original linear trajectory



α’ (string tension) and the intercept.

- The trajectories of the u, d and s quarks admit α ‘ (string tension)

$$\alpha' = 0.913 \quad T_{st} = 0.17 \quad Gev^2$$

- For the heavy quarks the best fit of α’ goes down to α ‘ = 0.78 for the ηc and α ‘ = 0.913 for $b\bar{b}$ mesons.
- The intercept of the light quark trajectories is

$$\alpha_0 = 0.6$$

- and for the heavy quakrs

$$\alpha_0 = 1.0$$

The string endpoint masses versus constituent ones

- For the b and the c quark the string endpoint masses found are the same as the constituent masses

$m_b = 4730 = M_{\Upsilon}/2 \qquad m_c = 1543 \sim M_{\Psi}/2$

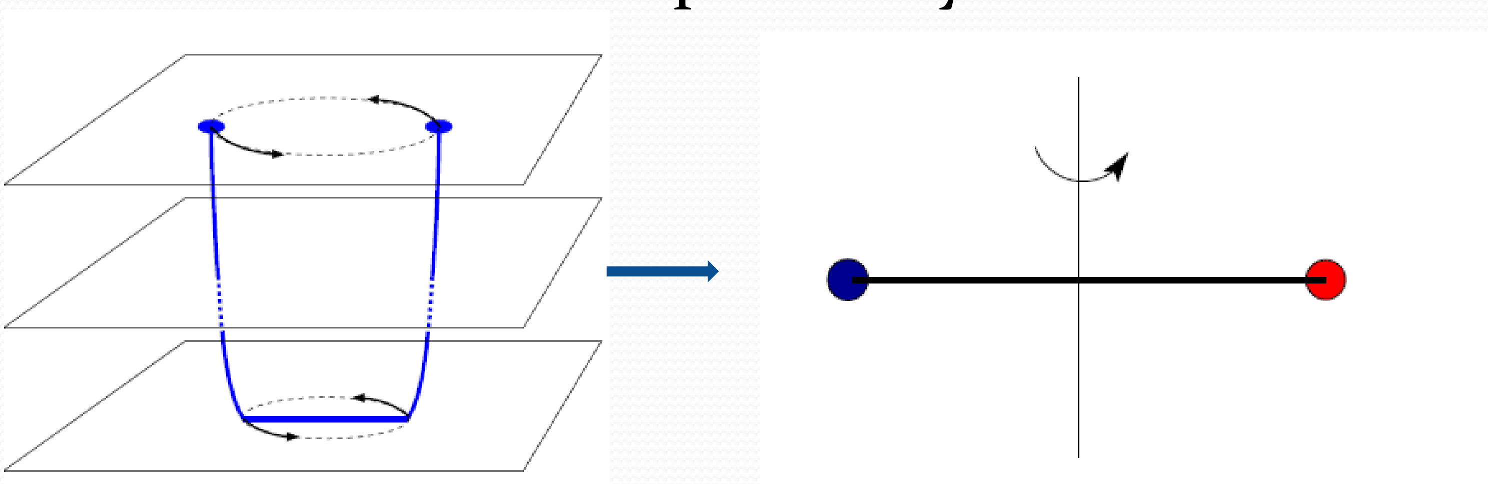
- However the string endpoint mass of s and definitely of the u and d quarks are different from the constituent masses

$m_{u/d} = 115$ versus $M_\rho/2 = 388$

$m_s = 410$ versus $M_{K^*} - M_\rho/2 = 504$

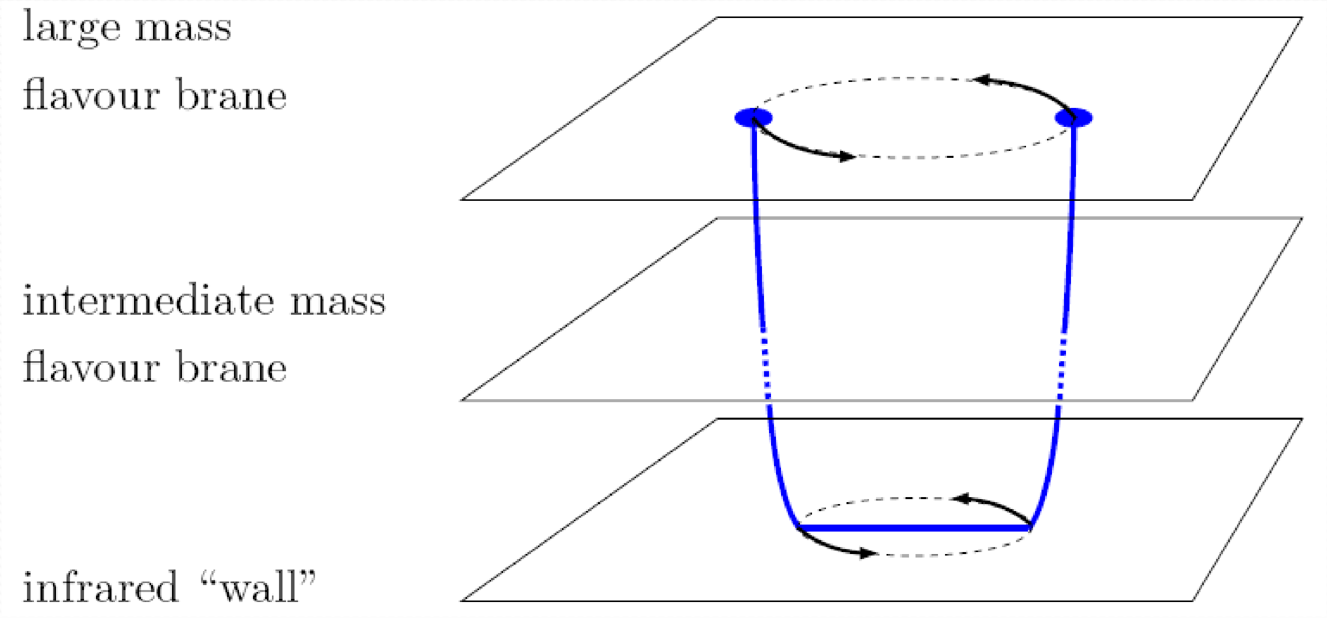
Holography versus massive endpoints toy model

- In the toy model of string with massive endpoints for vanishing orbital angular momentum $J=0$ the length of the string vanishes and hence only the quarks at the endpoints constitute the meson mass
- In holography we get non trivial contribution of the string even with no angular momentum
- Thus the comparison with data favors holography over the massive endpoints toy model .

