

# Super-Big J

Simeon Hellerman

Kavli Institute for the Physics and Mathematics of the Universe  
Tokyo University Institutes for Advanced Study

S.H., D. Orlando, S. Reffert, M. Watanabe, [arXiv:1505.01537](#)  
S.H., S. Maeda, M. Watanabe, [to appear](#)

String Theory in Greater Tokyo 5  
Tokyo Metropolitan University  
Hachiōji, Tokyo Metropolis, Japan  
December 1, 2016

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea**

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory**

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled**



# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density**

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large**

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale**

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In



# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization**

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization** when the

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization** when the **total charge** is large,

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization** when the **total charge is large**, the

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization** when the **total charge** is large, the **lowest state with a given charge**

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization** when the **total charge is large**, the **lowest state with a given charge** is always

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization** when the **total charge** is large, the **lowest state with a given charge** is always automatically in this regime

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization** when the **total charge** is large, the **lowest state with a given charge** is always **automatically in this regime** when the charge is much



# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization** when the **total charge** is large, the **lowest state with a given charge** is always automatically in this **regime** when the charge is much **greater than one**.

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization** when the **total charge** is large, the **lowest state with a given charge** is always automatically in this **regime** when the charge is much **greater than one**.

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Let us review the **general idea** .
- ▶ It's that we write a **conformally invariant effective field theory** that is **weakly coupled** in the limit where the **charge density** is **large** compared to the **infrared scale** .
- ▶ In **radial quantization** when the **total charge** is large, the **lowest state with a given charge** is always automatically in this **regime** when the charge is much **greater than one**.

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the *Wilsonian cutoff*  $\Lambda$

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always)

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally)

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action,

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables,

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed**



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the **denominator**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the **denominator**.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the **denominator**.
- ▶ Thus,

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the **denominator**.
- ▶ Thus, **operator dimensions**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the **denominator**.
- ▶ Thus, **operator dimensions** and

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the **denominator**.
- ▶ Thus, **operator dimensions** and **other properties**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the **denominator**.
- ▶ Thus, **operator dimensions** and **other properties** of ground states and other low-lying states of large  $J$ , are calculable perturbatively in



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the **denominator**.
- ▶ Thus, **operator dimensions** and **other properties** of ground states and other low-lying states of large  $J$ , are calculable perturbatively in  $\frac{1}{J}$

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the **denominator**.
- ▶ Thus, **operator dimensions** and **other properties** of ground states and other low-lying states of large  $J$ , are calculable perturbatively in  $\frac{1}{J}$ .

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ We then choose the **Wilsonian cutoff**  $\Lambda$  to be (as always) much **larger** than the infrared energy scale but (importantly but somewhat unconventionally) much **smaller** than the inverse mean distance between charges  $\rho^{-\frac{1}{2}}$ .
- ▶ In this limit both **higher derivative operators** in the effective action, and **quantum loop** corrections to observables, are both **parametrically suppressed** by powers of the density in the **denominator**.
- ▶ Thus, **operator dimensions** and **other properties** of ground states and other low-lying states of large  $J$ , are calculable perturbatively in  $\frac{1}{J}$ .

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude** of the complex scalar gets a

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude** of the complex scalar gets a **mass**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude** of the complex scalar gets a **mass** proportional to the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude** of the complex scalar gets a **mass** proportional to the **charge density**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude** of the complex scalar gets a **mass** proportional to the **charge density** and can be



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude** of the complex scalar gets a **mass** proportional to the **charge density** and can be **integrated out**,

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude** of the complex scalar gets a **mass** proportional to the **charge density** and can be **integrated out**, leaving us with an effective theory for

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude** of the complex scalar gets a **mass** proportional to the **charge density** and can be **integrated out**, leaving us with an effective theory for  **$\chi$  alone**.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude** of the complex scalar gets a **mass** proportional to the **charge density** and can be **integrated out**, leaving us with an effective theory for  **$\chi$  alone**.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ In the case of **effective string theory** the effective theory is simply that of  $D$  Poincaré-invariant goldstone bosons.
- ▶ In the case of the three-dimensional **critical  $O(2)$**  model, the theory is that of a **free periodically identified boson  $\chi$** .
- ▶ The boson  $\chi$  can be thought of as the **phase variable** of the **complex scalar field** of the  **$O(2)$  model**. The **magnitude** of the complex scalar gets a **mass** proportional to the **charge density** and can be **integrated out**, leaving us with an effective theory for  **$\chi$  alone**.

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension**  $\Delta_J$

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension**  $\Delta_J$  with charge  $J$  in the  $O(2)$  model,

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension**  $\Delta_J$  with charge  $J$  in the  $O(2)$  model, in terms of certain

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension  $\Delta_J$**  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension**  $\Delta_J$  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension  $\Delta_J$**  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian for  $\chi$** .

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension  $\Delta_J$**  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian for  $\chi$** .

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension  $\Delta_J$**  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian for  $\chi$** .
- ▶ We obtain an



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension  $\Delta_J$**  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian** for  $\chi$ .
- ▶ We obtain an **asymptotic expansion**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension  $\Delta_J$**  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian** for  $\chi$ .
- ▶ We obtain an **asymptotic expansion** of the form

$$\Delta_J = c_{\frac{3}{2}} J^{+\frac{3}{2}} + c_{\frac{1}{2}} J^{+\frac{1}{2}} - 0.0108451 + O(J^{-\frac{1}{2}}).$$

where  $c_{\frac{3}{2}}$  and  $c_{\frac{1}{2}}$  correspond to the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension  $\Delta_J$**  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian for  $\chi$** .
- ▶ We obtain an **asymptotic expansion** of the form

$$\Delta_J = c_{\frac{3}{2}} J^{+\frac{3}{2}} + c_{\frac{1}{2}} J^{+\frac{1}{2}} - 0.0108451 + O(J^{-\frac{1}{2}}).$$

where  $c_{\frac{3}{2}}$  and  $c_{\frac{1}{2}}$  correspond to the **two most relevant**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension  $\Delta_J$**  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian for  $\chi$** .
- ▶ We obtain an **asymptotic expansion** of the form

$$\Delta_J = c_{\frac{3}{2}} J^{+\frac{3}{2}} + c_{\frac{1}{2}} J^{+\frac{1}{2}} - 0.0108451 + O(J^{-\frac{1}{2}}) .$$

where  $c_{\frac{3}{2}}$  and  $c_{\frac{1}{2}}$  correspond to the **two most relevant** unknown Lagrangian coefficients.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension**  $\Delta_J$  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian** for  $\chi$ .
- ▶ We obtain an **asymptotic expansion** of the form

$$\Delta_J = c_{\frac{3}{2}} J^{+\frac{3}{2}} + c_{\frac{1}{2}} J^{+\frac{1}{2}} - 0.0108451 + O(J^{-\frac{1}{2}}).$$

where  $c_{\frac{3}{2}}$  and  $c_{\frac{1}{2}}$  correspond to the **two most relevant** unknown Lagrangian coefficients.

- ▶ Locally, the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension**  $\Delta_J$  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian** for  $\chi$ .
- ▶ We obtain an **asymptotic expansion** of the form

$$\Delta_J = c_{\frac{3}{2}} J^{+\frac{3}{2}} + c_{\frac{1}{2}} J^{+\frac{1}{2}} - 0.0108451 + O(J^{-\frac{1}{2}}).$$

where  $c_{\frac{3}{2}}$  and  $c_{\frac{1}{2}}$  correspond to the **two most relevant** unknown Lagrangian coefficients.

- ▶ Locally, the  $c_{\frac{3}{2}}$

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension  $\Delta_J$**  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian for  $\chi$** .
- ▶ We obtain an **asymptotic expansion** of the form

$$\Delta_J = c_{\frac{3}{2}} J^{+\frac{3}{2}} + c_{\frac{1}{2}} J^{+\frac{1}{2}} - 0.0108451 + O(J^{-\frac{1}{2}}).$$

where  $c_{\frac{3}{2}}$  and  $c_{\frac{1}{2}}$  correspond to the **two most relevant** unknown Lagrangian coefficients.

- ▶ Locally, the  $c_{\frac{3}{2}}$  coefficient corresponds to a leading-order equation of state of the form

$$\mathcal{H} \simeq (4\pi)^{+\frac{1}{2}} c_{\frac{3}{2}} \rho^{+\frac{3}{2}}.$$

which on general spatial geometries gives a leading-order relation

$$E \simeq \sqrt{\frac{4\pi}{\mathcal{A}}} c_{\frac{3}{2}} J^{+\frac{3}{2}}$$

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Quantizing the effective theory gives a **large- $J$**  expansion of the **lowest operator dimension  $\Delta_J$**  with charge  $J$  in the  $O(2)$  model, in terms of certain **unknown coefficients** in the **effective Lagrangian for  $\chi$** .
- ▶ We obtain an **asymptotic expansion** of the form

$$\Delta_J = c_{\frac{3}{2}} J^{+\frac{3}{2}} + c_{\frac{1}{2}} J^{+\frac{1}{2}} - 0.0108451 + O(J^{-\frac{1}{2}}).$$

where  $c_{\frac{3}{2}}$  and  $c_{\frac{1}{2}}$  correspond to the **two most relevant** unknown Lagrangian coefficients.

- ▶ Locally, the  $c_{\frac{3}{2}}$  coefficient corresponds to a leading-order equation of state of the form

$$\mathcal{H} \simeq (4\pi)^{+\frac{1}{2}} c_{\frac{3}{2}} \rho^{+\frac{3}{2}}.$$

which on general spatial geometries gives a leading-order relation

$$E \simeq \sqrt{\frac{4\pi}{\mathcal{A}}} c_{\frac{3}{2}} J^{+\frac{3}{2}}$$



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ ,

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is **metrically flat**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is **metrically flat**, then all curvature-dependent terms in the effective theory drop out as well, and the theory becomes



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is **metrically flat**, then all curvature-dependent terms in the effective theory drop out as well, and the theory becomes **more universal**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is **metrically flat**, then all curvature-dependent terms in the effective theory drop out as well, and the theory becomes **more universal** to higher order in  $J$ .

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is **metrically flat**, then all curvature-dependent terms in the effective theory drop out as well, and the theory becomes **more universal** to higher order in  $J$ .
- ▶ This makes the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is **metrically flat**, then all curvature-dependent terms in the effective theory drop out as well, and the theory becomes **more universal** to higher order in  $J$ .
- ▶ This makes the **torus geometry**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is **metrically flat**, then all curvature-dependent terms in the effective theory drop out as well, and the theory becomes **more universal** to higher order in  $J$ .
- ▶ This makes the **torus geometry** a good arena in which to isolate the leading coefficient

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is **metrically flat**, then all curvature-dependent terms in the effective theory drop out as well, and the theory becomes **more universal** to higher order in  $J$ .
- ▶ This makes the **torus geometry** a good arena in which to isolate the leading coefficient  $c_{+\frac{3}{2}}$

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is **metrically flat**, then all curvature-dependent terms in the effective theory drop out as well, and the theory becomes **more universal** to higher order in  $J$ .
- ▶ This makes the **torus geometry** a good arena in which to isolate the leading coefficient  $c_{+\frac{3}{2}}$ .

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For general spatial geometries, the subleading corrections are proportional to  $J^{+\frac{1}{2}}$  times the Euler number of the spatial slice.
- ▶ For a spatial slice with the topology of a torus  $T^2$ , the Euler number correction vanishes. If the torus is **metrically flat**, then all curvature-dependent terms in the effective theory drop out as well, and the theory becomes **more universal** to higher order in  $J$ .
- ▶ This makes the **torus geometry** a good arena in which to isolate the leading coefficient  $c_{+\frac{3}{2}}$ .



# Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus with

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus with circumference

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus with circumference (not radius)

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus with circumference (not radius)  $\ell$ ,

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus with circumference (not radius)  $\ell$ , the ground state energy

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus with circumference (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus with circumference (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus with circumference (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the charge- $J$

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus with circumference (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the charge- $J$  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a flat square torus with circumference (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the charge- $J$  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action.



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**,

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly **UV-divergent two-loop diagram**



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly **UV-divergent two-loop diagram** in

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly **UV-divergent two-loop diagram** in **finite volume**.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly **UV-divergent two-loop diagram** in **finite volume**. My collaborators and I have

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly **UV-divergent two-loop diagram** in **finite volume**. My collaborators and I have **not**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly **UV-divergent two-loop diagram** in **finite volume**. My collaborators and I have **not** calculated it yet,

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly **UV-divergent two-loop diagram** in **finite volume**. My collaborators and I have **not** calculated it yet, and I don't know if we

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly **UV-divergent two-loop diagram** in **finite volume**. My collaborators and I have **not** calculated it yet, and I don't know if we **ever will**.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly **UV-divergent two-loop diagram** in **finite volume**. My collaborators and I have **not** calculated it yet, and I don't know if we **ever will**.



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ For a **flat square torus** with **circumference** (not radius)  $\ell$ , the ground state energy  $E_J^{(0)}$  in the **charge- $J$**  sector has the expansion

$$\ell E_J^{(0)} = c_{\frac{3}{2}} \sqrt{4\pi} J^{+\frac{3}{2}} + K_{\text{Casimir}} + \frac{K_{\text{two loop}}}{c_{\frac{3}{2}}} J^{-\frac{3}{2}}$$

where  $K_{\text{Casimir}} \equiv -0.718873$ .

- ▶ The number  $K_{\text{two loop}}$  is a theory-independent coefficient describing the conformal part of the two-loop vacuum energy of the leading effective action. Despite being **universal** it is nonetheless tricky to actually **calculate**, as it describes the **UV-finite part** of a highly **UV-divergent two-loop diagram** in **finite volume**. My collaborators and I have **not** calculated it yet, and I don't know if we **ever will**.

# Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector,



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.
- ▶ This is

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.
- ▶ This is **slightly unfortunate**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.
- ▶ This is **slightly unfortunate** since these differences are



## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.
- ▶ This is **slightly unfortunate** since these differences are **insensitive**

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.
- ▶ This is **slightly unfortunate** since these differences are **insensitive** to the

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.
- ▶ This is **slightly unfortunate** since these differences are **insensitive** to the **universal order  $O(J^0)$  term**,

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.
- ▶ This is **slightly unfortunate** since these differences are **insensitive** to the **universal order  $O(J^0)$  term**, but we're going to focus on the glass being

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.
- ▶ This is **slightly unfortunate** since these differences are **insensitive** to the **universal order  $O(J^0)$  term**, but we're going to focus on the glass being **half-full**

## Critical $O(2)$ model in $D=3$ at large charge

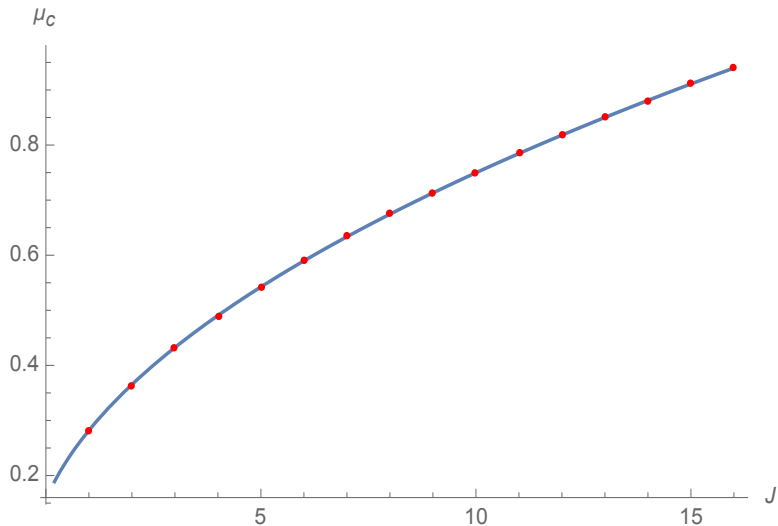
- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.
- ▶ This is **slightly unfortunate** since these differences are **insensitive** to the **universal order  $O(J^0)$  term**, but we're going to focus on the glass being **half-full** here.

## Critical $O(2)$ model in $D=3$ at large charge

- ▶ Despite our ignorance of the coefficient of the  $J^{-\frac{3}{2}}$  term in the energy, we should expect that the spectrum of charged ground states on the torus is a very **useful** setting for getting reasonably precise numerical estimates of  $c_{\frac{3}{2}}$  via **Monte Carlo** methods.
- ▶ I will now transmit some **not yet published results** due to D. Orlando and S. Reffert and their lattice colleagues. Apparently it is not so easy to get the **absolute additive normalization** of the energy of the ground state in the finite-charge sector, but it is **relatively easy** to get the **energy differences** between ground states in adjacent charge sectors.
- ▶ This is **slightly unfortunate** since these differences are **insensitive** to the **universal order  $O(J^0)$  term**, but we're going to focus on the glass being **half-full** here.

# Critical $O(2)$ model in $D=3$ at large charge

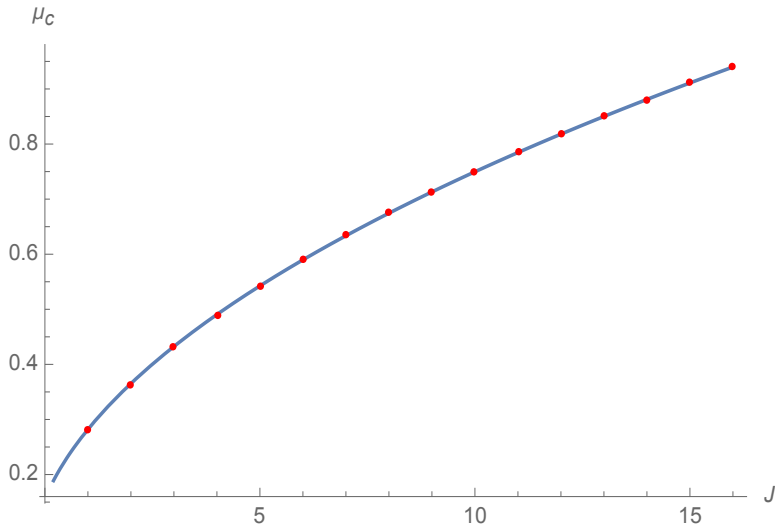
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$





# Critical $O(2)$ model in $D=3$ at large charge

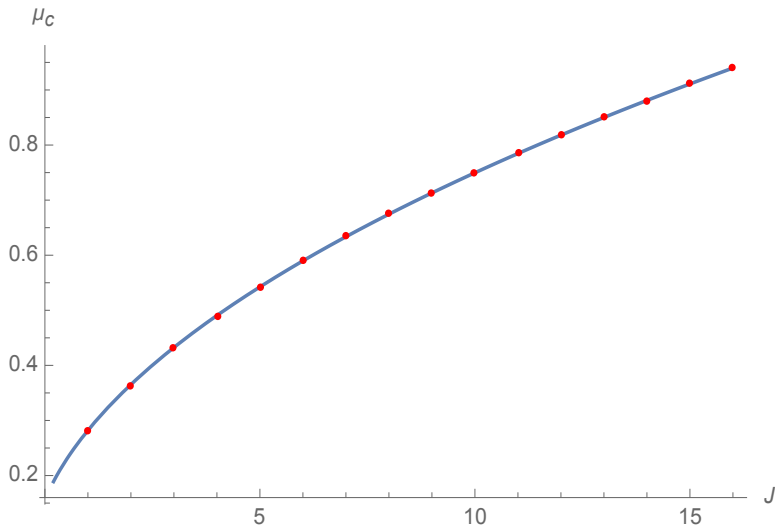
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Here, we are plotting

# Critical $O(2)$ model in $D=3$ at large charge

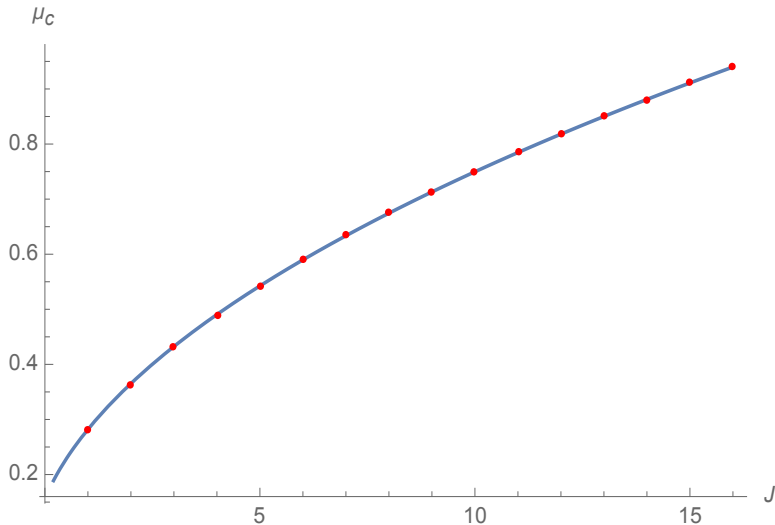
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Here, we are plotting  $\Delta_{J+1} - \Delta_J$

# Critical $O(2)$ model in $D=3$ at large charge

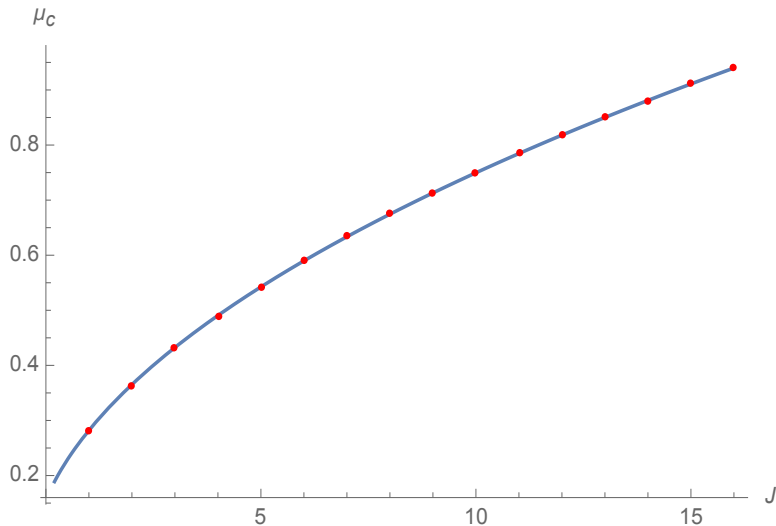
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Here, we are plotting  $\Delta_{J+1} - \Delta_J$  on the

# Critical $O(2)$ model in $D=3$ at large charge

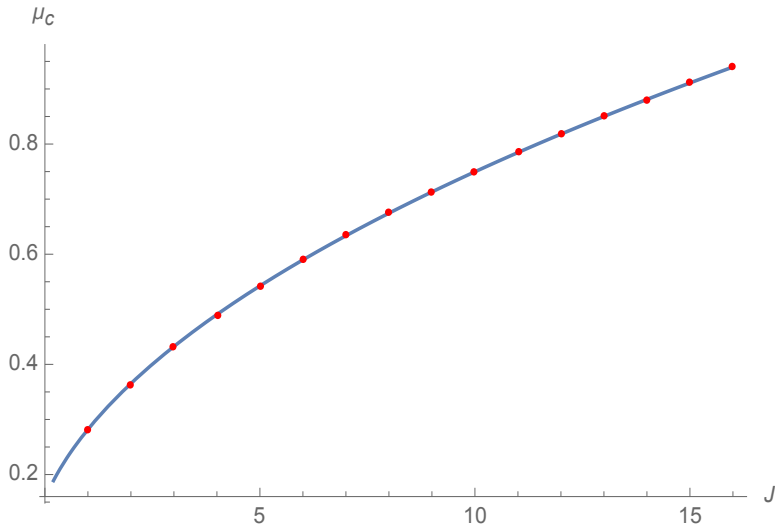
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Here, we are plotting  $\Delta_{J+1} - \Delta_J$  on the **vertical**

# Critical $O(2)$ model in $D=3$ at large charge

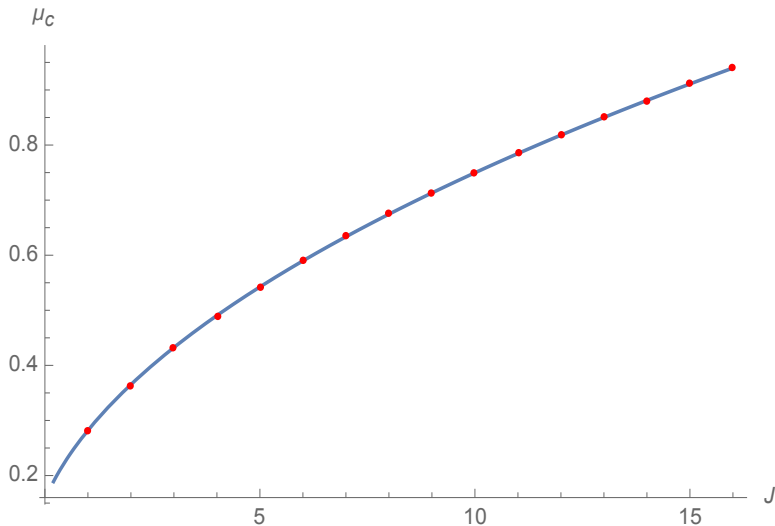
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Here, we are plotting  $\Delta_{J+1} - \Delta_J$  on the **vertical** axis,

# Critical $O(2)$ model in $D=3$ at large charge

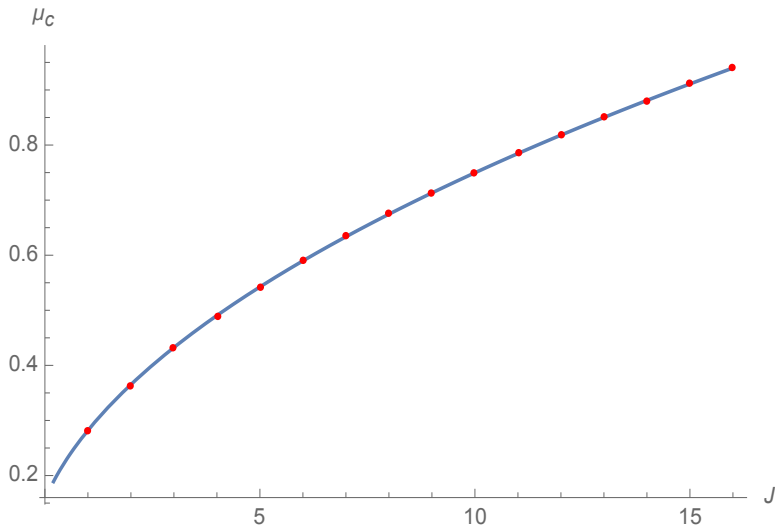
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Here, we are plotting  $\Delta_{J+1} - \Delta_J$  on the vertical axis, against  $J$  on the

# Critical $O(2)$ model in $D=3$ at large charge

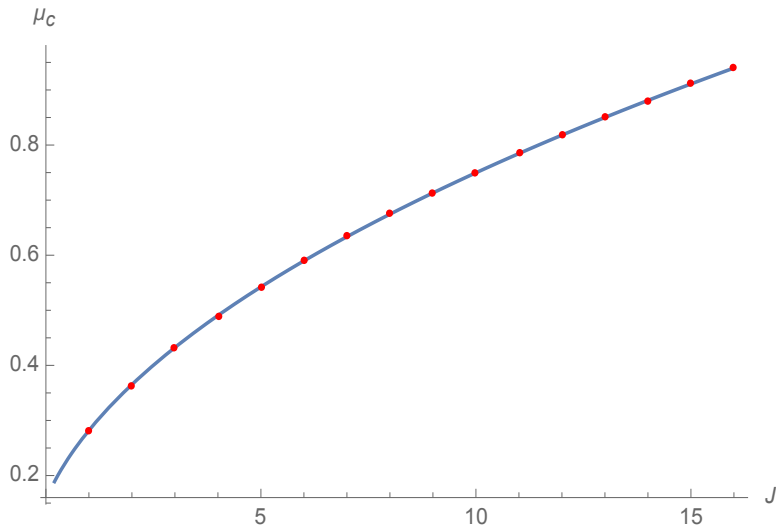
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Here, we are plotting  $\Delta_{J+1} - \Delta_J$  on the **vertical** axis, against  $J$  on the **horizontal**

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

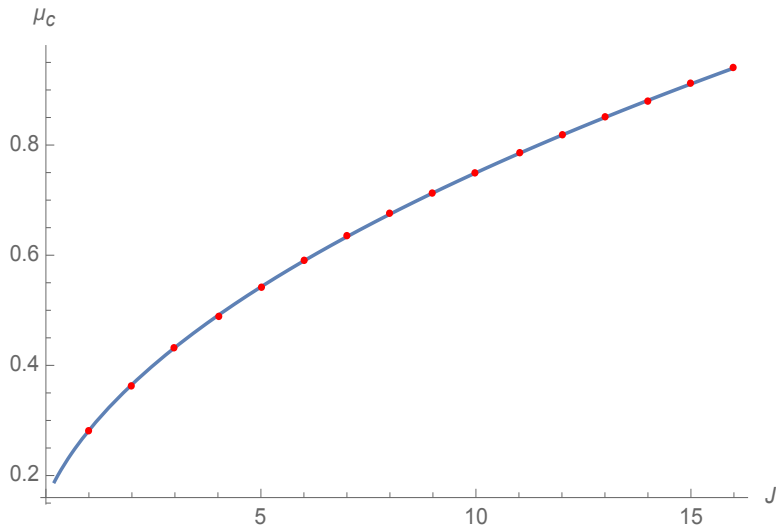


Here, we are plotting  $\Delta_{J+1} - \Delta_J$  on the **vertical** axis, against  $J$  on the **horizontal** axis.



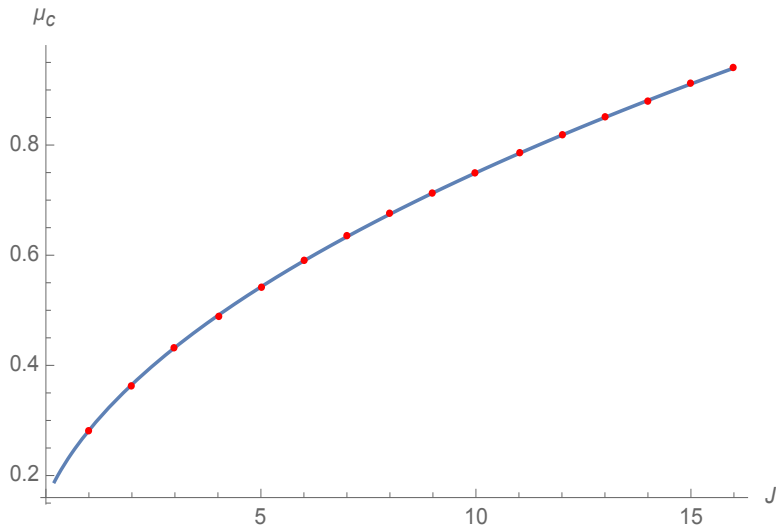
# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



# Critical $O(2)$ model in $D=3$ at large charge

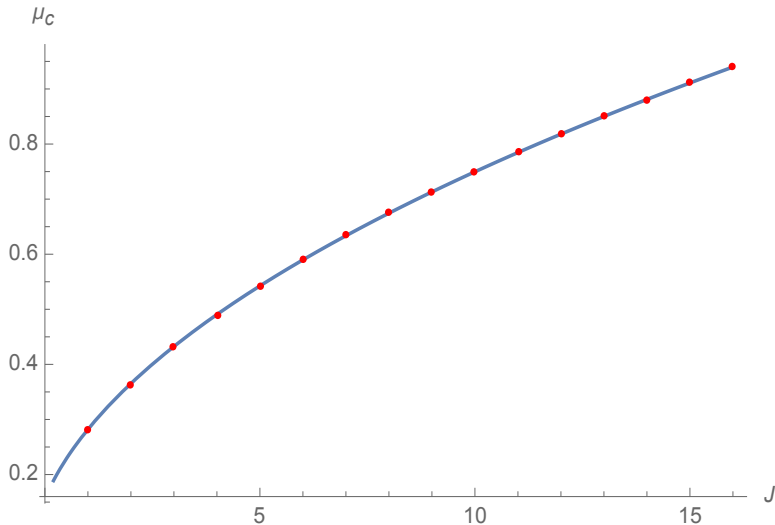
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The

# Critical $O(2)$ model in $D=3$ at large charge

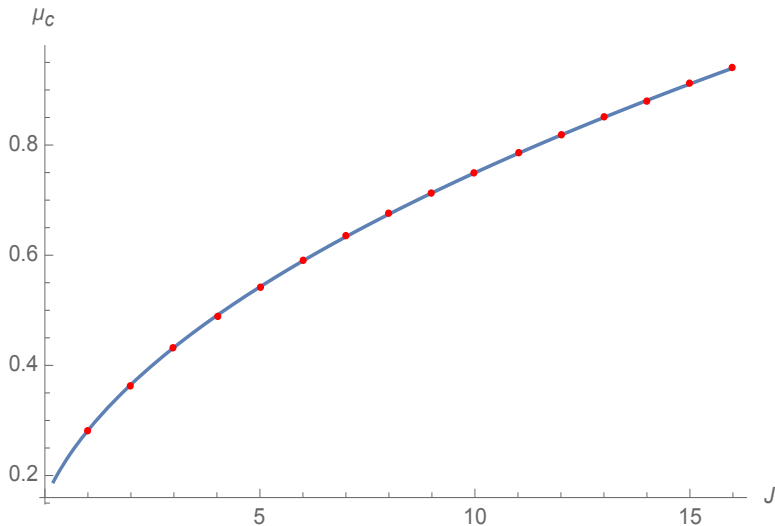
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The red dots

# Critical $O(2)$ model in $D=3$ at large charge

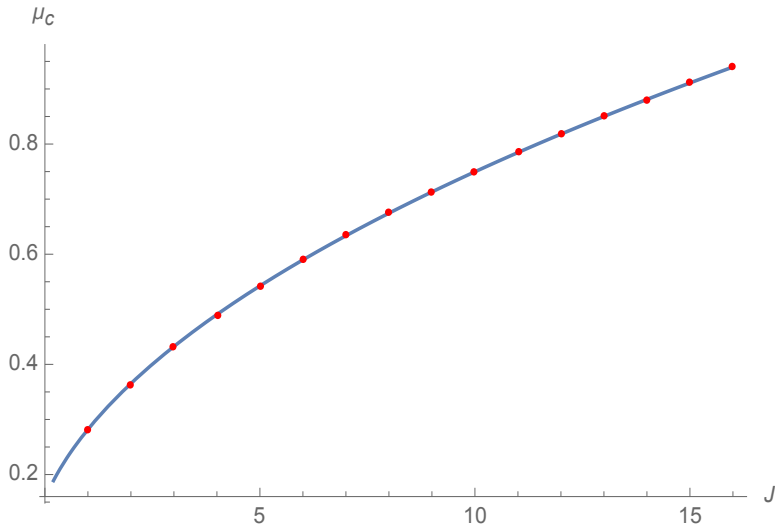
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The red dots are the

# Critical $O(2)$ model in $D=3$ at large charge

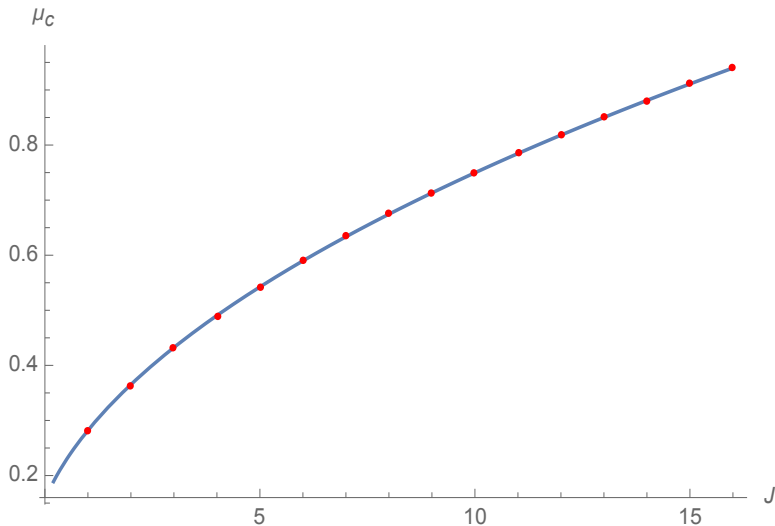
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The red dots are the lattice data

# Critical $O(2)$ model in $D=3$ at large charge

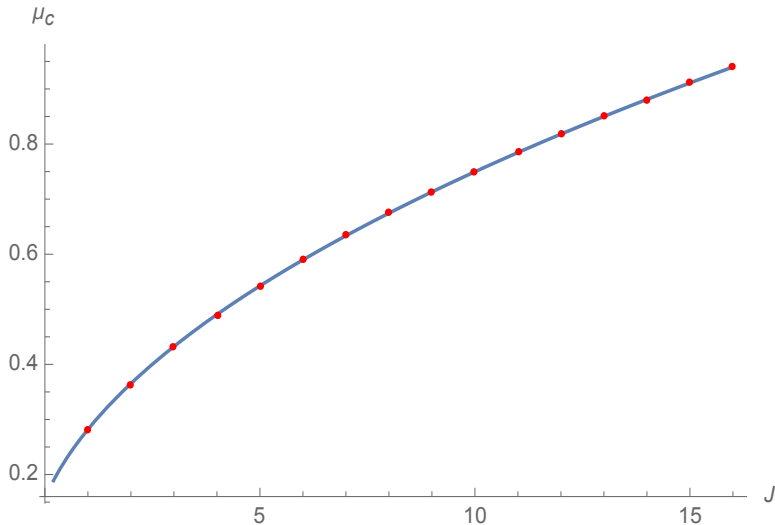
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The red dots are the lattice data for the differences between adjacent

# Critical $O(2)$ model in $D=3$ at large charge

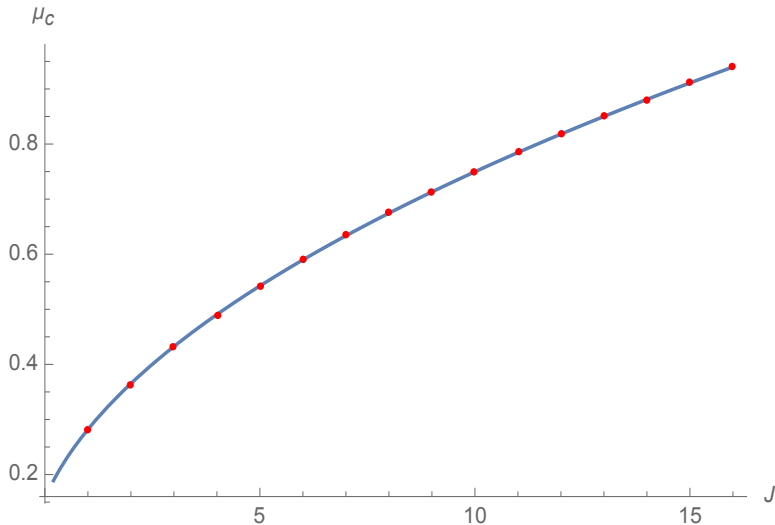
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The red dots are the lattice data for the differences between adjacent charged ground states.