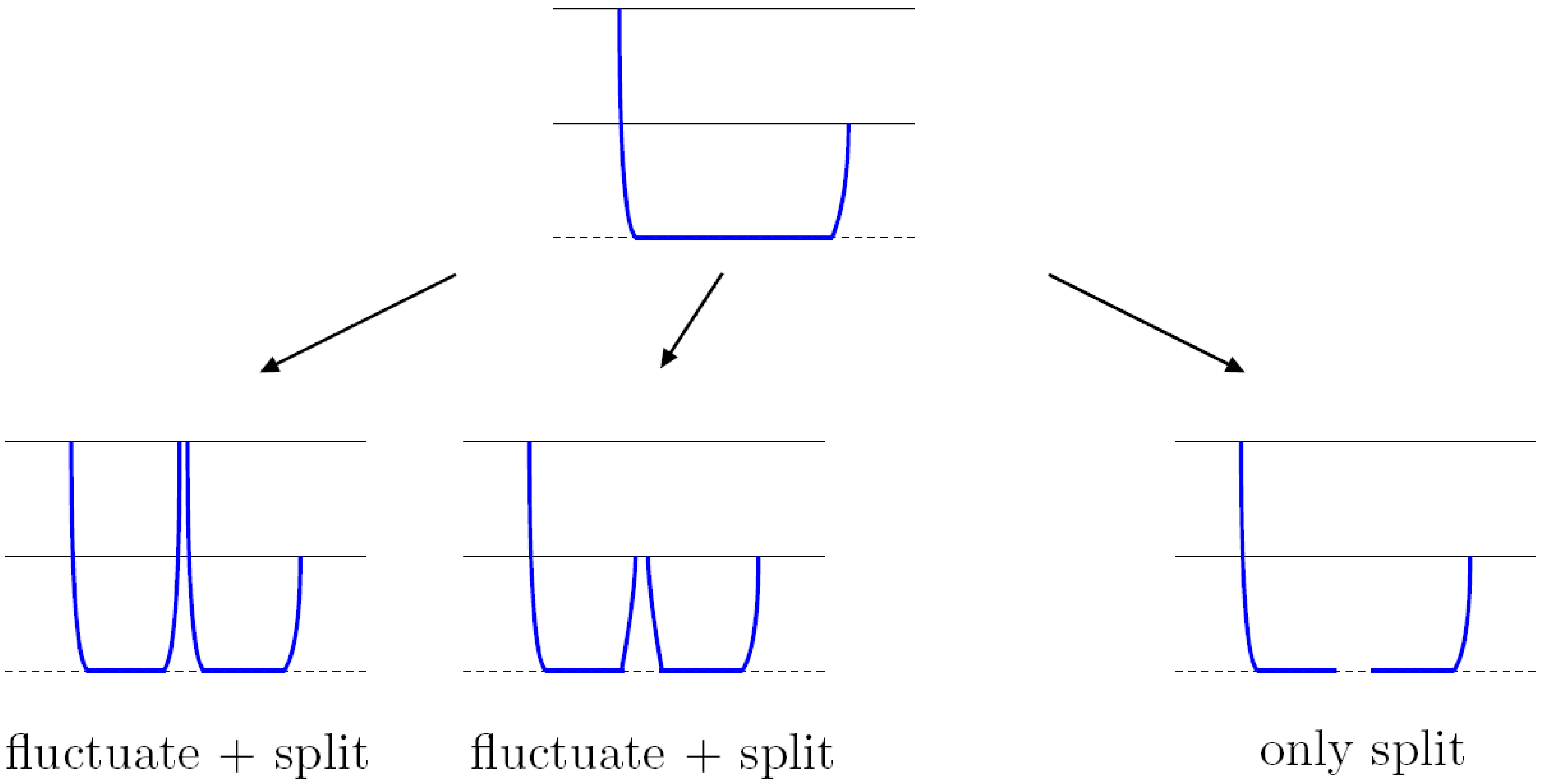


Decay width of Stringy holographic Mesons

The structure of a rotating holographic string



A cartoon of possible decays of a (h,m) meson



Holographic decay- qualitative picture

- Quantum mechanically the stringy meson is unstable.

- Fluctuations of endpoints   splitting of the string

- The string has to split in such a way that the new endpoints are on a flavor brane.

- The decay probability= (to split at a given point)   X (that the split point is on a flavor brane)

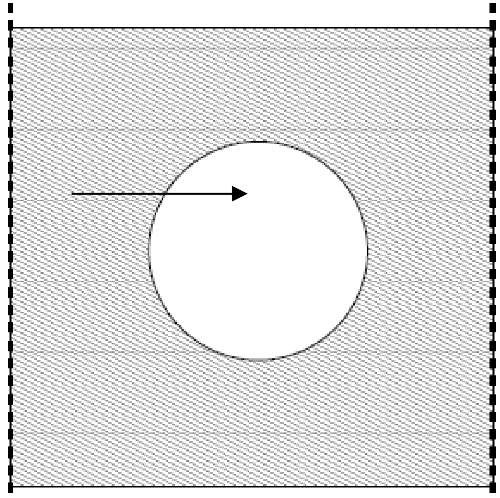
The probability to split of an open string in flat space time was • computed by Dai and Polchinski and by Turok et al.

$$\Gamma = \frac{1}{\pi^{23}\sqrt{2^{45}}}M$$

The split of an open sting in flat space time

An exercise in one loop string calculation

- Intuitively the string can split at any point and hence we expect width~ L
- The idea is to use the optical theorem and compute the total rate by computing the imaginary part of the self energy diagram
- Consider a string stretched around a long compact spatial direction. A winding state splits and joins. In terms of vertex operators it translates to a disk with two closed string vetex operators



•The corresponding amplitude takes the form

$$i\mathcal{A} = \frac{iTN}{g^2} L \left[\frac{\kappa}{2\pi\sqrt{L}}\right]^2 \int_{|z|<1} \mathrm{d}^2 z \, \langle : e^{ip_0 X(0)} :: e^{-ip_0 X(z)} : \rangle$$

where k is the gravitational coupling, g the coefficient of the open string tachyon, the factor L comes from the zero mode.

•Using the ope's we get

$$\langle : e^{ip_0 X(0)} :: e^{-ip_0 X(z)} : \rangle = |z\bar{z}|^{-2} (1 - z\bar{z})^{-\gamma}$$

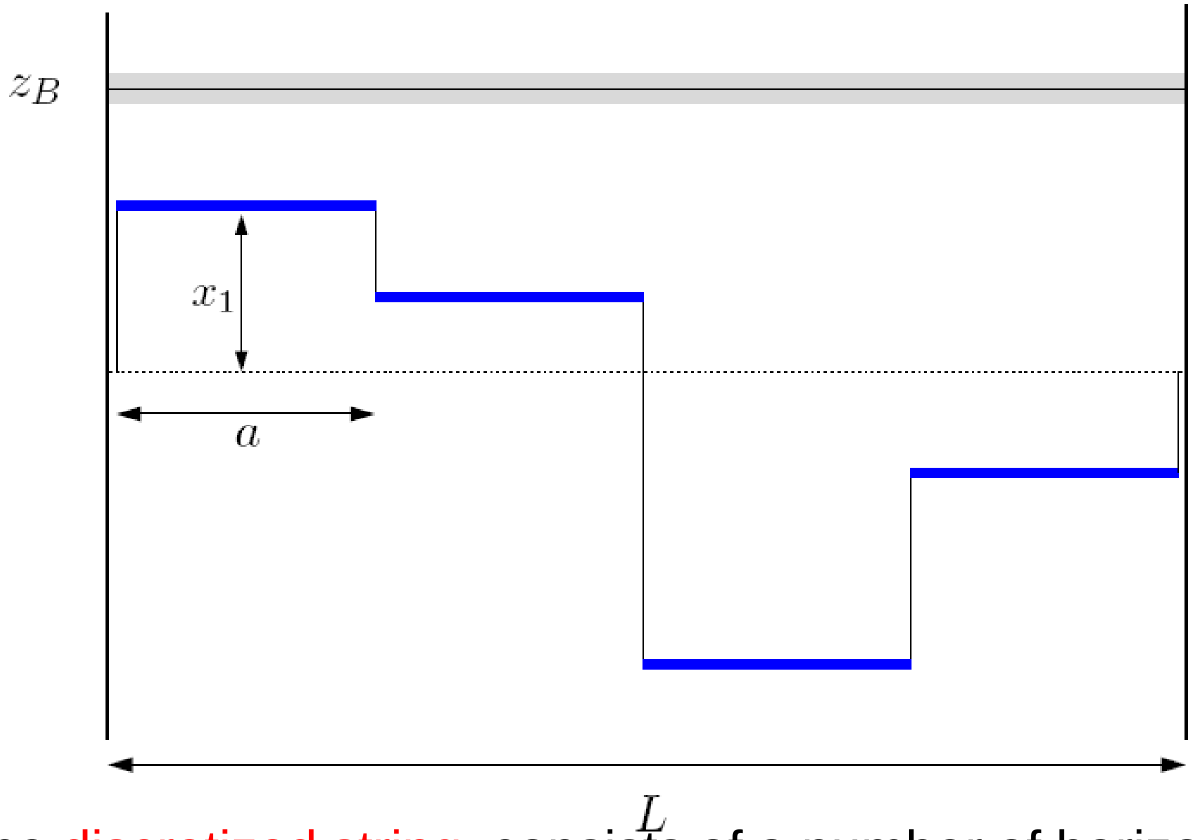
where $\gamma = \frac{L^2 T}{2\pi} - 2.$

•Performing the integral, taking the imaginary part

$$\Gamma = -\operatorname{Im} \delta m = -\frac{1}{2m} \operatorname{Im} \mathcal{A} = \frac{TN\kappa^2}{4g^2 E} \gamma \to \frac{TN\kappa^2}{8\pi g^2} L = \frac{g^2 T^{13}}{2^{26}\pi^{12}} NL$$

String bit approximation

•Using a string bit model the integration over the right subset of configurations becomes easier.



•The discretized string consists of a number of horizontal rigid rods connected by vertical springs.

- The mass of each bead is M, the length is L=(N+1)a and the action is

$$S = \frac{1}{2}\int \mathrm{d}t \left(\sum_{n=1}^{N} M\,\dot{x}_n^2 - \frac{T_{\text{eff}}}{a}\sum_{n=1}^{N+1}(x_n - x_{n-1})^2\right)$$

- The normal modes and their frequencies are

$$y_m = \frac{1}{N+1}\sum_{n=1}^{N}\sin\left(\frac{mn\pi}{N+1}\right)x_n\,, \qquad \omega_m^2 = \frac{4\,T_{\text{eff}}\,N(N+1)}{M_{\text{tot}}\,L}\sin^2\left(\frac{m\pi}{2(N+1)}\right)$$

- In the relativistic limit and large N $\omega_m^2 = m^2\pi^2/L^2$

- The action now is of N decoupled normal modes

$$S = (N+1)M\int \mathrm{d}t \sum_{m=1}^{N}\left(\dot{y}_m^2 - \omega_m^2 y_m^2\right)$$

- The wave function is a product of the wave functions of the normal modes

$$\Psi(\{y_1, y_2, \ldots\}) = \prod_{m=1}^{N}\left(\frac{2(N+1)M\omega_m}{\pi}\right)^{1/4}\exp\left(-(N+1)M\omega_m\,y_m^2\right)$$

•Note that the width of the Gaussian depends on T_{eff} and not on L

$$\lim_{N\to\infty}(N+1)M\omega_m = \lim_{N\to\infty}(N+1)\frac{T_{\text{eff}}L}{N}\frac{m\pi}{L} = T_{\text{eff}}\pi m$$

•The integration interval is when the bead is "at the brane" defined by

$$I_{\text{brane}} : \left[-z_B-\Delta,\, -z_B\right] \cup \left[z_B,\, z_B+\Delta\right],$$

$$I_{\text{space}} : \left\langle -\infty,\, -z_B-\Delta\right] \cup \left[-z_B,\, z_B\right] \cup \left[z_B+\Delta,\, \infty\right\rangle$$

•By computing the decay width for various values of N and extrapolating to large N we find that the decay rate is approximated by

$$\Gamma_{\text{beads}} = \text{const.}\cdot\exp\left(-1.0\,\frac{z_B^2}{\alpha'_{\text{eff}}}\right)\ \Gamma_{\text{open}}$$

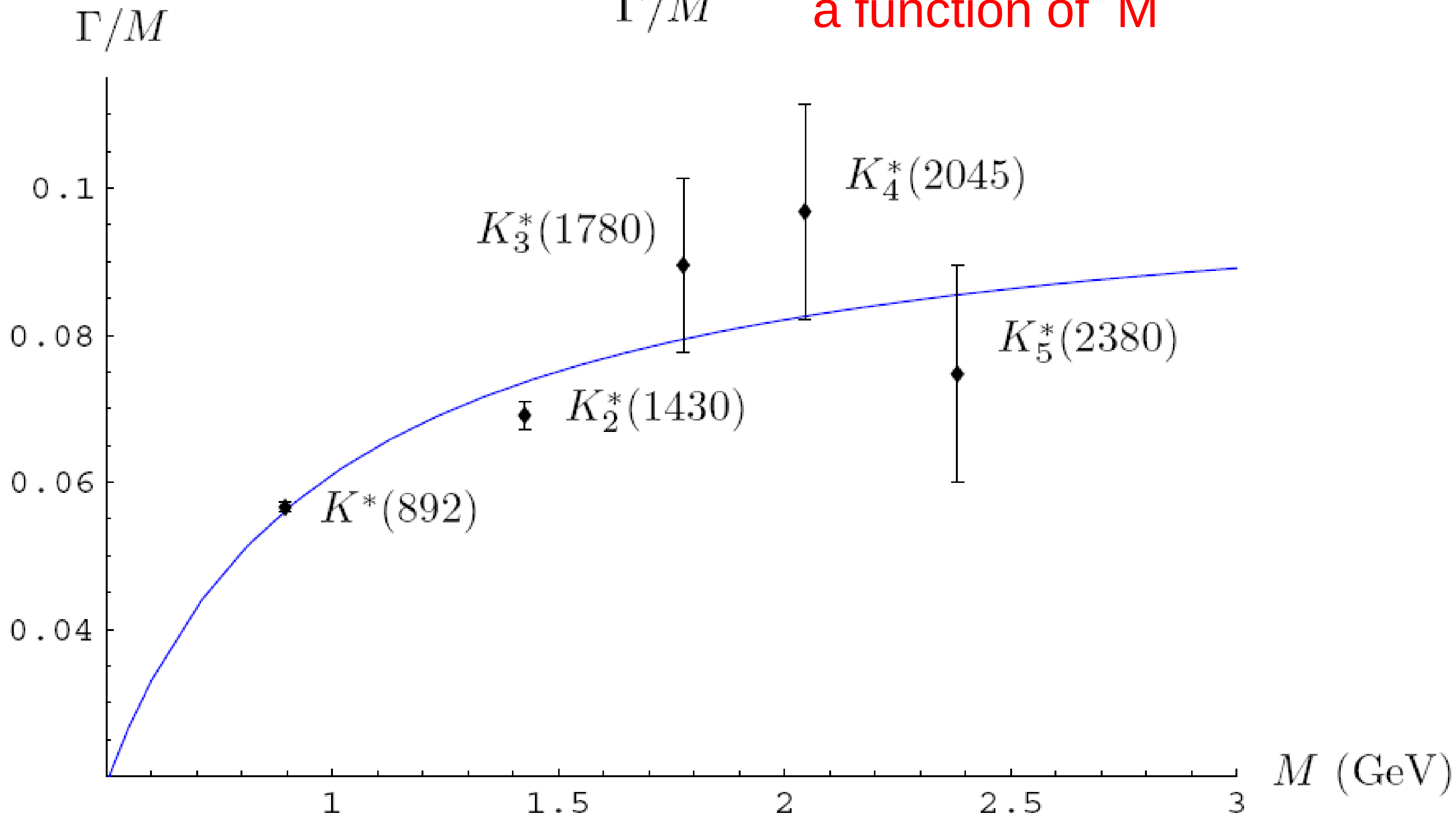
$$\Gamma = Const\ \cdot exp\left(\frac{m_{sep}^2}{T_{st}}\right)\Gamma_{open}$$

Γ/M- Correction to the decay width due to msep

- The basic CNN model predicts $\Gamma/M = \text{const.}$ In fact Γ/L is *constant*

and hence incorporating the corrections due to the massive endpoints we find the following blue curve which fits the data points of the K* mesons

Γ/M a function of M



Stringy holographic Baryons

Stringy Baryons in hologrphy

- How do we identify a baryon in holography ?

- Since a quark corresponds to a string, the baryon has to be a structure with N_c strings connected to it.

- Witten proposed a baryonic vertex in $AdS_5 x S^5$ in the form of a wrapped D5 brane over the S^5.