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

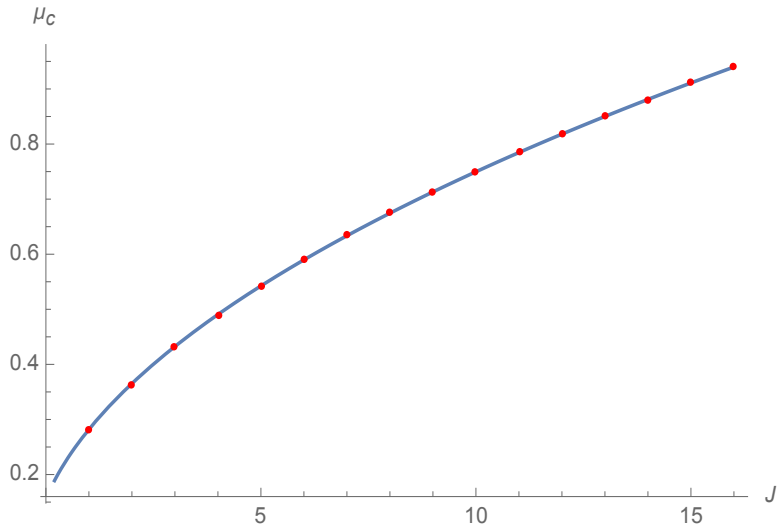


The red dots are the lattice data for the differences between adjacent charged ground states.



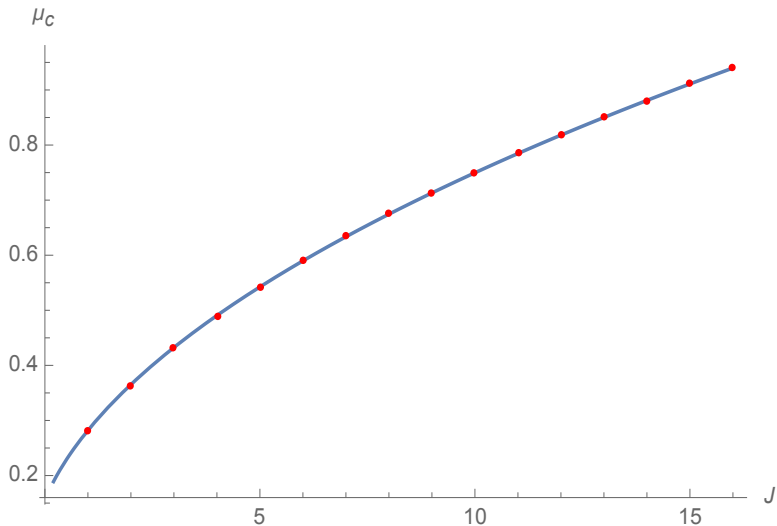
# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



# Critical $O(2)$ model in $D=3$ at large charge

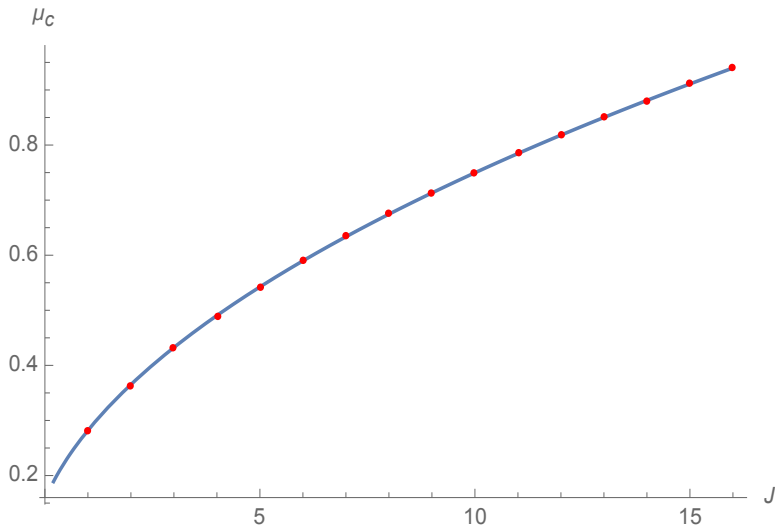
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The

# Critical $O(2)$ model in $D=3$ at large charge

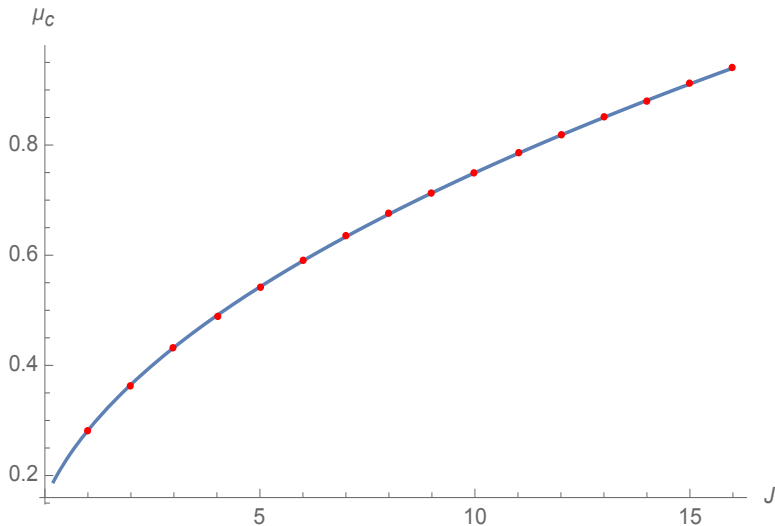
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The blue curve

# Critical $O(2)$ model in $D=3$ at large charge

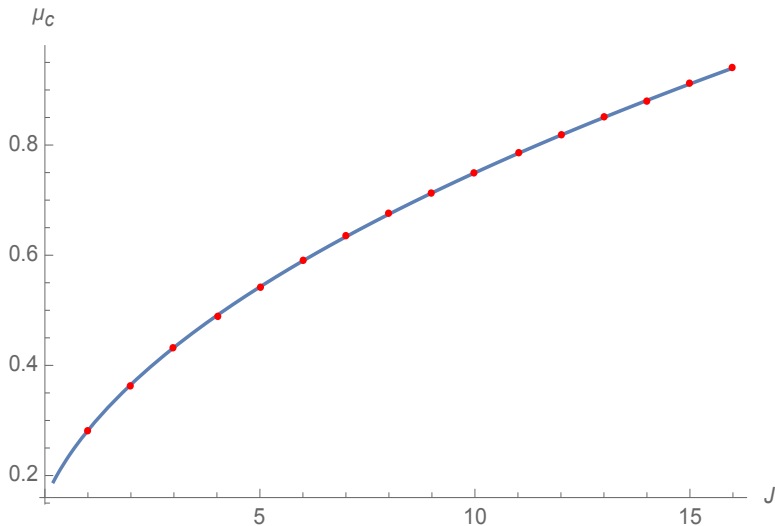
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The blue curve is the

# Critical $O(2)$ model in $D=3$ at large charge

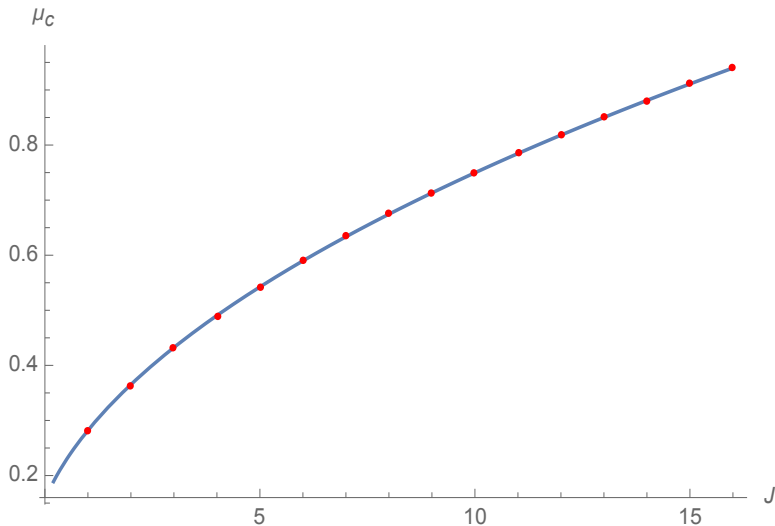
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The blue curve is the theoretical prediction

# Critical $O(2)$ model in $D=3$ at large charge

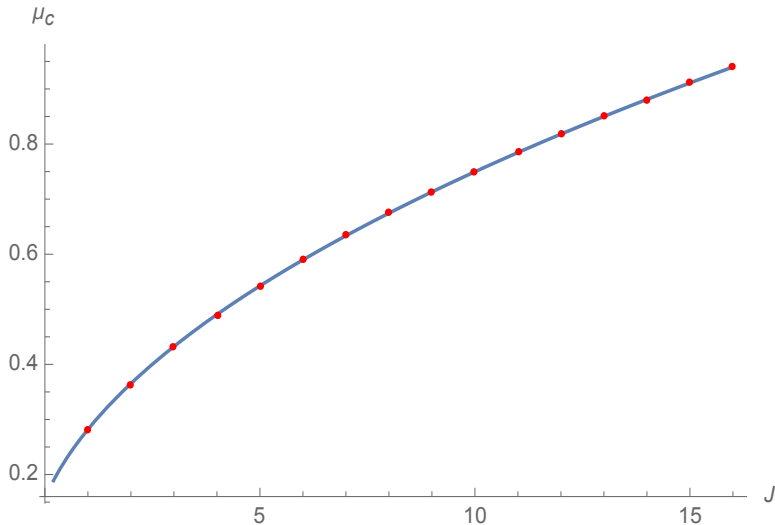
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The **blue curve** is the **theoretical prediction** for the energies at

# Critical $O(2)$ model in $D=3$ at large charge

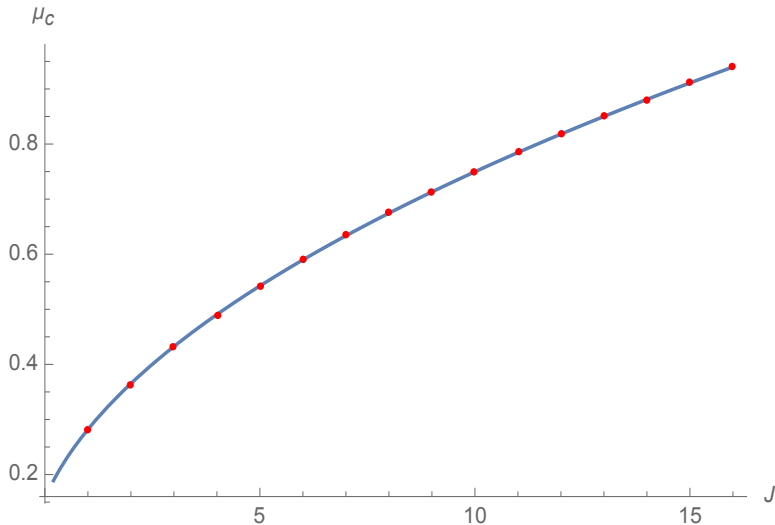
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The blue curve is the theoretical prediction for the energies at leading order in  $J$ .

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

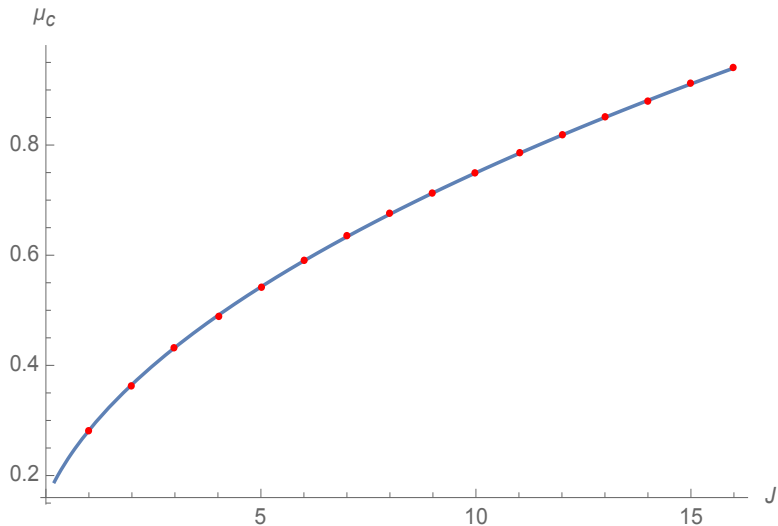


The blue curve is the theoretical prediction for the energies at leading order in  $J$ .



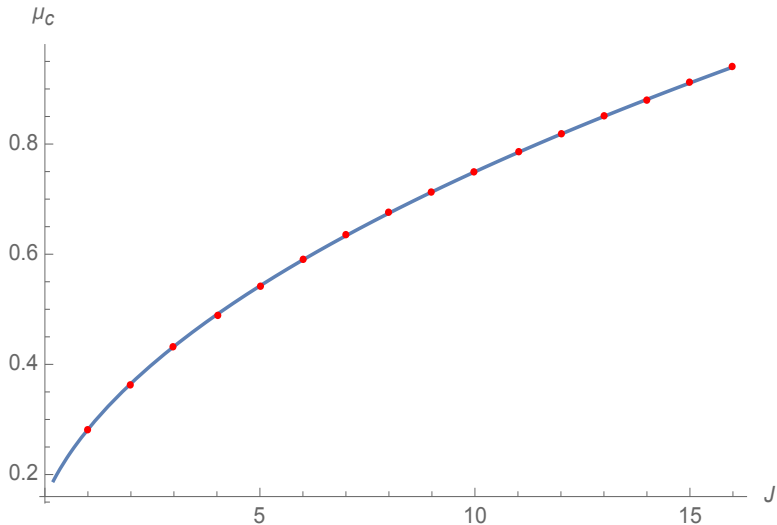
# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



# Critical $O(2)$ model in $D=3$ at large charge

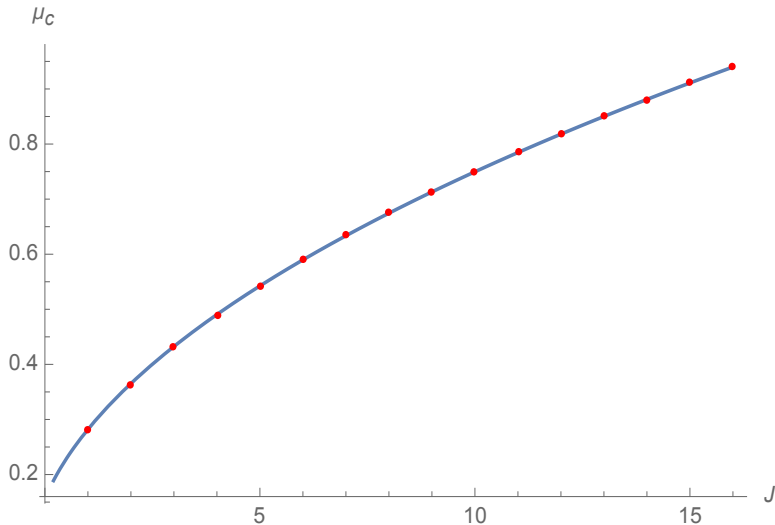
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



So

# Critical $O(2)$ model in $D=3$ at large charge

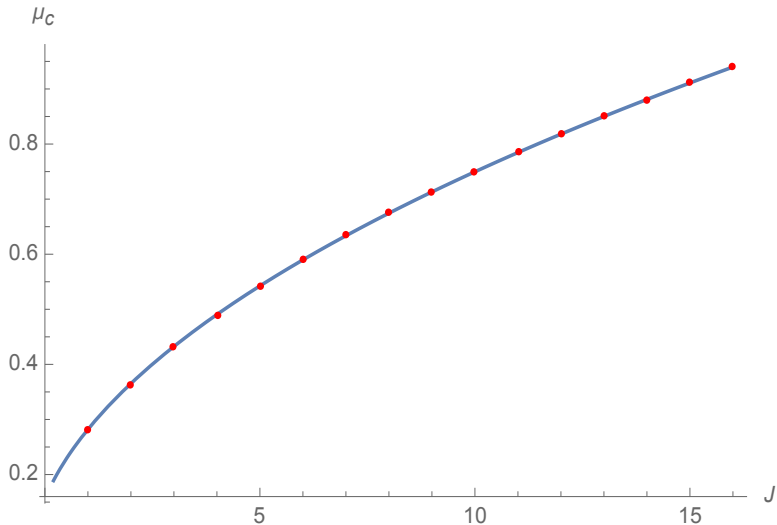
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



So ○ ○ ○

# Critical $O(2)$ model in $D=3$ at large charge

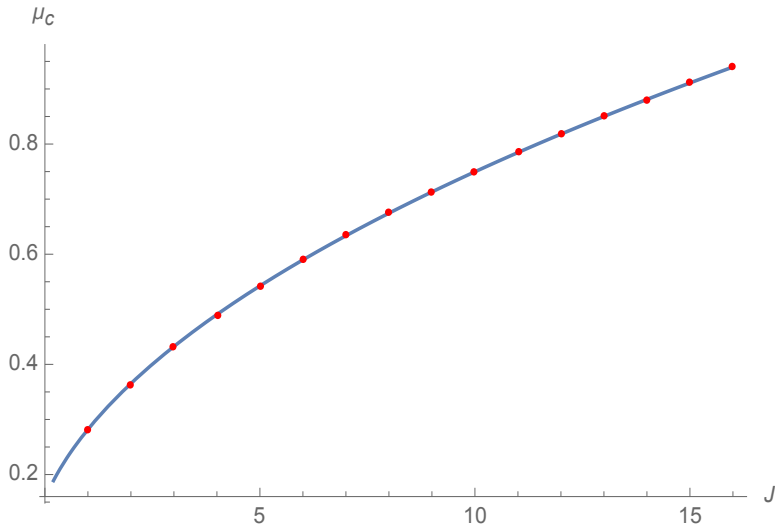
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



So ○ ○ ○ ○ ○ ○ ○ do you

# Critical $O(2)$ model in $D=3$ at large charge

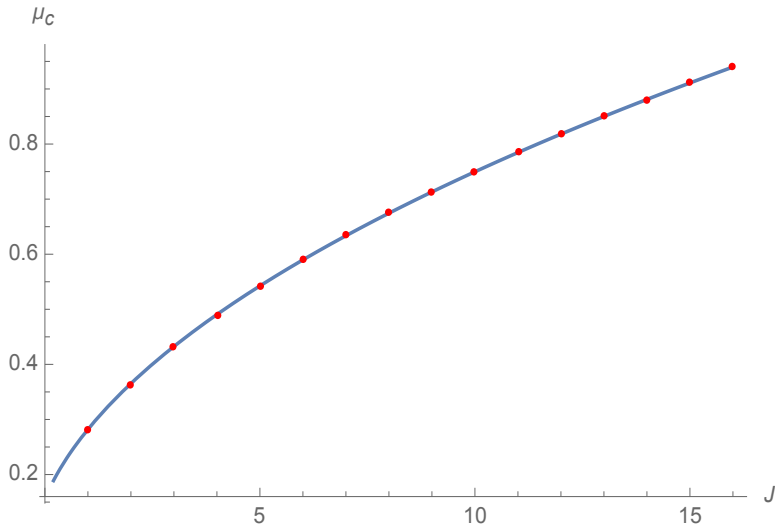
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



So  $\circ \circ \circ \circ \circ \circ \circ \circ$  do you like it?

# Critical $O(2)$ model in $D=3$ at large charge

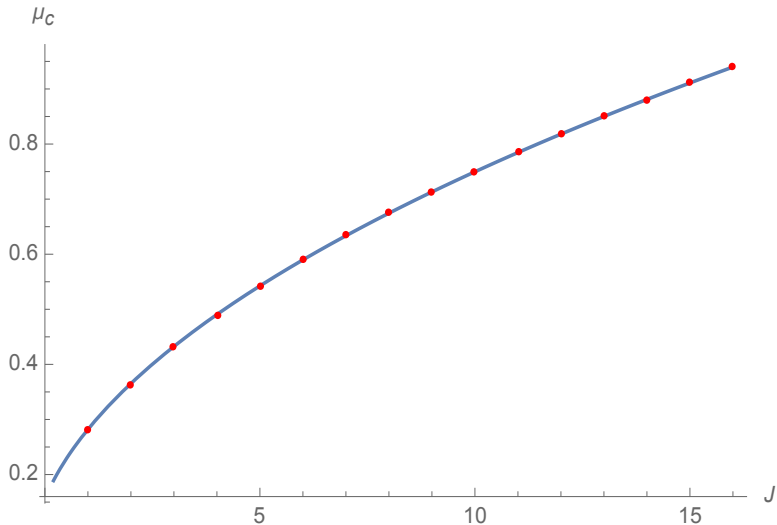
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



So ○ ○ ○ ○ ○ ○ ○ ○ do you like it?

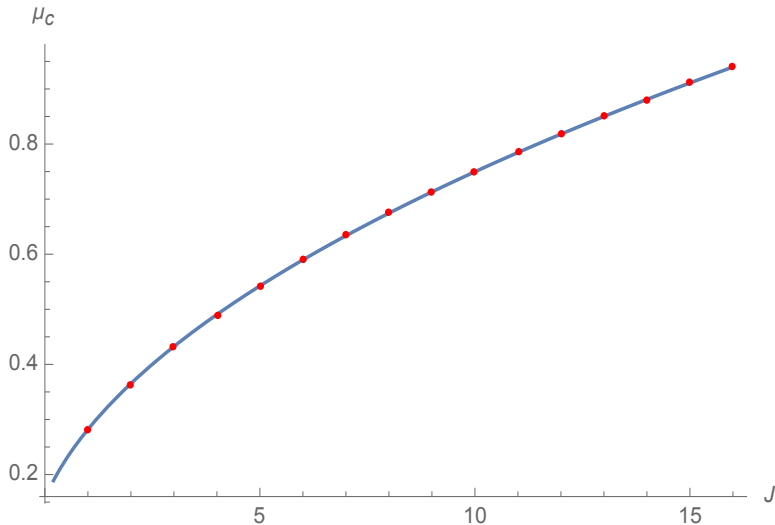
# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

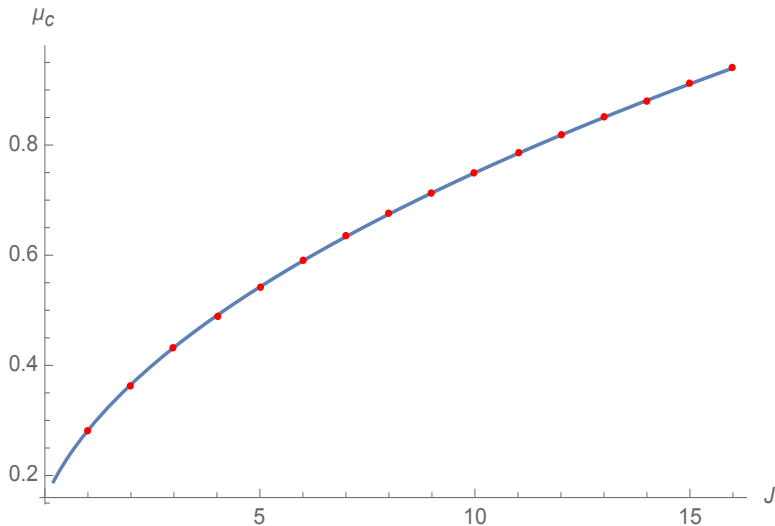


Because I'm pretty sure



# Critical $O(2)$ model in $D=3$ at large charge

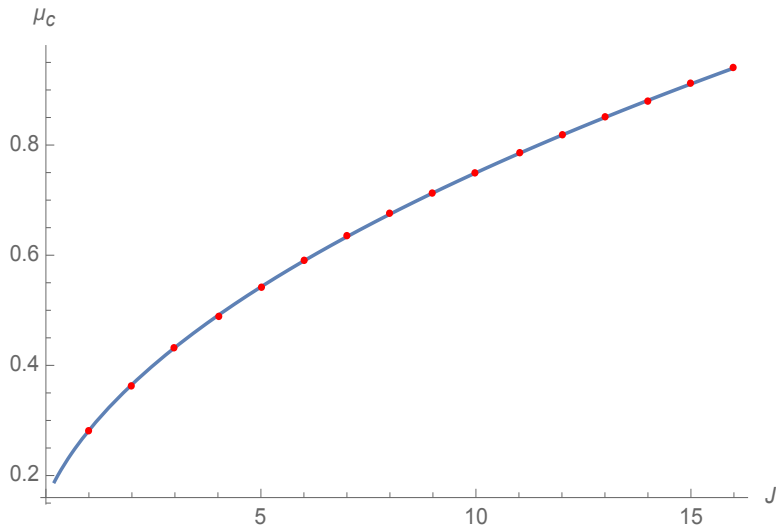
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Because I'm pretty sure I do.

# Critical $O(2)$ model in $D=3$ at large charge

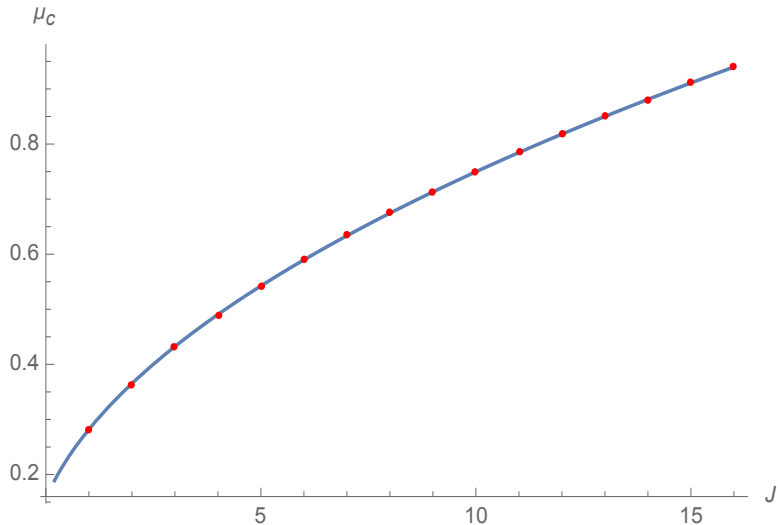
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Because I'm pretty sure I do.

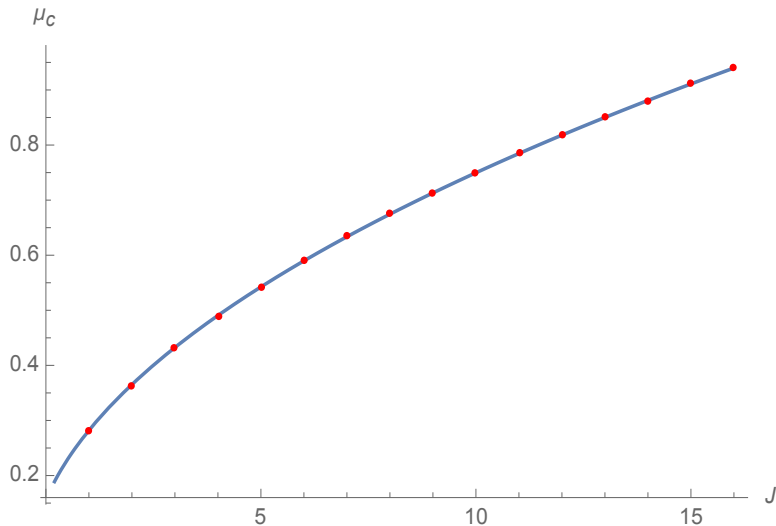
# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



# Critical $O(2)$ model in $D=3$ at large charge

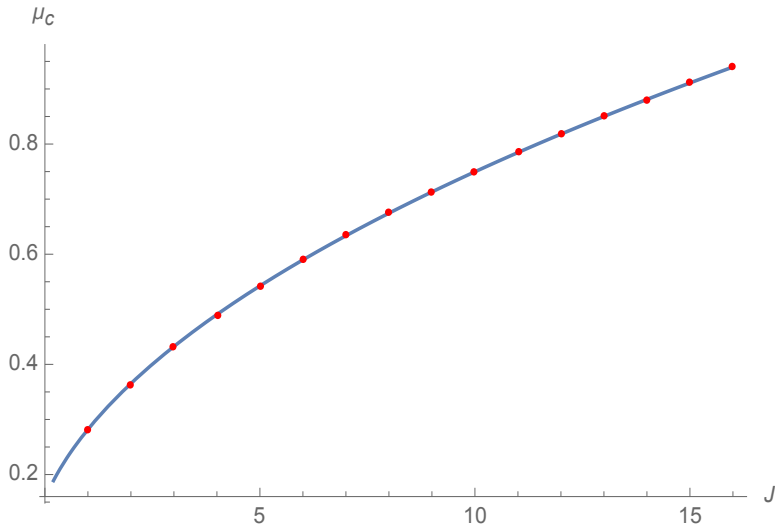
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



I want to emphasize that

# Critical $O(2)$ model in $D=3$ at large charge

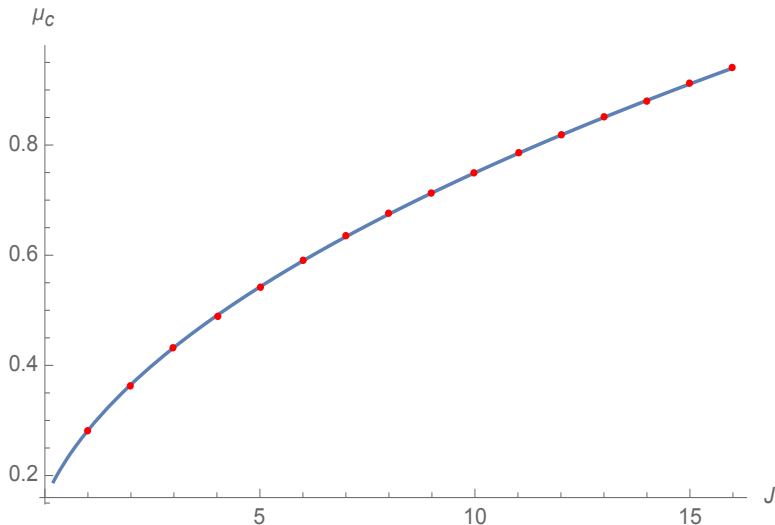
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



I want to emphasize that higher-derivative

# Critical $O(2)$ model in $D=3$ at large charge

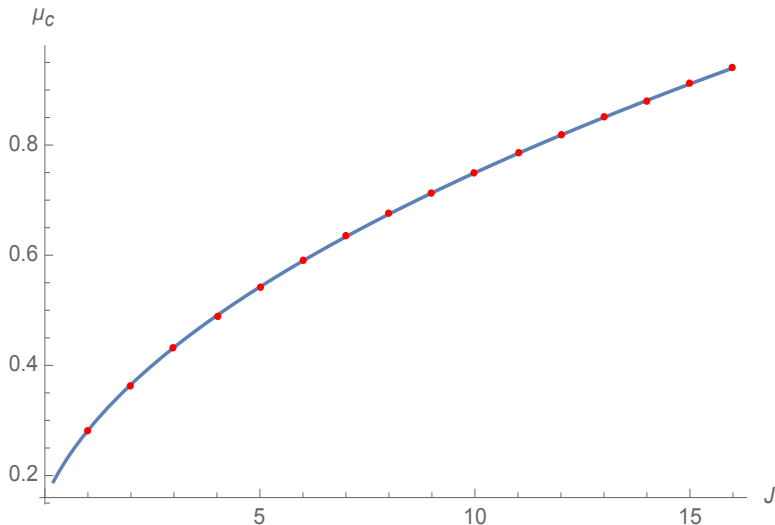
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



I want to emphasize that **higher-derivative** operators do not even enter the vacuum energy until order

# Critical $O(2)$ model in $D=3$ at large charge

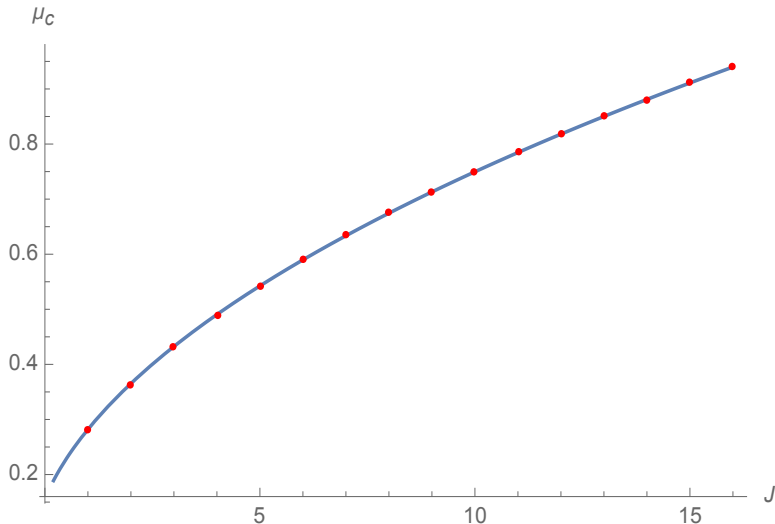
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



I want to emphasize that **higher-derivative** operators do not even enter the vacuum energy until order  $1/J$ ,

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

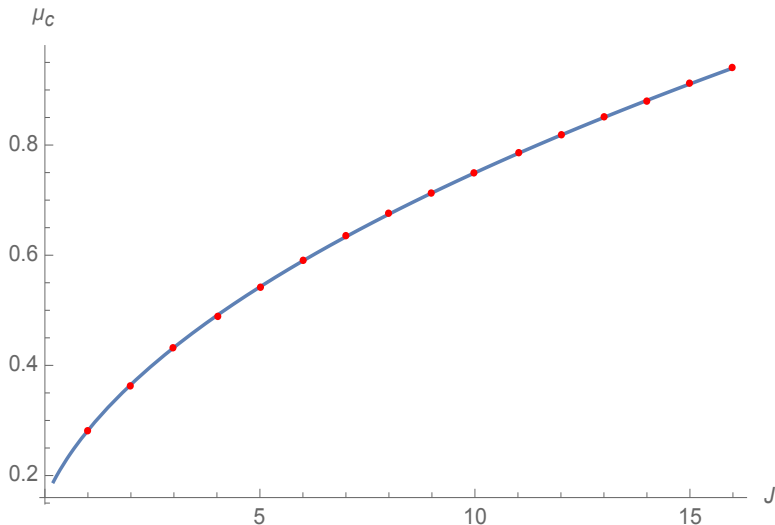


I want to emphasize that **higher-derivative** operators do not even enter the vacuum energy until order  $1/J$ , which is probably responsible for the



# Critical $O(2)$ model in $D=3$ at large charge

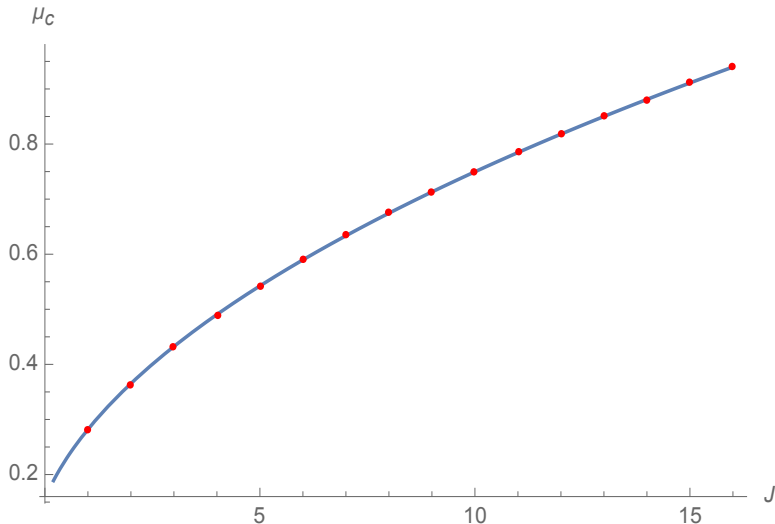
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



I want to emphasize that **higher-derivative** operators do not even enter the vacuum energy until order  $1/J$ , which is probably responsible for the **accuracy**

# Critical $O(2)$ model in $D=3$ at large charge

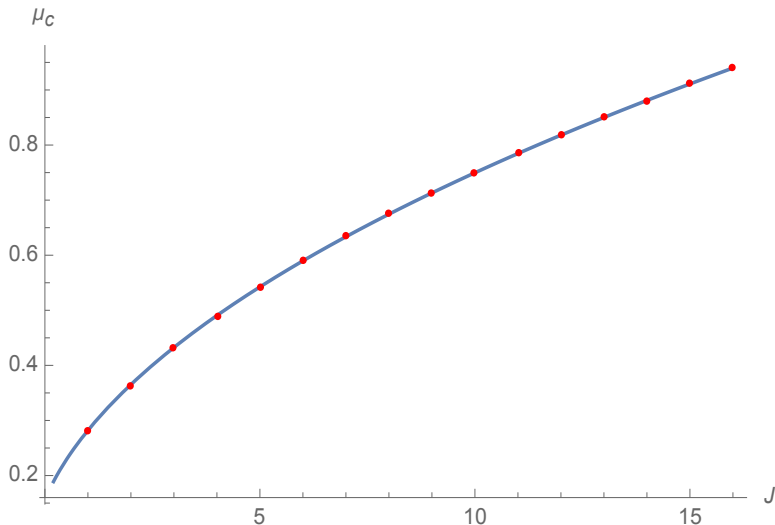
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



I want to emphasize that **higher-derivative** operators do not even enter the vacuum energy until order  $1/J$ , which is probably responsible for the **accuracy** of the

# Critical $O(2)$ model in $D=3$ at large charge

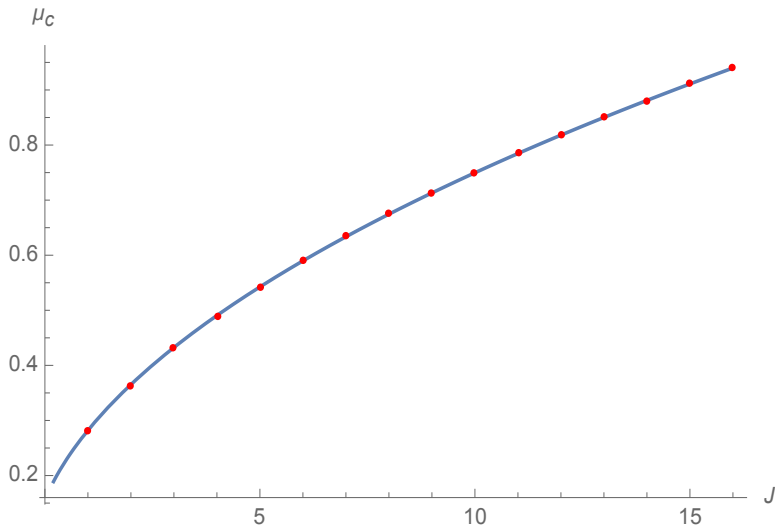
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



I want to emphasize that **higher-derivative** operators do not even enter the vacuum energy until order  $1/J$ , which is probably responsible for the **accuracy** of the **leading-order formula**.

# Critical $O(2)$ model in $D=3$ at large charge

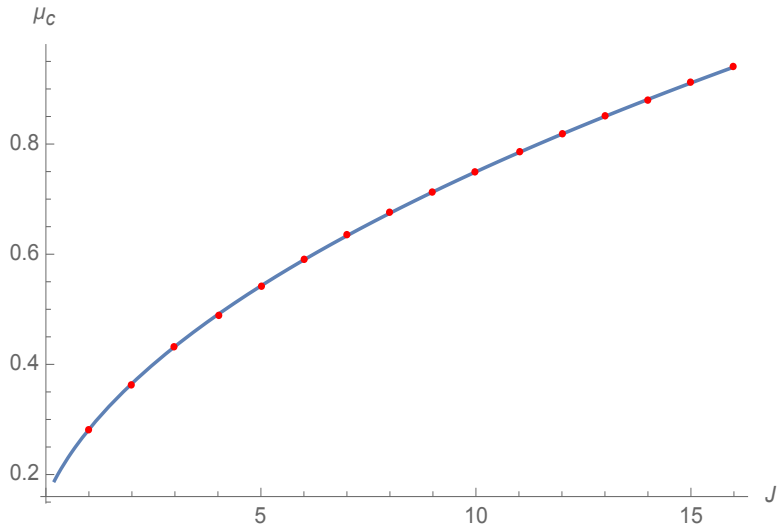
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



I want to emphasize that **higher-derivative** operators do not even enter the vacuum energy until order  $1/J$ , which is probably responsible for the **accuracy** of the **leading-order formula**.