- On the world volume of the wrapped D5 brane there is a CS term of the form

$$S_{cs} = \int_{\mathbf{S}^5 \times \mathbf{R}} a \wedge \frac{G_5}{2\pi}.$$

Baryonic vertex

- The flux of the five form is

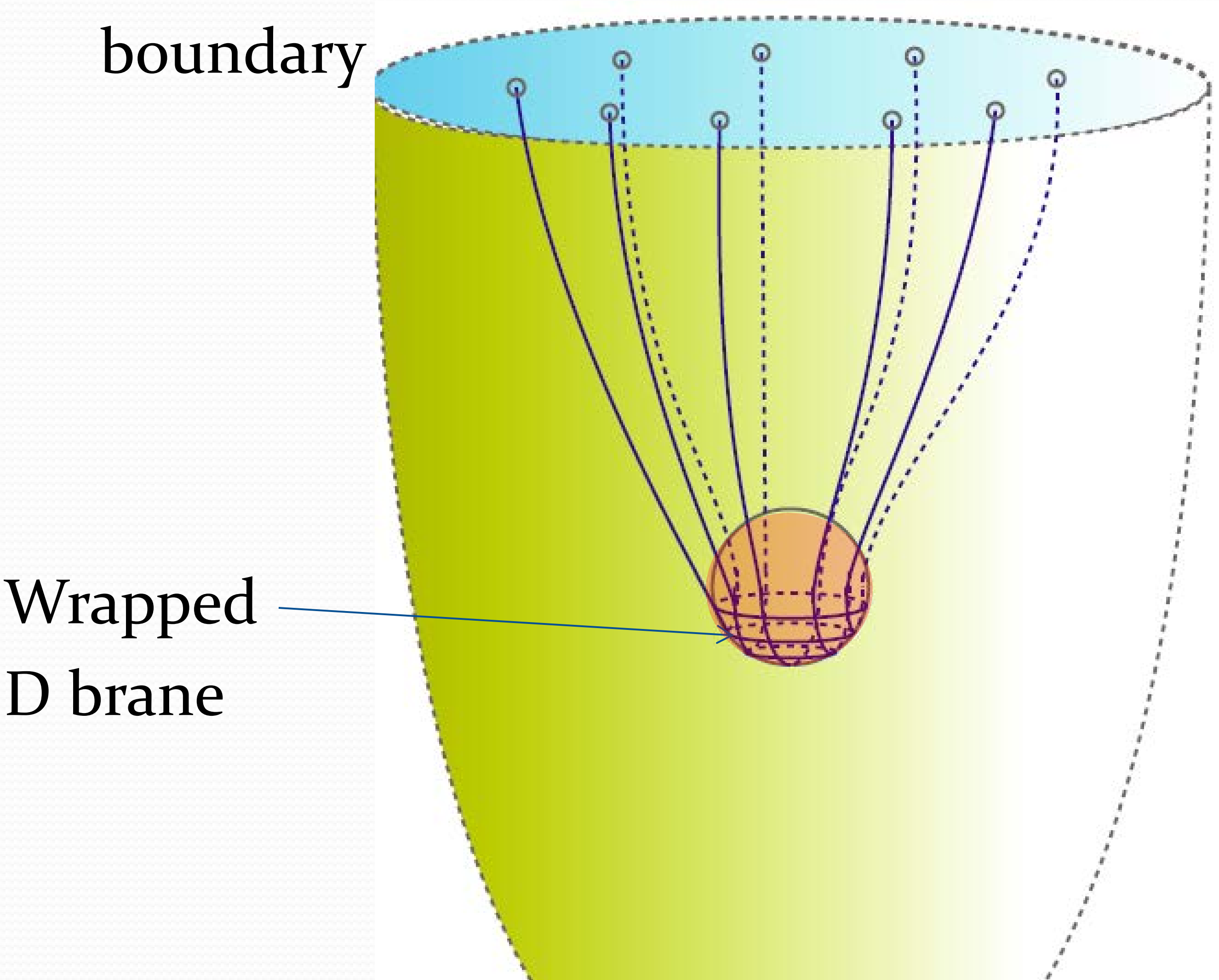
$$\int_{S^5} \frac{G_5}{2\pi} = N_c$$

- This implies that there is a charge N_c for the abelian gauge field. Since in a compact space one cannot have non-balanced charges there must be

N_c strings attached to it.

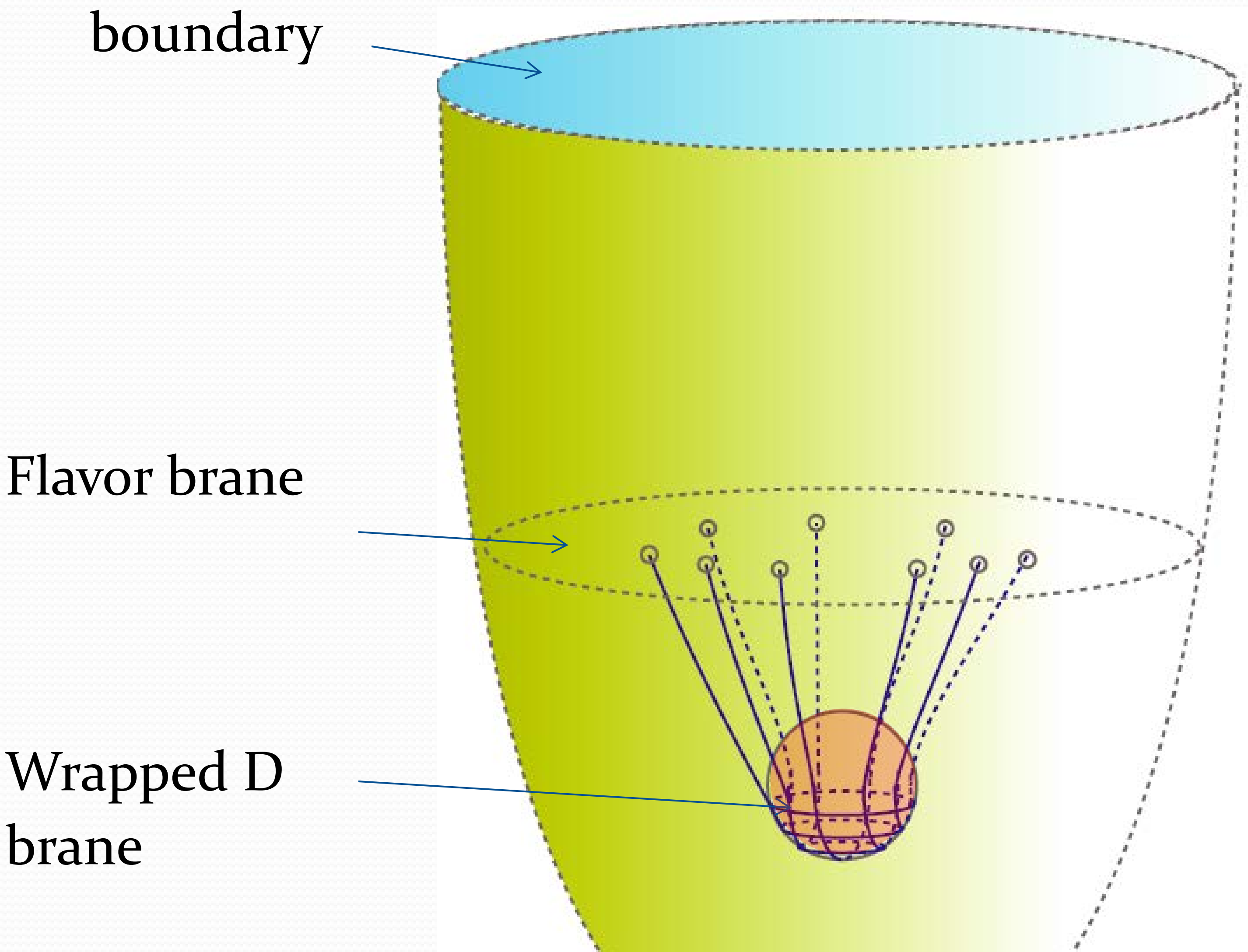
External baryon

- External baryon – Nc strings connecting the baryonic vertex and the boundary



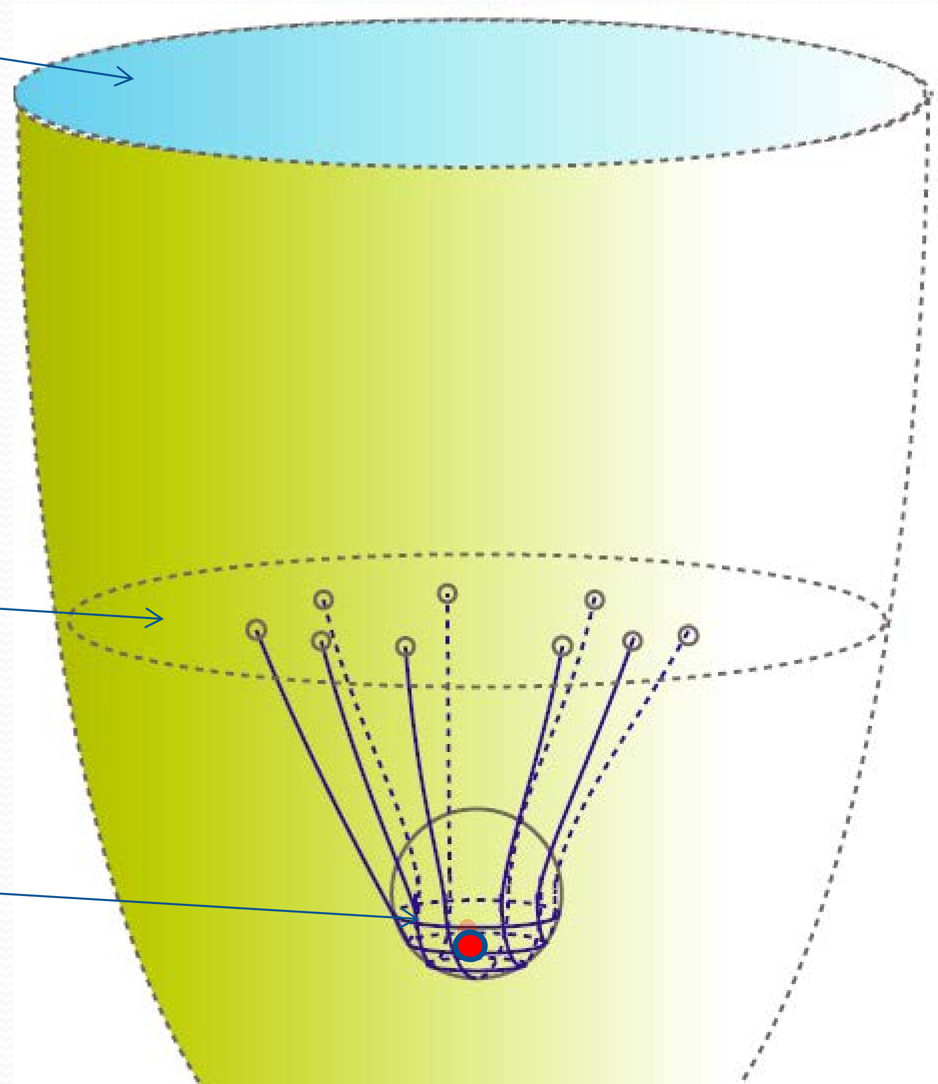
Dynamical baryon

- Dynamical baryon – Nc strings connecting the baryonic vertex and flavor branes



Dynamical baryon in the n-c Ads6 model

- In this model the baryonic vertex is a Do brane of the non-critical compact D4 brane background.



boundary

Flavor brane

unwrapped Do brane

The location of the baryonic vertex

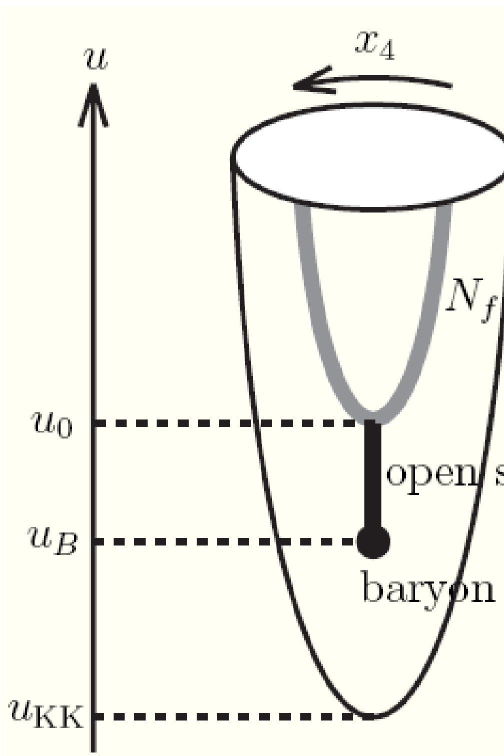
- We need to determine the location of the baryonic vertex in the radial direction.
- In the leading order approximation it should depend on the wrapped brane tension and the tensions of the Nc strings.
- We can do such a calculation in a background that corresponds to confining (like gSS) and to deconfining gauge theories. Obviously we expect different results for the two cases.

- The location of the baryonic vertex in the radial direction is determined by ``static equillibrium”.

$$S = -T_4 \int dt d\Omega_4 e^{-\phi} \sqrt{-\det g_{\mathrm{D4}}} - N_c T_f \int dt du \sqrt{-\det g_{\mathrm{string}}}$$

- The energy is a decreasing function of x=uB/uKK and hence it will be located at the tip of the flavor brane

$$\mathcal{E}_{\mathrm{conf}}(x; x_0) = \frac{1}{3}x + \int_x^{x_0} \frac{dy}{\sqrt{1 - y^{-3}}}$$

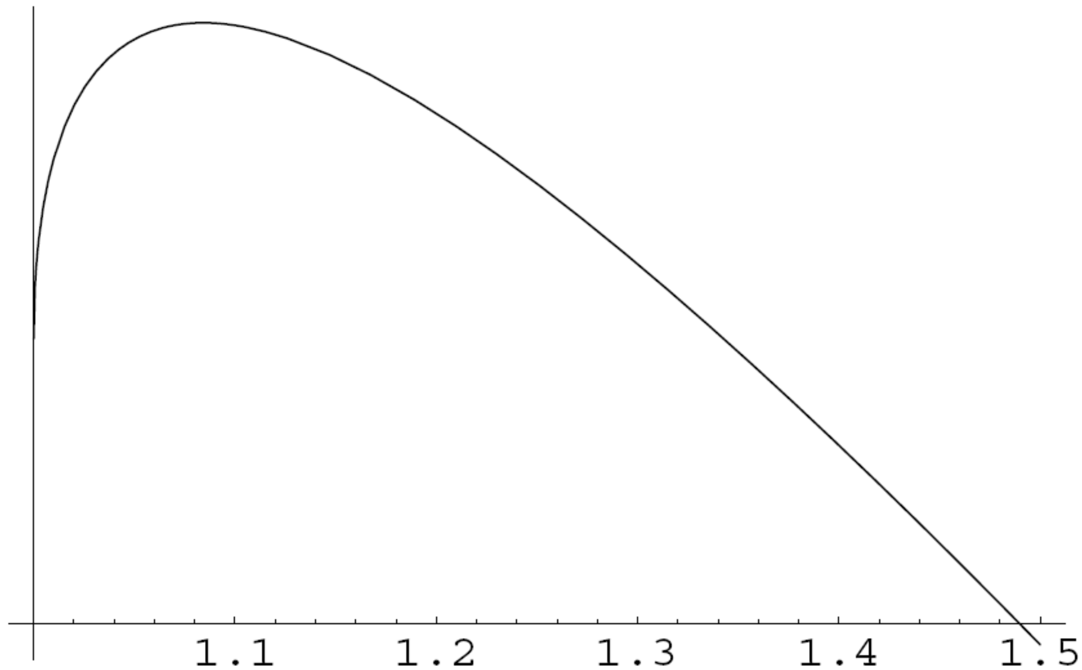


- It is interesting to check what happens in the deconfining phase.
- For this case the result for the energy is