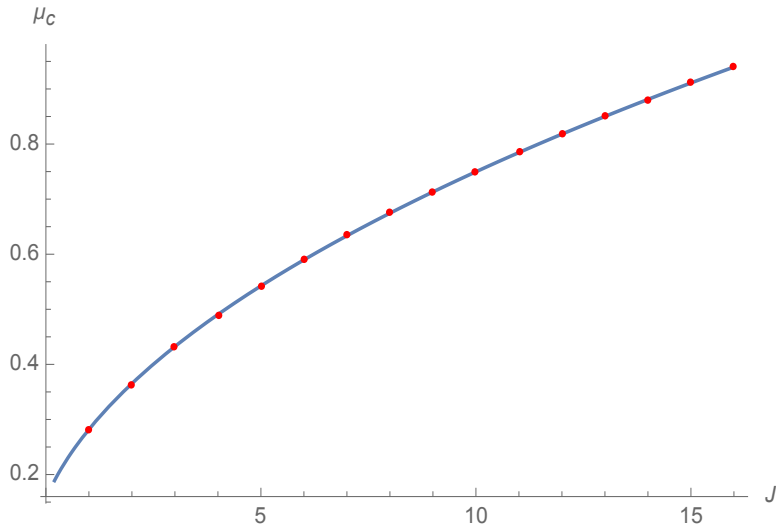
# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



# Critical $O(2)$ model in $D=3$ at large charge

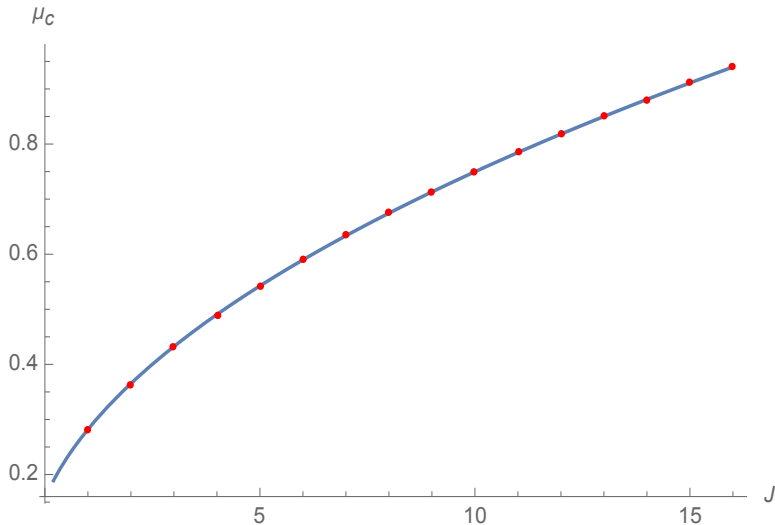
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Also:

# Critical $O(2)$ model in $D=3$ at large charge

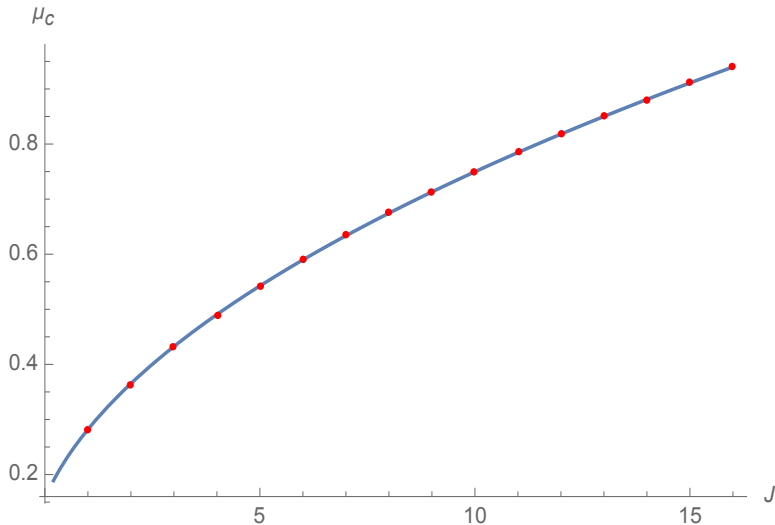
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Also: The plot above is

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

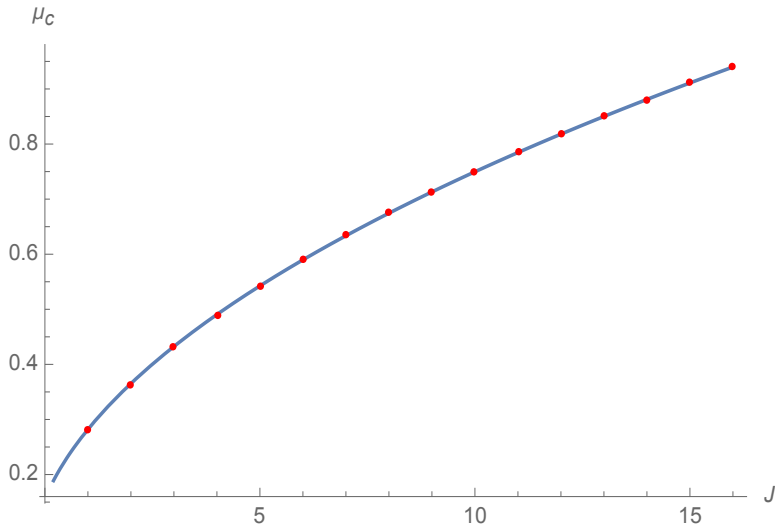


Also: The plot above is **not**



# Critical $O(2)$ model in $D=3$ at large charge

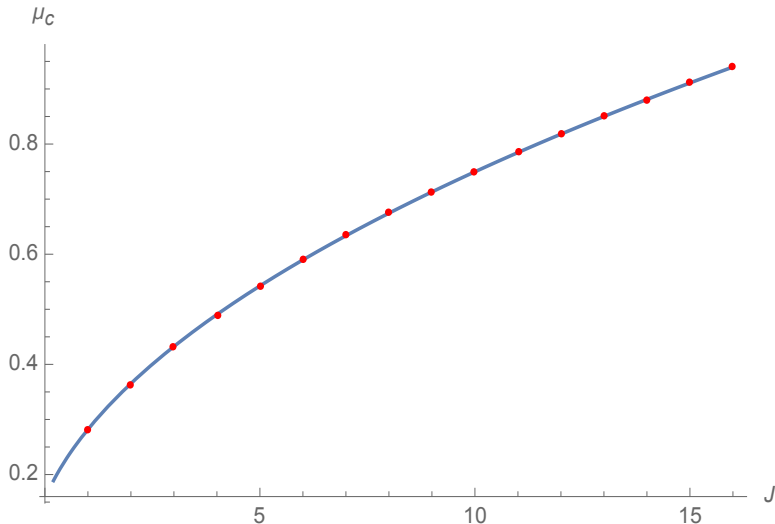
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Also: The plot above is **not** "fit"

# Critical $O(2)$ model in $D=3$ at large charge

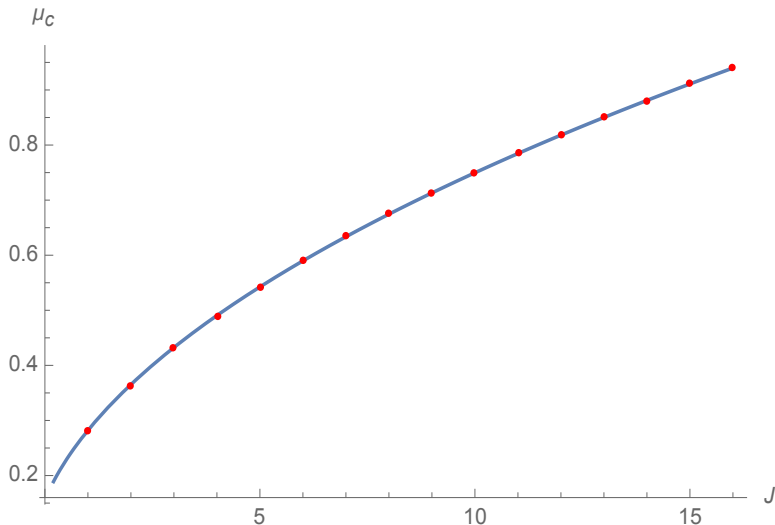
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Also: The plot above is **not** "fit" with any low- $J$  data at all.

# Critical $O(2)$ model in $D=3$ at large charge

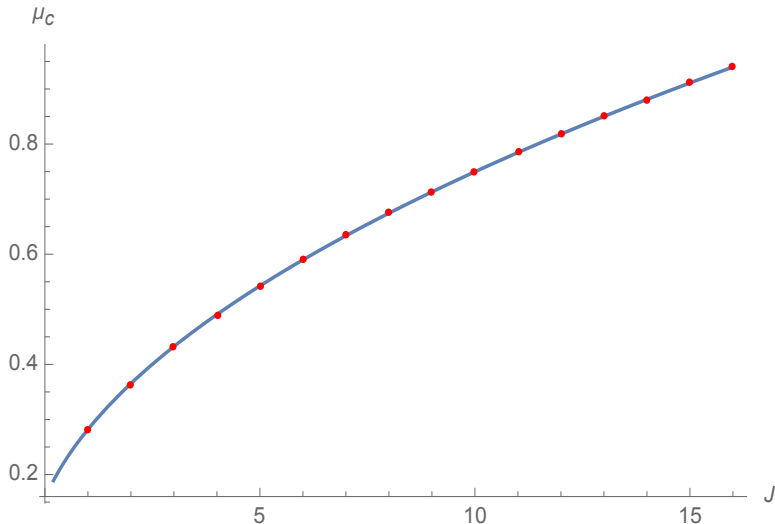
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Also: The plot above is **not** "fit" with any low- $J$  data at all. It is simply fit choosing the

# Critical $O(2)$ model in $D=3$ at large charge

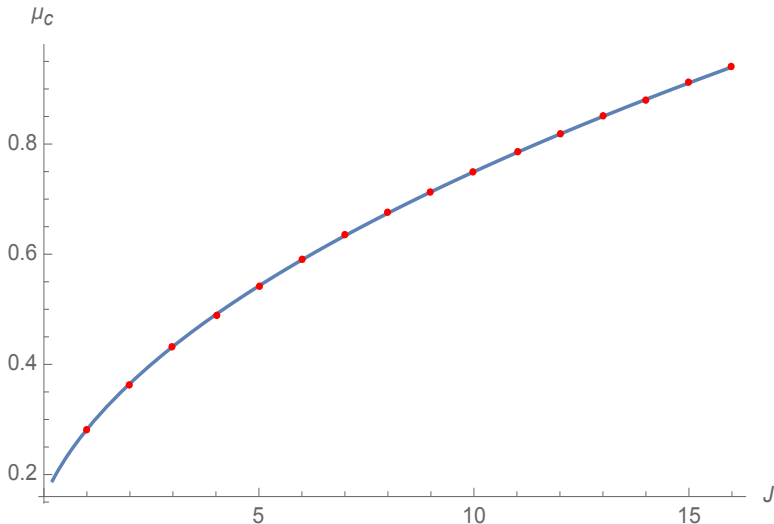
$$0.154253(J+1)^{3/2} - 0.154253J^{3/2}$$



Also: The plot above is **not** "fit" with any low- $J$  data at all. It is simply fit choosing the  $c_{\frac{3}{2}}$

# Critical $O(2)$ model in $D=3$ at large charge

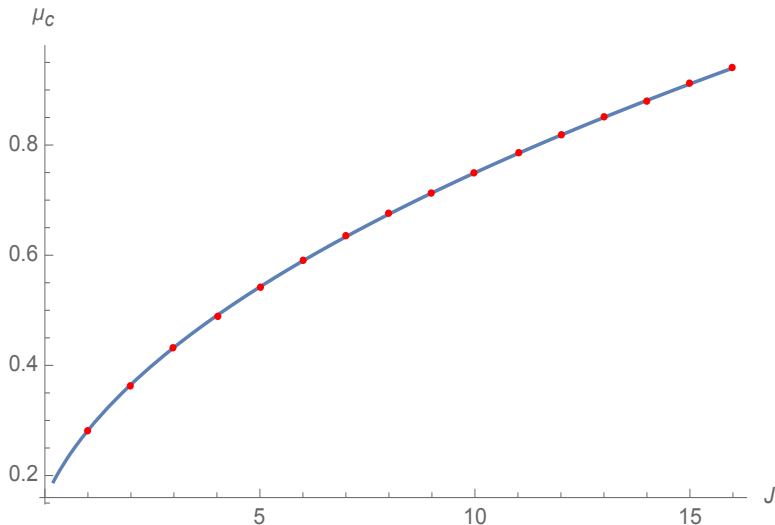
$$0.154253(J+1)^{3/2} - 0.154253J^{3/2}$$



Also: The plot above is **not** "fit" with any low- $J$  data at all. It is simply fit choosing the  $c_{\frac{3}{2}}$  coefficient so that the

# Critical $O(2)$ model in $D=3$ at large charge

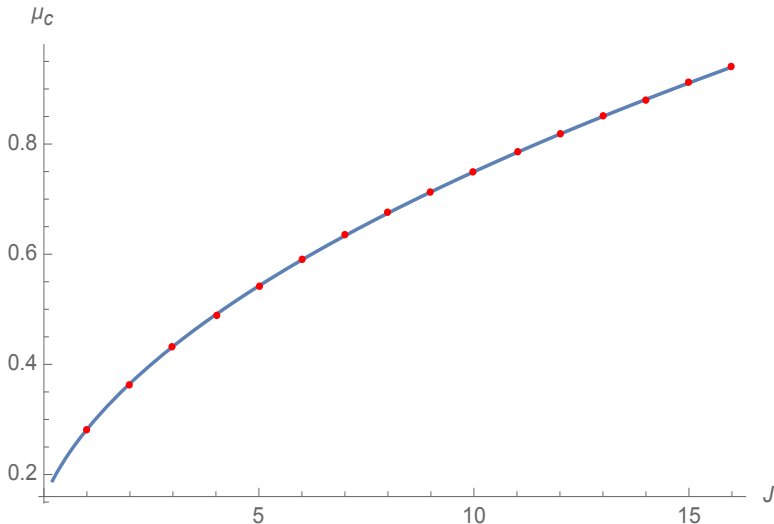
$$0.154253(J+1)^{3/2} - 0.154253J^{3/2}$$



Also: The plot above is **not** "fit" with any low- $J$  data at all. It is simply fit choosing the  $c_{\frac{3}{2}}$  coefficient so that the  $J = 15$  data point

# Critical $O(2)$ model in $D=3$ at large charge

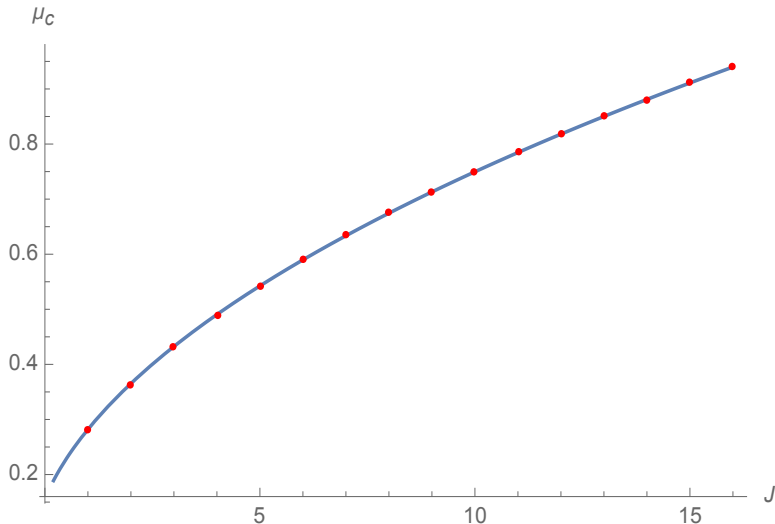
$$0.154253(J+1)^{3/2} - 0.154253J^{3/2}$$



Also: The plot above is **not** "fit" with any low- $J$  data at all. It is simply fit choosing the  $c_{\frac{3}{2}}$  coefficient so that the  $J = 15$  data point lies

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253(J+1)^{3/2} - 0.154253J^{3/2}$$

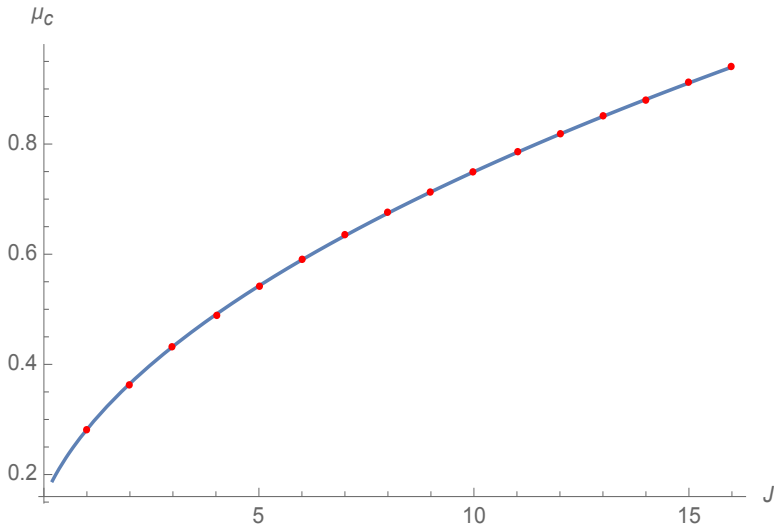


Also: The plot above is **not** "fit" with any low- $J$  data at all. It is simply fit choosing the  $c_{\frac{3}{2}}$  coefficient so that the  $J = 15$  data point lies **exactly on the curve**.



# Critical $O(2)$ model in $D=3$ at large charge

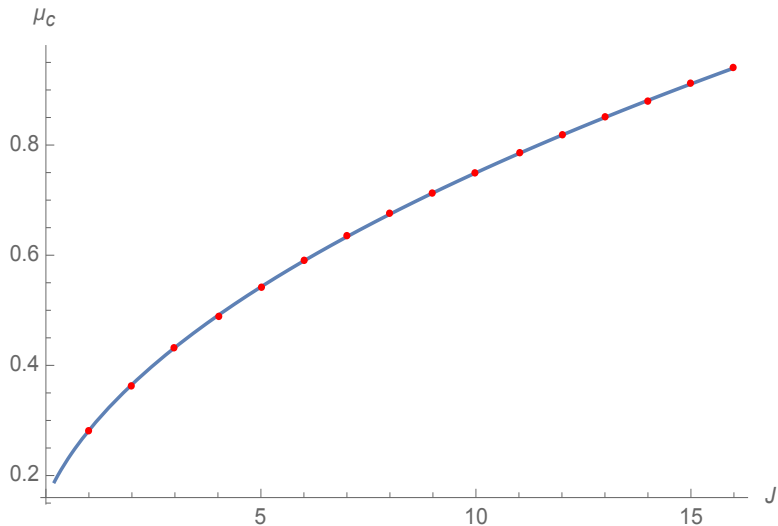
$$0.154253(J+1)^{3/2} - 0.154253J^{3/2}$$



Also: The plot above is **not** "fit" with any low- $J$  data at all. It is simply fit choosing the  $c_{\frac{3}{2}}$  coefficient so that the  $J = 15$  data point lies **exactly on the curve**.

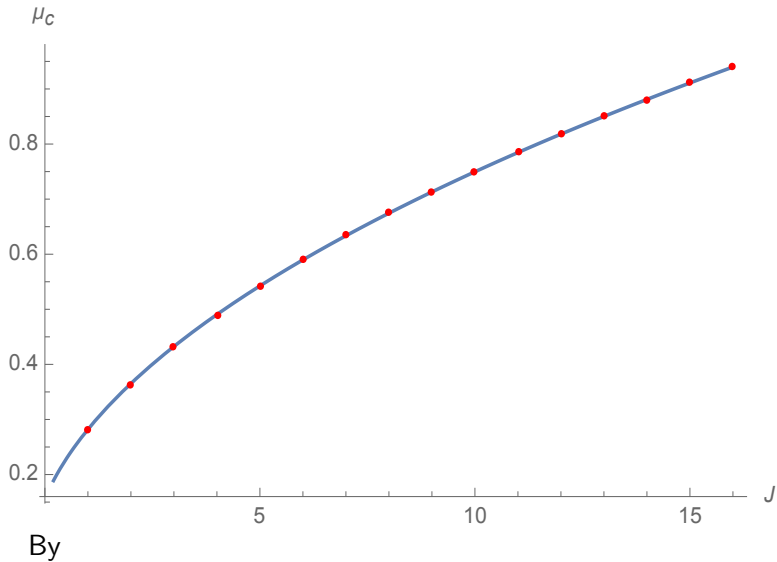
# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



# Critical $O(2)$ model in $D=3$ at large charge

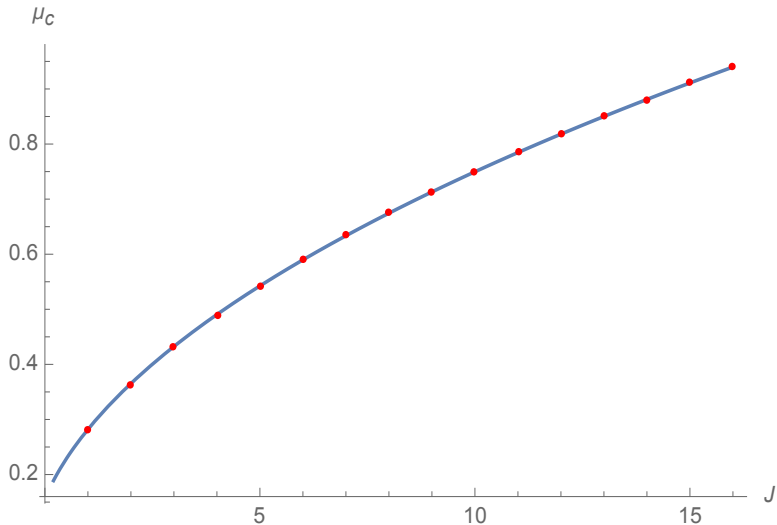
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



By

# Critical $O(2)$ model in $D=3$ at large charge

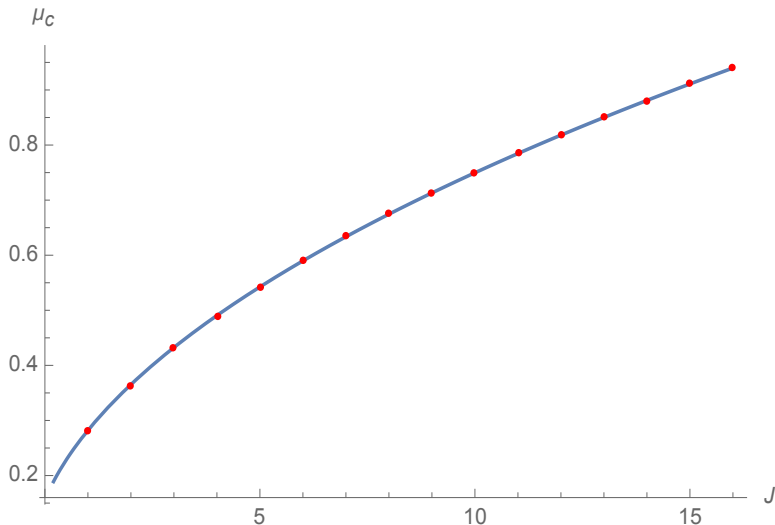
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



By fitting that single data point,

# Critical $O(2)$ model in $D=3$ at large charge

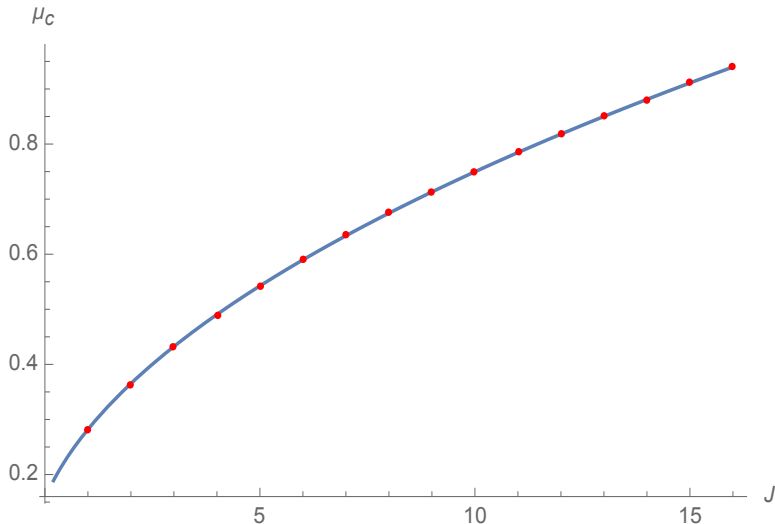
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



By fitting that single data point, the curve for the

# Critical $O(2)$ model in $D=3$ at large charge

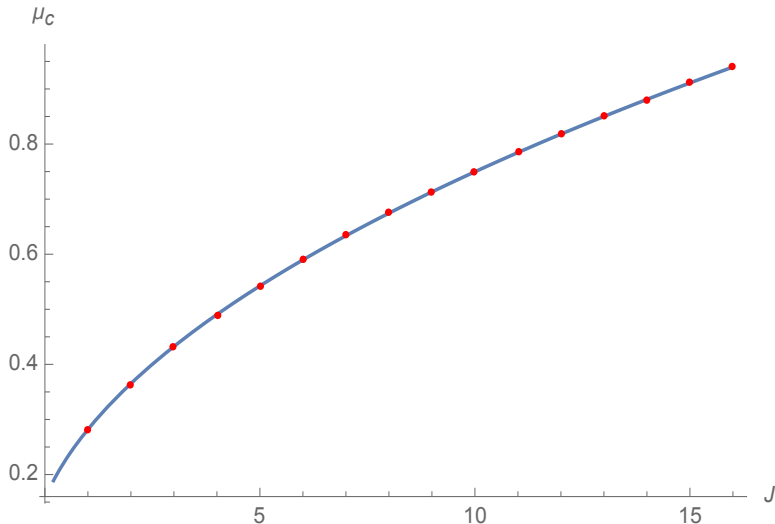
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



By fitting that single data point, the curve for the leading-order prediction

# Critical $O(2)$ model in $D=3$ at large charge

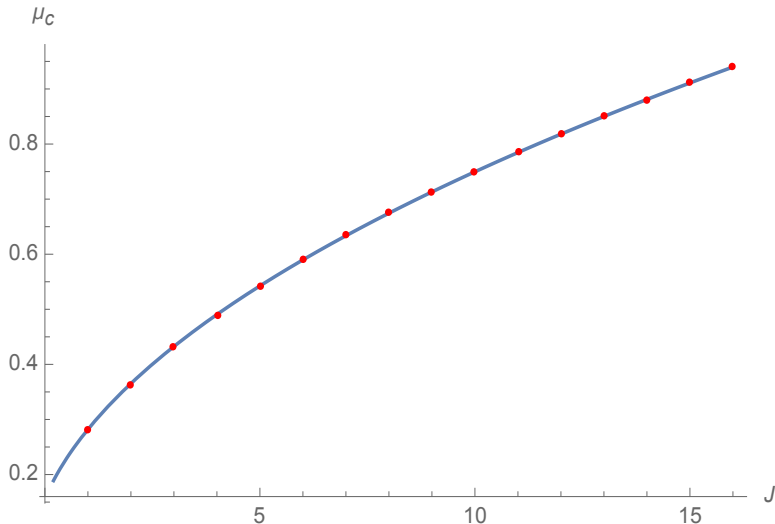
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



By fitting that single data point, the curve for the leading-order prediction runs more or less

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

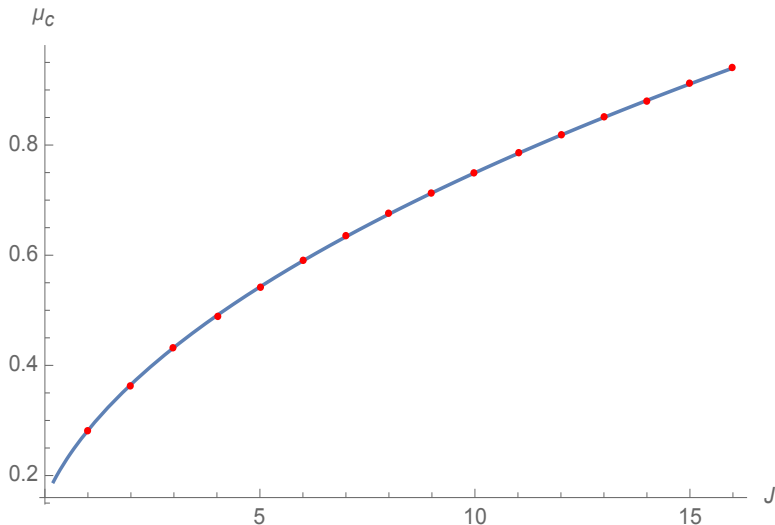


By fitting that single data point, the curve for the leading-order prediction runs more or less\*



# Critical $O(2)$ model in $D=3$ at large charge

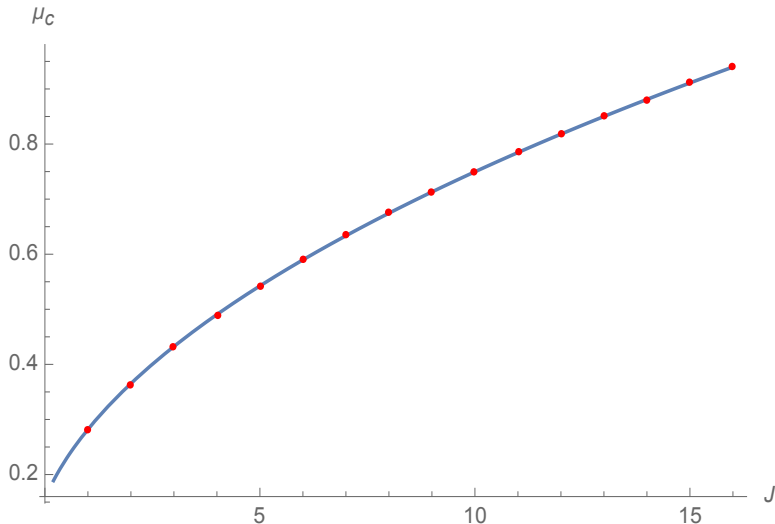
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



By fitting that single data point, the curve for the leading-order prediction runs more or less\* perfectly

# Critical $O(2)$ model in $D=3$ at large charge

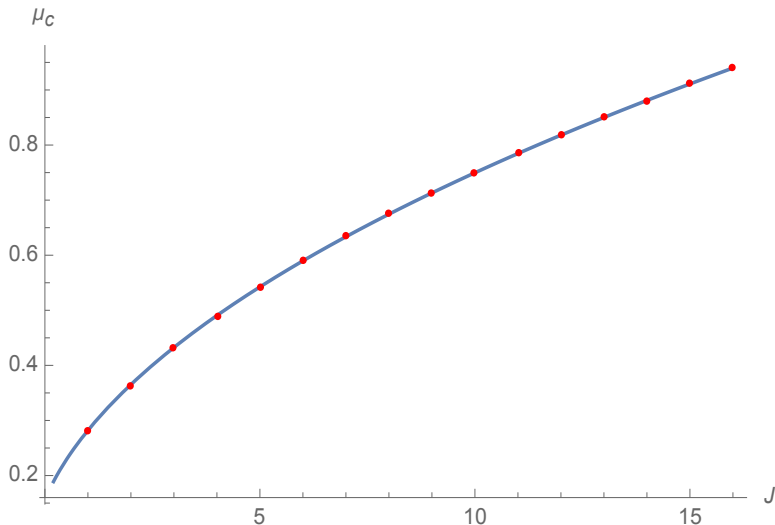
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



By fitting that single data point, the curve for the leading-order prediction runs more or less\* perfectly through all the existing data,

# Critical $O(2)$ model in $D=3$ at large charge

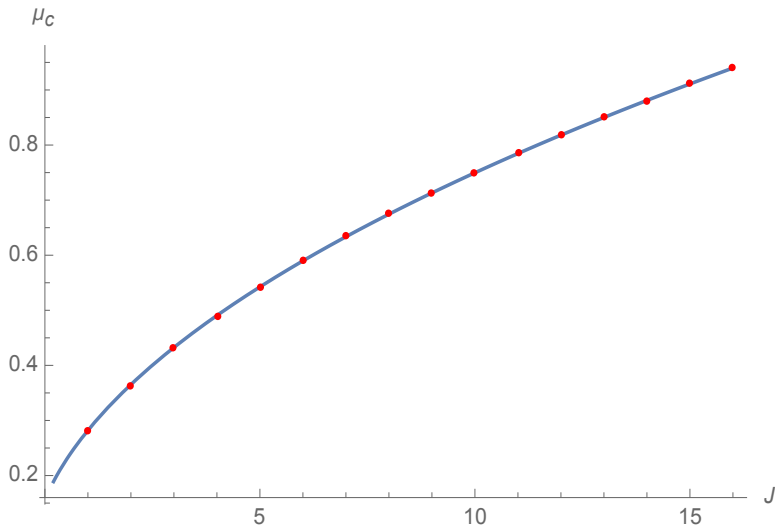
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



By fitting that single data point, the curve for the leading-order prediction runs more or less\* perfectly through all the existing data, down to

# Critical $O(2)$ model in $D=3$ at large charge

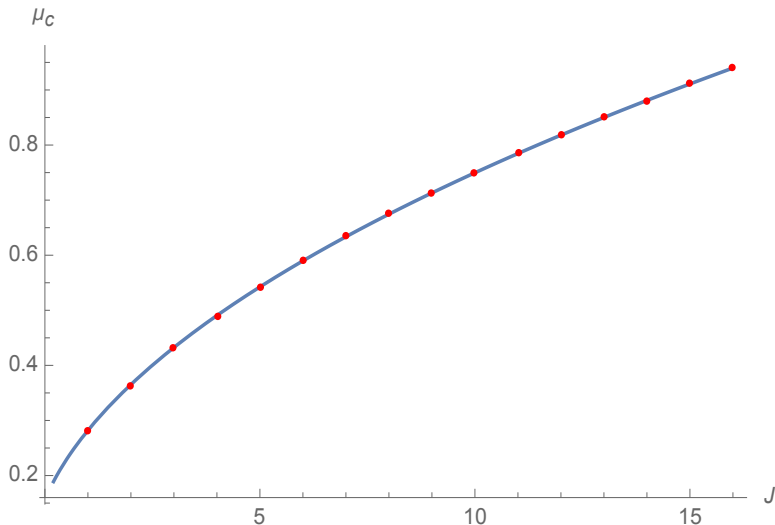
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



By fitting that single data point, the curve for the leading-order prediction runs more or less\* perfectly through all the existing data, down to  $J = 1$ .

# Critical $O(2)$ model in $D=3$ at large charge

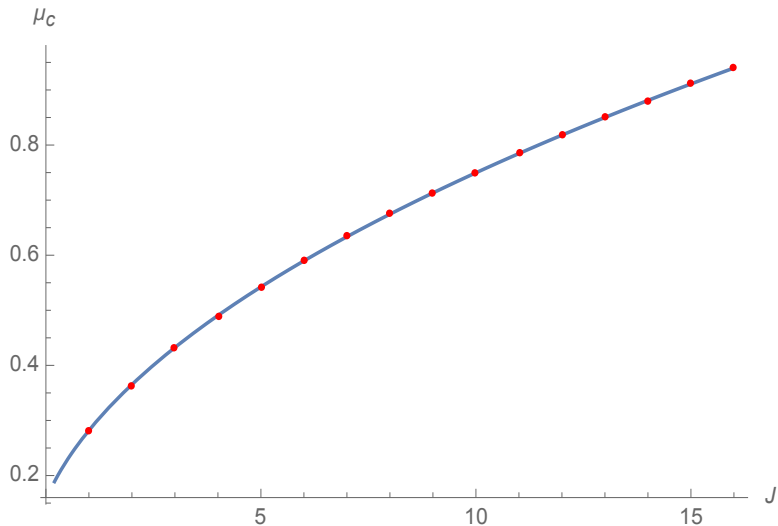
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



By fitting that single data point, the curve for the leading-order prediction runs more or less\* perfectly through all the existing data, down to  $J = 1$ .