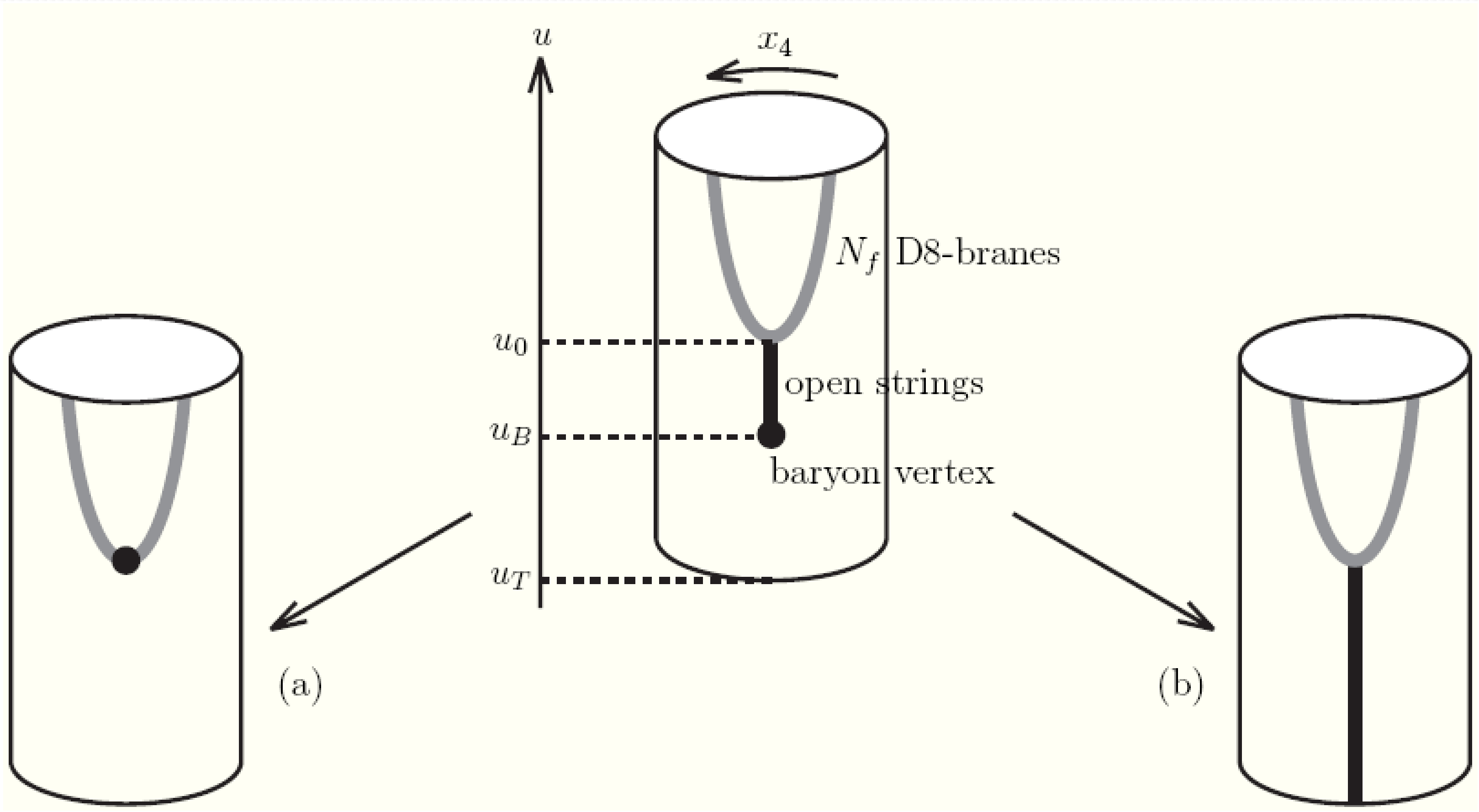
$$\mathcal{E}_{\text{deconf}}(x;x_0)=\frac{1}{3}x\sqrt{1-\frac{1}{x^3}}+(x_0-x)$$

- For $x>x_{cr}$ low temperature stable baryon
- For $x<x_{cr}$ high temperature dissolved baryon

The baryonic vertex falls into the black hole

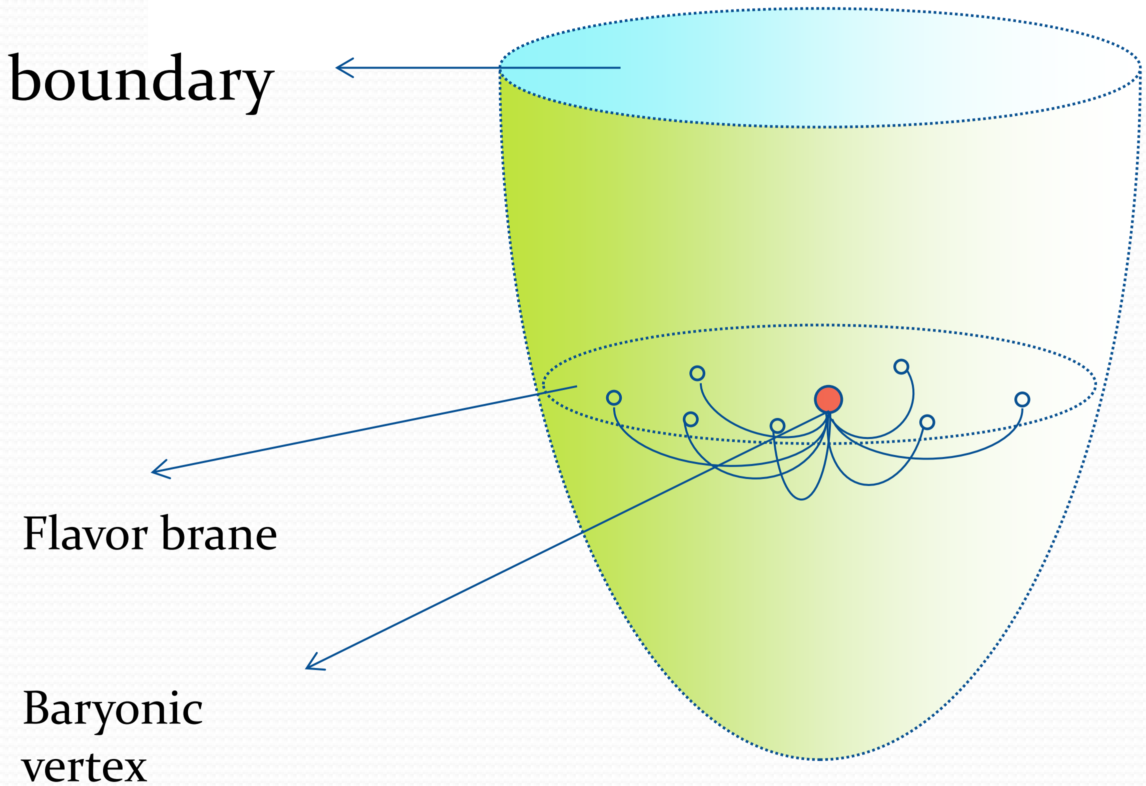


The location of the baryonic vertex at finite temperature



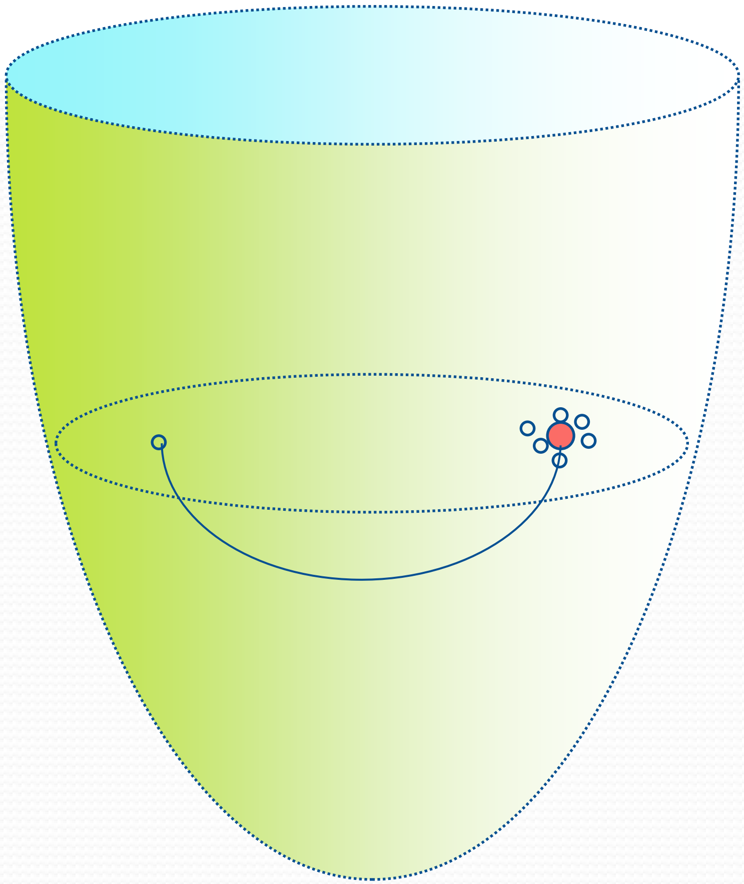
A possible baryon layout

- A possible dynamical baryon is with Nc strings connected to the flavor brane and to the BV which is also on the flavor brane.



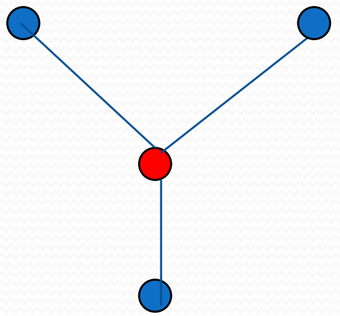
Nc-1 quarks around the Baryonic vertex

- Another possible layout is that of one quark connected with a string to the BV to which the rest of the Nc-1 quarks are attached.