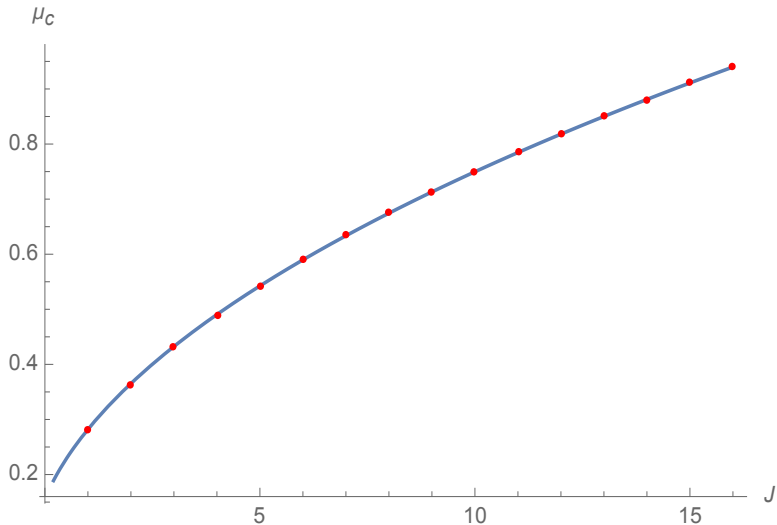
# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



# Critical $O(2)$ model in $D=3$ at large charge

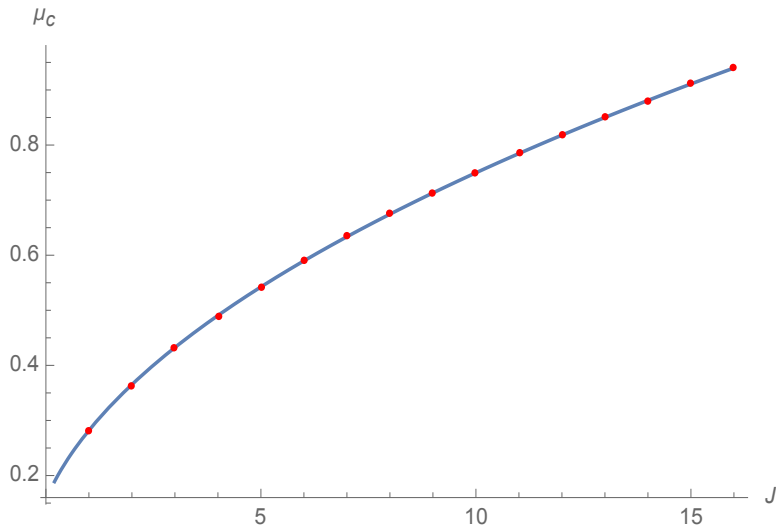
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



(

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

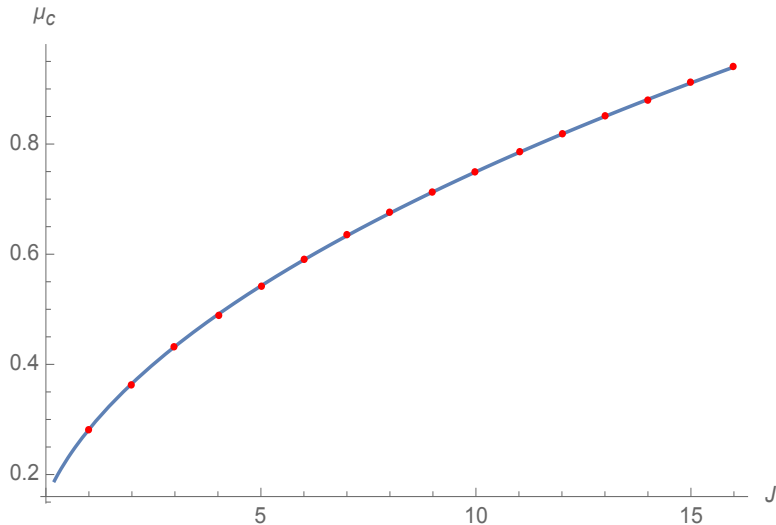


(\*)



# Critical $O(2)$ model in $D=3$ at large charge

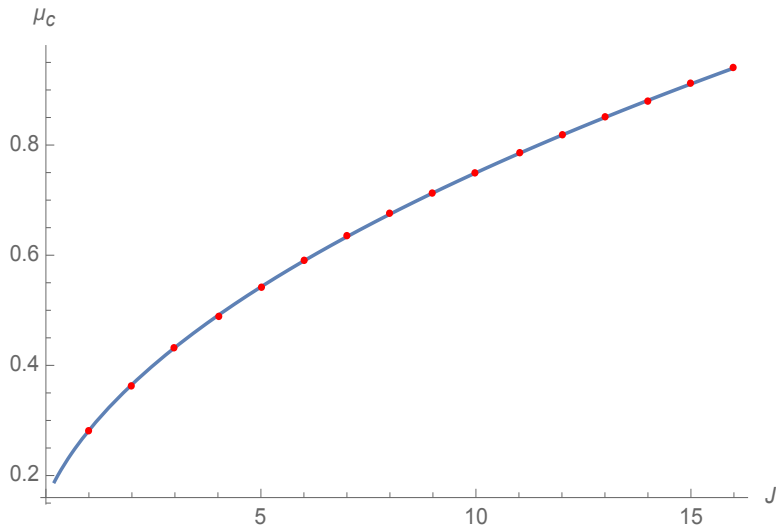
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



(\* I only say

# Critical $O(2)$ model in $D=3$ at large charge

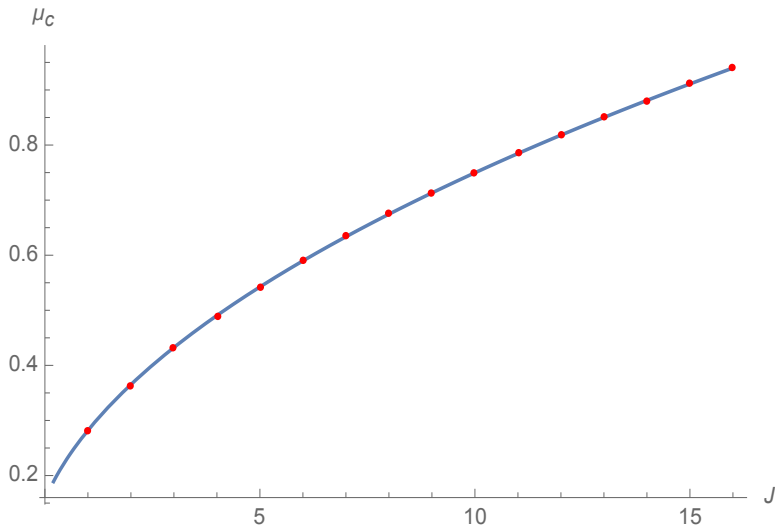
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



(\* I only say "

# Critical $O(2)$ model in $D=3$ at large charge

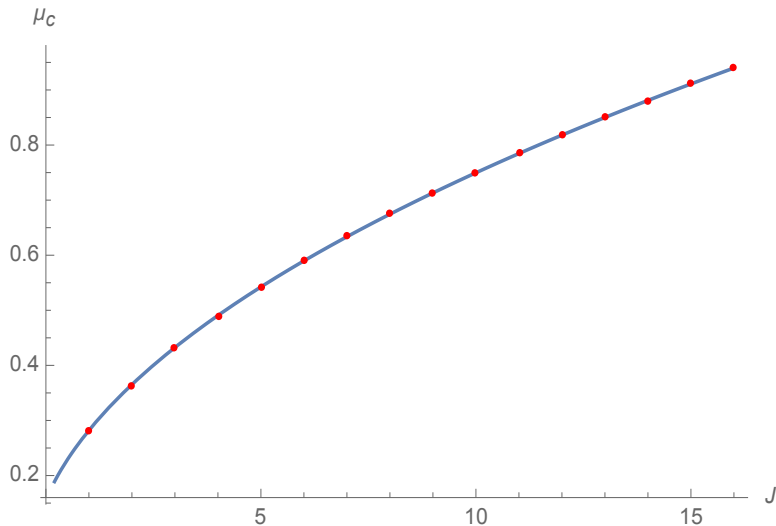
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



(\* I only say "more or less

# Critical $O(2)$ model in $D=3$ at large charge

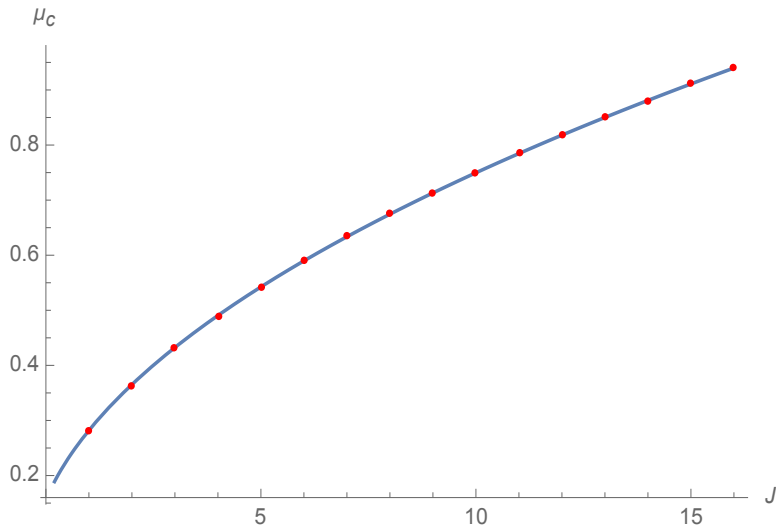
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



(\* I only say "more or less perfectly)

# Critical $O(2)$ model in $D=3$ at large charge

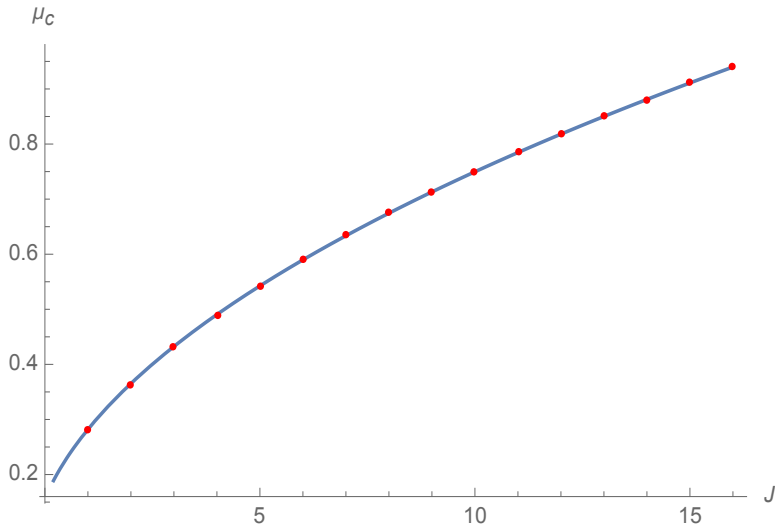
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



(\* I only say "more or less perfectly")

# Critical $O(2)$ model in $D=3$ at large charge

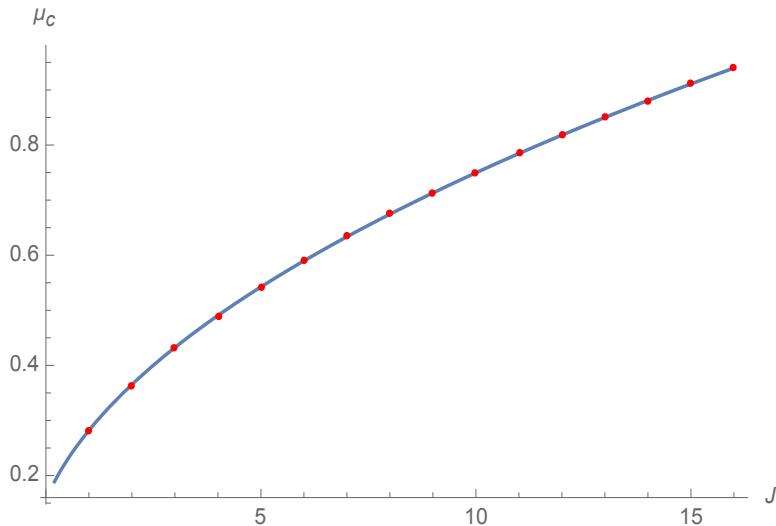
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



(\* I only say "more or less perfectly" because I don't have a graph with

# Critical $O(2)$ model in $D=3$ at large charge

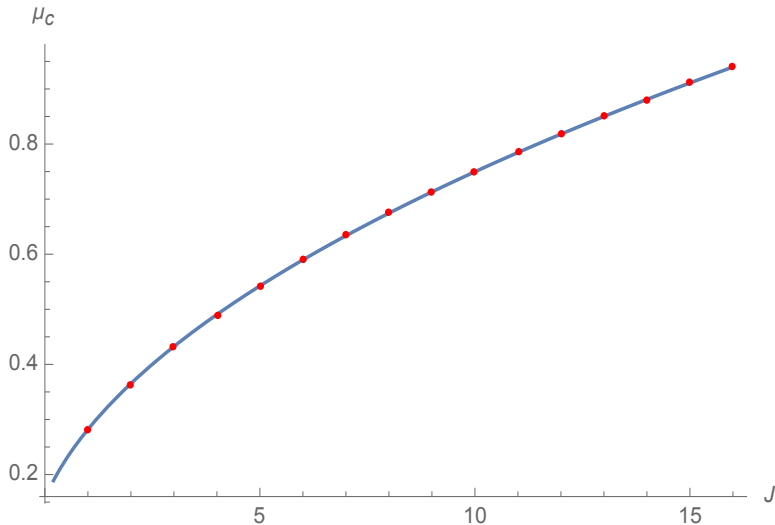
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



(\* I only say "more or less perfectly" because I don't have a graph with error bars.

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

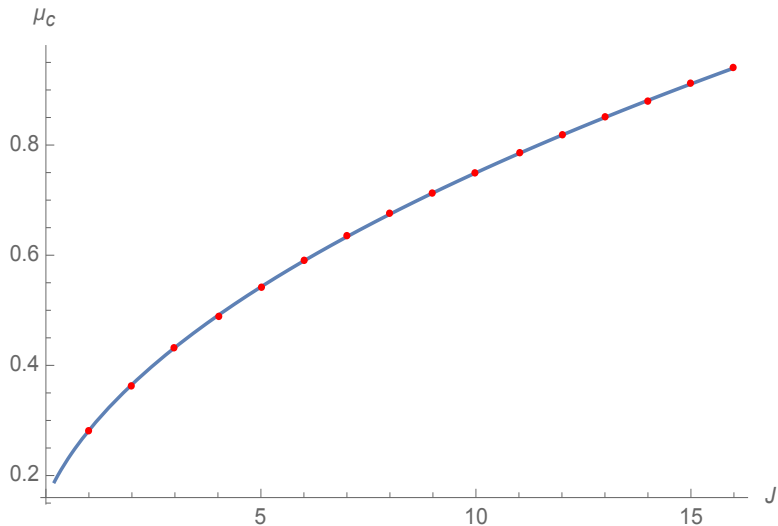


(\* I only say "more or less perfectly" because I don't have a graph with error bars. )



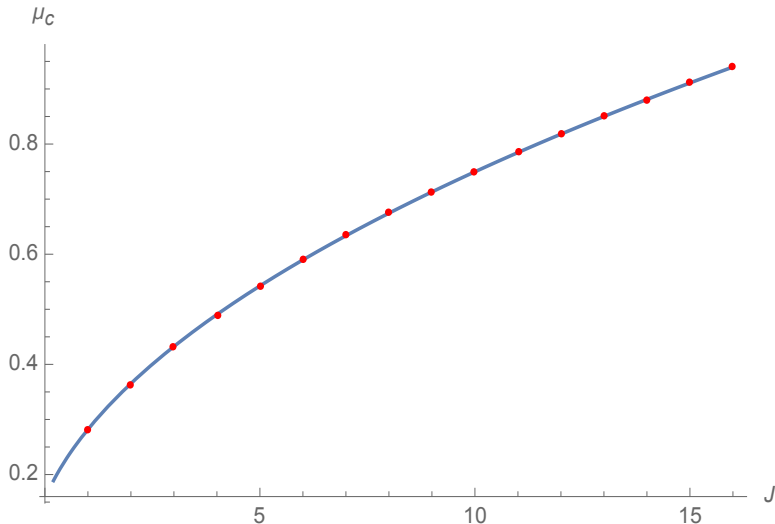
# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



# Critical $O(2)$ model in $D=3$ at large charge

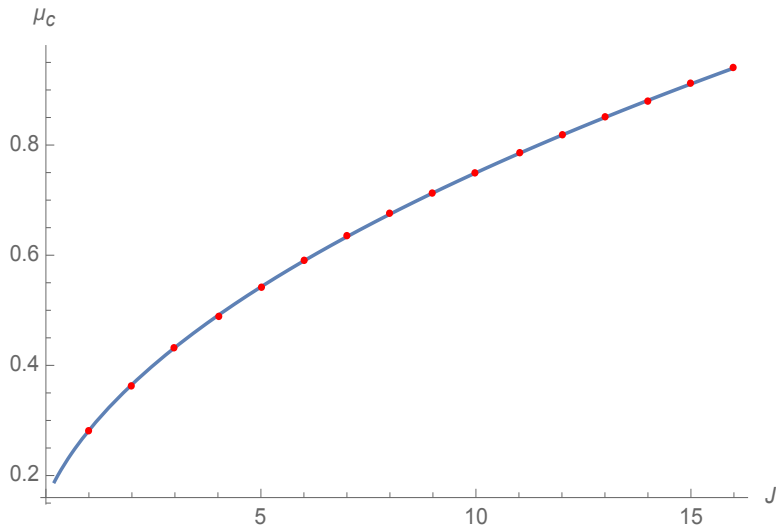
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The value obtained by fitting the  $J = 15$  point gives

# Critical $O(2)$ model in $D=3$ at large charge

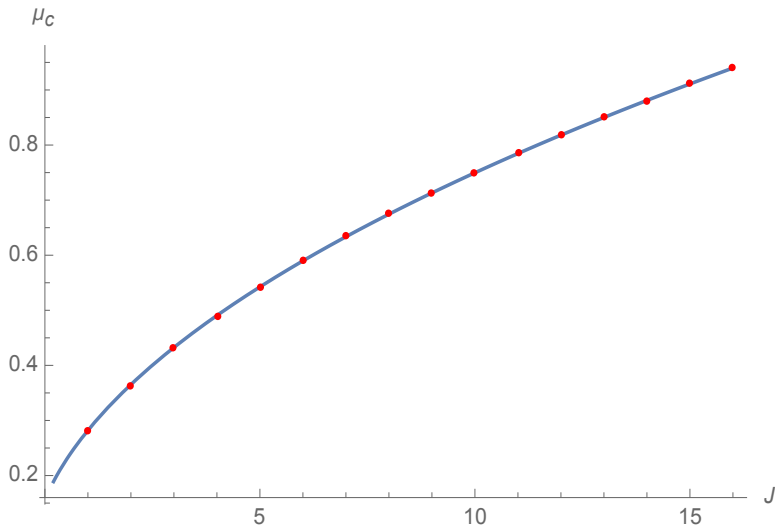
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The value obtained by fitting the  $J = 15$  point gives  
 $E_J^{(0)} \sim 1.23402 \ell^{-1} J^{3/2}$ ,

# Critical $O(2)$ model in $D=3$ at large charge

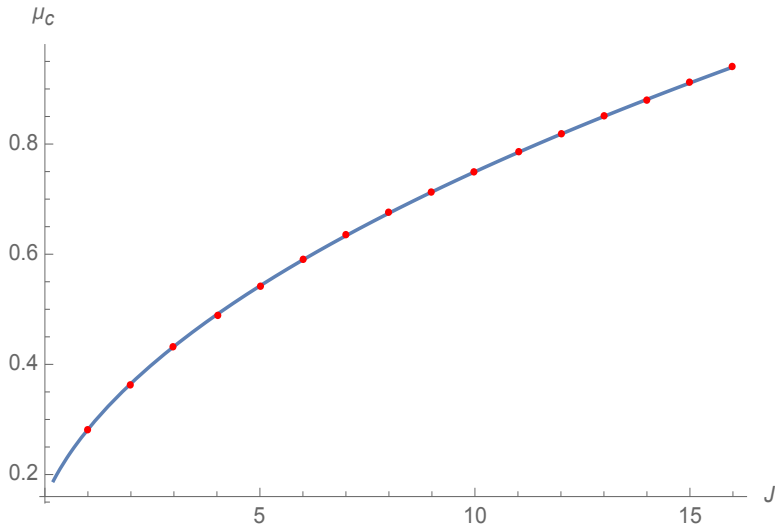
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The value obtained by fitting the  $J = 15$  point gives  $E_J^{(0)} \sim 1.23402 \ell^{-1} J^{3/2}$ , where

# Critical $O(2)$ model in $D=3$ at large charge

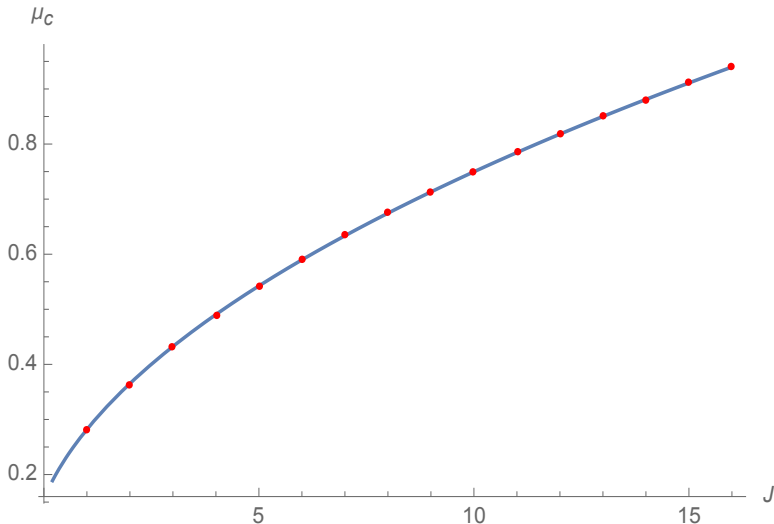
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The value obtained by fitting the  $J = 15$  point gives  $E_J^{(0)} \sim 1.23402 \ell^{-1} J^{3/2}$ , where  $\ell$

# Critical $O(2)$ model in $D=3$ at large charge

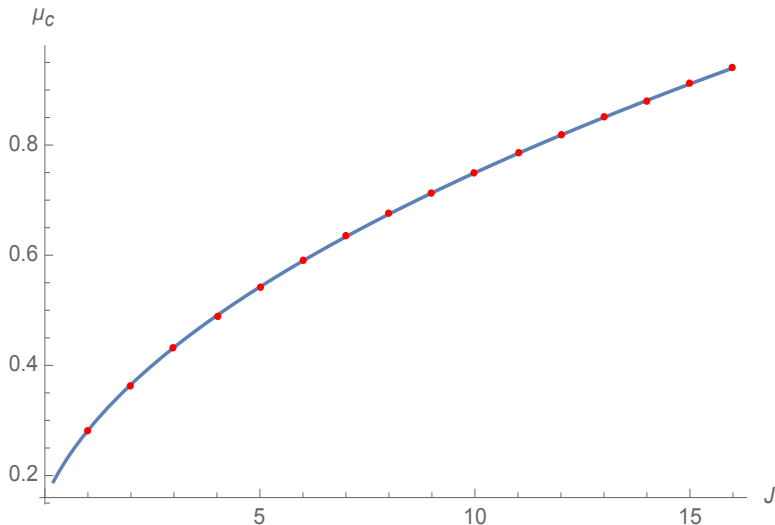
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The value obtained by fitting the  $J = 15$  point gives  $E_J^{(0)} \sim 1.23402 \ell^{-1} J^{3/2}$ , where  $\ell$  is the

# Critical $O(2)$ model in $D=3$ at large charge

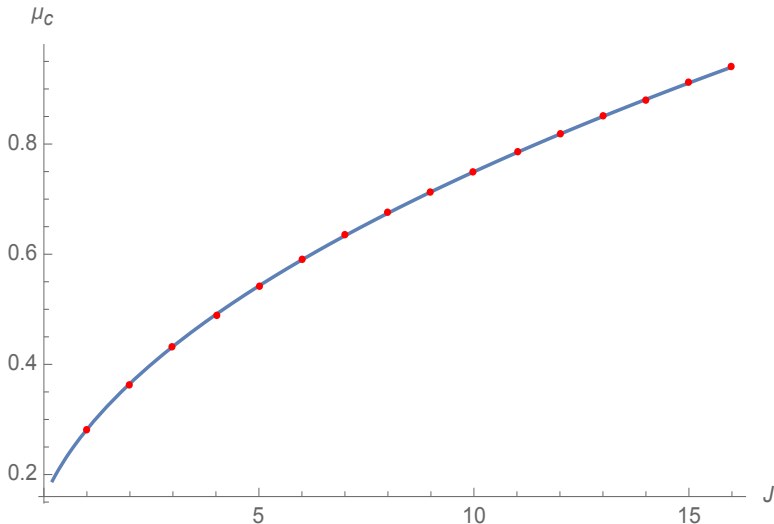
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The value obtained by fitting the  $J = 15$  point gives  $E_J^{(0)} \sim 1.23402 \ell^{-1} J^{3/2}$ , where  $\ell$  is the circumference

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

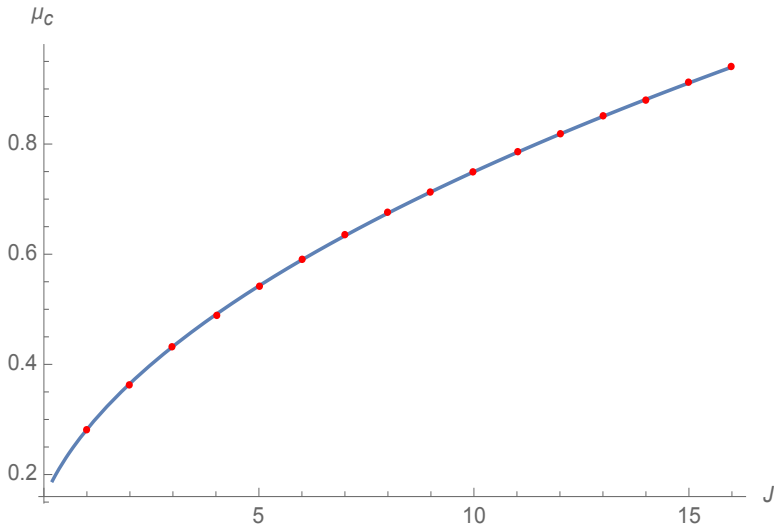


The value obtained by fitting the  $J = 15$  point gives  $E_J^{(0)} \sim 1.23402 \ell^{-1} J^{3/2}$ , where  $\ell$  is the circumference of each cycle of the



# Critical $O(2)$ model in $D=3$ at large charge

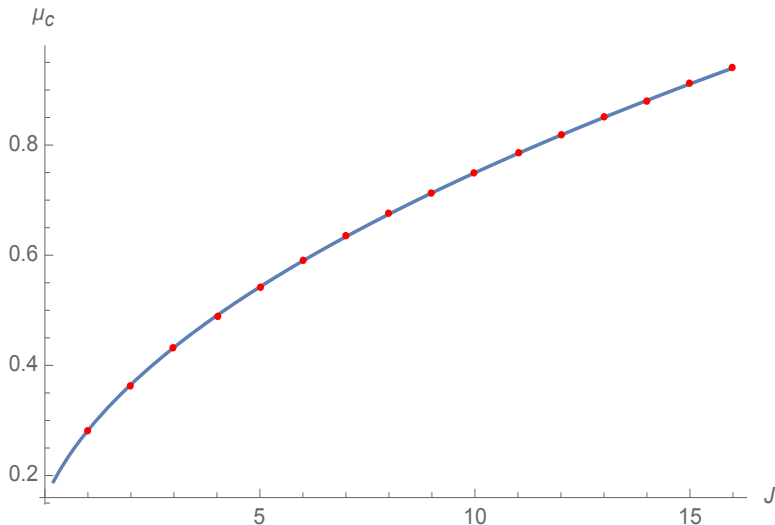
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The value obtained by fitting the  $J = 15$  point gives  $E_J^{(0)} \sim 1.23402 \ell^{-1} J^{3/2}$ , where  $\ell$  is the circumference of each cycle of the square torus.

# Critical $O(2)$ model in $D=3$ at large charge

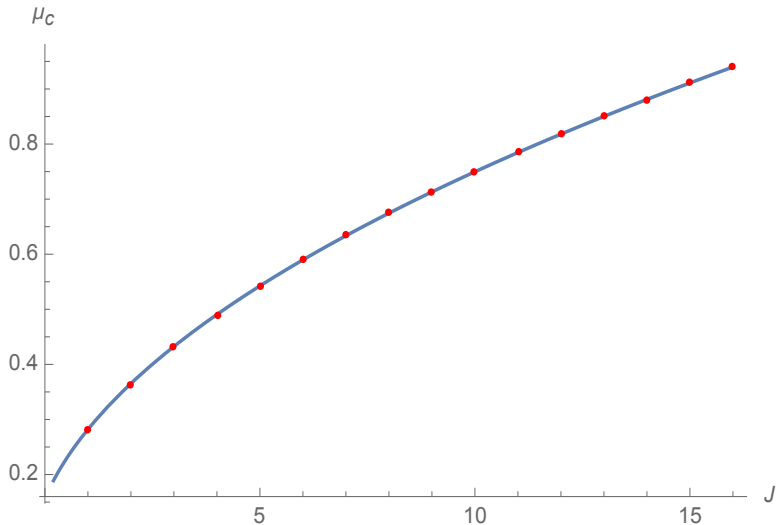
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



The value obtained by fitting the  $J = 15$  point gives  $E_J^{(0)} \sim 1.23402 \ell^{-1} J^{3/2}$ , where  $\ell$  is the circumference of each cycle of the square torus.

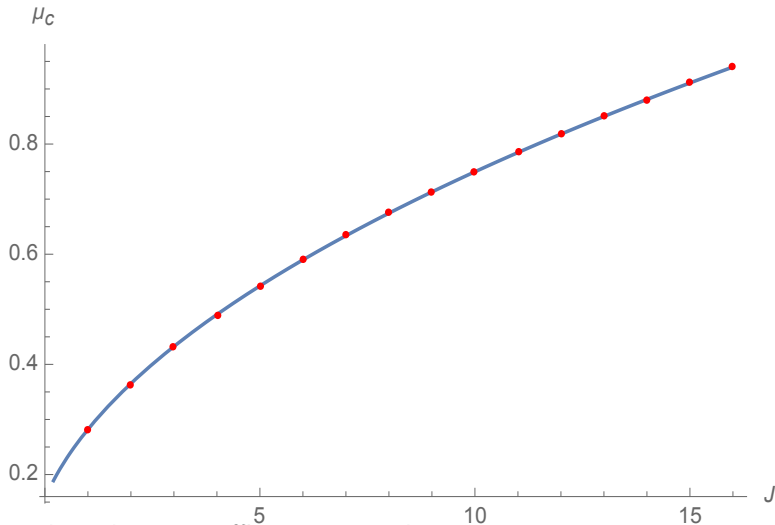
# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



# Critical $O(2)$ model in $D=3$ at large charge

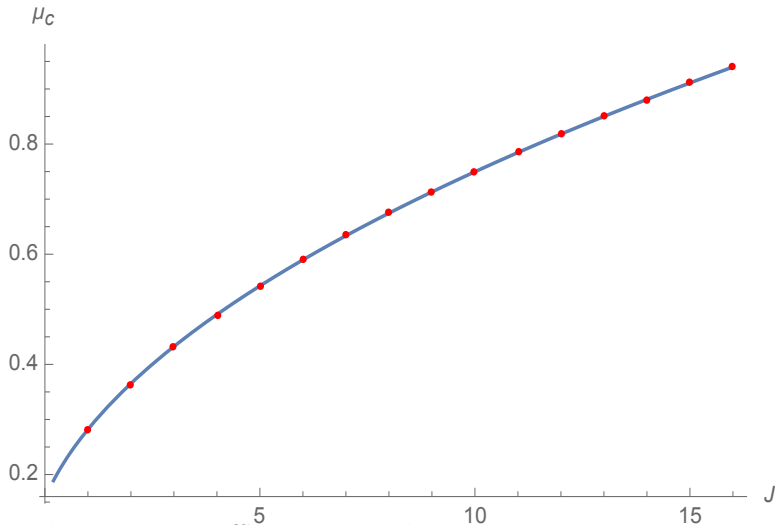
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Then the  $c_{\frac{3}{2}}$  coefficient is equal to

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

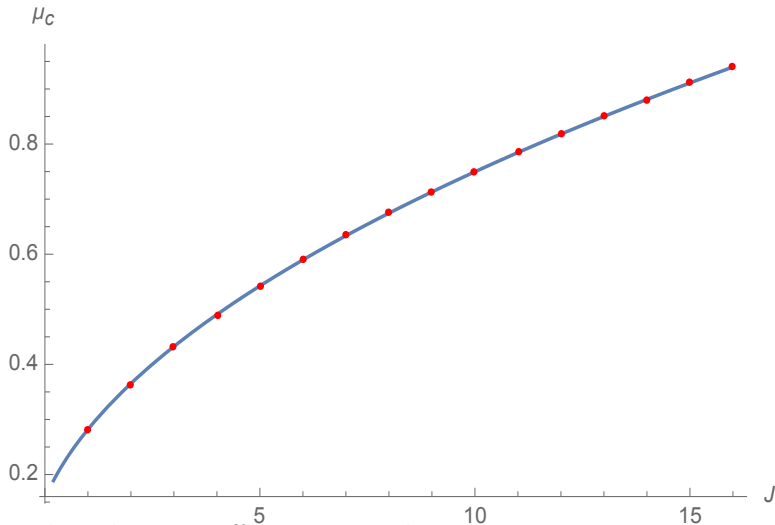


Then the  $c_{\frac{3}{2}}$  coefficient is equal to

$$c_{\frac{3}{2}} \equiv \left( \frac{\text{Area}}{4\pi} \right)^{\frac{1}{2}} \cdot \left( \text{coefficient of } J^{\frac{3}{2}} \text{ in } E_J \right) = \frac{1.23402}{\sqrt{4\pi}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

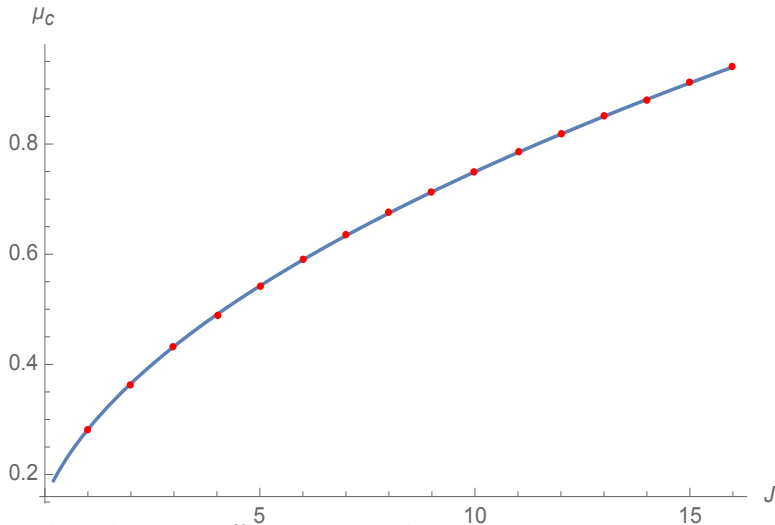


Then the  $c_{\frac{3}{2}}$  coefficient is equal to

$$c_{\frac{3}{2}} \equiv \left( \frac{\text{Area}}{4\pi} \right)^{\frac{1}{2}} \cdot \left( \text{coefficient of } J^{\frac{3}{2}} \text{ in } E_J \right) = \frac{1.23402}{\sqrt{4\pi}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



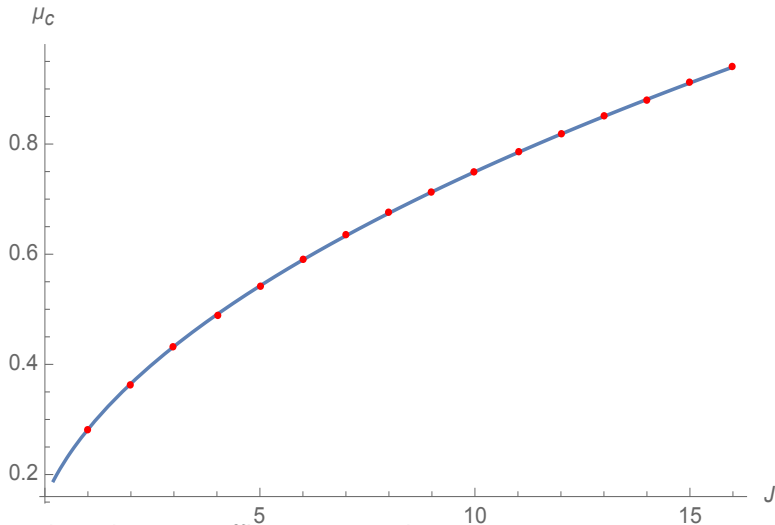
Then the  $c_{\frac{3}{2}}$  coefficient is equal to

$$c_{\frac{3}{2}}$$

$$= 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



Then the  $c_{\frac{3}{2}}$  coefficient is equal to

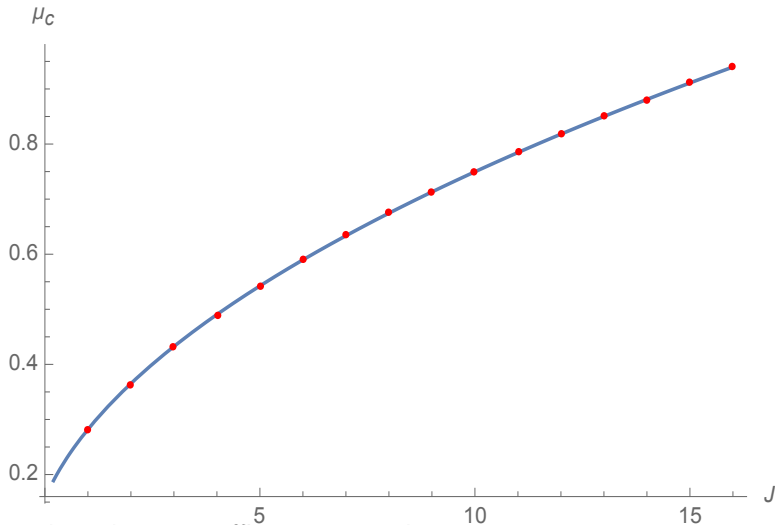
$$c_{\frac{3}{2}}$$

$$= 0.348111 .$$



# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



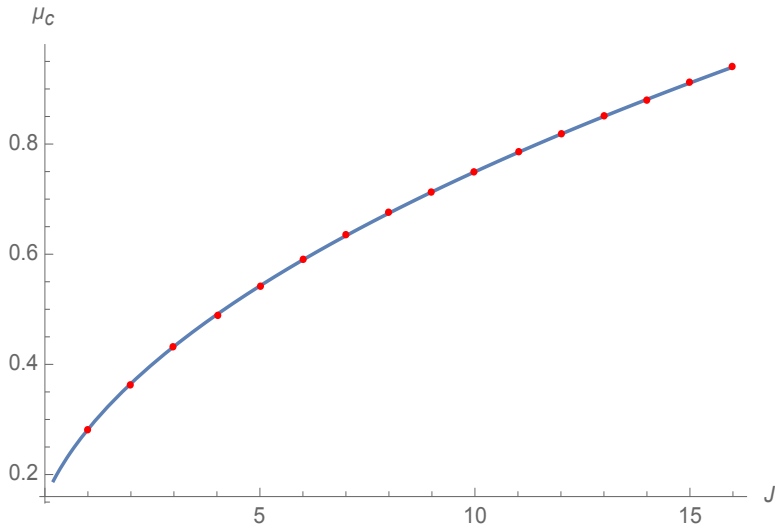
Then the  $c_{\frac{3}{2}}$  coefficient is equal to