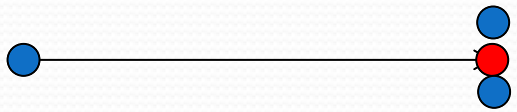


From large Nc to three colors

- Naturally the analog at Nc=3 of the symmetric configuration with a central baryonic vertex is the old Y shape baryon



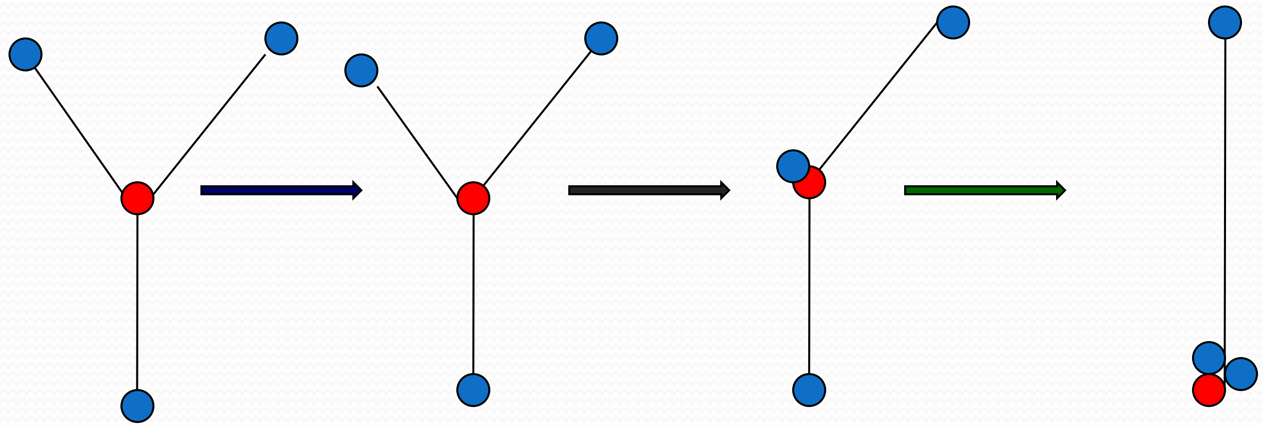
- The analog of the asymmetric setup with one quarks on one end and Nc-1 on the other is a straight sting with quark and a di-quark on its ends.



Stability of an excited baryon

- ‘t Hooft showed that the classical Y shape three string configuration is unstable. An arm that is slightly shortened will eventually shrink to zero size.
- We have examined Y shape strings with massive endpoints and with a massive baryonic vertex in the middle.
- The analysis included numerical simulations of the motions of mesons and Y shape baryons under the influence of symmetric and asymmetric disturbance.
- We indeed detected the instability
- We also performed a perturbative analysis where the instability does not show up.

Baryonic instability



The conclusion from both the simulations and the qualitative analysis is that indeed the Y shape string configuration is unstable to asymmetric deformations.

Thus an excited baryon is an unbalanced single string with a quark on one side and a di-quark and the baryonic vertex on the other side.

Stringy holographic Baryons versus experimental data

Baryons are straigh strings!

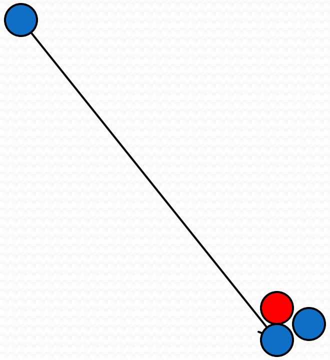
- It is straightforward to realize that the Y shape structure has

$$\alpha_Y' = 2/3\, \alpha_l'$$

A quick glance on the baryon trajectories shows that they admit the roughly (10%) the same α' as that of the mesons. Thus we conclude that baryons are straight strings and not Y shape strings

Excited baryon as a single string

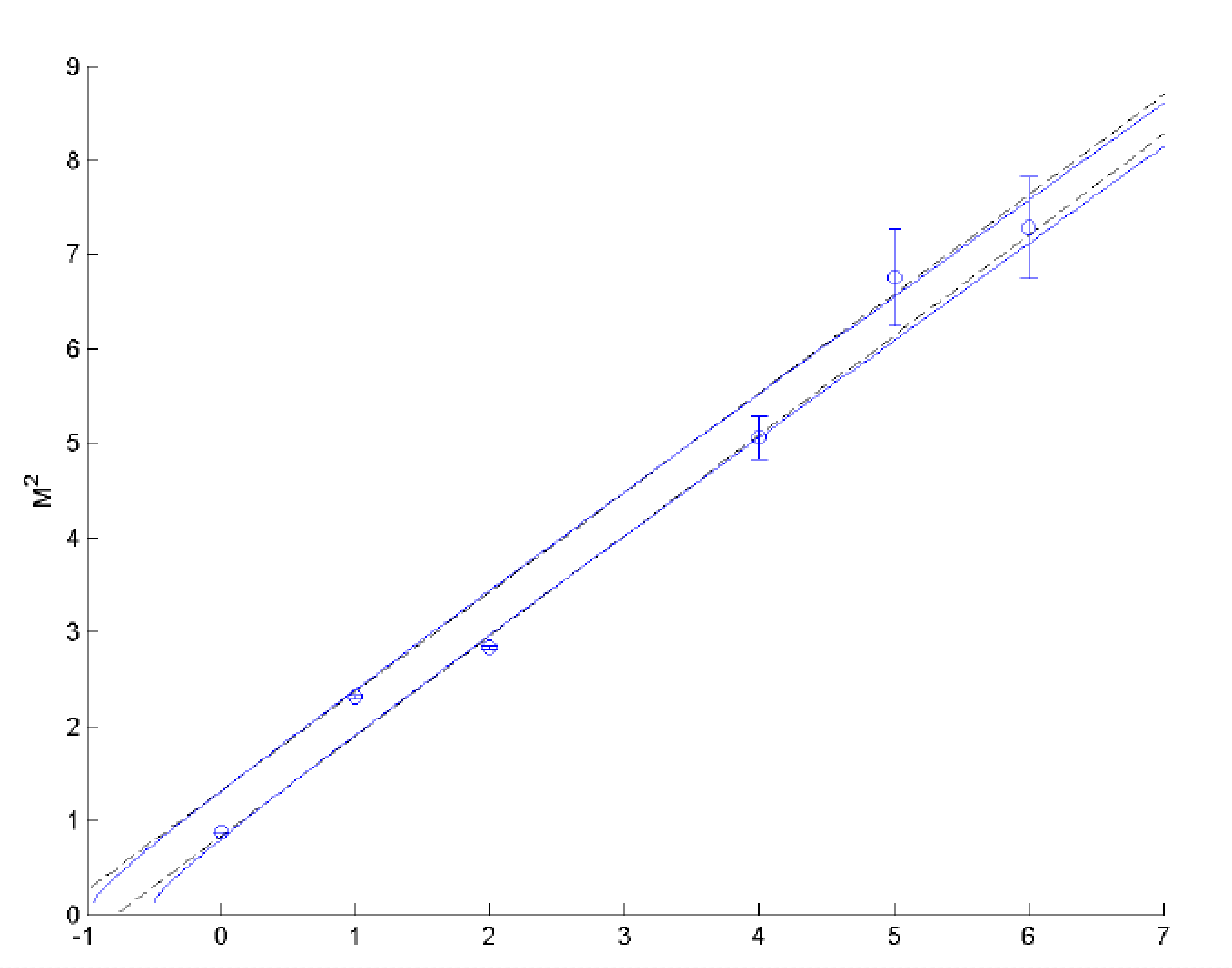
- Thus we are led to a picture where the baryon is a single string with a quark on one end and a di-quark (+ a baryonic vertex) at the other end.