$$c_{\frac{3}{2}}$$

$$= 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

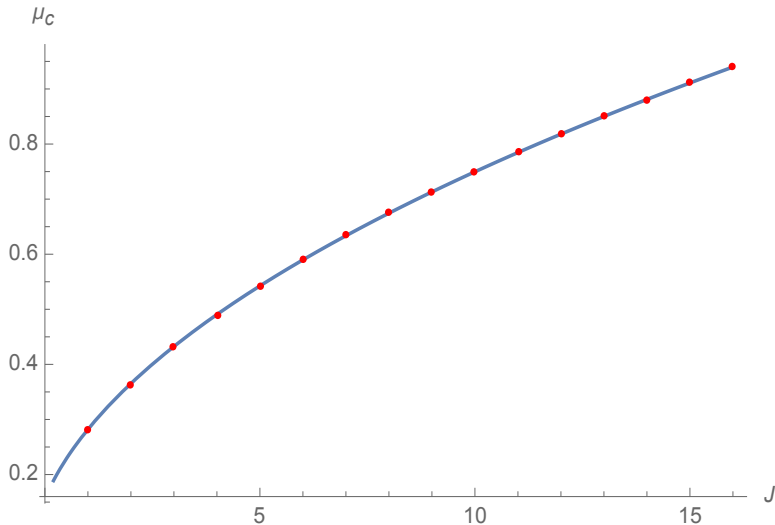


$C_{\frac{3}{2}}$

$= 0.348111$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

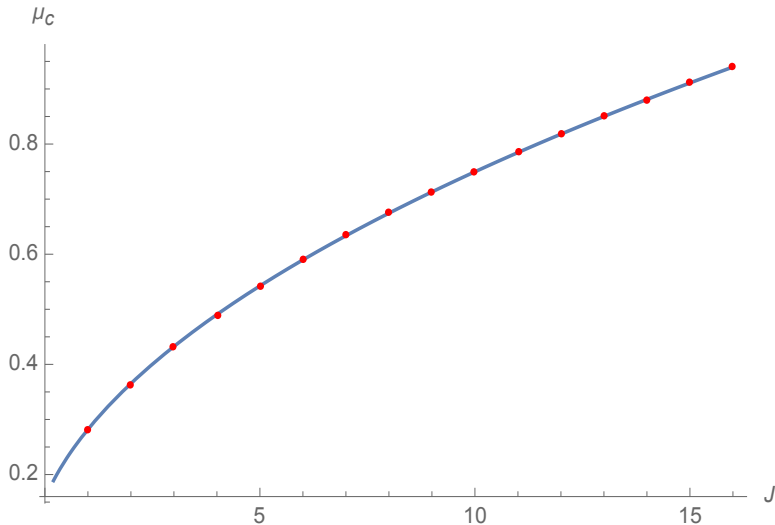


$C_{\frac{3}{2}}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

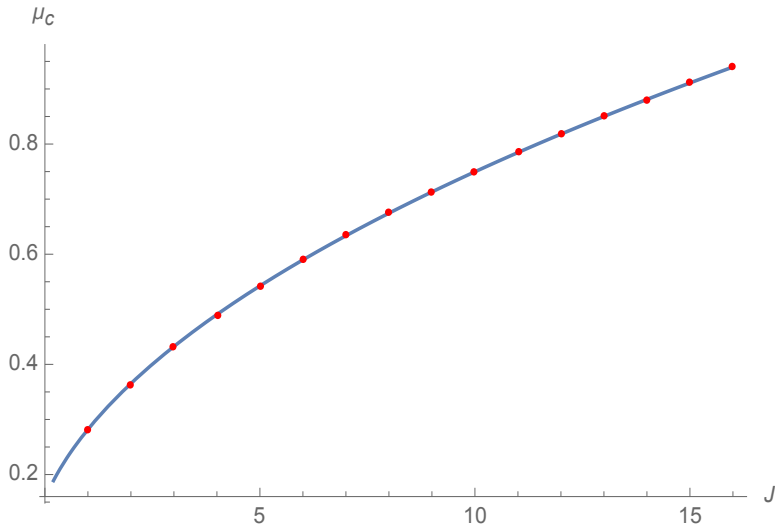


$C_{\frac{3}{2}}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

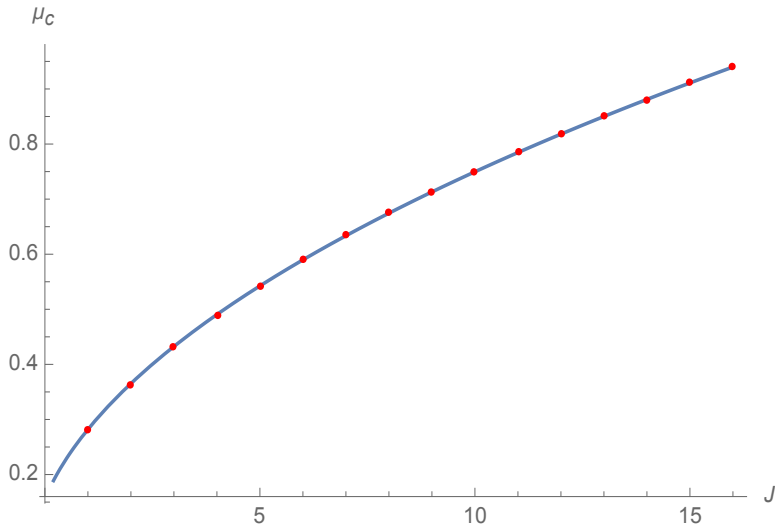


$C_{\frac{3}{2}}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

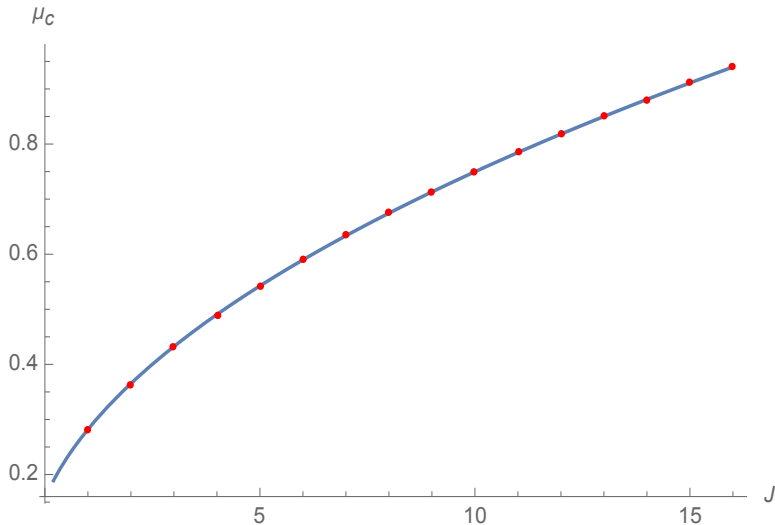


$C_{\frac{3}{2}}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

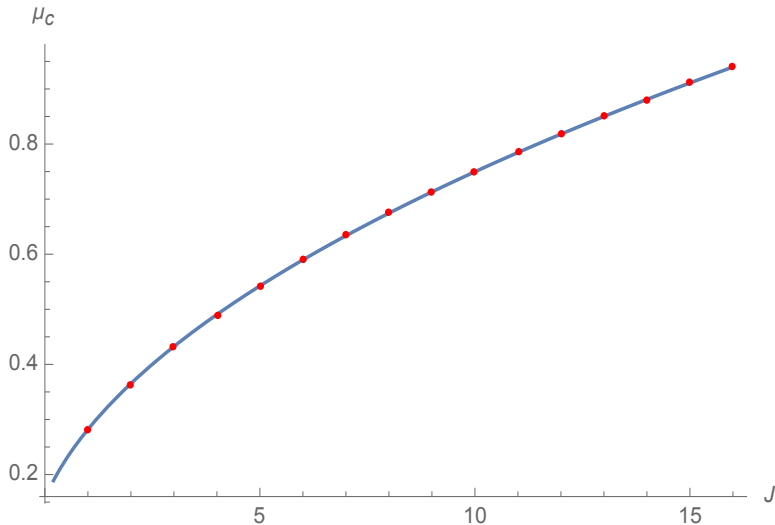


$C_{\frac{3}{2}}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



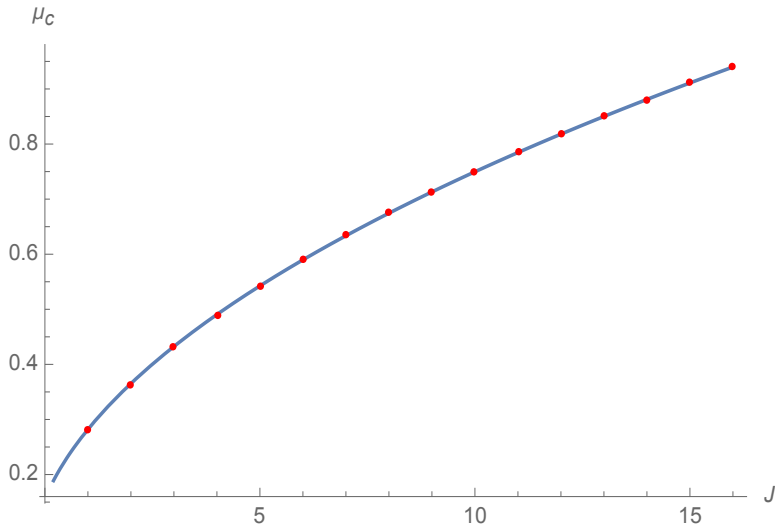
$C_{\frac{3}{2}}$

$= 0.348111 .$



# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

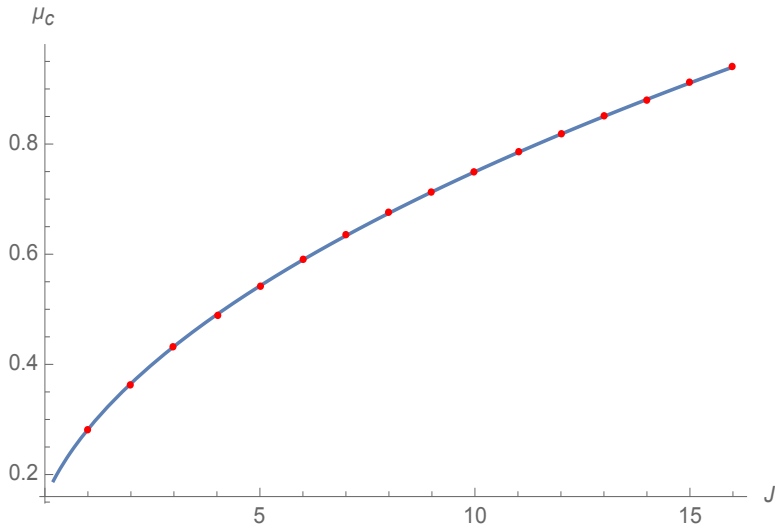


$C_{\frac{3}{2}}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

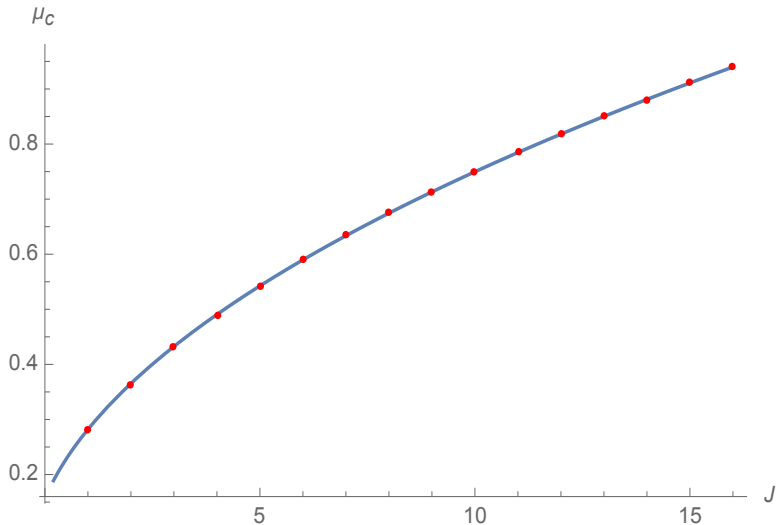


$\frac{c_3}{2}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

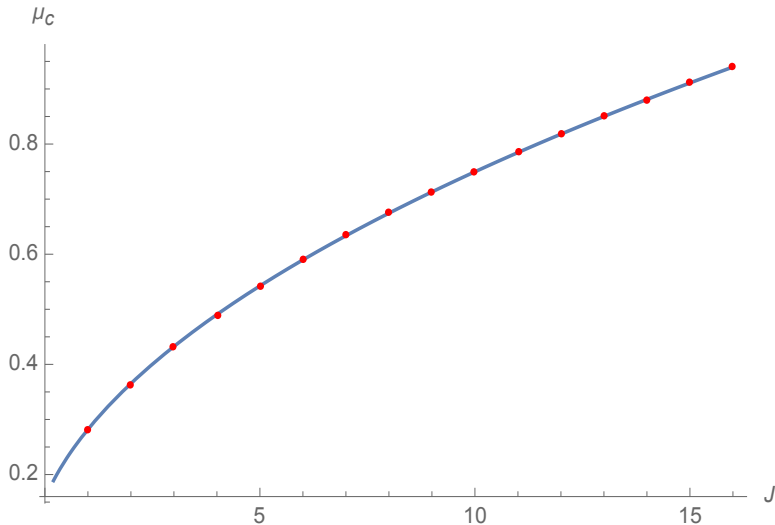


$\frac{c_3}{2}$

$= 0.348111$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

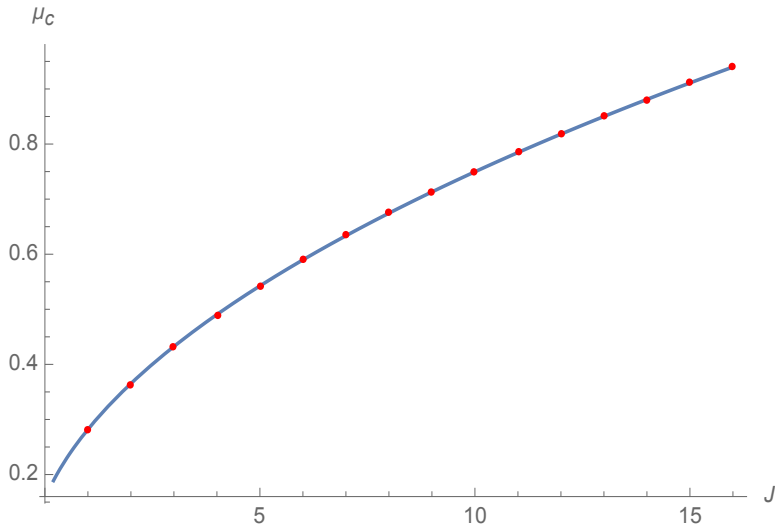


$C_{\frac{3}{2}}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

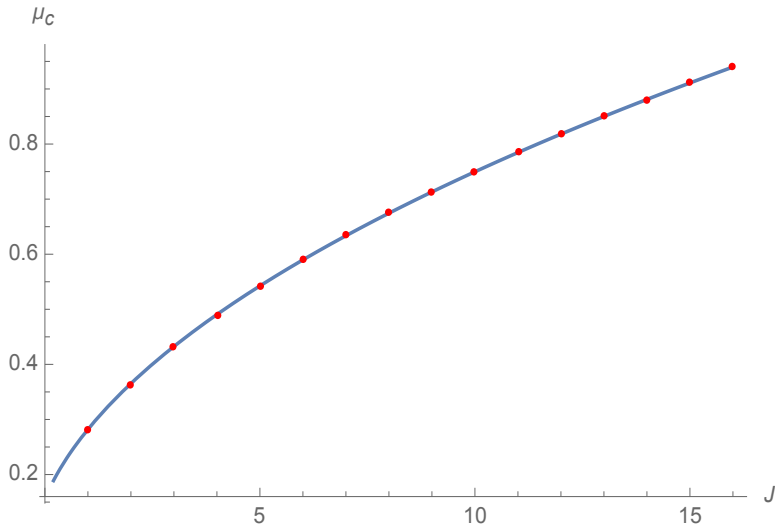


$$\frac{C_3}{2}$$

$$= 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

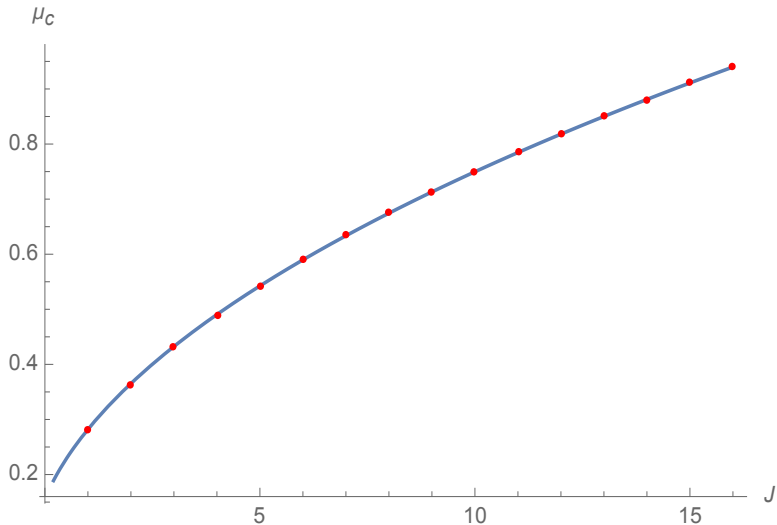


$C_{\frac{3}{2}}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

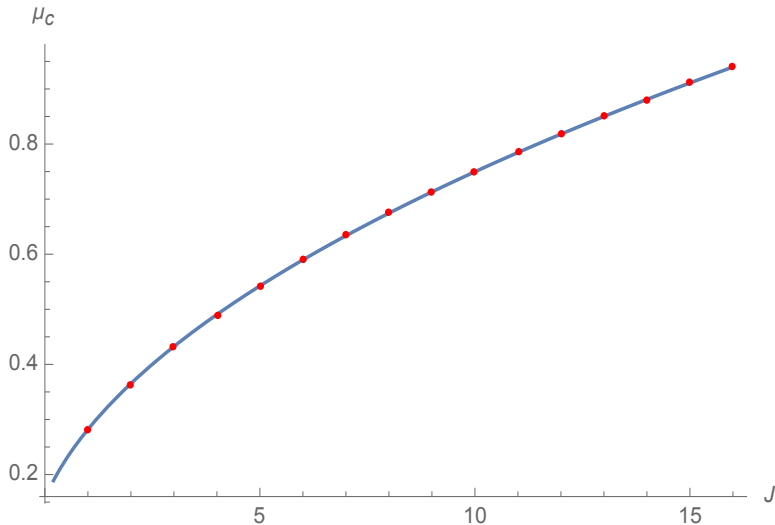


$\frac{C_3}{2}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



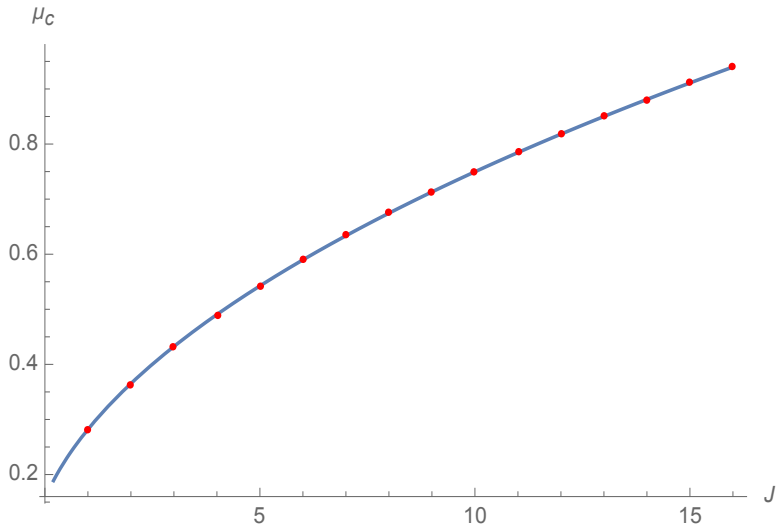
$C_{\frac{3}{2}}$

$= 0.348111 .$



# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

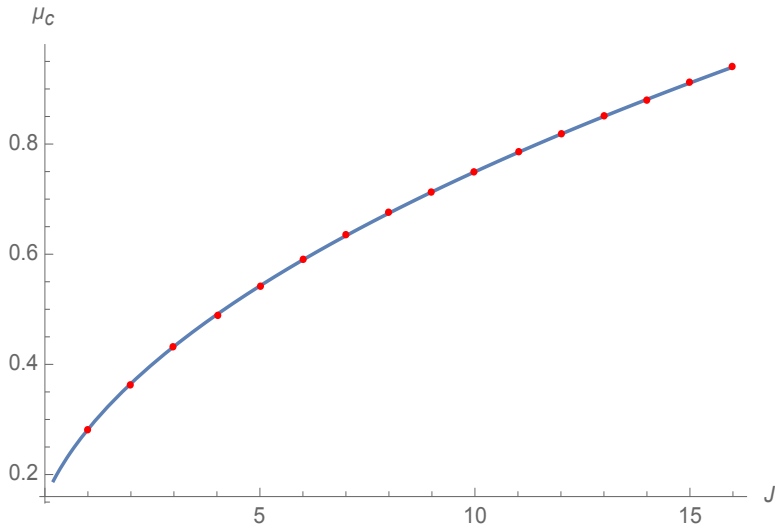


$$C_{\frac{3}{2}}$$

$$= 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

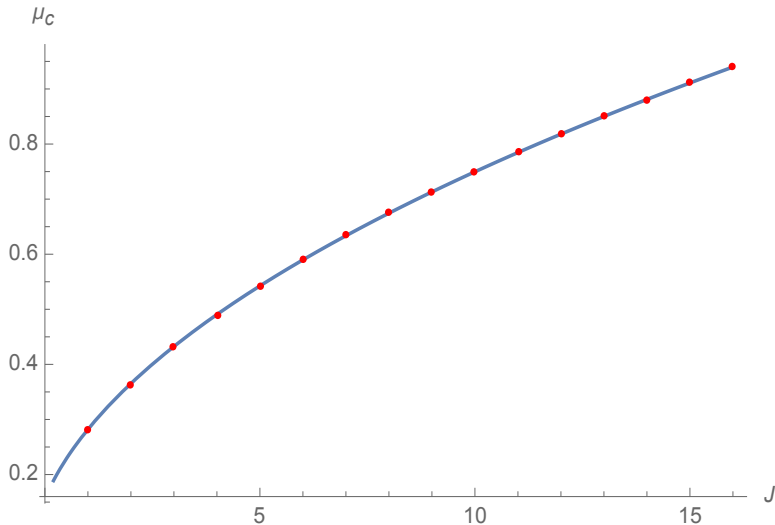


$C_{\frac{3}{2}}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

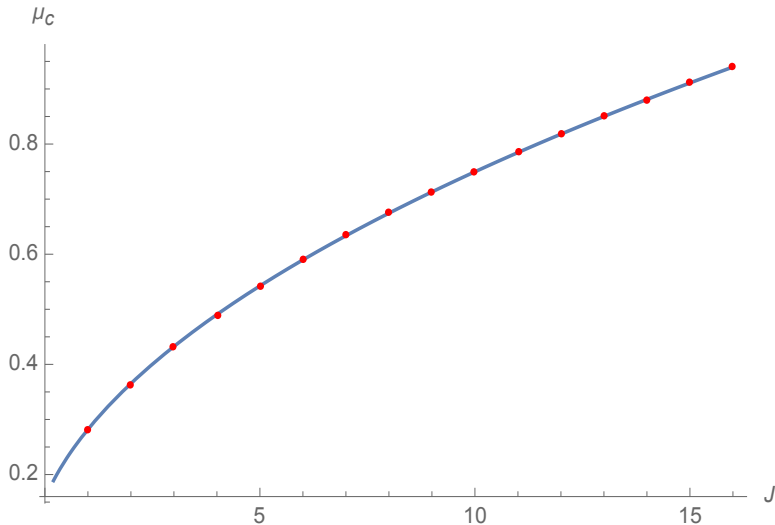


$C_{\frac{3}{2}}$

$= 0.348111 .$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



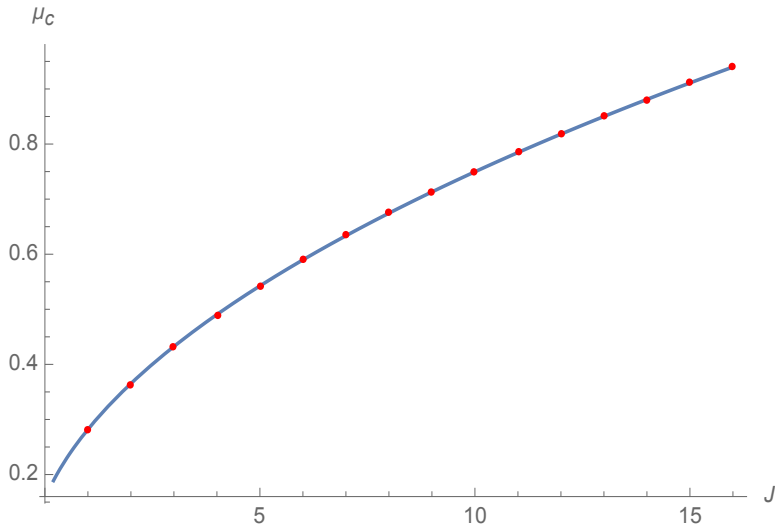
$$C_3$$

$$\frac{3}{2}$$

$$= 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

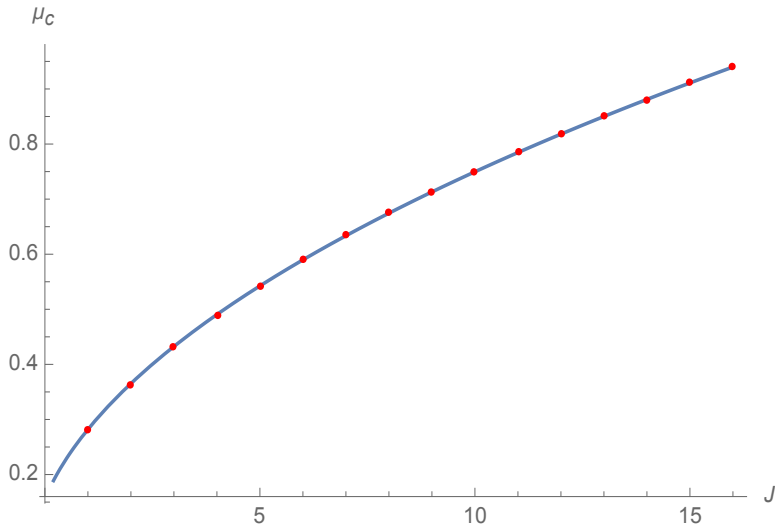


$c_{\frac{3}{2}}$

$= 0.348111$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

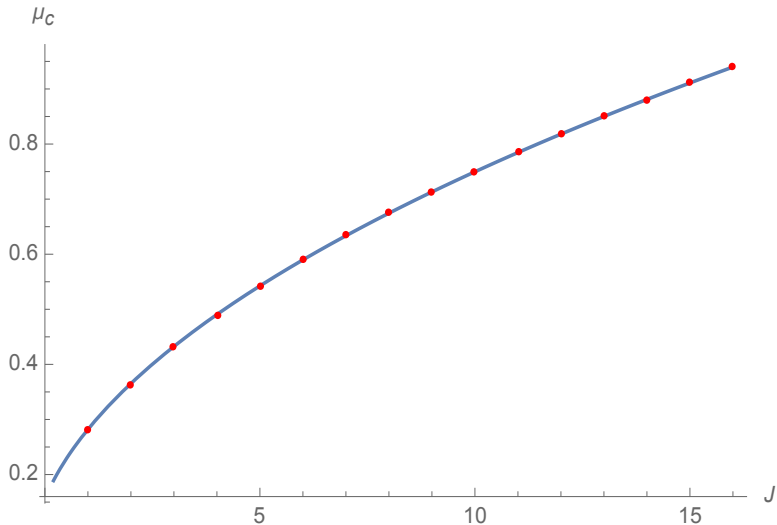


$$\frac{C_3}{2}$$

$$= 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

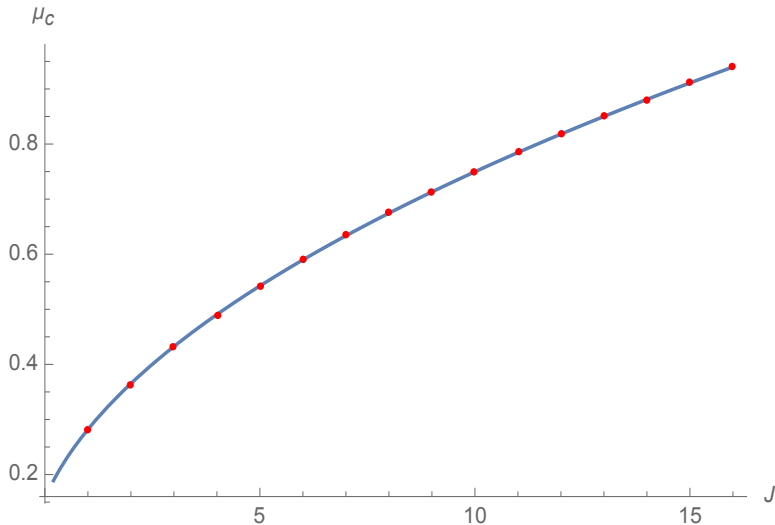


$$c_{\frac{3}{2}}$$

$$= 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



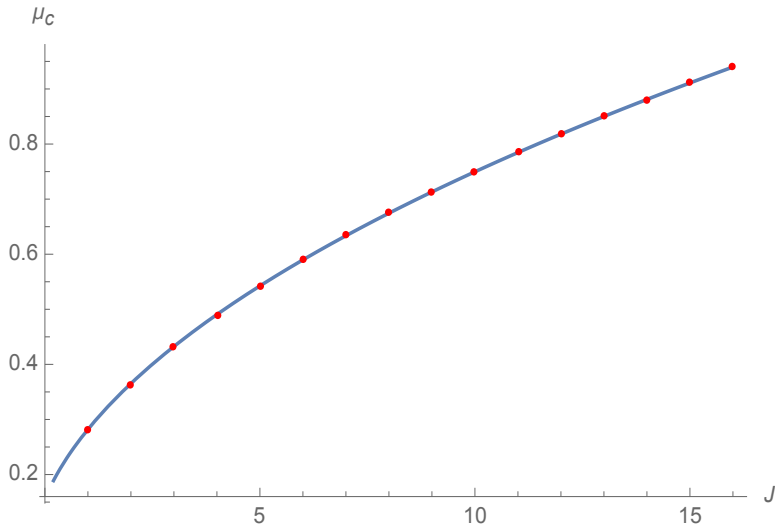
$$c_{\frac{3}{2}}$$

$$= 0.348111 .$$



# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

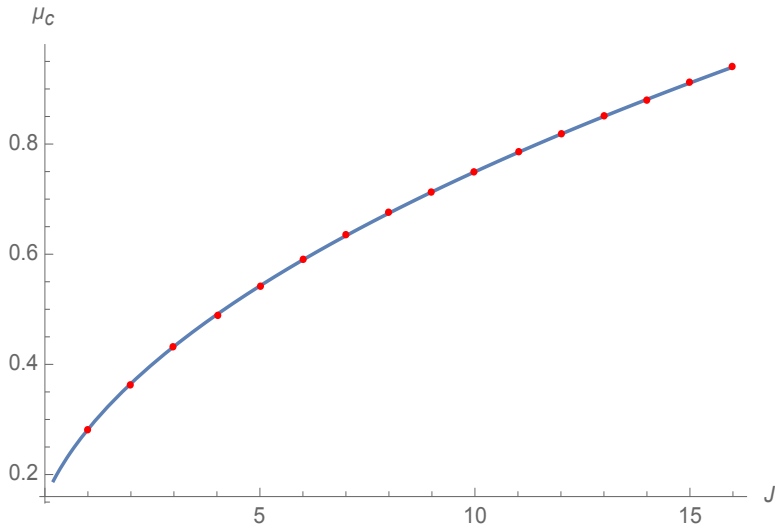


$$c_{\frac{3}{2}}$$

$$= 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

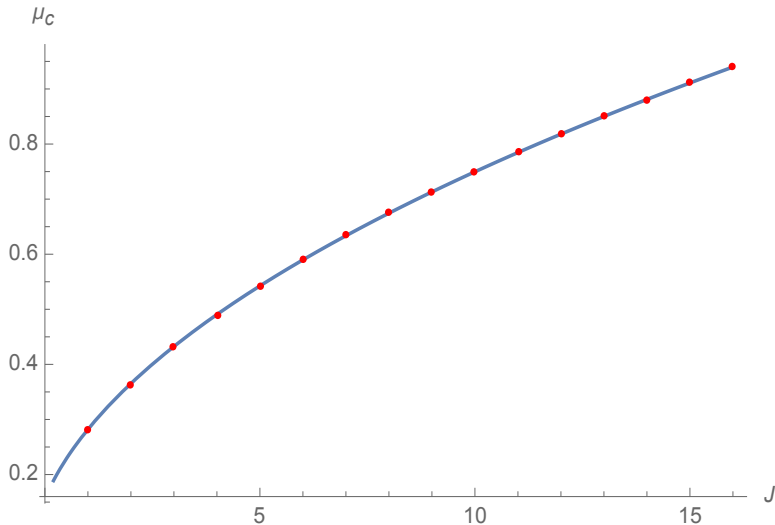


$$c_{\frac{3}{2}}$$

$$= 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

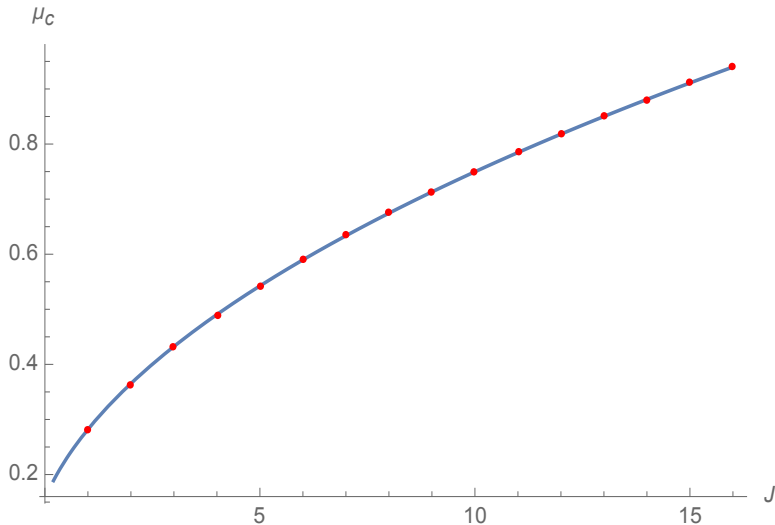
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

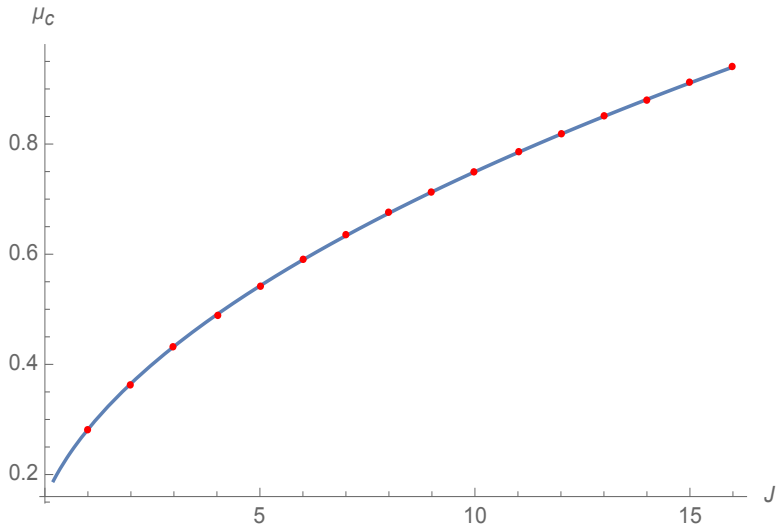
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

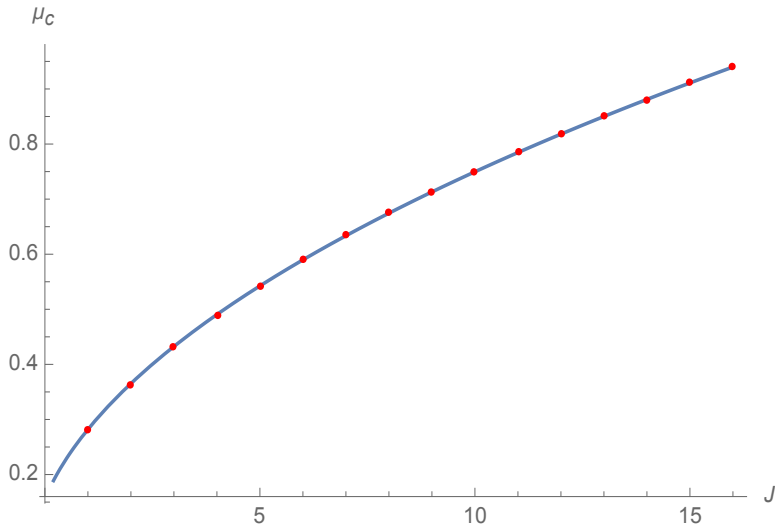
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

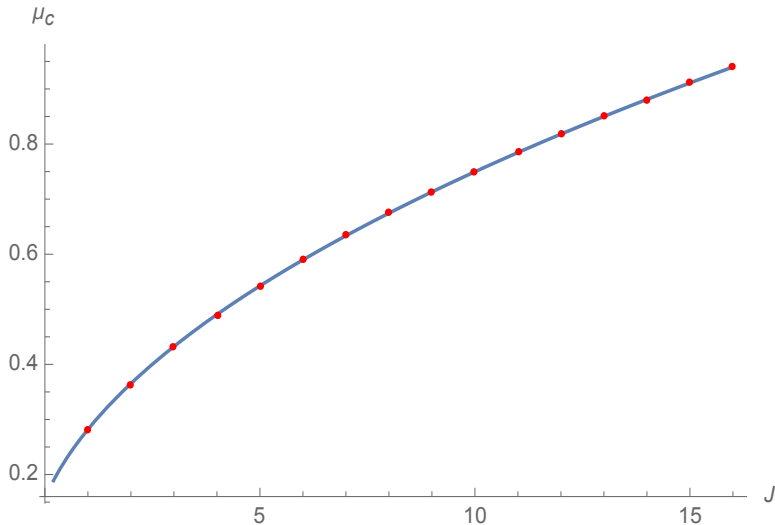
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

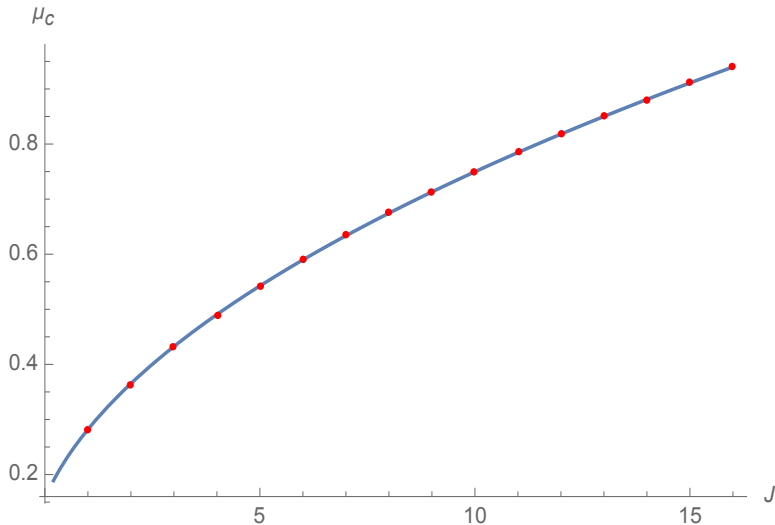
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$

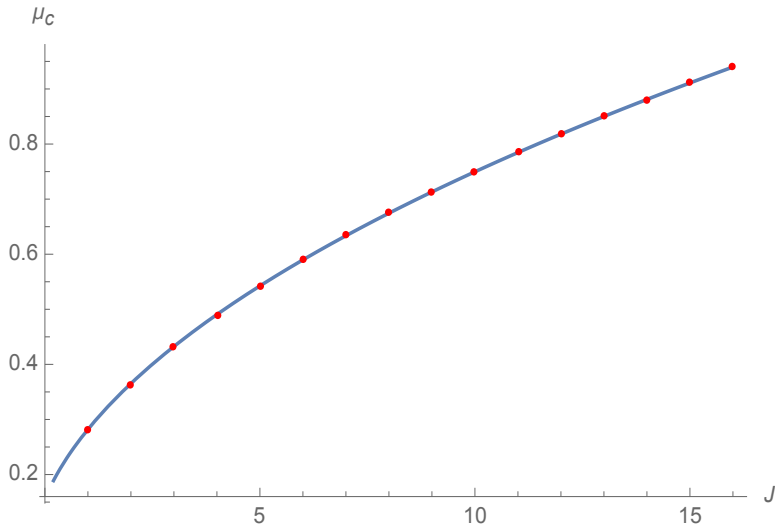


$$c_{\frac{3}{2}} = 0.348111 .$$



# Critical $O(2)$ model in $D=3$ at large charge

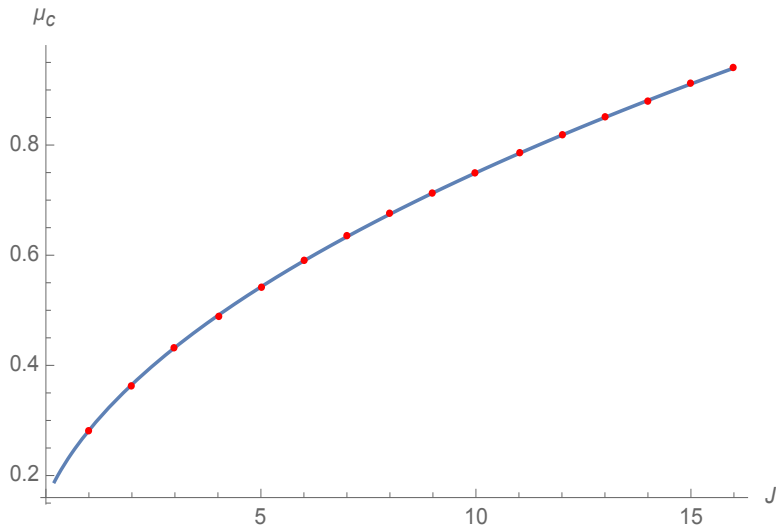
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

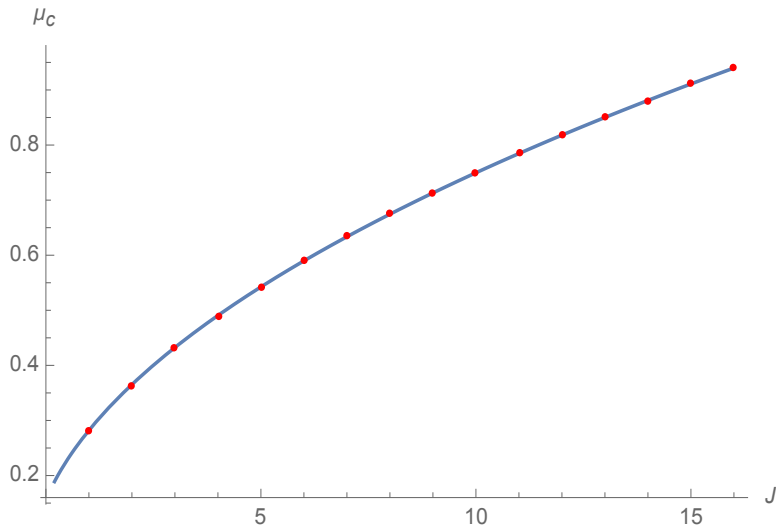
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

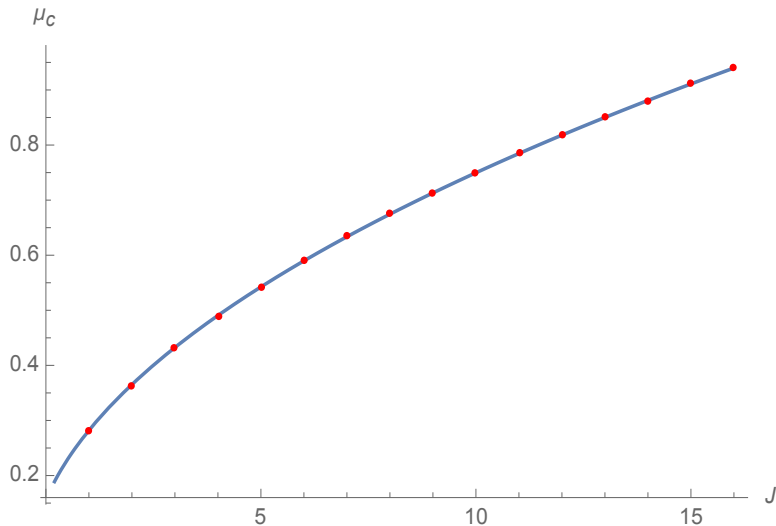
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

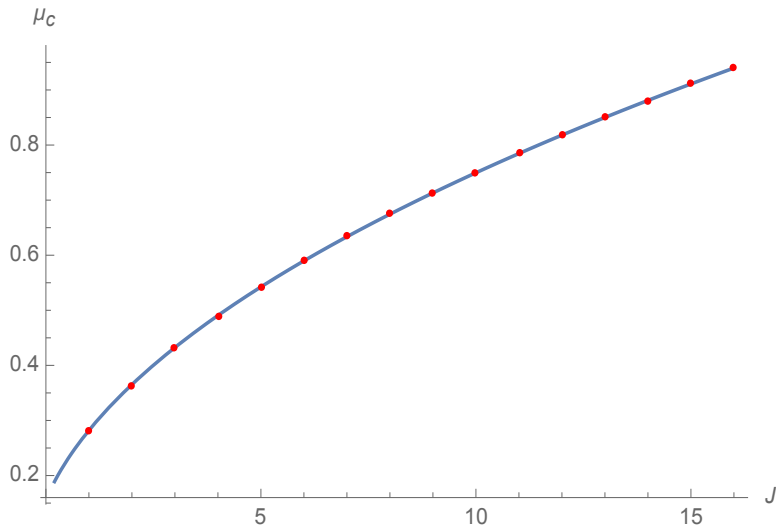
$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$0.154253 (J+1)^{3/2} - 0.154253 J^{3/2}$$



$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$c_{\frac{3}{2}} = 0.348111 .$$

# Critical $O(2)$ model in $D=3$ at large charge

$$c_{\frac{3}{2}} = 0.348111 .$$



# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the



# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory** .

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory** .
- ▶ The mass of the

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions**



# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory**.
- ▶ The mass of the **fermions** is exactly proportional

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory**.
- ▶ The mass of the **fermions** is exactly proportional (with a

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge**



# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$



# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are **weak**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  $\chi$ -**charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are **weak** because there is in general no

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory**.
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**) to the **chemical potential** for  $\chi$ -**charge**, which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$ .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory**.
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**) to the **chemical potential** for  **$\chi$ -charge**, which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$ .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$  -charge greater than



# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the **O(2) model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  $\chi$ -**charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$  -charge greater than  $\frac{1}{2}$  :

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the **O(2) model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  $\chi$ -**charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$  -charge greater than  $\frac{1}{2}$  :  
The

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the **O(2) model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory**.
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**) to the **chemical potential** for  $\chi$ -**charge**, which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$ .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$ -charge greater than  $\frac{1}{2}$ :  
The **chiral ring**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$  -charge greater than  $\frac{1}{2}$  :  
The **chiral ring** of the theory

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$  -charge greater than  $\frac{1}{2}$  :  
The **chiral ring** of the theory **truncates**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$  -charge greater than  $\frac{1}{2}$  :  
The **chiral ring** of the theory **truncates** and its

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model** .
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory** .
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient** ) to the **chemical potential** for  **$\chi$ -charge** , which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$  .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$  -charge greater than  $\frac{1}{2}$  : The **chiral ring** of the theory **truncates** and its **only elements**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory**.
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**) to the **chemical potential** for  **$\chi$ -charge**, which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$ .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$ -charge greater than  $\frac{1}{2}$ : The **chiral ring** of the theory **truncates** and its **only elements** are



# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the **O(2) model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory**.
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**) to the **chemical potential** for  $\chi$ -charge, which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$ .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$ -charge greater than  $\frac{1}{2}$ : The **chiral ring** of the theory **truncates** and its **only elements** are **1**

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the  **$O(2)$  model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  **$\chi$  theory**.
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**) to the **chemical potential** for  **$\chi$ -charge**, which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$ .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$ -charge greater than  $\frac{1}{2}$ : The **chiral ring** of the theory **truncates** and its **only elements** are **1** and

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the **O(2) model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory**.
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**) to the **chemical potential** for  $\chi$ -charge, which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$ .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$ -charge greater than  $\frac{1}{2}$ : The **chiral ring** of the theory **truncates** and its **only elements** are  $1$  and  $\phi$ .

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the **O(2) model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory**.
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**) to the **chemical potential** for  $\chi$ -charge, which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$ .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$ -charge greater than  $\frac{1}{2}$ : The **chiral ring** of the theory **truncates** and its **only elements** are  $1$  and  $\phi$ .

# Large charge with vs. without a vacuum manifold

- ▶ In the case of the  $\mathcal{N} = 2$  superconformal fixed point, the large-charge universality class is the **same** as that of the **O(2) model**.
- ▶ The only additional information contributed by **supersymmetry** is the size of the **gap** of the **massive excitations** above the effective  $\chi$  **theory**.
- ▶ The mass of the **fermions** is exactly proportional (with a **specific coefficient**) to the **chemical potential** for  $\chi$ -charge, which is **bounded** by the **splitting** between  $\Delta_{J+1}$  and  $\Delta_J$ .
- ▶ The constraints of SUSY are **weak** because there is in general no **BPS scalar primary** state at  $R$ -charge greater than  $\frac{1}{2}$ : The **chiral ring** of the theory **truncates** and its **only elements** are  $1$  and  $\phi$ .

# Large charge with vs. without a vacuum manifold

- ▶ The

# Large charge with vs. without a vacuum manifold

- ▶ The truncation of the chiral ring

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on



# Large charge with vs. without a vacuum manifold

- ▶ The truncation of the chiral ring corresponds on general grounds

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor**



# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the



# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"**



# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the



# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  **model**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  **model** is

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  **model** is **purely nilpotent** and correspondingly it has

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent** and correspondingly it has **no vacuum manifold**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent** and correspondingly it has **no vacuum manifold** .

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent** and correspondingly it has **no vacuum manifold** .
- ▶ The latter property is

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent** and correspondingly it has **no vacuum manifold** .
- ▶ The latter property is **pretty obvious**



# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent** and correspondingly it has **no vacuum manifold** .
- ▶ The latter property is **pretty obvious** from the fact there is a

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent** and correspondingly it has **no vacuum manifold** .
- ▶ The latter property is **pretty obvious** from the fact there is a **potential**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent** and correspondingly it has **no vacuum manifold** .
- ▶ The latter property is **pretty obvious** from the fact there is a **potential** for the only

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent** and correspondingly it has **no vacuum manifold** .
- ▶ The latter property is **pretty obvious** from the fact there is a **potential** for the only **complex scalar field**

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent** and correspondingly it has **no vacuum manifold** .
- ▶ The latter property is **pretty obvious** from the fact there is a **potential** for the only **complex scalar field** in the theory.

# Large charge with vs. without a vacuum manifold

- ▶ The **truncation of the chiral ring** corresponds on **general grounds** to the absence of a **manifold of Poincare-invariant vacua** .
- ▶ This comes from an **old observation** due to **Luty and Taylor** .
- ▶ In **general** supersymmetric theories with **four or more supercharges** , the **holomorphic coordinate ring** on the **vacuum manifold** corresponds precisely to the **radical** of the **ring of BPS scalar primaries** of the theory.
- ▶ The **"radical"** of a commutative ring is the **ring** modulo its ideal of **nilpotent elements** .
- ▶ The **chiral ring** of the  $W = \Phi^3$  model is **purely nilpotent** and correspondingly it has **no vacuum manifold** .
- ▶ The latter property is **pretty obvious** from the fact there is a **potential** for the only **complex scalar field** in the theory.

# Large charge with vs. without a vacuum manifold

- ▶ A theory with

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold**



# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**



# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on



# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.
- ▶ The existence of a

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .



# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density**



# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H}$



# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H}$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In



# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state of finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state of finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state of finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J$



# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state of finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension  $\Delta_J$**  of its **lowest primary** of **charge  $J$**  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state of finite charge density  $\rho$** .
- ▶ In **such a state** the relationship between the **energy density  $\mathcal{H}$**  and the **charge density  $\rho$**  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density  $\rho$**

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit on flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density**  $\rho$  is much larger than the

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density**  $\rho$  is much larger than the **scalar curvature**  $\text{Ric}_3$ ,

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density**  $\rho$  is much larger than the **scalar curvature**  $\text{Ric}_3$ , which holds when

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density**  $\rho$  is much larger than the **scalar curvature**  $\text{Ric}_3$ , which holds when  $J$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density**  $\rho$  is much larger than the **scalar curvature**  $\text{Ric}_3$ , which holds when  $J$



# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density**  $\rho$  is much larger than the **scalar curvature**  $\text{Ric}_3$ , which holds when  $J \gg$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density**  $\rho$  is much larger than the **scalar curvature**  $\text{Ric}_3$ , which holds when  $J \gg$

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density**  $\rho$  is much larger than the **scalar curvature**  $\text{Ric}_3$ , which holds when  $J \gg 1$ .

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density**  $\rho$  is much larger than the **scalar curvature**  $\text{Ric}_3$ , which holds when  $J \gg 1$ .

# Large charge with vs. without a vacuum manifold

- ▶ A theory with **no vacuum manifold** must necessarily have a  **$J$ -scaling** for the **operator dimension**  $\Delta_J$  of its **lowest primary** of **charge**  $J$  that goes as  $\Delta_J \propto J^{+\frac{3}{2}}$ .
- ▶ This follows from **scale invariance** and the **existence of a thermodynamic limit** on **flat space**.
- ▶ The existence of a **thermodynamic limit on flat space** means that there is a unique **lowest state** of **finite charge density**  $\rho$ .
- ▶ In **such a state** the relationship between the **energy density**  $\mathcal{H}$  and the **charge density**  $\rho$  must be of the form  $\mathcal{H} = f(\rho)$ .
- ▶ The only allowed **scale invariant relationship** is of the form  $\mathcal{H} \propto \rho^{3/2}$ .
- ▶ In **radial quantization** this leads immediately to the relation  $\Delta_J \propto J^{3/2}$  when the **charge density**  $\rho$  is much larger than the **scalar curvature**  $\text{Ric}_3$ , which holds when  $J \gg 1$ .

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is



# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different** .

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different**



# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at



# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is



# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is **singular**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is **singular**.

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is **singular**.
- ▶ As a result these theories do

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is **singular**.
- ▶ As a result these theories do **not**

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is **singular**.
- ▶ As a result these theories do **not** have to obey

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is **singular**.
- ▶ As a result these theories do **not** have to obey  $\Delta_J \propto J^{+3/2}$

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is **singular**.
- ▶ As a result these theories do **not** have to obey  $\Delta_J \propto J^{+3/2}$  and in fact they do

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is **singular**.
- ▶ As a result these theories do **not** have to obey  $\Delta_J \propto J^{+3/2}$  and in fact they do **not**



# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is **singular**.
- ▶ As a result these theories do **not** have to obey  $\Delta_J \propto J^{+3/2}$  and in fact they do **not**.

# Large charge with vs. without a vacuum manifold

- ▶ The case of a theory with an **infinite** chiral ring is **quite different**. These theories have **flat directions** and their  **$J$ -scaling** is **entirely different** from theories that do **not** have flat directions.
- ▶ Theories with flat directions do **not** have a good thermodynamic limit in **flat space** at **zero temperature** because the **Legendre transform** between **charge density** and **chemical potential** is **singular**.
- ▶ As a result these theories do **not** have to obey  $\Delta_J \propto J^{+3/2}$  and in fact they do **not**.

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent**

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring**

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state**

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the



# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate**



# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound**

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions,

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling**

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in



# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in  $\mathcal{N} = 2$  **superconformal theories**

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in  $\mathcal{N} = 2$  **superconformal theories** with

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in  $\mathcal{N} = 2$  **superconformal theories** with **vacuum manifolds**

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in  $\mathcal{N} = 2$  **superconformal theories** with **vacuum manifolds** .

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in  $\mathcal{N} = 2$  **superconformal theories** with **vacuum manifolds**.
- ▶ This formula follows from a

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in  $\mathcal{N} = 2$  **superconformal theories** with **vacuum manifolds**.
- ▶ This formula follows from a **multiplet-shortening condition**

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in  $\mathcal{N} = 2$  **superconformal theories** with **vacuum manifolds**.
- ▶ This formula follows from a **multiplet-shortening condition** and therefore the dimension of the BPS operators receives

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in  $\mathcal{N} = 2$  **superconformal theories** with **vacuum manifolds**.
- ▶ This formula follows from a **multiplet-shortening condition** and therefore the dimension of the BPS operators receives **no corrections**.



# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in  $\mathcal{N} = 2$  **superconformal theories** with **vacuum manifolds** .
- ▶ This formula follows from a **multiplet-shortening condition** and therefore the dimension of the BPS operators receives **no corrections** .

# Large charge with vs. without a vacuum manifold

- ▶ For theories with a **non-nilpotent chiral ring** there is at least **one state** satisfying the **BPS condition**  $\Delta_J = J_R$  for **arbitrarily high**  $J_R$ .
- ▶ These states **saturate** the **BPS bound** on scalar operator dimensions, so the **large  $J$  scaling** is exactly  $\Delta_J = J$  in  $\mathcal{N} = 2$  **superconformal theories** with **vacuum manifolds** .
- ▶ This formula follows from a **multiplet-shortening condition** and therefore the dimension of the BPS operators receives **no corrections** .

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the



# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second-

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the **second-** and

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients



# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .



# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be more naturally reformulated

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be more naturally reformulated as questions about the

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be more naturally reformulated as questions about the norms

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be more naturally reformulated as questions about the norms of certain states.

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be more naturally reformulated as questions about the norms of certain states. (This may sound like it can be fully

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be more naturally reformulated as questions about the norms of certain states. (This may sound like it can be fully absorbed



# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be more naturally reformulated as questions about the norms of certain states. (This may sound like it can be fully absorbed into a

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be more naturally reformulated as questions about the norms of certain states. (This may sound like it can be fully absorbed into a convention

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be more naturally reformulated as questions about the norms of certain states. (This may sound like it can be fully absorbed into a convention but in fact this is not the case in the context we will study.)

# Large charge with vs. without a vacuum manifold

- ▶ So it seems the scaling of the lowest state is actually kind of boring.
- ▶ However we can still ask about things like the dimensions of near-BPS states such as the second- and third-lowest operators of a given large  $R$ -charge  $J_R$ .
- ▶ There are also interesting questions about OPE coefficients involving BPS and near-BPS operators of large  $R$ -charge .
- ▶ Some such questions can be more naturally reformulated as questions about the norms of certain states. (This may sound like it can be fully absorbed into a convention but in fact this is not the case in the context we will study.)

# The XYZ Model

- ▶ We will address

# The XYZ Model

- ▶ We will address **all these questions**

# The XYZ Model

- ▶ We will address **all these questions** in the

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**,



# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.



# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model,

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**'

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our



# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X$ ,

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$



# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X$ ,  $Y$ , and  $Z$ ,

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X$ ,  $Y$ , and  $Z$ , and giving them a superpotential

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X$ ,  $Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and  $Z,$  and giving them a superpotential  $W = XYZ$  .

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and  $Z,$  and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and  $Z,$  and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and  $Z,$  and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of



# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and  $Z,$  and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and  $Z,$  and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and  $Z,$  and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches.

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches.  
So,



# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches.  
So,without

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and  $Z,$  and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches. So, without **losing any generality**

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches. So, without **losing any generality** we will study the

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches. So, without **losing any generality** we will study the  $X$  **branch**,

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$ .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin**, where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches. So, without **losing any generality** we will study the  $X$  **branch**, whose coordinate ring consists of all the operators of the form

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and  $Z,$  and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches. So, without **losing any generality** we will study the  **$X$  branch**, whose coordinate ring consists of all the operators of the form  $X$

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y,$  and  $Z,$  and giving them a superpotential  $W = XYZ$  .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin** , where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches. So, without **losing any generality** we will study the  **$X$  branch**, whose coordinate ring consists of all the operators of the form  $X$

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$ .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin**, where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches. So, without **losing any generality** we will study the  $X$  **branch**, whose coordinate ring consists of all the operators of the form  $X^J$



# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$ .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin**, where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches. So, without **losing any generality** we will study the  $X$  **branch**, whose coordinate ring consists of all the operators of the form  $X^J$ .

# The XYZ Model

- ▶ We will address **all these questions** in the **simplest possible case**, which is the case of an  $\mathcal{N} = 2$  **superconformal** field theory in **three dimensions** with a moduli space of complex dimension **one**.
- ▶ The most familiar model of this type is the  $W = XYZ$  super-model, not to be confused with the '**XYZ model**' of lattice statistical mechanics.
- ▶ Our  $W = XYZ$  **model** is simply the **superconformal fixed point** obtained by starting with **three free chiral superfields**  $X, Y$ , and  $Z$ , and giving them a superpotential  $W = XYZ$ .
- ▶ This model has a **vacuum manifold** consisting of **three branches connected at the origin**, where each one has a **vev** for one of the three chiral superfields.
- ▶ There is an obvious  $S_3$  **symmetry** permuting the branches. So, without **losing any generality** we will study the  $X$  **branch**, whose coordinate ring consists of all the operators of the form  $X^J$ .

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case.

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT**



# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out** the

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out** the **heavy states**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out** the **heavy states** on moduli space, we then obtain an



# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out** the **heavy states** on moduli space, we then obtain an **even simpler description** in terms of a

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out** the **heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out** the **heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out** the **heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at



# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}} ,$$

where

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}} ,$$

where  $c_{[K]}$

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}} ,$$

where  $c_{[K]}$  is a coefficient that we do not

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}},$$

where  $c_{[K]}$  is a coefficient that we do not **know how to calculate**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}},$$

where  $c_{[K]}$  is a coefficient that we do not **know how to calculate** without knowing the



# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}},$$

where  $c_{[K]}$  is a coefficient that we do not **know how to calculate** without knowing the **full form**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}} ,$$

where  $c_{[K]}$  is a coefficient that we do not **know how to calculate** without knowing the **full form** of the

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}} ,$$

where  $c_{[K]}$  is a coefficient that we do not **know how to calculate** without knowing the **full form** of the **Wilsonian action**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}} ,$$

where  $c_{[K]}$  is a coefficient that we do not **know how to calculate** without knowing the **full form** of the **Wilsonian action** at the

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}},$$

where  $c_{[K]}$  is a coefficient that we do not **know how to calculate** without knowing the **full form** of the **Wilsonian action** at the **fixed point**

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}} ,$$

where  $c_{[K]}$  is a coefficient that we do not **know how to calculate** without knowing the **full form** of the **Wilsonian action** at the **fixed point** .

# The XYZ Model

- ▶ Our approach will not be to constrain high-dimension operators directly with the conformal bootstrap.
- ▶ Rather, we will work in parallel with our approach in the **nonsupersymmetric** case. That is, we will **assume that the CFT** can be described by a **Wilsonian fixed point** with the appropriate symmetries.
- ▶ By **integrating out the heavy states** on moduli space, we then obtain an **even simpler description** in terms of a **conformally invariant effective action** on a given branch of moduli space.
- ▶ That is, the **X-branch** is described at **leading order in derivatives** by the unique **conformally invariant Kahler potential** one can write down:

$$K = c_{[K]} |X^\dagger X|^{+\frac{3}{4}} ,$$

where  $c_{[K]}$  is a coefficient that we do not **know how to calculate** without knowing the **full form** of the **Wilsonian action** at the **fixed point** .

# The XYZ Model

- ▶ This action



# The XYZ Model

- ▶ This action **by itself**

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat**

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness**



# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field  $\Phi$ .

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field  $\Phi$ .

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field  $\Phi$ . This will be

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field  $\Phi$ . This will be **mostly innocuous**

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field  $\Phi$ . This will be **mostly innocuous** and result mainly in a



# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field  $\Phi$ . This will be **mostly innocuous** and result mainly in a **nontrivial quantization rule**

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field  $\Phi$ . This will be **mostly innocuous** and result mainly in a **nontrivial quantization rule** for the number of

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field  $\Phi$ . This will be **mostly innocuous** and result mainly in a **nontrivial quantization rule** for the number of  **$\Phi$ -excitations**.

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field  $\Phi$ . This will be **mostly innocuous** and result mainly in a **nontrivial quantization rule** for the number of  **$\Phi$ -excitations**.

# The XYZ Model

- ▶ This action **by itself** produces trivial dynamics.
- ▶ One can see this by performing the change of variables

$$\Phi \equiv c_{[K]}^{+\frac{1}{2}} X^{+\frac{3}{4}} \qquad X = c_{[K]}^{-\frac{2}{3}} \Phi^{+\frac{4}{3}} ,$$

and similarly for the conjugate superfields.

- ▶ The Kahler potential then becomes the canonical one for  $\Phi$ :

$$K = \Phi^\dagger \Phi .$$

- ▶ The **only caveat** here is the **non-single-valuedness** of the map between  $X$  and the canonical field  $\Phi$ . This will be **mostly innocuous** and result mainly in a **nontrivial quantization rule** for the number of  **$\Phi$ -excitations**.

# The XYZ Model

- ▶ So at

# The XYZ Model

- ▶ So at **leading order**

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the



# The XYZ Model

- ▶ So at **leading order** this effective description description predicts that the physics of the **X-branch**

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself**

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the **states with large**

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the **states with large R-charge**  $J_R$



# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the **states with large R-charge  $J_R$  and large**

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the **states with large R-charge  $J_R$  and large X-charge  $J_X$**

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the **states with large R-charge  $J_R$  and large X-charge  $J_X$**  , with

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the **states with large R-charge  $J_R$  and large X-charge  $J_X$**  , with  $J_R/J_X \sim \frac{3}{2}$

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the **states with large R-charge  $J_R$  and large X-charge  $J_X$**  , with  $J_R/J_X \sim \frac{3}{2}$  and

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the **states with large R-charge  $J_R$  and large X-charge  $J_X$**  , with  $J_R/J_X \sim \frac{3}{2}$  and  $\Delta_{J_X}/J_R \sim 1$

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the **states with large R-charge  $J_R$  and large X-charge  $J_X$**  , with  $J_R/J_X \sim \frac{3}{2}$  and  $\Delta_{J_X}/J_R \sim 1$  .

# The XYZ Model

- ▶ So at **leading order** this effective description predicts that the physics of the **X-branch** is described by a free field theory with a nontrivial quantization condition.
- ▶ More precisely, what does this theory describe, other than the **chiral ring itself** ?
- ▶ It describes the **states with large R-charge  $J_R$  and large X-charge  $J_X$**  , with  $J_R/J_X \sim \frac{3}{2}$  and  $\Delta_{J_X}/J_R \sim 1$  .



# The XYZ Model

- ▶ Let us now get

# The XYZ Model

- ▶ Let us now get **more concrete**

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question:

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-**

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X$ "

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X$ "



# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X =$

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X =$

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R$ "

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R$ "

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg$

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg$

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ "

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"



# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations**

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations** such as the

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations** such as the **Y** and **Z** fields

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations** such as the **Y and Z fields** have their mass scale set by

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations** such as the **Y** and **Z fields** have their mass scale set by  $\sqrt{\rho} = \frac{\sqrt{J}}{\sqrt{4\pi r^2}}$

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations** such as the **Y and Z fields** have their mass scale set by  $\sqrt{\rho} = \frac{\sqrt{J}}{\sqrt{4\pi r^2}}$  which is much greater than the mass scale of the

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations** such as the **Y and Z fields** have their mass scale set by  $\sqrt{\rho} = \frac{\sqrt{J}}{\sqrt{4\pi r^2}}$  which is much greater than the mass scale of the **X-excitations**,



# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations** such as the **Y and Z fields** have their mass scale set by  $\sqrt{\rho} = \frac{\sqrt{J}}{\sqrt{4\pi r^2}}$  which is much greater than the mass scale of the **X-excitations**, which are of order

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations** such as the **Y and Z fields** have their mass scale set by  $\sqrt{\rho} = \frac{\sqrt{J}}{\sqrt{4\pi r^2}}$  which is much greater than the mass scale of the **X-excitations**, which are of order  $\frac{1}{r}$ .

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations** such as the **Y and Z fields** have their mass scale set by  $\sqrt{\rho} = \frac{\sqrt{J}}{\sqrt{4\pi r^2}}$  which is much greater than the mass scale of the **X-excitations**, which are of order  $\frac{1}{r}$ .

# The XYZ Model

- ▶ Let us now get **more concrete** and ask the question: "What is the **second-** lowest-dimension primary operator with  $J_X = \frac{2}{3}J_R \gg 1$ ?"
- ▶ The **massive excitations** such as the **Y and Z fields** have their mass scale set by  $\sqrt{\rho} = \frac{\sqrt{J}}{\sqrt{4\pi r^2}}$  which is much greater than the mass scale of the **X-excitations**, which are of order  $\frac{1}{r}$ .

# The XYZ Model

- ▶ To calculate the low-lying spectrum of

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states**

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$



# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\Phi$  variables.
- ▶ The bosonic  $\phi$

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\Phi$  variables.
- ▶ The bosonic  $\phi$  field carries

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and  $X$

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge:



# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\Phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and  $X$ -charge: It has  $J_X = \frac{3}{4}$

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and  $X$ -charge: It has  $J_X = \frac{3}{4}$  and

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and  $X$ -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$ .

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also**

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and  $X$ -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$ .
- ▶ Its fermionic superpartner **also** carries



# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state**

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state**



# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two**

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited,

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel**

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.
- ▶ The lowest-energy way to do this is to add

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.
- ▶ The lowest-energy way to do this is to add **one more excitation**



# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.
- ▶ The lowest-energy way to do this is to add **one more excitation** of

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.
- ▶ The lowest-energy way to do this is to add **one more excitation** of  $\phi$

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$  -charge and  $X$  -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$  .
- ▶ Its fermionic superpartner **also** carries  $R$  -charge and  $X$  -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.
- ▶ The lowest-energy way to do this is to add **one more excitation** of  $\phi$  in the

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and  $X$ -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$ .
- ▶ Its fermionic superpartner **also** carries  $R$ -charge and  $X$ -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.
- ▶ The lowest-energy way to do this is to add **one more excitation** of  $\phi$  in the **zero mode**

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and  $X$ -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$ .
- ▶ Its fermionic superpartner **also** carries  $R$ -charge and  $X$ -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.
- ▶ The lowest-energy way to do this is to add **one more excitation** of  $\phi$  in the **zero mode** and cancel its charges with an excitation of

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and  $X$ -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$ .
- ▶ Its fermionic superpartner **also** carries  $R$ -charge and  $X$ -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.
- ▶ The lowest-energy way to do this is to add **one more excitation** of  $\phi$  in the **zero mode** and cancel its charges with an excitation of  $\bar{\phi}$

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and  $X$ -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$ .
- ▶ Its fermionic superpartner **also** carries  $R$ -charge and  $X$ -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.
- ▶ The lowest-energy way to do this is to add **one more excitation** of  $\phi$  in the **zero mode** and cancel its charges with an excitation of  $\bar{\phi}$  in the zero mode.

# The XYZ Model

- ▶ To calculate the low-lying spectrum of **excited states** we use the  $\phi$  variables.
- ▶ The bosonic  $\phi$  field carries  $R$ -charge and  $X$ -charge: It has  $J_X = \frac{3}{4}$  and  $J_R = \frac{1}{2}$ .
- ▶ Its fermionic superpartner **also** carries  $R$ -charge and  $X$ -charge.
- ▶ Therefore the **second-lowest state** with the same quantum numbers as the **BPS ground state** must have at least **two** modes excited, in order to **cancel** the charges.
- ▶ The lowest-energy way to do this is to add **one more excitation** of  $\phi$  in the **zero mode** and cancel its charges with an excitation of  $\bar{\phi}$  in the zero mode.



# The XYZ Model

- ▶ Likewise, the

# The XYZ Model

- ▶ Likewise, the **third-**

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional**

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of  $\bar{\phi}$



# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of  $\bar{\phi}$ .

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of  $\bar{\phi}$ .
- ▶ In the

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of  $\bar{\phi}$ .
- ▶ In the **free approximation**

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of  $\bar{\phi}$ .
- ▶ In the **free approximation** then, the dimensions of the states are simply

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of  $\bar{\phi}$ .
- ▶ In the **free approximation** then, the dimensions of the states are simply  $\Delta_{J_X}^{(+1)} = \frac{3J_X}{2} + 1$

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of  $\bar{\phi}$ .
- ▶ In the **free approximation** then, the dimensions of the states are simply  $\Delta_{J_X}^{(+1)} = \frac{3J_X}{2} + 1$  and

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of  $\bar{\phi}$ .
- ▶ In the **free approximation** then, the dimensions of the states are simply  $\Delta_{J_X}^{(+1)} = \frac{3J_X}{2} + 1$  and  $\Delta_{J_X}^{(+2)} = \frac{3J_X}{2} + 2$

# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of  $\bar{\phi}$ .
- ▶ In the **free approximation** then, the dimensions of the states are simply  $\Delta_{J_X}^{(+1)} = \frac{3J_X}{2} + 1$  and  $\Delta_{J_X}^{(+2)} = \frac{3J_X}{2} + 2$  respectively.



# The XYZ Model

- ▶ Likewise, the **third-** lowest scalar superconformal primary is obtained by adding **two additional** zero mode excitations of  $\phi$  and two of  $\bar{\phi}$ .
- ▶ In the **free approximation** then, the dimensions of the states are simply  $\Delta_{J_X}^{(+1)} = \frac{3J_X}{2} + 1$  and  $\Delta_{J_X}^{(+2)} = \frac{3J_X}{2} + 2$  respectively.

# The XYZ Model

- ▶ So, in the

# The XYZ Model

- ▶ So, in the leading large- $J$  approximation

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine**

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring**



# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** .

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading**

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections**

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential.

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So,



# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries**

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries** of the full CFT, including

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries** of the full CFT, including  **$X$ -symmetry**,



# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation** , the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring** . Let us now investigate what can be said about the **subleading** large- $J$  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries** of the full CFT, including  **$X$ -symmetry**,

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation**, the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring**. Let us now investigate what can be said about the **subleading large- $J$**  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries** of the full CFT, including  **$X$ -symmetry,  $R$ -symmetry,**

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation**, the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring**. Let us now investigate what can be said about the **subleading large- $J$**  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries** of the full CFT, including  **$X$ -symmetry,  $R$ -symmetry,**

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation**, the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring**. Let us now investigate what can be said about the **subleading large- $J$**  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries** of the full CFT, including  **$X$ -symmetry**,  **$R$ -symmetry**,  **$\mathcal{N} = 2$  supersymmetry**,

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation**, the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring**. Let us now investigate what can be said about the **subleading large- $J$**  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries** of the full CFT, including  **$X$ -symmetry**,  **$R$ -symmetry**,  **$\mathcal{N} = 2$  supersymmetry**, and, most constrainingly,

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation**, the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring**. Let us now investigate what can be said about the **subleading large- $J$**  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries** of the full CFT, including  **$X$ -symmetry**,  **$R$ -symmetry**,  **$\mathcal{N} = 2$  supersymmetry**, and, most constrainingly, **Weyl invariance**.

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation**, the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring**. Let us now investigate what can be said about the **subleading large- $J$**  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries** of the full CFT, including  **$X$ -symmetry**,  **$R$ -symmetry**,  **$\mathcal{N} = 2$  supersymmetry**, and, most constrainingly, **Weyl invariance**.

# The XYZ Model

- ▶ So, in the **leading large- $J$  approximation**, the conformal dimensions are calculable in free field theory.
- ▶ This is **fine** but it's a little bit **boring**. Let us now investigate what can be said about the **subleading large- $J$**  corrections to the operator spectrum.
- ▶ There are no possible **conformal corrections** to the form of the Kahler potential. So, any corrections to the free spectrum must come from **higher derivative  $D$ -terms**.
- ▶ These higher terms must be invariant under **all symmetries** of the full CFT, including  **$X$ -symmetry**,  **$R$ -symmetry**,  **$\mathcal{N} = 2$  supersymmetry**, and, most constrainingly, **Weyl invariance**.



# The XYZ Model

- ▶ Even Weyl symmetry alone is

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension

# The XYZ Model

- ▶ Even Weyl symmetry alone is **remarkably constraining**.
- ▶ The leading **conformally invariant higher-derivative** term for a scalar field with a nonzero **scaling dimension** is the



# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz four-derivative term,

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz four-derivative term, which takes the form

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz four-derivative term, which takes the form  $\partial\phi|^4/|\phi|^6 + (\text{fermions})$

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz four-derivative term, which takes the form  $\partial\phi|^4/|\phi|^6 + (\text{fermions})$

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz four-derivative term, which takes the form  $\partial\phi|^4/|\phi|^6 + (\text{fermions})$
- ▶ This term has a

# The XYZ Model

- ▶ Even Weyl symmetry alone is **remarkably constraining**.
- ▶ The leading **conformally invariant higher-derivative** term for a scalar field with a nonzero **scaling dimension** is the **Fradkin-Tseytlin-Paneitz** four-derivative term, which takes the form  $\partial\phi|^4/|\phi|^6 + (\text{fermions})$
- ▶ This term has a **unique supersymmetrization**

# The XYZ Model

- ▶ Even Weyl symmetry alone is **remarkably constraining**.
- ▶ The leading **conformally invariant higher-derivative** term for a scalar field with a nonzero **scaling dimension** is the **Fradkin-Tseytlin-Paneitz** four-derivative term, which takes the form  $\partial\phi|^4/|\phi|^6 + (\text{fermions})$
- ▶ This term has a **unique supersymmetrization** that was written down only a



# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz four-derivative term, which takes the form  $\partial\phi|^4/|\phi|^6 + (\text{fermions})$
- ▶ This term has a unique supersymmetrization that was written down only a year ago

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz four-derivative term, which takes the form  $\partial\phi|^4/|\phi|^6 + (\text{fermions})$
- ▶ This term has a unique supersymmetrization that was written down only a year ago by

# The XYZ Model

- ▶ Even Weyl symmetry alone is **remarkably constraining**.
- ▶ The leading **conformally invariant higher-derivative** term for a scalar field with a nonzero **scaling dimension** is the **Fradkin-Tseytlin-Paneitz** four-derivative term, which takes the form  $\partial\phi|^4/|\phi|^6 + (\text{fermions})$
- ▶ This term has a **unique supersymmetrization** that was written down only a **year ago** by **Kuzenko**

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz four-derivative term, which takes the form  $\partial\phi|^4/|\phi|^6 + (\text{fermions})$
- ▶ This term has a unique supersymmetrization that was written down only a year ago by Kuzenko .

# The XYZ Model

- ▶ Even Weyl symmetry alone is remarkably constraining.
- ▶ The leading conformally invariant higher-derivative term for a scalar field with a nonzero scaling dimension is the Fradkin-Tseytlin-Paneitz four-derivative term, which takes the form  $\partial\phi|^4/|\phi|^6 + (\text{fermions})$
- ▶ This term has a unique supersymmetrization that was written down only a year ago by Kuzenko .

# The XYZ Model

# The XYZ Model



# The XYZ Model

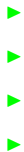




# The XYZ Model



# The XYZ Model



# The XYZ Model

- ▶
- ▶
- ▶
- ▶
- ▶

# The XYZ Model

- ▶
- ▶
- ▶
- ▶
- ▶
- ▶

# The XYZ Model



# The XYZ Model

- ▶
- ▶
- ▶
- ▶
- ▶
- ▶
- ▶
- ▶

# The XYZ Model

- ▶ Integration over superspace is equivalent to acting with four superderivatives and evaluating at  $\theta = \bar{\theta} = 0$ ,

# The XYZ Model

- ▶ Integration over superspace is equivalent to acting with four superderivatives and evaluating at  $\theta = \bar{\theta} = 0$ , or equivalently,



# The XYZ Model

- ▶ Integration over superspace is equivalent to acting with four superderivatives and evaluating at  $\theta = \bar{\theta} = 0$ , or equivalently, evaluating at  $\theta = \bar{\theta} = 0$ , and acting with four **supercharges**.