- This is in accordance with stability analysis which shows that a small instability in one arm will cause it to shrink so that the final state is a single string

Fit to Regge trajectories of Nucleons

- Fit of the Regge trajectories of the Nucleons



Fitting the Nucleon trajectories

- Notice that there are separate trajectories for even L and for odd L.

$$a_o = -0.95 \qquad a_e = -0.7, \alpha' = 0.966,$$

- Assuming that m1=115 Mev the best fit for m2= 57Mev with $\chi_c^2/\chi_l^2 = 0.564$

- The fit with m2=240 Mev is much worth

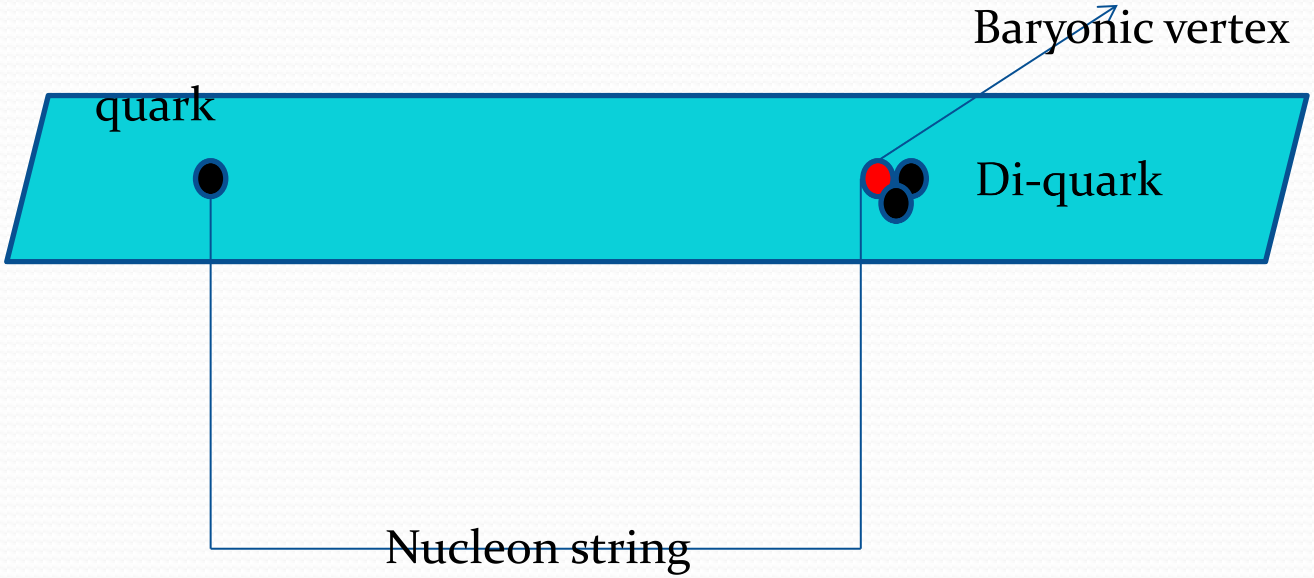
$$\chi_c^2/\chi_l^2 = 1.850$$

The sign of the intercept

- The intercept for the baryons is negative. The intercept associated with a bosonic string has to be positive (when defined as M^2 = 2πTs L -α ' it is negative)
- Both for odd and even L it is negative.
- In fact if we determine the trajectories for mesons for L and not for J we get also a negative intercept.
- Recall that we also did not account for the spin of the hadrons

The structure of the stringy nucleon

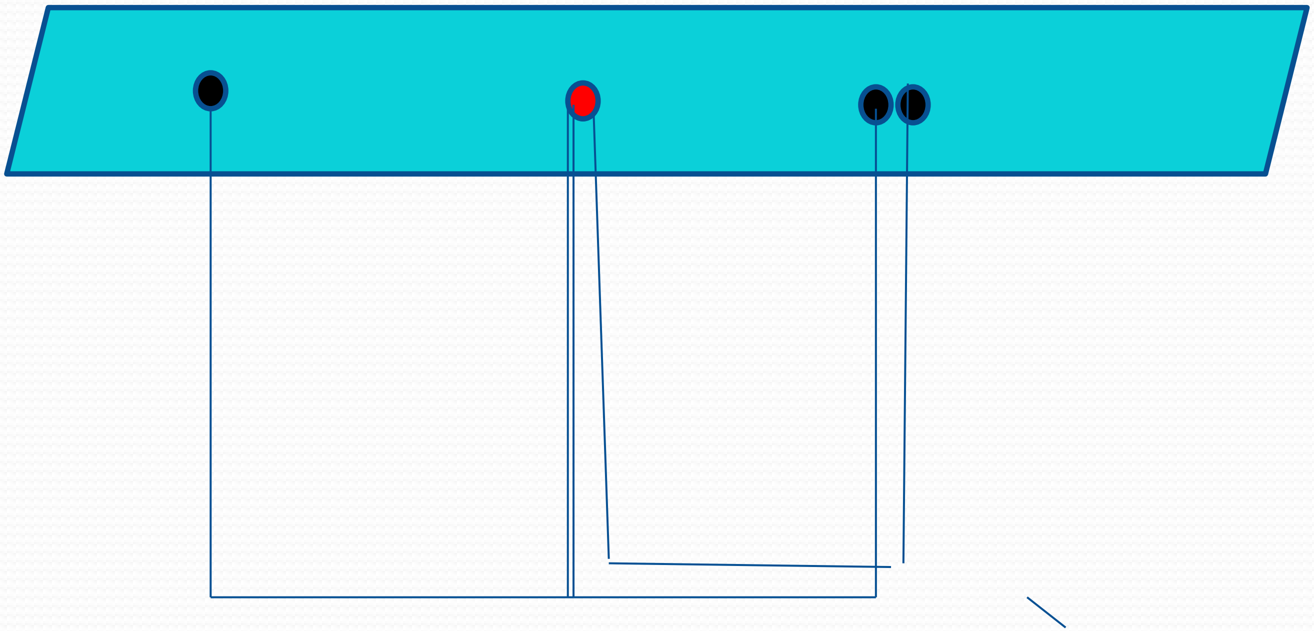
- We conclude that the setup is



- So in the right hand side we have m_q and not $2m_q$
- There does not seem to be a contribution to the mass from the Baryonic vertex

Central baryonic vertex is excluded

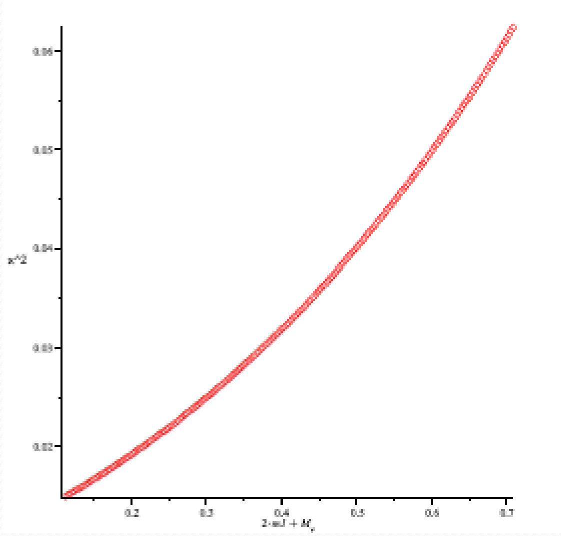
- The fit analysis definitely prefers the previous setup over a one with a central baryonic vertex



- A fit to such a scenario yields zero mass to the bayonic vertex but fails to see a 2m$_{sep}$ on the rhs

A baryonic vertex of vanishing mass !

- It follows from the fits that the Baryonic vertex has as an optimum vanishing mass .
- This favors the Do of the non-critical sting model over the wrapped D4 brane of the Sakai-Sugimoto model.



Summary and Outlook

Summary and outlook

- Hadron spectra fits much better strings in holographic backgrounds rather than the spectrum of bulk fields like fluctuations of flavor branes.
- Holographic Regge trajectories can be mapped into trajectories of strings with massive endpoints.
- Heavy quark mesons are described in a much better way by the holographic trajectories (or massive) than the original linear trajectories.
- Even for the u and d quark there is a non vanishing string endpoint mass of ~115 Mev.
- The stringy holographic mesons admit decay width

Summary and outlook

- Which is in accordance with the CNN model or Lund model.
- Baryons are also straight strings with tension which the same as the one of mesons.
- The baryonic vertex is still mysterious since data prefers it to be massless. It is not clear how could it be a D brane
- Open questions using the HSH scheme:
- Quantizing a string with massive endpoints
- Accounting for the spin and for the intercept
- Scattering amplitudes of mesons and baryons like (proton-proton scattering)
- Nuclear interaction and nuclear matter
- Incorporating leptons….