# The XYZ Model

- ▶ Integration over superspace is equivalent to acting with four superderivatives and evaluating at  $\theta = \bar{\theta} = 0$ , or equivalently, evaluating at  $\theta = \bar{\theta} = 0$ , and acting with four **supercharges**.
- ▶ In components, the super-FTP term contains operators such as

# The XYZ Model

- ▶ Integration over superspace is equivalent to acting with four superderivatives and evaluating at  $\theta = \bar{\theta} = 0$ , or equivalently, evaluating at  $\theta = \bar{\theta} = 0$ , and acting with four **supercharges**.
- ▶ In components, the super-FTP term contains operators such as  $(\bar{\psi}\nabla\psi)^2/|\phi|^6$

# The XYZ Model

- ▶ Integration over superspace is equivalent to acting with four superderivatives and evaluating at  $\theta = \bar{\theta} = 0$ , or equivalently, evaluating at  $\theta = \bar{\theta} = 0$ , and acting with four **supercharges**.
- ▶ In components, the super-FTP term contains operators such as  $(\bar{\psi}\nabla\psi)^2/|\phi|^6$  and

# The XYZ Model

- ▶ Integration over superspace is equivalent to acting with four superderivatives and evaluating at  $\theta = \bar{\theta} = 0$ , or equivalently, evaluating at  $\theta = \bar{\theta} = 0$ , and acting with four **supercharges**.
- ▶ In components, the super-FTP term contains operators such as  $(\bar{\psi}\nabla\psi)^2/|\phi|^6$  and  $|\nabla\phi|^4/|\phi|^6$ .

# The XYZ Model

- ▶ Integration over superspace is equivalent to acting with four superderivatives and evaluating at  $\theta = \bar{\theta} = 0$ , or equivalently, evaluating at  $\theta = \bar{\theta} = 0$ , and acting with four **supercharges**.
- ▶ In components, the super-FTP term contains operators such as  $(\bar{\psi}\nabla\psi)^2/|\phi|^6$  and  $|\nabla\phi|^4/|\phi|^6$ .

# The XYZ Model

- ▶ Integration over superspace is equivalent to acting with four superderivatives and evaluating at  $\theta = \bar{\theta} = 0$ , or equivalently, evaluating at  $\theta = \bar{\theta} = 0$ , and acting with four **supercharges**.
- ▶ In components, the super-FTP term contains operators such as  $(\bar{\psi}\nabla\psi)^2/|\phi|^6$  and  $|\nabla\phi|^4/|\phi|^6$ .

# The XYZ Model

- ▶ The naive



# The XYZ Model

- ▶ The naive  $J$ -scaling

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .

# The XYZ Model

- ▶ The naive  **$J$ -scaling** of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**,

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations**

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling



# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction**

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling  $J^{-2}$

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling  $J^{-2}$ .

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling  $J^{-2}$ . The

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling  $J^{-2}$ . The **cubic and quartic**



# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling  $J^{-2}$ . The **cubic and quartic** vertices have  $J$ -scaling

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling  $J^{-2}$ . The **cubic and quartic** vertices have  $J$ -scaling  $J^{-\frac{5}{2}}$

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling  $J^{-2}$ . The **cubic and quartic** vertices have  $J$ -scaling  $J^{-\frac{5}{2}}$  and

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling  $J^{-2}$ . The **cubic and quartic** vertices have  $J$ -scaling  $J^{-\frac{5}{2}}$  and  $J^{-3}$ ,

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling  $J^{-2}$ . The **cubic and quartic** vertices have  $J$ -scaling  $J^{-\frac{5}{2}}$  and  $J^{-3}$ , respectively.

# The XYZ Model

- ▶ The naive  $J$ -scaling of the term is  $J^{-1}$ , since each field  $\phi, \bar{\phi}$  scales as  $J^{+\frac{1}{2}}$ .
- ▶ When expanded as **vev and fluctuations**, the term with  $k$  **fluctuations** has  $J$ -scaling  $J^{-(1+\frac{k}{2})}$ .
- ▶ So the **propagator correction** has  $J$ -scaling  $J^{-2}$ . The **cubic and quartic** vertices have  $J$ -scaling  $J^{-\frac{5}{2}}$  and  $J^{-3}$ , respectively.

The

# The XYZ Model

- ▶ Since the super-FTP term is



# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller**

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ ,

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest**,

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest**,



# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest, second-lowest,**

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest, second-lowest**, and

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest, second-lowest, and third-lowest**

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest, second-lowest, and third-lowest** scalar primary states with

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest, second-lowest, and third-lowest** scalar primary states with  $J \equiv J_\phi \equiv 2 J_R = \frac{4}{3} J_X$

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest, second-lowest, and third-lowest** scalar primary states with  $J \equiv J_\phi \equiv 2 J_R = \frac{4}{3} J_X$  at

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest, second-lowest, and third-lowest** scalar primary states with  $J \equiv J_\phi \equiv 2 J_R = \frac{4}{3} J_X$  at large  $J$ .

# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest, second-lowest, and third-lowest** scalar primary states with  $J \equiv J_\phi \equiv 2 J_R = \frac{4}{3} J_X$  at large  $J$ .



# The XYZ Model

- ▶ Since the super-FTP term is **parametrically smaller** than the free term at large  $J$ , we can treat the effect of this term in **perturbation theory**.
- ▶ We will now calculate its effects in perturbation theory on the **lowest, second-lowest, and third-lowest** scalar primary states with  $J \equiv J_\phi \equiv 2 J_R = \frac{4}{3} J_X$  at large  $J$ .

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number**

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states**

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by



# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory**

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**,

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator**

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.



# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate**

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state**

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.



# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.
- ▶ The

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.
- ▶ The **subleading corrections**

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.
- ▶ The **subleading corrections** can be calculated systematically as

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.
- ▶ The **subleading corrections** can be calculated systematically as **fluctuation corrections**

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.
- ▶ The **subleading corrections** can be calculated systematically as **fluctuation corrections** around the saddle point.

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.
- ▶ The **subleading corrections** can be calculated systematically as **fluctuation corrections** around the saddle point. But,

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.
- ▶ The **subleading corrections** can be calculated systematically as **fluctuation corrections** around the saddle point. But, they are



# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.
- ▶ The **subleading corrections** can be calculated systematically as **fluctuation corrections** around the saddle point. But, they are **suppressed by powers of  $J$**

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.
- ▶ The **subleading corrections** can be calculated systematically as **fluctuation corrections** around the saddle point. But, they are **suppressed by powers of  $J$**  so we will not need them at the order of interest.

# Energy shift in large- $J$ perturbation theory

- ▶ The leading energy shift is given by the negative of the expectation value of the interaction Lagrangian in the free-field state  $|J\rangle$ .
- ▶ Expectation values in free-field states of large occupation number  $J$  are well-approximated by expectation values in a coherent state whose **average occupation number** is  $J$ .
- ▶ Expectation values in **free coherent states** are in turn given by **time-ordered perturbation theory** in a **vev**, where the **fluctuation propagator** satisfies **Feynman boundary conditions**.
- ▶ It can be shown by a **saddle-point estimate** that **large- $J$  Fock state** expectation values are equal at **leading order in  $J$**  to **coherent state eigenvalues**.
- ▶ The **subleading corrections** can be calculated systematically as **fluctuation corrections** around the saddle point. But, they are **suppressed by powers of  $J$**  so we will not need them at the order of interest.

# Energy shift in large-J perturbation theory

► So,

# Energy shift in large-J perturbation theory

- ▶ So, the

# Energy shift in large-J perturbation theory

- ▶ So, the **leading correction**

# Energy shift in large-J perturbation theory

- ▶ So, the **leading correction** to the energy of the

# Energy shift in large-J perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state**



# Energy shift in large-J perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the

# Energy shift in large-J perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term**

# Energy shift in large-J perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the

# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**

# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**  for the zero mode.

# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**  for the zero mode.
- ▶ The order  $O(J^{-1})$  term comes from the

# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**  for the zero mode.
- ▶ The order  $O(J^{-1})$  term comes from the **bosonic FTP term**

# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**  for the zero mode.
- ▶ The order  $O(J^{-1})$  term comes from the **bosonic FTP term** evaluated in the



# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**  for the zero mode.
- ▶ The order  $O(J^{-1})$  term comes from the **bosonic FTP term** evaluated in the **classical solution**.

# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**  for the zero mode.
- ▶ The order  $O(J^{-1})$  term comes from the **bosonic FTP term** evaluated in the **classical solution**.

# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**  for the zero mode.
- ▶ The order  $O(J^{-1})$  term comes from the **bosonic FTP term** evaluated in the **classical solution**.
- ▶ It

# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**  for the zero mode.
- ▶ The order  $O(J^{-1})$  term comes from the **bosonic FTP term** evaluated in the **classical solution**.
- ▶ It **vanishes**.

# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**  for the zero mode.
- ▶ The order  $O(J^{-1})$  term comes from the **bosonic FTP term** evaluated in the **classical solution**.
- ▶ It **vanishes**.

# Energy shift in large- $J$ perturbation theory

- ▶ So, the **leading correction** to the energy of the **lowest state** is given by the expectation value of the **FTP term** in the **classical solution with amplitude  $J$**  for the zero mode.
- ▶ The order  $O(J^{-1})$  term comes from the **bosonic FTP term** evaluated in the **classical solution**.
- ▶ It **vanishes**.

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation**



# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion**

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams**

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a



# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble**

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion**

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic** and a **fermionic**

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic and a fermionic** loop contribution.

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic and a fermionic** loop contribution.
- ▶ These contributions



# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic and a fermionic** loop contribution.
- ▶ These contributions **cancel**

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic and a fermionic** loop contribution.
- ▶ These contributions **cancel** against one another,

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic and a fermionic** loop contribution.
- ▶ These contributions **cancel** against one another, giving

# Energy shift in large-J perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic and a fermionic** loop contribution.
- ▶ These contributions **cancel** against one another, giving **zero**

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic and a fermionic** loop contribution.
- ▶ These contributions **cancel** against one another, giving **zero** energy shift at order

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic and a fermionic** loop contribution.
- ▶ These contributions **cancel** against one another, giving **zero** energy shift at order  $O(J^{-2})$ .

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic and a fermionic** loop contribution.
- ▶ These contributions **cancel** against one another, giving **zero** energy shift at order  $O(J^{-2})$ .

# Energy shift in large- $J$ perturbation theory

- ▶ The order  $O(J^{-1})$  term comes from the expectation value of the **two-fluctuation** terms in the **expansion** of the FTP term around the **classical solution**.
- ▶ In terms of **Feynman diagrams** this looks like a **one-loop vacuum bubble** with a single **insertion** .
- ▶ This piece has a **bosonic and a fermionic** loop contribution.
- ▶ These contributions **cancel** against one another, giving **zero** energy shift at order  $O(J^{-2})$ .



# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a **two-loop vacuum diagram**

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a **two-loop vacuum diagram** with a

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a **two-loop vacuum diagram** with a **single quartic vertex** and the

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram



# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus fermion-fermion

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a **two-loop vacuum diagram** with a **single quartic vertex** and the **topology of a figure eight**.
- ▶ The diagram **vanishes** due to a **nontrivial cancellation** between **boson-boson** plus **fermion-fermion** contributions on the one hand,



# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus fermion-fermion contributions on the one hand, and

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus fermion-fermion contributions on the one hand, and boson-fermion

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus fermion-fermion contributions on the one hand, and boson-fermion contributions on the other hand.

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus fermion-fermion contributions on the one hand, and boson-fermion contributions on the other hand.
- ▶ The

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus fermion-fermion contributions on the one hand, and boson-fermion contributions on the other hand.
- ▶ The figure-eight

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a **two-loop vacuum diagram** with a **single quartic vertex** and the **topology of a figure eight**.
- ▶ The diagram **vanishes** due to a **nontrivial cancellation** between **boson-boson** plus **fermion-fermion** contributions on the one hand, and **boson-fermion** contributions on the other hand.
- ▶ The **figure-eight** diagram is the

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus fermion-fermion contributions on the one hand, and boson-fermion contributions on the other hand.
- ▶ The figure-eight diagram is the only

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus fermion-fermion contributions on the one hand, and boson-fermion contributions on the other hand.
- ▶ The figure-eight diagram is the only contribution at



# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus fermion-fermion contributions on the one hand, and boson-fermion contributions on the other hand.
- ▶ The figure-eight diagram is the only contribution at  $O(J^{-3})$ ;

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a **two-loop vacuum diagram** with a **single quartic vertex** and the **topology of a figure eight**.
- ▶ The diagram **vanishes** due to a **nontrivial cancellation** between **boson-boson** plus **fermion-fermion** contributions on the one hand, and **boson-fermion** contributions on the other hand.
- ▶ The **figure-eight** diagram is the **only** contribution at  $O(J^{-3})$ ; the two-loop diagram with two

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a two-loop vacuum diagram with a single quartic vertex and the topology of a figure eight.
- ▶ The diagram vanishes due to a nontrivial cancellation between boson-boson plus fermion-fermion contributions on the one hand, and boson-fermion contributions on the other hand.
- ▶ The figure-eight diagram is the only contribution at  $O(J^{-3})$ ; the two-loop diagram with two cubic vertices

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a **two-loop vacuum diagram** with a **single quartic vertex** and the **topology of a figure eight**.
- ▶ The diagram **vanishes** due to a **nontrivial cancellation** between **boson-boson** plus **fermion-fermion** contributions on the one hand, and **boson-fermion** contributions on the other hand.
- ▶ The **figure-eight** diagram is the **only** contribution at  $O(J^{-3})$ ; the two-loop diagram with two **cubic vertices** is already down at order

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a **two-loop vacuum diagram** with a **single quartic vertex** and the **topology of a figure eight**.
- ▶ The diagram **vanishes** due to a **nontrivial cancellation** between **boson-boson** plus **fermion-fermion** contributions on the one hand, and **boson-fermion** contributions on the other hand.
- ▶ The **figure-eight** diagram is the **only** contribution at  $O(J^{-3})$ ; the two-loop diagram with two **cubic vertices** is already down at order  $O(J^{-5})$ .

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a **two-loop vacuum diagram** with a **single quartic vertex** and the **topology of a figure eight**.
- ▶ The diagram **vanishes** due to a **nontrivial cancellation** between **boson-boson** plus **fermion-fermion** contributions on the one hand, and **boson-fermion** contributions on the other hand.
- ▶ The **figure-eight** diagram is the **only** contribution at  $O(J^{-3})$ ; the two-loop diagram with two **cubic vertices** is already down at order  $O(J^{-5})$ .

# Energy shift in large-J perturbation theory

- ▶ The  $O(J^{-3})$  piece is given by a **two-loop vacuum diagram** with a **single quartic vertex** and the **topology of a figure eight**.
- ▶ The diagram **vanishes** due to a **nontrivial cancellation** between **boson-boson** plus **fermion-fermion** contributions on the one hand, and **boson-fermion** contributions on the other hand.
- ▶ The **figure-eight** diagram is the **only** contribution at  $O(J^{-3})$ ; the two-loop diagram with two **cubic vertices** is already down at order  $O(J^{-5})$ .

So the



# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**;

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS**

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a



# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition**

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was **inevitable**

# Energy shift in large-J perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was **inevitable** that the subleading

# Energy shift in large- $J$ perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was **inevitable** that the subleading **large- $J$**



# Energy shift in large- $J$ perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was **inevitable** that the subleading **large- $J$**  corrections would have to

# Energy shift in large- $J$ perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was **inevitable** that the subleading **large- $J$**  corrections would have to **vanish**

# Energy shift in large- $J$ perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was **inevitable** that the subleading **large- $J$**  corrections would have to **vanish** for the lowest state;

# Energy shift in large- $J$ perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was **inevitable** that the subleading **large- $J$**  corrections would have to **vanish** for the lowest state; we have now seen this

# Energy shift in large- $J$ perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was **inevitable** that the subleading **large- $J$**  corrections would have to **vanish** for the lowest state; we have now seen this **directly**

# Energy shift in large- $J$ perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was **inevitable** that the subleading **large- $J$**  corrections would have to **vanish** for the lowest state; we have now seen this **directly** at leading nontrivial order in large- $J$  perturbation theory.

# Energy shift in large- $J$ perturbation theory

- ▶ These perturbative cancellations have a simple explanation in terms of **supersymmetry**; the lowest state is after all **BPS** and corresponds to the operator  $X^{Jx} = X^{\frac{3J}{4}} = \phi^J$ .
- ▶ BPS states obey a **multiplet-shortening condition** and **cannot be lifted in perturbation theory**.
- ▶ Therefore it was **inevitable** that the subleading **large- $J$**  corrections would have to **vanish** for the lowest state; we have now seen this **directly** at leading nontrivial order in large- $J$  perturbation theory.

# Energy shift in large-J perturbation theory

- ▶ Now let us consider the



# Energy shift in large-J perturbation theory

- ▶ Now let us consider the **second-lowest**

# Energy shift in large-J perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.

# Energy shift in large-J perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the

# Energy shift in large-J perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free**

# Energy shift in large-J perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J+1\rangle .$$

# Energy shift in large-J perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J+1\rangle .$$

- ▶ Expectation values in this state are again

# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J+1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**

# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J+1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**  by expectation values in the



# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J+1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**  by expectation values in the **coherent state**,

# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J+1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**  by expectation values in the **coherent state**, which are given by

# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J + 1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**  by expectation values in the **coherent state**, which are given by **time-ordered perturbation theory**

# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J + 1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**  by expectation values in the **coherent state**, which are given by **time-ordered perturbation theory** in the same

# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J + 1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**  by expectation values in the **coherent state**, which are given by **time-ordered perturbation theory** in the same **vev**

# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J + 1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**  by expectation values in the **coherent state**, which are given by **time-ordered perturbation theory** in the same **vev** for the positive energy modes of the

# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J + 1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**  by expectation values in the **coherent state**, which are given by **time-ordered perturbation theory** in the same **vev** for the positive energy modes of the  $\phi$ -field

# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J + 1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**  by expectation values in the **coherent state**, which are given by **time-ordered perturbation theory** in the same **vev** for the positive energy modes of the  $\phi$ -field .



# Energy shift in large- $J$ perturbation theory

- ▶ Now let us consider the **second-lowest** state with the same quantum numbers.
- ▶ At the **free** level, this just corresponds to the state

$$(a^\dagger_0)^{\bar{\phi}} |J + 1\rangle .$$

- ▶ Expectation values in this state are again **approximated at leading order in  $J$**  by expectation values in the **coherent state**, which are given by **time-ordered perturbation theory** in the same **vev** for the positive energy modes of the  $\phi$ -field .

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the)

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.



# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected**

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not**

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected**



# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected** piece, where

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected** piece, where **two**

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected** piece, where **two** fluctuations of the FTP vertex

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected** piece, where **two** fluctuations of the FTP vertex **self-contract**,

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected** piece, where **two** fluctuations of the FTP vertex **self-contract**, and

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected** piece, where **two** fluctuations of the FTP vertex **self-contract**, and **two**

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected** piece, where **two** fluctuations of the FTP vertex **self-contract**, and **two** contract with the

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected** piece, where **two** fluctuations of the FTP vertex **self-contract**, and **two** contract with the **external lines**.



# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected** piece, where **two** fluctuations of the FTP vertex **self-contract**, and **two** contract with the **external lines**.

# Energy shift in large-J perturbation theory

- ▶ The energy shift at leading order is again given by the (negative of the) expectation value of the **FTP term**.
- ▶ The expectation value has **two pieces**.
- ▶ The first is a **disconnected** piece, where the four fluctuations in the vertex do **not** contract with the external lines.
- ▶ The second is a **connected** piece, where **two** fluctuations of the FTP vertex **self-contract**, and **two** contract with the **external lines**.

# Energy shift in large-J perturbation theory

- ▶ The

# Energy shift in large-J perturbation theory

- ▶ The disconnected piece

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function**

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams**



# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop**

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop** diagram.

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop** diagram.
- ▶ The vacuum factor

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially**

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**,

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the



# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece**

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a

# Energy shift in large- $J$ perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction**

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction to a bosonic propagator**



# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a

# Energy shift in large- $J$ perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.

# Energy shift in large- $J$ perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.

# Energy shift in large- $J$ perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The

# Energy shift in large- $J$ perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop**

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a

# Energy shift in large- $J$ perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic**

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a



# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic**

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic** contribution.

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic** contribution.
- ▶ These two contributions

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic** contribution.
- ▶ These two contributions **cancel nontrivially**

# Energy shift in large- $J$ perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic** contribution.
- ▶ These two contributions **cancel nontrivially** against one another.

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic** contribution.
- ▶ These two contributions **cancel nontrivially** against one another. Therefore the order

# Energy shift in large- $J$ perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic** contribution.
- ▶ These two contributions **cancel nontrivially** against one another. Therefore the order  **$O(J^{-3})$**

# Energy shift in large- $J$ perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic** contribution.
- ▶ These two contributions **cancel nontrivially** against one another. Therefore the order  **$O(J^{-3})$**  correction to the



# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic** contribution.
- ▶ These two contributions **cancel nontrivially** against one another. Therefore the order  **$O(J^{-3})$**  correction to the **second-**

# Energy shift in large-J perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic** contribution.
- ▶ These two contributions **cancel nontrivially** against one another. Therefore the order  **$O(J^{-3})$**  correction to the **second-** lowest state cancels as well.

# Energy shift in large- $J$ perturbation theory

- ▶ The **disconnected piece** is literally the product of a **free two-point function** times the sum of **vacuum diagrams** we have discussed, up to and including the **figure eight two-loop diagram**.
- ▶ The vacuum factor **cancels nontrivially** as we have **already seen**, so the **disconnected piece** vanishes.
- ▶ The **connected piece at order  $O(J^{-3})$**  has the topology of a **one loop correction** to a **bosonic propagator** with a **single quartic vertex**.
- ▶ The **internal loop** has a **bosonic** and a **fermionic** contribution.
- ▶ These two contributions **cancel nontrivially** against one another. Therefore the order  **$O(J^{-3})$**  correction to the **second-** lowest state cancels as well.

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest**

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state,

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation**



# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field**

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary**

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .



# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet**

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven**

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven**  $Q$ -

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven**  $Q$ -  
and

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven**  $Q_-$  and  $\bar{Q}_-$

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven**  $Q-$  and  $\bar{Q}-$  descendants



# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven  $Q$ - and  $\bar{Q}$ - descendants** instead of the usual

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven**  $Q$ - and  $\bar{Q}$ - **descendants** instead of the usual **fifteen**

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven**  $Q$ - and  $\bar{Q}$ - **descendants** instead of the usual **fifteen** .

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven  $Q-$  and  $\bar{Q}-$  descendants** instead of the usual **fifteen**. For instance, its

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven  $Q-$  and  $\bar{Q}-$  descendants** instead of the usual **fifteen**. For instance, its  $\bar{Q}^2$

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven  $Q-$  and  $\bar{Q}-$  descendants** instead of the usual **fifteen**. For instance, its  $\bar{Q}^2$ -descendant is

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven  $Q-$  and  $\bar{Q}-$  descendants** instead of the usual **fifteen**. For instance, its  $\bar{Q}^2$ -descendant is **absent**

# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven  $Q-$  and  $\bar{Q}-$  descendants** instead of the usual **fifteen** . For instance, its  $\bar{Q}^2$  -descendant is **absent** .



# Energy shift in large-J perturbation theory

- ▶ Just as in the case of the **lowest** state, there is again a **superalgebraic explanation** for the cancellation.
- ▶ At the **free-field** level, the second-lowest state is a **scalar primary** with dimension  $\Delta_J^{(+1)} = \frac{J}{2} + 1$ .
- ▶ Thus it lies in a **semi-short multiplet** with only **eleven  $Q-$  and  $\bar{Q}-$  descendants** instead of the usual **fifteen** . For instance, its  $\bar{Q}^2$  -descendant is **absent** .

# Energy shift in large-J perturbation theory

- ▶ Thus the

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest**

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected**

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join**

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a



# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long**

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate**

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state.

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state**

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge



# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ ,

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension**



# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge**

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$

# Energy shift in large-J perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -charge,

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  **$\phi$ -charge**, and at any rate would be

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -**charge**, and at any rate would be **BPS**



# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -**charge**, and at any rate would be **BPS** and could not have its

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -charge, and at any rate would be **BPS** and could not have its **energy lifted**

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -charge, and at any rate would be **BPS** and could not have its **energy lifted**.

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -**charge**, and at any rate would be **BPS** and could not have its **energy lifted**.
- ▶ So we have once more a

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  **$\phi$ -charge**, and at any rate would be **BPS** and could not have its **energy lifted**.
- ▶ So we have once more a **super-algebraic**

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -charge, and at any rate would be **BPS** and could not have its **energy lifted**.
- ▶ So we have once more a **super-algebraic** explanation for the vanishing of the

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -charge, and at any rate would be **BPS** and could not have its **energy lifted**.
- ▶ So we have once more a **super-algebraic** explanation for the vanishing of the **subleading large- $J$**

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  **$\phi$ -charge**, and at any rate would be **BPS** and could not have its **energy lifted**.
- ▶ So we have once more a **super-algebraic** explanation for the vanishing of the **subleading large- $J$**  correction to the operator dimension,



# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -charge, and at any rate would be **BPS** and could not have its **energy lifted**.
- ▶ So we have once more a **super-algebraic** explanation for the vanishing of the **subleading large- $J$**  correction to the operator dimension, which agrees with our

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -charge, and at any rate would be **BPS** and could not have its **energy lifted**.
- ▶ So we have once more a **super-algebraic** explanation for the vanishing of the **subleading large- $J$**  correction to the operator dimension, which agrees with our **explicit diagrammatic calculation**.

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  $\phi$ -charge, and at any rate would be **BPS** and could not have its **energy lifted**.
- ▶ So we have once more a **super-algebraic** explanation for the vanishing of the **subleading large- $J$**  correction to the operator dimension, which agrees with our **explicit diagrammatic calculation**.

# Energy shift in large- $J$ perturbation theory

- ▶ Thus the **second-lowest** state again **cannot have its energy corrected** unless it has another multiplet or multiplets with which to **join** into a **long** multiplet.
- ▶ There is **no candidate** for such a state. Any such state would have to be a **scalar state** with R-charge  $J_R = \frac{J}{2} + 2$ , with  $\phi$ -charge  $J_\phi = J$  and with energy  $\Delta_J^{(\text{candidate})} = \frac{J}{2} + 2$ .
- ▶ The only state with the correct **dimension** and **R-charge** is the state  $\phi^{J+2}$  which has the wrong  **$\phi$ -charge**, and at any rate would be **BPS** and could not have its **energy lifted**.
- ▶ So we have once more a **super-algebraic** explanation for the vanishing of the **subleading large- $J$**  correction to the operator dimension, which agrees with our **explicit diagrammatic calculation**.

# Energy shift in large-J perturbation theory

- ▶ This agreement is giving us some nice

# Energy shift in large-J perturbation theory

- ▶ This agreement is giving us some nice **confidence**

# Energy shift in large-J perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**



# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods,

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes**

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term**

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.



# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring**

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish**

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish**. We'd really like to find a case where the subleading large- $J$  correction is

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish** . We'd really like to find a case where the subleading large- $J$  correction is **nonzero**

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish**. We'd really like to find a case where the subleading large- $J$  correction is **nonzero** so we can make some sort of interesting

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish** . We'd really like to find a case where the subleading large- $J$  correction is **nonzero** so we can make some sort of interesting **prediction**



# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish**. We'd really like to find a case where the subleading large- $J$  correction is **nonzero** so we can make some sort of interesting **prediction** for an

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish**. We'd really like to find a case where the subleading large- $J$  correction is **nonzero** so we can make some sort of interesting **prediction** for an **operator dimension**

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish** . We'd really like to find a case where the subleading large- $J$  correction is **nonzero** so we can make some sort of interesting **prediction** for an **operator dimension** .

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish** . We'd really like to find a case where the subleading large- $J$  correction is **nonzero** so we can make some sort of interesting **prediction** for an **operator dimension** .
- ▶ So now we will look for the correction to the

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish** . We'd really like to find a case where the subleading large- $J$  correction is **nonzero** so we can make some sort of interesting **prediction** for an **operator dimension** .
- ▶ So now we will look for the correction to the **third-lowest**

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish** . We'd really like to find a case where the subleading large- $J$  correction is **nonzero** so we can make some sort of interesting **prediction** for an **operator dimension** .
- ▶ So now we will look for the correction to the **third-lowest** state with the same quantum numbers.

# Energy shift in large- $J$ perturbation theory

- ▶ This agreement is giving us some nice **confidence** in our **large- $J$**  methods, as well as confidence that we didn't **make any coefficient mistakes** in writing down the **super-FTP term** in **components**.
- ▶ On the other hand, this is getting a little bit **boring** having everything **vanish** . We'd really like to find a case where the subleading large- $J$  correction is **nonzero** so we can make some sort of interesting **prediction** for an **operator dimension** .
- ▶ So now we will look for the correction to the **third-lowest** state with the same quantum numbers.

# Energy shift in large-J perturbation theory

- ▶ Once again, we the evaluate matrix element of the FTP term in the state



# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory**

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.



# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected** and **disconnected**

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop**

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state,

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction**



# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest**

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state,

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state, both of which

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state, both of which **vanish**.

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger \bar{\phi})^2 ((a_0^\dagger \phi)^{J+2} |0\rangle)$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger \bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state, both of which **vanish**. So the only

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger \bar{\phi})^2 ((a_0^\dagger \phi)^{J+2} |0\rangle)$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger \bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state, both of which **vanish**. So the only **nonvanishing correction**

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger \bar{\phi})^2 ((a_0^\dagger \phi)^{J+2} |0\rangle)$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger \bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state, both of which **vanish**. So the only **nonvanishing correction** comes from the



# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state, both of which **vanish**. So the only **nonvanishing correction** comes from the **connected tree-level diagram**

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger \bar{\phi})^2 ((a_0^\dagger \phi)^{J+2} |0\rangle)$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger \bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state, both of which **vanish**. So the only **nonvanishing correction** comes from the **connected tree-level diagram** with a

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger \bar{\phi})^2 ((a_0^\dagger \phi)^{J+2} |0\rangle)$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger \bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state, both of which **vanish**. So the only **nonvanishing correction** comes from the **connected tree-level diagram** with a **single quartic vertex**.

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state, both of which **vanish**. So the only **nonvanishing correction** comes from the **connected tree-level diagram** with a **single quartic vertex**.

# Energy shift in large-J perturbation theory

- ▶ Once again, we evaluate matrix element of the FTP term in the state  $|2; J+2\rangle \equiv \frac{1}{\sqrt{2}\sqrt{(J+2)!}} ((a_0^\dagger)\bar{\phi})^2 ((a_0^\dagger)\phi)^{J+2} |0\rangle$  which we approximate by  $\frac{1}{\sqrt{2}} ((a_0^\dagger)\bar{\phi})^2 |[J+2]\rangle$  and evaluate the latter matrix element by **time ordered perturbation theory** in a **nontrivial vev**.
- ▶ Again the matrix element breaks up into a sum of **connected and disconnected** diagrams.
- ▶ The disconnected pieces contain factors proportional to the **two-loop** correction to the lowest state, and to the **one-loop correction** to the **second-lowest** state, both of which **vanish**. So the only **nonvanishing correction** comes from the **connected tree-level diagram** with a **single quartic vertex**.

# Energy shift in large-J perturbation theory

- ▶ This diagram does

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish**

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has



# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**,

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet**

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization**



# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign**

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction,

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign**

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term**

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the



# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.
- ▶ Denote the normalization of the super-FTP term in the action as

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.
- ▶ Denote the normalization of the super-FTP term in the action as  $\Delta\mathcal{L} \equiv \kappa_{\text{super-FTP}} \mathcal{O}_{\text{super-FTP}}$

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.
- ▶ Denote the normalization of the super-FTP term in the action as  $\Delta\mathcal{L} \equiv \kappa_{\text{super-FTP}} \mathcal{O}_{\text{super-FTP}}$

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.
- ▶ Denote the normalization of the super-FTP term in the action

$$\Delta\mathcal{L} \equiv \kappa_{\text{super-FTP}} \mathcal{O}_{\text{super-FTP}}$$
$$\mathcal{O}_{\text{super-FTP}} \equiv \int d^4\theta \mathcal{I}_{\text{super-FTP}}$$

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.
- ▶ Denote the normalization of the super-FTP term in the action

$$\Delta\mathcal{L} \equiv \kappa_{\text{super-FTP}} \mathcal{O}_{\text{super-FTP}}$$
$$\mathcal{O}_{\text{super-FTP}} \equiv \int d^4\theta \mathcal{I}_{\text{super-FTP}}$$

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.
- ▶ Denote the normalization of the super-FTP term in the action

$$\text{as } \Delta\mathcal{L} \equiv \kappa_{\text{super-FTP}} \mathcal{O}_{\text{super-FTP}}$$

$$\mathcal{O}_{\text{super-FTP}} \equiv \int d^4\theta \mathcal{I}_{\text{super-FTP}}$$

$$\mathcal{I}_{\text{super-FTP}} = \frac{|\partial\Phi|^2}{|\Phi|^4}$$



# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.

- ▶ Denote the normalization of the super-FTP term in the action

$$\Delta\mathcal{L} \equiv \kappa_{\text{super-FTP}} \mathcal{O}_{\text{super-FTP}}$$

$$\mathcal{O}_{\text{super-FTP}} \equiv \int d^4\theta \mathcal{I}_{\text{super-FTP}}$$

$$\mathcal{I}_{\text{super-FTP}} = \frac{|\partial\Phi|^2}{|\Phi|^4} \quad \text{where the superspace}$$

measure is normalized so that

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.

- ▶ Denote the normalization of the super-FTP term in the action

$$\Delta\mathcal{L} \equiv \kappa_{\text{super-FTP}} \mathcal{O}_{\text{super-FTP}}$$

$$\mathcal{O}_{\text{super-FTP}} \equiv \int d^4\theta \mathcal{I}_{\text{super-FTP}}$$

$$\mathcal{I}_{\text{super-FTP}} = \frac{|\partial\Phi|^2}{|\Phi|^4} \quad \text{where the superspace}$$

measure is normalized so that

$$\mathcal{L} \equiv \kappa_{\text{free}} = 1 |\nabla\phi|^2 + \dots = (\circ \circ \circ) \int d^4\theta \Phi^\dagger \Phi .$$

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.

- ▶ Denote the normalization of the super-FTP term in the action

$$\Delta\mathcal{L} \equiv \kappa_{\text{super-FTP}} \mathcal{O}_{\text{super-FTP}}$$

$$\mathcal{O}_{\text{super-FTP}} \equiv \int d^4\theta \mathcal{I}_{\text{super-FTP}}$$

$$\mathcal{I}_{\text{super-FTP}} = \frac{|\partial\Phi|^2}{|\Phi|^4} \quad \text{where the superspace}$$

measure is normalized so that

$$\mathcal{L} \equiv \kappa_{\text{free}} = 1 |\nabla\phi|^2 + \dots = (\circ \circ \circ) \int d^4\theta \Phi^\dagger \Phi .$$

# Energy shift in large-J perturbation theory

- ▶ This diagram does **not vanish** and it has **no reason to vanish**, since the third-lowest state lies in a **long multiplet** whose dimension is **not constrained super-algebraically**.
- ▶ Let us now be careful about the **normalization** and **sign** of this correction, relative to the **normalization and sign** of the **FTP term** in the **action**.

- ▶ Denote the normalization of the super-FTP term in the action

$$\Delta \mathcal{L} \equiv \kappa_{\text{super-FTP}} \mathcal{O}_{\text{super-FTP}}$$

$$\mathcal{O}_{\text{super-FTP}} \equiv \int d^4\theta \mathcal{I}_{\text{super-FTP}}$$

$$\mathcal{I}_{\text{super-FTP}} = \frac{|\partial\phi|^2}{|\phi|^4} \quad \text{where the superspace}$$

measure is normalized so that

$$\mathcal{L} \equiv \kappa_{\text{free}} = 1 |\nabla\phi|^2 + \dots = (\circ \circ \circ) \int d^4\theta \phi^\dagger \phi .$$

# Energy shift in large-J perturbation theory

- ▶ With

# Energy shift in large-J perturbation theory

- ▶ With this normalization

# Energy shift in large-J perturbation theory

- ▶ With **this normalization** for the FTP term,

# Energy shift in large-J perturbation theory

- ▶ With **this normalization** for the FTP term, the shift in energy of the third-lowest scalar primary is

$$\delta\Delta_J^{(+2)} = -\frac{12 r \kappa_{\text{FTP}}}{\pi |\phi_0|^6} .$$



# Energy shift in large-J perturbation theory

- ▶ With **this normalization** for the FTP term, the shift in energy of the third-lowest scalar primary is

$$\delta\Delta_J^{(+2)} = -\frac{12 r \kappa_{\text{FTP}}}{\pi |\phi_0|^6} .$$

- ▶ Using

# Energy shift in large- $J$ perturbation theory

- ▶ With **this normalization** for the FTP term, the shift in energy of the third-lowest scalar primary is

$$\delta\Delta_J^{(+2)} = -\frac{12 r \kappa_{\text{FTP}}}{\pi |\phi_0|^6} .$$

- ▶ Using  $J = 4\pi r |\phi_0|^2$

# Energy shift in large-J perturbation theory

- ▶ With **this normalization** for the FTP term, the shift in energy of the third-lowest scalar primary is

$$\delta\Delta_J^{(+2)} = -\frac{12 r \kappa_{\text{FTP}}}{\pi |\phi_0|^6}.$$

- ▶ Using  $J = 4\pi r |\phi_0|^2$  we have

$$\delta\Delta_J^{(+2)} = -\frac{192 \pi^2 \kappa_{\text{FTP}}}{J^3}.$$

# Energy shift in large-J perturbation theory

- ▶ With **this normalization** for the FTP term, the shift in energy of the third-lowest scalar primary is

$$\delta\Delta_J^{(+2)} = -\frac{12 r \kappa_{\text{FTP}}}{\pi |\phi_0|^6}.$$

- ▶ Using  $J = 4\pi r |\phi_0|^2$  we have

$$\delta\Delta_J^{(+2)} = -\frac{192 \pi^2 \kappa_{\text{FTP}}}{J^3}.$$

# Energy shift in large-J perturbation theory

- ▶ One can

# Energy shift in large-J perturbation theory

- ▶ One can also calculate

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the



# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -descendants

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -descendants of the state, and check that they are the same.
- ▶ At the

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level**

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and



# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$  -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -descendants of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values**

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$  -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$  -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator**

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$  -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states,

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states, and computed by



# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states, and computed by **tree level diagrams**

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$  -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states, and computed by **tree level diagrams** with a

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$  -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states, and computed by **tree level diagrams** with a **quartic vertex**

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$  -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states, and computed by **tree level diagrams** with a **quartic vertex** with

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states, and computed by **tree level diagrams** with a **quartic vertex** with **two**

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states, and computed by **tree level diagrams** with a **quartic vertex** with **two** and

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$ -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states, and computed by **tree level diagrams** with a **quartic vertex** with **two** and **four fermionic lines**,

# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$  -descendants of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states, and computed by **tree level diagrams** with a **quartic vertex** with **two** and **four fermionic lines**, respectively.



# Energy shift in large-J perturbation theory

- ▶ One can **also calculate** the energy shifts of the  $\bar{Q}$  -**descendants** of the state, and check that they are the same.
- ▶ At the **free level** the unperturbed states are of the form  $\phi^{J+2}\bar{\phi}\bar{\psi}$  and  $\phi^{J+2}\bar{\psi}^2$ .
- ▶ Their energy shifts are given by **expectation values** of the **TPP operator** in the appropriate states, and computed by **tree level diagrams** with a **quartic vertex** with **two** and **four fermionic lines**, respectively.

# The energy shifts

# Energy shift in large-J perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is

# Energy shift in large-J perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to**  $J^{-3}$

# Energy shift in large-J perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to**  $J^{-3}$  with a coefficient given in terms of the

# Energy shift in large-J perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term**

# Energy shift in large-J perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.



# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for **all other nonprotected**

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for **all other nonprotected** multiplets.

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for **all other nonprotected** multiplets. Their dimensions will

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for **all other nonprotected** multiplets. Their dimensions will **shift by an amount**

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for **all other nonprotected** multiplets. Their dimensions will **shift by an amount** proportional to

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for **all other nonprotected** multiplets. Their dimensions will **shift by an amount** proportional to  $\kappa_{\text{FTP}}/J^3$ ,



# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for **all other nonprotected** multiplets. Their dimensions will **shift by an amount** proportional to  $\kappa_{\text{FTP}}/J^3$ , with a coefficient depending on the Fock representation of the state in the

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for **all other nonprotected** multiplets. Their dimensions will **shift by an amount** proportional to  $\kappa_{\text{FTP}}/J^3$ , with a coefficient depending on the Fock representation of the state in the **infinite- $J$  limit**.

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for **all other nonprotected** multiplets. Their dimensions will **shift by an amount** proportional to  $\kappa_{\text{FTP}}/J^3$ , with a coefficient depending on the Fock representation of the state in the **infinite- $J$  limit**.

# Energy shift in large- $J$ perturbation theory

- ▶ So we have found that the leading nontrivial shift in dimension is **proportional to  $J^{-3}$**  with a coefficient given in terms of the **coefficient of the leading interaction term** in the **effective theory on moduli space**.
- ▶ The same must of course be true for **all other nonprotected** multiplets. Their dimensions will **shift by an amount** proportional to  $\kappa_{\text{FTP}}/J^3$ , with a coefficient depending on the Fock representation of the state in the **infinite- $J$  limit**.

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the



# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound**



# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range.

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient**



# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed**,

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi**,

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky**,



# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis**,

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in



# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**,

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{FTP}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{FTP}^{(\text{flat, bosonic})} = +4 \kappa_{FTP} \frac{|\partial\phi|^4}{|\phi|^6}.$$

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{FTP}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{FTP}^{(\text{flat}, \text{bosonic})} = +4 \kappa_{FTP} \frac{|\partial\phi|^4}{|\phi|^6}.$$

*Arkani-Hamed et al.*

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{FTP}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{FTP}^{(\text{flat}, \text{bosonic})} = +4 \kappa_{FTP} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{FTP}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{FTP}^{(\text{flat, bosonic})} = +4 \kappa_{FTP} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{\text{FTP}}^{(\text{flat, bosonic})} = +4 \kappa_{\text{FTP}} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive** in a consistent effective field theory.

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{\text{FTP}}^{(\text{flat}, \text{bosonic})} = +4 \kappa_{\text{FTP}} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive** in a consistent effective field theory. A

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{\text{FTP}}^{(\text{flat, bosonic})} = +4 \kappa_{\text{FTP}} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive** in a consistent effective field theory. A **negative coefficient**



# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{FTP}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{FTP}^{(\text{flat, bosonic})} = +4 \kappa_{FTP} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive** in a consistent effective field theory. A **negative coefficient** would lead to

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{\text{FTP}}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{\text{FTP}}^{(\text{flat}, \text{bosonic})} = +4 \kappa_{\text{FTP}} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive** in a consistent effective field theory. A **negative coefficient** would lead to **violations of unitarity**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{FTP}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{FTP}^{(\text{flat}, \text{bosonic})} = +4 \kappa_{FTP} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive** in a consistent effective field theory. A **negative coefficient** would lead to **violations of unitarity** in low-energy moduli scattering,

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{FTP}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{FTP}^{(\text{flat}, \text{bosonic})} = +4 \kappa_{FTP} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive** in a consistent effective field theory. A **negative coefficient** would lead to **violations of unitarity** in low-energy moduli scattering, as well as

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{FTP}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{FTP}^{(\text{flat, bosonic})} = +4 \kappa_{FTP} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive** in a consistent effective field theory. A **negative coefficient** would lead to **violations of unitarity** in low-energy moduli scattering, as well as **superluminal propagation**

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{FTP}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{FTP}^{(\text{flat}, \text{bosonic})} = +4 \kappa_{FTP} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive** in a consistent effective field theory. A **negative coefficient** would lead to **violations of unitarity** in low-energy moduli scattering, as well as **superluminal propagation** in backgrounds where a scalar gradient gets an expectation value.

# Energy shift in large-J perturbation theory

- ▶ A few comments on the FTP coefficient:
  - ▶ The coefficient  $\kappa_{FTP}$  is a "non-universal" coefficient in the **effective action**, that we don't know how to compute.
  - ▶ It would of course be **VERY interesting** to know how to solve for this term by **bootstrap** or at least **bound** it in some range. At present we do not know how to do **even that**.
  - ▶ However we **can** bound the **FTP coefficient** below at **zero** by a **non-bootstrap** argument due to **Arkani-Hamed, Rattazzi, Dubovsky, Nicolis, and Adams**.
  - ▶ If we examine the **purely bosonic** component of the **FTP term** in **flat space**, we find that it is equal to

$$\mathcal{L}_{FTP}^{(\text{flat, bosonic})} = +4 \kappa_{FTP} \frac{|\partial\phi|^4}{|\phi|^6}.$$

**Arkani-Hamed *et al.*** have pointed out that the coefficient of this term must **always be positive** in a consistent effective field theory. A **negative coefficient** would lead to **violations of unitarity** in low-energy moduli scattering, as well as **superluminal propagation** in backgrounds where a scalar gradient gets an expectation value.

# Energy shift in large-J perturbation theory

- ▶ The coefficient



# Energy shift in large-J perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$

# Energy shift in large-J perturbation theory

- ▶ The coefficient  $\kappa_{FTP}$  enters the formula for the

# Energy shift in large-J perturbation theory

- ▶ The coefficient  $\kappa_{FTP}$  enters the formula for the

# Energy shift in large-J perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$

# Energy shift in large-J perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift

# Energy shift in large-J perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the

# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary

# Energy shift in large-J perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a



# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.

# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.

# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.
- ▶ So we know the

# Energy shift in large-J perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.
- ▶ So we know the coefficient of this term

# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.
- ▶ So we know the coefficient of this term is

# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.
- ▶ So we know the coefficient of this term is negative definite

# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.
- ▶ So we know the coefficient of this term is negative definite by some consistency condition that is

# Energy shift in large-J perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.
- ▶ So we know the coefficient of this term is negative definite by some consistency condition that is entirely obscure



# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.
- ▶ So we know the coefficient of this term is negative definite by some consistency condition that is entirely obscure from the point of view of the

# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.
- ▶ So we know the coefficient of this term is negative definite by some consistency condition that is entirely obscure from the point of view of the underlying CFT

# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.
- ▶ So we know the coefficient of this term is negative definite by some consistency condition that is entirely obscure from the point of view of the underlying CFT .

# Energy shift in large- $J$ perturbation theory

- ▶ The coefficient  $\kappa_{\text{FTP}}$  enters the formula for the  $O(J^{-3})$  energy shift of the first unprotected scalar primary with a negative sign.
- ▶ So we know the coefficient of this term is negative definite by some consistency condition that is entirely obscure from the point of view of the underlying CFT .

# Energy shift in large-J perturbation theory

- ▶ We note that the

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion**

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel**



# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion**

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov**

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).**)

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).**)

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).**)
- ▶ The



# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (Komargodsky-Zhiboedov and Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).)
- ▶ The **large-spin expansion**

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).**)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).**)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).**)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).**)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).**)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin)**.)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin)**.)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient**



# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin)**.)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient**

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).**)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin)**.)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope**

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin)**.)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin)**.)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin)**.)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (Komargodsky-Zhiboedov and Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).)

- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is **independent**

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (**Komargodsky-Zhiboedov** and **Kaplan-Fitzpatrick-Poland-(Simmons-Duffin)**.)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is **independent** of the spin and depends on the details of the operators.
  - ▶ The



# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (Komargodsky-Zhiboedov and Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).)

- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is **independent** of the spin and depends on the details of the operators.
  - ▶ The **third term**

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (Komargodsky-Zhiboedov and Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).)

- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is **independent** of the spin and depends on the details of the operators.
  - ▶ The **third term** scales as a

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (Komargodsky-Zhiboedov and Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).)
- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is **independent** of the spin and depends on the details of the operators.
  - ▶ The **third term** scales as a **specific negative**

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (Komargodsky-Zhiboedov and Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).)

- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is **independent** of the spin and depends on the details of the operators.
  - ▶ The **third term** scales as a **specific negative** power of the quantum number,

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (Komargodsky-Zhiboedov and Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).)

- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is **independent** of the spin and depends on the details of the operators.
  - ▶ The **third term** scales as a **specific negative** power of the quantum number, with a coefficient of

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (Komargodsky-Zhiboedov and Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).)

- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is **independent** of the spin and depends on the details of the operators.
  - ▶ The **third term** scales as a **specific negative** power of the quantum number, with a coefficient of **negative definite**.

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (Komargodsky-Zhiboedov and Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).)

- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is **independent** of the spin and depends on the details of the operators.
  - ▶ The **third term** scales as a **specific negative** power of the quantum number, with a coefficient of **negative definite sign**.

# Energy shift in large-J perturbation theory

- ▶ We note that the **large-charge expansion** of this operator dimension is **entirely formally parallel** to the **large-spin expansion** of the dimensions of the primaries appearing in the OPE of two scalar primaries. (Komargodsky-Zhiboedov and Kaplan-Fitzpatrick-Poland-(Simmons-Duffin).)

- ▶ The **large-spin expansion** takes the form

$$\Delta_s = s + 2\delta_0 - \frac{(\#)}{s^\tau}$$

where the number  $\#$  is positive and  $\tau$  is the lowest twist exchanged in the cross-channel.

- ▶ The structure is entirely parallel with our own:
  - ▶ The leading term is linear in the large quantum number with a **particular coefficient** corresponding to the **slope** defined by the **unitarity bound**.
  - ▶ The second term is **independent** of the spin and depends on the details of the operators.
  - ▶ The **third term** scales as a **specific negative** power of the quantum number, with a coefficient of **negative definite sign**.



# Energy shift in large-J perturbation theory

- ▶ The parallel suggests there may be some sort of

# Energy shift in large-J perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof**

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel**

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin**

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment,



# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however,

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however, we do not know how to formulate such a bootstrap derivation.

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however, we do not know how to formulate such a bootstrap derivation. In particular we do not know what would play the role of the

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however, we do not know how to formulate such a bootstrap derivation. In particular we do not know what would play the role of the **light-cone limit**

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however, we do not know how to formulate such a bootstrap derivation. In particular we do not know what would play the role of the **light-cone limit** that plays a key role in the derivation of the large-spin formulae.

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however, we do not know how to formulate such a bootstrap derivation. In particular we do not know what would play the role of the **light-cone limit** that plays a key role in the derivation of the large-spin formulae.
- ▶ Since we don't know how to proceed,

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however, we do not know how to formulate such a bootstrap derivation. In particular we do not know what would play the role of the **light-cone limit** that plays a key role in the derivation of the large-spin formulae.
- ▶ Since we don't know how to proceed, that is all we will say about the possibility of a bootstrap derivation

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however, we do not know how to formulate such a bootstrap derivation. In particular we do not know what would play the role of the **light-cone limit** that plays a key role in the derivation of the large-spin formulae.
- ▶ Since we don't know how to proceed, that is all we will say about the possibility of a bootstrap derivation...



# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however, we do not know how to formulate such a bootstrap derivation. In particular we do not know what would play the role of the **light-cone limit** that plays a key role in the derivation of the large-spin formulae.
- ▶ Since we don't know how to proceed, that is all we will say about the possibility of a bootstrap derivation... **for now.**

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however, we do not know how to formulate such a bootstrap derivation. In particular we do not know what would play the role of the **light-cone limit** that plays a key role in the derivation of the large-spin formulae.
- ▶ Since we don't know how to proceed, that is all we will say about the possibility of a bootstrap derivation... **for now.**

# Energy shift in large- $J$ perturbation theory

- ▶ The parallel suggests there may be some sort of **bootstrap proof** for the large- $J$  expansion discussed in this talk **in parallel** with the bootstrap derivation of the **large-spin** formulae.
- ▶ At the moment, however, we do not know how to formulate such a bootstrap derivation. In particular we do not know what would play the role of the **light-cone limit** that plays a key role in the derivation of the large-spin formulae.
- ▶ Since we don't know how to proceed, that is all we will say about the possibility of a bootstrap derivation... **for now.**

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .

## Two- and three-point functions

- ▶ We can now about other observables at large  $J$ .
- ▶ One set of observables is about

# Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**.

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason**



## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that

# Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions**

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ ,

# Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point**

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three**

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have



## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one**

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation**)

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation** .)

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation** .)
- ▶ As long as

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation** .)
- ▶ As long as **at least two**

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point functions** are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation** .)
- ▶ As long as **at least two** of the operators have large  $J$ , you can treat these as

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation** .)
- ▶ As long as **at least two** of the operators have large  $J$ , you can treat these as **initial and final**



## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation** .)
- ▶ As long as **at least two** of the operators have large  $J$ , you can treat these as **initial and final** states, and then you can

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation**.)
- ▶ As long as **at least two** of the operators have large  $J$ , you can treat these as **initial and final** states, and then you can **evaluate the third**

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation**.)
- ▶ As long as **at least two** of the operators have large  $J$ , you can treat these as **initial and final** states, and then you can **evaluate the third** operator in the

## Two- and three-point functions

- ▶ We can now about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation** .)
- ▶ As long as **at least two** of the operators have large  $J$ , you can treat these as **initial and final** states, and then you can **evaluate the third** operator in the **moduli space effective theory**.

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point functions** are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation**.)
- ▶ As long as **at least two** of the operators have large  $J$ , you can treat these as **initial and final** states, and then you can **evaluate the third** operator in the **moduli space effective theory**.

## Two- and three-point functions

- ▶ We can now talk about other observables at large  $J$ .
- ▶ One set of observables is about **three-point functions**. For the **same reason** that **dimensions** are calculable at large  $J$ , **three-point** functions are calculable as well when **two or three** of the operators have large  $J$ .
- ▶ (You can't have **just one** operator carry large  $J$ , because **charge conservation** .)
- ▶ As long as **at least two** of the operators have large  $J$ , you can treat these as **initial and final** states, and then you can **evaluate the third** operator in the **moduli space effective theory**.

# Two- and three-point functions

► The

# Two- and three-point functions

- ▶ The **only** issue



# Two- and three-point functions

- ▶ The **only issue** we want to be take

# Two- and three-point functions

- ▶ The **only issue** we want to be take **special care**

# Two- and three-point functions

- ▶ The **only issue** we want to be take **special care** with here has to do with the

# Two- and three-point functions

- ▶ The **only issue** we want to be take **special care** with here has to do with the **norms**

# Two- and three-point functions

- ▶ The **only issue** we want to be take **special care** with here has to do with the **norms** of the states

# Two- and three-point functions

- ▶ The **only issue** we want to be take **special care** with here has to do with the **norms** of the states  $|J\rangle$

## Two- and three-point functions

- ▶ The **only issue** we want to be take **special care** with here has to do with the **norms** of the states  $|J\rangle$  .

## Two- and three-point functions

- ▶ The **only issue** we want to be take **special care** with here has to do with the **norms** of the states  $|J\rangle$  .



# Two- and three-point functions

- ▶ The **only issue** we want to be take **special care** with here has to do with the **norms** of the states  $|J\rangle$  .
- ▶

## Two- and three-point functions

- ▶ Let us define the three-point function as

$$\begin{aligned} & \langle X^A(\sigma_A) X^B(\sigma_B) \bar{X}^C(\sigma_C) \rangle \\ & \equiv \delta_{C,A+B} C^{AB\bar{C}} |\sigma_A - \sigma_C|^{-\frac{4}{3}A} |\sigma_B - \sigma_C|^{-\frac{4}{3}B} \end{aligned}$$

## Two- and three-point functions

- ▶ Let us define the three-point function as

$$\begin{aligned} & \langle X^A(\sigma_A) X^B(\sigma_B) \bar{X}^C(\sigma_C) \rangle \\ & \equiv \delta_{C,A+B} C^{AB\bar{C}} |\sigma_A - \sigma_C|^{-\frac{4}{3}A} |\sigma_B - \sigma_C|^{-\frac{4}{3}B} \end{aligned}$$

- ▶ And defined the norm of the state

## Two- and three-point functions

- ▶ Let us define the three-point function as

$$\begin{aligned} & \langle X^A(\sigma_A) X^B(\sigma_B) \bar{X}^C(\sigma_C) \rangle \\ & \equiv \delta_{C, A+B} C^{AB\bar{C}} |\sigma_A - \sigma_C|^{-\frac{4}{3}A} |\sigma_B - \sigma_C|^{-\frac{4}{3}B} \end{aligned}$$

- ▶ And defined the norm of the state  $X^{Jx}$

## Two- and three-point functions

- ▶ Let us define the three-point function as

$$\begin{aligned} & \langle X^A(\sigma_A) X^B(\sigma_B) \bar{X}^C(\sigma_C) \rangle \\ & \equiv \delta_{C,A+B} C^{AB\bar{C}} |\sigma_A - \sigma_C|^{-\frac{4}{3}A} |\sigma_B - \sigma_C|^{-\frac{4}{3}B} \end{aligned}$$

- ▶ And defined the norm of the state  $X^{Jx}$  to be

$$\mathcal{N}_C \equiv \sqrt{|\sigma|^{\frac{4C}{3}} \langle X^C(\sigma) \bar{X}^C(0) \rangle}$$

## Two- and three-point functions

- ▶ Let us define the three-point function as

$$\begin{aligned} & \langle X^A(\sigma_A) X^B(\sigma_B) \bar{X}^C(\sigma_C) \rangle \\ & \equiv \delta_{C,A+B} C^{AB\bar{C}} |\sigma_A - \sigma_C|^{-\frac{4}{3}A} |\sigma_B - \sigma_C|^{-\frac{4}{3}B} \end{aligned}$$

- ▶ And defined the norm of the state  $X^{Jx}$  to be

$$\mathcal{N}_C \equiv \sqrt{|\sigma|^{-\frac{4C}{3}} \langle X^C(\sigma) \bar{X}^C(0) \rangle}$$

- ▶ Then by taking  $\sigma_B \rightarrow \sigma_A$  we can relate the

## Two- and three-point functions

- ▶ Let us define the three-point function as

$$\begin{aligned} & \langle X^A(\sigma_A) X^B(\sigma_B) \bar{X}^C(\sigma_C) \rangle \\ & \equiv \delta_{C,A+B} C^{AB\bar{C}} |\sigma_A - \sigma_C|^{-\frac{4}{3}A} |\sigma_B - \sigma_C|^{-\frac{4}{3}B} \end{aligned}$$

- ▶ And defined the norm of the state  $X^{Jx}$  to be

$$\mathcal{N}_C \equiv \sqrt{|\sigma|^{\frac{4C}{3}} \langle X^C(\sigma) \bar{X}^C(0) \rangle}$$

- ▶ Then by taking  $\sigma_B \rightarrow \sigma_A$  we can relate the **three point functions**

## Two- and three-point functions

- ▶ Let us define the three-point function as

$$\begin{aligned} & \langle X^A(\sigma_A) X^B(\sigma_B) \bar{X}^C(\sigma_C) \rangle \\ & \equiv \delta_{C,A+B} C^{AB\bar{C}} |\sigma_A - \sigma_C|^{-\frac{4}{3}A} |\sigma_B - \sigma_C|^{-\frac{4}{3}B} \end{aligned}$$

- ▶ And defined the norm of the state  $X^{Jx}$  to be

$$\mathcal{N}_C \equiv \sqrt{|\sigma|^{-\frac{4C}{3}} \langle X^C(\sigma) \bar{X}^C(0) \rangle}$$

- ▶ Then by taking  $\sigma_B \rightarrow \sigma_A$  we can relate the **three point functions** directly to the



## Two- and three-point functions

- ▶ Let us define the three-point function as

$$\begin{aligned} & \langle X^A(\sigma_A) X^B(\sigma_B) \bar{X}^C(\sigma_C) \rangle \\ & \equiv \delta_{C, A+B} C^{AB\bar{C}} |\sigma_A - \sigma_C|^{-\frac{4}{3}A} |\sigma_B - \sigma_C|^{-\frac{4}{3}B} \end{aligned}$$

- ▶ And defined the norm of the state  $X^{Jx}$  to be

$$\mathcal{N}_C \equiv \sqrt{|\sigma|^{-\frac{4C}{3}} \langle X^C(\sigma) \bar{X}^C(0) \rangle}$$

- ▶ Then by taking  $\sigma_B \rightarrow \sigma_A$  we can relate the **three point functions** directly to the **two-point functions**:

## Two- and three-point functions

- ▶ Let us define the three-point function as

$$\begin{aligned} & \langle X^A(\sigma_A) X^B(\sigma_B) \bar{X}^C(\sigma_C) \rangle \\ & \equiv \delta_{C,A+B} C^{AB\bar{C}} |\sigma_A - \sigma_C|^{-\frac{4}{3}A} |\sigma_B - \sigma_C|^{-\frac{4}{3}B} \end{aligned}$$

- ▶ And defined the norm of the state  $X^{Jx}$  to be

$$\mathcal{N}_C \equiv \sqrt{|\sigma|^{-\frac{4C}{3}} \langle X^C(\sigma) \bar{X}^C(0) \rangle}$$

- ▶ Then by taking  $\sigma_B \rightarrow \sigma_A$  we can relate the **three point functions** directly to the **two-point functions**:

$$C^{AB\bar{C}} = \delta_{C,A+B} (\mathcal{N}_C)^2$$

## Two- and three-point functions

- ▶ Let us define the three-point function as

$$\begin{aligned} & \langle X^A(\sigma_A) X^B(\sigma_B) \bar{X}^C(\sigma_C) \rangle \\ & \equiv \delta_{C,A+B} C^{AB\bar{C}} |\sigma_A - \sigma_C|^{-\frac{4}{3}A} |\sigma_B - \sigma_C|^{-\frac{4}{3}B} \end{aligned}$$

- ▶ And defined the norm of the state  $X^{Jx}$  to be

$$\mathcal{N}_C \equiv \sqrt{|\sigma|^{-\frac{4C}{3}} \langle X^C(\sigma) \bar{X}^C(0) \rangle}$$

- ▶ Then by taking  $\sigma_B \rightarrow \sigma_A$  we can relate the **three point functions** directly to the **two-point functions**:

$$C^{AB\bar{C}} = \delta_{C,A+B} (\mathcal{N}_C)^2$$

# Two- and three-point functions

- ▶ So the

# Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral**

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point**

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the



## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral**

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact**

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point**

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily**



## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory**

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function**

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the



## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function**

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  **$\delta$ -function** sources proportional to

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  **$\delta$ -function** sources proportional to  $J_X \ln(X)$

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  **$\delta$ -function** sources proportional to  $J_X \ln(X)$  and

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .



## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that

$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that

$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that
$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$
- ▶ If desired, we can even



## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that  $|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}$ .
- ▶ If desired, we can even **smear the sources**

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that
$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$
- ▶ If desired, we can even **smear the sources** over a scale

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  $\delta$ -function sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that
$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$
- ▶ If desired, we can even **smear the sources** over a scale
$$|\sigma|^{-1} \ll \ell_{\text{source}} \ll \Lambda,$$

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  **$\delta$ -function** sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that
$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$
- ▶ If desired, we can even **smear the sources** over a scale  $|\sigma|^{-1} \ll \ell_{\text{source}} \ll \Lambda$ , though this will not

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  **$\delta$ -function** sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that
$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$
- ▶ If desired, we can even **smear the sources** over a scale  $|\sigma|^{-1} \ll \ell_{\text{source}} \ll \Lambda$ , though this will not **affect the leading result**

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  **$\delta$ -function** sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that
$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$
- ▶ If desired, we can even **smear the sources** over a scale  $|\sigma|^{-1} \ll \ell_{\text{source}} \ll \Lambda$ , though this will not **affect the leading result** but only serve to bound the

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  **$\delta$ -function** sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that
$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$
- ▶ If desired, we can even **smear the sources** over a scale  $|\sigma|^{-1} \ll \ell_{\text{source}} \ll \Lambda$ , though this will not **affect the leading result** but only serve to bound the **corrections**.

## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  **$\delta$ -function** sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that
$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$
- ▶ If desired, we can even **smear the sources** over a scale  $|\sigma|^{-1} \ll \ell_{\text{source}} \ll \Lambda$ , though this will not **affect the leading result** but only serve to bound the **corrections**.



## Two- and three-point functions

- ▶ So the **chiral-chiral-antichiral** three-point function depends only on the **two-point** function of the **antichiral** scalar and its conjugate.
- ▶ This is a **useful fact** because the **two-point** functions at large  $J$  can be **estimated easily** in the **effective field theory** on **moduli space**.
- ▶ Computing the **two-point function** amounts to calculating the **partition function** with  **$\delta$ -function** sources proportional to  $J_X \ln(X)$  and  $J_X \ln(\bar{X})$ .
- ▶ If we separate the points by a **unit distance**  $\sigma$  then we can compute the two-point function in the  $X$ -branch effective theory with Wilsonian cutoff  $\Lambda$  such that
$$|\sigma|^{-1} \ll \Lambda \ll \frac{J^{\frac{1}{2}}}{\sigma}.$$
- ▶ If desired, we can even **smear the sources** over a scale  $|\sigma|^{-1} \ll \ell_{\text{source}} \ll \Lambda$ , though this will not **affect the leading result** but only serve to bound the **corrections**.

# Two- and three-point functions

- ▶ At leading order the

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources**

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative**

## Two- and three-point functions

- ▶ At leading order the **log** of the partition function with sources is the **negative** of the

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum**

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the



## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**

$$S[J_X] = S_{\text{CFT}} + S_{\text{sources}}.$$

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**

$$S[J_X] = S_{\text{CFT}} + S_{\text{sources}}.$$

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**  
 $\mathcal{S}[J_X] = \mathcal{S}_{\text{CFT}} + \mathcal{S}_{\text{sources}}$ .
- ▶ After changing to the more convenient  $\Phi$ -variables, the terms in the action take the form

$$\mathcal{S}_{\text{CFT}} = |\nabla\phi|^2 + (\text{fermions}) + \mathcal{S}_{\text{FTP}} + \text{higher order in } \frac{1}{J},$$

$$\mathcal{S}_{\text{sources}} = -J \ln(\phi(\sigma_\phi)/\sqrt{c_{[K]}}) - J \ln(\bar{\phi}(\sigma_{\bar{\phi}})/\sqrt{c_{[K]}})$$

where

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**  
 $\mathcal{S}[J_X] = \mathcal{S}_{\text{CFT}} + \mathcal{S}_{\text{sources}}$ .
- ▶ After changing to the more convenient  $\Phi$ -variables, the terms in the action take the form

$$\mathcal{S}_{\text{CFT}} = |\nabla\phi|^2 + (\text{fermions}) + \mathcal{S}_{\text{FTP}} + \text{higher order in } \frac{1}{J},$$

$$\mathcal{S}_{\text{sources}} = -J \ln(\phi(\sigma_\phi)/\sqrt{c_{[K]}}) - J \ln(\bar{\phi}(\sigma_{\bar{\phi}})/\sqrt{c_{[K]}})$$

where  $J \equiv \frac{4}{3}J_X$

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**  
 $S[J_X] = S_{\text{CFT}} + S_{\text{sources}}$ .
- ▶ After changing to the more convenient  $\Phi$ -variables, the terms in the action take the form

$$S_{\text{CFT}} = |\nabla\phi|^2 + (\text{fermions}) + S_{\text{FTP}} + \text{higher order in } \frac{1}{J},$$

$$S_{\text{sources}} = -J \ln(\phi(\sigma_\phi)/\sqrt{c_{[K]}}) - J \ln(\bar{\phi}(\sigma_{\bar{\phi}})/\sqrt{c_{[K]}})$$

where  $J \equiv \frac{4}{3}J_X$  as before.

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**  
 $S[J_X] = S_{\text{CFT}} + S_{\text{sources}}$ .
- ▶ After changing to the more convenient  $\Phi$ -variables, the terms in the action take the form

$$S_{\text{CFT}} = |\nabla\phi|^2 + (\text{fermions}) + S_{\text{FTP}} + \text{higher order in } \frac{1}{J},$$

$$S_{\text{sources}} = -J \ln(\phi(\sigma_\phi)/\sqrt{c_{[K]}}) - J \ln(\bar{\phi}(\sigma_{\bar{\phi}})/\sqrt{c_{[K]}})$$

where  $J \equiv \frac{4}{3}J_X$  as before.

- ▶ At leading order this is simply the free-field two-point function



## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**  
 $S[J_X] = S_{\text{CFT}} + S_{\text{sources}}$ .
- ▶ After changing to the more convenient  $\Phi$ -variables, the terms in the action take the form

$$S_{\text{CFT}} = |\nabla\phi|^2 + (\text{fermions}) + S_{\text{FTP}} + \text{higher order in } \frac{1}{J},$$

$$S_{\text{sources}} = -J \ln(\phi(\sigma_\phi)/\sqrt{c_{[K]}}) - J \ln(\bar{\phi}(\sigma_{\bar{\phi}})/\sqrt{c_{[K]}})$$

where  $J \equiv \frac{4}{3}J_X$  as before.

- ▶ At leading order this is simply the free-field two-point function  
 $\mathcal{N}_J^2 = Z \sim c_{[K]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[K]}^{-J} \langle \phi \bar{\phi} \rangle^J$

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**  
 $S[J_X] = S_{\text{CFT}} + S_{\text{sources}}$ .
- ▶ After changing to the more convenient  $\Phi$ -variables, the terms in the action take the form

$$S_{\text{CFT}} = |\nabla\phi|^2 + (\text{fermions}) + S_{\text{FTP}} + \text{higher order in } \frac{1}{J},$$

$$S_{\text{sources}} = -J \ln(\phi(\sigma_\phi)/\sqrt{c_{[K]}}) - J \ln(\bar{\phi}(\sigma_{\bar{\phi}})/\sqrt{c_{[K]}})$$

where  $J \equiv \frac{4}{3}J_X$  as before.

- ▶ At leading order this is simply the free-field two-point function  
 $\mathcal{N}_J^2 = Z \sim c_{[K]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[K]}^{-J} \langle \phi \bar{\phi} \rangle^J$  which is generated by the (complex) classical solution

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**  
 $S[J_X] = S_{\text{CFT}} + S_{\text{sources}}$ .
- ▶ After changing to the more convenient  $\Phi$ -variables, the terms in the action take the form

$$S_{\text{CFT}} = |\nabla\phi|^2 + (\text{fermions}) + S_{\text{FTP}} + \text{higher order in } \frac{1}{J},$$

$$S_{\text{sources}} = -J \ln(\phi(\sigma_\phi)/\sqrt{c_{[K]}}) - J \ln(\bar{\phi}(\sigma_{\bar{\phi}})/\sqrt{c_{[K]}})$$

where  $J \equiv \frac{4}{3}J_X$  as before.

- ▶ At leading order this is simply the free-field two-point function  
 $\mathcal{N}_J^2 = Z \sim c_{[K]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[K]}^{-J} \langle \phi \bar{\phi} \rangle^J$  which is generated by the (complex) classical solution

$$\phi = \frac{q}{|\sigma - \sigma_\phi|}, \bar{\phi} = \frac{q}{|\sigma - \sigma_{\bar{\phi}}|}, q \equiv \sqrt{\frac{J}{4\pi|\sigma_\phi - \sigma_{\bar{\phi}}|}}.$$

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**  
 $S[J_X] = S_{\text{CFT}} + S_{\text{sources}}$ .
- ▶ After changing to the more convenient  $\Phi$ -variables, the terms in the action take the form

$$S_{\text{CFT}} = |\nabla\phi|^2 + (\text{fermions}) + S_{\text{FTP}} + \text{higher order in } \frac{1}{J},$$

$$S_{\text{sources}} = -J \ln(\phi(\sigma_\phi)/\sqrt{c_{[K]}}) - J \ln(\bar{\phi}(\sigma_{\bar{\phi}})/\sqrt{c_{[K]}})$$

where  $J \equiv \frac{4}{3}J_X$  as before.

- ▶ At leading order this is simply the free-field two-point function  
 $\mathcal{N}_J^2 = Z \sim c_{[K]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[K]}^{-J} \langle \phi \bar{\phi} \rangle^J$  which is generated by the (complex) classical solution

$$\phi = \frac{q}{|\sigma - \sigma_\phi|}, \bar{\phi} = \frac{q}{|\sigma - \sigma_{\bar{\phi}}|}, q \equiv \sqrt{\frac{J}{4\pi|\sigma_\phi - \sigma_{\bar{\phi}}|}}.$$

## Two- and three-point functions

- ▶ At leading order the **log of the partition function with sources** is the **negative** of the **minimum** of the **action with sources**  
 $S[J_X] = S_{\text{CFT}} + S_{\text{sources}}$ .
- ▶ After changing to the more convenient  $\Phi$ -variables, the terms in the action take the form

$$S_{\text{CFT}} = |\nabla\phi|^2 + (\text{fermions}) + S_{\text{FTP}} + \text{higher order in } \frac{1}{J},$$

$$S_{\text{sources}} = -J \ln(\phi(\sigma_\phi)/\sqrt{c_{[K]}}) - J \ln(\bar{\phi}(\sigma_{\bar{\phi}})/\sqrt{c_{[K]}})$$

where  $J \equiv \frac{4}{3}J_X$  as before.

- ▶ At leading order this is simply the free-field two-point function  
 $\mathcal{N}_J^2 = Z \sim c_{[K]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[K]}^{-J} \langle \phi \bar{\phi} \rangle^J$  which is generated by the (complex) classical solution

$$\phi = \frac{q}{|\sigma - \sigma_\phi|}, \bar{\phi} = \frac{q}{|\sigma - \sigma_{\bar{\phi}}|}, q \equiv \sqrt{\frac{J}{4\pi|\sigma_\phi - \sigma_{\bar{\phi}}|}}.$$

# Two- and three-point functions

- ▶ The first

# Two- and three-point functions

- ▶ The first **correction**

# Two- and three-point functions

- ▶ The first **correction** comes from inserting the



## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP**

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient**

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts**



## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts**

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms**

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions**

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\mathcal{K}]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\mathcal{K}]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\mathcal{K}]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\mathcal{K}]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are



## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results**

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\mathcal{K}]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\mathcal{K}]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving **two BPS**

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\mathcal{K}]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\mathcal{K}]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving **two BPS** operators and one

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving **two BPS** operators and one **semi-short**

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\kappa]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\kappa]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving **two BPS** operators and one **semi-short** scalar primary.

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\mathcal{K}]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\mathcal{K}]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving **two BPS** operators and one **semi-short** scalar primary. These use the fact that the

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\mathcal{K}]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\mathcal{K}]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving **two BPS** operators and one **semi-short** scalar primary. These use the fact that the **scalar semi-shorts**



## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\mathcal{K}]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\mathcal{K}]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving **two BPS** operators and one **semi-short** scalar primary. These use the fact that the **scalar semi-shorts** form a

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\mathcal{K}]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\mathcal{K}]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving **two BPS** operators and one **semi-short** scalar primary. These use the fact that the **scalar semi-shorts** form a **module**

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\mathcal{K}]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\mathcal{K}]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving **two BPS** operators and one **semi-short** scalar primary. These use the fact that the **scalar semi-shorts** form a **module** over the chiral ring.

## Two- and three-point functions

- ▶ The first **correction** comes from inserting the **FTP** term into the classical action, which leads to a **large- $J$**  correction of the form

$$\mathcal{N}_J^2 = Z \sim c_{[\mathcal{K}]}^{-J} \langle \phi^J \bar{\phi}^J \rangle = J! c_{[\mathcal{K}]}^{-J} \langle \phi \bar{\phi} \rangle^J \left[ 1 + \frac{(\text{const.}) \kappa_{\text{FTP}}}{J} \right]$$

- ▶ The **same coefficient** controlling the **leading energy shifts** also controls the **leading shifts** of the **norms** and thus of the **three-point functions** of chiral primaries.
- ▶ There are **similar results** for three-point functions involving **two BPS** operators and one **semi-short** scalar primary. These use the fact that the **scalar semi-shorts** form a **module** over the chiral ring.

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the  $XYZ$

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the  $XYZ$  model simplify at large global charge.
- ▶ Both



# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.
- ▶ Both **corrections to nonprotected operator dimensions**

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.
- ▶ Both **corrections to nonprotected operator dimensions** as well as

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the  $XYZ$  model simplify at large global charge.
- ▶ Both corrections to nonprotected operator dimensions as well as three point functions

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.
- ▶ Both **corrections to nonprotected operator dimensions** as well as **three point functions** can be calculated in terms of the

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the  $XYZ$  model simplify at large global charge.
- ▶ Both corrections to nonprotected operator dimensions as well as three point functions can be calculated in terms of the leading irrelevant operator

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.
- ▶ Both **corrections to nonprotected operator dimensions** as well as **three point functions** can be calculated in terms of the **leading irrelevant operator** in the effective theory on moduli space.

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.
- ▶ Both **corrections to nonprotected operator dimensions** as well as **three point functions** can be calculated in terms of the **leading irrelevant operator** in the effective theory on moduli space.
- ▶ There are many **key issues** remaining to be **understood**

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.
- ▶ Both **corrections to nonprotected operator dimensions** as well as **three point functions** can be calculated in terms of the **leading irrelevant operator** in the effective theory on moduli space.
- ▶ There are many **key issues** remaining to be **understood**, such as the **computation** or **bootstrap constraints** on the **large-J** Lagrangian parameters.



# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.
- ▶ Both **corrections to nonprotected operator dimensions** as well as **three point functions** can be calculated in terms of the **leading irrelevant operator** in the effective theory on moduli space.
- ▶ There are many **key issues** remaining to be **understood**, such as the **computation** or **bootstrap constraints** on the **large-J** Lagrangian parameters.
- ▶ For those reasons, we will **continue to investigate it**.

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.
- ▶ Both **corrections to nonprotected operator dimensions** as well as **three point functions** can be calculated in terms of the **leading irrelevant operator** in the effective theory on moduli space.
- ▶ There are many **key issues** remaining to be **understood**, such as the **computation** or **bootstrap constraints** on the **large-J** Lagrangian parameters.
- ▶ For those reasons, we will **continue to investigate it**.
- ▶ **You** should work on it **too**.

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.
- ▶ Both **corrections to nonprotected operator dimensions** as well as **three point functions** can be calculated in terms of the **leading irrelevant operator** in the effective theory on moduli space.
- ▶ There are many **key issues** remaining to be **understood**, such as the **computation** or **bootstrap constraints** on the **large-J** Lagrangian parameters.
- ▶ For those reasons, we will **continue to investigate it**.
- ▶ **You** should work on it **too**.
- ▶ Thank you.

# Conclusions

- ▶ We have seen interesting strongly coupled dynamics of the **XYZ** model simplify at **large global charge**.
- ▶ Both **corrections to nonprotected operator dimensions** as well as **three point functions** can be calculated in terms of the **leading irrelevant operator** in the effective theory on moduli space.
- ▶ There are many **key issues** remaining to be **understood**, such as the **computation** or **bootstrap constraints** on the **large-J** Lagrangian parameters.
- ▶ For those reasons, we will **continue to investigate it**.
- ▶ **You** should work on it **too**.
- ▶ Thank you.